

“High Energy, Cosmology and strings”

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*Graviton cloning,  
light massive gravitons  
and  
gauge theory/gravity  
correspondence*

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# Bibliography

- The work has appeared in

E. Kiritsis

**hep-th/0608088**

- Related work by:

O. Aharony, A. Clark and A. Karch

**hep-th/0608089**

# Introduction

- Gravity is the oldest known interaction.
- There is widespread feeling that it is probably the least understood.
- The first signals stem from failed attempts to construct the quantum theory due to non-renormalizability.
- Further signals emerged from the presence of black-hole solutions, the associated thermodynamics, and the ensuing information paradox
- The cosmological constant problem hounds physicists for the past few decades.
- And the latest surprise is that the universe seems to accelerate due a 70% component of dark energy.

These are good reasons to advocate that we do not understand gravity very well.

# The gauge theory/string-theory correspondence

One of the most promising approaches to such problems has been the gauge-theory/string theory correspondence.

- It provides a set of microscopic degrees of freedom for gravity
- It defines a non-perturbative quantum theory of gravity
- It explains BH thermodynamics and provides a resolution to the information paradox.
- It has not provided a breakthrough on the cosmological constant yet, but the verdict is still out.

## Some questions for gravity

- Are there consistent and UV complete theories of multiple interacting massless gravitons?
- Are there consistent and UV complete theories of multiple interacting massive gravitons?

In string theory there are massive stringy modes that are spin-2 but their mass cannot be made light without bringing down the full spectrum

A similar remark applies to KK gravitons.

- Is it always, the gravitational dual of a large-N  $CFT_d$ , a string theory on  $AdS_{d+1} \times X$  or a warped product?

♠ The plan is to answer these questions using the tools of gauge-theory/gravity correspondence

## The quick answers

- No more than one interacting massless gravitons are possible. This is in agreement with previous studies in field theory and string theory.

- ♠ There can be many massive interacting gravitons in a theory. The light ones can have masses proportional to the string coupling  $\mathcal{O}(g_s)$ , or equivalently in the large N theory,  $N_c^{-1}$ .

This provides an UV completion to theories with light massive gravitons

- ♣ There are conformal large-N gauge theories, whose gravitational duals are defined on a product of two (or more)  $\text{AdS}_5$  manifolds (bearing internal manifolds).

The associated theories are tensor products of large N theories coupled by multiple-trace deformations.

This is probably the most general type of geometry that can describe the duals of large-N conformal theories.

# Massive gravitons at low energy

- Massive gravitons have been effectively described very early.

$$\frac{\mathcal{L}_{\text{FP}}}{M_P^2} = \int d^4x \left[ \sqrt{-g} R + \sqrt{-\eta} \left[ k_1 h^{\mu\nu} h_{\mu\nu} + k_2 (h^\mu{}_\mu)^2 \right] \right], \quad \sqrt{-g} g^{\mu\nu} = \sqrt{-\eta} (\eta^{\mu\nu} + h^{\mu\nu}) \quad \text{Fierz+Pauli}$$

$$m_g^2 = 4k_1, \quad m_0^2 = -\frac{2k_1(k_1 + 4k_2)}{k_1 + k_2} \quad (\text{ghost} \rightarrow k_1 + k_2 = 0)$$

- Effectively massive gravitons (resonances) arise in induced brane gravity.

*Dvali+Gabadadze+Porrati*

♠ All such theories are **VERY sensitive** in the UV. There are intermediate thresholds where the theory is strongly coupled or depends on UV details.

*Vainshtein, Kiritsis+Tetradis+Tomaras, Luty+Porrati+Ratazzi,Rubakov*

♠ In the FP theory, there is a strong coupling threshold at

$$\Lambda_V \sim (m_g^4 M_P)^{\frac{1}{5}}, \quad \Lambda_{\text{tuned}} \sim (m_g^2 M_P)^{\frac{1}{3}}$$

*Arkani-Hamed+Georgi+Schwartz*

- It is suspected that the most improved threshold is  $\Lambda \sim \sqrt{m_g M_P}$
- If one aspires to use 4d gauge theories to describe observable gravity, he then is forced to have at best a massive, albeit VERY LIGHT 4d graviton.

*Kiritsis+Nitti*

# Massive graviton cosmology

- We consider the cosmology of a Fierz-Pauli theory  $\mathcal{L} = \mathcal{L}_{\text{FP}} + \mathcal{L}_{\text{matter}}$  and a cosmological ansatz  
*Babak+Grishchuk, Damour+Kogan+Papazoglou*

$$g_{00} = -b^2 \quad , \quad g_{ij} = a^2 \delta_{ij} \quad , \quad d\tau = b dt$$

$$G_{\mu\nu} + M_{\mu\nu} = T_{\mu\nu} \quad , \quad M_{\mu\nu} = \frac{m_g^2}{4} \left( 2\delta_\mu^\alpha \delta_\nu^\beta - g^{\alpha\beta} g_{\mu\nu} \right) \left[ h_{\alpha\beta} - h^\gamma{}_\gamma \eta_{\alpha\beta} \right]$$

- The equations map to  $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_P^2} + \rho_m$

$$\rho_m = \frac{m_g^2}{4} \left( \frac{2b}{a} + \frac{1}{b^2} - \frac{3}{a^2} \right) \quad , \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad , \quad a^2 b^3 - (a^4 + 2)b + 2a^3 = 0$$

- Solving we find a late-time positive effective cosmological constant

$$\rho_m = \frac{m_g^2}{2} + \mathcal{O}\left(\frac{1}{a^2}\right)$$

*Kiritsis*

♠ Assuming  $m_g \sim H_0^{-1}$ , the effective vacuum energy is what we measure today. But... the cutoffs are very low, except  $\sqrt{m_g M_P} \sim 10^{-3} - 10^{-4} \text{ eV}$ .

♣ There are still signals of the peculiar UV-IR effects here also: higher terms in the potential for the graviton give very sensitive IR contributions.



# Massive gravitons in $\text{AdS}_{d+1}/\text{CFT}_d$

The massless gravitons are typically dual to the CFT stress tensor

$$e^{-W(h)} = \int \mathcal{D}A e^{-S_{\text{CFT}} + \int d^4x h_{\mu\nu} T^{\mu\nu}}$$

Energy conservation translates into (linearized) diffeomorphism invariance:

$$x^\mu \rightarrow x^\mu + \epsilon^\mu \quad \rightarrow \quad \partial_\mu T^{\mu\nu} = 0 \quad \rightarrow \quad W(h_{\mu\nu} + \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) = W(h_{\mu\nu})$$

$h_{\mu\nu}$  is promoted to a massless 5d graviton. If

$$\partial_\mu T^{\mu\nu} = J^\nu \neq 0$$

then  $\Delta_T > d$  and  $J^\nu$  corresponds to a bulk vector  $A^\nu$ . This will be massive

$$\partial_\mu J^\mu = \Phi \neq 0 \quad \Delta(\Phi) = d + 2$$

in order to the degrees of freedom to match. **This is the gravitational Higgs effect**

$$M_{\text{grav}}^2 = d(\Delta_T - d)$$

There is no vDVZ discontinuity for gravitons in AdS

*Porrati, Kogan+Mouslopoulos+Papazoglou*

# Conserved and non-conserved stress tensors

- An example of a non-conserved stress tensor can be obtained by introducing a  $(d - 1)$ -dimensional defect in a  $\text{CFT}_d$

*Karch+Randall*

The graviton is massive due to the fact that energy is not conserved (it can leak to the bulk via the defect).

This theory however is not translationally invariant.

- Other (trivial) examples exist typically in any CFT. In  $\mathcal{N}=4$  SYM all operators of the type

$$\text{Tr}[\Phi^i \Phi^j \dots \Phi^k D_\mu D_\nu \Phi^l]$$

give rise to massive gravitons, albeit with large (string-scale) masses.

- Non-trivial examples appear in perturbations of product CFTs

In  $CFT_1 \times CFT_2$  both stress tensors are conserved.

$$\partial_\mu T_1^{\mu\nu} = \partial_\mu T_2^{\mu\nu} = 0$$

This should correspond to two massless gravitons that are however non-interacting.

- The dual theory is gravity on  $(AdS_{d+1} \times C_1) \times (AdS_{d+1} \times C_2)$
- The two spaces are necessarily distinct

♠ The central idea in the following will be to consider products of large-N CFTs that are coupled in the UV.

# Massless interacting gravitons

- Have been argued to be impossible in the context of FT  
*Aragone+Deser, Boulanger+Damour+Gualtieri+Henneaux*
- Have been argued to not be possible in the context of asymptotically flat string theory  
*Bachas+Petropoulos*

Assume that we have a  $CFT_2$  (dual to an asymptotically  $AdS_3$  theory of gravity) with two conserved stress tensors. This was analyzed in 2d in detail with the following results:

- It is at the heart of the coset construction  
*Goddard+Kent+Olive*
- It is the key to the generalizations, that use this to factorize the CFT into a product:  
*Kiritsis, Dixon+Harvey*  
*Halpern+Kiritsis*

The strategy is to diagonalize the two commuting hamiltonians as well as the action of the full conformal group.

- The product theory can have discrete correlations between the two factors.  
*Douglas, Halpern+Obers*
- These remarks generalize to other dimensions although they are less rigorous.
- We conclude: two or more massless gravitons are necessarily non-interacting

# Interacting product CFTs

It is now obvious that if we couple together (at the UV) two large- $N$  CFTs, one of the two gravitons will become massive

$$S = S_{CFT_1} + S_{CFT_2} + h \int d^d x O_1 O_2$$

with  $O_i \in CFT_i$  be scalar single-trace operators of dimension  $\Delta_i$ , with  $\Delta_1 + \Delta_2 = d$ , and  $\langle OO \rangle \sim \mathcal{O}(1)$

- This is necessarily a double-trace perturbation
- When  $h \sim \mathcal{O}(1)$ ,  $\beta_{O_2} \sim \mathcal{O}\left(\frac{1}{N}\right)$  and the perturbation is marginal to leading order in  $1/N_c$ .
- When  $h \sim \mathcal{O}(N)$ , generically  $(O_1)^2$  and  $(O_2)^2$  perturbations are also generated, and the perturbation is **marginally relevant**

*Witten, Dymarksy+Klebanov+Roiban*

The relevant perturbations are:

$$\delta\langle T^1(x)T^1(y)\rangle = \frac{h^2}{2!} \int d^4z_1 d^4z_2 \langle T^1(x)T^1(y)O(z_1)O(z_2)\rangle_c \langle \tilde{O}(z_1)\tilde{O}(z_2)\rangle_c$$

$$\delta\langle T^1(x)T^2(y)\rangle = \frac{h^2}{2!} \int d^4z_1 d^4z_2 \langle T^1(x)O(z_1)O(z_2)\rangle_c \langle T^2(y)\tilde{O}(z_1)\tilde{O}(z_2)\rangle_c$$

$$\delta\langle T^2(x)T^2(y)\rangle = \frac{h^2}{2!} \int d^4z_1 d^4z_2 \langle T^2(x)T^2(y)\tilde{O}(z_1)\tilde{O}(z_2)\rangle_c \langle O(z_1)O(z_2)\rangle_c$$

- All are of order  $\mathcal{O}\left(\frac{h^2}{N^2}\right)$ . The subleading corrections scale as higher powers of  $h$  but are always  $\sim N^{-2}$ . Therefore

$$M_{grav}^2 = \frac{h^2}{N^2} [a_1 + a_2 h + a_3 h^2 + \dots] + \mathcal{O}\left(\frac{h^3}{N^4}\right)$$

- The graviton mass is a one-loop effect on the gravitational side.
- $\delta\langle T^1(x)T^2(y)\rangle$  is a trivial correction because it is spacetime independent. The same applies to  $\delta\langle O_1(x)O_2(x)O_1(y)O_2(y)\rangle$ .
- The corrections to the higher couplings are  $\delta\langle T^n\rangle \sim \frac{h^2}{N^2}\langle T^n\rangle$

# The graviton mass

The conserved stress tensor is

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} - \frac{h}{2} g_{\mu\nu} O_1 O_2$$

The orthogonal linear combination is

$$\tilde{T}^{\mu\nu} = c_2 T_1^{\mu\nu} - c_1 T_2^{\mu\nu} - \frac{h}{2d} [c_1 \Delta_2 - c_2 \Delta_1] g_{\mu\nu} O_1 O_2$$

with  $\langle T_{\mu\nu}^i T_{\rho\sigma}^i \rangle = c_i (g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - \frac{2}{d} g_{\mu\nu} g_{\rho\sigma})$  To leading order in  $h$  it satisfies

$$\partial^\mu \tilde{T}_{\mu\nu} = h (c_1 + c_2) \left[ \frac{\Delta_2}{d} (\partial_\nu O_1) O_2 - \frac{\Delta_1}{d} (\partial_\nu O_2) O_1 \right]$$

Using

$$|\partial_\mu O|^2 = 2\Delta |O|^2 \quad , \quad |\partial_\mu T^{\mu\nu}|^2 = 2c \frac{(d+2)(d-1)}{d} (\Delta_{\tilde{T}} - d) > 0$$

we finally obtain

$$M_{grav}^2 = d (\Delta_{\tilde{T}} - d) = h^2 \left( \frac{1}{c_1} + \frac{1}{c_2} \right) \frac{d}{(d+2)(d-1)} \Delta_1 \Delta_2 \sim \mathcal{O} \left( \frac{h^2}{N^2} \right)$$

*Aharony+Clark+Karch*

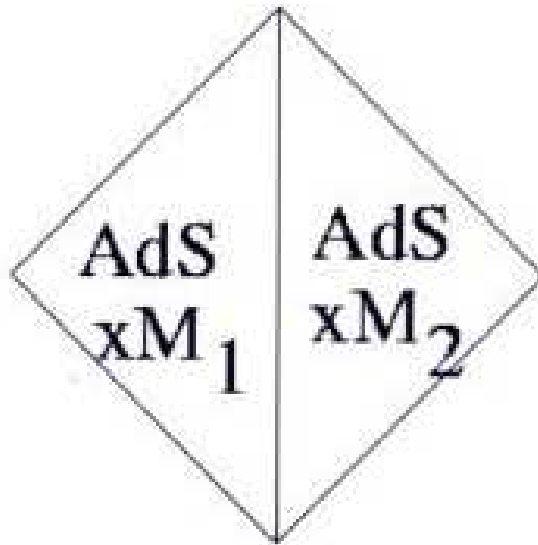
# The spacetime picture

What is the spacetime picture?

- A small deformation of the product geometry

$$( AdS_{d+1} \times M_1 ) \quad \times \quad ( AdS_{d+1} \times M_2 )$$

- As the two CFTs are defined on the same spacetime  $R^d$ , the boundaries of the two  $AdS_{d+1}$  should be identified. In particular the two holographic directions are distinct.



- In the non-conformal (relevant) case, the AdS spaces are replaced by asymptotically AdS spaces.



# Correlated boundary conditions

Description of the perturbation  $O_1 O_2$ , that couples the two CFTs?

- It is a double-trace perturbation and it is implemented by the "canonical" formalism

*Witten*

$$\Phi_1 \leftrightarrow O_1 \quad , \quad \Phi_2 \leftrightarrow O_2 \quad , \quad m_1^2 \ell_1^2 = m_2^2 \ell_2^2$$

because  $\Delta_1 + \Delta_2 = d$ . Their asymptotic behavior is ( $\Delta_1 < d/2$ )

$$\Phi_{\Delta_1} \sim q_1(x) r_1^{\Delta_1} + p_1(x) r_1^{d-\Delta_1} \quad , \quad \tilde{\Phi}_{4-\Delta} \sim p_2(x) r_2^{\Delta_1} + q_2(x) r_2^{d-\Delta_1}$$

$p_1(x)$  and  $p_2(x)$  correspond to the expectation values of the associated operators while  $q_1(x)$  and  $q_2(x)$  correspond to sources.

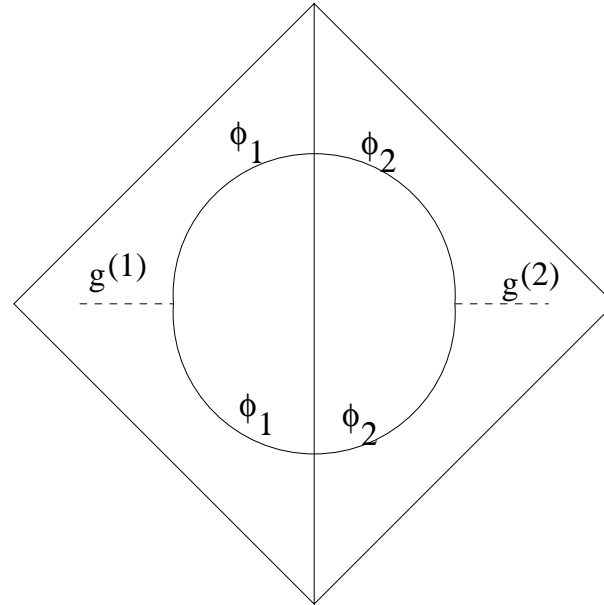
- The perturbation is generated by the bc

$$q_1(x) + h p_2(x) = 0 \quad , \quad q_2(x) + h p_1(x) = 0$$

The full canonical formalism

# The gravitational loop calculation

- On the gravity side the graviton mass arises from the loop corrections of the scalars  $\Phi_1$  and  $\Phi_2$  associated to the perturbing operators  $O_{1,2}$ .



- The “half calculation” corresponds to giving  $\Phi_1$  “transparent” boundary conditions and was done already by [Porrati, and Duff+Liu+Sati](#). It does give the graviton a mass.

- The induced mass agrees with the CFT formula

*Aharony+Clark+Karch*

# Power counting

Reinstating dimensionfull quantities we have

$$M^3 \ell_{\text{AdS}}^3 \sim N^2, \quad m_g \sim \frac{1}{N \ell_{\text{AdS}}}, \quad \ell_{\text{string}} = \frac{\ell_{\text{AdS}}}{\lambda^{\frac{1}{4}}}$$

- Define the following “cutoff” lengths

$$L_n \equiv (M m_g^n)^{-\frac{1}{n+1}} = \ell_{\text{AdS}} N^{\frac{3n-2}{3(n+1)}}$$

- $L_0 \rightarrow L_P$ ,  $L_4 \rightarrow$  Vainshtein cutoff,  $L_2 \rightarrow$  Arkani-Hamed, Georgi, Schwartz cutoff,  $L_1 \rightarrow$  maximal cutoff.

$$L_P \ll \ell_{\text{string}} \ll \ell_{\text{AdS}} \ll L_1 \ll L_2 \ll L_4$$

# $\mathcal{N} = 4$ d=4 Super Yang Mills

Consider  $\text{CFT}_1 = \text{CFT}_2 = \mathcal{N} = 4$  SYM

- The only operators that can be used to deform are the **20**-plet

$$O = \frac{1}{N} \sum_{I=1}^6 \text{Tr}[\Phi^I \Phi^I] \quad , \quad O_{IJ} \equiv \frac{1}{N} \left[ \text{Tr}[\Phi^I \Phi^J] - \frac{1}{6} \delta^{IJ} O \right]$$

so that

$$S_{\text{interaction}} = h_{IJ,KL} \int d^4x \ O_{IJ} \tilde{O}_{KL}$$

- This is generically a marginal perturbation when  $h_{IJKL} \sim \mathcal{O}(1)$  but...
- It is non-perturbatively unstable: the resulting potential is unbounded below (easily visible on the Cartan  $\Phi^I \rightarrow \Phi_i^I$ ,  $i = 1, 2, \dots, 6$ )

$$S_{\text{interaction}} = \frac{h_{IJ,KL}}{N_1 N_2} \int d^4x \left[ \Phi^I \cdot \Phi^J - \frac{1}{6} \delta^{IJ} \Phi \cdot \Phi \right] \left[ \tilde{\Phi}^I \cdot \tilde{\Phi}^J - \frac{1}{6} \delta^{IJ} \tilde{\Phi} \cdot \tilde{\Phi} \right]$$

# The conifold CFT

- This is the  $\mathcal{N} = 1$   $SU(N) \times SU(N)$  quiver CFT dual to  $AdS_5 \times T^{1,1}$ , with two bi-fundamentals  $A_i$  and two anti-bifundamentals  $B_i$  and an  $SU(2) \times SU(2) \times U(1)_R$  global symmetry. There is a line of fixed points.

*Klebanov+Witten*

- It is known that the theory can be deformed keeping conformality and preserving the R-symmetry by

$$W = Tr[A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1]$$

leading to a two-parameter family of CFTs

*Klebanov+Witten*

- It is also known that the double-trace perturbation generated by

$$W = Tr[A_1 B_1] Tr[A_2 B_2] - Tr[A_1 B_2] Tr[A_2 B_1]$$

preserves both conformal invariance and R-symmetry.

*Aharony+Berkooz+Silverstein*

This implies that the deformation of the product of two conifold CFTs (at the same moduli point):  $CFT_c \times CFT'_c$  by the double-trace operator

$$W = \text{Tr}[A_1 B_1] \text{Tr}[A'_2 B'_2] - \text{Tr}[A_1 B_2] \text{Tr}[A'_2 B'_1]$$

is exactly marginal

- The R symmetry is broken to the diagonal one

$$(SU(2)^2 \times U(1)_R) \times (SU(2)^2 \times U(1)_R)' \rightarrow (SU(2)^2 \times U(1)_R)_{\text{diagonal}}$$

The fate of the axial combination is as the gravitons'. **The bulk gauge bosons get masses at one-loop.**

- The geometry remains  $(AdS_5 \times T^{1,1})^2$  pasted back-to-back.
- Nothing is known about the non-perturbative stability of this deformation.

# Multiply connected CFTs

- Several copies of a CFT can be coupled together two at a time

$$S = \sum_{i=1}^M S_i + \sum_{i<j}^M h_{ij} \int d^d x O_i O_j \quad , \quad \Delta_i + \Delta_j = d$$

- The combined theory is an asymptotically free theory and in special cases conformal.
- It contains M copies of an  $AdS_{d+1}$  coupled via boundary conditions in their common boundary.
- It contains 1 massless and M-1 massive gravitons.

Can we have more than two theories coupled together via a "cubic" or higher "vertex" eg.

$$S = \sum_{i=1}^M S_i + \int d^d x \prod_{i=1}^M O_i \quad , \quad \sum_{i=1}^M \Delta_i \leq d$$

- The answer to this question is dimension dependent and we need the unitarity bounds  $\Delta_{scalar} \geq \frac{d-2}{2}$ ,  $\Delta_{vector} \geq d-1$ ,  $\Delta_{s=2} \geq d$ .

♠ In  $d=6$ , the maximum possible is a cubic vertex, and  $Dim(O) = 2 \rightarrow O$  is a free scalar. This leads to an unstable potential.

♣ In  $d=4$  a quartic vertex has a similar fate. But there can be a non-trivial cubic vertex using CFTs with scalar operators with  $\Delta \leq 4/3$   
 SQCD in conformal window  $\frac{1}{3} < \frac{N}{N_f} < \frac{2}{3}$ .

$$\text{Meson operators} \rightarrow \Delta_{meson} = 3 - 3\frac{N}{N_f}$$

We take the Veneziano limit

$$N \rightarrow \infty, \quad N_f \rightarrow \infty, \quad x = \frac{N}{N_f} = \text{fixed}, \quad \frac{1}{3} \leq x \leq \frac{2}{3}$$

We may now take the product  $SQCD_{x_1} \times SQCD_{x_2} \times SQCD_{x_3}$  with  $x_1 + x_2 + x_3 = \frac{5}{3}$

- This cubic vertex can be used also in tandem to connect several CFTs as advocated earlier.



♠ In  $d=2$  the unitarity bound squeezes to zero, and this allows any possible vertex coupling these theories.

- In the example we studied, the 't Hooft couplings can be chosen so that

$$\Delta_{\square, \bar{\square}; 1} = \frac{1}{k}, \quad k = 1, 2, 3 \dots$$

Moreover, fixed points can be found in weak coupling perturbation theory.

- Again the non-perturbative stability of such deformations is not understood.

# The relationship to multi-throat geometries

- It is known that multi-throat geometries can arise in the IR of string compactifications
- A prototype of this is the breaking of  $U(2N) \rightarrow U(N) \times U(N)$  by Higgs vevs

*Klebanov+Witten*

Here the two large- $N$  throats, are coupled in the IR, **but not in the UV**

- The dual geometrical picture is very different: one space (with two throats), one graviton (with two localisations).
- There is a simplified RS-like picture where two AdS slices (with in general different cosmological constants) are separated by a RS brane.

*Padilla, Gabadadze+Grisa+Shang*

It involves a RS graviton and a massive DGP-like bound state.

However, in this case the two cutoff-AdS spaces communicate via the RS brane. This is not the case in the backgrounds that are coupled in the UV. There is an infinite barrier in between.

## Directions and open problems

- Are products of AdS spaces the most general dual geometry of large-N CFTs?
- What are other "frame-independent" characteristics of perturbed product large-N CFTs? (beyond  $\Delta_{\tilde{T}} = 4 + \mathcal{O}(N^{-2})$ .)?
- Analysis of concrete examples where 3 or more CFTs are coupled together. Structure of graviton mass matrix.
- These are examples of UV complete theories of massive gravitons. It is interesting to see how they resolve the problems of Pauli-Fierz truncations, what is the effective UV cutoff, and what is the effective resolution of the strong coupling puzzles of massive graviton theories.
- The question of thermalization of coupled products is correlated with the existence of black-holes in the product space-times. This may shed light in the process of equilibration between coupled reservoirs.

- There seems to be a structure reminding cobordism, but it is certainly distinct. What are the precise rules, and is that interesting mathematically?
- Are such product geometries non-perturbatively stable?
- How much of this survives at small  $N$ ?
- There are indications that massive gravitons with  $mass \sim H_0^{-1} \sim 10^{-33}$  eV can produce today's acceleration. Can the theories here help implement this idea?
- One may extend these ideas to asymptotically flat string backgrounds. This produces clone universes interacting at their asymptotic boundaries. Can this be responsible of what we see in our universe?

# Double trace couplings and the RG flow

We normalize single trace operators as

$$\langle O(x)O(y) \rangle = \frac{1}{|x-y|^{2\Delta}} \quad , \quad \langle O^n \rangle \sim \frac{1}{N^{n-2}}$$

We label by  $O_i$  operators in  $\text{CFT}_1$  and  $O_I$  operators in  $\text{CFT}_2$  (all of dimension  $d/2$ ) and perturb

$$\delta S = f_{ij} \int O_i O_j + \tilde{f}_{IJ} \int O_I O_J + g_{iI} \int O_i O_I$$

We may now compute the flow equations by considering

$$\langle O_i O_j \delta S \delta S \rangle \quad , \quad \langle O_I O_J \delta S \delta S \rangle \quad , \quad \langle O_i O_I \delta S \delta S \rangle$$

to obtain

$$\dot{f}_{ij} = -8(f^2)_{ij} - 2(gg^T)_{ij}$$

$$\dot{\tilde{f}}_{IJ} = -8(\tilde{f}^2)_{IJ} - 2(g^T g)_{IJ}$$

$$\dot{g}_{iI} = -2(g\tilde{f})_{iI} - 2(fg)_{iI}$$

Generically, the couplings are asymptotically free (marginally relevant).

RETURN

# The full canonical formalism

Witten, Mück

The perturbed CFT action:

$$I^W = I_{CFT} + \int d^4x W(O) \quad , \quad W(O) \rightarrow \text{local}$$

The CFT action is related to the bulk supergravity action as

$$\langle \exp \left[ - \int d^4x \alpha O \right] \rangle = \exp [-I_{sugra}(q)]$$

The source  $\alpha(x)$  is related to the asymptotic form of the bulk field  $\Phi$

$$\lim_{r \rightarrow 0} \Phi(x, r) \sim r^\Delta q(x) + r^{4-\Delta} p(x) + \dots \quad , \quad q(x) + \alpha(x) = 0$$

In the Hamilton-Jacobi formalism,  $p$  and  $q$  are conjugate variables with

$$p = -\frac{\delta I_{sugra}(q)}{\delta q} \quad , \quad q = \frac{\delta J(p)}{\delta p} \quad , \quad J(p) = I_{sugra} - \int d^4x qp$$

The bulk generating functional for the perturbed theory is

$$I_{sugra}^W(\alpha) = I_{sugra}(q) + \int d^4x \left( W(p) - p \frac{\delta W}{\delta p} \right) \quad , \quad \frac{\delta I_{sugra}^W}{\delta p} = q + \frac{\delta W(p)}{\delta p} + \alpha = 0$$

The bulk/boundary correspondence translates to:

$$\langle \exp \left[ - \int d^4x \alpha O \right] \rangle_W = \exp [-I_{sugra}^W(\alpha) + I_{sugra}^W(0)]$$

For the case of interest the perturbed CFT action is

$$I^W = I_{CFT_1} + I_{CFT_2} + \int d^4x W(O_\Delta, \tilde{O}_{4-\Delta}) \quad , \quad W(O_\Delta, \tilde{O}_{4-\Delta}) = h O_\Delta \tilde{O}_{4-\Delta}$$

The canonical variables are

$$p_i = -\frac{\delta I_{sugra}^i(q_i)}{\delta q_i} \quad , \quad q_i = \frac{\delta J^i(p_i)}{\delta p_i} \quad , \quad J^i(p) = I_{sugra}^i - \int d^4x q_i p_i \quad , \quad i = 1, 2$$

The bulk generating functional for the perturbed theory is

$$I_{sugra}^W(\alpha_1, \alpha_2) = I_{sugra}^1(q_1) + I_{sugra}^2(q_2) + \int d^4x \left( W(p_1, p_2) - \sum_{i=1}^2 p_i \frac{\delta W}{\delta p_i} \right)$$

with  $p_i, q_i$  determined by the sources  $\alpha_i$

$$\frac{\delta I_{sugra}^W}{\delta p_i} = q_i + \frac{\delta W(p_1, p_2)}{\delta p_i} + \alpha_i = q_i + g (\sigma^1)^{ij} p_j + \alpha_i = 0$$

The bulk/boundary correspondence recipe is

$$\langle \exp \left[ - \int d^4x \left( \alpha_1 O_\Delta + \alpha_2 \tilde{O}_{4-\Delta} \right) \right] \rangle_W = \exp \left[ -I_{sugra}^W(\alpha_1, \alpha_2) + I_{sugra}^W(0, 0) \right]$$

RETURN

# Transversality and the graviton mass

*Porrati, Duff+Liu+Sati*

Most general graviton self-energy in AdS satisfying the Ward identities is

$$\Sigma_{\mu\nu;\alpha\beta} = \beta(\Delta)\Pi_{\mu\nu;\alpha\beta} + \gamma(\Delta)K_{\mu\nu;\alpha\beta} \quad , \quad \Delta \rightarrow \text{Lichnerowicz}$$

$$\Pi_{\mu\nu}{}^{\alpha\beta} = \delta_{\mu}^{\alpha}\delta_{\nu}^{\beta} - \frac{1}{3}g_{\mu\nu}g^{\alpha\beta} + 2\nabla_{\mu} \left( \frac{\delta_{\nu}^{\beta} + \nabla_{\nu}\nabla^{\beta}/2\Lambda}{\Delta - 2\Lambda} \right) \nabla^{\alpha} - \frac{\Lambda}{3} \left( g_{\mu\nu} + \frac{3}{\Lambda}\nabla_{\mu}\nabla_{\nu} \right) \frac{(g_{\alpha\beta} + \frac{3}{\Lambda}\nabla_{\alpha}\nabla_{\beta})}{3\Delta - 4\Lambda}$$

$$K_{\mu\nu}{}^{\alpha\beta} = \frac{\Delta - \Lambda}{3\Delta - 4\Lambda}d_{\mu\nu}d^{\alpha\beta} \quad , \quad d_{\mu\nu} = g_{\mu\nu} + \frac{1}{\Delta - \Lambda}\nabla_{\mu}\nabla_{\nu} \quad , \quad \Lambda = -\frac{3}{\ell_{AdS}^2}$$

Consider the kinetic graviton operator, and the linearized equation of motion

$$\left[ \frac{1}{16\pi G}D_{\mu\nu}{}^{\alpha\beta} + \Sigma_{\mu\nu}{}^{\alpha\beta} \right] h_{\alpha\beta} = 0 \quad , \quad \Sigma_{\mu\nu}{}^{\alpha\beta} = \frac{c}{2\ell_{AdS}^4}\Pi_{\mu\nu}{}^{\alpha\beta}$$

Using  $K * h = -\frac{M^2}{2}h$  we obtain

$$M_{grav}^2 = (16\pi G)\frac{c}{\ell_{AdS}^4}$$



## Ad<sub>d+1</sub> Scalar propagators

We will use homogeneous coordinates,  $X^\mu$ , to embed AdS<sub>d+1</sub> in  $R^{(2,d-1)}$ :

$$X \cdot X = -\ell_{AdS}^2$$

The propagator from  $X$  to  $Y$  is a function of  $Z = X^\mu Y_\mu$  and satisfies

$$\left[ (1 - Z^2) \partial_Z^2 - (d + 1) Z \partial_Z + L(L - d) \right] D_L = 0 \quad , \quad m^2 \ell_{AdS}^2 = L(L - d)$$

Boundary conditions are parametrized by  $\alpha$  and  $\beta$  as follows

$$D_{1,d-1}(Z) = \frac{1}{(Z^2 - 1)^{\frac{d-1}{2}}} \left[ \alpha + \beta Z F \left( \frac{1}{2}, \frac{3-d}{2}, \frac{3}{2}, Z^2 \right) \right]$$

Here  $\alpha = 1, \beta = 0$  are the boundary conditions conserving energy and momentum across the boundary. On the other hand  $\alpha = \beta = 1$  are “transparent” boundary conditions.

In the case of the double trace perturbation the full propagator is

*Mück, Aharony+Berkooz+Katz*

$$G = \frac{1}{1 + \hat{h}^2} \begin{pmatrix} D_1 + \hat{h}^2 D_2 & \hat{h}(D_1 - D_2) \\ \hat{h}(D_1 - D_2) & D_2 + \hat{h}^2 D_1 \end{pmatrix}, \quad \hat{h} = (2\Delta_1 - d) h$$

for  $\text{CFT}_1 = \text{CFT}_2$

- This can be used to calculate the  $2 \times 2$  matrix  $\langle g_i^{\mu\nu}(x) g_j^{\rho\sigma}(y) \rangle$  and from this extract the graviton mass

RETURN

# Examples in two dimensions

The simplest coupling between two distinct CFTs in 2d is a current-current coupling

$$S = S_1 + S_2 + g \int d^2z J_1 \bar{J}_2 \quad , \quad \partial \bar{J}_2 = \bar{\partial} J_1 = 0$$

and this is always an exactly marginal perturbation. It provides a boost of the Charge lattice  $Q_1 \times Q_2$

- This may not have a large-N interpretation generically, but it has the basic property of the double-trace perturbation: only disconnected correlators survive.

- A good example of a solvable large-N CFT in 2d is the conformal coset

$$\frac{SU(N)_{k_1} \times SU(N)_{k_2}}{SU(N)_{k_1+k_2}}$$

- It is the IR limit of an  $SU(N)$  gauge theory. The 't Hooft coupling constants are

$$\lambda_1 = \frac{N}{k_1} \quad , \quad \lambda_2 = \frac{N}{k_2}$$

- The large N limit involves

$$N \rightarrow \infty \quad , \quad \lambda_{1,2} = \text{fixed}$$

$$c = \frac{(\lambda_1 + \lambda_2 + 2\lambda_1\lambda_2)}{(1 + \lambda_1)(1 + \lambda_2)(\lambda_1 + \lambda_2 + \lambda_1\lambda_2)}(N^2 - 1)$$

- One single trace operator is  $\Phi_{\square, \bar{\square}; 1}$  with

$$\Delta_{\square, \bar{\square}; 1} = \frac{1}{2} \left[ \frac{\lambda_1}{(1 + \lambda_1)} + \frac{\lambda_2}{(1 + \lambda_2)} \right] + \mathcal{O}\left(\frac{1}{N}\right)$$

It can be used to couple together two such theories, provided the  $\lambda_i$  are appropriately chosen.

- For  $\lambda_i = 1 - \epsilon_i$ ,  $\epsilon_i \ll 1$ , there is a fixed point in perturbation theory.

# Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 3 minutes
- The gauge theory/string-theory correspondence 4 minutes
- Some questions for gravity 6 minutes
- The quick answers 8 minutes
- Massive gravitons at low energy 10 minutes
- Massive graviton cosmology 13 minutes
- Massive gravitons in  $\text{AdS}_{d+1}/\text{CFT}_d$  15 minutes
- Conserved and non-conserved stress tensors 18 minutes
- Massless interacting gravitons 21 minutes
- Interacting product CFTs 23 minutes
- The graviton mass 26 minutes
- The spacetime picture 28 minutes
- Correlated boundary conditions 30 minutes
- The gravitational loop calculation 32 minutes
- Power counting 34 minutes

- $\mathcal{N} = 4$  d=4 Super Yang Mills 36 minutes
- The conifold CFT 39 minutes
- Multiply connected CFTs 44 minutes
- The relationship to multithroat geometries 46 minutes
- Directions and open problems 51 minutes
- The double trace couplings and the RG flow 53 minutes
- The full canonical formalism 55 minutes
- Transversality and the graviton mass 57 minutes
- AdS Scalar Propagators 59 minutes
- Examples in 2d 63 minutes