Cosmology, Strings, and Black Holes,
Copenhagen, 18-21 April 2006

A GAUGE THEORY FOR GRAVITY

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hep-th/0310001 version 1 and work in progress

SURGEON'S WARNING!

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- I will present speculative ideas about the UV nature of gravity and how it might affect the IR.

- They are backed up by very few hard calculations (so far).

- Some might find the ideas irreverent or provocative.

Plan of the talk

- Introduction and motivations
- Gravitons and glueballs
- Gravity from gauge theory
- Fitting the Standard Model
- 4d Gravitational engineering
- Unification
- Conclusions and open problems

Introduction

• String theory has became popular in 1984 because it was thought that it would provide a unique theory of fundamental forces.

• The argument that string theory provides a perturbative quantization of a gravity theory was not important in the beginning.

• After 20 years and several "revolutions" we now realize that the strongest argument for string theory is that it goes some way in treating quantum effects in gravity theories (several "gravity" theories depending on vacua).

• It is instructive to look back at how string theory bypasses the problems of QFT in dealing with gravity. This is inherent in the treatment of UV asymptotics in string theory.

♣ Closed strings have an effective UV cutoff at the *string scale* M_s. This is evident at one-loop. Its geometrical implementation is responsible for consistency.

♠ Open strings, if tadpoles are cancelled, can have their UV divergences interpreted as IR closed string divergences in the cross channel.

• Although perturbative string theory works very well at energies up to the string scale, it fails at higher energies.

At weak string coupling:

$$R \quad < \quad M_s \quad < \quad M_P$$

By choosing $g_s << 1$ we may make $M_P >> M_s$ and extend the validity of the perturbative theory for much more.

String perturbation theory breaks down when $E \to M_P$. In view of the "M_s cutoff" this is not unexpected.

Amati+Ciafaloni+Veneziano, Gross+Mende

The relevant dimensionless coupling is $\frac{E^8}{M_P^8}$.

Non-perturbative dualities do not help with this state of affairs: $E < M_P$ is a duality-invariant statement.

• Do we ever care about such high energies? It seems that the answer is yes:

(a) Black hole singularities

(b) Cosmological singularities

It is hard to see how singularities caused by gravitational collapse can be resolved without understanding the short distance (Planck length) behavior of gravity. (contrast with time-like singularities)

• There has been speculation, in the past about new UV degrees of freedom in string theory, that would allow access to a well-behaved high-energy behavior.

Witten+Attick,Veneziano, Gross+Mende

Gauge Theory

• 't Hooft has argued that strings arise in gauge theory, in a potentially controllable way at large N.

• This is a property that seems to be independent on the space-time dimension the gauge theory lives on.

• It does depend crucially on the gauge representations. They must be bi-fundamentals at best. (Dimension $\sim \mathcal{O}(N^2)$)

$\Longrightarrow$ It is an intriguing problem to try to obtain higher branes from representations with (Dimension $\sim \mathcal{O}(N^3)$ or higher)

• The details of how such a string theory is realized, started to be uncovered in 1997, with AdS/CFT correspondence and its avatars.

Maldacena

Witten, Gubser+Klebanov+Polyakov

• All asymptotically-free or asymptotically conformal theories are expected to have a string theory/gravity dual. Some duals will be somewhat bizarre (see below). The string is the flux-tube. No confinement is necessary (qf. N=4 sYM).

• The string theory dual to a gauge theory lives in a dimension at least by one higher from that of the gauge theory

• The string theory graviton is the 2^{++} glueball. (the dilaton is the 0^{++} glueball, the axion the 0^{+-}). The operators that create them are (in pure gauge theory)

$$g_{\mu\nu} \sim Tr[F^2_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}F^2] \quad , \quad \Phi \sim Tr[F^2] \quad , \quad a \sim Tr[F \wedge F]$$

If there are adjoint fermions, more low-lying fields are generated:

$$Tr[\bar{\psi}(1,\gamma^5)\psi] \sim \chi \quad , \quad Tr[\bar{\psi}\gamma^{\mu}\psi] \sim A_{\mu}$$

$Tr[\bar{\psi}\gamma^{\mu\nu}\psi]$ seems to correspond to stringy states.

Gravitons out of glue?

• The central idea is that gravitons are composite: they are made out of glue.

• An asymptotically free gauge theory, at low energy has dynamics described in terms of strings (gravitons, dilatons etc).

At high energies ($E >> \lambda$) the right degrees of freedom are gluons.

• The associated string theory has low energy Regge trajectories. They disappear at high energy.

• For the same reason such a string theory is not expected to have a Hagedorn transition. The density of states at $g_s \sim \frac{1}{N} = 0$, evaporates, at non-zero string coupling.

• What is the appropriate dimension of the gauge theory?The best bet is 4:

(a) Four dimensions is the preferred dimension for gauge theories

(b)We seem to live in four dimensions

(c) It "unifies" nicely with the SM gauge interactions.

• Why N must be large? To ensure that observable gravity is essentially classical on most length scales.

• The observable space-time geometry is not a fundamental concept but rather a derived one ("emerging" to use some "in" terms)

• It is a quantity, similar to the thermodynamic ones: averaging over "color" is needed! This matches the emergence of thermodynamic properties and entropies of gravitational singularities (black holes) as we learned in the past 10 years.

There are several obvious problems and a bonus:

♣ problem 1: We know that 4d gravitons are massless. A generic gauge theory has massive 2^{++} "gravitons". Massive gravitons have been also theoretically problematic so far.

♠ problem 2: The gravity generated by the gauge theory strings lives in 5 (or more dimensions). The background is curved at large scales and it is typically asymptotically AdS. How do we recover 4d gravity?

♣ problem 3: How is the $SU(3)\times SU(2)\times U(1)$ standard model incorporated in this picture?

Expected Bonus: the cosmological constant

If the 4d graviton is a bound-state of glue, the cosmological constant problem changes aspect:

- It is not anymore true that the "vacuum energy" is the cosmological constant.

- The "observable cosmological constant can be calculated for the graviton two-point function.

An electron loop couples to the graviton at low energy in the standard way. However, now the graviton coupling has a form factor.

The form factor $f(p)$ turns-off at the "compositeness scale", Λ_c: In the simplest gauge theory $\Lambda_c \sim \Lambda$ but it can get more complicated depending on the structure.

We therefore expect that the effective cosmological constant will be

$$\int d^4p \ \log(p^2 + m_e^2) \ f(p) \quad \sim \quad \Lambda_c^4$$

If $\Lambda_c \sim 10^{-3} eV$ that could do the job! We may correlate the compositeness scale with the graviton mass as $m_g = \frac{\Lambda_c^2}{M_P} \sim 10^{-38} eV$.

This argument is not novel. It has been used recently to construct a toy model which could handle the CC problem.

Sundrum

The proposal here differs in that the SM particles will not be charged under the strong (gravity-generating) gauge group, and we are at large N

Similar directions

Gauge theories have been used before to describe string theory.

- Old matrix models $\to D \leq 2$ string theory (bosonic+type 0).
  Douglas+Shenker, Bezin+Kazakov, Gross+Migdal

- BFSS+IKKT: a matrix model for DLCQ D=11 supergravity.
  Banks+Fischler+Shenker+Susskind, Ishibashi+Kawai+Kitazawa+Tsutchiya

- Matrix string theory: An avatar of the above.
  Dijkgraaf+Verlinde+Verlinde

- Higher-d gauge theories $\to$ lower dimensional compactifications.
  W. Taylor, N. Seiberg

There are also similarities with "deconstruction":

A geometrical extra S^1 is simulated by a quiver $SU(k)^N$ at large N.
Arkani-Hamed+Cohen+Georgi

How many dimensions?

There are two related questions here:

(a)What is the maximal (effective) spacetime dimension?

In the context of theories that are in the "connected class" of $\mathcal{N} = 4$ sYM, the dimension of spacetime can be 10 or less. In particular, there are many non-critical AdS_5 related IIA,B vacua, that can be in this class.
Kuperstein+Sonnenschein, *Klebanov+Maldacena*, *Bigazzi+Casero+Cotrone+Kiritsis+Paredes*

There are other "connected classes" with different maximal dimension with the highest one being 26. There are also product space-times. (they may or may not be relevant for implementing observable gravity.

The important common ingredient is the presence of at least the 5-th (holographic direction). Supersymmetry seems not necessary in the "gravitational sector".

(b) How do we obtain the observable four-dimensional gravitational force?

Where is the Standard Model?

• The Standard model is an extra (”splinter”) gauge-group, $SU(3) \times SU(2) \times U(1)$, with the SM particles NOT charged under the large-N gauge group.

• The pictorial representation in the dual gravity theory is: probe branes inside the bulk geometry. In the gauge theory → just the SM.

• A messenger sector is needed to transfer the large-N theory force to the SM, and therefore generate the gravitational interactions of matter. (this is the analogue of the stretched strings)

These should be “bifundamentals”: charged under the large-N gauge group, and the SM gauge group. (Q^i_a, with i is the fundamental of G_{SM}).

♣ In order to couple both the SM fermions and bosons, we need both fermionic and bosonic messengers.

Bosons Q^i_a, fermions χ^{i_a}.

Marginal couplings to the SM fields $A^{ij}_\mu, \psi^{ij}, H^{ij}$, will transfer the gravitational interaction:

$$\bar{\chi}^i_a \gamma^\mu \chi^j_a \; A^{ij}_\mu \quad , \quad (Q^*)^i_a (\overleftrightarrow{\partial})^\mu \chi^j_a \; A^{ij}_\mu \quad , \quad \bar{\psi}^{ij} \chi^i_a Q^j_a \quad , \quad \bar{\chi}^i_a \chi^j_a \; H^{ij}$$

♣ For this to be possible, the SM fields must be "bi-fundamentals". Is this achievable? :
Yes, (in more than one ways).

Antoniadis+Kiritsis+Tomaras, Ibanez+Marchesano+Rabadan
Aldazabal+Ibanez+Quevedo+Uranga, Antoniadis+Dimopoulos

particle	$U(3)_c$	$SU(2)_w$	$U(1)$
$Q(\mathbf{3},\mathbf{2},+\frac{1}{6})$	V	V	0
$U^c(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$\bar{V}$	0	V
$D^c(\bar{\mathbf{3}},\mathbf{1},+\frac{1}{3})$	$\bar{V}$	0	$\bar{V}$
$L\ (\mathbf{1},\mathbf{2},-\frac{1}{2})$	0	$\bar{V}$	V
$e^c(\mathbf{1},\mathbf{1},+1)$	0	0	$\bar{S}$
$\nu^R(\mathbf{1},\mathbf{1},0)$	0	A	0
$H(\mathbf{1},\mathbf{2},-\frac{1}{2})$	0	$\bar{V}$	V

$$Y = \frac{1}{6}Q_3 - \frac{1}{2}Q_1$$

This is an "un-oriented" configuration. There are also "oriented" configurations. The leftover mixed anomaly can be cancelled by an axion.

Anastasopoulos+Dijkstra+Kiritsis+Schellekens, to appear

• the couplings to the large-N theory:

$$\chi^i_a \gamma^\mu \chi^i_b \; A^{ab}_\mu \quad , \quad (Q^*)^i_a (\overleftrightarrow{\partial})^\mu \chi^i_b \; A^{ab}_\mu$$

• The messengers must be massive:

$$(Q^*)^i_a Q^i_a \to M^B_I \quad , \quad \bar{\chi}^i_a \chi^i_a \to M^F_I \quad , \quad I \in \{U(3), SU(2), U(1)\}$$

The masses $M^{B,F}_{3,2,1}$ roughly correspond to the (inverse) "positions" of U(3), U(2), U(1) "branes" in the emerging geometry.

• Integrating out the messengers, induces the gravitational couplings to the SM particles. The graviton couples correctly to the SM stress-tensor (via the "induced" metric).

This follows from translation invariance of the underlying gauge theory.

• Most of the parameters of the gauge theory are dimensionless:

Large-N: g^2_N, λ_{IJ}, g_{ij}

SM: g^2_I, Yukawa's, λ_{Higgs}.

Some are dimensionfull: $M^{B,F}_I$, m_{Higgs}.

four dimensional gravity

We may now re-address the question: what type of gravitational force is felt by the SM particles?

- Without any engineering, it will be 5-dimensional.

- Well-known mechanisms like compactification or RS reduction seem inappropriate.

♣ Brane-induced gravity seems to be the right mechanism

Dvali+Gabadadze+Porrati

- SM loops induce a four-dimensional Einstein term for the induced metric.

$$S \sim M^3 \int d^5x \ \sqrt{g} R_5 + \delta(r - r_0) \Lambda_c^2 \int d^4x \ \sqrt{\hat{g}} R_4$$

The coefficient is

$$\Lambda_c \sim \Lambda_{\text{gauge theory}} \sim 10^{-3} \text{ eV}$$

At energies above Λ_c, there is no gravitational interaction: it is the gauge force that is the appropriate description → the SM "fuses" with the large-N gauge theory.

The R_4 term induces a four-dimensional gravitational force with scale

$$M_P = \frac{L}{r_0}\,\Lambda_c \quad , \quad r_0 \to 0 \quad , \quad M_5 = \frac{L}{r_0}\,M$$

$r_0 \sim M_I^{-1}$ denotes collectively the "position" of SM branes in the holographic direction.

• This dominates and characterizes the gravitational interaction in the UV. The interaction is four-dimensional from

$$\frac{N}{r_0^2 M_P} \quad \leq \quad E \quad \leq M_P$$

Compatibility with cosmology implies that

$$\frac{N}{r_0^2 M_P} \leq 10^{-33}\ \text{eV}$$

It does not suffer from the non-decoupling properties of DGP setup (UV completion known).
Kiritsis+Tetradis+Tomaras, *Luty+Porrati+Ratazzi*, *Rubakov*

4d graviton quasi-massive. Correlated with late time acceleration. *Defayet+Dvali+Gabadadze*

The UV structure

We would like to have an UV complete gauge theory. This will work out if the theory is asymptotically free or conformal.

• In the large-N sector this poses a bound on the adjoint scalars and fermions. The messenger fields give subleading contributions to the large-N β-functions.

• They do however give dominant contributions to the SM β-functions. There are two options:

♣ To preserve asymptotic freedom, we must package messengers in $\mathcal{N} = 4$ multiplets.

♠ Or, we must "fuse" the SM model group and the large-N group above the messenger scale $\sim M_P$. This requires some detailed model building (to be done).

Questions of hierarchy

Why is the Higgs vev $\mathcal{O}$(200 GeV) and not M_P?

- A possible solution: the Higgs is a colorless bound-state, that gets a vev

$$H_{ij} \sim \bar{\chi}_i^a \; \chi_j^a$$

The vacuum condensate in the SU(2) sector will break the weak symmetry.

- Why this happens only in the SU(2) sector?

- What sets the size of the vev?

The ultimate Unification?

It is tempting to expect that in this context:

- All dimensionfull parameters are given by vevs
- At high energy the gauge group is unified:

$$U(N) \times U(3) \times U(2) \times U(1) \times \cdots \quad \rightarrow \quad U(N')$$

- We even entertain the case of large N_i

$$U(N_1) \times U(N_2) \times U(N_3) \times \cdots$$

and the associated presence of "parallel" hyper-universes coupled via their boundaries.

- The only natural choice is

$$N_{total} = \infty$$

Outlook and open problems

There are several important questions and loose ends:

♣ What makes the other large-N "glueballs" (dilaton, axion etc) unobservable so far?

• What is the detailed structure of the large-N theory and messenger sector?

♣ Is there a representation of the SM as bi-fundamentals that is preferred?

• What is the interpretation of some of the dimensionless interaction couplings?

♣ How is the Higgs symmetry breaking induced and protected?

♠ Are we giving up the predictability we always wished from the fundamental theory?

In string theory we believe in a unique UV theory, but in a large number of IR incarnations (vacua)

In gauge theory, we have a (probably large) number of UV options, but also the idea of universality: different UV theories → same IR physics.

♠ How much of the physics we are interested in is really computable?

Links to transparencies

- Massive gravitons and cosmological acceleration 3 minutes

- AdS_5 propagators 3 minutes

- Induced gravity in AdS_5 3 minutes

What are the allowed dual geometries of large-N FTs?

A related question: Can the graviton be non-trivially cloned?

Every large-N CFT has a conserved traceless $T_{\mu\nu}$, therefore a dual massless graviton.

If it has two independent ones, then it can be factorized into a product of non-interacting CFTs $\to$ We have two AdS_5 space-times and two non-interacting massless gravitons.

We may try to couple the product $CFT_1 \times CFT_2$ by a marginal or relevant operator. Such operators do not exist if $CFT_1 \sim CFT_2 \sim \mathcal{N} = 4$ sYM.

But they do exist for example for $CFT_i \sim$ Seiberg $\mathcal{N} = 1$ fixed points.

The interpretation of the modified theory is : a product space-time ($AdS_5 \times AdS_5$ if marginal) two independent string theories on each, interacting via coupled boundary conditions (relevant for multi-trace perturbations).

Aharony et al. to appear
Kiritsis to appear

We may think of the AdSs as being back-to-back.

- One of the two gravitons has acquired a mass (combined stress tensor conserved).

$$\partial^\mu T'_{\mu\nu} = J_\nu \quad , \quad \partial^\mu J_\mu = \Phi$$

• There may be "triple intersections" in $D = 4$, as there are large-N theories with scalar operators of dimension $< \frac{4}{3}$. There are no quadruple intersections.

• Any kind of intersection is possible for D=2 large-N theories.

• Considering several double or triple intersections we may construct theories with an arbitrary number of interacting gravitons. Only one is however massless.

• The space of dual geometries decomposes into a direct sum of several single components coupled via their boundaries.

• These observation indicate the proper interpretation of "double trace" (non-local) deformations of ordinary asymptotically flat string theories.

• Can product geometries be relevant for real four-dimensional gravity or cosmology?

We may entertain the existence of many "correlated" parallel universes. It is just that the correlation is at an AdS/like boundary rather than an initial time slice.

Back

Massive gravitons and cosmological acceleration

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Grischuk
Kiritsis

Consider a Pauli-Fierz massive graviton

$$\sqrt{g}\ g^{\mu\nu} = \sqrt{\eta}(\eta^{\mu\nu} + h^{\mu\nu}) \quad , \quad L = L_{GR} + L_{mass} + L_{matter}$$

$$L_{GR} = -\frac{1}{2\kappa^2}\sqrt{-g}\ R \quad , \quad L_{mass} = -\frac{1}{2\kappa^2}\sqrt{-\eta}\left[k_1 h^{\mu\nu}h_{\mu\nu} + k_2(h^{\mu\nu}\eta_{\mu\nu})^2\right]$$

$$k_1 = \frac{m_g^2}{4} \quad , \quad k_2 = -\frac{m_g^2}{8}\ \frac{m_g^2 + 2m_0^2}{2m_g^2 + m_0^2} \quad , \quad \zeta = \frac{m_0^2}{m_g^2}$$

The equation of motion are

$$G_{\mu\nu} + M_{\mu\nu} = T_{\mu\nu} \quad , \quad M_{\mu\nu} = \left(\delta^a_\mu \delta^b_\nu - \frac{1}{2}g^{ab}g_{\mu\nu}\right)\left[2k_1 h_{ab} + 2k_2(h^{cd}\eta_{cd})\eta_{ab}\right]$$

The cosmological ansatz is now $g_{00} = b^2$, $g_{11} = g_{22} = g_{33} = -a^2$ and the cosmological time $\frac{d}{d\tau} = \frac{1}{b}\frac{d}{dt}$

The FRW equations are:

$$3\left(\frac{\dot a}{a}\right)^2 + M^0_0 = \kappa^2\ \rho \quad , \quad M^0_0 = \frac{3m_g^2}{8(\zeta+2)}\frac{a^5 + (4\zeta - 1)ab^4 + 2\zeta\ a^2 b - 6\zeta\ b^3}{a^2 b^3}$$

$$\dot\rho + 3\frac{\dot a}{a}(\rho + p) = 0 \quad , \quad 3\frac{a^6}{b^2} + (4\zeta - 1)\ a^2 b^2 - 2(2\zeta+1)a^4 + 8\zeta\frac{a^3}{b} - 8\zeta = 0$$

We choose $\zeta = \infty$ for consistency (Pauli-Fierz) and solve

$$b = -a - \frac{1}{a} + \mathcal{O}\left(\frac{1}{a^3}\right) \quad , \quad \rho_{\text{grav}} = -\frac{1}{3}M^0_0 = m_g^2\left[\frac{1}{2} + \frac{1}{a^2} + \mathcal{O}\left(\frac{1}{a^4}\right)\right]$$

The effective cosmological constant is $\Lambda = m^2_{\text{grav}}$.

For $(M_P\Lambda)^2 = (10^{-3}\ eV)^4$, $m_g^{-1} = 10^{-33} eV$

Back

AdS_5 propagators

Consider 5-d gravity in AdS_5 space with metric and action

$$ds^2 = \frac{L^2}{r^2}(dr^2 + dx^i dx^i) \quad , \quad S_5 = M^3 \int d^5 y \sqrt{g} \left[R_5 - \frac{12}{L^2} \right]$$

The equation for the propagator with a unit source at $r = r_0$, $x^i = 0$ is

$$M^3 \left[r^3 \partial_r \; \frac{1}{r^3} \; \partial_r + \Box \right] G(r, x^i; r_0) = \frac{r^3}{L^3} \delta(r - r_0) \delta^{(4)}(x)$$

and after Fourier transform

$$M^3 L^3 \left[r^3 \partial_r \; \frac{1}{r^3} \; \partial_r - p^2 \right] G(r, p; r_0) = r^3 \delta(r - r_0)$$

$$G(r, p; r_0) = \begin{cases} -\dfrac{r_0^2 \; r^2}{M^3 L^3} \; K_2(p r_0) \; I_2(p r), & r < r_0, \\[2ex] -\dfrac{r_0^2 \; r^2}{M^3 L^3} \; I_2(p r_0) \; K_2(p r), & r > r_0. \end{cases}$$

We would now like to calculate the static potential between two sources, both at $r = r_0$ and one at $x = 0$ the other at arbitrary x.

The potential is given by

$$V(|\vec{x}|) = \int \frac{d^3 p}{(2\pi)^3} \; e^{ip \cdot x} \; G(r_0, p; r_0) = -\frac{15 r_0^8}{32 \pi M^3 L^3 \; |\vec{x}|^7} \; {}_1F_2 \left[\frac{5}{2}, \frac{7}{2}; 5, -\frac{4 r_0^2}{|\vec{x}|^2} \right]$$

$$V(|\vec{x}|) = \begin{cases} -\frac{15 r_0^8}{32\pi M^3 L^3\ |\vec{x}|^7} + \mathcal{O}\left(\frac{1}{|\vec{x}|^9}\right), & |\vec{x}| >> r_0 \\ -\frac{r_0^3}{4\pi M^3 L^3\ |\vec{x}|^2}\left[1 + \mathcal{O}\left(\frac{|\vec{x}|^2}{r_0^2}\right)\right], & |\vec{x}| << r_0 \end{cases}$$

The effective five dimensional Planck mass is

$$M_5 = M\frac{L}{r_0}$$

Induced gravity in AdS_5

$$S = S_5 + \delta(r - r_0)\Lambda_c^2 \int d^4x\sqrt{\hat{g}}\hat{R}_4$$

The propagator equation now becomes

$$M^3\left[r^3\partial_r\,\frac{1}{r^3}\,\partial_r + \Box_4 - r_c\;\delta(r-r_0)\Box_4\right]G(r,x^i;r_0) = \frac{r^3}{L^3}\delta(r-r_0)\delta^{(4)}(x) \quad , \quad r_c = \frac{r_0}{L}\frac{\Lambda_c^2}{M^3}$$

$$G(r,p;r_0) = \begin{cases} -\dfrac{r_0^2\;r^2}{M^3L^3}\,\dfrac{K_2(pr_0)\;I_2(pr)}{1+p^2r_0r_c\;I_2(pr_0)K_2(pr_0)}, & r < r_0, \\[2ex] -\dfrac{r_0^2\;r^2}{M^3L^3}\,\dfrac{I_2(pr_0)\;K_2(pr)}{1+p^2r_0r_c\;I_2(pr_0)K_2(pr_0)}, & r > r_0. \end{cases}$$

The exact form of the potential is

$$V(|\vec{x}|) = -\frac{r_0^2}{2\pi^2M^3L^3\;|\vec{x}|}\int_0^\infty qdq\;\sin\frac{q|\vec{x}|}{r_0}\;\frac{I_2(q)K_2(q)}{1+\frac{r_c}{r_0}q^2\;I_2(q)K_2(q)}$$

The Fourier transform of the static potential at $r = r_0$ is

$$G(r_0,p;r_0) = -\frac{r_0^4}{M^3L^3}\,\frac{K_2(pr_0)\;I_2(pr_0)}{1+p^2r_0r_c\;I_2(pr_0)K_2(pr_0)} \simeq \begin{cases} -\dfrac{r_0^4}{M^3L^3}\,\dfrac{1}{4+(pr_0)(pr_c)}, & pr_0 << 1 \\[2ex] -\dfrac{r_0^4}{M^3L^3}\,\dfrac{1}{pr_0(2+pr_c)}, & pr_0 >> 1. \end{cases}$$

- $r_0 << r_c \Rightarrow \Lambda_c^2 >> M^3 L.$

$$\simeq \begin{cases} -\frac{r_0^4}{M^3 L^3} \left[\frac{1}{4} - \frac{p^2 r_0 r_c}{16} + \cdots \right], & 1/p >> \sqrt{r_0 r_c} \\ -\frac{r_0^3}{M^3 L^3 r_c \; p^2}, & 1/p << \sqrt{r_0 r_c} \end{cases}$$

At length scales shorter than $\sqrt{r_c r_0} = r_0 \frac{\Lambda_c}{\sqrt{M^3 L}}$ we have four-dimensional behavior.

- $r_0 >> r_c \Rightarrow \Lambda_c^2 << M^3 L.$

$$\simeq \begin{cases} -\frac{r_0^4}{M^3 L^3} \left[\frac{1}{4} - \frac{p^2 r_0 r_c}{16} + \cdots \right], & 1/p >> r_0 \\ -\frac{r_0^2}{2 M^3 L^3 \; p}, & r_c << 1/p << r_0 \\ -\frac{r_0^3}{M^3 L^3 r_c \; p^2}, & 1/p << r_c. \end{cases}$$

At length scales shorter than $r_c = \frac{r_0}{M^3} \frac{\widehat{M}_P^2}{L}$ we have 4d behavior. The effective 4d planck scale in both cases is $M_P = \frac{L}{r_0} \widehat{M}_P$