String Cosmology Workshop
24-28 April 2005, Uppsala

Holography and Cosmological Brane-Bulk Energy Exchange

Sources and Credits

Based on:

hep-th/0207060 : with G. Kofinas, N. Tetradis, T. Tomaras, N. Zarikas

hep-th/0503189

hep-th/0504ddd to appear soon

hep-th/05mmddd to appear with R. Guedens

Review

hep-th/0310001 to appear in Physics Reports

Preview

I will investigate cosmological brane-bulk energy exchange in the context of brane worlds, and indicate that it can be responsible for interesting cosmological effects like tracking and acceleration

A dual holographic description of the framework will be worked out.

Surgeon's Warning:

This is speculative physics! No comparisons of numbers with Data.

Plan of the presentation

- Introduction
- Brane-bulk energy exchange
- The aim and context
- Randall Sundrum geometry
- RS Cosmology
- RS Cosmology II
- Approximations and Solutions
- Solutions
- Outflow
- Inflow
- Tracking
- General Properties
- The holographic dual
- Derivation
- Derivation II
- Generalization
- Cosmological Solutions
- Cosmological Solutions II
- Sources of acceleration
- A gauge theory for gravity?
- Concluding Remarks

Introduction

The brane-world idea originates in string theory:

- Heterotic M-theory
- Orientifolds and D-branes

Basic characteristic: brane physics (typically associated to the SM) is geometrically distinct from bulk physics (associated with gravity).

Compactification is indicating that gravity is 4d in the IR but higher-dimensional in the UV.

Therefore, early universe cosmology is expected to differ considerably from 4d cosmology.

In all models, there is a generic effect, that is very interesting:

Brane-Bulk interaction $\Leftrightarrow$ Energy Exchange

brane-bulk energy exchange

It involves interactions essentially between the standard model fields (directly observable in experiments) with hidden matter of various sorts:

- Other bulk light fields
- KK descendants of bulk fields

It is very important because:

♡ It is mostly unavoidable in a large class of models.

♣ It provides stringent constraints of models (e.g. energy loss from Supernovae)

♢ It puts an "early" limit to conventional cosmology.

♠ It may be responsible for interesting cosmological effects (acceleration for example)

The aim and context

My aim is to study Brane-Bulk Energy Exchange (BBEE) in the cosmological context.

The framework will be the Randall-Sundrum paradigm.

There are alternative frameworks (large extra dimensions, perturbative heterotic compactifications, heterotic M-theory, etc)

It is expected that qualitative features are similar.

The RS setup will allow me to eventually use holographic ideas.

Randall Sundrum geometry

The setup:
4+1 dimensions $(z,\ x^\mu)$ a brane at $z = 0$ and a Z_2 symmetry $z \to -z$.

There is some "observable" matter on the brane, and some general matter, including the graviton in the bulk.

$$S = M^3 \int d^5x \sqrt{g} \left[R_{(5)} - \Lambda + \mathcal{L}_{bulk}\right] +$$

$$+\delta(z) \int d^4x \sqrt{\hat{g}} \left[-V + \mathcal{L}_{brane}\right]$$

V is the brane tension, the simplest solution is a slice of AdS:

$$\ell_{\rm ads} = k^{-1} = \frac{M^3}{V} \quad , \quad \lambda = \Lambda + \frac{V^2}{M^3} = 0$$

RS Cosmology

We find cosmological solutions via an ansatz

Binetruy, Defayet, Langlois

$$ds^2 = -n^2(t,z)dt^2 + a^2(t,z)\zeta_{ij}dx^i dx^j + b^2(t,z)dz^2,$$

and equations

$$G_{AC} = \frac{1}{2M^3} T_{AC}$$

where T_{AC} is the total energy-momentum tensor.

Assuming a perfect fluid on the brane and a general energy-momentum tensor $T^A_C|_{m,B}$ in the bulk

$$T = T|_{\mathrm{v},b} + T|_{m,b} + T|_{\mathrm{v},B} + T|_{m,B}$$

$$T|_{\mathrm{v},b} = \frac{\delta(z)}{b}\mathrm{diag}(-V,-V,-V,-V,0)$$

$$T|_{\mathrm{v},B} = \mathrm{diag}(-\Lambda,-\Lambda,-\Lambda,-\Lambda,-\Lambda)$$

$$T|_{\mathrm{m},b} = \frac{\delta(z)}{b}\mathrm{diag}(-\rho,p,p,p,0),$$

where ρ and p are the energy density and pressure on the brane.

RS Cosmology II

Gauge fixing, imposing the Israel conditions and projecting the equations of motion on the brane we obtain:

$$\dot\rho + 3\frac{\dot a}{a}(\rho + p) = -T^0{}_5$$

$$\frac{\ddot a}{a}+\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2} = -\frac{(V(3p-\rho)+\rho(3p+\rho))}{144M^6}-\frac{T^5{}_5}{6M^3}$$

$T^0{}_5, T^5{}_5$ is the bulk stress tensor, evaluated at the position of the brane. $T^0{}_5$ is the rate of energy loss from the brane, due to its interaction with bulk.

The second order equation can be written as first order by introducing an new variable (energy density) χ:

$$H^2+\frac{k}{a^2}=\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2}=\frac{\rho^2}{M^6}+\frac{1}{M_P^2}(\rho+\chi)$$

$$\dot\chi + 4H\chi = \left(\frac{\rho}{V}+1\right)\ T^0{}_5 - \frac{M^3}{V}H\ T^5{}_5$$

E. K., Kofinas, Tetradis, Tomaras, Zarikas

In the standard case $T^0{}_5 = T^5{}_5 = 0$, χ is the dark/mirage radiation.

Approximations and Solutions

Energy loss in the minimal RS solution due to radiation of KK gravitons from brane matter.

Kiritsis, Tetradis, Tomaras
Langlois, Sorbo
Maartens, Leeper, Sopuerta

It is described by the Vadya metric for which

$$T^0{}_5 = A\ \rho^2 \quad , \quad T^5{}_5 = B\ \rho^2$$

Exact solutions are rare. We need an approximation:

$$\frac{T^5{}_5}{\Lambda} \quad << \quad \frac{\rho}{V}$$

It amounts to neglecting $T^5{}_5$ from the equations. The only unknown remains the energy loss rate $T^0{}_5$

This can be calculated from microphysics considerations as a function of the model It will depend on particle densities, and temperatures, that typically scale as powers.

To explore further we reexpress as $T^0{}_5(\rho)$ or $T^0{}_5(a)$ .....

Solutions

We must therefore solve (we scaled all densities $\to$ dimensionless)

$$H^2 + \frac{k}{a^2} = \rho^2 + \rho + \chi - \frac{k}{a^2}$$

$$\dot{\chi} + 4H\chi = (2\rho + 1)\ \ T^0_{\ 5}(\rho)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -T^0_{\ 5}(\rho)$$

with $T^0_{\ 5}(\rho)$ a known function of ρ. We will take it to be a power (scaling)

$$T^0_{\ 5} = A\ \rho^\nu$$

Some typical exponents

- $\nu = 2$ RS radiative loss
- $\nu = 3/2$ Tracking
- $\nu = 1$ Decay

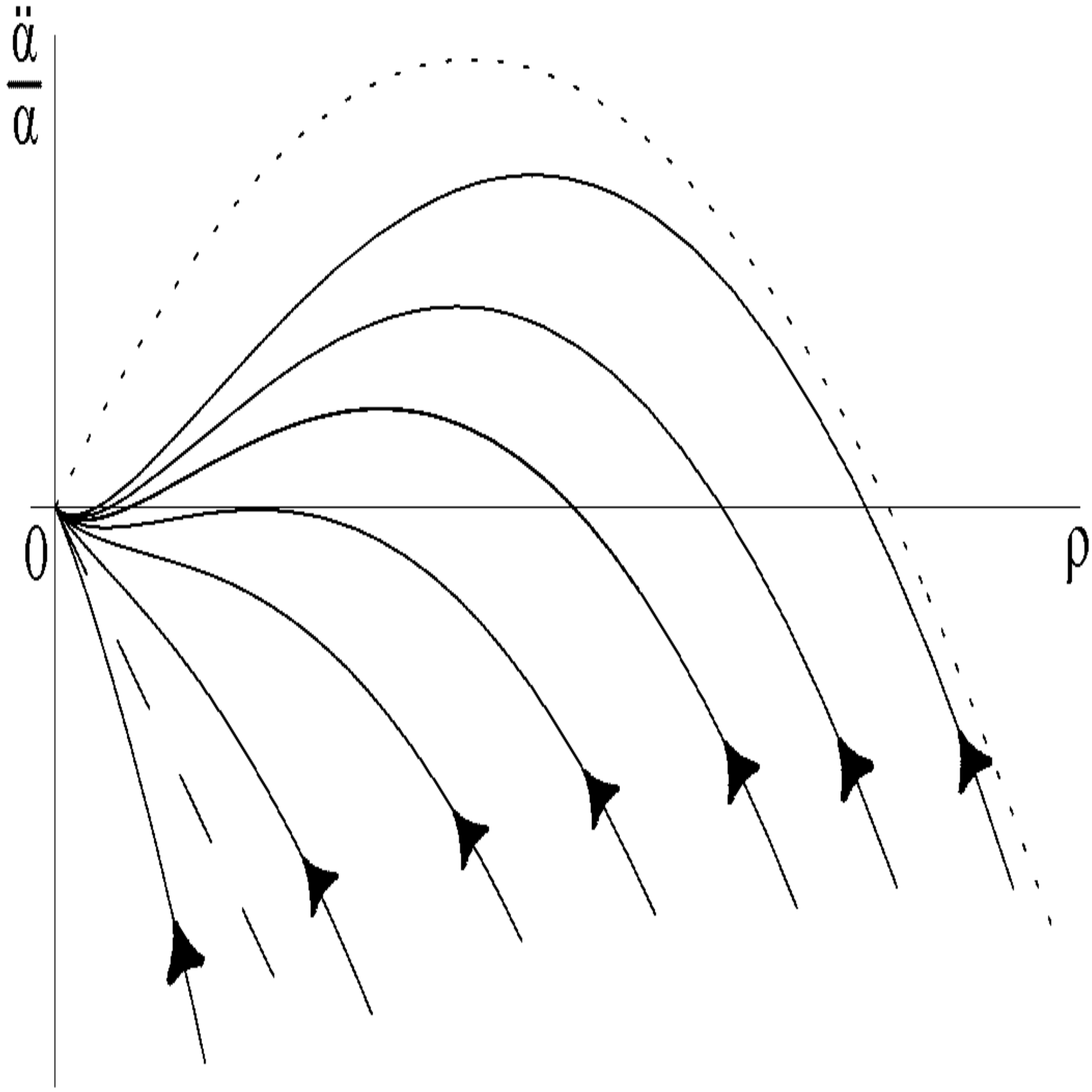
In general for a $2 \to 2$ process plus radiation in the bulk we get

$$\nu = \frac{2 + w_1 + w_2 + n}{1 + w_1} \quad , \quad < \sigma\ v\ E > \sim T^{3n}$$

where w_1 corresponds to the "driving" energy density.

Outflow

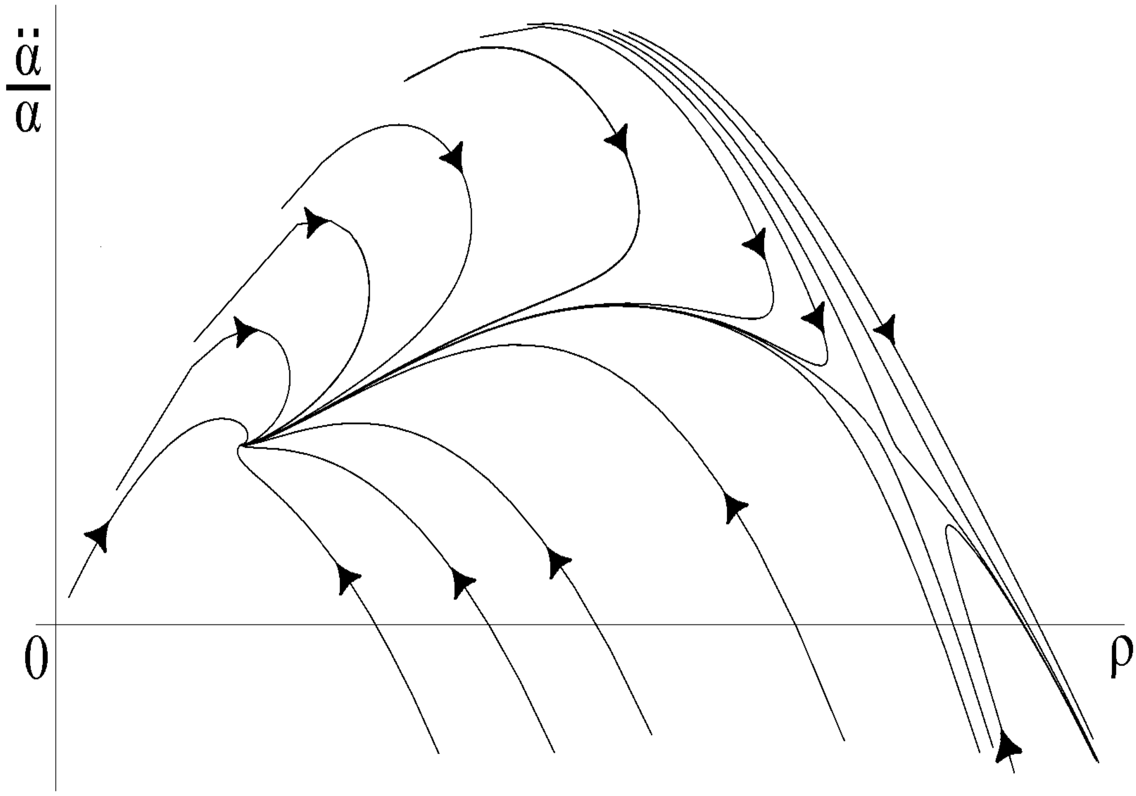
We consider $\nu = 1$ (unstable matter decaying), $A > 0$, a flat brane (k=0) and non-relativistic matter (w=0).



Intermediate acceleration (non-linear effect)

Inflow

We consider $\nu = 1$ (unstable matter decaying), $A < 0$, a flat brane (k=0) and non-relativistic matter (w=0).



Two fixed points, one stable, the other unstable.

$$H_* = A/3$$

Tracking

Assume $w = 0$, $k = 0$ and

$$T^0{}_5 = \left(\mu - \frac{1}{\mu}\right) \rho^{\frac{3}{2}}$$

Can be solved exactly

we obtain at late times

$$\rho \sim a^{-4+\frac{1}{\mu^2}} \quad , \quad \chi \sim (\mu^2 - 1)\ \rho$$

Mirage density tracks the observable energy density. Could be used for dark matter?

Similar behavior found by Gasperini, Piazza, Veneziano. from dilaton interactions

General Properties

♣ There are generic fixed points (inflation) when $T^0{}_5 < 0$. They are stable in the low density regime.

Some become saddles in the high-density regime.

♠ Acceleration can also appear during out-flow.

There are several exact solutions that although not exactly realistic, test the structure and the approximation in some regimes

Langlois, Sorbo
Maartens, Leeper, Sopuerta
Tomaras
Tetradis

The Holographic dual

5D physics in AdS_5 is dual to a 4D strongly coupled CFT

Maldacena

A slice of AdS_5 is relevant for the RS setup. This is obtained by cutting off a portion that contains the boundary, and gluing in, a similar copy.

This corresponds to introducing a UV cutoff Λ_{UV} in the dual theory as well as allowing 4d gravity to propagate (with $M_P \sim \Lambda_{UV}$)

Maldacena, Verlinde, Witten, Gubser

- This will generate a new arena for model building

- Shed further light into approximations

- Bypass the problem of "hidden" bulk singularities in five dimensions.

- It is possible that observable gravity is the avatar of a (hidden, large-N) gauge theory.

Derivation

$$e^{-S_{\text{bulk}}} \sim Z_{\text{string}}[\phi_i(x)] = e^{-W_{CFT}(\phi_i)} \equiv \langle\, e^{\,\sum_i \int d^4x\ \phi_i(x)\ O_i(x)}\ \rangle$$

renormalized at $r = \epsilon$ near the boundary.

$$S_{\text{bulk}} = S_{EH_5} + S_{GH_4} - S_{\text{counter}}$$

$$S_{\text{counter}} = \int d^4x \sqrt{-g_{(0)}} \left[\frac{1}{\epsilon^2} L_0 + \frac{1}{\epsilon} L_1 + \log\epsilon\ L_2 \right]$$
$$= S_0 + S_1 + S_2$$

$$S_0 = 6M^3\ell^2 \int_{r=\epsilon} d^4x \sqrt{-\gamma}$$

$$S_1 = -\frac{M^3\ell}{2} \int_{r=\epsilon} d^4x \sqrt{-\gamma}\ R[\gamma]$$

$$S_2 = \frac{\log\epsilon}{4} M^3\ell^3 \int_{r=\epsilon} d^4x \sqrt{-\gamma}\ \left[R_{ij}[\gamma] R^{ij}[\gamma] - \frac{1}{3} R[\gamma]^2 \right] -$$
$$-\frac{b}{6} \int d^4x \sqrt{-\gamma}\ R[\gamma]^2$$

$\gamma_{\mu\nu}$ is the induced metric at $r = \epsilon$.

We may use this to map

$$S_{RS} = S_{EH_5} + S_{GH_4} - 2S_0 + S_{\text{matter}}$$

to the gauge theory side:

Hawking, Chablin, Reall/ de Haro, Skenderis, Solodukhin

Derivation II

$$S_{\text{dual}} = S_{CFT} + S_R + S_{R^2} + S_{\text{matter}}$$

$$S_{CFT} = 2W_{CFT} \quad , \quad S_P = 2S_1 = -M_P^2 \int d^4x\sqrt{-\gamma}\ R$$

$$S_{R^2} = b' \int d^4x\sqrt{-\gamma}\ \left[R_{ij}R^{ij} - \frac{1}{3}R^2\right] - \frac{b}{3}\int d^4x\sqrt{-\gamma}\ R^2$$

with $M_P^2 = M^3\ell$

Comments

- This derivation neglected higher (bulk) derivative terms on the 5d side

- This map is not an exact duality. The reason is that the RS solution does not directly lift to ten dimensions, due to the Z_2 orbifold action. This duality is good at low energy. It can be taken as the DEFINITION of the 4-d cosmology

- The type of four dimensional theory obtained is a mild generalization of the Starobinsky model It will be generalized further soon.

Notice that the scheme dependent non-linear terms needed in the Starobinsky model, are absent here.

Generalization

$$S_{\text{dual}} = S_{\text{hidden}} + S_R + S_{R^2} + S_{\text{matter}} + S_{\text{interaction}}$$

• The CFT will be replaced by a general hidden gauge theory (at strong coupling).

• It may be interacting with observable matter.

The above are dual to general bulk content in the 5d case.

• If the hidden QFT is not directly observable, it may also have weak coupling.

This is the generalized Starobinsky model

Cosmological Solutions

$$M_P^2 G_{\mu\nu} = T^m_{\mu\nu} + V_{\mu\nu}$$

$$T^m_{\mu\nu} = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_{\rm matter}}{\delta \gamma^{\mu\nu}}$$

$$V_{\mu\nu} = \frac{1}{\sqrt{-\gamma}} \frac{\delta (S_{\rm hidden} + S_{\rm interaction} + S_{R^2})}{\delta \gamma^{\mu\nu}}$$

$$\nabla^\mu T^m_{\mu\nu} = -\nabla^\mu V_{\mu\nu} \equiv T \quad , \quad V^\mu{}_\mu = -2a\, C^2 - 2c\, G + D\, R^2 + X$$

C^2 is the Weyl tensor (irrelevant in cosmology), G is the Gauss-Bonnet density.

Only the graviton, gravitini and non-conformally coupled scalars contribute to D

Parametrize T^m in terms of ρ, p and V in terms of σ, σ_p and using a cosmological ansatz we obtain

$$3M_P^2 \left(H^2 + \frac{k}{a^2} \right) = \rho + \sigma$$

$$\dot\rho + 3H(\rho + p) = T \quad , \quad \dot\sigma + 3H(\sigma + \sigma_p) = -T$$

$$\sigma - 3\sigma_p = 48c \frac{\ddot a}{a} \left[\frac{\dot a^2}{a^2} + \frac{k}{a^2} \right] + X$$

X collects the contributions of non-conformal invariance due to (self)-interactions

$$X = \sum_{ij} (\beta_{ij} < O_i > \Omega_j + R\ \beta^R_{ij} < O_i > \Omega_j)$$

It is a function of the metric, curvature, and observable matter fields

Cosmological Solutions II

We solve for σ_p and redefine

$$\sigma = \chi + 12c\left[H^2 + \frac{k}{a^2}\right]^2$$

to finally obtain:

$$\dot{\chi} + 4H\chi = T + H\ X \quad , \quad \dot{\rho} + 3H(\rho + p) = -T$$

$$3M_P^2\left[H^2 + \frac{k}{a^2}\right] - 12c\left[H^2 + \frac{k}{a^2}\right]^2 = \rho + \chi$$

$$\Rightarrow H^2 + \frac{k}{a^2} = \frac{H_0^2}{2}\left[1 + \epsilon\sqrt{1 - \frac{\rho + \chi}{E_0}}\right]$$

At small energy densities (compared to $E_0 = \frac{3M_P^4}{16c}$) and choosing the smooth branch $\epsilon = -1$ this agrees with 5d setup:

$$M_P^2 = 24\frac{M^6}{V} \quad , \quad M^6 = \frac{M_P^6}{64c}$$

$$T \simeq -2{T^0}_5 \quad , \quad X \simeq -24\frac{M^3}{V}{T^5}_5$$

The difference appears in the quadratic terms ρ^2 and its ρT_{05} avatar.

We may redefine the radiation density so that the equations match exactly at the expense of redefining also the X density:

$$\chi_{\text{new}} = \chi + 12c\left(H^2 + \frac{k}{a^2}\right)^2 - \frac{M_P^2}{48M^6}\rho^2$$

$$X_{\text{new}} = X - \frac{\rho(\rho+p)M_P^2}{8M^6} - 24c\left(H^2 + \frac{k}{a^2}\right)\left(\dot{H} - \frac{k}{a^2}\right)$$

However, this is "scheme dependent" and does not change the fact that the 4d framework has an upper bound on the energy density.

Sources of Acceleration

We may write the acceleration as

$$q = -\frac{1}{6M_P^2}\ \frac{(\rho + 3p) + (\rho_\sigma + 3p_\sigma)}{1 - \frac{8c}{M_P^2}\left[H^2 + \frac{k}{a^2}\right]}$$

• ρ_σ, p_σ are the energy density and pressure of the hidden gauge theory, without the contribution of the conformal anomaly.

• The denominator is due to the conformal anomaly. It is positive in the smooth branch and negative in the Starobinsky branch.

In this form, the conditions for acceleration are clear:

♣ In the smooth branch, $\rho + 3p$ or $\rho_\sigma + 3p_\sigma$ should be sufficiently negative.

♡ In the Starobinsky branch, any standard matter with $\rho + 3p > 0$ is accelerating.

A gauge theory for gravity?

The natural conclusion of this line of thinking is that observable four-dimensional gravity is a manifestation of a "hidden" large N gauge theory. Such a realization of 4d gravity provides a number of advantages:

• This implies again that there is a "string theory" behind 4d gravity. But the short distance degrees of freedom are now known: gluons. The graviton is a bound state of glue.

• Because of this it has a form factor. The cosmological constant problem here is different. It can be solved if the scale of this form factor is 10^{-3} eV. gravity does not couple to ordinary vacuum energy.

• This setup is mostly inspired from the fact that gauge theory is solving the problem of black hole microstates.

• The SM is a splinter of the large gauge group. Gravitational interactions are transferred to SM particles by bifundamental matter charged under the large gauge group and the SM group.

Problems to be solved:

• Make the graviton extremely light while keeping the other universal scalar glueballs (dilaton, axion) heavier.

• Make gravity four-dimensional rather than 5d. Possible solution: brane induced gravity.

• Generate the right scale for the effective cosmological constant.

Concluding Remarks

• Brane-bulk energy exchange is a necessary consequence of the higher dimensional nature of gravity in string theory and related setups.

• It provides constraints on models, but has also the potential of altering early universe cosmology and be a source of cosmological acceleration of brane-bulk locking

• Its cosmological equations can be mapped via holography to generalized Starobinsky models (without the standard non-linear terms)

• Investigation of cosmological model-building in this context seems worth-while