Introduction to string theory and the gauge theory/string theory correspodence.

Plan

- String theory
- Relations among string theories
- D-branes
- Gauge-gravity correspondence. (general considerations)
- AdS / CFT
- Another view: the matrix model.

String theory

A theory introduced because it incorporates (quantized) gravity

- Unlike other interactions the short-distance infinities of gravity are un-controlable

- $\longrightarrow$ Non-renormalizability

String theory predicts (perturbatively) quantized gravity.

fundamental objects of string theory are strings (open and closed

- closed string always provide a graviton
- The theory contains gauge interactions (SM)
- Existence of fermions implies supersymmetry
- The theory is UV finite.

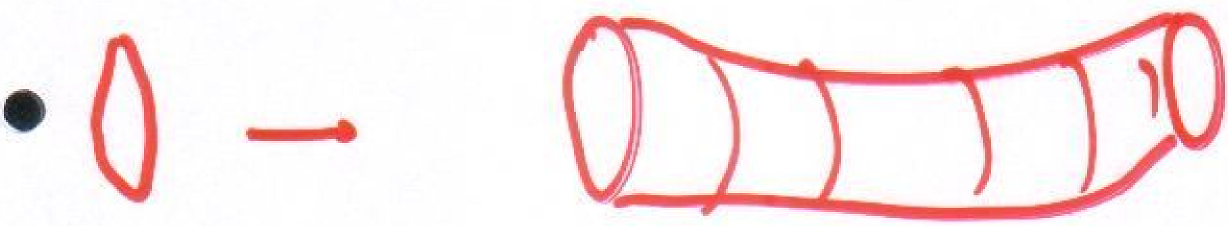
In field theory, fields (particles) are point-like.

world-lines

• quantum-mechanics

$$\sim \int(\text{paths})\, e^{-\text{length}}$$

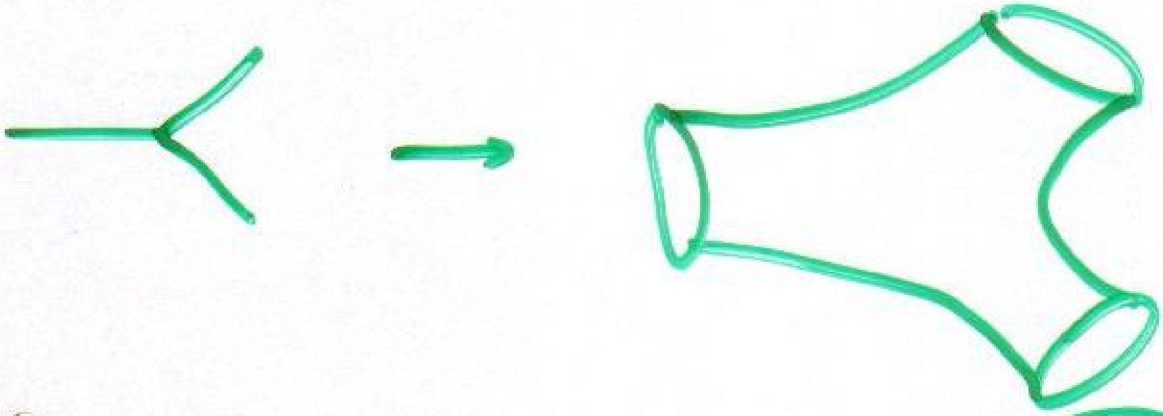
world-lines



2-d world-volume

$$\sim \int(\text{wv})\, e^{-\text{Area}}$$

Unlike FT, in ST the interactions are unique:



$\Longrightarrow$ geometry of Riemann surfaces

The theory is tightly constrained.

(resembles a theory with a smart cut-off)

The theory has one scale : $M_s \to$ string scale

and $l_s \sim M_s^{-1} \sim \sqrt{\alpha'}$

$T \sim \frac{1}{\alpha'}$

A string gives rise to an infinite ladder of particles with

$$M^2 \sim n M_s^2$$

When $E \ll M_s$ they are "invisible"

$\Rightarrow$ We must look at $l \sim l_s$ to see a string

Other "parameters" depend on the background.

(Superstring theory lives in 9+1 dimensions.)

- the string coupling constant g_s is an expectation value:

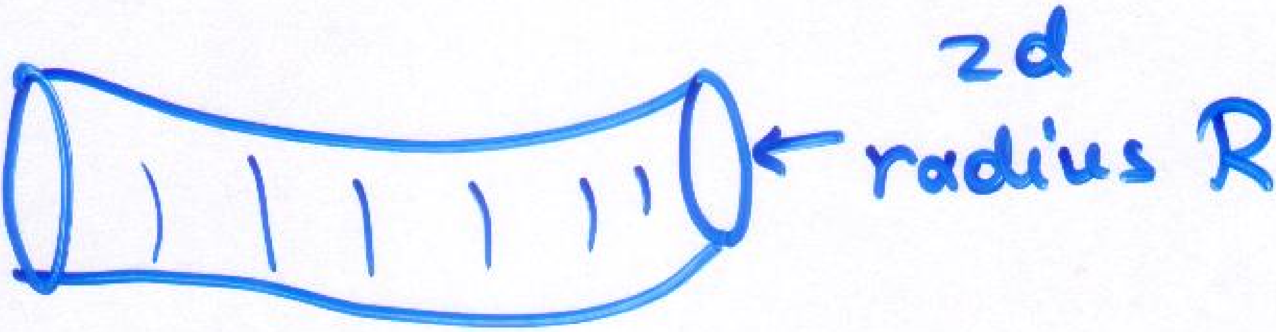
- $g_s = \langle e^{\Phi} \rangle$ → dilaton scalar

$$g_s^{-\chi} \sim g_s^{-2+2g}$$

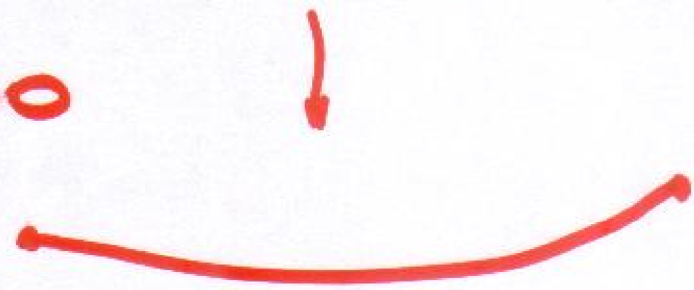
Compactification

How are ten dimensions, compatible with observations?

Kaluza-Klein idea

- 

$R \to 0$



- $R \to \infty$

Example: 4+1 dimensions

circle of radius R.

and a scalar Φ (massless)

$$\Box \Phi = 0$$

- $\Rightarrow \quad P_0^2 - \vec{P}^2 - P_5^2 = 0$

wave function $\quad e^{i P_5 \cdot x^5}$

must be invariant under $x^5 \to x^5 + 2\pi R$

- $$\Downarrow$$

$$P_5 = \frac{n}{R} \qquad n \in \mathbb{Z}$$

$$\longrightarrow \quad P_0^2 - \vec{P}^2 = \frac{n^2}{R^2}$$

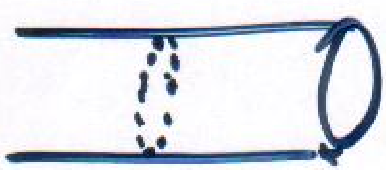
Infinite collection of particles (KK part.)

with $\quad M_n^2 = \dfrac{n^2}{R^2}$

T-duality

In string theory there are other configurations.

- a string can wrap a compact dimension (m times)



energy cost

$= T \cdot (2\pi m R)$

- mass formula:

$$M^2 = \frac{n^2}{R^2} + T^2 (2\pi m R)^2$$

invariant under:

$$R \longrightarrow \frac{1}{2\pi T \cdot R}$$

$\partial_\sigma X^\mu \leftrightarrow \partial_\tau X^\mu$

$\partial_{\sigma+\tau} X^\mu \to \text{inv}$

$\partial_{\sigma-\tau} X^\mu \to -(\,)$

Consistent supersymmetric string theories in D=10

- Closed strings (type II)

 $\longrightarrow$ $\overset{\text{(super)}}{x^\mu} \rightarrow 10D$

left movers ~ Right movers

Subtle diffence in fermion sector $\rightarrow$ II A, B.

- For both,

NSNS $\rightarrow$ $G_{\mu\nu}$, $B_{\mu\nu}$, Φ

RR $\rightarrow$

II A: A_μ, $C_{\mu\nu\rho}$

II B: α, $C_{\mu\nu}$, $C^{+}_{\mu\nu\rho\sigma}$

IIA : massless sector

= $N=2$ supergravity

(non-chiral)

IIB : massless sector

- $N=2$ supergravity (IIB)

  chiral (+ anomaly free)

- Heterotic super-string theory

Closed strings :

left movers : superstring

x^μ, ψ^μ

$\mu = 0, 1, \ldots, 9$

Right movers → non-susy

x^μ, $X^{I=1,..16}$

X^I compactified on even self-dual lattice $\left[\begin{array}{l} E_8 \times E_8 \\ O(32) \end{array}\right.$

- massless fields:

$G_{\mu\nu}$, $B_{\mu\nu}$, ϕ + fermions

N=1 supergravity multiplet in d=10

- A_μ^a + fermions

S Yang Mills multiplet for SO(32), $E_8 \times E_8$

Type I string

Closed unoriented strings.

projected by orientation reversal $\Omega : L \leftrightarrow R$

- $G_{\mu\nu}$, ϕ, $C_{\mu\nu}$ remain
  ($B_{\mu\nu}$, α, $C^{+}_{\mu\nu\rho\sigma}$) $\rightarrow$ projected out.

  $N=1$, $d=10$ sugra multiplet.

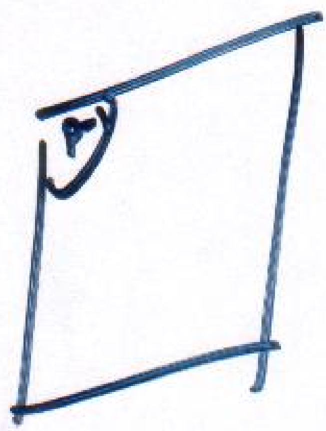
- Open (unoriented) strings

  $O(32)$, $N=1$ $d=10$ SYM multiplet.

Antsymetric tensors and p-branes.

$A_{\mu_1 \dots \mu_p} \rightarrow$ is a massles gauge field

- it couples minimally to a (p-1)-brane



- $$Q_{p-1} \int_{M_p} A_{\mu_1 \dots \mu_p} \epsilon^{\mu_1 \dots \mu_p}$$

generalization of the EM coupling of point particles.

$$\sim e \int dx^\mu A_\mu$$

All string theories (except-I) have a $B_{\mu\nu}$ → couples to a 1-brane ≈ string

This is the fundamental string itself (F_1)

There are no perturbative states that couples to the R-R forms in type-II string theory.

such configurations must be p-brane-like

IIA : A_μ → 0-brane (electric)
→ 6-brane (magnetic

$C_{\mu\nu\rho}$ → 2-brane (E)
→ 4-brane (M)

IIB α → (−1)-brane (?)
→ 7-brane (M)

$C^{+}_{\mu\nu\rho\sigma}$ → 3-brane
→

$C_{\mu\nu}$ → 1-brane (strin
→ 5-brane (M)

Such p-brane solutions arise as quasi-solitonic solutions of the low-energy ~~eff~~ effective supergravity.

However, such solutions are

- generically singular

(Dirac, vs, t'Hooft monopole

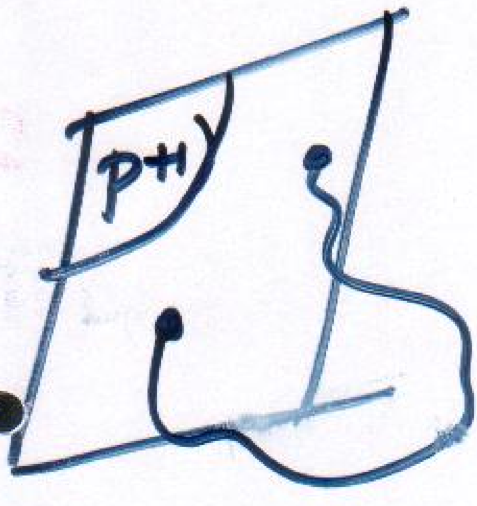
Do they correspond to

- states in the quantum theory?

Non-perturbative duality "symmetries" indicate that they should

D-branes

Consider a $(p+1)$-dimensional subspace (plane) of 10-d spacetime. We will describe open strings "stuck" on this subspace



$X^\mu \rightarrow$ longitudinal

$X^I \rightarrow$ transverse

$X^\mu \rightarrow$ Neuman boundary conditions

(free end points)

$$\partial_\sigma X^\mu \Big|_{\text{end point}} = 0$$

$X^I \to$ Dirichlet bc
(fixed end)

$$\partial_\tau X^I \Big|_{\text{end point}} = 0$$

$$\sim \quad X^I \Big|_{\text{end point}} = 0 \to \text{(fixed)}$$

Nbc allow momentum only
D-bc " winding only
(in compact cases)

Spectrum:

a vector $\to \psi^\mu_{-1/2}|0\rangle$
(transverse) scalar $\to \psi^I_{-1/2}|0\rangle$
+ fermions.

Special case $p=9$

$\Rightarrow$ Neuman only

$\longrightarrow$ one vector $A_\mu(x)$

one MW spinor Ψ_α

$\longrightarrow$ $N=1$ $D=10$ vector multiplet

arbitrary p:

one vector $(D=p+1)$ $A_\mu(x)$

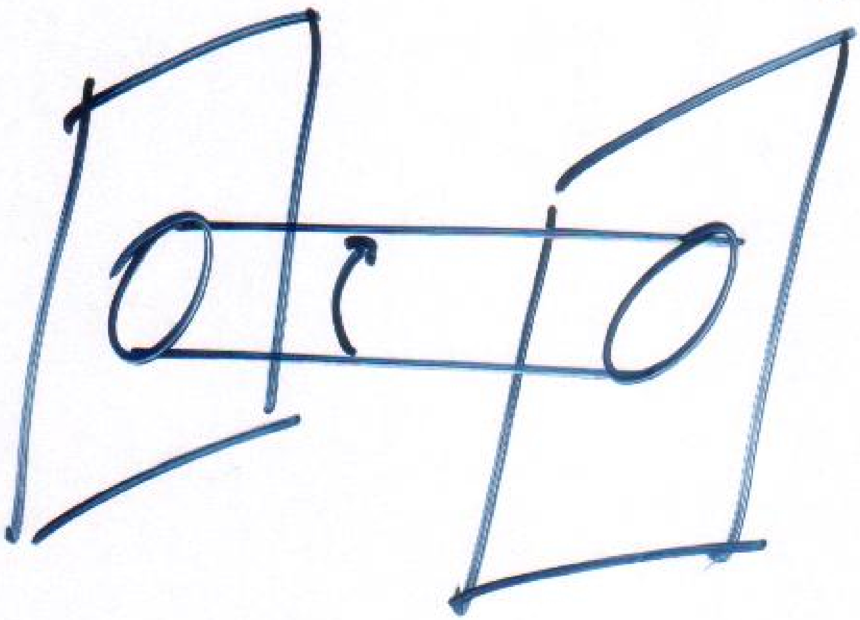
$9-p$ scalars $\Phi^I(x)$

plus the fermions.

Dimensional reduction to $D=p+1$ of $D=10$ vector multiplet.

D-branes

Force between D-branes
(at distance $\vec{d}$)



one-loop open string amplit

= tree-level exchange of closed strings.

massless contribution due to $g_{\mu\nu}$, ϕ (attractive)

and RR field (repulsive)

total = 0

D-branes carry RR charge. → a stringy description of such solitons.

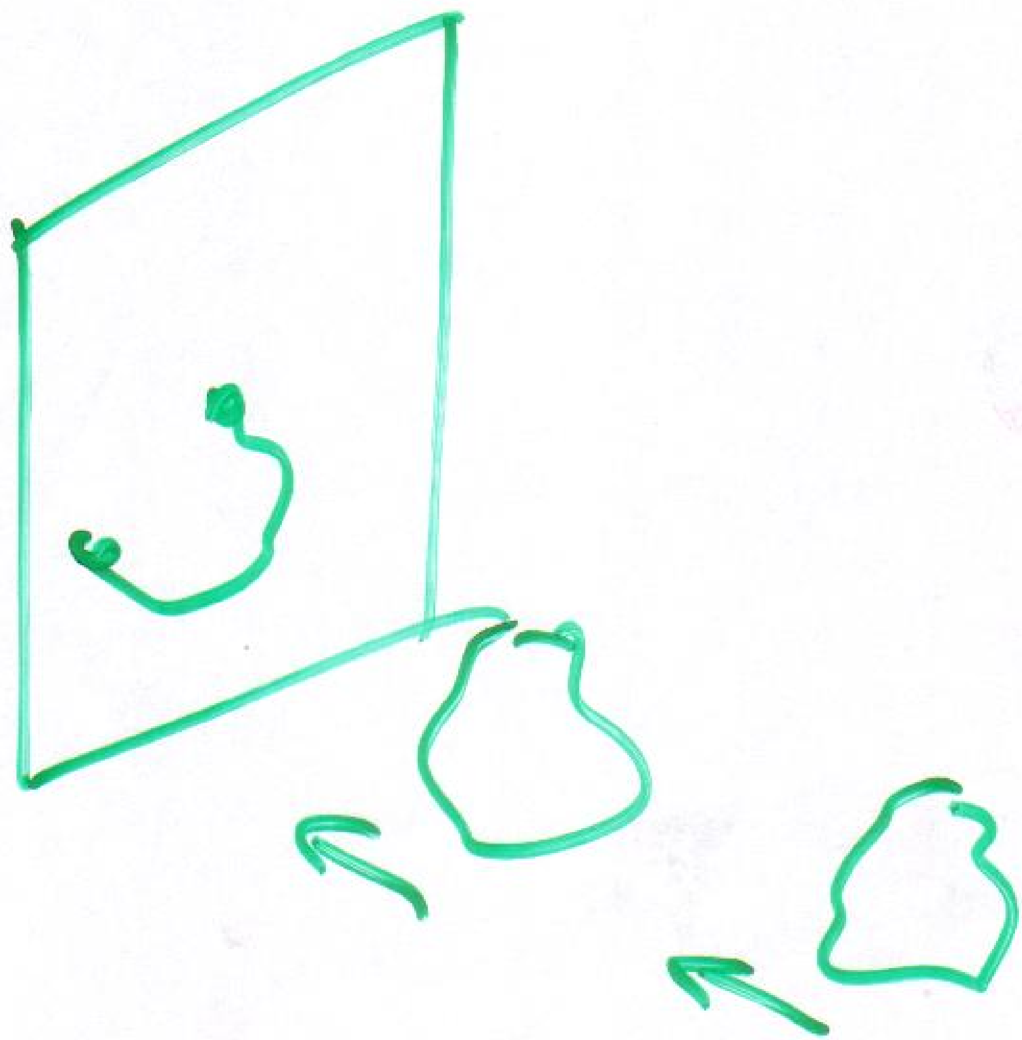
- So...

D-branes are stringy solitons. Their tension

- $$T_p \sim \frac{M_s^{p+1}}{g_s} = Q_p \quad \text{(BPS)}$$

Their fluctuating modes are the open strings with endpoints on the brane.

They must interact withe the closed super-string modes (graviton dilaton, RR-forms) since they are charged



IIB self duality

Scalars: ϕ (dilaton)

a (axion) RR

Define: $S = a + i e^{\phi}$

Classical equations inv. under:

$$S \to \frac{aS + b}{cS + d}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is an $SL(2)$ matrix

exchanges weak with strong coupling

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \longrightarrow S \to -\frac{1}{S}$$

at $\alpha = 0 \Rightarrow g_s = e^{\phi} \to \frac{1}{g_s}$

two 2-forms B_2, C_2

fundmental string $F_1 \to B_2$

D_1 string $D_1 \to C_2$

if $SL(2)$ a symmetry we must include D_1 Stings (solitons, heavy as $g_s \to 0$)

$$T_1 \sim \frac{l_s^2}{g_s}$$

$$SL(2,R) \longrightarrow SL(2,Z)$$

M-theory

What is the strong coupling limit of IIA string?

- → M-theory ($E \to 0$, $D=11$ supergravity)

  → G_{AB}, C_{ABC}

- Compactification on circle of radius R

⇒ IIA with coupling

$$g_s \sim R^{3/2}$$

$G_{AB} \rightarrow G_{\mu\nu} \rightarrow$ metric

$G_{\mu 11} \rightarrow A_\mu$ RR 1-form

$G_{11} \rightarrow \Phi \rightarrow$ dilaton

- $C_{ABC} \rightarrow C_{\mu\nu\rho} \rightarrow$ RR 3-form

$C_{\mu\nu 11} \rightarrow B_{\mu\nu}$

- M5 → NS5, D4

M2 → D2, F1

KK-gravitons → D0 branes

Heterotic M-theory

Orbifold M-theory on S^1 by Parity

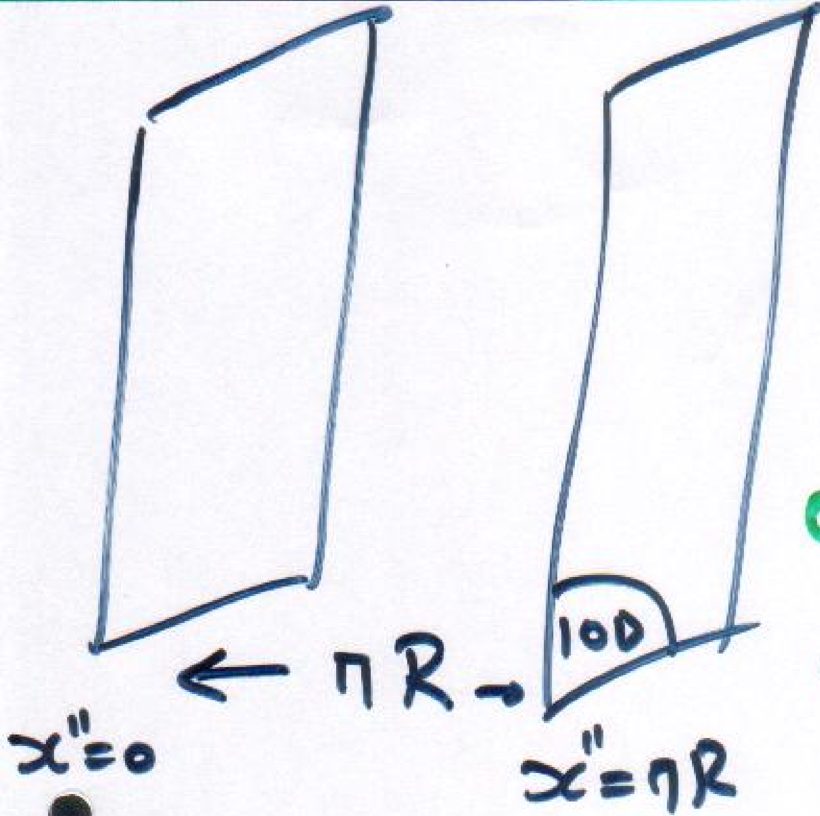
$$x^{11} \to -x^{11}$$

- $$g_{\mu\nu} \to g_{\mu\nu} \qquad C_{\mu\nu\rho} \to -C_{\mu\nu\rho}$$
  $$g_{\mu,11} \sim A_\mu \to -A_\mu \qquad B_{\mu\nu} \to B_{\mu\nu}$$
  $$\phi \to \phi$$

- $N=1$, $D=10$ supergravity (anomalous)

must have "twisted sector"

Fixed points: $x^{11} = 0$
$x^{11} = \pi R$



For anomaly cancellation

one E_8 multiplet on each 10-plane

This corresponds to the $E_8 \times E_8$ heterotic string with

$$g_s \sim R^{3/2}$$

Heterotic/Type-I duality

Both Theories have the same massless spectrum ($O(32)$).

- $$S_{het} \sim \int \sqrt{g}\, e^{-2\varphi} \left(R + (\nabla\varphi)^2 - \frac{1}{12} H^2 - \frac{1}{4} F^2 + \dots \right)$$

- $$S_{I} \sim \int \sqrt{g}\, e^{-2\varphi} \left(R + (\nabla\varphi)^2 \right) - \frac{1}{12} H^2 - \frac{1}{4} e^{-\varphi} F^2)$$

scale $g \to g\,e^{+\varphi/2}$

to go to Einstein frame

$$S_{het} \sim \int \sqrt{g} \left(R - (\nabla\varphi)^2 - \frac{e^{-\frac{\varphi}{2}}}{4} F^2 - \frac{1}{12} e^{-\varphi} H^2 + \dots \right)$$

$$S_{I} \sim \int \sqrt{g} \left(R - (\nabla\varphi)^2 - \frac{1}{4} e^{\frac{\varphi}{2}} F^2 - \frac{1}{12} e^{\varphi} H^2 + \dots \right)$$

$$\Phi \to -\Phi$$

strong/weak coupling duality.

T-duality and D-branes

Consider a D_p brane and the end-point of an open string

- $\partial_\sigma X^\mu \big|_{end} = 0$ Neumann

$\partial_\tau X^I \big|_{end} = 0$

- T-duality along direction X^i :

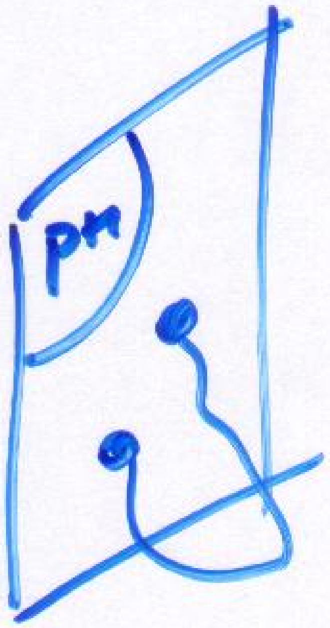
$$\partial_\sigma X^i \longleftrightarrow \partial_\tau X^i$$

Along longitudinal: $D_p \to D_{p-1}$

" transverse: $D_p \to D_{p+1}$

$$(2\pi\alpha') A_i \longleftrightarrow X^i$$

D-brane effective action



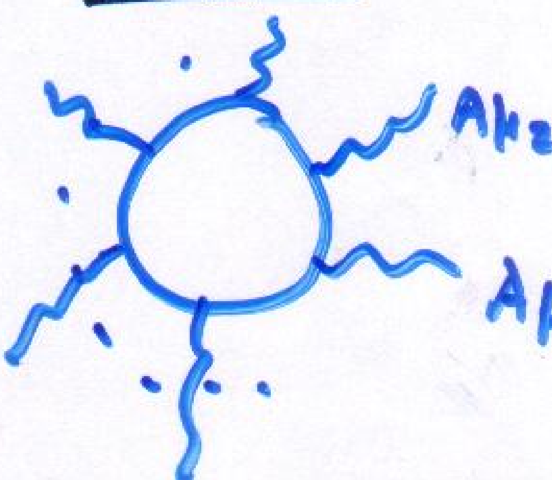
Spectrum : A_μ, Φ^I
fermions

(Dim. red. of $\mathcal{N}=1$ $D=10$
SYM multiplet)

Invariance under 16 super-
charges

- Leading contributions come from disk (tree-level)

in $\boxed{p=9}$ $\rightarrow$ only A_μ



Direct* calculation of string amplitudes in flat space

$$S_D = T_9 \int d^{10}x \, e^{-\phi} \sqrt{\det\left(\delta_{\mu\nu} + 2\pi\alpha' \times F_{\mu\nu}\right)}$$

disk $\leftarrow \frac{1}{g_s}$

- Dirac-Born-Infeld action

Weak field limit

$$S_D \sim \int T_9 e^{-\phi} \left\{ 1 + \frac{1}{2}\left(2\pi\alpha' F\right)^2 + \ldots \right.$$

- What happens when $p < 9$?

$\longrightarrow$ Dimensional reduction

$$\left.\begin{array}{l} A_\mu(x) \longrightarrow (p+1) \\ A_I(x) \end{array}\right\} \partial_I \to 0$$

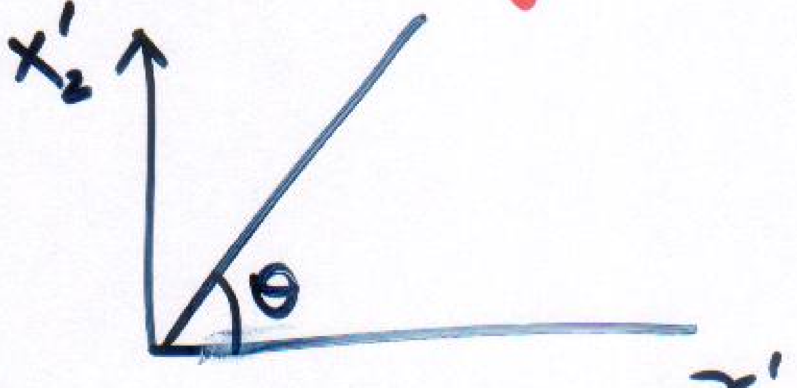
Another "quick argument"

D2 brane with constant F_{12}

$$\Rightarrow \quad A_2 = X^1 F_{12}$$

T-dualize $x^2 \Longrightarrow \quad X'^2 = (2\pi\alpha') X^1 F_{12}$

this is a D1 brane at an angle ($x^1 x^2$ axes)



$$\tan\theta = (2\pi\alpha') F_{12}$$

$$S_{D1} = \int ds = \int dx^1 \sqrt{1 + (\partial_1 X'_2)^2}$$

$$= \int dx^1 \sqrt{1 + (2\pi\alpha' F_{12})^2}$$

Generalizes to other dimensions

$$\left(\mathbb{1} + (2\pi\alpha') F_{\mu\nu} \right)$$

$\partial_I \to 0$

$$\downarrow$$

- $$\begin{array}{c|c} & \begin{array}{cc} \nu & J \end{array} \\ \begin{array}{c} \mu \\ I \end{array} & \left(\begin{array}{c:c} \mathbb{1} + 2\pi\alpha' F_{\mu\nu} & \partial_\mu A_J \\ \hdashline -\partial_\mu A_I & \mathbb{1} \end{array} \right) \end{array}$$

- $$\det \left(\quad \right) =$$

$$= \det \left(\delta_{\mu\nu} + (2\pi\alpha') \partial_\mu A^I \partial_\nu A^I + (2\pi\alpha') F_{\mu\nu} \right)$$

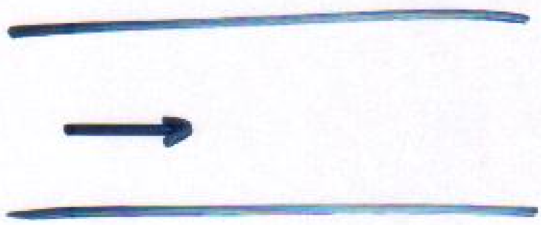
$$= \det \left(\delta_{\mu\nu} + \partial_\mu X^I \partial_\nu X^I + (2\pi\alpha') F_{\mu\nu} \right)$$

The B-dependence.

SKIP

Comes from gauge-invariance and boundaries.

open string:



•

→ conf. map →

disk

σ model:

•

$$\sim \frac{1}{2\pi\alpha'} \int_{M} d^2\sigma \; \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}$$

$$+ \int_{\partial M} d\tau \; A_\mu \partial_\tau x^\mu$$

gauge invariance $A_\mu \to A_\mu + \partial_\mu \epsilon$

$$\delta S \sim \int_{\partial M} d\tau \; \partial_\tau \epsilon = 0$$

B-gauge invariance.

$$B \to B + d\Lambda$$

$$\frac{1}{2\pi\alpha'}\int_{\mathcal{M}} \hat{B} \to \frac{1}{2\pi\alpha'}\int_{\mathcal{M}} \hat{B} + \frac{1}{2\pi\alpha'}\int_{\partial\mathcal{M}} \hat{\Lambda}$$

- For no boundaries this is zero

→ this is cancelled

by $A \to A - \Lambda$

- $\Rightarrow$ $\hat{B}_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}$

is gauge-invariant

$B_{IJ}\,\partial_\mu X^I \partial_\nu X^J$

SKIP

Introducing also a spacetime metric $G_{\mu\nu}$

$$S_D^{even} = T_{p+1} \int d^{p+1}\xi \, e^{-\Phi}$$

- $$\times \sqrt{\det\left(\hat{G}_{\alpha\beta} + \hat{B}_{\alpha\beta} + (2\pi\alpha')F_{\alpha\beta}\right)}$$

ξ^a are world volume coordinates

- $$\hat{G}_{\alpha\beta} = G_{\mu\nu}\,\partial_\alpha x^\mu\,\partial_\beta X^\nu$$
$$\hat{B}_{\alpha\beta} = B_{\mu\nu}\,\partial_\alpha X^\mu\,\partial_\beta X^\nu$$

This gives the couplings of $G_{\mu\nu}$, $B_{\mu\nu}$, Φ to the D-brane a p-branes couples minimally to a (p+1)-form

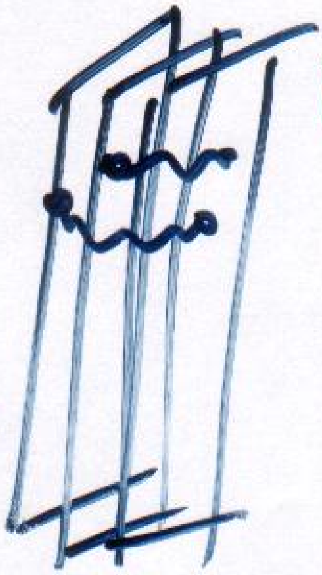
$$S_D^{odd} = Q_p \int_{M_{p+1}} \hat{C}_{p+1}$$

There are also subleading terms for gauge invariance that depend on $C_{q<p}$, F, B, metric

Non-abeliam symmetry:

We will consider N, identical parallel, coinciding D_p-branes

Only way to distinguish → index $i = 1, \ldots, N$



N^2 open strings → (i,j)

Massless spectrum

A_μ^{ij}, Φ_I^{ij} → $N \times N$ matrices

+ fermions

string interactiom have now a $U(N)$ non-abelian gauge symmetry

$$E_{\alpha\beta} \equiv (G_{\mu\nu} + B_{\mu\nu}) \text{ on the brane}$$

$$\equiv (G_{\mu\nu} + B_{\mu\nu}) D_\alpha X^\mu D_\beta X^\nu$$

$$D_\alpha X^\mu = \partial_\alpha X^\mu + [A_a, X^\mu]$$

$\hookrightarrow X^i \equiv \Phi^i$

- $$\mathcal{L}_N \sim \mathrm{STr} \sqrt{\det \left(E_{\alpha\beta} + 2\pi\alpha' F^{+}_{\alpha\beta} + E_{\alpha i} \left(Q^{-1} - 1 \right)^{ij} E_{j\beta} \right] \times \det(Q^i{}_j)}$$

- $$Q^i{}_j = \delta^i{}_j + 2\pi\alpha' i \, [\Phi^i, \Phi^j] E_{kj}$$

$$E_{ij} = G_{ij} + B_{ij}$$

$$\boxed{\frac{1}{g_{YM}^2} = \frac{T_p}{g_s} (2\pi\alpha')^{\frac{1}{2}}}$$

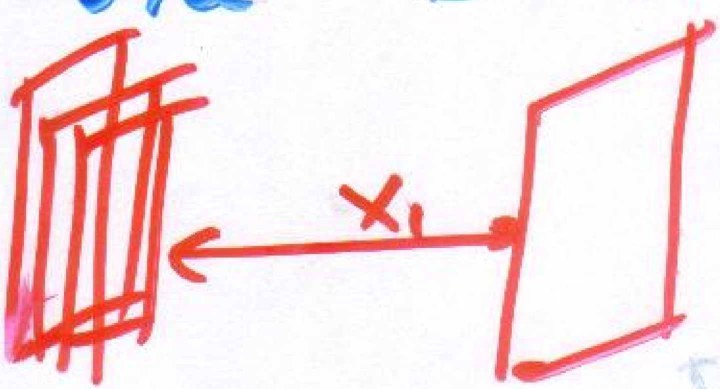
Expanding:

$$\mathcal{L} \sim -\frac{(2\pi\alpha')^2 T_p}{4} \int e^{-\phi} \, \mathrm{Tr} \left(F^2 + 2 D\Phi^i D\Phi^i + [\Phi^i, \Phi^j]^2 \right)$$

By a $U(N)$ rotation we can diagonalize Φ^I (hermitian)

$$\Phi^I \sim \begin{pmatrix} x_1^I & & & \\ & x_2^I & & 0 \\ & & \ddots & \\ & 0 & & x_N^I \end{pmatrix}$$

- x_α^i can be thought of as the transvers coordinates of the N D-branes
- Giving x_1^i an non-zero expectation value amounts to pulling one D-brane away



There is no energy cost for such an expectation value.

From $N=4$ $U(N)$ SYM action we see that

- $U(N) \rightarrow U(N-1) \times U(1)$ } Higgs
- From $D_\alpha \Phi^i D_\alpha \Phi^i$ the $2 \times N$ gauge bosons pick a mass $\sim |x_1|$

(energy of the strings streched between the branes)

"Geometrization" of gauge dynamics.

A new viewpoint on the description of Spacetime via non-commutative coordinates.

The EFT viewpoint

$$S \sim \int e^{-2\Phi}\left\{R + 4(\nabla\Phi)^2 - \frac{1}{2(p+2)!}(dC_{p+1})^2\right\}$$

Find spherically symmetric (in transverse space) solution with RR charge $N\,\mu_p$

p-brane extends along x^μ cordinates ($\mu = 0, 1, \ldots p$)

transverse to x^i coordinates

$$ds^2 = \frac{\eta_{\mu\nu} dx^\mu dx^\nu}{\sqrt{H_p}} + \sqrt{H_p}\, dx^i dx^i$$

$$e^{2\Phi} = g_s^2 \, H_p^{\frac{3-p}{2}}$$

- $$C_{p+1} = \frac{1}{g_s}\left(1 - \frac{1}{H_p}\right) dx^0 \wedge \dots \wedge dx^p$$

$$H_p = 1 + \left(\frac{L}{r}\right)^{7-p}$$

- $r^2 = x^i x^i \longrightarrow$ transverse distance

$$L^{7-p} = \underbrace{\alpha_p}_{\text{constant}} (2\pi)^{-2} g_s N \, l_s^{7-p}$$

$r=0$ is a horizon (of zero$^{(p>3)}$ area, except at $p=3$)

The solution is BPS (preserves half of spacetime susy)

The N branes can be put elsewhere by

$$H_p = 1 + \sum_{i=1}^{N} \frac{L^{7-p}}{|\vec{r}-\vec{r}_i|^{7-p}}$$

The solution generalises to a Black-brane sol.

$$ds^2 = \frac{-dt^2 \cdot f + d\vec{x}^2}{\sqrt{H_p}} + \sqrt{H_p}\left(\frac{dr^2}{f} + r^2 d\Omega^2\right)$$

$$f \sim 1 - (r_0/r)^{7-p}$$

The near-horizon limit

$p=3$: take $r \to 0$

$$H_3 \to \left(\frac{L}{r}\right)^4$$

$$ds^2 = \frac{r^2}{L^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu\right) + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2$$

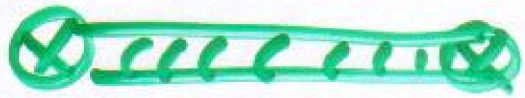
$AdS_5 \times S_5$ geometry

(high symmetry)

Gauge theories at large N_c

It is expected that AF gauge theories confine e-flux

- into flux tubes



closed and open string-like objects.

- Find altertative low energy description.

At large N this picture becomes plausible

Consider an $N \times N$ hermitian matrix M.

$$\mathcal{L} = \frac{1}{g^2} \operatorname{Tr}\left\{ (\partial M)^2 + M^2 + M^3 + \dots \right.$$

$$\equiv \frac{1}{g^2} \operatorname{Tr}\left\{ (\partial M)^2 + V(M) \right\}$$

$U(N)$ invariance: $M \to U M U^{+}$

similar to YMills.

Double line notation:

$M_i{}^j \to$  $\sim g^2$ propagator

 vertices.

$\sim \frac{1}{g^2}$

Each closed line

$\sim N$ (because of sum)

Diagram: $\sim (g^2)^{\text{Prop - vertices}}$

- $\times\; N^{\text{Closed lines}}$

$$\sim N^{\underbrace{\text{Faces} - \text{Edges} + \text{vertices}}_{\chi = 2-2h}}$$

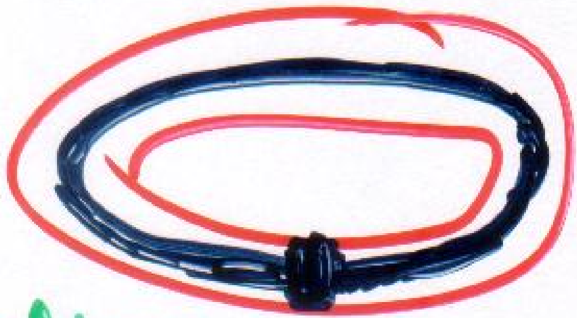
- $\times\, (g^2 N)^{\text{Edges - vertices}}$

planar

$\sim N^3 g^2 \sim N^2 (g^2 N)$

sphere

 $\sim N g^2 \sim N^0 (g^2 N)$

torus: 

Non-planar diagram

- leading at large-N $\to$ planar

$$N^2 \left(c_0 + c_1 (g^2 N) + \cdots \right)$$

- $\sim N^2 f(g^2 N)$

Full generating function

$$\log Z = \sum_{h=0}^{\infty} N^{2-2h} f_h (g^2 N)$$

't Hooft limit:

$$N \to \infty \qquad \lambda = g^2 N = \text{fixed}$$

λ = large , large diagrams are expected to contribute.

generate the Riemann surfaces of some string theory (with $g_s \sim \frac{1}{N}$) (closed)

if there are fundamentals

$\rightarrow$ boundaries $\rightarrow$ open strings

$\rightarrow$ mesons $\leftrightarrow$ $q\bar{q}$

QCD at large-N has strings, Regge trajectories, $q\bar{q}$ mesons which are weakly coupled etc

What could the associated string theory look like?

4-D YM $\longrightarrow$ 4D string theory

but no-Weyl invariance

$\Rightarrow$ Liouville mode ϕ is dynamical

What is the associated 5d background?

$$ds^2 = W(z)^2 \left(d\vec{x}^2 + dz^2 \right)$$

if YM has a scaling symmetry : $x \to \lambda x$

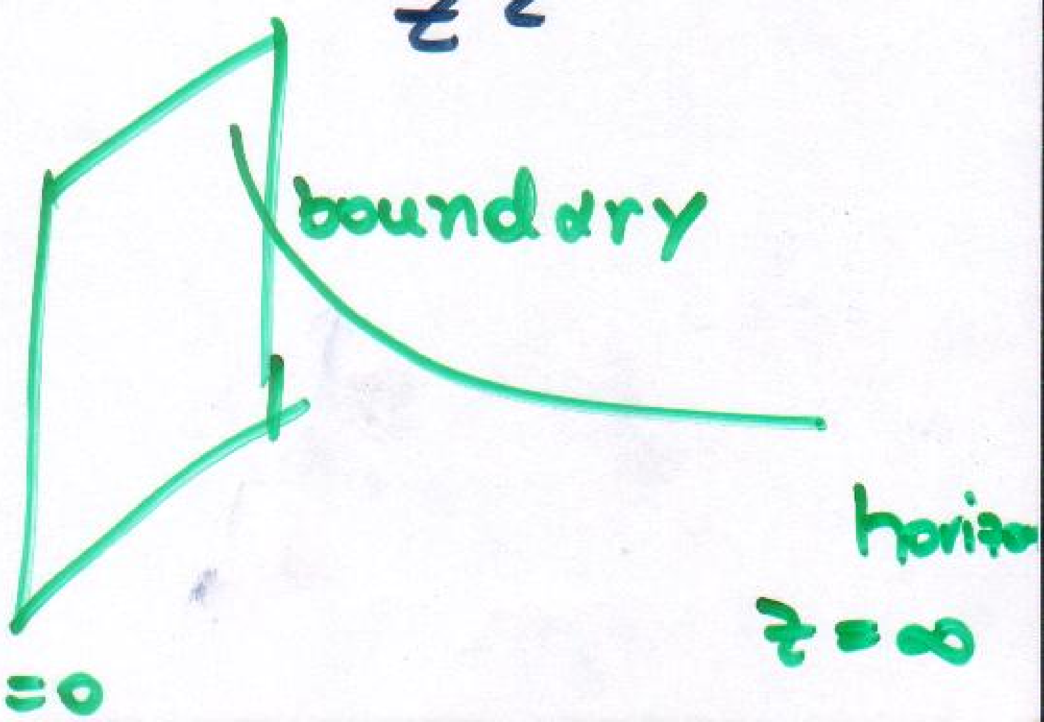
this should be a symmetry in string theory:

- $z \to \lambda z$

$$W(z) = \frac{R}{z}$$

- $\Rightarrow ds^2 = R^2 \frac{d\vec{x}^2 + dz^2}{z^2}$

$\to AdS_5$



Back to D3 branes and the near-horizon limit

$$ds^2 = \frac{d\vec{x}^2}{\sqrt{H}} + \sqrt{H}\left(dr^2 + r^2 d\Omega_5^2\right)$$

$$H = 1 + \frac{L^4}{r^4} \quad , L^4 = g_s N \ell_s^4$$

gravity is a good aproximation iff $\underset{\substack{\uparrow \\ \text{curvature}}}{L} \gg \ell_s$

$\Rightarrow \quad g_s N \gg 1$

pertrbative $g_s \ll 1$ ok iff $N \to \infty$

We now take $l_s \to 0$

keeping $\frac{r}{l_s^2} \equiv u$ fixed

$u \to$ W-boson mass on a probe

- $H = \cancel{1 + g_s N \left(\frac{l}{r}\right)^4}$

$= 1 + g_s N \left(\frac{l_s^2}{r}\right)^4 l_s^{-4}$

- $\simeq \frac{g_s N}{l_s^4} \frac{1}{u^4}$

$$ds^2 = \frac{r^2}{L^2} d\bar{x}^2 + \frac{L^2}{r^2} dr^2$$

$$+ L^2 d\Omega_5^2 \qquad L^4 = g_s N l_s^4$$

$$S = S_{brane} + S_{bulk} + S_{inter.}$$

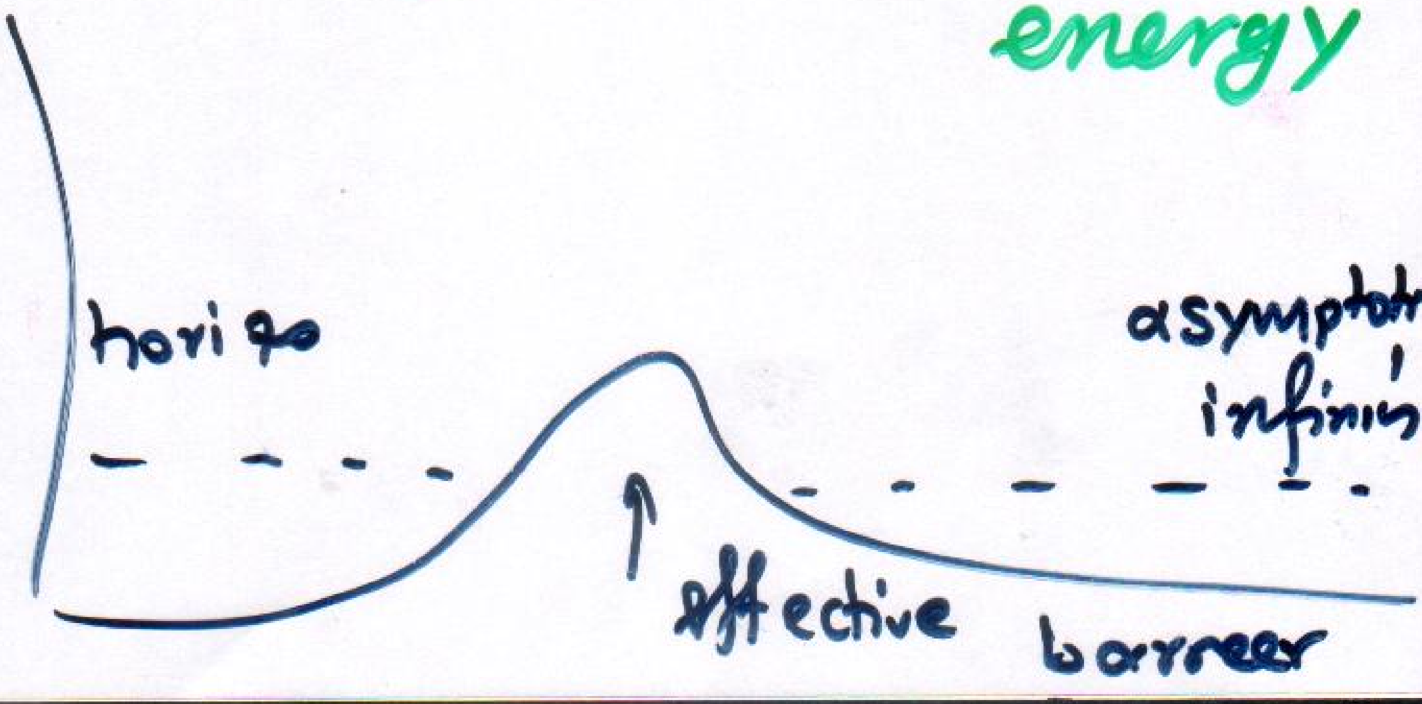
as $l_s \to 0$ $\quad S_{bulk} \to$ free

$M_p \to \infty$

- $S_{interaction} \to 0$

(Non-trivial) calkulation of greybody factors

- $\sigma \sim \omega^3 L^8 \to 0$ with energy



Since $g_s = g_{YM}^2$

$\lambda = g_s N$:

- When $\lambda \ll 1 \rightarrow$ gauge theory description perturbative

$\lambda \gg 1 \Rightarrow L \gg \ell_s$

$\Rightarrow$ gravitational description

- (stringy corrections $\sim \left(\frac{L}{\ell_s}\right)^{-1}$

$\sim \lambda^{-\frac{1}{4}}$ supressed

at $\lambda \rightarrow \infty$

ex: $M_p^2 \left\{ R + \ell_s^6 R^4 + \ldots \right.$

$\sim \left(\frac{\ell_s}{L}\right)^6 \cdot R$

AdS/CFT correspondence

$\mathcal{N}=4$ SYM	AdS_5 string
$\lambda = g^2 N$	$\frac{\ell_s}{L} = \lambda^{-\frac{1}{4}}$
• N	$g_s = \frac{\lambda}{N} \sim \frac{1}{N}$
θ	

$$M_5^3 = M_{10}^8 \cdot R^5$$

• 5d Planck scale

$$= \frac{\ell_s^{-8}}{g_s^2} \cdot (g_s N)^{\frac{5}{4}} \ell_s^5$$

$$= \frac{N^2}{R^3}$$

This is a duality

$$\underset{\text{YM}}{0} \quad | \quad \underset{\text{SGR}}{\infty} \quad \lambda$$

Symmetries

$\mathcal{N}=4$ SYM is invariant under:

- 4D conformal symmetry*
  $\sim O(2,4)$ (Minkowski)
- $O(6)$ R-symmetry
  $\phi^i \to$ vector
  $\chi^a \to$ spinor
  $A_\mu \to$ singlet
- $\mathcal{N}=4$ supersymmetry

* Conformal symmetry

Scale invariance usually
$\Rightarrow$ conformal invariance

- $$\delta S = \int T^{\mu\nu}\,\delta g_{\mu\nu}$$

$x^\mu \to x^\mu + \xi^\mu$: $\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

$x^\mu \to \lambda x^\mu \Rightarrow \delta g_{\mu\nu} = 2\,\delta\lambda \cdot g_{\mu\nu}$

- $\Rightarrow \delta S = 0$ iff : $T^\mu{}_\mu = 0$

[any $\delta g_{\mu\nu} = h(x)\, g_{\mu\nu}$ is a symmetry)

Conformal group = Poincare'
+ scale transf + $\left(\vec{x} \to -\frac{\vec{x}}{|x|^2} \right)$

$$\begin{array}{c} \equiv \\ O(2,4) \end{array}$$

$AdS_5 \longrightarrow \mathbb{R}^{2,4}$

$$-X_{-1}^2 - X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = -R^2$$

(one time is orthogonal)

- Solve by $X_{-1} + X_4 = \frac{R}{z}$

$$X_\mu = \frac{R x_\mu}{z}$$

Ex: solve for x_4, X_{-1} to derive from $-dX_{-1}^2 - dX_0^2 + dX_i^2 \rightarrow \frac{dx_\mu^2 + dz^2}{z^2}$

- Not global coordinates.

Another set

$$X_1^2 + X_2^2 + X_3^2 + X_4^2 = R^2 \sinh^2 \rho$$

$$x_{-1}^2 + x_0^2 \qquad = R^2 \cosh^2 \rho$$

$$x_{-1} = R \cosh\rho \cos\tau$$
$$X_0 = R \cosh\rho \sin\tau$$

$$ds^2 = R^2\left(-\cosh^2\rho\, d\tau^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2\right)$$

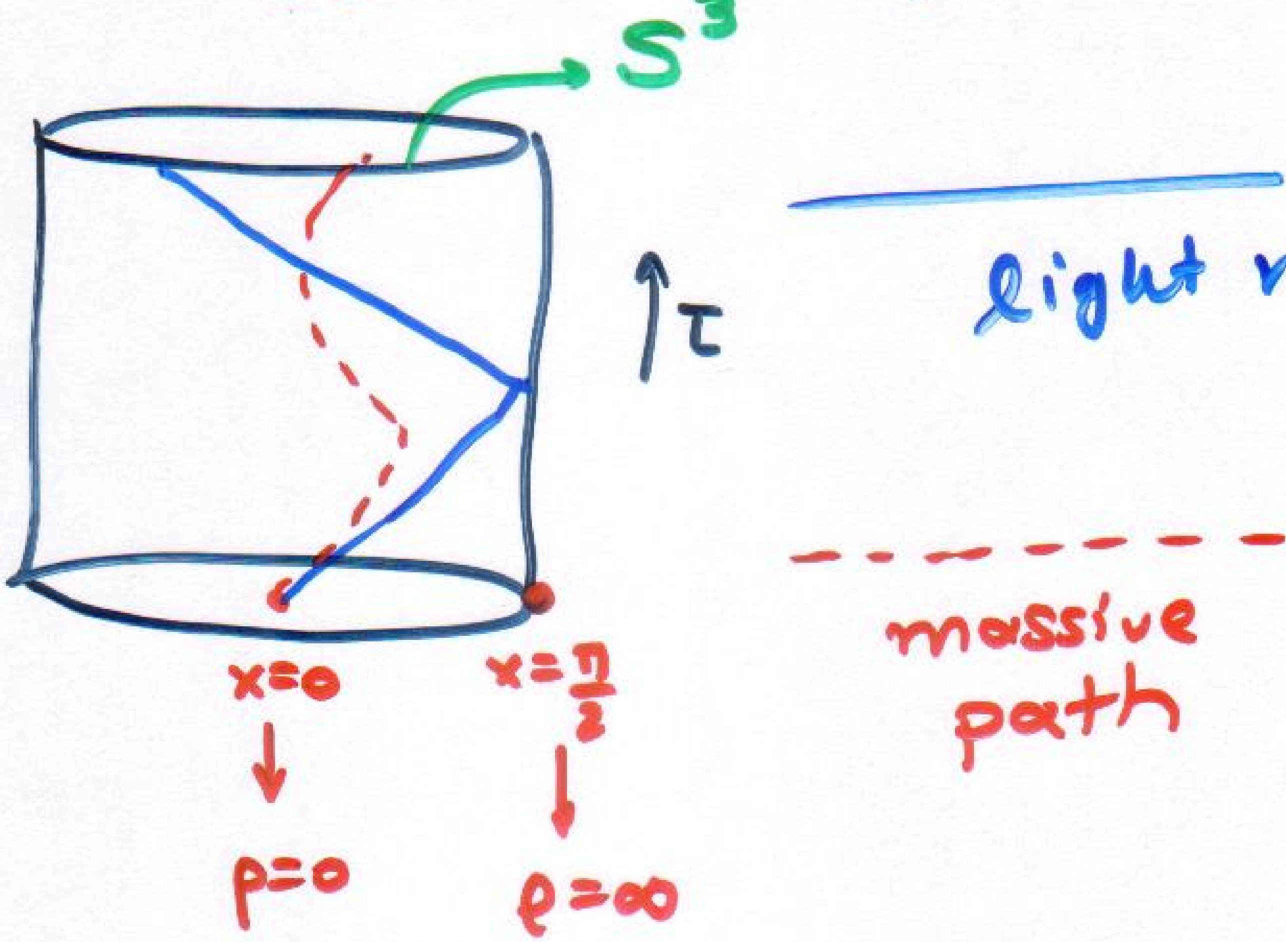
we may go to the covering space : $-\infty < \tau < \infty$

def: $dx = \dfrac{d\rho}{\cosh\rho} \Rightarrow \tan\frac{x}{2} = \tanh\frac{\rho}{2}$

$0 < x < \frac{\pi}{2}$

$$ds^2 = R^2 \underbrace{\cosh^2\rho}_{\frac{1}{\cos^2 x}} \left\{ -d\tau^2 + dx^2 + \sin^2 x\, d\Omega_3^2 \right\}$$

Penrose diagrams

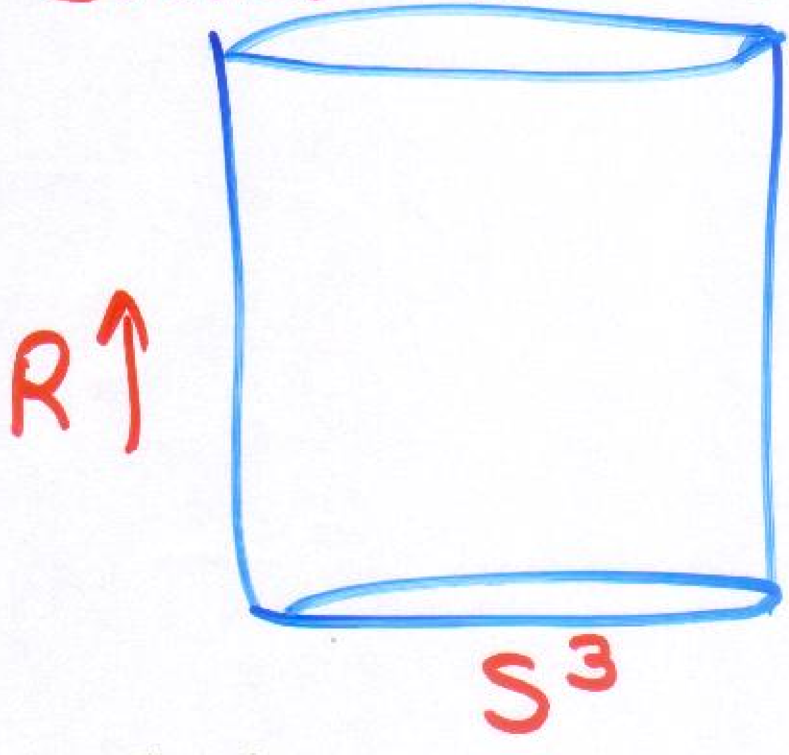


- boundary is $\rho=\infty$ (the surface of cylider)

proper distance to boundary is finite

The $O(2,4)$ isometries of the AdS act as the standard conformal transformations on the boundary coordinates.

States and operators in CFT



States on $S^3 \times R$ $\longrightarrow$ Operators on $\mathbb{R}^4$

$E_{cylinder} = \Delta_{plane}$

The correspondence.

In AdS we can consider the string theory partition function as a functional of boundary sources

$$Z_{Bulk}\left(\phi(z,\bar{x})|_{z=0} = \phi_0(\bar{x})\right) =$$

$$= e^{-N^2 S_{class.}(\phi_0) + O(\alpha')}$$

$$\left[1 + \text{Quantum Corrections}\right]$$

$$\alpha' \sim \frac{1}{\lambda^{1/2}}$$

$$g_s \sim \frac{1}{N}$$

Each field in the bulk $\phi(z, \vec{x})$ corresponds to a specific operator in boundary theory.

(via bulk boundary coupling)

e.g.:

$$g_{\mu\nu} \longleftrightarrow T_{\mu\nu}$$

$$\phi \longleftrightarrow \mathcal{L}$$

$$A_\mu^a \longleftrightarrow J_\mu^a$$

the correspoolence

$$Z_{BULK}\left(\phi_0(\vec{x})\right) =$$

$$= \left\langle e^{\int d^4x \, \phi_0(\vec{x})\, O(\vec{x})} \right\rangle_{FT}$$

<u>Example:</u> scalar

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + d\vec{x}^2\right)$$

$$S = N^2 \int \frac{d^4x \, dz}{z^5}\left[z^2(\partial\phi)^2 + m^2R^2\phi^2\right]$$

Equations:

$$z^3 \partial_z\left(\frac{1}{z^3}\partial_z\phi\right) - p^2 z^2\phi - m^2R^2\phi = 0$$

(after Fourier Tr. in 4D)

Near the boundary: $z=0$

$$\phi(z) \sim z^{\alpha}$$

$$\alpha(\alpha-4) - m^2R^2 = 0$$

$$\Rightarrow \quad \alpha_{\pm} = 2 \pm \sqrt{4+m^2R^2}$$

- $\alpha_- \rightarrow$ dominates at $z \rightarrow 0$
$\Rightarrow$ determines the b.c.

$$\phi(\vec{x}, z)\Big|_{z=\epsilon} = \epsilon^{\alpha_-}\phi_0(\vec{x})$$

- (regularisation and renormalisation)

a scale transformation
$z \rightarrow \lambda z$, $\vec{x} \rightarrow \lambda \vec{x}$ leaves
ϕ invariant $\Rightarrow$

$$\Rightarrow \boxed{\Delta = 4 - \alpha_- = \alpha_+}$$

from $e^{\int d^4x \, \phi_0(\vec{x}) \, O(\vec{x})}$

So $\Delta = 2 + \sqrt{4 + (mR)^2}$

- For massles fields $\Delta = 4$ like $\mathcal{L}$ or $T_{\mu\nu}$

To make this comparison we must reduce the 10-d fields to 5d by compactifying on S^5

For a 10-d massless field The KK masses are $m_\ell^2 = \frac{\ell(\ell+4)}{R^2}$

$\Rightarrow \quad \Delta = 4 + l$

For $\phi_0 \sim \mathrm{Tr}(F^2 + \dots)$

$\longrightarrow \mathrm{Tr}\left(F^2 \phi^{[I_1} \dots \phi^{I_l]}\right)$

$\hookrightarrow$ BPS multiplets

$\rightarrow$ protected

All SYM BPS operators correspont to the KK modes of 10-d massless fields.

$\rightarrow$ matching is not trivial

Others $m \sim \frac{1}{l_s} \sim \sqrt{g^2 N} \rightarrow \infty$

Singletons

The representation with $\Delta = 1$ is special

Unitarity $\rightarrow$ free field.

- Come from $U(1)$ factors

ex: $\mathrm{Tr}[\phi^I]$ for $U(N)$

No corresponding bulk

- field.

FT → distane + time → $\vec{x}$.

Not equal to bulk distancs due to "warp" factor $\frac{R^2}{z^2}$

- $ds^2 = W^2 \left(dz^2 + d\vec{x}^2 \right)$

$$W = \frac{R}{z}$$

- α mass scale of energy in bulk gets scaled

$$E_{FT} = W(z)\, E_{proper}$$

$$(\text{Size})_{FT} = \frac{1}{W(z)} (\text{proper size})$$

$z \to 0 \Rightarrow \left(\begin{array}{c} \text{small size} \\ \text{high energy} \end{array} \right) \to UV$

from bulk point of view

$z \to 0 \quad \to$ large distance

$\Rightarrow \quad$ UV $\longleftrightarrow$ IR

FT $\qquad\qquad$ string

consequence of

open-closed string

duality.

Another instance of bulk - boundary correspodence:

D=2 (good laboratory)

- simple background:

$$G_{\mu\nu}=\eta_{\mu\nu}, \quad B=0 \quad \Phi=\frac{Q}{2}x^1$$

bosonic $\quad Q^2=\frac{24}{3} \Rightarrow C=2+3Q^2 = 26.$

superstring $\quad Q^2=8/3$

- need to "block" strong coupli at $x^1\to\infty$ | $g_s\to\infty$

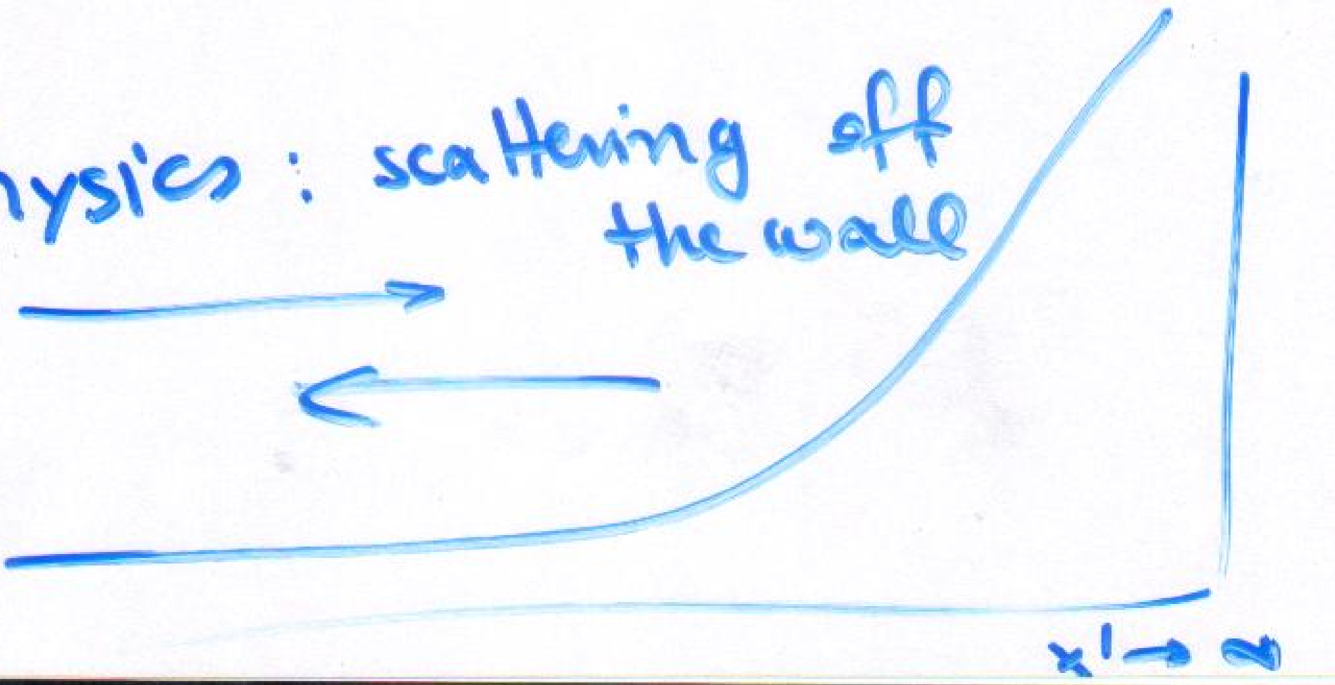
One way is to turn on the "tachyon"

$$S = \frac{1}{8\pi}\int d^2\sigma \left\{ \sqrt{g}\, g^{\alpha\beta}\eta_{\mu\nu}\,\partial_\alpha X^\mu \partial_\beta X^\nu + Q\sqrt{g}\,R^{(2)} X^1 + \underbrace{\mu e^{\alpha X^1}}_{\text{Liouville wall}} \right\}$$

One propagating (*) degree of freedom (massless

"Tachyon" $\sim e^{i\bar{k}\cdot\vec{x} + \frac{1}{2}QX^1}$

Physics: scattering off the wall



Could this be dual to
a 1-d "boundary" theory?

"Old" matrix model!

new light: world volume
theory of
unstable D-brane

- $M_{ij} \sim$ "tachyon" (open). $N \times N$ Hermitian

$A_\mu^{ij} \rightarrow$ imposes gauss' law

- $$S = 6N \int dt \left\{ \frac{1}{2} \mathrm{Tr}\, \dot{M}^2 + \mathrm{Tr}\, V(M) \right\}$$

The large-N expansion
generates the sum over
Riemann surfaces

$$H = -\frac{1}{2\beta N}\sum_{ij}\frac{\partial}{\partial M_{ij}}\frac{\partial}{\partial M_{ji}} + \beta N_{\times}$$
$$\times \, \mathrm{Tr}\left(V(M)\right)$$

unitary invariance $\quad M \to U M U^{-1}$

- $A_0 \longrightarrow$ implies singlet condition

use invariance to

- $M \to \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \\ & & & \lambda_N \end{pmatrix} \to \lambda \leftrightarrow \lambda'$

$$H\,\Psi(\lambda) = \left\{ \sum_k -\frac{1}{2\beta N}\frac{\partial^2}{\partial \lambda_k^2} + \frac{1}{\beta N}\sum_{\ell \neq k}\frac{1}{\lambda_\ell - \lambda_k} \right.$$
$$\left. \times \frac{\partial}{\partial \lambda_k} + \beta N\, V(\lambda_k) \right\} \Psi(\lambda)$$

$$H\,\Psi(\lambda) = \frac{1}{\Delta} H' \Delta \Psi ,$$

$$\Delta(\lambda) = \prod_{i<j} (\lambda_i - \lambda_j)$$

- $$H' = \beta N \sum_k \left[-\frac{1}{2\beta^2 N^2} \frac{\partial^2}{\partial \lambda_k^2} + V(\lambda_k) \right]$$

absorb Δ :

$$\Psi(\lambda) \cdot \Delta(\lambda) = P(\lambda)$$

- N coordinates λ_k, non-interacting

and fermionic

or a non-relativistic 1-d fermion χ :

$$H' = \beta N \int dx \left\{ \frac{1}{2\beta^2 N^2} \partial \chi^+ \partial \chi + \chi^+ \chi \, V(\lambda) \right.$$

large N limit is controled by sphere

$$\lambda = g \cancel{N}$$

standard large N limit

analogue of 't Hooft coupl.

$$N \to \infty \, , \quad g \to \text{fixed}$$

Filling the Fermi-sea

$$N = g N \int \frac{dp \, d\lambda}{2\pi} \quad \frac{1}{\hbar}$$

$$\times \, \Theta\left(\epsilon_F - \frac{p^2}{2} - V(\lambda) \right)$$

$$* \quad \frac{\pi}{\eta g} = \int_{\lambda_-}^{\lambda_+} d\lambda \, \sqrt{2(\epsilon_F - V(\lambda))}$$

We will need to take $b \to \infty$ to get continuous surfaces (b controls the interaction)

as $b \to \infty$ there is

- a critical point b_c so that $\langle 0|H'|0\rangle$ are non-analytic (singular)

$$b - b_c \sim (\epsilon_* - \epsilon_F) \log|\epsilon_* - \epsilon_F|$$

- $$\langle 0|H|0\rangle \sim (\quad)^2 \log|\quad|$$

Double scaling:

$N \to \infty$
$b \to b_c$

$\mu = bN(\epsilon_c - \epsilon_F)$ fixed

The topologies sum is controled now from $\to \mu^{x}$ (related to string coupling)

- At the lelvel of hamiltonianian:

$$\lambda - \lambda_{max} = (\beta N)^{-\frac{1}{2}} x$$

$$J = (\beta N)^{\frac{1}{4}} \psi$$

-

$$H' - \beta N \epsilon_F = \int dx \left\{ \frac{1}{2} \partial_x \psi^{+} \partial_x \psi - \frac{x^2}{2\alpha'} \psi^{+} \psi + \mu \psi^{+} \psi \right\}$$

In terms of the rescaled variables:

$\mu \sim \frac{1}{g_s}$

the physical degrees of freedom are ripples of the Fermi sea ("tachyon"

— · —

The string theory has t and $\varphi \to$ Liouville
the tachyons are the bosonized fermions.