(Thermo)dynamics of D-brane probes

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Abstract: We discuss the dynamics and thermodynamics of particle and D-brane probes moving in non-extremal black hole/brane backgrounds. When a probe falls from asymptotic infinity to the horizon, it transforms its potential energy into heat, TdS, which is absorbed by the black hole in a way consistent with the first law of thermodynamics. We show that the same remains true in the near-horizon limit, for BPS probes only, with the BPS probe moving from AdS infinity to the horizon. This is a quantitative indication that the brane-probe reaching the horizon corresponds to thermalization in gauge theory. It is shown that this relation provides a way to reliably compute the entropy away from the extremal limit (towards the Schwarzschild limit).

1. Introduction and conclusions

The black D-brane solutions of type II supergravity [1] and their near-horizon geometry [2] are the central elements of CFT/Anti-deSitter (AdS) correspondence [3, 4, 5]. In particular, the (3+1)-dimensional world-volume of N coinciding, extremal D3-branes is the arena of $\mathcal{N}$=4 supersymmetric $SU(N)$ Yang-Mills (SYM) theory which in the large N limit, according to the Maldacena conjecture [3], is linked to type IIB superstrings propagating on (the near-horizon) $AdS_5 \times S^5$ background geometry. Recently, there has been an interesting proposal [6] linking the thermodynamics of large N, $\mathcal{N}$=4 supersymmetric $SU(N)$ Yang-Mills theory with the thermodynamics of Schwarzschild black holes embedded in the AdS space [7]. The classical geometry of black holes with Hawking temperature T encodes the magnetic confinement, mass gap and other qualitative features of large N gauge theory heated up to the same temperature. At the computational level, the quantity that has been discussed to the largest extent [8, 6, 9] is the Bekenstein-Hawking entropy which, in the near-horizon limit, should be related to the entropy of Yang-Mills gas at $N \to \infty$ and large 't Hooft coupling $g^2_{\rm YM}N$.

The SYM/AdS correspondence and its thermal black hole generalizations emerge in a particular limit of N D-branes coinciding at one point in the transverse space; this corresponds to a conformal point in the SYM moduli space, with zero vacuum expectation values (vevs) of all scalar fields. Before taking the near-horizon limit one could also consider some other configurations, obtained for instance by placing a number of D-branes in the bulk of AdS; this corresponds to switching on some scalar vevs on the Higgs branch of the gauge theory. In this work, we consider the case of a single D-brane probe in the background of a *near*-extremal black hole with a large number of coinciding D-branes. We consider a probe moving from asymptotic infinity towards the black hole horizon. As the probe moves through the horizon, the black hole receives the quantity of heat that is determined by the first law of thermodynamics. The corresponding change in black hole entropy is consistent with thermodynamical identities. Hence from the *thermodynamical* point of view, the D-brane gas is physically located at the black hole horizon.

We can then consider a similar process in the near-horizon limit description. That is, a

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probe brane is falling from the boundary of the appropriate space (AdS_5 for D3-branes) rather than from spatial infinity, to the horizon. We find again, for BPS probes, that the potential energy released is equal to the heat, TdS, absorbed by the black hole. Thus, this process can be used to calculate the entropy of gauge theory in an alternative way by integrating back the potential energy. Moreover, this process has now an interpretation in terms of the spontaneously broken $SU(N+1) \to SU(N) \times U(1)$ gauge theory. At $T = 0$, any Higgs expectation value is stable, since it is a modulus protected by supersymmetry. Once $T > 0$, supersymmetry is broken, and the Higgs acquires a potential with an absolute minimum at zero expectation value. Starting with the theory at the top of the potential (very large expectation value) the Higgs will start rolling down. The Higgs field reaching an expectation value of the order of the electric mass corresponds in supergravity to the probe reaching the horizon. At this point, the energy stored in the Higgs field is thermalized and equal to the overall heat received by the thermal Yang-Mills system. This makes more precise a similar picture described in [10]. From the supergravity point of view the brane continues to move, crossing the horizon. It is not obvious what this motion corresponds to in gauge theory (but see [10] for a proposal).

We consider also the effects of higher order α' corrections to the probe action. Such corrections have been partially computed for the appropriate backgrounds directly in the near-horizon limit [11]. We argue that that although there are α' corrections to the D-brane world-volume action, they do not influence the calculation of the heat in our argument.

In all of the examples analyzed here, the probe method can be viewed as an independent method of "measuring the entropy" by integrating the heat, TdS. In particular, it provides a first order partial differential equation for the entropy in terms of mass M and charge Q. α'-corrections do not modify the leading equation. The only source of stringy corrections to the entropy equation comes from bulk α'-corrections to the RR gauge potential at the horizon. This equation is important since it can be used in conjunction with entropy calculations, done near the limit where supersymmetry is restored (and thus the calculation is reliable) and then extrapolated to the Schwarzschild region where supersymmetry is completely broken and calculations are difficult to control.

This talk is based on results obtained in collaboration with Constantin Bachas [12]. We will first describe in detail the case of Reissner-Nordström black holes in four dimensions and their near-horizon limit. Then we will describe arbitrary black Dp-branes and finally focus on the near-horizon limit of D3-branes.

2. Reissner-Nordström black holes

In order to see how probes can be used to study black hole thermodynamics, it is instructive to consider first the case of a point particle propagating in the background geometry of a charged black hole. The Reissner-Nordström (RN) metric for a charged black hole with ADM mass M and electric charge Q reads

$$ds^2 = -\left(1 - \frac{2l_p^2 M}{r} + \frac{l_p^2 Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2l_p^2 M}{r} + \frac{l_p^2 Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_2^2 \tag{2.1}$$

where $G_N = l_p^2$ is Newton's constant. There is also a static electromagnetic potential, which can be obtained from Gauss' law, $A_0 = \frac{Q}{r}$. The outer and inner horizons are located at r_+ and r_- with $r_\pm = l_p^2 \left(M \pm \sqrt{M^2 - Q^2/l_p^2}\right)$. The standard expression for the Bekenstein-Hawking entropy,

$$S(M,Q) = \frac{\text{Area}}{4G_N} = \frac{\pi r_+^2}{l_p^2} , \tag{2.2}$$

can be thought of as the 'equation of state' in the microcanonical ensemble. From it we can obtain the temperature and 'chemical potential' of the black hole

$$\frac{1}{T} \equiv \frac{\partial S}{\partial M}\bigg|_Q = \frac{4\pi r_+^2}{r_+ - r_-} , \quad \mu \equiv -T\frac{\partial S}{\partial Q}\bigg|_M = \frac{Q}{r_+} . \tag{2.3}$$

The free energy and grand canonical potential can also be obtained by the standard thermodynamic expressions, $F \equiv M - TS$ and $A \equiv M - TS - \mu Q$.

Consider next the process in which a probe particle with mass m and electric charge q falls inside the black hole. The black hole plus particle form an isolated system, with total mass $M+m$ and total charge $Q+q$, which will eventually reach thermal equilibrium. The entropy will therefore change by an amount

$$\delta S = \left.\frac{\partial S}{\partial M}\right|_Q m + \left.\frac{\partial S}{\partial Q}\right|_M q \ . \tag{2.4}$$

Using the explicit form of the chemical potential, derived from the 'equation of state', we find

$$T\delta S = m - \frac{Qq}{r_+} \ . \tag{2.5}$$

This equation admits a simple interpretation as the heat released by the probe particle while falling inside the black hole. The action for such a particle indeed reads

$$\Gamma = m\int d\tau\sqrt{-G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} + q\int d\tau A_\mu\dot{x}^\mu \ . \tag{2.6}$$

In the static gauge, $t=\tau$, it takes the form

$$\Gamma = \int dt\ V(r) + \text{velocity terms} \tag{2.7}$$

with V the static potential,

$$V(r) = m\sqrt{\left(1-\frac{2l_p^2M}{r}+\frac{l_p^2Q^2}{r^2}\right)} + \frac{qQ}{r} \ . \tag{2.8}$$

Notice that the potential includes the self-energy m of the probe, and is constant in the extremal limit for both source ($M = Q/l_p$) and probe ($m = q/l_p$), consistently with the absence of a static force in this case. Now as the particle moves from spatial infinity to the outer horizon, the difference in potential energy, $\delta V = V(\infty) - V(r_+)$, is converted to kinetic energy and then eventually dissipated as heat. This is precisely the content of eq. (2.5).

The argument can also be run backwards. Starting from the static potential eq. (2.8), and assuming that the potential energy of the probe is converted to heat at the outer horizon, leads to eq. (2.5). Comparing with eq. (2.4) then gives

$$\mu = A_0(r_+) = \frac{Q}{r_+} \iff -\left.\frac{\partial S}{\partial Q}\right|_M = A_0(r_+)\left.\frac{\partial S}{\partial M}\right|_Q \ . \tag{2.9}$$

This partial differential equation can be integrated for the equation of state $S(M,Q)$, provided we know already the answer on some (initial) curve in the (M,Q) plane, such as for Schwarzschild black holes $Q=0$ or extremal black holes, $M = Q/l_p$. Notice that even though the thermodynamic properties of a Schwarzschild black hole are in an essential way quantum ($\hbar$ enters in the expressions for both temperature and entropy), the extension to charged black holes follows from the simple classical argument outlined here. Such a differential equation may turn out to be practically important. Several thermodynamic properties seem to be more easily computable on or close to the extremal limit. There, supersymmetry is of help as evidenced by recent D-brane/black hole calculations [13]. Being able to calculate at the extremal boundary of the (M,Q) plane, one can use (2.9) to extrapolate the calculation to the whole plane, most importantly in the region $Q=0$ where supersymmetry is completely broken. It should be noted though that imposing a boundary condition at the extremal boundary $M=Q$ maybe problematic since in most cases the partial derivatives diverge there ($T=0$). This is the case here as well as for the Dp-branes with $p<5$. We could however impose boundary conditions on a line just outside the extremal boundary (at near-extremality) where computations are still reliable. This point is conceptually important and needs further investigation.

The reader may of course object that these considerations depend on our choice of a minimal probe action. Quantum gravity effects or stringy effects (controlled respectively by l_p and l_s) can give rise to curvature terms and/or non-minimal electromagnetic couplings, which would modify the potential (2.8). Nevertheless, *assuming thermodynamic equilibrium*, the relation

$$T\delta S = V(\infty) - V(r_+) = m - qA_0(r_+) \tag{2.10}$$

continues, as we will now argue, to be valid. This is of course consistent with the fact that for a neutral particle $T\delta S = m$ is simply the first law of thermodynamics.

To see why the potential difference is always given by eq.(2.10), consider possible corrections to the action Γ of the particle. These must be of the form

$$\delta\Gamma = \int ds\ f(R, F, \dot{x})\ , \qquad (2.11)$$

with $ds \equiv d\tau\sqrt{\dot{x}^\mu \dot{x}_\mu}$ the invariant proper time element, and f a scalar made out of the electromagnetic field strength, the Riemann tensor, the probe velocity, and covariant derivatives thereof. Note that corrections of the form $\int dx^\mu A_\mu \tilde{f}$ are not allowed, because gauge invariance forces the scalar function $\tilde{f}$ to be a constant.[1] Now in the quasi-static limit $x^\mu = (\tau, 0\cdots 0)$, the invariant element ds tends to dt at spatial infinity, and vanishes at the horizon, when expressed in terms of the asymptotic time. The scalar f on the other hand must vanish at spatial infinity where both F and R go to zero. Thus, *provided f stays smooth at the event horizon*, these higher-order terms will not contribute to the static potential either at $r = r_+$ or at $r = \infty$, as advertised.

To show that f cannot indeed diverge at the horizon, note first that this is automatic for a scalar function of the background fields, since the horizon singularity is a coordinate artifact. Suppose next that f is the pull back on the world-line of a space-time tensor

$$f = T_{\mu_1\cdots\mu_n}\frac{dx^{\mu_1}}{ds}\cdots\frac{dx^{\mu_n}}{ds} = T_{0\cdots 0}\frac{dt}{ds}^n + \cdots \qquad (2.12)$$

with T a function of F, R and their covariant derivatives and the dots stand for kinetic terms. Near the horizon $(\frac{dt}{ds})^2 = G^{00}$ diverges. Nevertheless f must remain regular, or else the scalar invariant $T_{\mu_1\ \cdots\mu_n}T^{\mu_1\ \cdots\mu_n}$ could not possibly be finite.

The only remaining possibility is that f depends on extrinsic invariants, or on other higher-derivative terms of the coordinate functions. They are constructed by using the covariant accelerations Ω^μ_n. For $n = 1$, this is the usual four-velocity, $\Omega^\mu_1 = \dot{x}^\mu$. For $n = 2$, this is the acceleration

$$\Omega^\mu_2 = \ddot{x}^\mu - \frac{1}{2}\partial_\tau \log(G_{\nu\rho}\dot{x}^\nu\dot{x}^\rho)\dot{x}^\mu + \Gamma^\mu_{\nu\rho}\dot{x}^\nu\dot{x}^\rho \qquad (2.13)$$

and so on. Notice that Ω^μ_n are tensors both of the target space diffeomorphisms as well as world-line reparametrizations. The piece of the acceleration that contributes to the potential is

$$\Omega^\mu_2\Big|_{pot} = -\frac{1}{2}\frac{G_{00,0}}{G_{00}}\delta^\mu_0 + \Gamma^\mu_{00} \qquad (2.14)$$

This in principle can give singular contributions on the horizons via invariants of the form $G_{\mu\nu}\Omega^\mu_m$ Ω^ν_n. We can show however that in the usual on-shell perturbative derivation of the higher α' corrections such invariants cannot appear. The reason is that the first non-trivial correction is computed by matching *on-shell* some scattering amplitude involving the particle with a combination of higher velocity or acceleration terms. However, on-shell $\Omega^\mu_2 \sim F^\mu{}_\nu\dot{x}^\nu$. Thus, the Ω_2 is redundant and does not appear on-shell in the first correction terms. Consequently the corrected action involves only velocities, and the corrected equations will equate Ω_2 to velocities again. Thus, to any finite order of perturbation theory, we do not have the dangerous acceleration terms. This is supported by a recent calculation of (part of) the $\mathcal{O}(\alpha'^2)$ terms for D0-branes [11].

The upshot of the previous argument is that the right-hand side of eq. (2.10) is universal, and hence so is the differential equation (2.9) which one can integrate for the equation of state. One immediate corollary, assuming the entropy stays smooth in the extremal limit where $T = 0$, is that $A_0(r_+)\Big|_{\text{extremal}} = 1/l_p$ always.

There is, to be sure, still a lot of room for string or quantum-gravity corrections to the thermodynamic functions. Both the chemical potential, $A_0(r_+)$, away from extremality, and the equation of state for, say, neutral holes $S(M, 0)$, are expected in general to receive such corrections. It is also conceivable that, like finite-size effects, string and/or quantum gravity corrections invalidate our thermodynamic treatment of the problem.

3. Near-extremal near-horizon limit

To discard quantum gravity effects we will now assume that l_p is vanishingly small compared to all other length scales in the problem. We will furthermore take the near-extremal limit, and

[1]This does not imply that the electromagnetic coupling must be minimal, since (2.11) may depend non-trivially on the field strength.

concentrate on the near-horizon geometry of the black hole,

$$l_p \ll \delta r \sim (r_+ - r_-) \ll r_+ \ . \qquad (3.1)$$

In order to analyze this limit, it is convenient to define the new coordinates

$$r \equiv l_p^2 M \ (1+u) \ \text{ and } \ t \equiv l_p^2 M \ \tilde{t} \ , \qquad (3.2)$$

in terms of which the metric reads

$$(l_p^2 M)^{-2} \ ds^2 \ = \ -\frac{f(u)}{(1+\frac{1}{u})^2} \ d\tilde{t}^2 + \left(1+\frac{1}{u}\right)^2 \times \times \left(\frac{du^2}{f(u)} + u^2 d\Omega_2^2\right) \ , \qquad (3.3)$$

where

$$f(u) = 1 - (\frac{u_0}{u})^2 \ \text{ and } \ r_\pm \equiv l_p^2 M \ (1 \pm u_0) \ . \ (3.4)$$

The outer and inner horizons are located in the new coordinates at $u = \pm u_0$, while the electric potential takes the form

$$A_0 \equiv (l_p^2 M)^{-1} \ \tilde{A}_0 = \frac{\sqrt{1-u_0^2}}{l_p(1+u)} \ . \qquad (3.5)$$

Finally the potential of a point probe can be worked out easily with the result

$$V(u) = m\frac{\sqrt{u^2-u_0^2}}{1+u} + \frac{q}{l_p}\frac{\sqrt{1-u_0^2}}{1+u} \ . \qquad (3.6)$$

Consider now the limit (3.1) which can be written equivalently as $(l_p M)^{-1} \ll u, u_0 \ll 1$. In this limit the metric simplifies to

$$(l_p^2 M)^{-2} \ ds^2 \ = \ -f(u)u^2 \ d\tilde{t}^2 + \frac{du^2}{f(u)u^2} + d\Omega_2^2 \ . \qquad (3.7)$$

The extremal case ($f = 1$) gives the $AdS_2 \times S^2$ space, also known as the Bertotti-Robertson universe. For finite u_0, on the other hand, one has a two-dimensional black hole embedded in this asymptotic geometry.[2] For the thermodynamic quantities, we obtain[3]

$$S = \pi Q^2 + 4\pi^2 Q^3 T l_p + \mathcal{O}(l_p^2) \qquad (3.8)$$

$$U = \frac{Q}{l_p} + \frac{Q^3}{2}(2\pi T)^2 \ l_p + 2Q^4(2\pi T)^3 l_p^2 + \mathcal{O}(l_p^3) \qquad (3.9)$$

$$\Phi = \frac{1}{l_p} - 2\pi T Q - 6\pi^2 Q^2 T^2 \ l_p + \mathcal{O}(l_p^2) \quad (3.10)$$

Note here that the leading contributions to U and Φ are singular. This will also be the case for Dp-branes.

The static potential of a point probe reads in this limit [16]

$$V(u) = \frac{q}{l_p} - \frac{q}{l_p}u + m\sqrt{u^2-u_0^2} \ . \qquad (3.11)$$

For a generic probe $m > q/l_p$, so that the potential grows linearly at the spatial infinity of AdS_2 space. For an extremal probe, on the other hand, the potential goes to a constant at infinity, and the potential difference $\delta V = V(\infty) - V(u_0) = \frac{q}{l_p}u_0$ is well defined. We can therefore use our thermodynamic argument to derive the expression for the chemical potential

$$\mu = \frac{1-u_0}{l_p} \ , \qquad (3.12)$$

in agreement with the near-extremal expansion of the electrostatic potential at the horizon $\mu = A_0(u_0)$. Note that, as with most other thermodynamic quantities, one has to keep the first subleading correction in the near-extremal expansion of μ, in order to find the leading temperature dependence.

A heuristic rephrasing of the main message of this section is as follows: a near-extremal black hole exerts no net force on an extremal probe at long distance. The potential energy, is thus converted to kinetic energy and eventually released as heat while the probe falls in the near-horizon geometry. To leading order in the extremality parameter one can therefore compute the chemical potential by ignoring the physics in the asymptotically-flat region.

4. Black Dp-branes

We consider now the background geometry (in the string frame) of a near-extremal black hole describing a number of coinciding Dp-branes [1]:

$$ds_{10}^2 = \frac{-f(r)dt^2 + d\vec{x}\cdot d\vec{x}}{\sqrt{H_p(r)}} + \sqrt{H_p(r)}\left(\frac{dr^2}{f(r)} + \right.$$

[2] This solution is different from the one discussed in [14].

[3] A similar result was obtained in [15].

$$+r^2 d\Omega_{8-p}^2) \quad (4.1)$$

$$H_p(r) = 1 + \frac{L^{7-p}}{r^{7-p}} \quad , \quad f(r) = 1 - \frac{r_0^{7-p}}{r^{7-p}} \quad (4.2)$$

The parameters L and r_0 determine the AdS throat size and the position of horizon, respectively. They are related to the ADM mass M and the (integer) Ramond-Ramond charge N in the following way:

$$M = \frac{\Omega_{8-p} V_p}{2\kappa_{10}^2} \left[(8-p) r_0^{7-p} + (7-p) L^{7-p} \right]$$

$$N = \frac{(7-p)\Omega_{8-p}}{2\kappa_{10}^2 T_p} L^{(7-p)/2} \sqrt{r_0^{7-p} + L^{7-p}} \quad (4.3)$$

where Ω_n is the volume of a unit n-dimensional sphere, and V_p is the common p-dimensional D-brane (flat) volume. The relations (4.3) involve the D-brane tension T_p and the 10-dimensional gravitational constant κ_{10}. The RR charge N is quantized, with each D-brane carrying a unit charge so that N is equal to the number of D-branes. Finally,

$$L^{7-p} = \sqrt{\left(\frac{2\kappa_{10}^2 T_p N}{(7-p)\Omega_{8-p}} \right)^2 + \frac{1}{4} r_0^{2(7-p)}} - \frac{1}{2} r_0^{7-p} \quad (4.4)$$

The RR charge is the source of the p-form field

$$C_{012\cdots p}(r) = \frac{2\kappa_{10}^2 T_p N}{\Omega_{8-p}(7-p)(r^{7-p} + L^{7-p})} =$$

$$= \sqrt{1 + \frac{r_0^{7-p}}{L^{7-p}}} \, \frac{H_p(r) - 1}{H_p(r)} \, . \quad (4.5)$$

All other components vanish, except in the case of $p = 3$, when the self-duality condition

$$F_{\mu_1 \cdots \mu_5} = \frac{1}{5! \sqrt{\det g}} \epsilon_{\mu_1 \cdots \mu_5 \nu_1 \cdots \nu_5} F^{\nu_1 \cdots \nu_5} \quad (4.6)$$

requires non-zero p-form components in the transverse directions. There is also a dilaton background (constant for $p = 3$): $e^\phi = H_p^{(3-p)/4}(r)$.

By using standard methods of black hole thermodynamics, it is straightforward to determine the Hawking temperature, entropy and chemical potential corresponding to the solution (4.1,4.5). They are respectively:

$$T = \frac{7-p}{4\pi} \frac{r_0^{(5-p)/2}}{\sqrt{r_0^{7-p} + L^{7-p}}} \quad (4.7)$$

$$\Phi = V_p T_p \frac{L^{(7-p)/2}}{\sqrt{r_0^{7-p} + L^{7-p}}} \quad (4.8)$$

$$S = \frac{4\pi \Omega_{8-p} V_p}{2\kappa_{10}^2} r_0^{(9-p)/2} \sqrt{r_0^{7-p} + L^{7-p}} \, , \quad (4.9)$$

It is easy to check that these quantities satisfy the thermodynamic identity$dU = TdS + \Phi dN$ with $U = M$.

We consider now a Dp-brane probing the above solution, with zero background values for all other fields. In this case, the D-brane probe action is

$$\Gamma_p = T_p \, e^{-\phi} \int \sqrt{\det \hat{g}} + T_p \int \hat{C} \quad (4.10)$$

where we have also set the world-volume $F_{\alpha\beta} = 0$. Using the solution above we obtain the static potential [17]

$$V(r) = V_p T_p \left[\frac{\sqrt{f(r)}}{H_p(r)} + C(r) \right] = V_p T_p \left[\frac{\sqrt{f(r)}}{H_p(r)} + \right.$$

$$\left. + \sqrt{1 + \frac{r_0^{7-p}}{L^{7-p}}} \frac{H_p(r) - 1}{H_p(r)} \right] \quad (4.11)$$

where $C(r) \equiv C_{012\cdots p}(r)$. Note that, like in the RN case, the horizon value $C(r_0)$ is equal to the chemical potential Φ. The values of the potential at infinity and at the horizon are, respectively, $V(\infty) = V_p T_p$ and $V(r_0) = \Phi$.

A Dp-brane probe is a BPS state with the mass $\Delta M = V_p T_p$ and charge $\Delta N = 1$. As it moves from infinity to the horizon, its potential energy changes by $\Delta E = V(r_0) - V(\infty)$, and the quantity of heat received by the black hole is again $dE = -\Delta E$. On the other hand, the black hole gains mass $dM = \Delta M = V_p T_p$ and charge $dN = \Delta N = 1$. This process is described by the equation

$$dE = V(\infty) - V(r_0) = dM - \Phi dN =$$

$$= dU - \Phi dN = TdS \quad (4.12)$$

which does indeed hold. As expected, the probe motion, governed by the background field action of eq.(4.10), is consistent with black hole thermodynamics and provides a similar partial equation of state as in the RN case. The argument concerning the absence of α' corrections here is more involved. Unlike the case of one-dimensional world-volumes, here the equations of motion do not set the accelerations (second fundamental forms) to be a function of velocities [11]. At this point we can argue that to order $\mathcal{O}(\alpha'^2)$ there are no corrections using the explicit results of [11]. It turns out that the second fundamental form enters in such a way that there are no extra corrections again on the horizon. Moreover the appearance of a non-zero five form is not expected to change the previous statement. Here we must also investigate the higher anomalous CP-odd couplings. The relevant case is that of D3 branes and the coupling

$$S_\theta = -T_3 \frac{(4\pi\alpha')^2}{48} \int a\,[p_1(\mathcal{T}) - p_1(\mathcal{N})] \quad (4.13)$$

where p_1 denotes the first Pontriagin class and $\mathcal{T},\mathcal{N}$ stand for the tangent and normal bundles to the brane respectively It can be shown that this vanishes for diagonal metrics with the required Poincaré symmetry. Thus, there is no correction to the heat up to order $\mathcal{O}(\alpha'^2)$ and we suspect that this is true to all orders. This would imply again that all corrections come from the corrected background fields.

5. D3-branes in the near-horizon limit

The case of D3-branes is particularly interesting because the world-volume action of N coinciding D-branes involves a four-dimensional $\mathcal{N}$=4 supersymmetric $SU(N)$ Yang-Mills theory. According to the Maldacena conjecture [3], the large N limit of this gauge theory is related to the near-horizon AdS geometry of the extremal ($r_0 = 0$) black D3-brane solution (4.1). Witten [6] has exploited the AdS/SYM correspondence in order to study the large N dynamics of non-supersymmetric SYM, with $\mathcal{N}$=4 supersymmetries broken by non-zero temperature effects. According to this proposal, the non-extremal solution (4.1) may be used to study SYM at T identified with the Hawking temperature (4.8) as long as $T \ll 1/L$, so that the metric remains near-extremal ($r_0 \ll L$). In the Maldacena limit, $\alpha' \equiv l_s^2 \to 0$ at $u \equiv r/\alpha'$ and T fixed, the solution (4.1) describes an AdS-Schwarzschild black hole [7]:

$$ds^2 = l_s^2 \Bigg[\frac{u^2}{R^2}(-f(u)dt^2 + d\vec{x}\cdot d\vec{x}) + R^2 \frac{du^2}{u^2 f(u)} + R^2 d\Omega_5^2 \Bigg] + \mathcal{O}(l_s^4) \ , \quad (5.1)$$

where

$$f(u) = 1 - \frac{u_0^4}{u^4} \ , \ \ R^4 \equiv 4\pi g_s N = \lambda \quad (5.2)$$

$$u_0 = \pi T R^2 \, , \quad (5.3)$$

where λ is the t'Hooft coupling. The limiting value of the four-form (4.5) is

$$C_{0123} = 1 + l_s^4 \left(\frac{(\pi T R)^4}{2} - \frac{u^4}{R^4} \right) + \mathcal{O}(l_s^8), \quad (5.4)$$

As in the near-horizon limit of the RN black-hole, here also the gauge field diverges at the boundary of AdS_5, $u \to \infty$.

A D3-brane probe in the bulk of the AdS space corresponding to N background D3-branes can be thought of as a realization of $SU(N+1)$ gauge theory in the $SU(N) \times U(1)$ symmetric Higgs phase. In the following, we examine some aspects of the probe dynamics and thermodynamics in the near-horizon limit, in order to show that it is well-defined and it leads to sensible results also in the Higgs phase. To that end, we will use the following expansions in the string length scale l_s:

$$L^4 = R^4 l_s^4 \left(1 - \frac{1}{2}\pi^4 R^4 T^4 l_s^4 \right) + \mathcal{O}(l_s^{12}) \quad (5.5)$$

$$r_0 = \pi T R^2 l_s^2 \left(1 + \frac{1}{4}\pi^4 T^4 R^4 l_s^4 + \mathcal{O}(l_s^8) \right) \quad (5.6)$$

which follow from relations written in the previous section. Similarly,

$$M = N V_3 T_3 + \frac{3}{8}\pi^2 V_3 N^2 T^4 + \mathcal{O}(l_s^4) \quad (5.7)$$

$$S = \frac{1}{2}\pi^2 V_3 N^2 T^3 + \mathcal{O}(l_s^4) \quad (5.8)$$

$$\Phi = V_3 T_3 - \frac{1}{4}\pi^2 V_3 N T^4 + \mathcal{O}(l_s^4) \quad (5.9)$$

Note that $T_3 \sim 1/l_s^4 \to \infty$. As pointed out before in ref.[9], the limiting entropy (5.8) is 3/4 of the corresponding quantity in the weakly coupled $SU(N)$ SYM.

Taking the limit in the static potential (4.11), we obtain[4]

$$V(u) = V_3 T_3 \Bigg\{ 1 + l_s^4 \frac{u^4}{R^4} \Bigg[\sqrt{1 - \left(\frac{\pi T R^2}{u}\right)^4} - 1 +$$

[4]We disagree with the potential obtained in the near-horizon limit in [18].

$$+\frac{1}{2}\left(\frac{\pi T R^2}{u}\right)^4\Bigg] + \mathcal{O}(l_s^8)\Bigg\} \qquad (5.10)$$

so that the interaction energy is

$$V^{\rm int}(u) = V(u) - V_3 T_3 = \frac{V_3}{(2\pi)^3 g_s}\frac{u^4}{R^4}\times$$

$$\times\left[\sqrt{1-\left(\frac{\pi T R^2}{u}\right)^4} - 1 + \frac{1}{2}\left(\frac{\pi T R^2}{u}\right)^4\right] + \mathcal{O}(l_s^4) \qquad (5.11)$$

and has a smooth limit as $l_s \to 0$. Since the probe is BPS, $V^{\rm int}(\infty) = 0$ as in the RN case.

The thermodynamic argument is still valid. We let the probe fall until it reaches the horizon. The heat supplied by the black hole to the probe is

$$dE = V(\infty) - V(u_0) = V_3 T_3 - V(u_0) =$$

$$= \Delta M - \Phi \Delta N = \frac{1}{4}\pi^2 V_3 N T^4 \qquad (5.12)$$

where $\Delta N = 1$. On the other hand, taking the limit of the equations of the previous section we obtain that the black hole cools down by $dT = -T/(2N)$ which is sufficient to prove directly that $dE = TdS$, with the entropy given by eq.(5.8). We thus find as before consistent thermodynamics, when we allow the probe brane to move from the boundary of AdS space to the horizon. This (gravitational) equality corresponds to the qualitative expectation that an ultraviolet fluctuation in N=4 SYM spreads until it reaches the size of the thermal wavelength and thus thermalizes [10].

Acknowledgments

We would like to thank C. Bachas for his collaboration and fruitful discussions. E. Kiritsis would like to thank the Ecole Polytechnique for hospitality and support while parts of this work was done. He is also grateful to the SISSA node for financially supporting his participation to the TMR midterm meeting. The work of T.R. Taylor was done while he was visiting Laboratoire de Physique Théorique et Hautes Energies at Université de Paris Sud, Orsay. He is grateful to Pierre Binétruy and all members of LPTHE for their kind hospitality and support.

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