

String Gravity and Cosmology: Some new ideas *

Elias Kiritsis and Costas Kounnas[†]

*Theory Division, CERN,
CH-1211, Geneva 23, SWITZERLAND*

ABSTRACT

String theory provides the only consistent framework so far that unifies all interactions including gravity. We discuss gravity and cosmology in string theory. Conventional notions from general relativity like geometry, topology etc. are well defined only as low energy approximations in string theory. At small distances physics deviates from the field theoretic intuition. We present several examples of purely stringy phenomena which imply that the physics at strong curvatures can be quite different from what one might expect from field theory. They indicate new possibilities in the context of quantum cosmology.

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[†]On leave from Ecole Normale Supérieure, 24 rue Lhomond, F-75231, Paris, Cedex 05, FRANCE.

1 Introduction

It is no accident that (super)string theory attracted the attention of theorists during the last decade (for a review see [1]). It is the only theory we have so far, that contains a consistent theory of quantum gravity, a feat impossible to reproduce using conventional quantum field theory.

String theory is based on the idea that the elementary building blocks of matter, instead of being point-like particles (with local interactions), are strings (one dimensional objects, either closed or open*). Conventional particles are identified with the eigenmodes of the string. Thus, the string vibrating in two different modes, corresponds to two distinct particles. Unlike field theory, there is automatically a mass scale inherent in string theory, namely the tension of the string. Since the theory always turns out to contain a graviton, it is natural to identify this scale with the Plank mass.

There are several attractive features of the theory. We list below some of them.

- String theory is a finite theory at short distances. This is due to the fact that string interactions are not localized at a point.

- At large distances (much bigger than the Plank length $\sim 10^{-33}$ cm) strings look like point-like objects and an ordinary particle description is valid. Gauge symmetries and gravity appear naturally at low energy.

- A consequence of the above is that string theory contains a consistent (UV-finite) theory of gravity. Moreover, it automatically unifies all interactions: gravitational, gauge and Yukawa. This, so far, is not possible to achieve in the context of field theory.

- In order that string theory contains fermions, some form of (broken) spacetime supersymmetry seems to be needed. This is nice since we know that (spontaneously broken) supersymmetry can help with hierarchy type of problems.

The most useful formulation of the theory so far is the first quantized formulation. Although there have been attempts to construct string field theory, the problem is far from solved. This explains our partial knowledge of the full symmetry of the theory.

The first quantized formulation of field theory as developed by Dirac and Feynman rests on representing field theory amplitudes as a sum over paths of point particles

$$\langle x|x' \rangle \sim \int_0^T d\tau \int_{x(0)=x}^{x(T)=x'} [Dx(\tau)] e^{iS[x(\tau)]} \quad (1.1)$$

with

$$S[x(\tau)] = \int_0^T d\tau [G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + A_\mu \dot{x}^\mu + \dots] \quad (1.2)$$

where τ parametrizes the path, $G_{\mu\nu}$ is the metric, A_μ a gauge field and the dots stand for other interactions. The action here is that of a one-dimensional field theory defined over the path of the particle.

*From now on, for concreteness we will consider closed strings.

In string theory, one can write a similar formula for the amplitude for string propagation. A closed string, when it propagates (classically) sweeps a two-dimensional cylinder, (called the world-sheet) and the amplitude now is a two-dimensional generalization of the field theory formula (1.1). A closed string is a ring whose position is described by $x^\mu(\sigma)$, where $0 \leq \sigma \leq 1$ parametrizes the ring. Then,

$$\langle x(\sigma) | x'(\sigma) \rangle \sim \int [Dx(\sigma, \tau)] e^{iS[x(\sigma, \tau)]} \quad (1.3)$$

$$S[x(\sigma, \tau)] = \frac{1}{4\pi\alpha'} \int d\sigma d\tau [G_{\mu\nu}(\dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu) + B_{\mu\nu}(\dot{x}^\mu x'^\nu - (\mu \rightarrow \nu)) + \dots] \quad (1.4)$$

where dot stands for derivative with respect to σ , $G_{\mu\nu}$ is the metric, $B_{\mu\nu}$ is an antisymmetric tensor, and the dots contain interactions with other massless fields (gauge fields etc.)

Perturbative string theory contains two parameters.

- The first is the string tension α' , which appears in the action of the two-dimensional σ -model (1.4). It has dimensions of inverse mass-squared. It sets the length (or mass) scale of the theory. Note also that it is the coupling constant of the world-sheet σ -model (1.3,1.4) which describes string propagation. For small values of α' the σ -model is semiclassical.

- The second parameter is the string coupling constant g_{string} , which is dimensionless, and governs the strength of string interactions. It is the loop-expansion parameter of perturbative string theory.

Thus, α' , the σ -model coupling constant, controls *stringy* effects, in the sense that when $\alpha' \rightarrow 0$ the theory becomes equivalent to a field theory. On the other hand, g_{string} , the string theory coupling constant, controls *quantum* effects (when $g_{\text{string}} \rightarrow 0$ the theory is classical).

At tree level, the Plank mass (or Newton's constant) is given in terms of these two parameters as

$$M_{\text{Planck}}^2 \sim \frac{1}{g_{\text{string}}^2 \alpha'} \quad (1.5)$$

Another interesting feature of (super)string theory is that, in principle, the dimension of spacetime can be any integer between zero and ten. This gives the possibility that the theory determines dynamically the dimension of spacetime to be four, although we do not understand the mechanism so far. All ground states with a four-dimensional spacetime contain some universal fields: The metric (graviton) $G_{\mu\nu}$, the antisymmetric tensor[†] $B_{\mu\nu}$ and a scalar field Φ , the dilaton. It is interesting that the string coupling g_{string} is related to the expectation value of the dilaton as

$$g_{\text{string}} = \langle e^\Phi \rangle \quad (1.6)$$

which indicates that the coupling constant of string theory, although undetermined in perturbation theory, could be determined by non-perturbative effects.

Since particles are in correspondence with the eigenmodes of the string, it is obvious, that string theory describes the interactions of an infinite number of particles. Some

[†]In four dimensions $B_{\mu\nu}$ is equivalent to a pseudoscalar, usually called the axion.

of them are massless. We mentioned already, that a flat four-dimensional ground state contains the “universal excitations”, $G_{\mu\nu}, B_{\mu\nu}, \Phi$ as well as gauge fields, fermions, and scalars whose quantum numbers and detailed interactions depend on the ground state. The theory also contains towers of massive states, most of them having masses of the order of, or bigger than the Plank mass.

The classical equations of motion of string theory turn out to be equivalent with the conformal invariance of the two-dimensional σ -model (1.3,1.4). Conformal invariance translates into the vanishing of the β -functions of the σ -model. For small α' , one can derive the σ -model one-loop β -function equations and show that they come from the following spacetime action[‡]

$$S_{st} = M_{\text{Plank}}^2 \int \sqrt{G} e^{-2\Phi} \left[R + 4(\nabla\Phi)^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \dots + \mathcal{O}(\alpha') \right] \quad (1.7)$$

with

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{cyclic permutations} \quad (1.8)$$

We have displayed only the bosonic universal fields, $G_{\mu\nu}, B_{\mu\nu}, \Phi$ (which we will call the gravitational sector). The dots stand for the rest of the fields. As indicated, there are higher order in α' (stringy) corrections with more than two derivatives. There are, for example, $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$ terms.

This two-derivative effective action is valid when all relevant length scales are much smaller than the string scale $\sim 1/\alpha'$. In particular, when curvatures become of the order of the string scale, the α' -corrections (neglected here) are large and we cannot trust (1.7). The natural question to address in this context is: Can we handle strong curvatures? Or, can we sum-up the α' -expansion?

The answer to this question is given by the concept of Conformal Field Theory. It is a formalism which is exact (non-perturbatively) in α' , and thus contains all (perturbative) stringy effects. It should be thought of as an infinite dimensional analogue of fields like the metric, the antisymmetric tensor, gauge and scalar fields etc. It also provides a handle on the perturbative symmetries of string theory.

Since string theory contains a massless graviton, we certainly have diffeomorphism invariance. Massless gauge fields indicate the presence of gauge invariance.

Is the symmetry of string theory the direct product of such field-theoretic symmetries?

Although we do not know the symmetries of string theory in their full glory, we certainly know that the answer to the previous question is *negative*. We can find extra symmetries that go beyond field theory. Such symmetries come under the name of *duality* symmetries. We have examples where two *different* effective field theory solutions correspond to the same string theory solution. Duality symmetries act in a non-perturbative way on the α' -expansion. In this respect, they are stringy symmetries. What accounts for the difference here is that our probes (strings) are extended objects.

[‡] To be distinguished from the world-sheet σ -model action (1.4).

It turns out that concepts like geometry, topology, gauge symmetry, dimension of space etc. are low energy approximations in string theory. In view of this we can ask: can every classical solution to string theory be described completely in terms of the standard fields, like the metric $G_{\mu\nu}$ etc.? The answer turns out again to be *negative*. We know of solutions which have semiclassical regions where geometry is well defined, as well as regions that are “fuzzy” (no geometric description). We also know of solutions where there is no semiclassical region at all.

When do we have a good geometrical description? This exists only if solutions contain parameters that can be varied in such a way that relevant scales (volume, curvature) become much larger than the string scale.

2 A Simple Example: String Theory on a Circle

We will consider here a closed string moving on a manifold that contains a circle of radius R . The part of the two-dimensional σ -model that describes the circle has the following action:

$$S_{\text{circle}} = \frac{R^2}{4\pi\alpha'} \int d^2\xi \partial_\mu\phi\partial^\mu\phi \quad (2.1)$$

where we have explicitly displayed the radius dependence of the action and ϕ is a two dimensional field that takes values in $[0, 2\pi]$. Note that R has dimensions of length since the angle ϕ is dimensionless.

In the field theory case, we know that for a particle moving on a circle the momentum is quantized

$$p = \frac{m}{R} \quad (2.2)$$

where m is an integer. In this case if an internal dimension is such a circle the 4-D mass of such momentum excitations has a term proportional to p^2 (in the Kaluza-Klein framework). Thus, in field theory

$$M^2 = \frac{m^2}{R^2} + \dots \quad (2.3)$$

where the dots summarize other contributions. Note that p^2 is the spectrum of the Laplacian operator $\frac{1}{R^2} \frac{\partial^2}{\partial\phi^2}$ on the circle. Knowing the spectrum of masses (or the Laplacian) we can reconstruct the manifold. Thus here geometry alone determines the spectrum and vice versa. In particular by measuring the low lying spectrum we can measure the size (radius) of the circle.

In the string case, there are excitations that are not of the momentum type. The string can also wrap (several times) around the circle. The “energy” of such winding excitations depends on the string tension α' and the total length of the winding string in a very simple (linear) fashion:

$$E_{\text{winding}} = n \frac{R}{\alpha'} \quad (2.4)$$

where n is the winding number (an integer) and $2\pi nR$ is the total length of the string. Such excitations give additional contributions to the 4-D masses and the full formula is

$$M_{\text{string}}^2 = \frac{m^2}{R^2} + \frac{n^2 R^2}{\alpha'^2} + \dots \quad (2.5)$$

Imagine that we have a circle of large radius, $R^2 \gg \alpha'$. Then, from (2.5), we observe that the low lying spectrum is given solely in terms of momentum (field theory-like) excitations. The states with non-zero winding numbers are comparatively very massive, namely of the order of the Plank mass. Thus an observation of the low energy spectrum is described by geometry and we will measure the (large) value of the radius.

Consider however, the case where $R^2 \sim \alpha'$. Then the low lying spectrum contains both winding and momentum excitations, and in fact is not described by any (one-dimensional) geometry. In this region, the geometric description breaks down.

Let us look now at the opposite limit, $R^2 \ll \alpha'$. Now the low energy spectrum is composed solely in terms of the winding excitations and low lying masses are

$$M_{\text{low-lying}}^2 \simeq \frac{n^2}{(\sqrt{\alpha'}/R)^2} \quad (2.6)$$

Now this spectrum is similar to the field theory spectrum (2.3), and there is again a geometric description in terms of a circle. However, by comparing (2.3) and (2.6) we will measure an effective radius

$$\tilde{R} = \frac{\sqrt{\alpha'}}{R} \quad (2.7)$$

which is obviously different from R . We can thus conclude that

$$R_{\text{eff}} \geq \sqrt{\alpha'} \quad (2.8)$$

and that there is an effective *minimum size* for the manifold in string theory. This minimum length, $\sqrt{\alpha'}$, is of the order of the Plank length.

The above discussion follows from the observation that there is a symmetry in the stringy spectrum (2.5) of the circle, namely

$$R \rightarrow \frac{\sqrt{\alpha'}}{R} \quad (2.9)$$

and a simultaneous exchange of winding and momentum excitations. Such a symmetry is known as “target space duality”. Another way to state the effect of this symmetry is: the two σ -models associated to two *distinct* geometries, namely a circle of radius R and a circle of radius $\sqrt{\alpha'}/R$ correspond to the *same* Conformal Field Theory, and thus to the same classical string solution. Target space duality symmetry is particular to the theory of strings but not to field theory, since the necessary ingredient, namely winding modes do not exist in field theory*. It is obvious from (2.9) that target space duality is

*Sometimes field theories have effective excitations which are stringy, for example Nielsen-Olesen vortices. In such cases one could in principle have such a behavior.

not a symmetry order by order in α' (the σ -model coupling constant). We need the exact solution of the theory to see the symmetry. Thus target space duality is a non-perturbative symmetry of the σ -model.

Another relevant observation concerns the self-dual radius $R = \sqrt{\alpha'}$. At this point, the symmetry of the σ -model is enhanced from $U(1)_L \times U(1)_R$ to $SU(2)_L \times SU(2)_R$. Chiral σ -model symmetries, are associated to gauge symmetries in string theory. Thus the theory with $R = \sqrt{\alpha'}$ has a $SU(2) \times SU(2)$ unbroken gauge symmetry. When R moves away from this value, the gauge symmetry is broken to $U(1) \times U(1)$. In this respect, R is like the expectation value of a Higgs field.

Although we indicated the simplest example of such a symmetry [2], it is more general, [3]-[6]. It can be shown to be an exact symmetry order by order in string perturbation theory [7]. In general, there are two parts of the spectrum which we will continue to call momentum and winding modes, which are interchanged by target space duality. The effective geometry seen by the momentum modes is in general different from the one seen by the winding modes (the dual geometry).

3 Implications for Effective Theories of Strings

The radius of the circle of the previous example was taken to be a (fixed) constant. In string theory however it can vary, and it really corresponds to a field. Let us consider a compactification of string theory, where part of the internal space is a circle with radius $R^2 \gg \alpha'$. As mentioned before, the low energy spectrum is composed of momentum modes only. Thus when we derive the low energy effective field theory we must integrate out the heavy (winding) modes.

Imagine now following this effective field theory, as the radius becomes smaller. It is obvious that when we reach the region with $R^2 \sim \alpha'$ we will encounter some strange behavior, namely non-unitary and/or singularities. The reason is that at this region we have integrated out fields that have comparable mass to the ones we kept, and this is inconsistent. There is no singularity in the full string spectrum, just our approximation broke down.

Such a change of R can happen in a cosmological context. There are exact solutions in string theory where R changes with time [8]. The functional behavior can have the forms

$$R(t) \sim t \quad , \quad R(r) \sim 1/t \quad , \quad R(t) \sim \tan(at) \tag{3.1}$$

$$R(t) \sim \coth(at) \quad , \quad R(t) \sim \tanh(at) \tag{3.2}$$

Thus we have an example of a cosmological situation where a single effective field theory is not enough to describe the entire evolution of the universe.

4 Topology Change and Black Hole Singularities

As we mentioned earlier not all string solutions have a conventional geometrical interpretation. Here we will examine situations in which the geometry of 3-space changes as a function of time. Of particular interest are situations in which at early times curvatures are weak in which there is a well defined geometry, at intermediate times curvature gets strong and thus the geometrical interpretation is strong, and at late times the universe flattens again so that geometry is again well defined but can be quite different from the original one. In such cases 3-d topology can change.

In general relativity, topology cannot change without going through a singularity. We will see that in string theory this can happen smoothly. If however we follow the evolution through the effective field theory we will encounter singularities in the region of strong curvatures. As argued however in the previous section this only indicates the breakdown of the effective field theory. There is no singularity in the context of the full string theory. Solutions exhibiting smooth topology change in string theory when some parameters are varied have been described in [9],[10]. Exact solutions for time-dependent topology change have been described in [8]. We will describe here such an example.

At time $t = 0$ the 3-space has the topology of a line times a disk. The disk is not flat and its metric is

$$ds^2 = dr^2 + \tanh^2 r d\theta^2 \quad (4.1)$$

The space evolves at intermediate time to arrive at $t = \infty$ to a space with the topology of a line times a cylinder with metric

$$ds^2 = dr^2 + \coth^2 r d\theta^2 \quad (4.2)$$

Although this metric looks singular, the associated string theory is regular.

This brings us to the other interesting question, which is concerned with the nature of singularities in string theory.

There are exact solutions in string theory which from the naive (effective field theory) point of view have point-like singularities and associated horizons. In fact such apparent singularities in string theory can be worse in the sense that we can have solutions with singularities on higher dimensional hypersurfaces. As we argued above, close to the singularity the curvature is large so the effective field theory breaks down. Can we use tools like duality to say something about what is the nature of such black holes in string theory? We will show first examples of euclidean manifolds which although geometrically singular are absolutely regular in string theory.

The first example is what is known as the $SU(2)/U(1)$ space. Its metric and dilaton are

$$ds^2 = d\beta^2 + \tan^2 \beta d\alpha^2, \quad \Phi = \log(1 + \cos 2\beta) \quad (4.3)$$

where $\alpha, \beta \in [0, 2\pi]$. The curvature and dilaton are singular at $\beta = \pi$. However the σ -model associated with this space can be solved exactly and all amplitudes are regular.

The next example is given by the metric (4.2) along with the dilaton $\Phi = \log(1 - \cosh 2r)$. Here also the curvature and dilaton are singular at $r = 0$. Although we cannot solve this model exactly it can be shown [10] that it is dual to the one with metric (4.1) and dilaton $\Phi = \log(1 + \cosh 2r)$ which is perfectly regular. This duality is relevant since analytic continuation of these metrics gives an exact solution to string theory where the four space has two flat directions while the other two have a metric (in Kruskal coordinates) and dilaton given by [11],

$$ds^2 = \frac{du dv}{1 - uv} \quad , \quad \Phi = \log(1 - uv) \quad (4.4)$$

The Penrose diagram for this black hole is similar to the standard Schwarzschild black hole. The duality between the Euclidean spaces we described above has some peculiar consequences for this black hole. It interchanges in particular the horizon and the singularity, [12]. This implies that the physics here is quite different from a field theoretic black hole.

Using our previous experience with the effects of duality we can speculate about the physics of such a black hole. An example of how this type of symmetry can affect string propagation, can be given (heuristically) as follows [13]. Consider a string background which is singular (semiclassically) in a certain region. In the asymptotic region, (which is obtained by some spacetime-dependent radius becoming very large), one has quantum numbers for asymptotic states that correspond roughly to windings and momenta. Momentum states are the only low energy states in this region. An experimenter sitting at the asymptotic region, far away from the black hole would like to probe its nature. He can do this using the low lying (momentum) modes available to him. Consider a momentum mode travelling towards the high curvature region. Its effective mass starts growing as it approaches large curvatures. At some point it becomes energetically possible for it to decay to winding states which, in this region, start having effective masses that are lower than momentum modes. In such backgrounds (unlike flat ones) winding and momentum are not separately conserved so that such a transition is possible. The reason for this is that there is a non-trivial dilaton field and thus, winding and momentum conservation is broken by the screening operators which transfer it to discrete states localized at the high curvature region.

In fact it is a general property of string theory that classical singularities have always associated with them states localized at the singularity. These can be interpreted as internal states of the (would be) singularity. A useful picture here is that of the hydrogen atom where the localized states are the bound states, while the scattering states above form the continuum. Thus we could say that particles interact with such localized states losing momentum (in discrete steps) and gaining winding number.

Once such a momentum to winding mode transition happens in the strongly curved region, the winding state sees a different geometry, namely the dual one and thus continues to propagate further into the strong curvature region since it feels only the (weak) dual curvature.

5 On Cosmological Singularities

We believe today that our universe underwent a big bang and continued expanding thereafter. We certainly do not trust Einstein's equations beyond a time of the order of the Plank scale. If we naively extrapolate however we will find a (time-like) singularity at $t=0$ where the universe squeezes at zero volume.

Using the duality ideas we can analyze a similar situation in the context of string cosmology.

Let us consider an expanding universe in string theory. For simplicity we will consider the spatial slice to be a three torus with a time dependent volume. A solution of this kind (to lowest order in α') is given by [14]

$$ds^2 = -dt^2 + \sum_{i=1}^3 a_i^2 t^{2b_i} d\sigma_i^2 \quad (5.1)$$

$$B_{\mu\nu} = 0 \quad , \quad \Phi = \frac{1}{2} \left(\sum_{i=1}^3 b_i - 1 \right) \log t \quad , \quad \sum_{i=1}^3 b_i^2 = 1 \quad (5.2)$$

where $\sigma_i \in [0, 2\pi]$ and we can choose $b_i > 0$ so the universe is expanding. As $t \rightarrow 0^+$ the universe is collapsing to zero volume, and there is a curvature singularity there.* We can also make the solution isotropic by choosing $a_i = a$ and $b_i = 1/\sqrt{3}$.

$$ds_{iso}^2 = -dt^2 + a^2 t^{\frac{2}{\sqrt{3}}} \sum_{i=1}^3 d\sigma_i^2 \quad (5.3)$$

$$\Phi = \frac{\sqrt{3} - 1}{2} \log t \quad (5.4)$$

In this example space is a torus with a radius that changes with time. At late times the radius is large and geometry is well defined with the low lying states being the momentum states. When however the volume becomes of the order of the string length, $t \sim a^{-\sqrt{3}}$, the geometrical picture breaks down since winding states have energies comparable to the momentum states. If we continued naively to $T \rightarrow 0$ we would think that the universe shrinks to zero volume, with a curvature singularity.

The correct approach however is that for $t < a^{-\sqrt{3}}$ to use the metric seen by the winding modes that now dominate the low energy spectrum.

$$d\tilde{s}_{iso}^2 = -dt^2 + a^{-2} t^{-\frac{2}{\sqrt{3}}} \left(\sum_{i=1}^3 d\sigma_i^2 \right) \quad (5.5)$$

$$\Phi = -\frac{\sqrt{3} + 1}{2} \log t \quad (5.6)$$

For these modes the universe is expanding as $t \rightarrow 0$. Thus the correct picture is the following: As we go back in time the universe shrinks until it becomes of Plank size at which point it starts re-expanding again. This idea is central in pre-big bang type of cosmologies in the context of string theory [15].

*There are solutions exact to all orders in α' with a similar behavior, but we chose one with the simplest interpretation.

6 Conclusions

We have presented some ideas on how physics concerning gravity, the structure of spacetime (black holes and cosmological singularities being the focus) can be quite different in string theory compared to point-particle field theory. We noted in particular the role played by stringy symmetries, known as dualities in establishing this non-field theoretic behavior.

However it should be obvious that we are in the beginning of a long road towards establishing string theory (or maybe a variant thereof) as the theory that describes nature. The low energy properties of string theory (in the matter sector) are now a subject of active investigation and we hope to be able soon to tell whether some ground state of string theory, looks like the standard model at low energy and agrees with the experimental data.

The gravitational sector gives another window both to test the theory, but also indicates the existence of a host of new phenomena (some of them described in this talk) which might indicate that Nature is always richer than we think.

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