The horizon problem

By writing the FRW metric in conformal coordinates it is easy to calculate the particle horizon: the maximal linear scale inside which communication is possible till today.

The comoving particle horizon is

$$r_H = \int_0^t \frac{dt}{a(t)}$$

while the physical particle horizon is $a(t)$ times larger.

In a flat universe, $r_H = t$ as expected. This horizon is finite if the universe expansion is driven by matter with $w > -1/3$ (which is the case till today).

At late times the integral is dominated by the $w = 0$ matter and the physical horizon today is

$$D_H = a_0 \; r_H \simeq \frac{6000}{\sqrt{\Omega z}} \; h^{-1} \; Mpc$$

The horizon at last scattering ($z \sim 10^3$) was only $\sim 100 MPc$ in size. This corresponds to patches in the sky with angular size

$$\theta_{\text{horizon}} = \frac{D_H}{r_a} \simeq 2^o \sqrt{\Omega \frac{10^3}{z}}$$

The sky is a collection of patches that are not causaly connected but CMBR is homogeneous to a high degree. Why? This is the **horizon problem.**

The flatness problem

Today $\Omega \simeq 1$. However $\Omega = 1$ is unstable:

$$1 - \frac{1}{\Omega(z)} = f(z)\left[1 - \frac{1}{\Omega}\right]$$

where $f = (1+z)^{-1}$ for matter domination and $f = (1+z_{eq})(1+z)^{-2}$ for radiation domination. Thus we need fine tuning in the past. Using $1+z = T_{init}/2.7\ K$ and $1+z_{eq} \simeq 10^4$ we find

$$|\Omega_{init} - 1| \simeq 10^{-22}\left(\frac{GeV}{E_{init}}\right)^2$$

At the Plack epoch $E_{init} \sim 10^{19}$ GeV and we need a **fine tuning** 1 in 10^{60}. This is the **flatness problem**.

This can be satisfied if $k = 0$

The antimatter problem

When $kT \geq m_p$ protons, antiprotons, and photons are in equilibrium.

Today $N_p/N_\gamma \simeq 10^{-9}$ and $N_{\bar{p}} \simeq 0$.

Conservation of baryon number would imply $N_p/N_{\bar{p}} \simeq 1$. Where did the **initial asymmetry** come from?

$\boxed{\text{The expansion problem}}$

Why the universe is expanding? Why not contracting?

$\boxed{\text{The structure problem}}$

What generated the fluctuations that were subsequently magnified by gravity to eventually form clusters and galaxies?

$\boxed{\text{The monopole problem}}$

During the GUT phase transition various topological defects (monopoles, strings etc) are produced copiously. Such large numbers should have been already detected but are not observed today. Why?

The Inflation idea

The expansion rate of the universe may have been much faster in the past.

In order for solving the causality (horizon) problem we must have had for a period $a(t) \sim t^a$ with $a > 1$ so that the expansion is superluminal.

The phase of accelerated expansion is the **inflationary universe**.

An easy way to achieve this is to assume that for a period the universe is dominated by a cosmological constant. Then the equations have as solution:

$$a \sim \begin{cases} \sinh Ht & k = -1 \\ \cosh Ht & k = +1 \\ \exp[Ht] & k = 0 \end{cases}$$

If there is a total expansion factor f during this period, the final Ω will be $\Omega = 1 + \mathcal{O} f^{-2}$. We can solve the flatness problem if $\log f \geq 60$.

First prediction of inflation

$$\boxed{k = 0 \quad \Leftrightarrow \quad \Omega = 1}$$

Inflation

Reheating

Inflation needs to **stop** after some time. A cosmological phase transition is needed. During the period that the vaccuum energy dominates, any energy is redshifted (diluted) away. Its temperature redshifts to zero. At the end of inflation the vaccuum energy must be transformed into normal matter and thus will generate a "hot" plasma with temperature equal to the initial temperature just before inflation.

Inflation generates **quantum fluctuations**.

De Sitter space has an event horizon: the comoving distance that particles can travel between t_0 and $t = \infty$ is finite:

$$r_{dS} = \int_{t_0}^{\infty} \frac{dt}{a(t)} = \text{finite}$$

The exponential expansion makes distant regions recede faster than light. Points separated by a distance $\geq H^{-1}$ can never communicate.

Like in the Black hole case, there is thermal radiation (Carter) produced during the expansion. These quantum fluctuations become the seeds for the subsequent generation of structure (galaxies, clusters etc.)

These fluctuations afect **all** fields (incuding gravitons).

Inflaton Dynamics

A quantum field can generate an energy density that mimics a cosmological constant.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\ \mathcal{L}$$

from which we can read the energy and pressure (assume $\vec{\nabla}\phi = 0$)

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi) \quad , \quad p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

Thus, if $\dot{\phi} = 0$ $\rho = -p$.

Take ϕ to be complex with $V(\phi) = -\mu^2|\phi|^2 + \lambda|\phi|^4$. In a heat bath there are temperature dependent quantum corrections to the potential:

$\Delta V = aT^2|\phi|^2$ so that $\mu^2_{eff} = \mu^2 - aT^2$.

At high T, the potential has a single minimum and no symmetry breaking (symmetric phase).

At $T \simeq 0$ the potential has two minima and the $\phi \rightarrow e^{ia}\phi$ symmetry is spontaneously broken (broken phase).

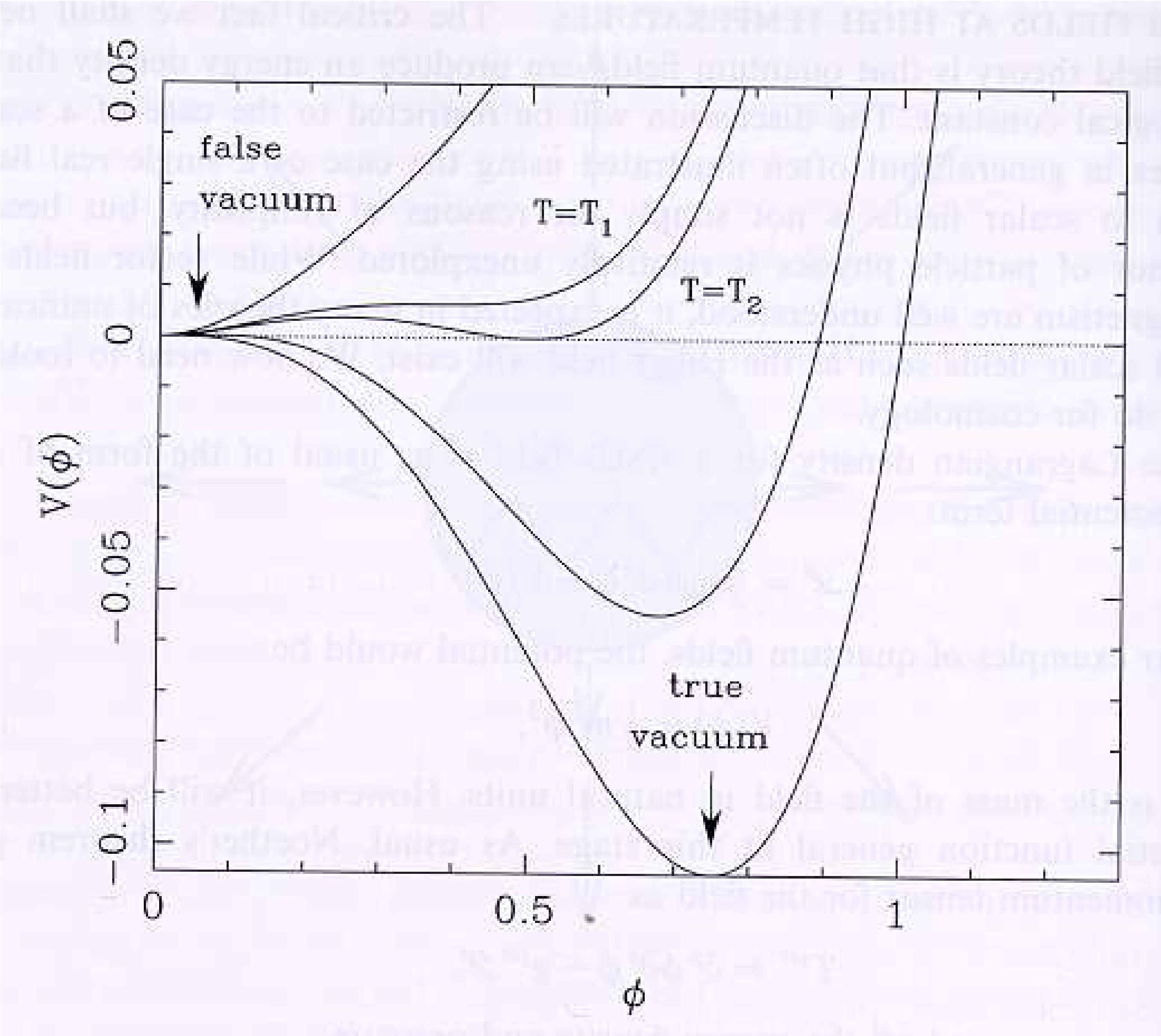
There is a **phase transition** separating the two phases at $T_{crit} \sim \mu/\sqrt{a}$.

The energy difference between the two vaccua is

$$\delta V = \frac{\mu^4}{2\lambda}$$

For a GUT scale phase transition mass$\sim 10^{15} GeV$ so that $\rho_{vac} \sim 10^{80}\ kgr\ m^{-3}$

Effective Potential



The temperature-dependent effective potential $T^2|\phi|^2 - |\phi|^3| + |\phi|^4$ illustrated at several temperatures $T^2 = 1/2, 9/32, 1/4, 1/10, 0$. For $T > T_1 = sqrt9/32$ only the false vacuum is available. For $T < T_2 = 1/2$ the true vacuum is energetically favored and the potential approaches the zero-temperature form.

Inflaton Dynamics

Classical Equation of motion

$$\ddot{\phi} + 3H(t)\dot{\phi} - \nabla^2\phi + V' = 0$$

We can solve this equation ignoring spatial variations and making the "slow roll" approximation: $\ddot{\phi}, |\nabla\phi| << H\dot{\phi}, V'$. and the equation becomes

$$3H(t)\dot{\phi} = -V'$$

Conditions for inflation

$$\epsilon \equiv \frac{m_P^2}{16\pi}\left(\frac{V'}{V}\right)^2 << 1$$

(slow roll)

$$\eta \equiv \frac{m_P^2}{8\pi}\frac{V''}{V} << 1$$

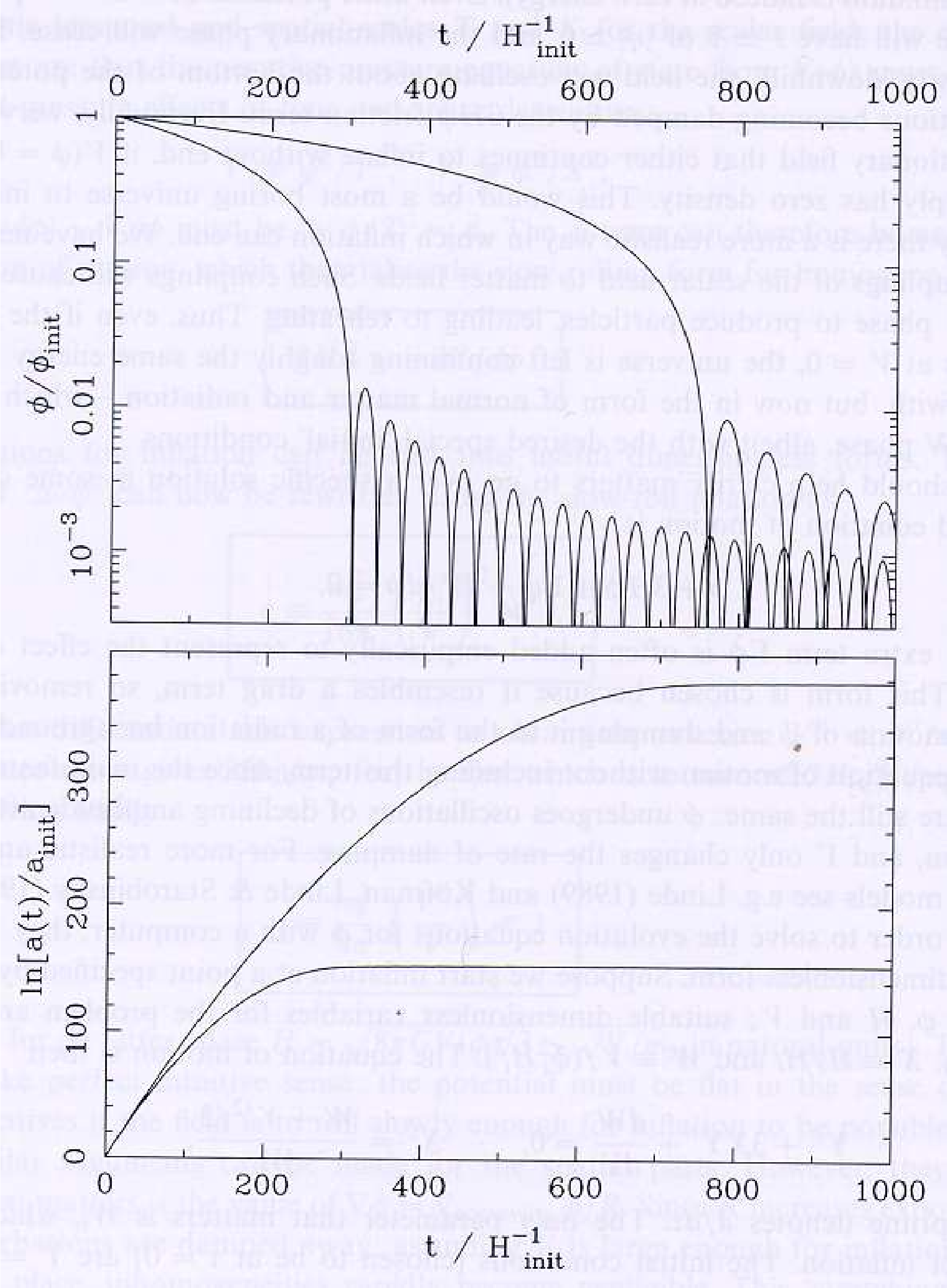
(flatness of the potential)

The spatial derivatives are less critical since they are rendered small by the exponential expansion. This is not the case with the time derivatives that tend to grow as the fields rolls down.

As the field rolls, eventualy $\epsilon \sim \eta \sim 1$ and inflation ends. The field will oscillate at the bottom of the potential, and through its coupling to other fields will give up its energy producing particles. This can be simulated by an extra drag term in the equation of motion:

$$\ddot{\phi} + 3H(t)\dot{\phi} + \Gamma\dot{\phi} - \nabla^2\phi + V' = 0$$

Numerical evolution



Plot of the exact solution for the scalar field in a model with $V \sim \phi^2$ potential

Inflationary Models

• Inflation due to a first order phase transition (Guth) Problematic due to bubble nucleation picture.

• Chaotic inflation. The field starts out somewhere away from the global minimum. The original position is random.

▴ **Polynomial Inflation**. If the potential behaves as $V \sim \phi^a$ then the expansion is close to exponential. This becomes less true as a increases.

▴ **Power-law inflation.** $a(t) \sim t^p$ with $p > 1$ suffices. A potential that can give this is

$$V \sim \exp\left[\sqrt{\frac{16\pi}{pm_P^2}}\phi\right]$$

▴ **Intermediate inflation**. Another time dependence that suffices for inflation is $a \sim \exp[(t/t_0)^f]$. In the slow roll approximation it is generated by $V \sim \phi^{-\beta}$ with $\beta = 4(f^{-1} - 1)$.

Inflation Criteria

We must obtain at least 60 e-foldings

$$N = \int H\ dt = -\frac{8\pi}{m_p^2}\int_{\phi_1}^{\phi_2} d\phi \frac{V}{V'} d\phi$$

For a smooth potential, $N \sim (\phi_{start}/m_P)^2$ so we need an initial field value that is larger than the Planck scale.

This is unnatural in the quantum theory.

Relic Fluctuations

For a black hole with horizon radius r_s the Hawking temperature is

$$kT_H = \frac{\hbar c}{4\pi r_s}$$

A similar temperature exists for the spectrum of fluctuations (emmited particles) in de Sitter space:

$$kT_{dS} = \frac{\hbar H}{2\pi}$$

We expect a spectrum of fluctuations that are nearly scale invariant.

$$\delta_H^2 = \Delta_\Phi^2 \equiv \frac{d\sigma^2(\Phi)}{d \log k} = \text{constant}$$

De Sitter space is invariant under all translations.

The horizon scale amplitude is given by

$$\delta_H = \frac{H^2}{2\pi\dot{\phi}}$$

that can be derived by $\delta t = \phi/\dot{\phi}$ and

$$\delta_H \sim H\ \delta t = H\frac{< \phi >}{< \dot{\phi} >} = \frac{H^2}{2\pi\dot{\phi}}$$

Inflaton coupling

From the number of e-foldings

$$N = \int H\ dt = \int H \frac{d\phi}{\dot{\phi}} = \int 3H^2 \frac{d\phi}{V'}$$

we obtain for $V = \lambda\phi^4$ that $N = 3H^2/(8\lambda\phi^2)$.

The density perturbations are

$$\delta_H \sim \frac{H^2}{\dot{\phi}} = \frac{3H^3}{V'} \sim \sqrt{\lambda N^3}$$

Asking for $N \geq 60$ and that $\delta_H \sim 10^{-5}$ demands that

$$\lambda \lesssim 10^{-15}$$

For $V = m^2\phi^2$, $\delta_H = 3H^3/(2m^2\phi)$. Since $H = \sqrt{V}/m_P$ we obtain

$$\delta_H \sim \frac{m\phi^2}{m_P^3} \sim 10^{-5}$$

Since we need $\phi > m_P$, we must have $m \lesssim 10^{-5} m_P$

Inflation **does not** naturally explain the size of fluctuations.