5D Superconformal Theories

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Introduction

Superconformal symmetry plays a central role in modern theoretical physics.

However, interacting SCFTs in $\mathrm{D} > 4$ were discovered (and appreciated) much later than for $\mathrm{D} \leq 4$.

$\Rightarrow$ YM coupling in 5D has dimensions of $M^{-1/2}$ and no Lagrangian descriptions of such SCFTs

$\Rightarrow$ Advent of 2nd superstring revolution in mid-90s acted as a catalyst

Begin with the classification of superconformal algebras [Nahm '78]

In 5D $\exists$ only one superconformal algebra, $F(4)$. This includes:

$\Rightarrow$ The bosonic subalgebra $\mathfrak{so}(5,2) \times \mathfrak{su}(2)_R$

$\Rightarrow$ 8 Poincaré and 8 superconformal supercharges ($\mathcal{N} = 1$)

Similarly for 6D $\exists$ only $D(4,1) \simeq \mathfrak{osp}(8|2)$ and $D(4,2) \simeq \mathfrak{osp}(8|4)$

$\Rightarrow$ These are the 6D (1,0) and (2,0) SCFTs

N.B. : Notation comes from classification of Lie superalgebras [Kac '77]

Such theories allowed but do they actually exist?

Consider the following IIA system:

$\Rightarrow$ An O8 plane

$\Rightarrow$ $N_f \leq 8$ D8-branes ($2N_f$ on the covering space)

$\Rightarrow$ A D4-brane probe

$\Rightarrow$ k D0 branes

Close to the orientifold:

$\Rightarrow$ The D4 probe hosts an Sp(1) gauge symmetry

$\Rightarrow$ The N_f D8s host an SO($2N_f$) gauge symmetry

$\Rightarrow$ The D0s are instanton solitons in 5D gauge theory

$\Rightarrow$ At low energies one expects an $\mathcal{N}=1$ Sp(1) gauge theory with N_f (massive) fundamental hypers plus instantons.

Instanton solitons carry a topological charge associated with the conserved U(1)$_I$ current

$$J=\frac{1}{8\pi^2}\mathrm{Tr}\star(F\wedge F)$$

$\Rightarrow$ Global symmetry is SO$(2N_f)\times$U(1)$_I$

These gauge theories have Coulomb branch $\mathbb{R}/\mathbb{Z}_2=\mathbb{R}^+$ and are not remormalisable.

The effective gauge coupling can be calculated to be

$$\frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + 16\phi - \sum_i^{N_f} |\phi - m_i| - \sum_i^{N_f} |\phi + m_i|$$

and $\phi > 0$ is the Coulomb branch parameter.

◇ When $N_f > 8$ there are singularities in moduli space.

◇ But: when $N_f < 8$ one can take $g_0 \to \infty$ and then $\phi \to 0$.
$\Rightarrow$ This leads to the prediction of a 5D SCFT

◇ For $N_f = 8$ the metric on the Coulomb branch vanishes and the 5D description is not meaningful.
$\Rightarrow$ The UV fixed point is a 6D SCFT

At the origin of the half line $\mathbb{R}^+$ there is a global symmetry enhancement predicted from Type I'/Type I/$E_8 \times E_8$ Heterotic string theory: SO($2N_f$)$\times$ U(1)$_I \Longrightarrow E_{N_f+1}$, where $N_f = 0, ..., 7$ and:

$$E_8 \supset \mathrm{SO}(14) \times \mathrm{U}(1)$$

$$E_7 \supset \mathrm{SO}(12) \times \mathrm{U}(1)$$

$$E_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)$$

$$E_5 = \mathrm{Spin}(10) \supset \mathrm{SO}(8) \times \mathrm{U}(1)$$

$$E_4 = \mathrm{SU}(5) \supset \mathrm{SO}(6) \times \mathrm{U}(1)$$

$$E_3 = \mathrm{SU}(3) \times \mathrm{SU}(2) \supset \mathrm{SO}(4) \times \mathrm{U}(1)$$

$$E_2 = \mathrm{SU}(2) \times \mathrm{U}(1) \supset \mathrm{SO}(2) \times \mathrm{U}(1)$$

$$E_1 = \mathrm{SU}(2) \supset \mathrm{U}(1)$$

The 5D E_{N_f+1} SCFTs can be coupled to a background U(1)$_I$ vector multiplet and this global symmetry can be gauged. The scalar component is the 5D gauge coupling $m_0 \sim 1/g_0^2$.

$\Rightarrow$ Turning on m_0 flow to 5D Sp(1) gauge theories with N_f massive hypermultiplets.

One can also turn on relevant deformations m_i for $i = p, ..., N_f$.

$\Rightarrow$ These flow to 5D E_p SCFTs

[Seiberg '96]

The E_{N_f+1} SCFTs can also be obtained from M-theory compactifications on singular CYs and all interacting SCFTs can be classified
[Morrison-Seiberg '96, Intriligator-Morrison-Seiberg '97]

These include Sp(N), as well as SU(N) generalisations, for which one also needs to turn on 5D Chern–Simons terms.

$\Rightarrow$ Existence of these theories is surprising. Even then, they are inherently strongly coupled $\Rightarrow$???

$\Rightarrow$ There should be a field-theoretic derivation of the symmetry enhancement...

AdS$_6$/CFT$_5$

There is a natural candidate for a gravity dual in terms of the near-horizon-limit of large-N D4s probing $N_f < 8$ D8s on the O8 plane. [Ferrara-Kehagias-Partouche-Zaffaroni '98]

This can be found to be a warped product of AdS$_6 \times S^4$, with an RR 6-form flux and isometries

$$\mathrm{SO}(5,2) \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)$$

This is a compactification of massive IIA SUGRA to the $F(4)$ gauged SUGRA in 6D. [Brandhuber-Oz '99]

(More) Recent Developments

The study of 5D (and 6D) SCFTs was greatly boosted by the introduction of nonperturbative techniques using:

$\Rightarrow$ The superconformal index

$\Rightarrow$ Supersymmetric localisation

$\Rightarrow$ Instanton operators

This allows the study of global symmetry structure purely from field theory and also gives a handle on the BPS spectrum.

The 5D Superconformal Index

The superconformal index is an essential tool in the study of SCFTs.

Consider such a theory in radial quantisation, such that

$$\mathcal{Q}^\dagger = \mathcal{S} \qquad \text{and} \qquad \mathcal{P}^\dagger = \mathcal{K}$$

The Hilbert space of the theory can be constructed using local operators.

These fall into irreps of the superconformal algebra, characterised by a higest weight state of the maximal compact subalgebra.

In 5D we have that

$$F(4) \supset \mathfrak{so}(5) \oplus \mathfrak{so}(2) \oplus \mathfrak{su}(2)_R$$

that is, each operator carries quantum numbers:

$$\mathcal{O}|0\rangle = |\Delta; l_1, l_2, k\rangle$$

$\Rightarrow$ Δ is the conformal dimension

$\Rightarrow$ $l_1 \geq l_2 > 0$ are Lorentz labels in the orthogonal basis

$\Rightarrow$ k is an R-symmetry label

The highest weights of the SCA are in 1-1 correspondence with the superconformal primaries, which are annihilated by the $\mathcal{S}$ and $\mathcal{K}$ generators.

The irrep can then be constructed by acting on the primary as

$$\prod_{\mathbf{A},a} (\mathcal{Q}_{\mathbf{A}a})^{n_{\mathbf{A},a}} \prod_{\mu} \mathcal{P}_{\mu}{}^{n_{\mu}} |\Delta; l_1, l_2; k\rangle^{hw}$$

The ensuing unitary multiplets are called long and have

$$\Delta > 3k + l_1 + l_2 + 4$$

For some values of the quantum numbers a subset of states can become null giving rise to short multiplets.

This is equivalent to imposing

$$\prod \mathcal{Q} |\Delta; l_1, l_2; k\rangle^{hw} = 0$$

The enumeration of shortening conditions and ensuing classification of multiplets gives:

Multiplet	Shortening Condition	Conformal Dimension
$\mathcal{A}[l_1, l_2; k]$	$\Lambda^4_{\mathbf{1}}\Psi = 0$	$\Delta = 3k + l_1 + l_2 + 4$
$\mathcal{A}[l_1, 0; k]$	$\Lambda^3_{\mathbf{1}}\Lambda^4_{\mathbf{1}}\Psi = 0$	$\Delta = 3k + l_1 + 4$
$\mathcal{A}[0, 0; k]$	$\Lambda^1_{\mathbf{1}}\Lambda^2_{\mathbf{1}}\Lambda^3_{\mathbf{1}}\Lambda^4_{\mathbf{1}}\Psi = 0$	$\Delta = 3k + 4$
$\mathcal{B}[l_1, 0; k]$	$\Lambda^3_{\mathbf{1}}\Psi = 0$	$\Delta = 3k + l_1 + 3$
$\mathcal{B}[0, 0; k]$	$\Lambda^2_{\mathbf{1}}\Lambda^3_{\mathbf{1}}\Psi = 0$	$\Delta = 3k + 3$
$\mathcal{D}[0, 0; k]$	$\Lambda^1_{\mathbf{1}}\Psi = 0$	$\Delta = 3k$

where

$$\Lambda^a_{\mathbf{1}} := \sum_{b=1}^{a} \mathcal{Q}_{\mathbf{1}b}\lambda^a_b$$

Long multiplets can split into short multiplets when

$$\Delta + \epsilon \to \frac{3}{2}K + d_1 + d_2 + 4$$

where we converted to the Dynkin basis

$$d_1 = l_1 - l_2 \ , \quad d_2 = 2l_2 \ , \quad K = 2k$$

according to the recombination rules

$$\mathcal{L}[\Delta + \epsilon; d_1, d_2; K] \xrightarrow{\epsilon \to 0} \mathcal{A}[d_1, d_2; K] \oplus \mathcal{A}[d_1, d_2 - 1; K + 1]$$
$$\mathcal{L}[\Delta + \epsilon; d_1, 0; K] \xrightarrow{\epsilon \to 0} \mathcal{A}[d_1, 0; K] \oplus \mathcal{B}[d_1 - 1, 0; K + 2]$$
$$\mathcal{L}[\Delta + \epsilon; 0, 0; K] \xrightarrow{\epsilon \to 0} \mathcal{A}[0, 0; K] \oplus \mathcal{D}[0, 0; K + 4]$$

This classification is complete and the multiplets have been constructed explicitly. [Minwalla ’97, Buican-Hayling-CP ’16, Córdova-Dumitrescu-Intriligator ’16]

We can finally define the superconformal index: Pick a supercharge and calculate

$$\delta := \{\mathcal{Q}_{\mathbf{1}4}, \mathcal{S}_{\mathbf{2}1}\} = \Delta - \frac{3}{2}K - d_1 - d_2$$

then write

$$\mathcal{I}(x, y, w_a) = \mathrm{Tr}_{\mathcal{H}}(-1)^F e^{-\beta\delta} x^{K+d_1+d_2} y^{d_1} \prod_a w_a^{\mathfrak{Q}_a}$$

This is a Witten-type index which:

⇒ Receives contributions only from $\delta = 0$ states

⇒ x, y are a maximal set of fugacities, the exponents of which commute amongst themselves, as well as with $\mathcal{Q}_{\mathbf{1}4}$ and $\mathcal{S}_{\mathbf{2}1}$

As a result:

$\Rightarrow$ Long multiplets do not contribute to the index
$\Rightarrow$ Short multiplets which recombine cancel out in the index
$\Rightarrow$ The index is invariant under continuous deformations that preserve the susy

For theories with free limits this leads to simple evaluation.

This enumeration of states in a BPS subsector of the theory can e.g. be cross-checked with AdS/CFT.

[Romelsberger '07, Kinney-Maldacena-Minwalla-Raju '07, Bhattacharya-Bhattacharyya-Minwalla-Raju '08]

However, for inherently strongly-coupled theories like our 5D SCFTs, such an approach is intractable. E.g. not clear which of the 5D multiplets appear in the dynamics.

There is another way of formulating the index using the state-operator map.

Operators in the radially quantised SCFT on $\mathbb{R}^D$ can be conformally mapped to states in the theory on $\mathbb{R} \times S^{D-1}$.

The index of the SCFT is then mapped to a Euclidean path integral with twisted boundary conditions for the fields.

Upon compactification of the Euclidean time direction

$$\mathcal{I} = \int_{S^1_\beta \times S^4} \mathcal{D}\Psi e^{-S_E[\Psi]}$$

where the fermions satisfy periodic boundary conditions.

The fields satisfy

$$\Psi(\tau+\beta) = e^{-(d_1+d_2+\frac{3}{2}K)\beta-(d_1+d_2K)\gamma_1-d_1\gamma_2-i\mathfrak{Q}_a m_a}\Psi(\tau)$$

with

$$x = e^{-\gamma_1}\ , \qquad y = e^{-\gamma_2}\ , \qquad w_a = e^{-im_a}$$

Then one can restore the usual boundary conditions $\Psi(\tau+\beta) = \Psi(\tau)$ upon replacing

$$\partial_\tau \to \partial_\tau + \frac{\beta-\gamma_1-\gamma_2}{\beta}d_1 + \frac{\beta-\gamma_1}{\beta}d_2 - \frac{\frac{3}{2}\beta-\gamma_1}{\beta}K - \frac{im_a}{\beta}\mathfrak{Q}_a$$

Supersymmetric Localisation

Evaluating this path integral exactly is usually not possible.

But for certain supersymmetric theories this can be done using the method of supersymmetric localisation [Pestun et al. '16]

Let $\mathcal{Q}$ be a fermionic symmetry of the theory, squaring to a bosonic symmetry and leaving the action invariant, $\mathcal{Q}S = 0$.

Consider a functional V, such that $\mathcal{Q}^2 V = 0$. Deforming the action by the $\mathcal{Q}$-exact term $\mathcal{Q}V$ is a total derivative and does not change the integral

$$\frac{d}{dt}\int e^{S+t\ \mathcal{Q}V} = \int \mathcal{Q}V e^{S+t\ \mathcal{Q}V} = \int \mathcal{Q}\Big(V e^{S+t\ \mathcal{Q}V}\Big) = 0$$

When $t \to \infty$ the one-loop approximation at the critical set of saddle-point solutions $\mathcal{Q}V = 0$ becomes exact.
$\Rightarrow$ the PI localises

The path integral reduces to evaluating S at the critical points and the 1-loop determinant! [Witten '88, Witten '91, Pestun '07]

By identifying an appropriate $\mathcal{Q}$, the localisation technique can be applied to various backgrounds and any dimension.

Made easier by an algorithm which constructs rigid susy theories on curved backgrounds from supergravity multiplets. [Festuccia-Seiberg '11]

Index + Localisation

Back to the 5D SCFTs, there is no action to work with...

So here one uses the properties of the index to make an educated guess about which quantity to calculate.

The index is independent of all continuous deformations. One such deformation is the 5D gauge coupling.

$\Rightarrow$ One should be able to calculate the PI in the weakly-coupled 5D gauge theory, while keeping track of all nonperturbative effects, i.e. instantons. [Kim^2-Lee '12]

Shut up and calculate:

- $\diamond$ Set up the index as a PI on $S^4 \times S^1$
- $\diamond$ Include global fugacities for SO($2N_f$) and U$(1)_I$
- $\diamond$ Include gauge fugacities
- $\diamond$ Identify the fermionic symmetry as $\mathcal{Q} + \mathcal{S}$
- $\diamond$ Deform the 5D Lagrangian by $t\delta_\epsilon((\delta_\epsilon\lambda)^\dagger\lambda)$
- $\diamond$ Find critical points; (anti)instantons localise on the (N) S pole
- $\diamond$ Calculate S on the critical points
- $\diamond$ Calculate one-loop determinants
- $\diamond$ Near the poles the space is locally $\mathbb{R}^4 \times S^1$. The instanton contributions become Nekrasov partition functions

After a long calculation, the result is remarkably simple. For the Sp(N) gauge theory with N_f flavours:

$$\mathcal{I}(x,y,m_a,\mathfrak{q}) = \int [d\alpha]\mathrm{PE}[f_{vec}(x,y,e^{i\alpha}) + f_{matt}(x,y,e^{im},e^{i\alpha})]\times$$
$$\times\, |Z_{Nek}((x,y,e^{im},e^{i\alpha},\mathfrak{q}))|^2$$

where

$$f_{vec} = -\frac{x(y+1/y)}{(1-xy)(1-x/y)}\sum_{\mathbf{R}} e^{-i\mathbf{R}\cdot\alpha}$$
$$f_{matt} = \frac{x}{(1-xy)(1-x/y)}\sum_{R,R'}\chi_R(e^{im})\chi_{R'}(e^{i\alpha})$$

and

$$\mathrm{PE}[g(t)] := \exp\Big[\sum_{n=1}^{\infty}\frac{1}{n}g(t^n)\Big]$$

The Nekrasov partition functions admit an expansion

$$Z_{Nek} = \sum_{k=1}^{\infty} \mathfrak{q}^k Z_{Nek}^{(k)}$$

and there exist closed-form expressions for $Z_{Nek}^{(k)}$.

The existence of these closed-form expressions suggests the guess was legitimate.

Moreover, for special values of N and N_f one can expand the answer in an x power series to get (say for $N = 1$ and $N_f = 3$):

$$\begin{aligned} \mathcal{I} &= 1 + (1 + \chi_{15}^{\mathrm{SO}(6)} + \mathfrak{q}\chi_{4}^{\mathrm{SO}(6)} + \mathfrak{q}^{-1}\chi_{\bar{4}}^{\mathrm{SO}(6)})x^2 + O(x^3) \\ &= 1 + \chi_{24}^{\mathrm{SU}(5)} x^2 + O(x^3) \end{aligned}$$

What we used here is that

$$\mathrm{SO}(6) \times \mathrm{U}(1)_I \subset \mathrm{SU}(5) = E_4$$
$$1_0 + 15_0 + 4_1 + \bar{4}_{-1} = 24$$

This answer organises itself into characters of the E_4 global symmetry!

$\Rightarrow$ The index calculation sees the UV global symmetry enhancement

This phenomenon holds for all values of N and N_f that can be calculated (but this is possible up to a limited order in x) [Kim^2-Lee '12, Hwang-Kim^2-Park '14]

One can argue for the global symmetry enhancement at all orders for Sp(1). For generic $N_f < 8$ one gets

$$\mathcal{I} = 1 + \chi_{adj}^{E_{N_f+1}} x^2 + O(x^3)$$

$\Rightarrow$ This contribution could only come from the supermultiplet of conserved currents [Bashkirov '12]

The global-symmetry enhancement can be seen in a few more cases. [Bergman-Rodríguez Gómez-Zafrir '13, Bao-Mitev-Pomoni-Taki-Yagi '13, Hayashi-Kim-Nishinaka '13, Taki '13]

Worth mentioning:

◇ The index of 5D n-node quiver SCFTs can be calculated and seen to agree with the large-N gravity calculation on AdS$_6 \times S^4/\mathbb{Z}_n$ [Bergman-Rodríguez Gómez-Zafrir '13]

◇ Z_{Nek} can be independently shown to exhibit the global symmetry enhancement using "fibre-base duality" [Mitev-Pomoni-Taki-Yagi '14]

◇ Susy 5D theories on S^5 [Hosomochi-Seong-Terashima '12, Källén-Zabzine '12, Källén-Qiu-Zabzine '12]

◇ The 5D free energy matches the entanglement entropy of AdS_6 duals and reproduces the expected $N^{5/2}$ scaling [Jafferis-Pufu '12]

◇ One also can use this to evaluate the vev of Wilson loops for SCFTs on S^5 [Assel-Estes-Yamazaki '12]

◇ The exact evaluation of the 5D $\mathcal{N}=2$ SYM path integral on S^5 using localisation is related to the superconformal index of the 6D (2,0) theory [Kim^2 '12, Kim^3 '12]

Instanton Operators

We have seen indirect ways for symmetry enhancement using index calculations.

Can we find a simpler way? $\Rightarrow$ Draw upon our knowledge of 3D theories where local monopole operators play important role.

$\Rightarrow$ Global symmetry and susy enhancement in the IR [Borokhov-Kapustin-Wu '02, Gaiotto-Witten '08, ABJM '08, ...]

Q: Is there an analogue in 5D?
A: We can construct local instanton operators [Lambert-CP-Schmidt Sommerfeld '14]

We have already seen that 5D SYM has conserved current

$$J = \frac{1}{8\pi^2} \mathrm{Tr} \star (F \wedge F)$$

Charged BPS-particle solutions: instanton solitons

Both global and Lorentz symmetry enhancement is associated with instanton charge.

Preliminary: An instanton operator is a local operator which creates instanton solitons out of vacuum

The OPE of this current with $\mathcal{I}_n(0)$ is given by

$$J^\mu(x)\mathcal{I}_n(0) \sim \frac{3n}{8\pi^2} \frac{x^\mu}{|x|^5} \mathcal{I}_n(0) + \cdots$$

More formally: Instanton operators, $\mathcal{I}_n(x)$, modify boundary conditions for gauge field in Euclidean path integral:

$$\langle \mathcal{I}_n(x)\mathcal{O}_{01}(x_1)\dots\mathcal{O}_{0k}(x_k)\rangle =$$
$$= \int_{\frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4_x} F\wedge F=n} [DXDADψ]\ \mathcal{O}_{01}(x_1)\dots\mathcal{O}_{0k}(x_k)e^{-S_E}$$

Fields need to satisfy classical eom near insertion point in $\mathbb{R}^5$

$$D^\mu F_{\mu\nu} = 0\ , \qquad D_{[\mu}F_{\nu\lambda]} = 0$$

but with non-vanishing

$$I = \frac{1}{8\pi^2}\mathrm{Tr}\oint_{S^4} F\wedge F$$

In spherical coordinates a simple solution is given by taking $A_r = F_{ri} = 0$ and the angular components satisfying

$$F = \pm \star_{S^4} F$$

A solution for $\mathrm{SU}(2)$ theory was found long ago by Yang, as static $\mathrm{SO}(5)$-symmetric particle in 6D ⇒ Yang monopole '78

A DBI generalisation for $\mathrm{SU}(N)$ later given by [Constable-Myers-Tafjord '01] in context of D1⊥D5 intersections

Alternatively: Instanton operators defined by the condition that the gauge field has a Yang monopole singularity at the insertion point

Instantons on S^4 can be straightforwardly constructed by stereographic projection from $\mathbb{R}^4$.

The solutions exhibit some amusing properties:

$$F \wedge F = \frac{8\rho^4 \ \sum_{i=1}^3 T_i^2}{\left(1 + \rho^2 + (1 - \rho^2)\cos\theta^1\right)^4} \sqrt{\gamma}\ d^4\theta$$

with $[T_i, T_j] = 2i\epsilon_{ijk}T_k$ an $N \times N$ representation of $\mathfrak{su}(2)$

When $\rho = 1$ this reduces to the $\mathrm{SO}(5)$-symmetric

$$F \wedge F = \frac{1}{2} \sum_{i=1}^{3} T_i^2 \sqrt{\gamma}\ d^4\theta$$

◇ Instanton operators are $\frac{1}{2}$-BPS for $\rho = 0$ (at the poles of the S^4) but generically not BPS [Bergman-Rodríguez Gómez '16]

◇ In $\mathcal{N} = 1$ theories we can also add Chern-Simons terms

$$S_{CS} = \frac{k}{24\pi^2} \mathrm{Tr} \int (F \wedge F \wedge A + \frac{i}{2} F \wedge A \wedge A \wedge A - \frac{1}{10} A \wedge A \wedge A \wedge A \wedge A)$$

$\Rightarrow$ In the presence of such a term the instanton operators are not always gauge invariant

Applications of Instanton Operators

Use them to construct the broken global symmetry currents in 5D IR theories with $\mathcal{N}=1$. [Tachikawa '15]

Consider the multiplet of conserved currents in the 5D E_{Nf+1} SCFT. This is the linear multiplet $\mathcal{D}[0,0;2]$. It contains the following conformal primary states

$$\mu^a_{(ij)}\,, \qquad \psi^a_{i\alpha}\,, \qquad J^a_\mu\,, \qquad M^a$$

with scaling dimensions 3, 3.5, 4, 4 and where $i,j=1,2$ are SU(2)$_R$-symmetry indices, a is an adjoint flavour index and α is a spinor index of SO(5).

The 5D gauge theory in the IR is obtained by turning on a mass deformation in the UV through a term $\delta L = h_a M^a$.

This leads to a partial breaking of the flavour current

$$\partial^\mu J^a_\mu \propto f^{ab}{}_c h_b M^c$$

The supermultiplet of broken currents can be identified as

$$\mu^+_{(ij)} \,, \qquad \psi^+_{i\alpha} \,, \qquad J^+_\mu$$

$\Rightarrow$ Take the Sp(1) gauge theory with no matter.

IR: in the presence of an instanton operator there are 8 fermion zero modes, which reconstruct the broken-current multiplet J_μ^+.

When combined with the IR instanton current

$$J_\mu^0 \propto \epsilon_{\mu\nu\kappa\lambda\rho} Tr F^{\nu\kappa} F^{\lambda\rho}$$

(under which J_μ^+ carries charge +1) these form the $E_1 = \mathrm{SU}(2)$ in the UV.

$\Rightarrow$ Consider the theory with N_f flavours.

The fermion zero modes additionally transform in spinor reps of SO($2N_f$).

When combined with the conserved SO($2N_f$) flavour currents and the instanton current they again form the E_{N_f+1} conserved currents in the UV.

$\Rightarrow$ For the 5D IR theories with $\mathcal{N} = 2$ the fermion zero modes associated with the instanton operator lead to a 2^8-dim multiplet: KK modes of 6D E-M supermultiplet

$\Rightarrow$ This indicates that the UV theory is a 6D theory [Tachikawa '15]

This method can be extended to show the UV symmetry enhancement for a variety of 5D SCFTs [Tachikawa '15, Yonekura '15, Zaffrir '15]

Other developments

◇ The structure of deformations in 5D SCFTs has been studied:

⇒ No marginal deformations.

⇒ The only relevant deformations are the mass deformations residing in the linear multiplet

[Córdova-Dumitrescu-Intriligator '16]

◇ Duality walls separating different IR fixed-point theories originating from the same UV theory constructed

[Gaiotto-Kim '15]

◇ Use Inst. Op.s to probe Higgs branch of $\mathcal{N} = 1$ SU(2) theory at infinite coupling [Cremonesi-Ferlito-Hanany-Mekareeya '15]

⇒ Employs Hilbert series and leads to modification of chiral operator relations

◇ Interesting relation between 5D index and BPS spectrum on the Coulomb branch found [CP-Pini-Rodríguez Gómez '16]

◇ Body of work using topological strings to calculate the 5D index [Iqbal-Vafa '14]

Summary

- Discussed the existence of 5D SCFTs
- Described superconformal index
- Related this to supersymmetric localisation
- Introduces instanton operators

Open questions

◇ Proof of UV index calculation from the IR theory

◇ Spectroscopy of 5D (and 6D) SCFTs

◇ Find additional applications of instanton operators

◇ Expand on the connection between the 5D index and the BPS spectrum on the Coulomb branch [Córdova-Shao '15, Córdova-Gaiotto-Shao '16]