Old and new aspects of the holographic renormalization group

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Outline

- Overview of the geometric description of the RG flow by gauge/gravity duality
- Focus on the flow of a single coupling in bottom-up models
- I will review and present in a unified setting many known facts and show some recent new results
- Part I: Characterisation and classification of holographic RG flows
- Part II: Generating functional and local RG.
- Part III: Perspectives and further directions

References

- Long list of contributions to this topic, e.g. (but not limited to) Freedman, Bianchi, Skenderis, De Wolfe, Gubser, Karch, Porrati, Starinets, Martelli, Miemiec, De Boer, Verlinde, Papadimitriou, Kiritsis, Niarchos, Faulkner, Liu, Rangamani, Polchinski, Heemskerk, Kuperstein, Mukhopadhyay, and more.

Setup

Simple setup: $d+1$-dimensional Einstein Gravity plus one scalar field:

$$S = M_p^{d-1} \int d^d x \int dr \sqrt{-g} \left[R - \frac{1}{2} (\partial \Phi)^2 - V(\Phi) \right] + S_{GH}$$

- Only one scalar $\leftrightarrow$ focus on a single operator O in the field theory.
- The potential $V(\phi)$ encodes the dimension of the operator and the way the coupling runs.
- Work with the fully backreacted system.
- Take $V(\Phi) < 0$ throughout (avoids transitions into cosmological solutions).

Field/Operator correspondence

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- $\Phi(x, 0)$ represents a source for O in the CFT:

$$\Phi(x, r) \sim \alpha(x) r^{(d-\Delta)} + \ldots \ r \to 0 \quad \Leftrightarrow \quad S_{CFT} = S_0 + \int d^4x \, \alpha(x) O(x)$$

- The on-shell action with boundary conditions fixed by $\alpha(x)$ is the QFT generating functional:

$$S\left[\Phi_\alpha(x, r)\right] = \text{funcional of } \alpha(x), \qquad \mathcal{Z}_{QFT}[\alpha(x)] = \exp iS[\Phi_\alpha(x, r)]$$

- The ation must be supplemented with appropriate local covariant boundary terms to obtain finite results (holographic renormalization).

Running away from AdS

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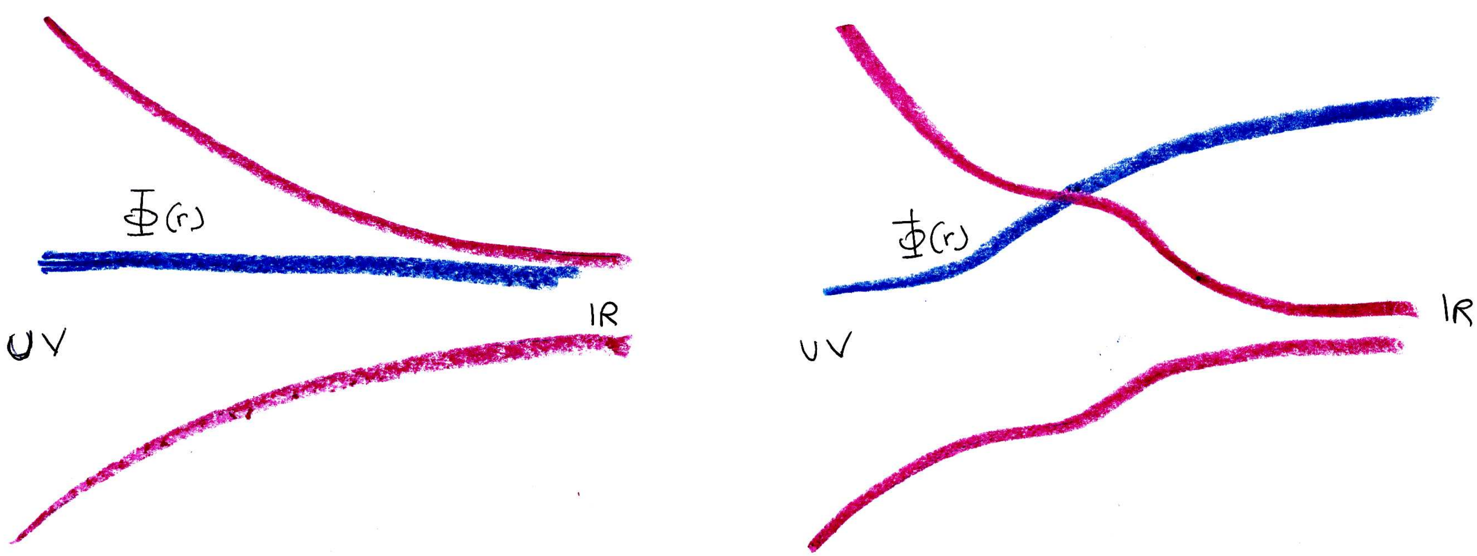

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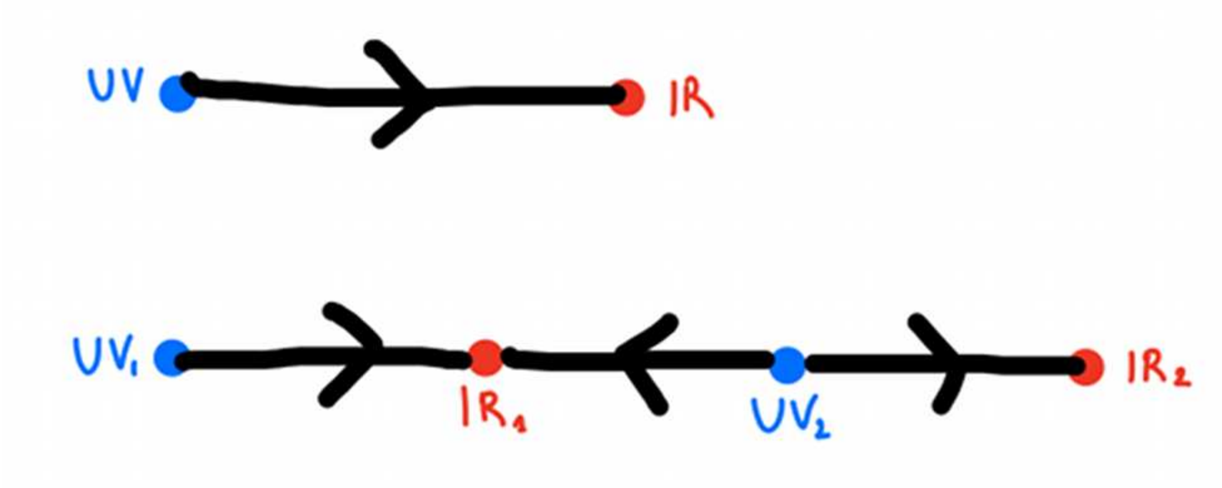
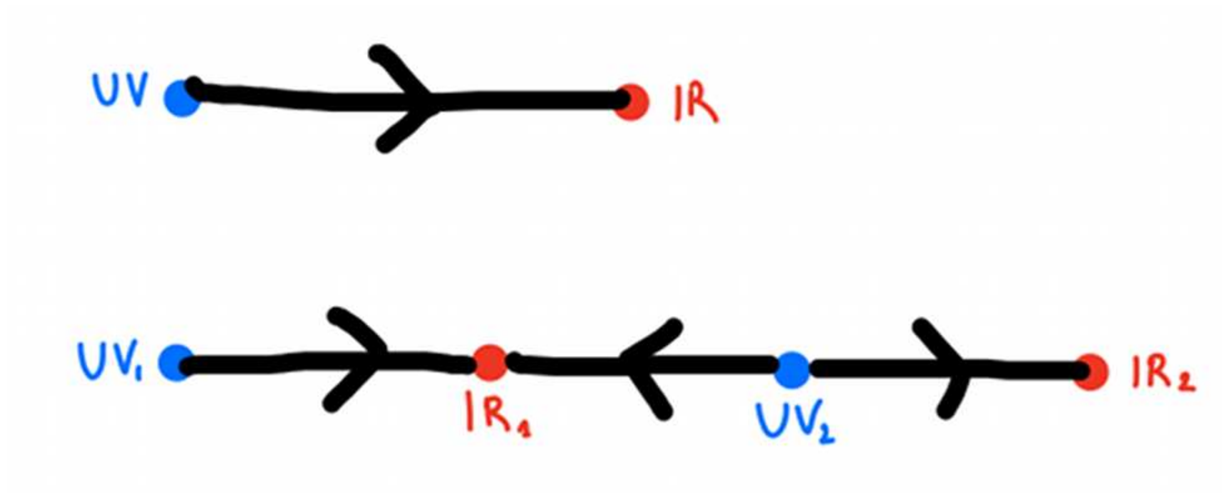
- Classify RG-flow solutions (in a scheme-independent way).
- One encounters standard flows expected in field theory, but also some exotic situations. Do these make sense from the FT point of view? Are they acceptable gravity solutions?
- "Exotic" flows due to second order nature of Einstein's equations.



Standard



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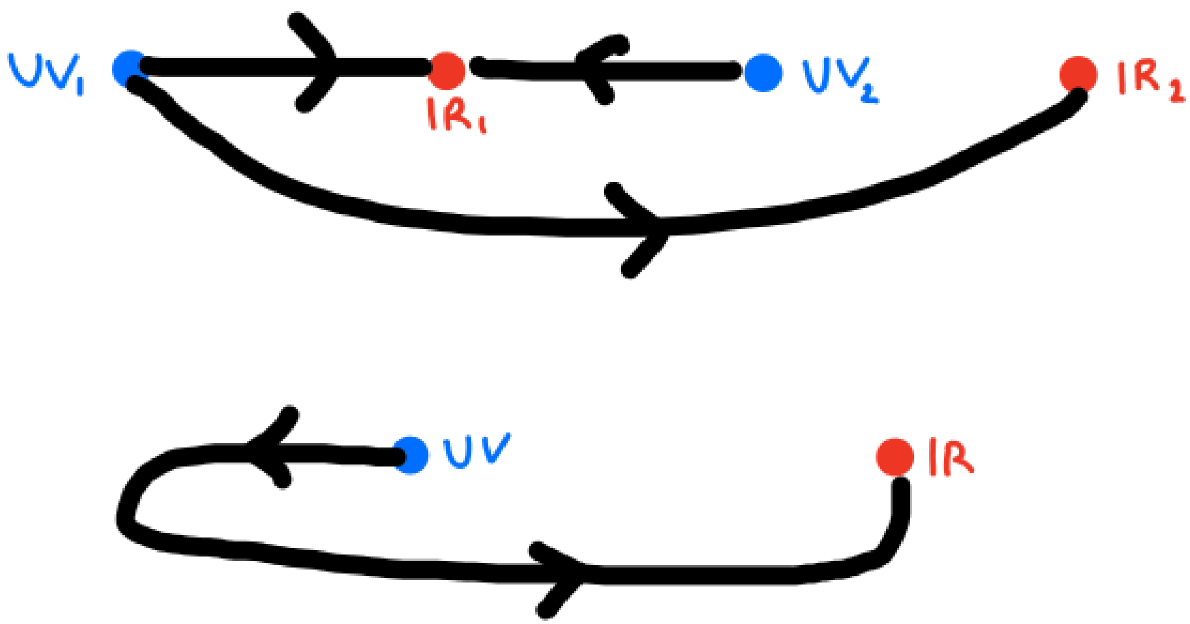
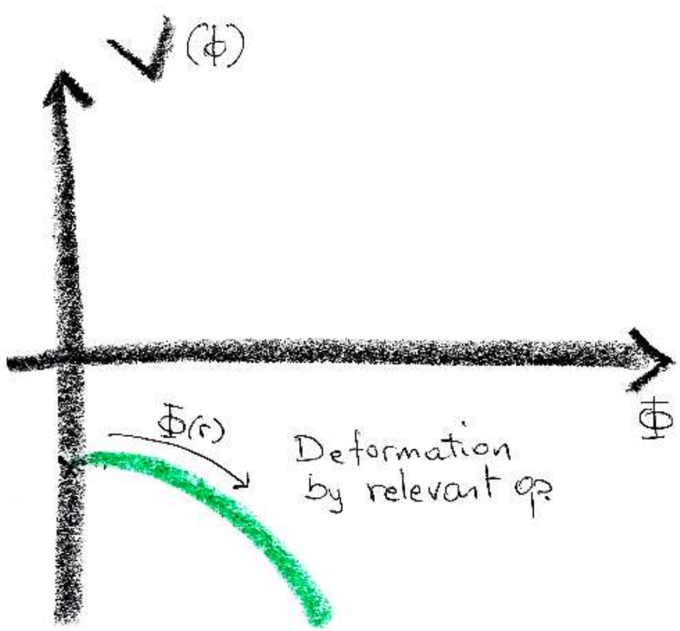


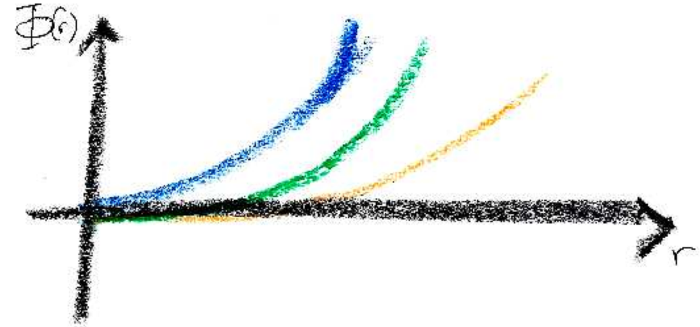
Exotic

Deformations of AdS

Generic Poincaré-invariant solution:

$$ds^2 = du^2 + e^{A(u)}\eta_{\mu\nu}dx^\mu dx^\nu, \quad \Phi = \Phi(u)$$





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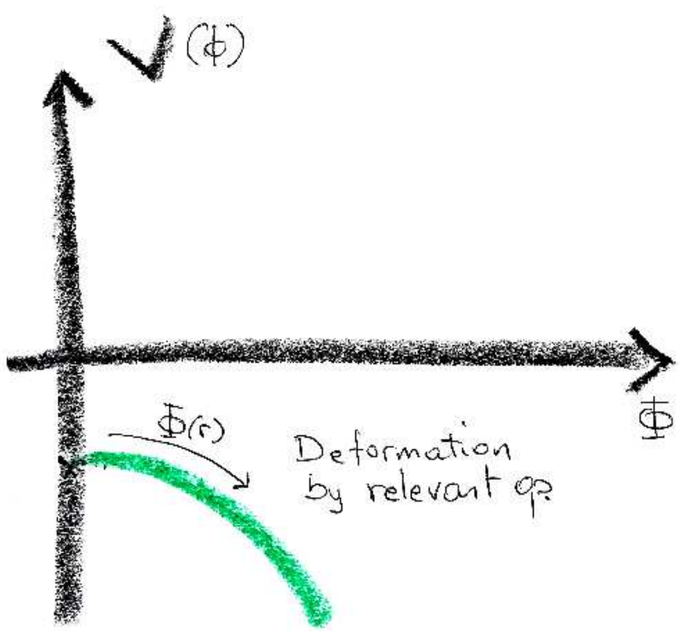
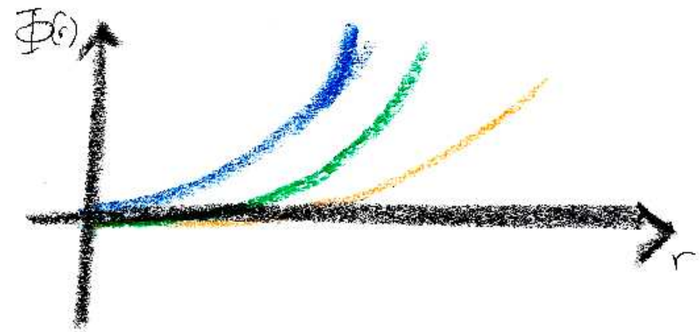
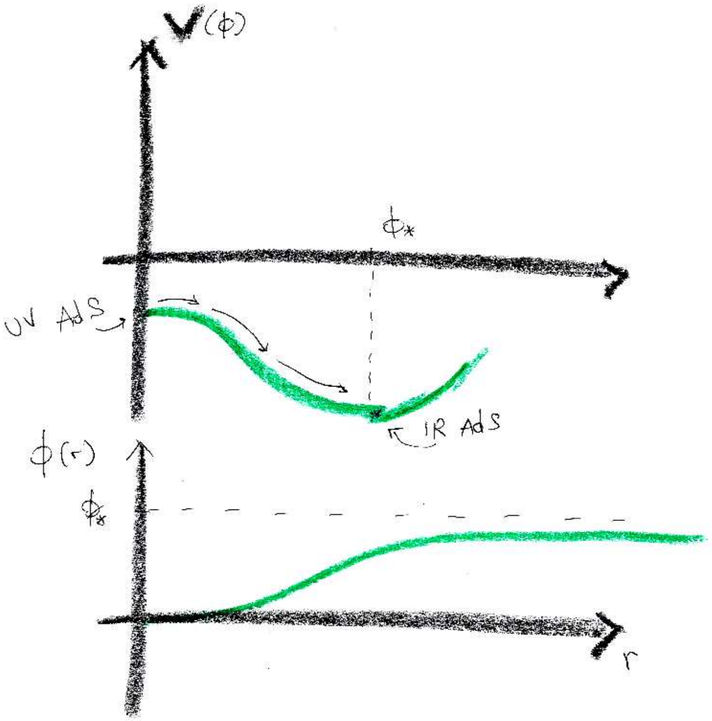




The UV (IR) is represented by the region where $e^{A(u)} \to +\infty$ ($\to 0$). Intuitively, we can think of e^A as the energy scale.

RG-flow solutions

A second extremum for $V(\Phi) \Rightarrow$ a different AdS solution $\Rightarrow$ a different conformal fixed point.



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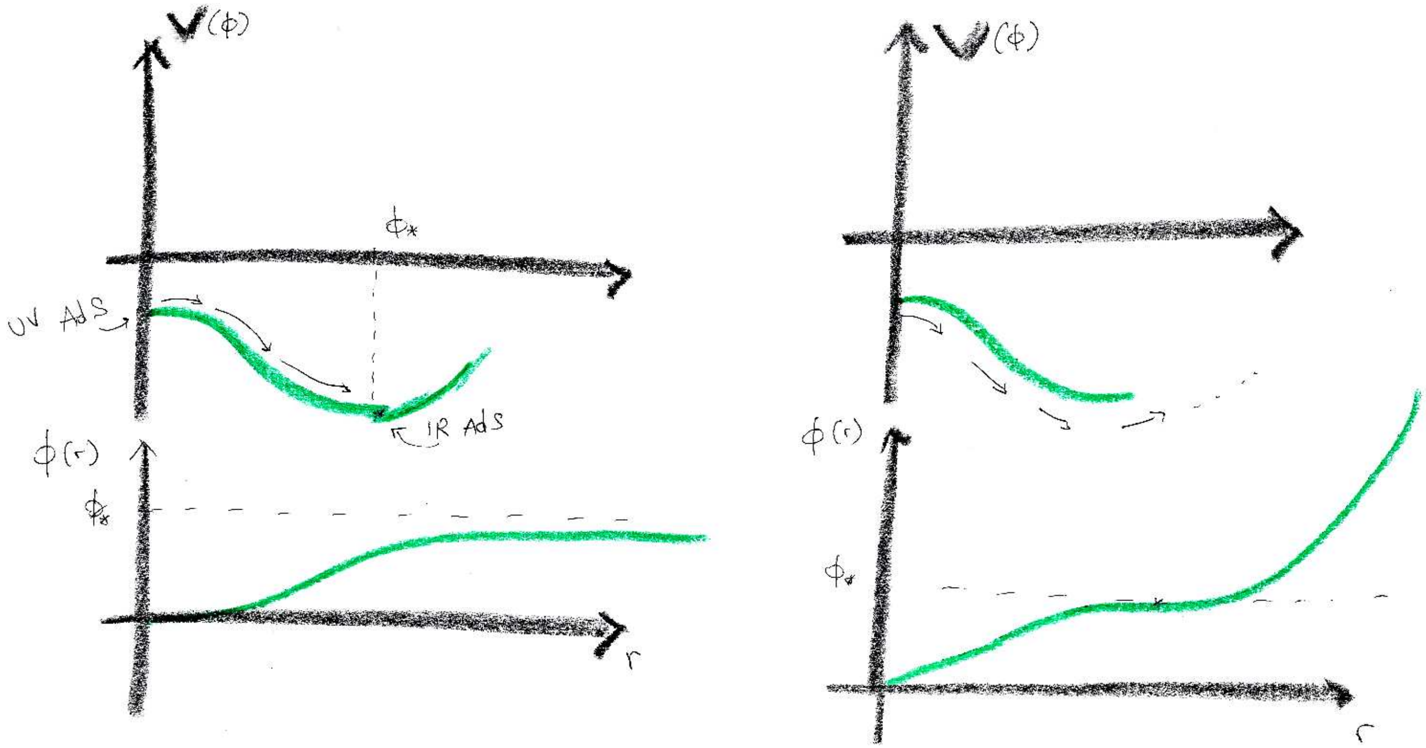

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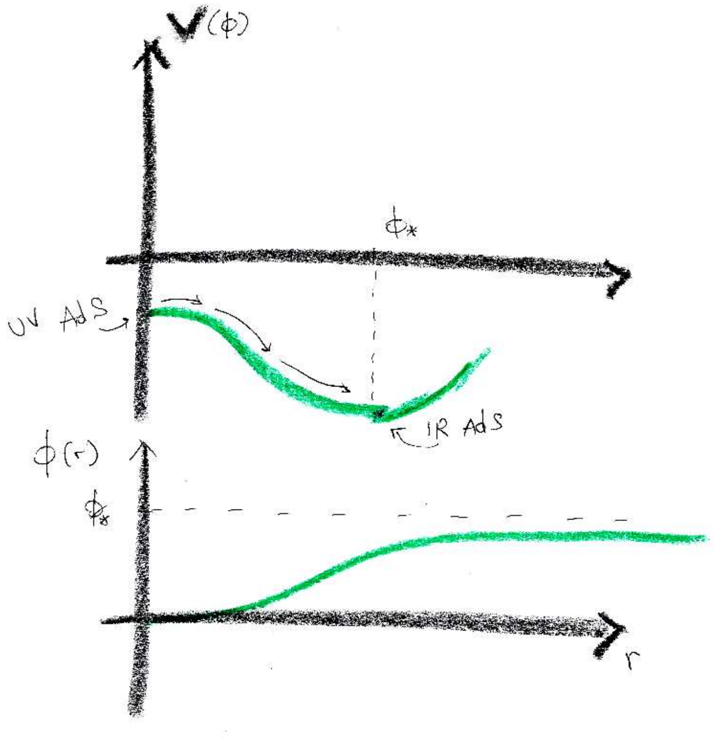

- For $V(\Phi)$ fixed there is a continous family of inequivalent flows. Only one hits the IR fixed point.
- the UV fixed point is an attractor, and is reached for a continuous class of initial conditions.

Superpotential

Write Einstein's equations as first order flow equations, with an auxiliary scalar function $W(\Phi)$ $\quad (' = d/d\Phi)$:

$$\dot{A} = -\frac{1}{2(d-1)} W(\Phi) \qquad \dot{\Phi} = W'(\Phi),$$

$$-\frac{d}{4(d-1)} W^2 + \frac{1}{2}\left(W'\right)^2 = V$$

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- This system is equivalent to usual Einstein equations.
- System has 3 integration constants. One picks $W(\Phi)$.
- Once $W(\Phi)$ is given, the other field equations integrate to:

$$A(\Phi) = A_0 - \frac{1}{2(d-1)}\int_{\Phi_0}^{\Phi} d\phi \frac{W(\phi)}{W'(\phi)},$$

- Different solutions with the same $W(\Phi)$ all look the same up to an additive consant in A.
- Last integration constant: choice of a reference point u_0.

Running coupling

$$A(\Phi) = A_0(\Phi_0) - \frac{1}{2(d-1)} \int_{\Phi_0}^{\Phi} d\phi \frac{W(\phi)}{W'(\phi)},$$

- Formally the same as:

$$\ln \mu(\Phi) = \ln \mu_0 + \int_{\Phi_0}^{\Phi} \frac{d\phi}{\beta(\phi)}, \qquad \beta \equiv -2(d-1)\frac{W'}{W}$$

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- Integrated version of the solution for the running coupling if we identify $\mu = \exp A$:

$$\frac{d\Phi}{d\log\mu} = \beta(\Phi)$$

- If we identify $\Phi(u)$ with the running coupling, then the identification $\mu = \exp A$ is the only consistent one (wait a few slides).

Flow direction

$$\dot{\Phi} = W', \qquad \dot{A} = -\frac{1}{2(d-1)} W$$

$$-\frac{d}{4(d-1)} W^2 + \frac{1}{2} \left(W'\right)^2 = V$$

- The geometry is completely encoded in the superpotential W. Classifying RG flows is the same as classifying solutions of the superpotential equation.

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- The geometry is completely encoded in the superpotential W. Classifying RG flows is the same as classifying solutions of the superpotential equation.
- For *generic* initial condition $W(\Phi_0) = W_0$ there are two solutions $W_\pm$ corresponding to the $\pm$ choice.
- Two W_+ or two W_- solutions cannot cross (uniqueness in each branch) at *generic* points.

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- The superpotential is monotonically increasing along the flow, for either the $W_\pm$ solution

$$\frac{dW}{du} = \dot{\Phi} \frac{dW}{d\Phi} = (\dot{\Phi})^2 \geq 0$$

- W provides a holographic c-function and an indication of the direction of the flow.
- Equations symmetric under $W \to -W, u \to -u$. Assume $W > 0$ for definiteness.

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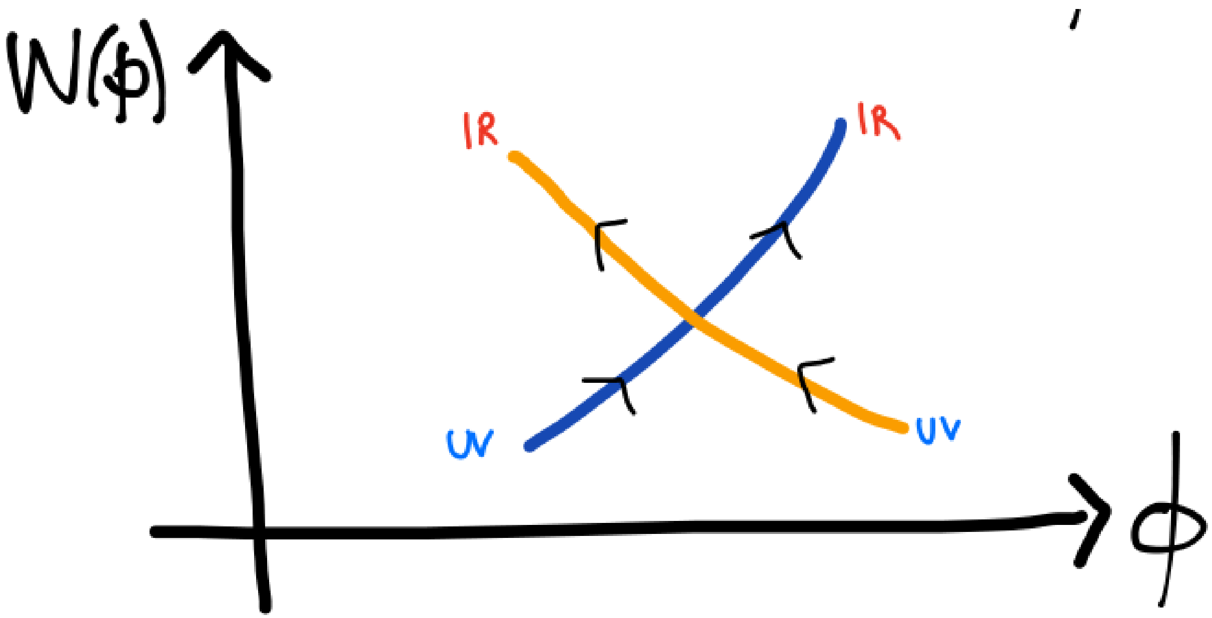

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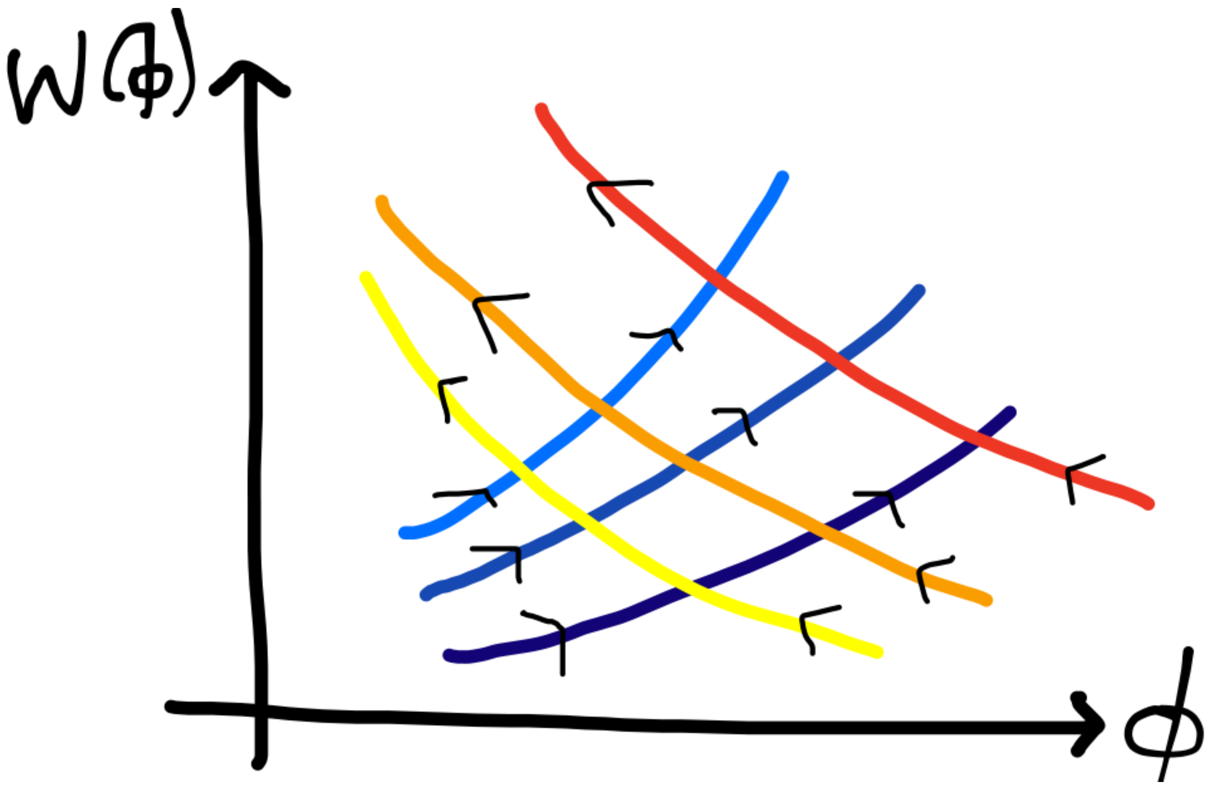

HRG taxonomy

- Extremal points of $W(\Phi)$
  - Exterma of $V(\Phi)$
  - Non-extremal points of $V(\Phi)$
- Asymptotic regions and regularity
- Bouncing solutions
- Examples of standard and exotic RG flows

The critical curve

$$W'(\Phi) = \pm Q\sqrt{W^2(\Phi) - B^2(\Phi)}, \qquad B \equiv \frac{1}{Q}\sqrt{-2V(\Phi)}$$

$$Q \equiv \sqrt{\frac{d}{2(d-1)}},$$

- Phase space bounded by $W(\Phi) \geq B(\Phi)$ (critical curve)
- On the critical curve, $W' = 0$

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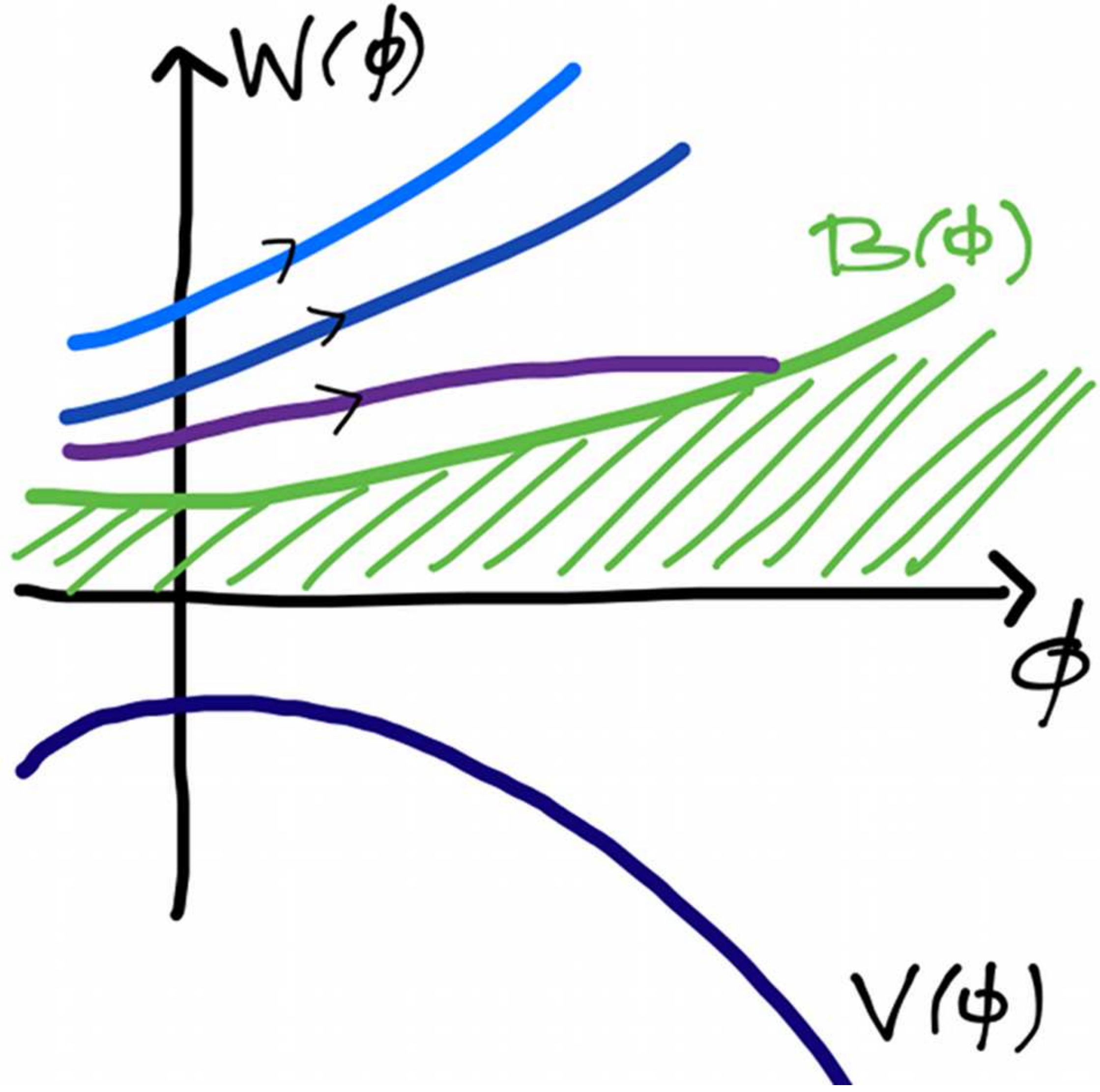

Maxima of V

$$V \simeq -d(d-1)+\frac{m^2}{2}\Phi^2+\ldots, \quad m^2 = \Delta(\Delta-d) < 0 \quad (\Delta < d), \quad V'(0) = 0.$$

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- A continuous family of solution such that $W(0) = B(0)$:

$$W(\Phi) = 2(d-1) + \frac{(d-\Delta)}{2}\Phi^2 + \ldots + C\Phi^{d/(d-\Delta)} + \ldots$$

source flow.

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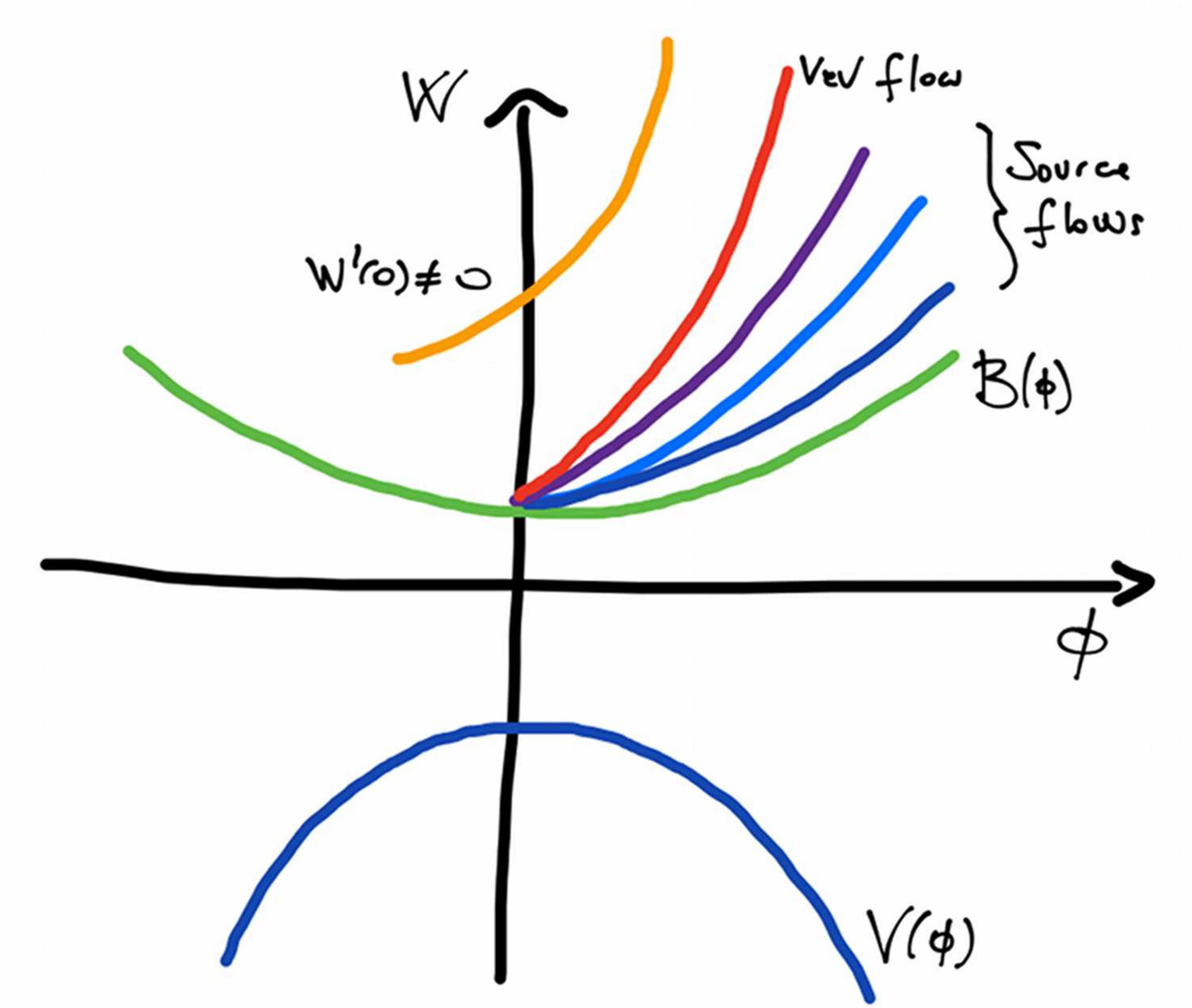
source flow.

- A single solution also arriving at $W(0) = B(0)$

$$W(\Phi) = 2(d-1) + \frac{\Delta}{2}\Phi^2 + \ldots \qquad \Delta > d - \Delta$$

vev flow. (can be reached in the limit $C \to \infty$):

Maxima of V



UV-Asymptotically AdS solutions

Continuous family:

$$W(\Phi) = 2(d-1) + \frac{(d-\Delta)}{2}\Phi^2 + \ldots + C\Phi^{d/(d-\Delta)} + \ldots$$

$$\Phi = \alpha e^{(d-\Delta)u} + \ldots + \frac{d-\Delta}{d} C e^{\Delta u} + \ldots, \qquad e^{A(u)} = e^{-u+A_0+\ldots}, \quad u \to -\infty$$

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- Solution describes the UV, with deformation by a relevant operator $(\Delta < d)$
- UV AdS is an *attractor* for these solutions, C is the integration constant for the superpotential equation.
- α fixes the initial condition for the flow. Not part of W.
- C controls the vev: $\langle O \rangle \propto C\alpha^{\frac{\Delta}{d-\Delta}}$

UV-Asymptotically AdS solutions

Special solution:

$$W(\Phi) = 2(d-1) + \frac{\Delta}{2}\Phi^2 + \ldots$$

$$\Phi = \alpha e^{\Delta u} + \ldots \qquad e^{A(u)} = e^{-u+A_0+\ldots}, \quad u \to -\infty$$

- Solution describes the UV, with deformation by the vev but no source

$$\langle O \rangle \propto \alpha^{\Delta}$$

- This solution lies above the continous family ($\Delta > d - \Delta$) but below the solutions which do not reach the fixed point.

Minima of V

$$V \simeq -d(d-1) + \frac{m^2}{2}\Phi^2 + \ldots, \qquad m^2 = \Delta(\Delta - d) > 0 \qquad (\Delta > d)$$

- A single solution reaching the critical curve at $W(0) = B(0)$:

$$W(\Phi) = 2(d-1) - \frac{\Delta}{2}\Phi^2 + \ldots$$

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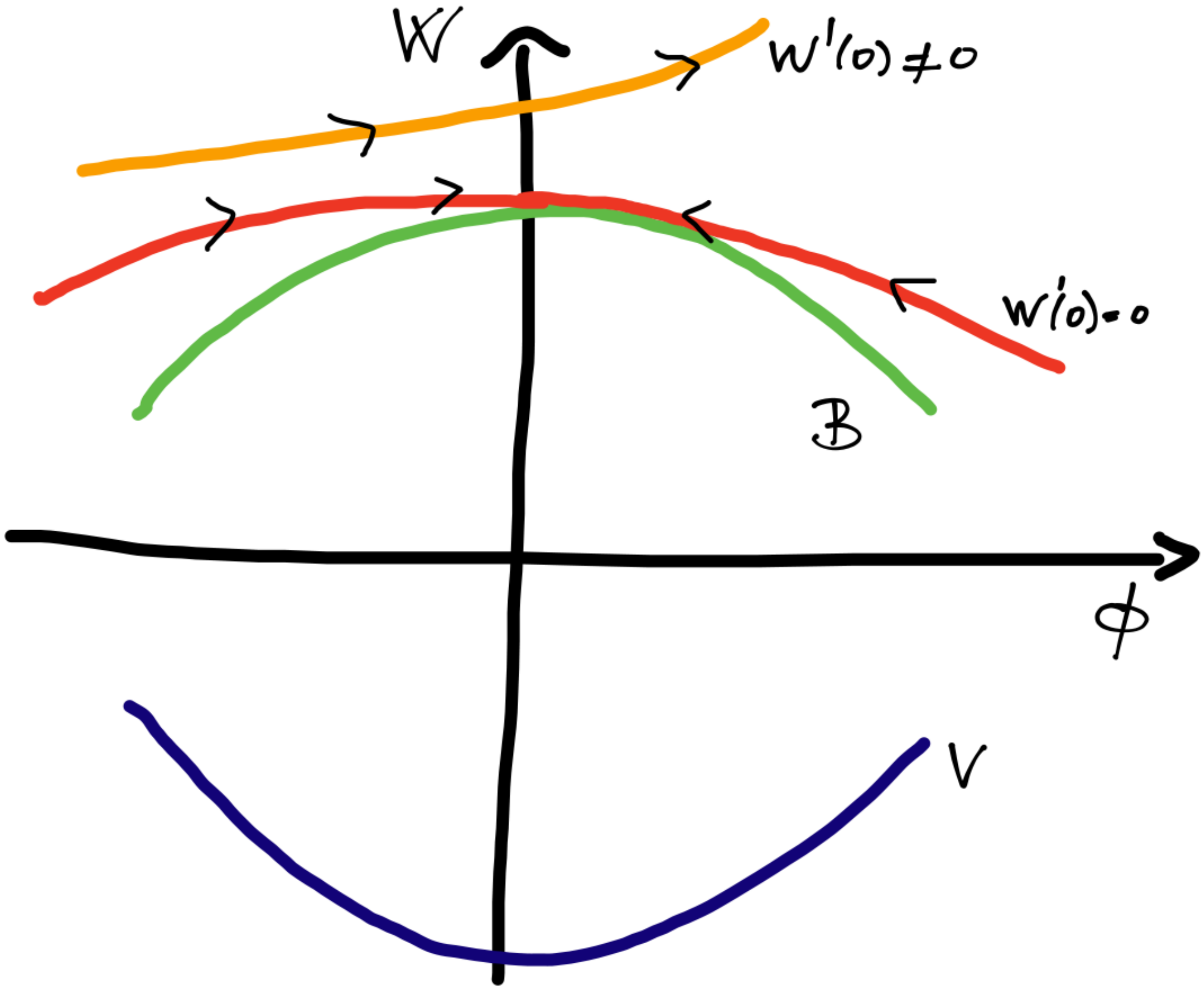
- The solution describes a vev in the IR:

$$\Phi = \alpha e^{-\Delta u} + \ldots \qquad e^{A(u)} = e^{u + A_0 + \ldots}, \quad u \to +\infty$$

- Operator is irrelevant (source not allowed).

Minima of V



Choosing the right W: IR regularity

- The superpotential equation fixes $W(\Phi)$ up to an integration constant.
- Close to a UV extremum of $V(\Phi)$ there is a one-paramenter family of $W(\Phi)$ that all correspond to the same UV AdS fixed point but have different IR behavior.

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  - AdS fixed point in the IR
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  - AdS fixed point in the IR
  - Potential dominated by a single exponential at large Φ.
- Strictly, generic solutions are singular except if the IR is AdS. But some singularities are *"good"* and may be accepted in holography.
- Roughly "regularity" means that the the solution does not need to specify IR boundary conditions, but the UV data completely specifies the theory.

Large field behavior

- A solution can reach the region $\Phi \to \infty$. Typically this leads to a singularity, where both W and V diverge.
- Curvature invariants:

$$R = aV + bW^2, \quad R_{\mu\nu}R^{\mu\nu} = a'V^2 + b'VW^2 + c'W^4 \quad \ldots$$

Assuming V is everywhere regular this can only diverge as $|\Phi| \to \infty$ (as we will see, it generically will)

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- Two main "good singularity" criteria:
  1. *Gubser:* The singularity can be cloacked by a horizon by turning on a black hole of arbitrarily small mass
  2. *Spectral Computability:* The small fluctuation spectrum is determined withour need of IR boundary condition beyond normalizability

Superpotential in the large Φ region

$$V(\Phi) \sim \exp[b\Phi] \qquad \Phi \to \infty$$

Two kinds of asymptotic solutions $W(\Phi)$:

- Continuos family:

$$W_C \simeq C \exp Q\Phi \qquad (W \gg \sqrt{V}) \quad \Phi \to +\infty$$

- Special solution (no free parameters)

$$W_* = W_0 \exp\left[\frac{b}{2}\Phi\right] \qquad (W \sim \sqrt{V}) \quad \Phi \to +\infty$$

These solutions exist only if $b < 2Q$.

IR Regularity, Gubser's bound and all that

Both regularity criteria are decided by the superpotential (not the potential):

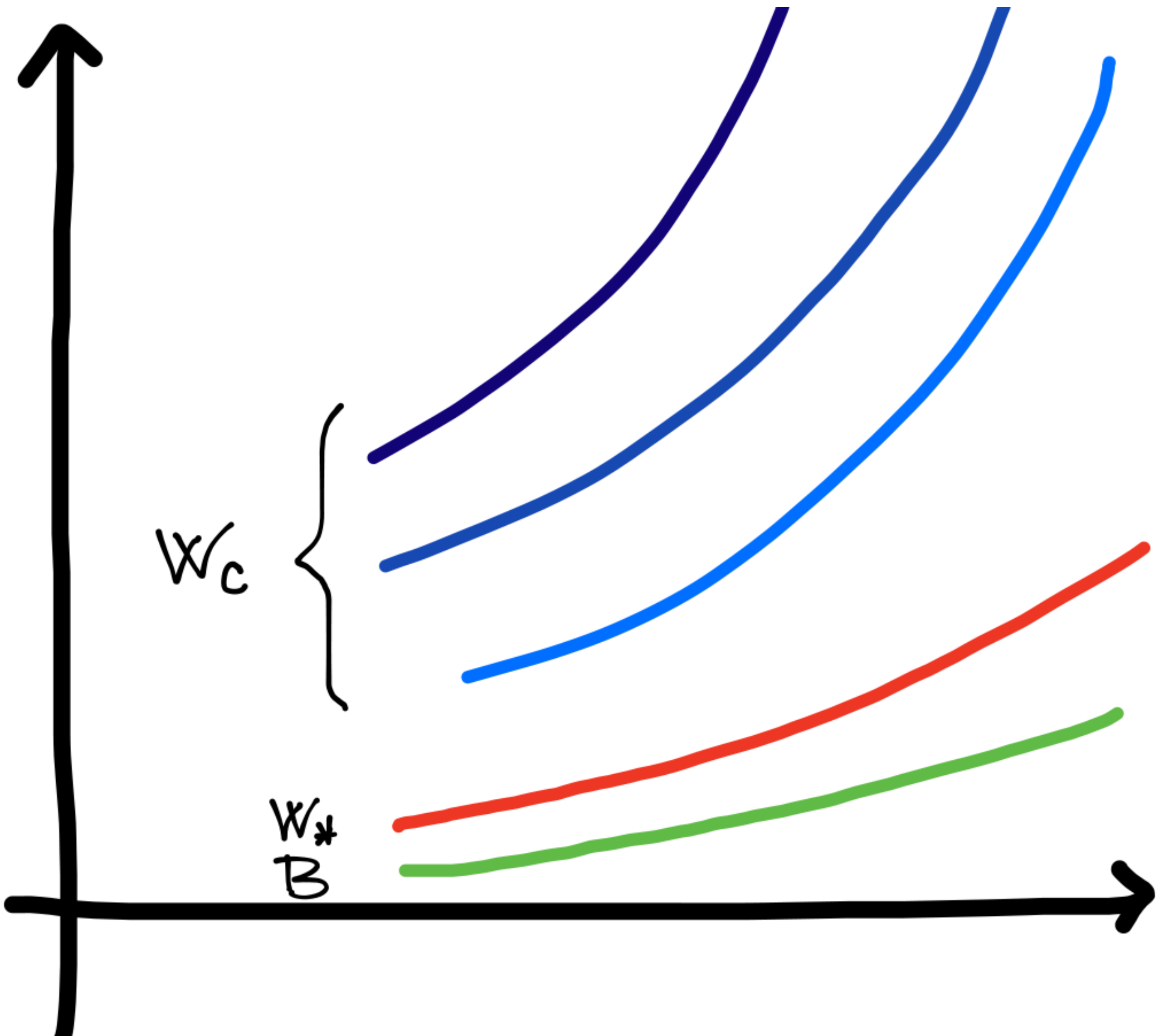
- *Gubser:* Only the $W_*(\Phi)$ solution can be approximated by small black holes. Hence the *Gubser bound* $b < 2Q$
- *Computability:* Spectrum is computable if the supertpotential grows no faster than

$$W \leq K \exp \frac{Q}{\sqrt{2}}$$

Only the $W_*(\Phi)$ solution can satifsy this, and *only* if $b < \sqrt{2}Q$ (*comutability bound*).

What are the endpoints of the flow if $b > 2Q$?

IR Regularity, Gubser's bound and all that



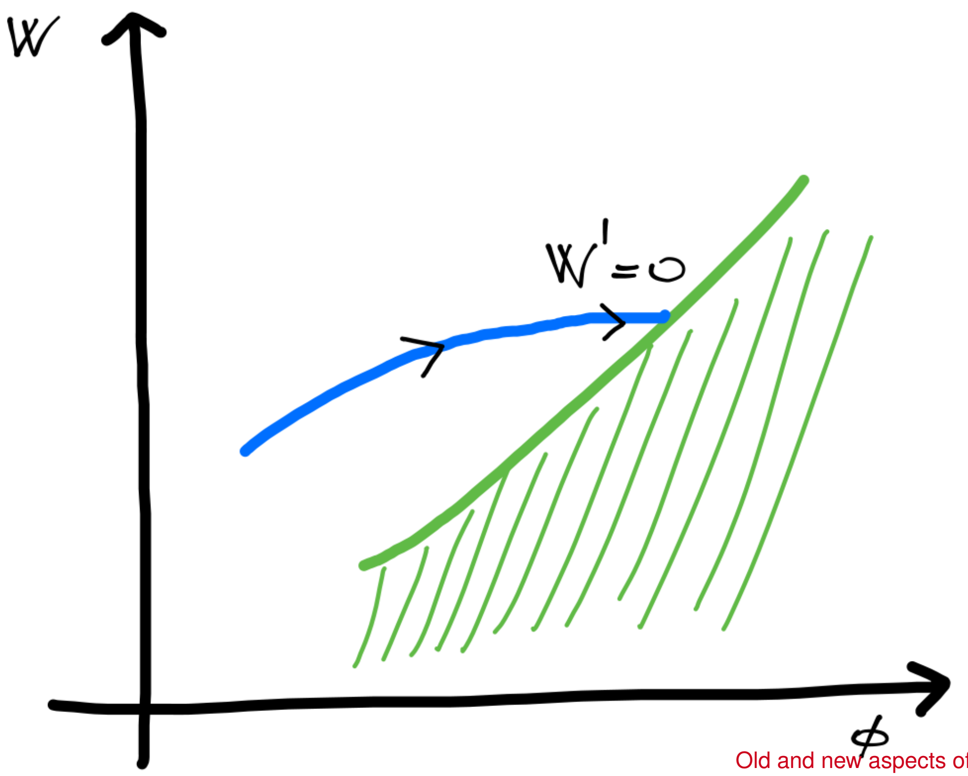
Bouncing off the critical curve

$W(\Phi)$ can reach the critical curve at a non-extremal point of V.

$$W(\Phi_b) = B(\Phi_b), \qquad W'(\Phi_b) = 0$$

Here the solution stops being analytic:

$$W \simeq B(\Phi) + (\Phi_b - \Phi)^{3/2} + \ldots \qquad \Phi < \Phi_b$$



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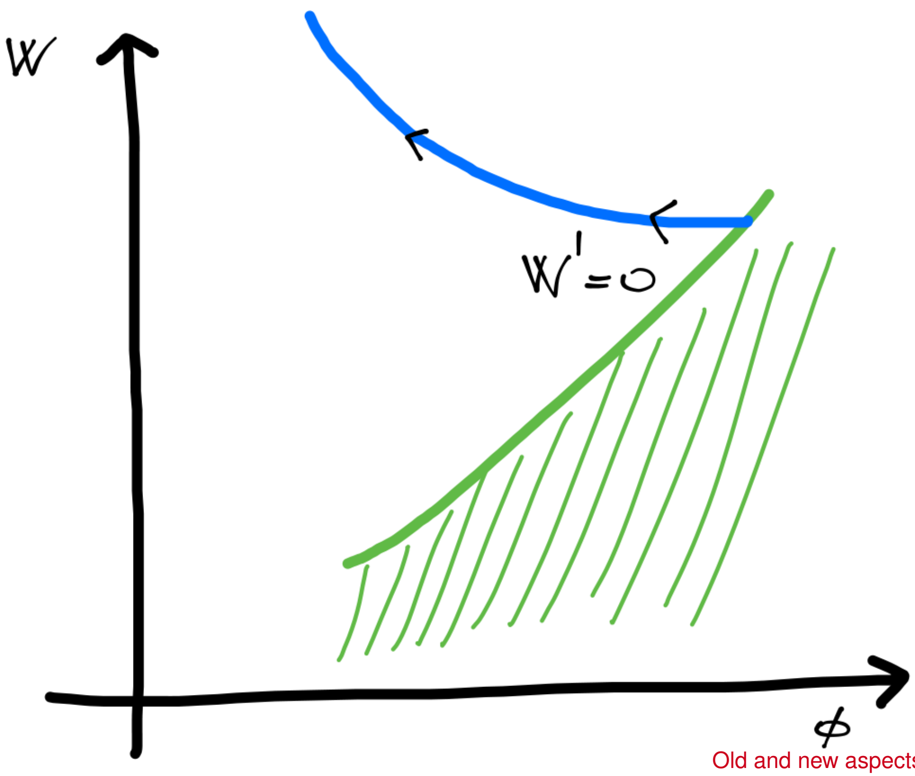

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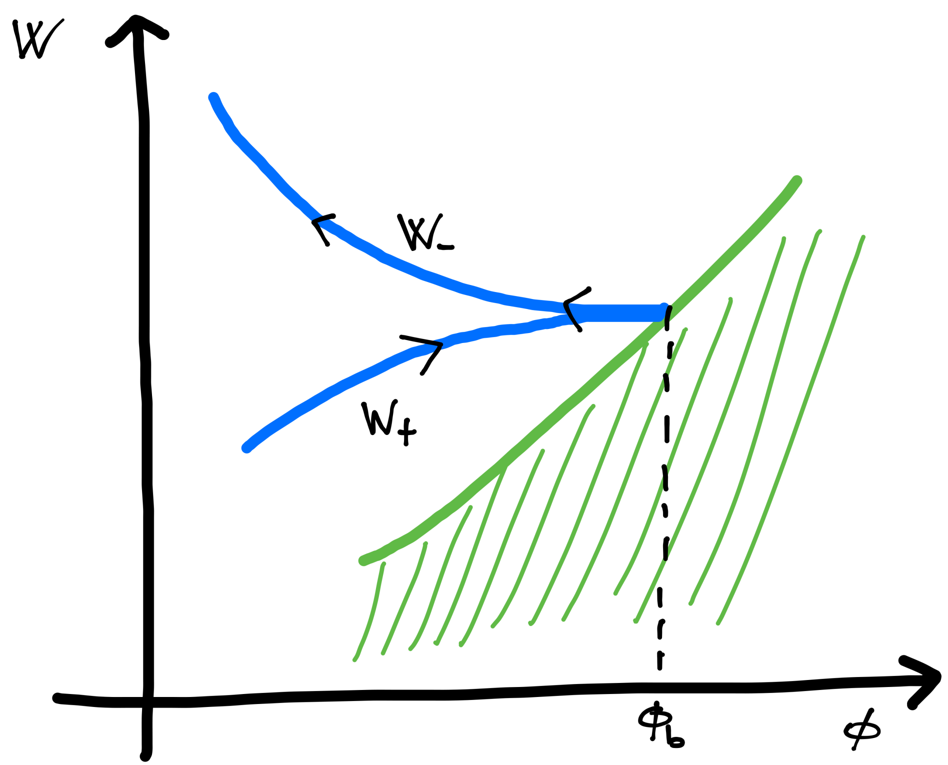
- What are the enpoints of the flow if $b > 2Q$?
  It never reaches $\Phi \to +\infty$, but it stops before.

Bouncing off the critical curve

At Φ_b the two branches can meet: can continue a W_+ type solution past Φ_b by glueing it with a W_- type solution:

$$W_\pm \simeq B(\Phi) \pm (\Phi_b - \Phi)^{3/2} \qquad \Phi < \Phi_b$$

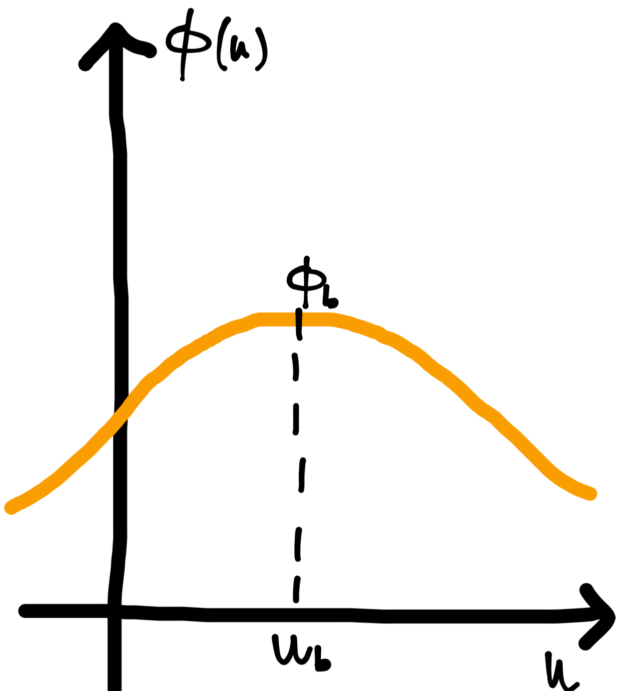


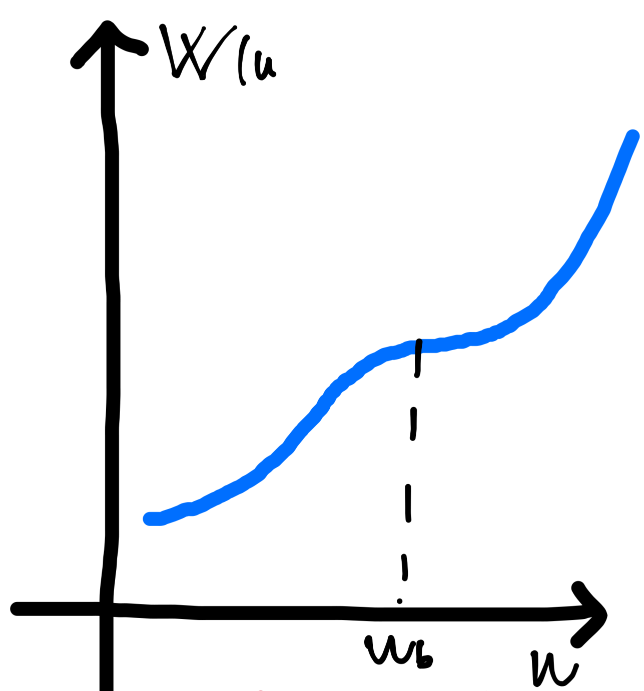
Regularity of bounces

- Although they look non-analytic in W, bounces are regular when written in terms of u

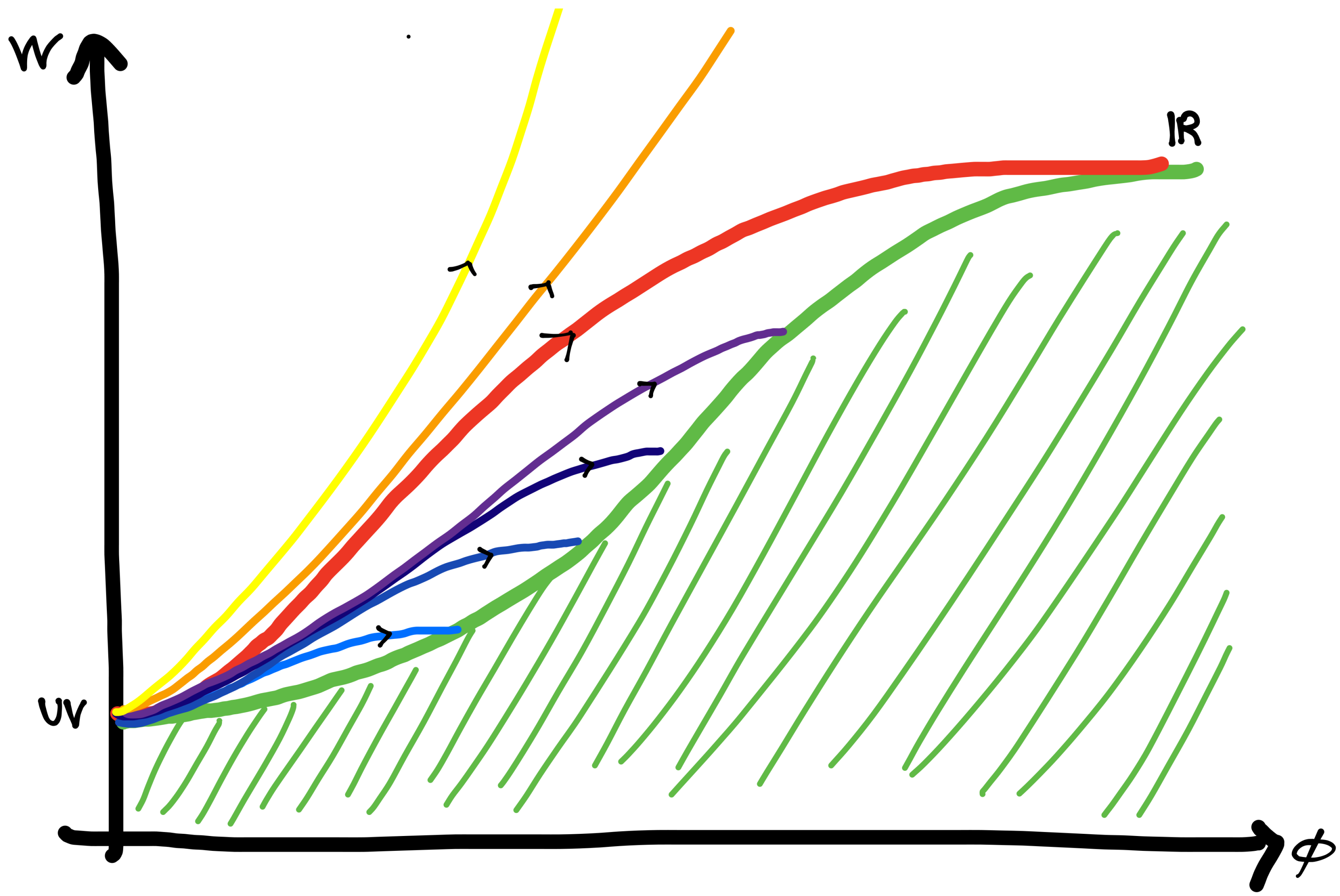
$$A(u) = A_0 + A_1(u - u_b) + \ldots, \quad \Phi(u) = \Phi_b + \frac{1}{2}\Phi_2(\Phi - \Phi_b)^2 + \ldots$$

- $\Phi(u)$ has an extremum at Φ_b where it turns around.
- $W(\Phi(u))$ is single-valued (and monotonic) as a function of u

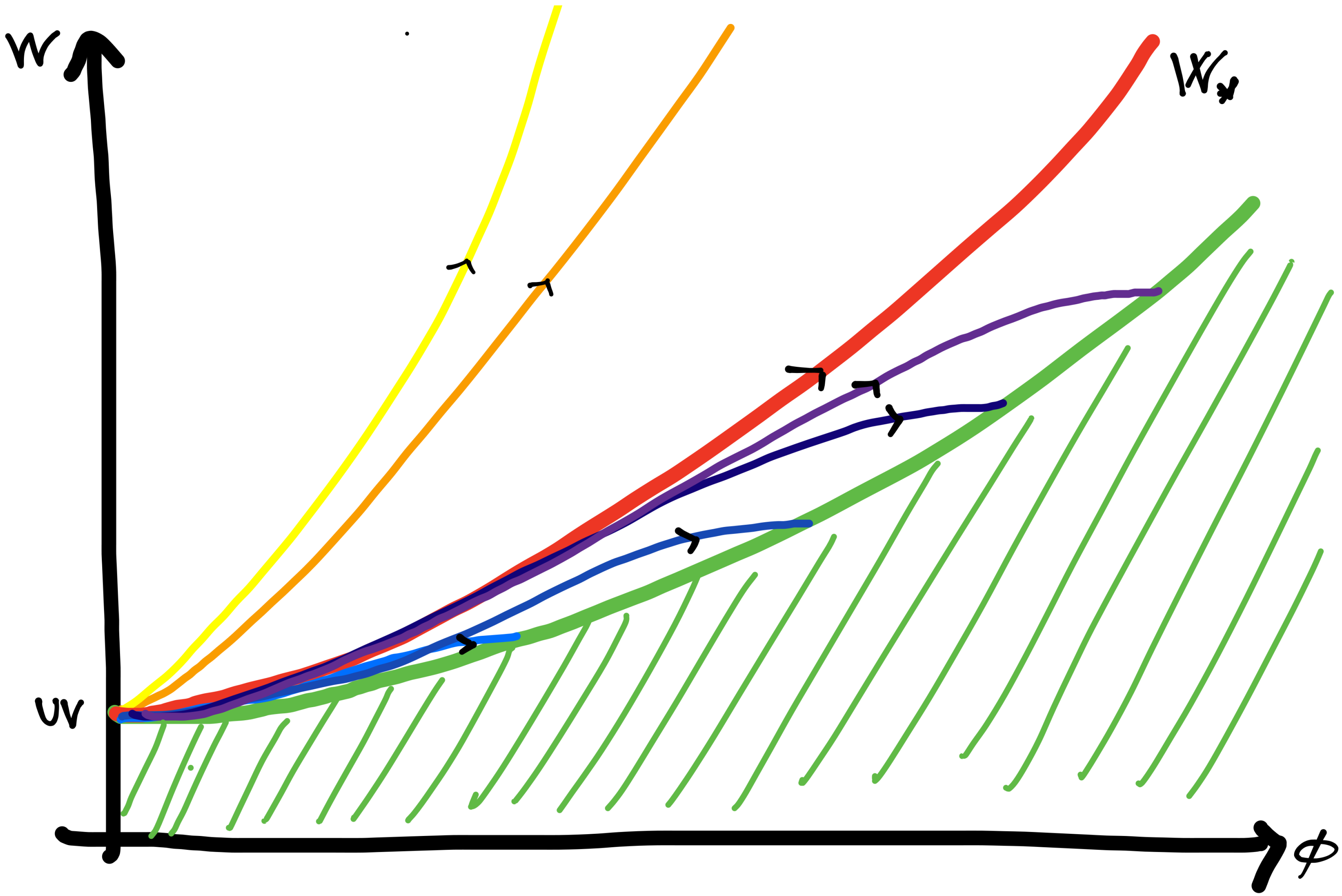




UV fixed point to IR fixed point



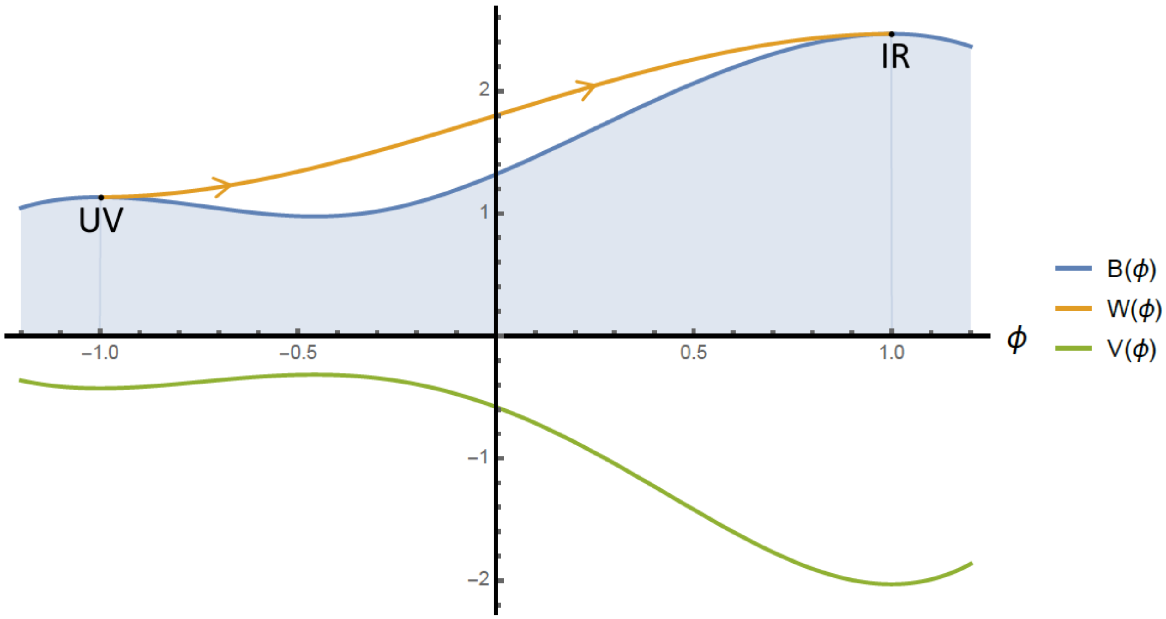
UV fixed point to good IR singularity



Exotic RG flows

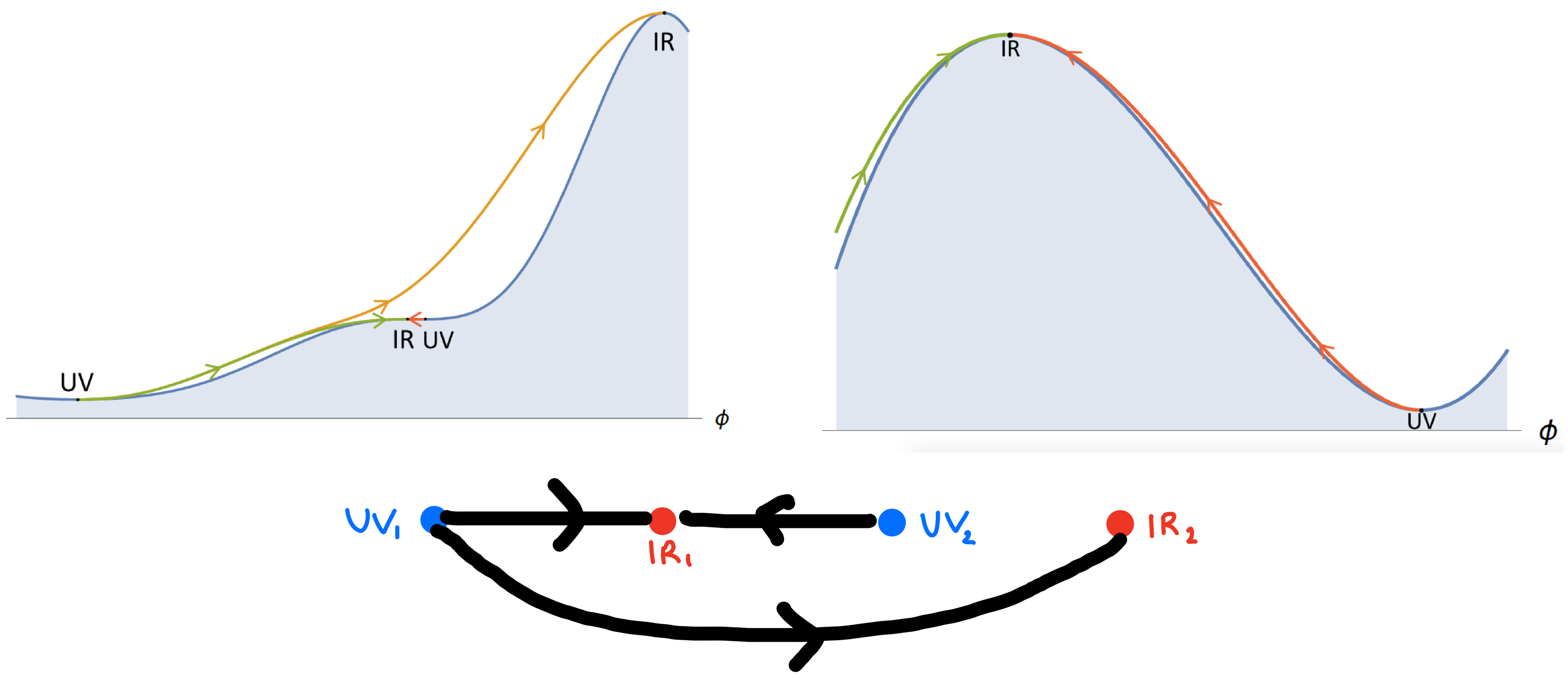
E.Kiritsis,FN,L.Silva Pimenta, to appear

Vev flow between maxima



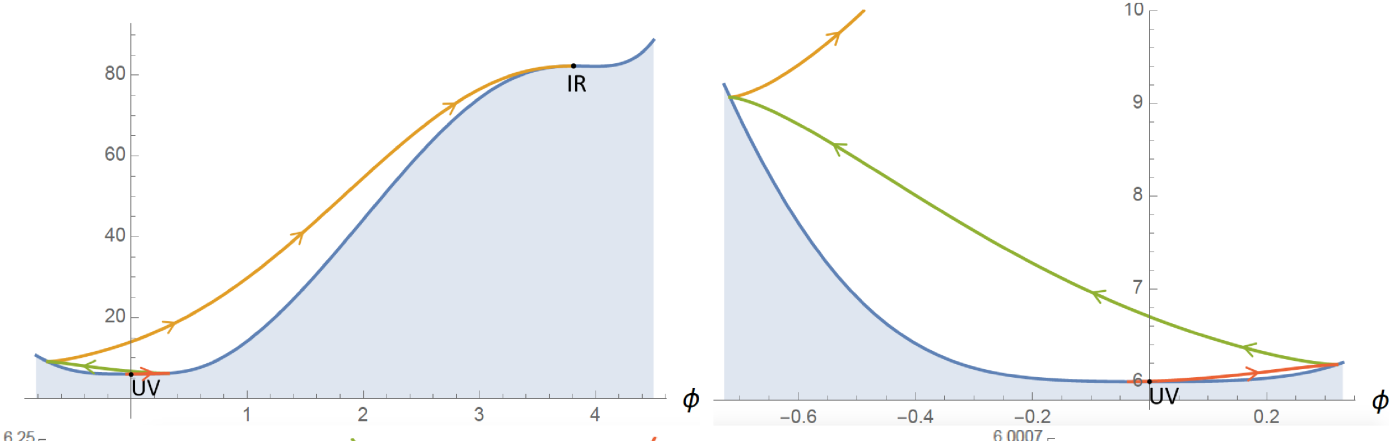
- Only the vev deformation flows out of the UV
- Potential needs to be fine-tuned

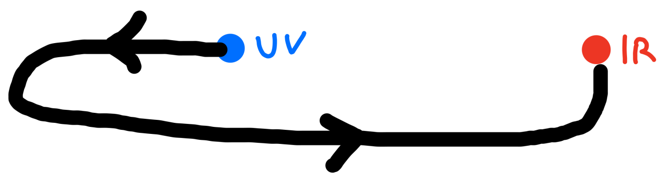
Skipping fixed points



- A flow can skip a fixed point and end up in a far IR extremum.
- The flow to the far IR point can exist if no flow from the near UV exists.
- The two solutions are dual to different vacua (different vevs) which affect the flow non-perturbatively.

Bouncing RG flows





- Solution bounces several times before reaching the IR

Bouncing in Field theory?

- Close to the bounce:

$$\beta(\Phi) \propto \sqrt{\Phi_b - \Phi}$$

- Locally the same as some proposed RG flows with *limit cycles*

$$\beta(g) \propto \sqrt{1 - g^2}$$

- Toy models with limit cycles exist (e.g. Wilson and Glazek, '93; Russian Doll model), but either they are non-unitary or they are not full field theories.
- Are there any full-fledge bouncing QFT examples?

Part II

Generating functional and Local RG

(based on work with E.Kiritsis and W.Li)

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Generating functional and Local RG

(based on work with E.Kiritsis and W.Li)

- Generating functional
- Local RG

On-shell action and counter-terms

For IR-regular solutions, the on-shell action is a UV-boundary term:

$$S_{on-shell} = \int d^d x \sqrt{-\gamma} W(\Phi)\Big|_{u_{UV}} \qquad \gamma_{\mu\nu} = e^{A(u)} \eta_{\mu\nu}$$

As $W \to W(0)$ in the UV, this is divergent as $A \to +\infty$. Using A as the radial coordinate:

$$S_{UV} \simeq \int d^d x e^{dA} W(\Phi(A)) + \ldots$$

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$$S_{UV} \simeq \int d^d x e^{dA} W(\Phi(A)) + \ldots$$

- The divergence can be cancelled by adding $S_{ct} = -\int \sqrt{\gamma} W_{ct}(\Phi)$ where W_{ct} is any solution of the superpotential equation. This works because the UV is an attractor for the superpotential equation.
- Different choices of W_{ct} correspond to different renormalization schemes.

Renormalized generating functional

We can define a UV-renormalized on-shell effective action by:

$$S^{(ren)} = \lim_{u_{UV} \to -\infty} \int d^d x \sqrt{\gamma} \left[W(\Phi) - W_{ct}(\Phi) \right] \Big|_{u_{UV}}$$

From the near-boundary expansion of $\gamma_{\mu\nu}$ and $W(\Phi)$:

$$S^{(ren)}[\alpha] = \int d^d x \, C^{(ren)} \alpha^{\frac{d}{d-\Delta}}, \qquad C^{(ren)} = C - C_{ct}$$

Papadimitriou ’07; Kititsis and Niarchos ’12

- To see the flow to the IR, one has to write it in terms of the field at a finite scale $\Phi(u)$
- For asymptotically non-AdS backgrounds, the UV source is not even well defined. But we can still proceed as above if we write $S^{(ren)}[\Phi(u)]$

Renormalized on-shell action revisited

$$S^{(ren)}[\alpha] = \int d^d x \, C^{(ren)} \, \alpha^{\frac{d}{d-\Delta}}$$

- Choosing α defines a particular flow solution $(A, \Phi_\alpha(A))$. We can invert this relation and write

$$\alpha = \alpha[A, \Phi(A)] \Rightarrow S^{(ren)}[A, \Phi(A)]$$

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$$\alpha = \alpha[A, \Phi(A)] \Rightarrow S^{(ren)}[A, \Phi(A)]$$

- For any fixed A, we can see $S^{(ren)}[A, \Phi]$ as an independent funcion of Φ.
- $S[A, \Phi]$ is constant along a radial flow:

$$\frac{d}{dA} S^{(ren)}[A, \Phi(A)] = \left[\frac{\partial}{\partial A} + \frac{d\Phi}{dA} \frac{\partial}{\partial \Phi} \right] S^{(ren)}[A, \Phi(A)] = 0$$

Renormalized on-shell action revisited

$$S^{(ren)}[A,\Phi] = \int d^dx e^{dA}[W - W_{ct}]_{A\to+\infty}$$

Renormalized on-shell action revisited

$$S^{(ren)}[A,\Phi] = C^{(ren)} \int d^d x\, e^{dA} \exp\left[-\frac{d}{2(d-1)}\int^{\Phi} \frac{W}{W'}\right]$$

Renormalized on-shell action revisited

$$S^{(ren)}[A,\Phi] = C^{(ren)} \int d^d x \, e^{dA} \exp\left[-\frac{d}{2(d-1)} \int^{\Phi} \frac{W}{W'} \right]$$

- This equation gives the explicit form of the renormalzied generating functional at a finite scale A and renormalized coupling Φ on any RG-glow trajectory.
- It can be used to derived trace identities for the renormalized operators

Trace identity

$$S^{(ren)}[A,\Phi] = C^{(ren)} \int d^d x \, e^{dA} \exp\left[-\frac{d}{2(d-1)} \int^{\Phi} \frac{W}{W'}\right]$$

- Scale transformation $x^\mu \to \lambda x^\mu$:

$$\langle T^\mu{}_\mu \rangle^{(ren)} = d \; C^{(ren)} e^{dA} \exp\left[-\frac{d}{2(d-1)} \int^{\Phi} \frac{W}{W'}\right]$$

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- Renormalized vev:

$$\langle O^{(ren)}\rangle = \frac{\partial S^{(ren)}}{\partial \Phi} = -\frac{d}{2(d-1)}\frac{W}{W'} C^{(ren)} e^{dA} \exp\left[-\frac{d}{2(d-1)}\int^{\Phi}\frac{W}{W'}\right]$$

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Renormalized trace identity:

$$\langle T^{\mu}{}_{\mu}\rangle = -2(d-1)\frac{W'}{W}\langle O^{(ren)}\rangle$$

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$$\beta(\Phi) = -2(d-1)\frac{W'}{W} = \frac{d\Phi}{dA},$$

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- identification $\mu \equiv e^{A}$ everywhere in the bulk.
- The β-function sees the whole geometry encoded in W.
- RG-invariance $\Leftrightarrow$ $S^{(ren)}[A,\Phi]$ constant along the radial flow.

Local RG flows

- The field theory data (coupling constants) Φ can be generalized to x^μ-dependent coupling *functions* $(\gamma_{\mu\nu}(x), \Phi(x))$ (non-homogeneous sources for GI operators).
- On the gravity side, Einstein's equations can be recast as first order flow equations for running local couplings $(\gamma_{\mu\nu}(x,u), \Phi(x,u))$ by using a derivative expansion, in terms of local covariant beta-functions:

$$\dot{\Phi}(x,u) = B_\Phi[\Phi,\gamma], \qquad \dot{\gamma}_{\mu\nu} = B_{\mu\nu}[\Phi,\gamma]$$

Local RG flows

Kiritsis, Li, F.N, 1401.8888

The data will be the d-dimensional metric $\gamma_{\mu\nu}(x,u)$ and scalar field $\Phi(x,u)$ evaluated on a space-time slice in the bulk.

Changing the slice corresponds to changing the RG scale.

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Changing the slice corresponds to changing the RG scale.

We take a solution with a general space-time metric $\gamma_{\mu\nu}(x,u)$, in ADM form:

$$ds^2 = N^2 du^2 + \gamma_{\mu\nu}(x)(dx^\mu + N^\mu du)(dx^\nu + N^\nu du), \qquad \Phi = \Phi(u,x)$$

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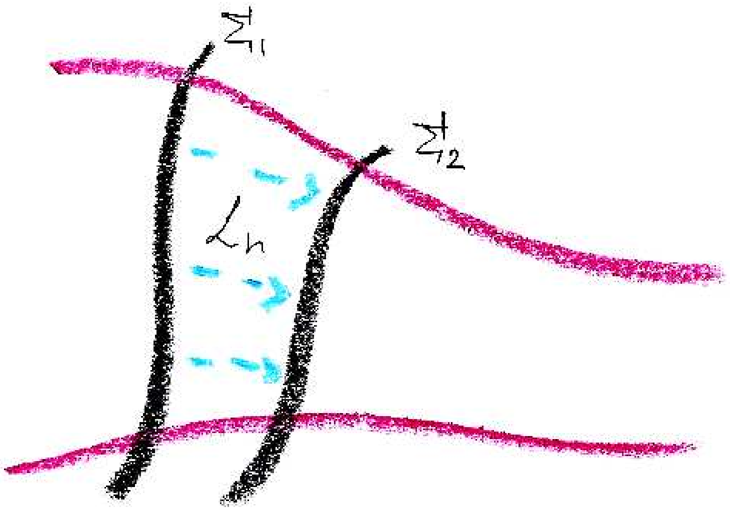
Set $N = 1$, $N_\mu = 0$ (FG gauge)

First order local flow equations



The flow equations tell how to go from one hypersurface at u to another one nearby at $u + \xi$, as a function only on the *invariants on the slice*.

$$\delta_\xi \gamma_{\mu\nu} = \xi\, \pounds_n \gamma_{\mu\nu} = \xi B_{\mu\nu}(\gamma_u, \Phi_u)$$
$$\delta_\xi \Phi = \xi\, \pounds_n \Phi = \xi B_\Phi(\gamma_u, \Phi_u)$$

Flow equations and derivative expansion

We write an ansatz for Lie derivative of the slice metric and scalar as a derivative expansion on the slice:

$$\dot{\gamma}_{\mu\nu} = g_1 \gamma_{\mu\nu} + g_2 R^{(\gamma)}_{\mu\nu} + g_3 \gamma_{\mu\nu} R^{(\gamma)} + g_4 \partial_\mu \Phi \partial_\nu \Phi + g_5 (\gamma^{\rho\eta} \partial_\rho \Phi \partial_\eta \Phi) \gamma_{\mu\nu}$$
$$+ g_6 \nabla_\mu \partial_\nu \Phi + g_7 (\gamma^{\rho\eta} \nabla_\rho \partial_\eta \Phi) \gamma_{\mu\nu} + \dots ,$$

$$\dot{\Phi} = h_1 + h_2 R^{(\gamma)} + h_3 \gamma^{\rho\eta} \partial_\rho \Phi \partial_\eta \Phi + h_4 \gamma^{\rho\eta} \nabla_\rho \partial_\eta \Phi + \dots ,$$

The functions $g_i(\Phi)$ and $h_i(\Phi)$ are determined using the constraints.

First order local flow equations

Imposing the constraints, the 2-derivative order flow equations are govenerd by only two functions $W(\Phi)$, $f(\Phi)$

$$\dot{\gamma}_{\mu\nu} = -\frac{W}{d-1}\gamma_{\mu\nu}$$

$$\dot{\Phi} = W'$$

First order local flow equations

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$$\begin{aligned}
\dot{\gamma}_{\mu\nu} &= -\frac{W}{d-1}\gamma_{\mu\nu} - \frac{1}{(d-1)}\left(fR + \frac{W}{2W'}f'(\gamma^{\rho\sigma}\partial_\rho\Phi\partial_\sigma\Phi)\right)\gamma_{\mu\nu} \\
&\quad +2fR_{\mu\nu} + \left(\frac{W}{W'}f' - 2f''\right)\partial_\mu\Phi\partial_\nu\Phi - 2f'\nabla_\mu\partial_\nu\Phi + \ldots \\
\dot{\Phi} &= W' - f'R + \frac{1}{2}\left(\frac{W}{W'}f'\right)'(\gamma^{\rho\eta}\partial_\rho\Phi\partial_\eta\Phi) + \frac{W}{W'}f'(\gamma^{\rho\eta}\nabla_\rho\partial_\eta\Phi) + \ldots
\end{aligned}$$

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\end{aligned}
$$

$W(\Phi)$ and $f(\Phi)$ are solutions of:

$$
\frac{d}{4(d-1)}W^2 - \frac{1}{2}W'^2 = -V, \qquad W'f' - \frac{d-2}{2(d-1)}Wf = 1
$$

Dynamical flow equations

- Flow equations controlled by the (zeroth-order) superpotential $W(\Phi)$, plus an additional scalar function $f(\Phi)$
- The new superpotential $f(\Phi)$ is otained from $W(\Phi)$, up to an integration constant, also fixed by IR regularity.
- The flow equations solve automaticaly the remaining (dynamical) Einstein's equation. Thus, the flow equation ansatz is consistent and encodes the full bulk dynamics.
- Does it describe all solutions which admit a derivative expansion around a Poincaré invariant background?

Radial flow vs. Local RG transformation

$$\pounds_n \gamma_{\mu\nu} = B_{\mu\nu}\left(\gamma_{\mu\nu}, \Phi\right), \qquad \pounds_n \Phi = B_{\Phi}\left(\gamma_{\mu\nu}, \Phi\right)$$

The Lie derivative says how the coupling change under a change in the slice. How is this related to a Weyl transformation?

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The change in the metric under $\pounds_n$ can be divided up into:

- a Weyl transfomation with parameter $\sigma(x) = (2d)^{-1}\gamma^{\mu\nu}B_{\mu\nu}$
- a volume-preserving transformation $\hat{B}_{\mu\nu}$

$$\pounds_n \gamma_{\mu\nu} = 2\sigma\gamma_{\mu\nu} + \hat{B}_{\mu\nu}, \quad \gamma^{\mu\nu}\hat{B}_{\mu\nu} = 0$$

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On a functional of $(\gamma_{\mu\nu}(x), \Phi(x))$:

$$\pounds_n = \int dx\, \sigma(x)\Delta(x), \quad \Delta(x) = 2\gamma_{\mu\nu}\frac{\delta}{\delta\gamma_{\mu\nu}} + \beta_{\mu\nu}\frac{\delta}{\delta\gamma_{\mu\nu}} + \beta_\Phi\frac{\delta}{\delta\phi}$$

$$\beta_{\mu\nu} = \frac{\hat{B}_{\mu\nu}}{\sigma}, \qquad \beta_\Phi = \frac{B_\Phi}{\sigma}$$

Beta-functions

$$\Delta\gamma_{\mu\nu} = 2\gamma_{\mu\nu} + \beta_{\mu\nu}, \qquad \Delta\Phi = \beta_\Phi$$

$$\beta_\Phi = -2(d-1)\frac{W'}{W} - \frac{2(d-1)}{W}\left(f' + \frac{W'}{W}f\right)R + \dots$$

$$\beta_{\mu\nu} = \frac{f}{W}\left[R_{\mu\nu} - \frac{1}{d}\gamma_{\mu\nu}R\right] + \dots$$

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- To zeroth order we recover the results of the homogeneous calculation
- The metric gets an anomalous change beyond a Weyl rescaling due to the curvature terms. This resembles the case of Ricci flows.

Generating functional for local sources

The on-shell action is a slice-covariant boundary term:

$$S = \int d^d x \int du \sqrt{g} \left(R^{(g)} - (\partial\Phi)^2 - V \right)$$

Papadimitriou '11

Generating functional for local sources

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Papadimitriou '11

The order-two terms introduce a new divergence $\sim e^{(d-2)A}$.

- We can cancel the UV divergences with three independent covariant boundary counterterms for the cosmological term, Einstein term and scalar kinetic term. They are specified by choosing three solutions $W_{ct}, f_{ct}^{(1)}, f_{ct}^{(2)}$ of the superpotential equations.
- The divergence is cancelled provided the UV is an attractor (so that a continous family of solutions with the same UV asymptotics exists).

Renormalized generating functional

- The generating functional has the local covariant form:

$$\log \mathcal{Z}^{(ren)}[\gamma,\Phi] = \int d^dx\sqrt{\gamma}\left[C_0 Z_0(\Phi) + C_1 Z_1(\Phi) R + C_2 Z_2(\Phi)(\partial\Phi)^2\right] + \ldots$$

- $Z_i(\Phi)$ are complicated but known functions of Φ, written in terms of W and f. Up to the three scheme-dependent multiplicative quantities C_i, the action is completely fixed and gives the full non-linear result up to second derivative order.

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- $Z_i(\Phi)$ are complicated but known functions of Φ, written in terms of W and f. Up to the three scheme-dependent multiplicative quantities C_i, the action is completely fixed and gives the full non-linear result up to second derivative order.
- By Legendre transform, this can be turned in the quantum effective action $\Gamma[\gamma_{\mu\nu}, O]$, that gives the dynamics of condensates of composite operators.

$$\Gamma^{(ren)}[\gamma_{\mu\nu}, O] = \int d^d x \sqrt{\gamma} \left[C_0 \Gamma_0(O) + C_1 \Gamma_1(O) R + C_2 \Gamma_2(O)(\partial O)^2 \right]$$

in terms of known functions $\Gamma_i(O)$ depending only on W.

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- $Z_i(\Phi)$ are complicated but known functions of Φ, written in terms of W and f. Up to the three scheme-dependent multiplicative quantities C_i, the action is completely fixed and gives the full non-linear result up to second derivative order.
- $\log \mathcal{Z}^{(ren)}$ obeys the local RG equation up to local Weyl anomalies

$$\int \left(2\gamma^{\mu\nu}\frac{\delta}{\delta\gamma^{\mu\nu}} - \beta_{\mu\nu}\frac{\delta}{\delta\gamma_{\mu\nu}} - \beta_\Phi \frac{\delta}{\delta\phi}\right)\mathcal{Z}^{(ren)} = 0$$

with the holographic β-functions appearing.

Applications and further developements

Pheno

- Use the holographic quantum action functional to model 4d strongly coupled sectors coupled to weakly interacting physics to do pheno (semi-holography).

$$S = S_{weak}[\phi] + S_{strong} + \int d^4x\, \phi\, O_{sc}$$

$$\Downarrow$$

$$S^{eff} = S_{weak}[\phi] + \log Z[\phi]$$

- The bulk is integrated out. Description purely 4d. $W(\Phi)$ only input in the model

Pheno

- Model building with condensates (as opposed to elementary fields) via the quantum effective action:

$$\Gamma^{(ren)}[\gamma_{\mu\nu}, O] = \int d^d x \sqrt{\gamma} \left[C_0 \Gamma_0(O) + C_1 \Gamma_1(O) R + C_2 \Gamma_2(O) (\partial O)^2 \right]$$

- e.g. inflation driven by a condensate: better UV properties.

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- e.g. inflation driven by a condensate: better UV properties.
- Example: Quantum effective potential for RG-invariant gluonic operator in holographic YM (Kiritsis, Li, FN '14)

$$\Gamma_0[\mathcal{T}] = \int d^4x \frac{\mathcal{T}}{4}\left(\log\frac{\mathcal{T}}{\Lambda^4} - 1\right) \qquad \mathcal{T} \equiv \frac{\beta(\lambda)}{\lambda^2} TrF^2$$

Non-vacuum RG-flows

- The solutions we have seen preseve Poincaré invariance. Can generalize the flow equation formalism to less symmetric situations:
  - Finite temperature and density
  - Time-dependent solutions
  - Curved slice geometries

Temperature

The vacuum (homogeneous) solution breaks Poincaré invariance

$$ds^2 = du^2 + \left[f(u)dt^2 + e^{A(u)}dx_i^2\right]$$

- Must have a different flow in the tt and ij directions.
- Three superporentials W_t, W_x, W_Φ, or a single one with A-dependence, $W(\Phi, A)$ (Papadimitriou, '15)

Temperature

The vacuum (homogeneous) solution breaks Poincaré invariance

$$ds^2 = du^2 + \left[f(u,x)dt^2 + \gamma_{ij}dx^i dx^j \right]$$

- Must have a different flow in the tt and ij directions.
- Three superporentials W_t, W_x, W_Φ, or a single one with A-dependence, $W(\Phi, A)$ (Papadimitriou, '15)
- Challenges: covariantize w.r.t. to the spatial metric γ_{ij}; Einstein's constraints imposing momentum conservation become non-trivial.
- A covariant approach with spacetime-dependent fields and a derivative expansion would generalize fluid/gravity beyond dynamics of (pseudo-)goldstone bosons.

Curvature

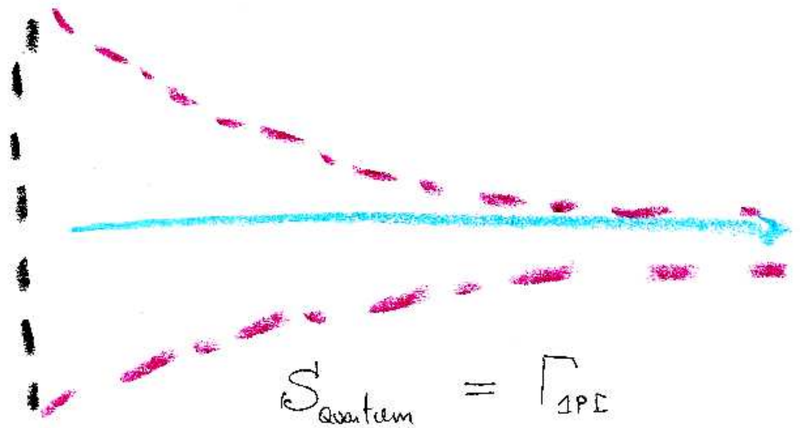
- Look at RG flows where the spacetime metric has a (not necessarily small) curvature

$$ds^2 = du^2 + \gamma_{\mu\nu}(u,x)dx^\mu dx^\nu$$

- Same symmetries as the black hole background.
- Possibly relevant for dual gravity duals of cosmologically active strongly coupled sectors
- Cosmology as an RG-flow ?

Wilsonian picture

So far we have computed the quantum effective action by integrating the solution from a UV cutoff to the IR.



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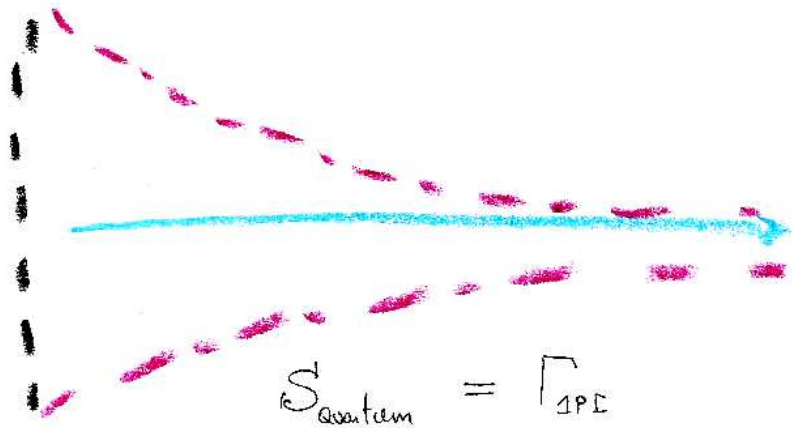
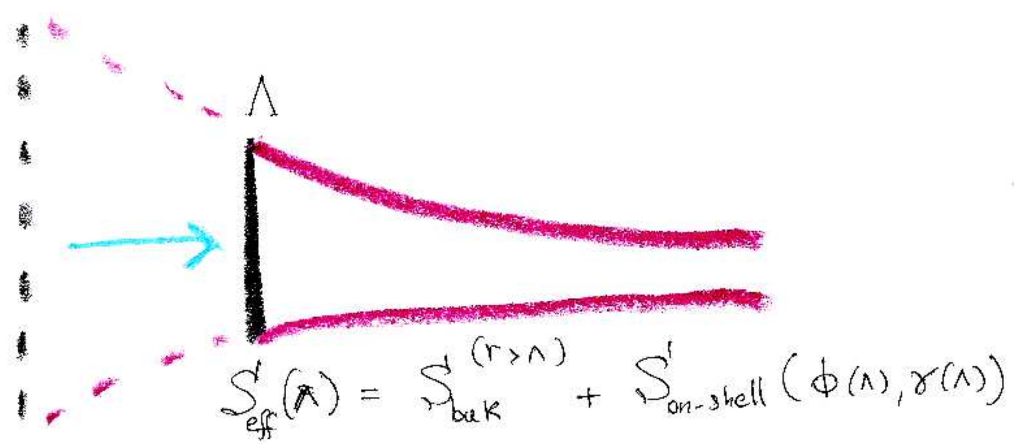


What about the Wilsonian action?



Wilsonian generating functional

We can indtruduce a variable IR cut-off slice at $u_{IR} = u(\Lambda)$

$$S = \int_{u_{UV}}^{u(\Lambda)} du\, \mathcal{L}_{grav} + S_{ct}\Big|_{u_{UV}}$$

The generating functional receives an extra contribution from $u(\Lambda)$:

$$S^{Wilson} = S^{(ren)} + S_{\Lambda}$$

Wilsonian generating functional

We can indtruduce a variable IR cut-off slice at $u_{IR} = u(\Lambda)$

$$S = \int d^dx \left[\sqrt{\gamma} \left(\tilde{W} - \tilde{f} R - \left(\frac{\tilde{W}}{\tilde{W}'} \tilde{f}' \right) \frac{1}{2} \gamma^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \ldots \right) \right]_{u_{UV}}^{u(\Lambda)} + S_{ct} \Big|_{u_{UV}}$$

The generating functional receives an extra contribution from $u(\Lambda)$:

$$S^{Wilson} = S^{(ren)} + S_\Lambda$$

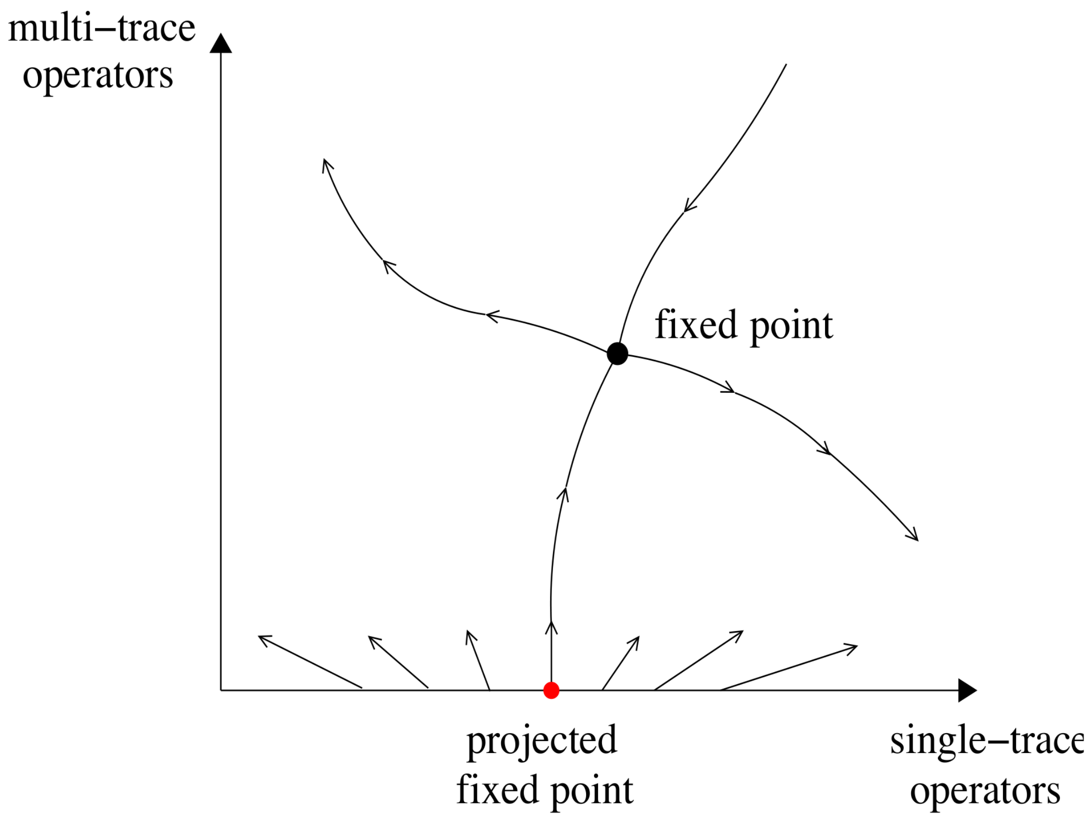
$$S_\Lambda = \int d^dx \sqrt{\gamma} \left(\tilde{W} - \tilde{f} R - \left(\frac{\tilde{W}}{\tilde{W}'} \tilde{f}' \right) \frac{1}{2} \gamma^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \ldots \right)_\Lambda$$

In this case the superpotentials are chosen based on IR boundary conditions at Λ. According to Polchinksi et Heemskerk we should integrate over possible W.

Emergent gravity from Quantum local RG

(S.S. Lee, 2013)

- Quantum RG: projecting the RG on the submanifold of single-trace couplings.



Emergent gravity from Quantum local RG

(S.S. Lee, 2013)

- Quantum RG: projecting the RG on the submanifold of single-trace couplings.
- The coupling constants acquire kinetic terms w.r.t. the RG-direction: sources become dynamical field in a higher dimensional space. $\Rightarrow$ Holography!
- How does one recover Einstein's gravity? How do large-N and strong coupling play a role (except for the fact that multitrace are suppressed) ?
- Study the structure of local holographic RG to connect with FT quantum RG
- How much can we reconstruct of the gravity theory from knowing the covatiant β-functions only?

Understanding regularity

- How exactly does the IR regularity condition select a solution?
  - Is the mechanism dynamical (suppression in the full bulk path integral)?
  - Is it related to UV completions (or lack thereof) ? E.g, only the regular solutions may have a string theory uplift.
  - How does this play out in the Wilsonian framework, where IR regularity is not imposed?
  - Possible role played by Gubser-like criteria for singularities.

A new type of σ-model?

Let us take another look at the flow equations:

$$\dot{\gamma}_{\mu\nu} = W\,\gamma_{\mu\nu} + f\,R_{\mu\nu} + g\partial_\mu\Phi\partial_\nu\Phi + \ldots$$
$$\dot{\Phi} = W' + hR + k(\partial\Phi)^2 + \ldots$$

- They have a striking similarity to the worldsheet β-function equations of perturbative string theory. There, setting the r.h.s. to zero enforces conformal invariance. Here, it enforces translation invariance along the space-time slices.
- $\gamma_{\mu\nu}(x, r)$ and $\Phi(x, \gamma)$ are *spacetime-dependent* coupling of the dual four-dimensional field theory. The r.h.s. of the flow eqs. are their $beta$-functions

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- The *fixed points* of these equations solve the equations of motion derived from the effective action. In string theory, they determine consistent string background. What is their meaning here? We already know some fixed points: they are the AdS fixed points corresponding to conformal theories. Are non-homogeneous solutions some kind of new CFTs ?
- Are we looking at a new type of sigma model whose “worldsheet” are gauge theories?

  (see work by E. Kiritsis ’14 for some related thoughts)