Holographic Theories in Strong Magnetic Fields (review)

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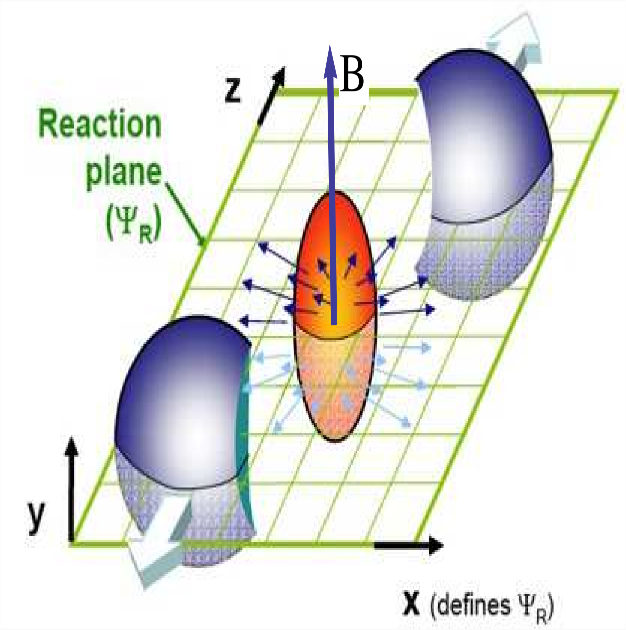
Rich physics for both QCD and CMT

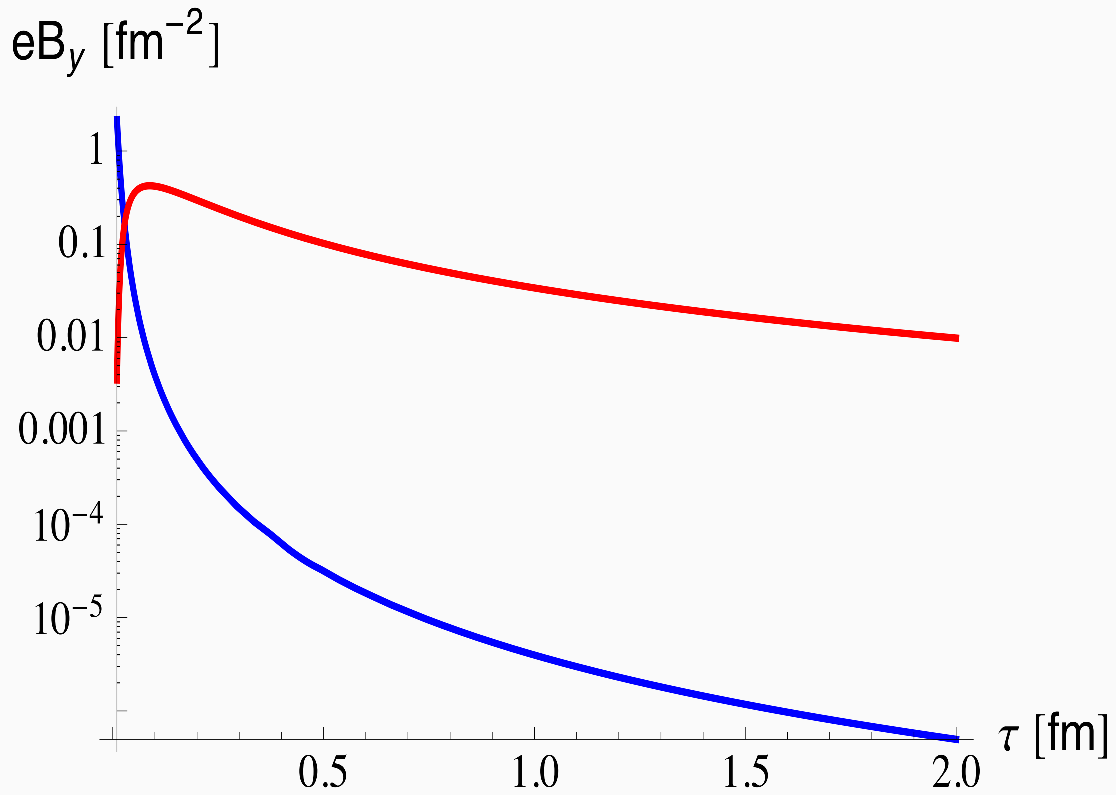
- Anomalous transport
- Magnetically induced currents in QGP
- Magnetic catalysis - inverse magnetic catalysis
- Equilibration/thermalization in strongly interacting matter
- Quantum critical points induced by B
- Rich phase diagram in the (T, B, μ) space

Heavy ion collisions and magnetic fields



- Initial magnitude of B
- Bio-Savart: $B_0 \sim \gamma Z e \frac{b}{R^3} \Rightarrow \sim 10^{18} (10^{19})$ G at RHIC (LHC).
- $B_0 \sim 10^{10} - 10^{13}$G (neutron stars), 10^{15} (magnetars)
- More relevantly $eB \approx 5 - 15 \times m_\pi^2$ RHIC (LHC).

Time profile of B at LHC



- with $\sigma = 0.023\text{fm}^{-1}$ and with $\sigma = 0$:

Anomalous transport

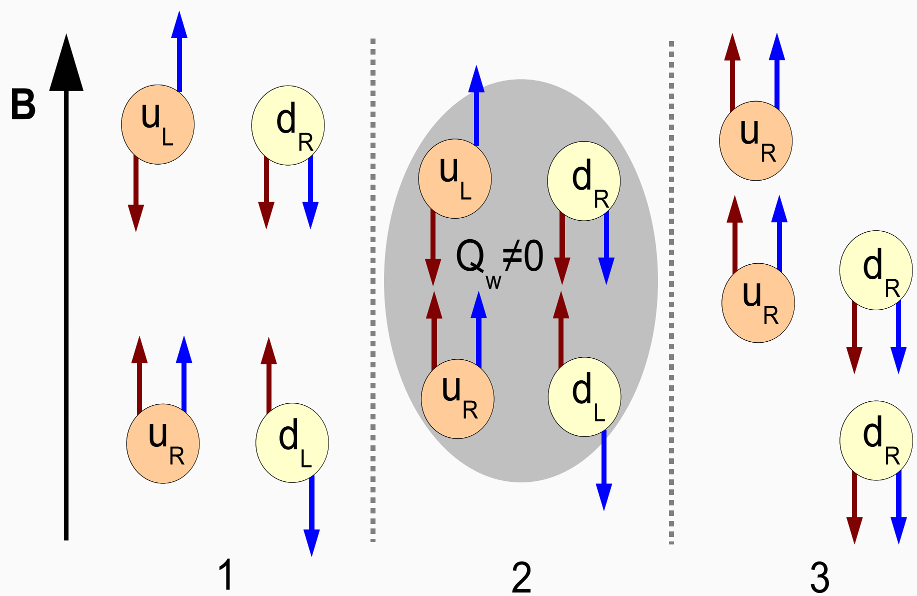
- Chiral magnetic effect
- (Non)-renormalization of anomalous transport
- Sphaleron decay rate and axial chemical potential

Chiral Anomaly in QCD

- *massless* fermions are chiral: left and right-handed quarks.
- *Classically* QGP chiral symmetric: $N_L = N_R$
  as $T \approx 500$ MeV $\gg m_u,\ m_d$
- Axial current $\partial_\mu J^{\mu 5} = \partial_\mu \left(\langle \bar{\psi} \gamma^\mu \psi \rangle_L - \langle \bar{\psi} \gamma^\mu \psi \rangle_R \right) = 0$
- However there is a QM anomaly: $\partial_\mu j^{\mu 5} = -\frac{N_f g^2}{16\pi^2} \mathrm{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})$.
- Due to topologically non-trivial gluon configurations
- Gluon winding number: $Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4 x\, F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \in \mathbb{Z}$.
- Atiyah-Singer index theorem: $\Delta(N_L - N_R) = 2 N_f Q_w$

Chiral Magnetic Current

- Under B spin degeneracy of quarks lifted due $H \sim -q\vec{s}\cdot\vec{B}$:



- Macroscopic manifestation of the axial anomaly
- Anomalous magnetohydrodynamics: $\vec{J} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$

  Kharzeev et al ’07
- μ_5 encodes the imbalance $N_L \neq N_R$

Anomalous transport in a chiral plasma

- A relativistic chiral plasma with velocity $\vec{u}(x)$

  magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$ and vorticity $\vec{\omega} = \langle \vec{\nabla} \times \vec{u} \rangle$.

- As a result of the chiral anomaly

$$\partial_\mu J^{5\mu} = a_1 \, F \wedge F + a_2 \, R \wedge R + a_3 \, \mathrm{Tr}(G \wedge G) \, ,$$

  Anomalous electric currents are produced:

$$\vec{J} = \sigma_B \vec{B} + \sigma_V \vec{w} \, .$$

  with $\sigma_B \sim a_1, \; a_3$ and $\sigma_V \sim a_2$.

- Coefficients a_1, a_2 , a_3 are one-loop exact Adler, Bardeen, '69
- Do σ_B and σ_V receive radiative corrections or not?
- Answer by AdS/CFT: Corrections due to a_3 at order $\mathcal{O}(N_f/N_c)$

Hydrodynamic description

A plasma with velocity u^μ, energy ϵ, pressure P, charge density n, axial charge density n_5, chemical potential μ, axial chemical potential μ_5, magnetic field B^μ and vorticity ω^μ and no gluonic anomaly $a_3 = 0$ (no gluonic contribution):

- Constitutive relations:

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} \\ J^\mu &= n\, u^\mu + \nu^\mu \,, \qquad \nu^\mu = \sigma_B B^\mu + \sigma_V \omega^\mu \\ J^{5\mu} &= n_5\, u^\mu + \nu_5^\mu \,, \qquad \nu_5^\mu = \sigma_{B,5} B^\mu + \sigma_{V,5} \omega^\mu \end{aligned}$$

with $\tau^{\mu\nu}$ and ν^μ the anomalous contributions.

- Equations of motion:

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\alpha} J_\alpha \,, \\ \partial_\mu J^{5\mu} &= a_1 \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} \,, \qquad \partial_\mu J^\mu = 0 \,. \end{aligned}$$

Anomalous conductivities

- Require positivity of the entropy current: $\partial_\mu (s u^\mu) \geq 0$:

  Son, Surowka '09

  $$\sigma_B = a_1 \, \mu_5 \, , \qquad \sigma_{B,5} = a_1 \, \mu$$
  $$\sigma_V = a_1 \, \mu \mu_5 \, , \qquad \sigma_{V,5} = \frac{a_1}{2}(\mu^2 + \mu_5^2) + C T^2$$

- C is due to the mixed axial-gravitational anomaly, thus $C = 0$ when $a_2 = 0$
- Value of C is undetermined. Neimann, Oz '09
- σ_B, $\sigma_{B,5}$ and σ_V is unrenormalized
- $\sigma_{V,5}$ may or may not be renormalized depending on C.

Field theory arguments

Kubo formulae:

- $\vec{B}$ and $\vec{\omega}$ in the x-direction
- Electric current in the x-direction:

$$\sigma_B = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^x J^z \rangle \,,$$

$$\sigma_V = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^x T^{0z} \rangle \,,$$

- Axial current in the x-direction:

$$\sigma_{B,5} = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^{5^x} J^z \rangle \,,$$

$$\sigma_{V,5} = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^{5^x} T^{0z} \rangle \,.$$

- Use these formulae in the field theory and holographic calculations.

Relation to anomalies Jensen '12, Buividovich '13

- Anomalous 2pf's at constant μ, μ_5 or g_{00} background related to the anomalous 3pfs. For example:
- $\frac{\partial}{\partial\mu}\langle J^x(0)J^{5z}(k_y)\rangle_\mu\Big|_{\mu=0} = -\Gamma^{VVA}_{x0z}(0,k_y)$
- $\frac{\partial}{\partial\mu_5}\langle J^x(0)J^{z}(k_y)\rangle_\mu\Big|_{\mu_5=0} = -\Gamma^{VVA}_{zx0}(k_y,-k_y)$
- Γ^{VVA} is strongly constrained by the vector and axial Ward identities
- Assuming CPT invariance,
- For $|k_y| \ll \mu,\ \mu_5$ anomalous conductivities fixed almost completely: $\sigma_B = a_1\,\mu_5\,, \qquad \sigma_{B,5} = a_1\,\mu$
  $\sigma_V = a_1\,\mu\mu_5\,, \qquad \sigma_{V,5} = \frac{a_1}{2}(\mu^2+\mu_5^2) + CT^2$
- Complete agreement with hydrodynamics
- C still unfixed

Effective field theory on the cone

Jensen, Loganayagam, Yarom '13

- Consider generic 4D theory on a cone $\times R^2$:
  $ds^2 = dr^2 + r^2 d\tau^2 + dR_2, \qquad \tau \sim \tau + 2\pi\delta$
  with $U(1)^3$ and mixed axial-GR anomaly.
- Assume finite static screening lengths:
  $\langle \mathcal{O}(\tau, x) \mathcal{O}(\tau, 0) \rangle \sim \exp(-|x|/\xi)\,.$
  $\Rightarrow W$ is analytic around $k = 0$.
- Euclidean effective action of static sources W Taylor expandable in k:
- $W = \int d^4x \sqrt{-g} \left(P(T, \mu) + C T X_1^0 + a_2 j_m^\mu + \cdots \right)$
  where $X_1^\mu = T(B^\mu + \mu_5 \omega^\mu)$ and $j_m = j_m(u^\mu, g_{\mu\nu}, \omega^\mu)$.
- Calculate $T^{\tau\, r}$ from W : $T^{\tau\, r} \propto B(C + 8\pi^2\delta^2 a_2)$
- Require translational invariance at $\delta \to 1$: $C = -8\pi^2 a_2$
- C is fixed completely!

Validity of the EFT arguments

Jensen, Loganayagam, Yarom '13

- The geometry cone $\times R^2$ is singular at $r = 0$
- The effective action W_δ may not be continuous as $\delta \to 1$:
  - Possible states localized at $r = 0$ such as twisted states for $\delta \neq 1$
  - Breakdown of the derivative expansion $W_\delta = W_0 + kW_1 + k^2 W_2 + \cdots$
- JLY argues non of these problems arise as continuity required away from $r = 0$.
- However the arguments are complicated
- Desirable to check this directly in holography

Holographic approach Landsteiner et al '11

- Let's illustrate the calculation in the conformal plasma:
- First we ignore dynamical glue i.e. set $a_3 = 0$.
- The action:

$$S = \frac{1}{16\pi G}\int_M \sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F^2\right] + a_1\, A\wedge F\wedge F + a_2\, A\wedge R\wedge R + \cdots$$

- Solution: AdS-RN blackhole with gauge field A
- Fluctuate $\Phi^I_k(r) = \Big(A_x(r),\, h^x_t(r),\, A_z(r),\, h^z_t(r)\Big)$, with $k = k_y$.
- Calculate the two-pfs $G_{IJ}(k)$, in the limit $k \to 0$

$$\sigma_B = \frac{\mu}{4\pi^2}, \qquad \sigma_V = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}.$$

- Confirms the generic form derived in FT and hydro above!
- Fixes $C = \frac{1}{24}$ and agrees with the EFT result $C = -8\pi^2 a_2$!

An example with phase transition

U.G., A. Jansen '14

- Want to check validity of the EFT arguments in a theory with conf/deconf. transition
- In flat space this requires an intrinsic scale "Λ_{QCD}"
- Break conformality by $\langle\mathcal{O}\rangle \neq 0 \Rightarrow$ non-trivial bulk scalar Φ
- $S = -\frac{1}{16\pi G}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\nabla\Phi)^2 - V(\Phi) - Z(\Phi)F^2\right) + \cdots$

  Gao, Zhang '06
- $V(\Phi) = -\frac{3}{(2+\alpha^2)^2}\left\{4\alpha^2(\alpha^2-1)e^{-\frac{8\Phi}{3\alpha}} + 4(4-\alpha^2)e^{\frac{4\alpha\Phi}{3}} + 24\alpha^2 e^{-\frac{2(2-\alpha^2)\Phi}{3\alpha}}\right\}$,

  $Z(\Phi) = e^{-\frac{4}{3}\alpha\Phi}$.
- For $\alpha = 0$ reduces to conformal plasma.
- Expand V near minimum $\Phi = 0 \Rightarrow m^2 = -\frac{32}{3}$.
- Deformation of $\mathcal{N} = 4$ $\langle\mathcal{O}\rangle$ with $\Delta_{\mathcal{O}} = 2$ regardless of α
- Analytic, dilatonic and charged, asymptotically AdS BH

An analytic BH: details

- $ds^2 = -N^2(r)\bar{f}^2(r)dt^2 + \frac{r^2 dr^2}{(r^2+b^2)\bar{f}^2(r)} + (r^2+b^2)R^2(r)dR_3\,,$
- with $N^2(r) = \Gamma^{-\gamma},\ \bar{f}^2(r) = \frac{r^2+b^2}{l^2}\Gamma^{2\gamma} - \frac{c^2}{r^2+b^2}\Gamma^{1-\gamma},$
  $\Phi(r) = -\frac{3}{4}\sqrt{\gamma(2-2\gamma)}\log\Gamma,\ R^2(r) = \Gamma^{\gamma},\ \Gamma = \frac{r^2}{r^2+b^2}\,.$
  $A_t = \mu - \frac{Q}{2(r^2+b^2)}, \qquad \gamma = \frac{\alpha^2}{2+\alpha^2}, \qquad Q = \sqrt{3(1-\gamma)}bc\,.$
- Integration constants: $c = 4\pi GTS + \mu Q$, and $b \propto \Lambda_{QCD}$

Thermodynamics: U.G., A. Jansen '14

- Corresponding thermal gas obtained by $c \to 0, b = finite.$
- Hawking-Page transition between BH and TG at finite T_c only for $\gamma = 2/3$:
- $\Delta G = M - \mu Q - TS \approx -\frac{2\pi^3 V_3}{3G} T_c^3 (T - T_c), \qquad T \to T_c\,,$
  with $T_c = b/2\pi\,.$

- Anomalous conductivities U.G., A. Jansen '14

$$\langle J^x J^z \rangle = -\frac{i\kappa k \rho_h}{\sqrt{2}\pi G}\sqrt{1-\xi}v\left(1-v^2\right)^{\xi},$$

$$\langle J^x {T_t}^z \rangle = \frac{i\kappa k v^2 \rho_h^2}{2\pi G}(1-\xi)\left(1-v^2\right)^{2\xi} + \frac{2ik\lambda\rho_h^2}{\pi G}\left(1-v^2\right)^{2\xi-1}\left((\xi-1)v^2+1\right)^2 \text{, etc.}$$

with ρ_h horizon location, $v \sim \Lambda_{QCD}$, $\xi = \frac{\alpha^2-1}{\alpha^2+2}$

- Yet, when expressed in $T = T(\rho_h, v, \alpha)$ and $\mu = \mu(\rho_h, v, \alpha)$:

$$\sigma_B = \frac{\mu}{4\pi^2}, \qquad \sigma_V = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}.$$

- A non-trivial check on the EFT and hydro arguments

Horizon universality

J. Tarrio, U.G. '14

- A generic background:
  $ds^2 = -g_{tt}(r)dt^2 + g_{xx}d\vec{x}^2 + g_{rr}dr^2, \; A = A_t(r), \; \Phi = \Phi(r)$
- Use a trick by A. Donos, J. Gauntlett '14
  $\delta A_x(y,r) = -B_z^5\, y + \alpha_x(r), \; \delta A_z(y,r) = B_x^5\, y + \alpha_z(r)$
  $\delta V_x(y,r) = -B_z\, y + \beta_x(r), \; \delta V_z(y,r) = B_x\, y + \beta_z(r)$
  $\delta g_{tx}(y,r) = g_{xx}\, \gamma_x(r), \; \delta g_{tz}(y,r) = g_{xx}\, \gamma_z(r)$
- where $\alpha_x, \alpha_z, \beta_x, \beta_z, \gamma_x, \gamma_z$ normalizable
- One reads off at the horizon:
  $\langle J_5^a \rangle = \frac{N_c}{2\pi^2} \left(\mu_5\, B_b^5 + \mu\, B_b \right) \delta^{ab}$,
  $\langle J^a \rangle = \frac{N_c}{2\pi^2} \left(\mu_5\, B_b + \mu\, B_b^5 \right) \delta^{ab}$.
- Holography predicts universal values for $\sigma_B, \sigma_{B5}, \sigma_V, \sigma_{V5}$

Now consider dynamical gluons: $a_3 \neq 0$

Anomalous conductivities with glue

- In QCD-like theories
  $\partial_\mu J^{5\mu} = a_1 \,\mathrm{Tr}(F \cdot \tilde{F}) + a_3 \,\mathrm{Tr}(G \cdot \tilde{G}) + a_2 \, R \cdot \tilde{R} \,,$
- So far we only considered $a_3 = 0$
- Non of the non-renormalization arguments above apply when $a_3 \neq 0$
- Direct FT calculation $\Rightarrow$ $\sigma_{V,5}$ receives perturbative corrections from dynamical glue loops Golkar, Son '12; Hou et al '12
- Lattice-QCD: both σ_B and $\sigma_{V,5}$ receive huge corrections Yamamoto '12, Braguta et al. '13
- Hydro arguments above do not apply $\Rightarrow$ need hydro d.o.f. for $\mathrm{Tr}(G \cdot \tilde{G})$
- Nor does the EFT argument!
- Can we find an alternative approach through holography?

Holography with dynamical glue

- How to compute σ_B, σ_V at strong coupling?
- First: how to realize $a_3 \neq 0$ situation in holography?
  Klebanov, Ouyang, Witten '02
- Anomalous breaking of the R-symmetry $U(1) \to Z_N$ in $\mathcal{N}=1$ Klebanov-Witten theory $\Leftrightarrow$
- Higgs the corresponding gauge field A_M in the bulk
- C_2 not invariant under $A \to A + d\epsilon$: $C_2 \to C_2 + \epsilon\omega_2$
- A eats the scalar $\Theta = \int_{S^2} C_2$ and gets a mass m_A^2
- Θ dual to the $\theta_{YM} \int \mathrm{Tr}\, G \wedge G$
- Thus, holography:

$$\partial_\mu J_R^\mu = m_A^2 \mathrm{Tr}\, G \wedge G$$

Bottom-up construction

Casero, Kiritsis, Paredes '07 generalized this construction.

- Consider a system of N_c D3 gauge branes and N_f D4 flavor branes. More relevant to QCD.
- Ignore GR anomaly for simplicity
- Let's first get the electromagnetic piece $\partial_\mu J^\mu = a_1 F \wedge F$ right:
- WZ term on the flavor branes: $S_{WZ,4} = T_4 \int d^5 x i F_0 \wedge \Omega_5 + \cdots$ with $F_0 = \star F_5 \propto N_c$, and $\Omega_5 \sim A \wedge F \wedge F$.
- $A \to A + d\epsilon \Rightarrow \partial_\mu J^\mu \propto N_c F \wedge F$
- Where is the $\mathrm{Tr}\, G \wedge G$ term?
- WZ term on the D3 branes: $S_{WZ,3} = T_3 \int d^4 x C_0(r) \mathrm{Tr}\, G \wedge G$
- Thus, $\theta_{YM} = C_0(\infty)$

- C_0 is NOT invariant under $A \to A + d\epsilon$:
- C_0 also couples to the flavor branes
  $S_{WZ,4} = T_4 \int d^5x \tilde{F}_4 \wedge A + \cdots$ with $\tilde{F}_4 = d\tilde{C}_3 = \star dC_0$
- Dualize $\tilde{C}_3$ to get the axion action:
  $S_a \sim \int d^5x \sqrt{g}\,(dC_0 - N_f A)^2$.
- Invariant only if $C_0 \to C_0 - N_f \epsilon$

All in all... under $A \to A + d\epsilon,\ C_0 \to C_0 - N_f \epsilon$:

$$\partial_\mu J^\mu_R \sim N_f \mathrm{Tr}\,(G \wedge G) + N_f\, F \wedge F$$

Bulk axion is the key to get dynamical gluon contribution.

Application to anomalous conductivities

U.G., A. Jansen '14

$$S_{tot} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 - \frac{1}{4}Z_1(\Phi)F^2 - V(\phi)\right)$$
$$+\int d^5x\left(a_1 A\wedge F\wedge F + a_2 A\wedge R\wedge R\right),$$
$$-\int d^5x\sqrt{-g}\left(\frac{Z_0(\Phi)}{2}(dC_0 - N_f\,A)^2\right) + \int d^4x\sqrt{-h}a_3\,C_0\,\mathrm{Tr}(G\wedge G)$$

- Now apply the same procedure to compute $\langle J_x J_z\rangle$, $\langle T_{0x}J_z\rangle$
- Two important differences:
  1. Gauge field gets a mass:
     $\partial_N\left(\sqrt{-g}Z_1(\Phi)F^{MN}\right) = \frac{N_f}{N_c}\sqrt{-g}\,Z_0(\Phi)\,A^M\,.$
  2. Fluctuation eqs for A_x, h^0_z etc has new terms
- We identified the holographic mechanism to produce the dynamical gluon contribution to the anomalous conductivities.

Summary

- Hydro and QFT —> non-renormalization of

  $\sigma_B, \sigma_{B5}, \text{and}\, \sigma_V$

- Possible radiative corrections to

  $\sigma_{V5} \quad \text{as} \quad C(\lambda)T^2$

- Both EFT and Holography suggests

  $C = -8\pi^2 a_2$

- Proved in holography at $\lambda = \infty$

  by horizon universality. Robust against $1/\lambda$

  Grozdanov and Poovittikul '16

- All corrected by dynamical gluons.

  In holography corrections at order N_f/N_c

Open questions

- All anomalous transport non-renormalized in the absence of dynamical glue? How about non-perturbative corrections?

- How to calculate gluonic corrections and compare with lattice? Assume Stueckelberg mass for C0 and calculate in effective model

  Landsteiner et al '14

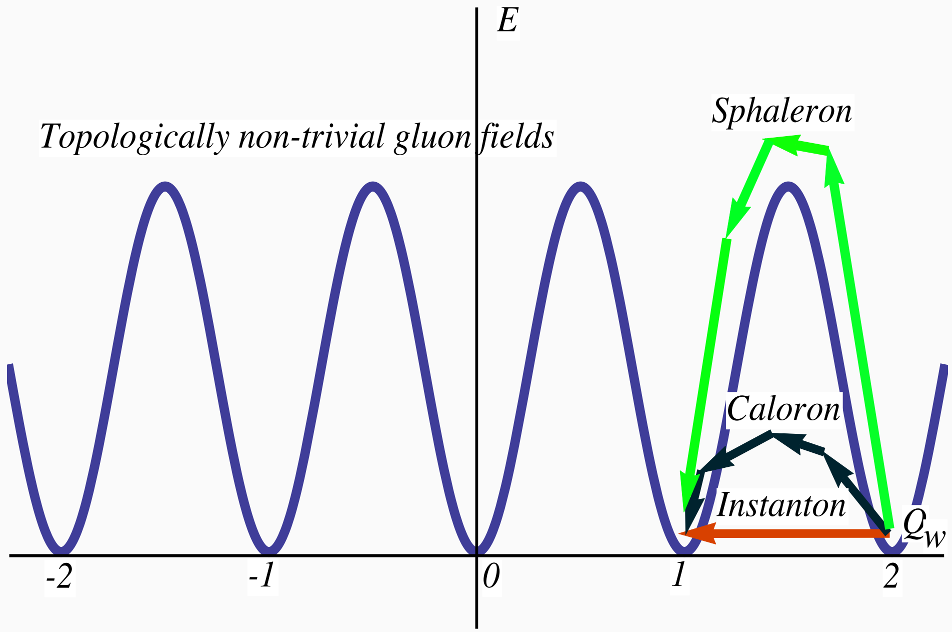
- But not possible to compare to lattice except in Veneziano limit, really hard!

  UG, Jansen, Tarrio (work in progress)

Sphaleron decay and axial potential

- How to determine μ_5 in holography?
- Sphaleron decay rate
- Dependence of the decay rate on B

How to produce Q_w in QGP?



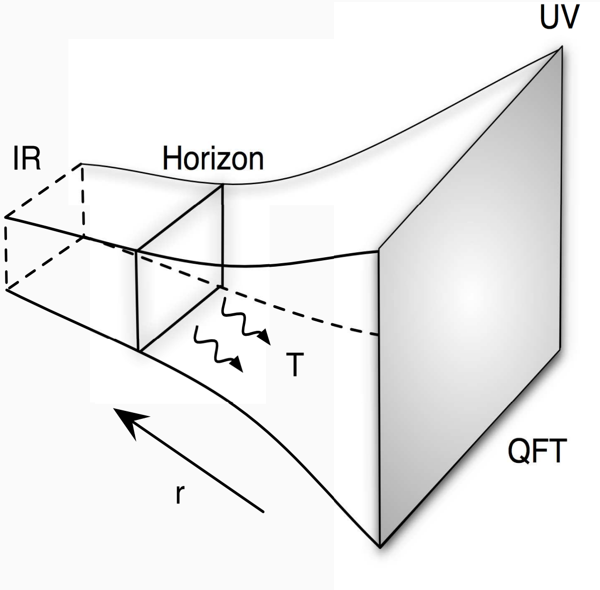
- Sphalerons: thermally induced changes in Q_w
- The most dominant Q_w decay Moore et al '97 due to sphalerons
- Sphaleron decay rate: $\Gamma_{CS} = \frac{d(N_L - N_R)}{dt d^3x} \approx 192.8\, \alpha_s^5\, T^4$

- If $Q_w \neq 0$ (or μ_5 finite)
- If there is an external magnetic field
- There exists $J^\mu \propto B$ due to chiral anomaly
- In QGP the main source of Q_w is sphalerons
- A effective μ_5 is generated by topological charge fluctuations due to Sphaleron decay:

$$\mu_5 \propto lim_{\omega \to 0} \langle \mathrm{Tr} F\tilde{F}(\omega) \mathrm{Tr} F\tilde{F}(\omega) \rangle$$

- However, QGP is strongly interacting, how to calculate μ_5?

Holographic calculation



Finite T, $N_c \gg 1, \alpha_s \gg 1$ QFT $\Leftrightarrow$ GR on black holes in 5D

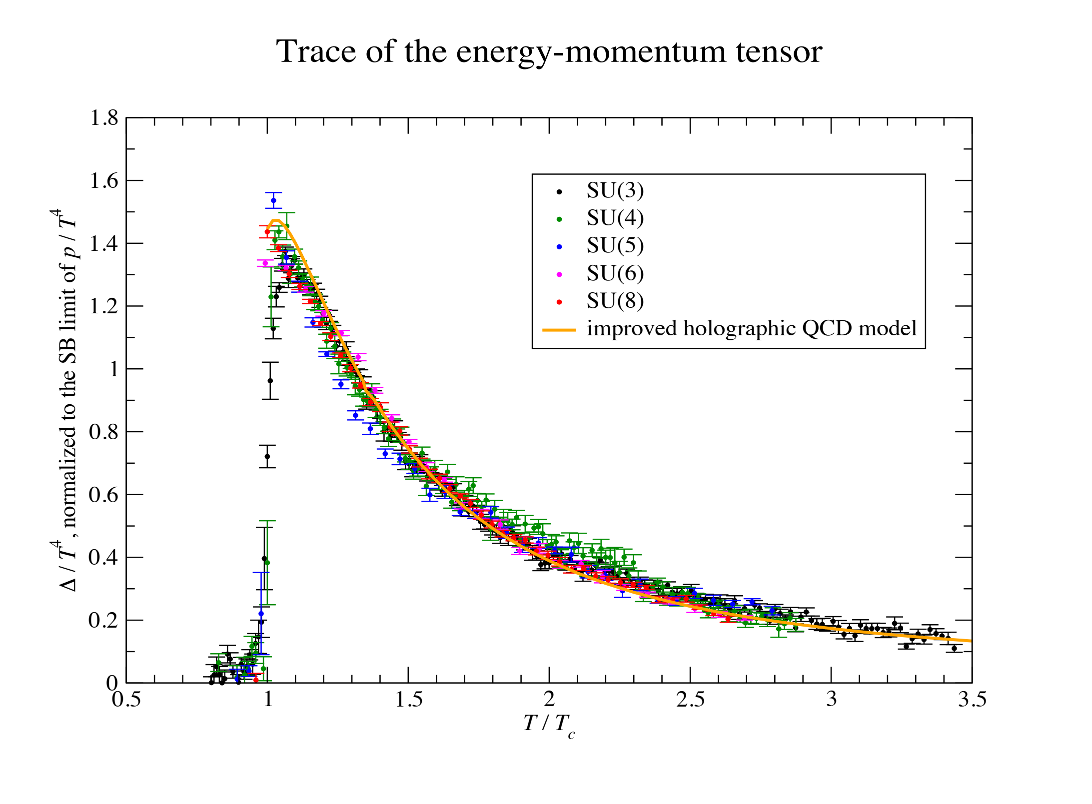
Maldacena '97; Witten; Gubser, Klebanov, Polyakov '98

1. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle$ computed from $\hat{\nabla}^2\phi = m^2\phi$ on the BH.
2. Recall $\omega J^{\mu 5}(\omega) \propto \mathrm{Tr} F\tilde{F}(\omega)$. Introduce CP odd axion $a(r,x)$
3. The source term $\int d^4x a_0(x) \mathrm{Tr} F\tilde{F}(x)$ with $a(r,x) \to a_0(x)$ at the boundary.

Holographic calculation of $\Delta(N_L - N_R)$

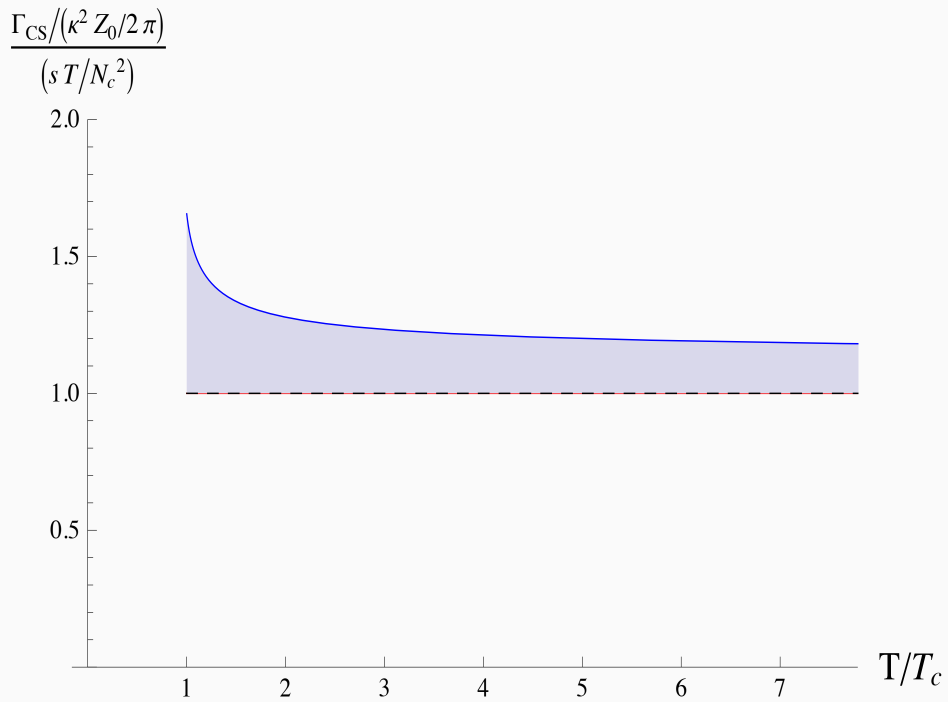
AdS/CFT: $\Gamma_{CS} = \frac{(g^2 N_c)^2}{256\pi^3} T^4$, Son, Starinets '02

Phenomenologically interesting region $T \approx T_c$ where conformality breaks down:



Improved holographic QCD U.G., Nitti, Kiritsis '07

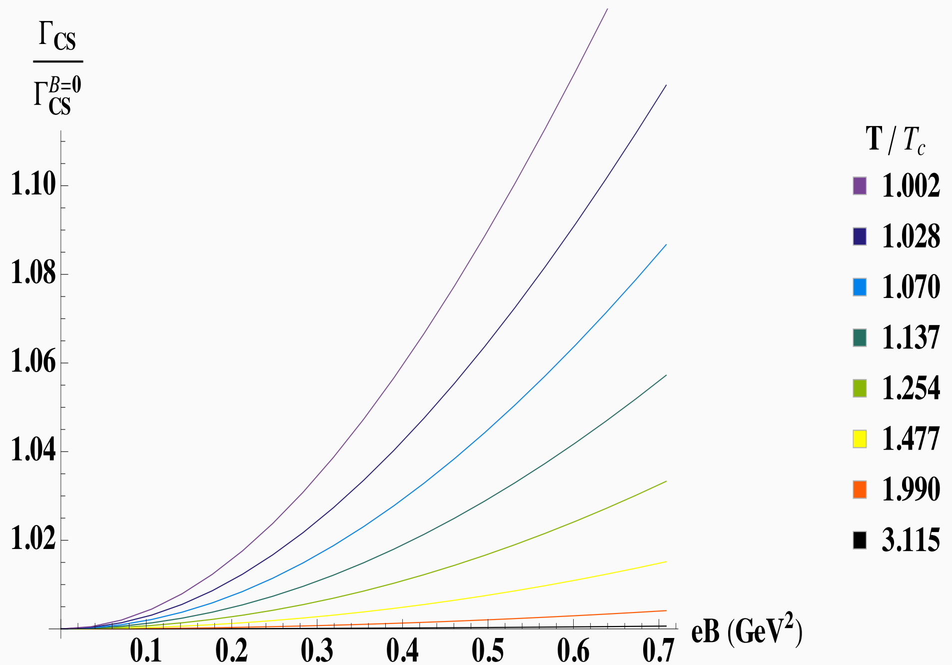
- $\frac{S_{GR}}{M_p^3 N_c^2} = \int d^5x \sqrt{-g} \left(R - (\partial\Phi)^2 + V(\Phi) - \frac{1}{2N_c^2} Z(\Phi)(\partial\alpha)^2 \right)$
- Parametrize $Z(\lambda) = Z_0 \left(1 + c_1\lambda + c_4\lambda^4\right)$
- Result: $\Gamma_{CS}(T_c) \geq C\, s(T_c) T_c \chi$ O'Bannon, U.G, Iatrakis, Kiritsis, Nitti '12
- where $\chi = \frac{\partial^2 \epsilon(\theta)}{\partial\theta^2}$ is the topological susceptibility



for ihQCD to reproduce lattice 0^{+-} glueball spectrum within 1σ.

Dependence of Γ on the magnetic field

T. Drwenski, I. Iatrakis, U.G., '15



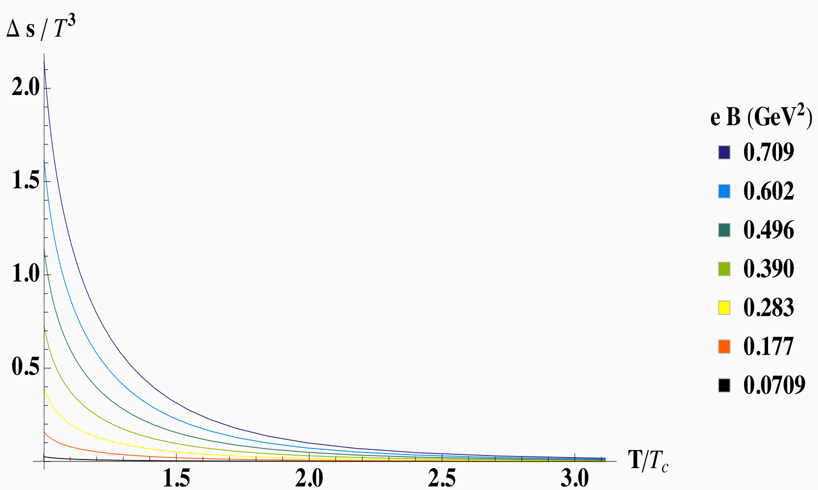
- Backreacted ihQCD with $N_f/N_c = 0.1$.

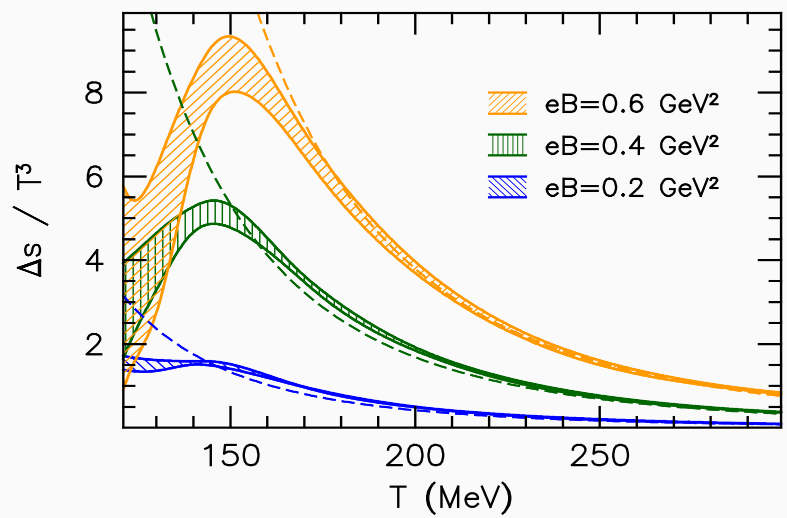
As an aside...

- Determine the entropy $S(B,T)$

  T. Drwenski, I. Iatrakis, U.G., '15

  Compare with lattice Bali et al, '14





- Backreacted ihQCD with $N_f/N_c = 0.1$.

Perturbative Magnetohydrodynamics

- Effects of EM interactions on the QGP flow
- New phenomena: magnetically induced electric currents
- Predictions for charged hadron spectra

UG, D. Kharzeev, K. Rajagopal ‘14

Perturbative Magnetohydrodynamics

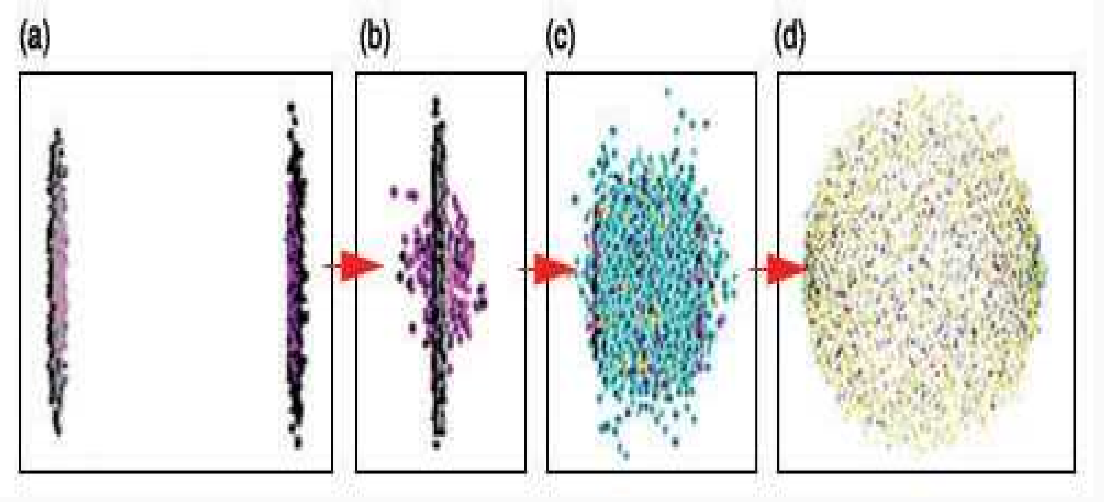
- Suppose $u^\mu(x)$ with no back reaction of electromagnetic fields and $\vec{E}$, $\vec{B}$ are known
- Go to the comoving frame by $\Lambda(-\vec{u})$ e.g. $F'_{\mu\nu} = (\Lambda \cdot F \cdot \Lambda)_{\mu\nu}$
- Compute the stationary velocity:

$$m\frac{d\langle \vec{v_B}\rangle}{dt} = q\langle \vec{v_B}\rangle \times \vec{B}' + q\vec{E}' - \mu m\langle \vec{v_B}\rangle = 0\,,$$

μm the drag coefficient; e.g. from AdS/CFT: $\frac{\pi\sqrt{\lambda}}{2}T^2$

- Go back to the center of mass frame: by $V^\mu = \Lambda(\vec{u})^\mu_\nu v^\nu_B$
- V^μ contains both u^μ and v^μ_B
  $\Rightarrow$ construct observables from V

Constructing u^μ for the expanding fluid



- Start from the Bjorken flow: Bjorken '83
  1. Boost invariance along z: $\xi = z\partial_t + t\partial_x$
  2. Rotation around z: $\xi = x\partial_y - y\partial_x$
  3. Translations in transverse plane: $\xi = \partial_x$ and $\xi = \partial_y$
- Solution to $[\xi, u] = 0$ is $u = \partial_\tau$ $(ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx_\perp^2 + x_\perp^2 d\phi^2)$
- Fine except transverse translations

Gubser's flow solution

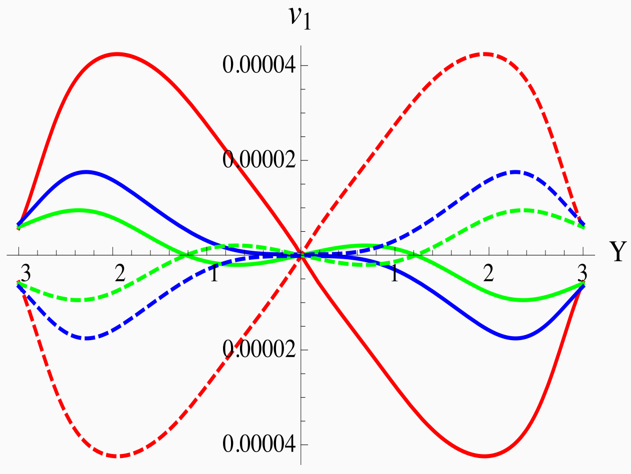
Gubser '10

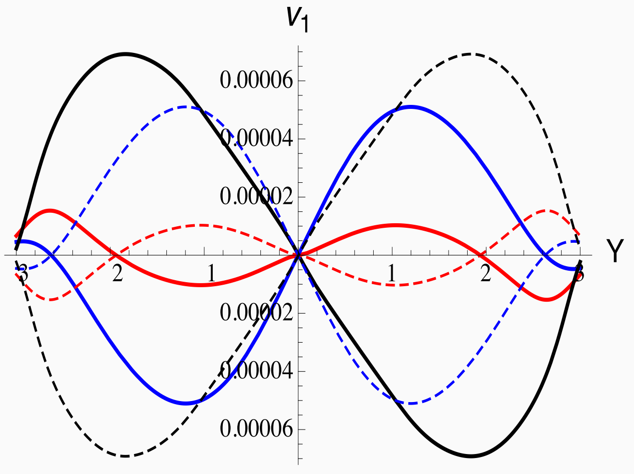
- Begin by Bjorken's flow
- Replace $\xi_i = \partial_x,\, \partial_y$ with $\xi_i = \partial_i + q^2 \left[2x^i x^\mu \partial_\mu - x^\mu x_\mu \partial_i\right]$
- Solution to $[\xi, u] = 0$ is $u = \cosh \kappa \partial_\tau + \sinh \kappa \partial_\perp$ with $\kappa = \frac{2q^2 \tau x_\perp}{1+q^2\tau^2+q^2x_\perp^2}$
- Solution to Hydrodynamics: $\nabla_\mu T^{\mu\nu} = 0$ with $\epsilon = \frac{\hat{\epsilon}_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1+2q^2(\tau^2+x_\perp^2)+q^4(\tau^2-x_\perp^2)^2\right]^{4/3}}$
- Also analytic dissipative correction with η/S.
- Two parameters to fix: Initial energy $\hat{\epsilon}_0$ and "system size" $1/q$

Results

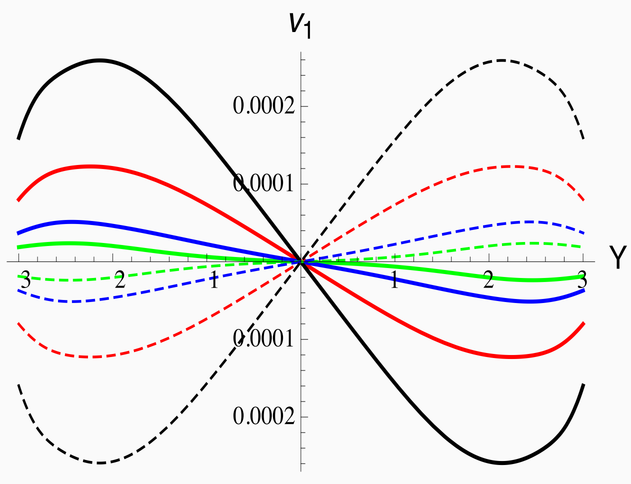
- Calculated B
- Fixed Gubser's flow parameters $\Rightarrow u^\mu$
- Solve classical force equation electromagnetic force = drag
- Do Cooper-Frye to calculate v_n
- The simplest and most direct effect: directed flow v_1:

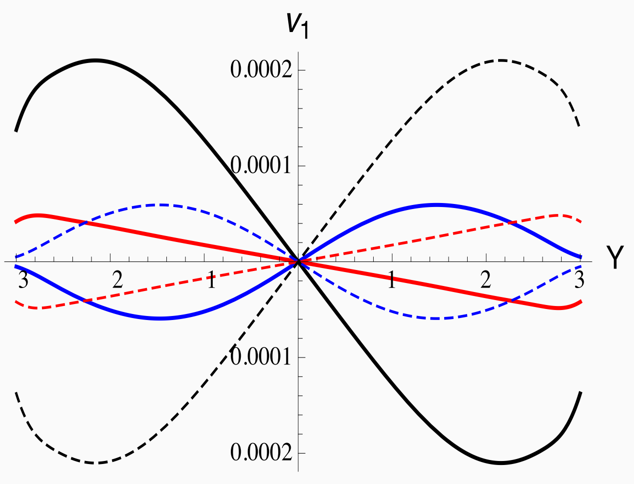
- Pions and protons at LHC



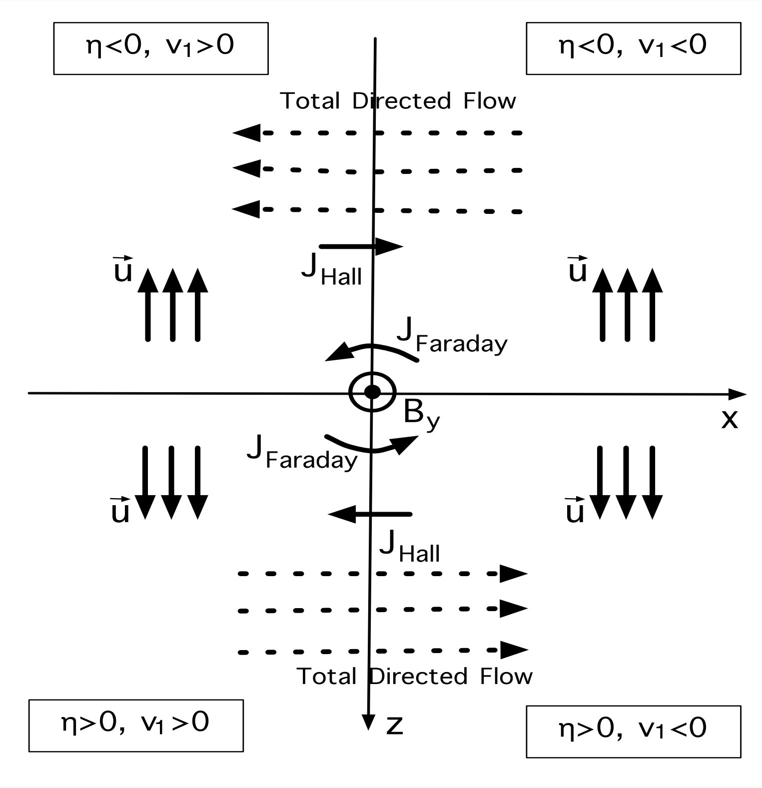


- Pions and protons at RHIC





Color coding: $p_T = 0.25$ (green), 0.5 (blue), 1 (red), 2 (black) GeV



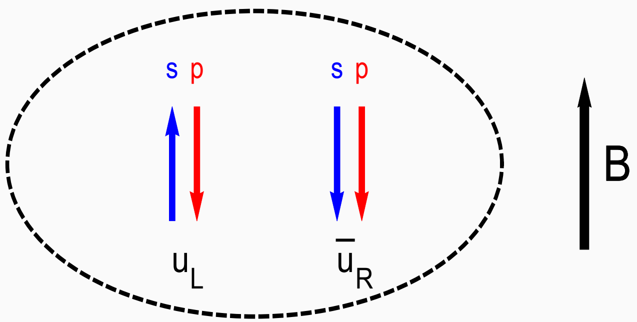
"Classical" currents in charged and expanding medium:

- Faraday currents $\vec{J}_F \sim \sigma \vec{E}_F$ with $\nabla \times \vec{E}_F = -\frac{\partial \vec{B}}{\partial t}$
- Hall currents $\vec{J}_H \sim \sigma \vec{E}_H$ with $\vec{E}_H = \vec{u} \times \vec{B}$

Magnetic Catalysis

- Effects of B on the quark condensate
- Use holography to understand the recent Lattice QCD results: "inverse magnetic catalysis"

Magnetic Catalysis Gusynin, Miransky, Shovkovy '94



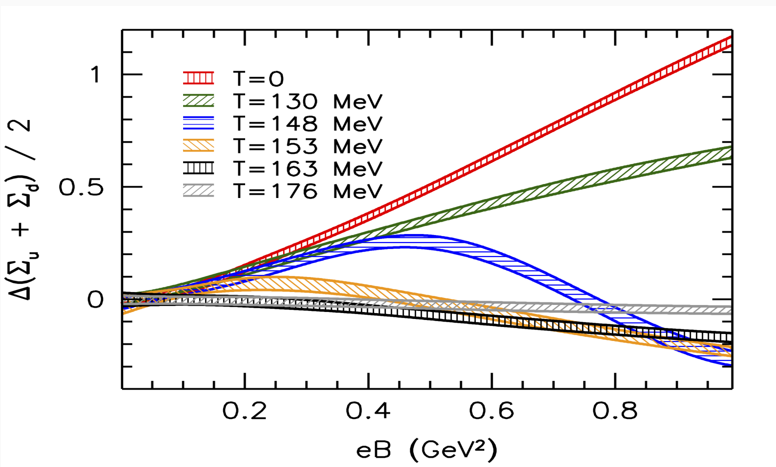
- B increases $\langle \bar{q}q \rangle$ at $T = 0$
- Seems counter-intiutive in the light of BCS: B destroys the Cooper pair $\langle ee \rangle$
- Opposite charges important

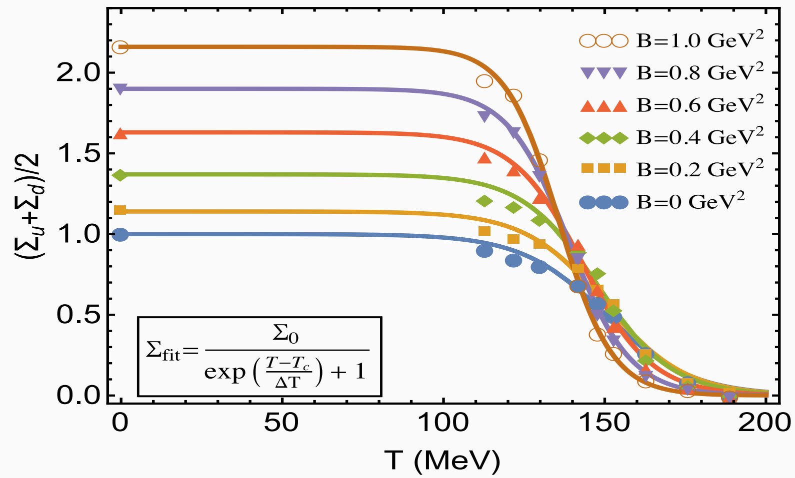
- Gusynin, Miransky, Shovkovy '94 studied in 3+1 NJL:
$$\langle \bar{q}q(B) \rangle^2 = \langle \bar{q}q(0) \rangle^2 \left(1 + \frac{|eB|^2}{3\langle \bar{q}q(0) \rangle^4 \ln(\Lambda / \langle \bar{q}q(0) \rangle^2} \right)$$
- Similar behavior for QCD at weak α_s
- Dimensional reduction $3 + 1 \to 1 + 1$ through Landau quantization $\Rightarrow$ IR dynamics stronger in lower D

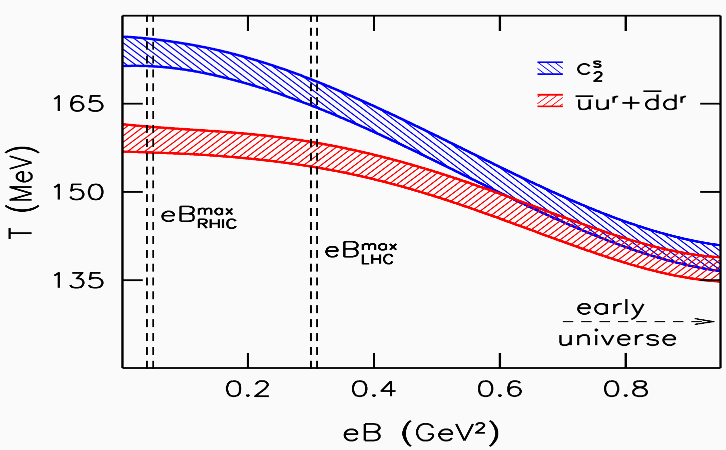
Inverse Magnetic Catalysis: a puzzle

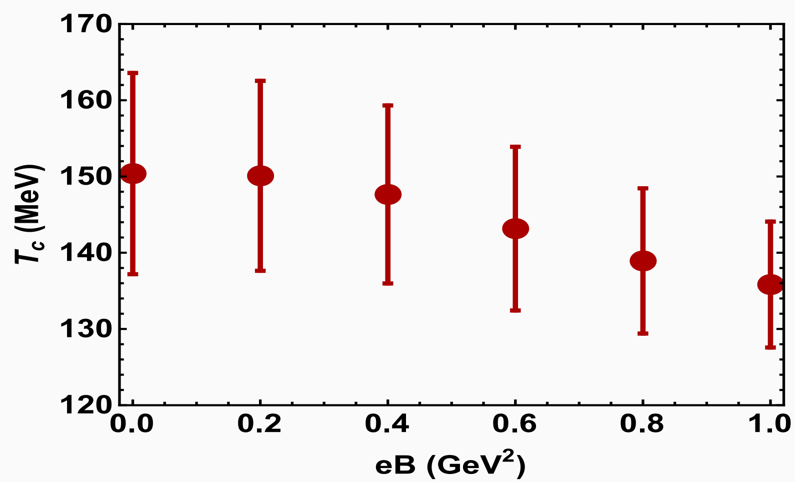
G. Bali et al '11, '12

- B destroys $\langle \bar{q}q \rangle$ around $T \sim T_c$
- T_c decreases with B









Possible Explanations

- "Valence" vs. "sea" quarks F. Bruckmann et al '13

$$\langle \bar{q}q \rangle = \int \mathcal{D}A e^{-S[A]} \mathrm{det}(D(A,B)+m) \mathrm{tr}(D(A,B)+m)^{-1}$$

  Valence $\Rightarrow$ enhances $\langle \bar{q}q \rangle$ for any B. Sea $\Rightarrow$ favors A configs. with larger Dirac eigenvalues $\Rightarrow$ suppresses $\langle \bar{q}q \rangle$ for larger B

- B effectively reduces $3+1 \to 1+1$, enhancing $\langle \bar{q}q \rangle$. But larger B reduces α_s by asymptotic freedom, diminishing $\langle \bar{q}q \rangle$.
- Clearly a result of non-trivial interplay between confinement and chiral symmetry breaking
- Should be able to observe in a phase deconfined with broken chiral symmetry
- This happens in holographic QCD in the Veneziano limit Jarvinen, Kiritsis '12

The gravitational action

$$S = M^3 N_c^2 \int \mathrm{d}^5x \left[\sqrt{g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right)\right.$$
$$\left. - xV_f(\lambda,\tau)\sqrt{\det(g_{\mu\nu} + w(\lambda)V_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau)}\right],$$

with $V_{\mu\nu}$ the EM field tensor, $x = N_f/N_c = 1$ in Veneziano limit.
Ansatz:

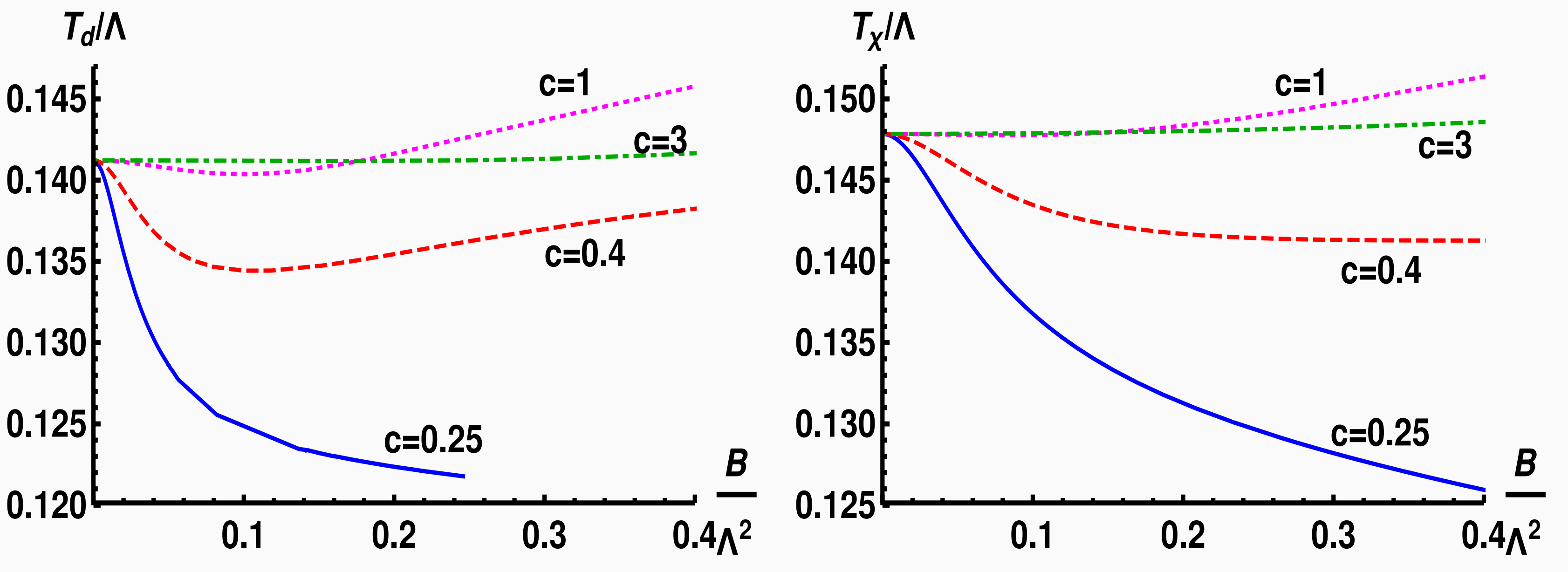
$$\mathrm{d}s^2 = e^{2A(r)}\left(\frac{\mathrm{d}r^2}{f(r)} + f(r)\,\mathrm{d}t^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + e^{2W(r)}\,\mathrm{d}x_3^2\right).$$

Tachyon:

$$\frac{\tau}{\mathcal{L}_{\mathrm{UV}}} = m_q r\,(-\log(\Lambda r))^{-\gamma_0/b_0} + \langle\bar{q}q\rangle r^3\,(-\log(\Lambda r))^{\gamma_0/b_0}$$

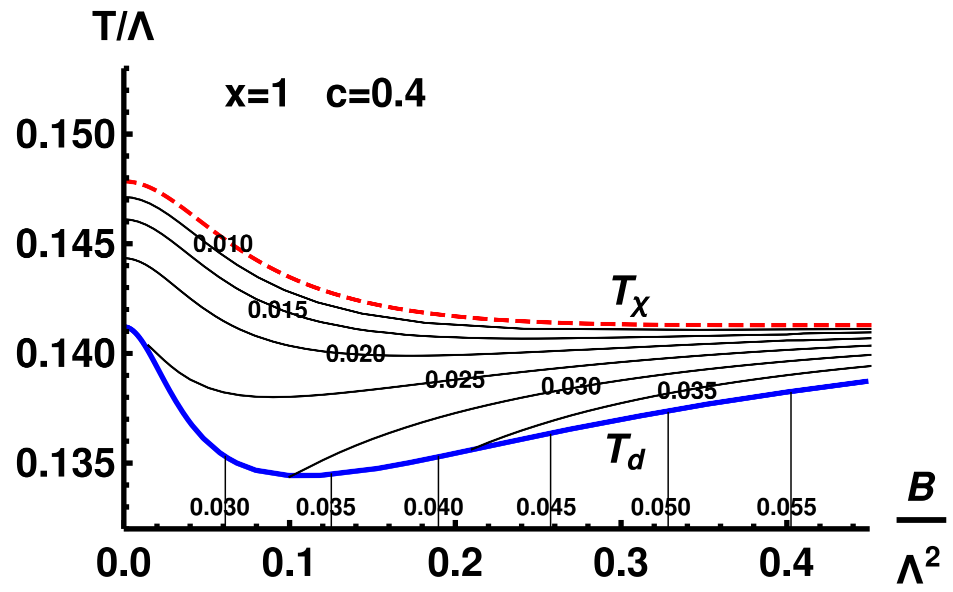
V_f, V_g, κ and w chosen to satisfy known properties of QCD. Set $m_q = 0$. Define $q = e^A \frac{dr}{dA}$

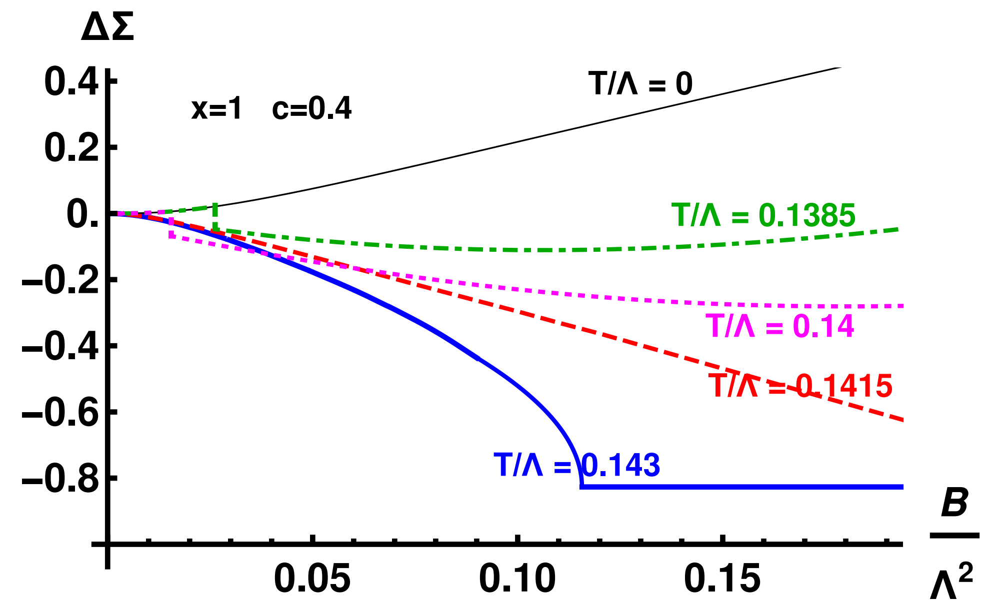
- Playing with c in $w(\lambda) = \frac{\sqrt{1+\log(1+c\,\lambda)}}{\left(1+\frac{3}{4}\left(\frac{115-16x}{27}+\frac{1}{2}\right)c\,\lambda\right)^{4/3}}$ produce different qualitative behavior

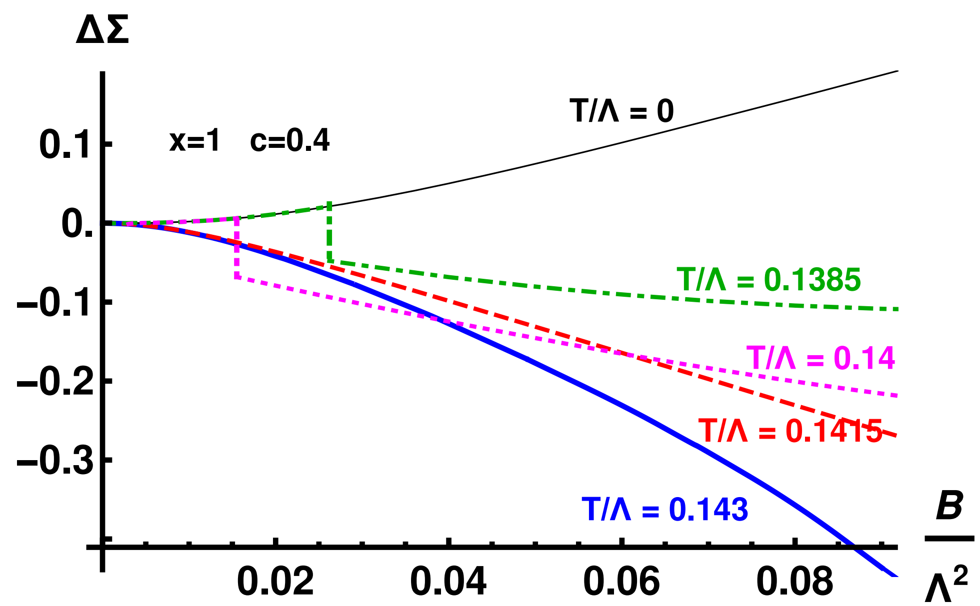


- Decreasing c reduces the effect of λ in the B-sector of the DBI action

Behavior of the condensate







A clear sign of inverse magnetic catalysis for sufficiently small c

Open questions

- Match with lattice —> fix unknown functions in V-QCD
- Understand the QFT reason for inverse MC
- How about dependence on B of $\langle \mathrm{tr}\, G^2 \rangle$

Thermalization and equilibration

- Large magnetic fields generated in off-central heavy ion collisions
- How do they affect the subsequent evolution ?
- First step: analyze equilibration of linear fluctuations on B —> quasi-normal modes
- Second: full non-linear analysis —> characteristic formulation of GR + EM

Linear fluctuations

- Equilibration profile of linear fluctuations
  <—> quasi-normal modes of the BH

- Quasi-normal modes: $\delta\phi_{\alpha\beta..} = h^{n}_{\alpha\beta..}(r)e^{-i\omega_n t+\vec{k}\cdot\vec{x}}$

$$\int_{\infty}^{r} \sqrt{-g}\, h^{n}_{\alpha\beta..} h_{n}^{\alpha\beta..} dr < \infty \qquad\qquad h^{n}_{\alpha\beta..} \sim (r-r_h)^{\frac{-i\omega_n}{4\pi T}}$$

- Solutions: discrete spectrum ω_n

$$\mathrm{Im}(\omega_n) : \text{ decay rate}$$

$$\mathrm{Re}(\omega_n) : \text{ wavelength}$$

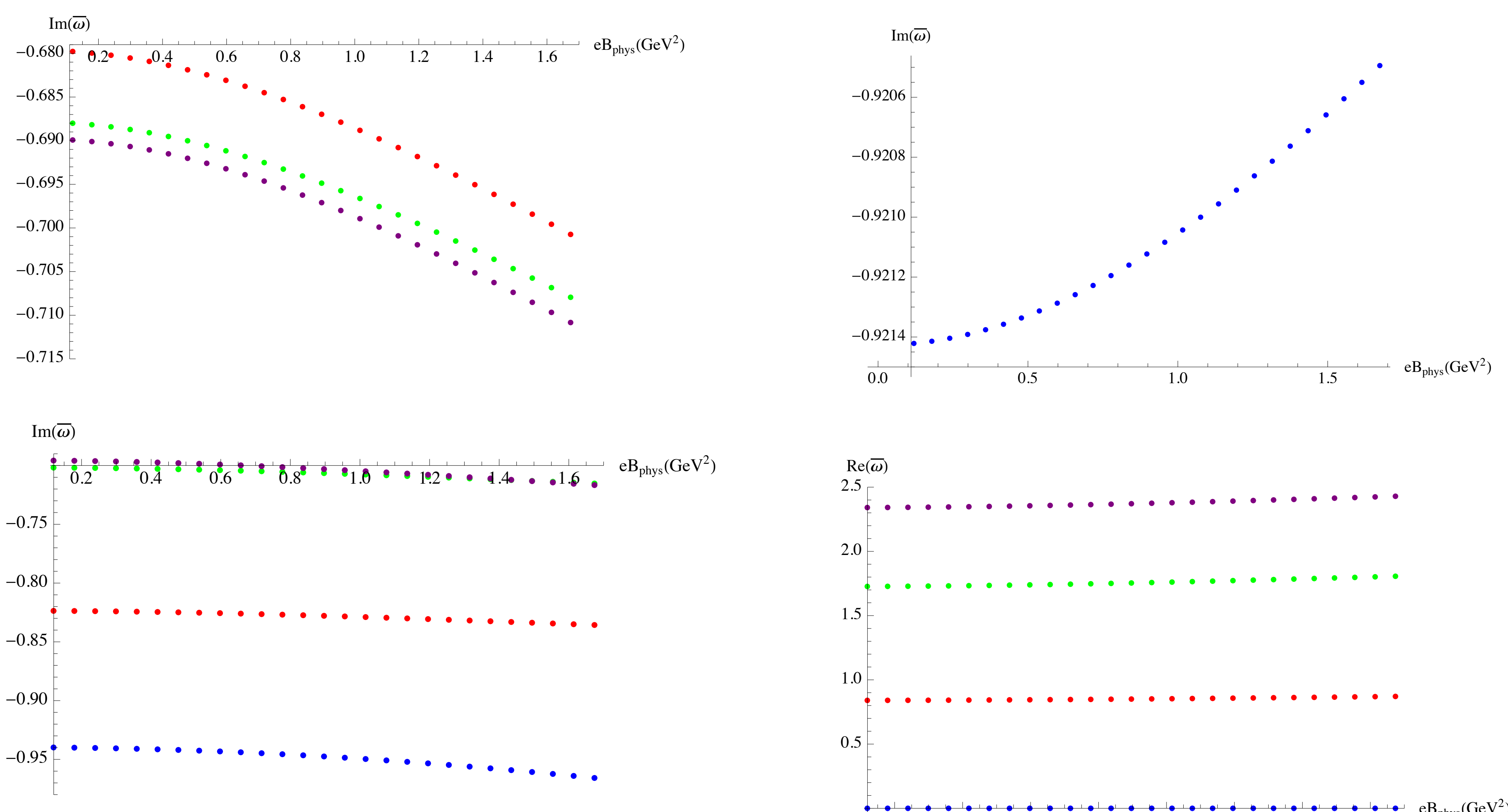
Dependence of equilibration on B

T. Demircik, UG '16

- B reduces the symmetry, choose $\vec{B} \parallel \vec{k}$

Only spin-2: $h_{\alpha\beta} - \delta_{\alpha\beta} h/2$ spin-1: $h_{t\alpha},\ h_{z,\alpha},\ v_\alpha$

$T = T_c,\ k \ll T_c$



Summary

- B affects little the spin-2 and spin-1 fluctuations
- There exists a purely imaginary mode. Behaves opposite to others under B

Open questions

Solve the entire time-dependent GR + EM problem

Other directions

- Anomalous transport and Weyl/Dirac semimetals
  CME recently observed in ZrTe5 Q. Li et al '14
- Possible technological applications? Quantum computing?

- Quantum critical fixed points induced by B?
  What can be learned from holography?
- Dependence of transport coefficients on B? ALICE has sufficient statistics to observe B-dependence in bulk and shear viscosity!

- Landau levels at strong coupling
- Construction of improved holographic magnetic QCD
  Fix the unknown functions by matching Euclidean correlators on lattice —> Predict real-time quantities that depend on B