Optical Conductivity from a Holographic Lattice

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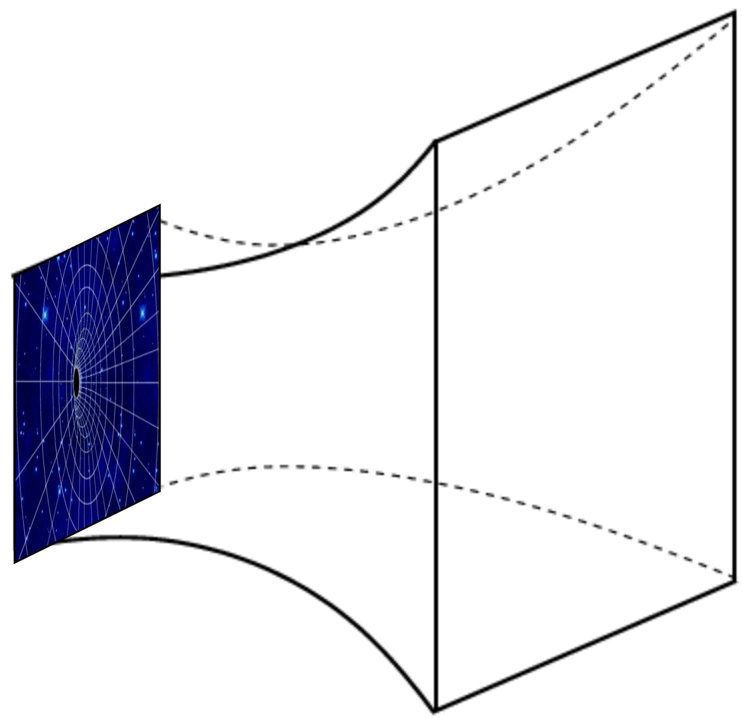
Based on work with Gary Horowitz and Jorge Santos

arXiv:1204.0519

UNIVERSITY OF CAMBRIDGE

Gravity Theories and Their Avatars, Crete 2012

Strongly Interacting Stuff



Boundary field theory
d=2+1

- Bulk d=3+1 black hole
- Hawking radiation = finite temperature
- Electrically charged (Reissner-Nordstrom) = finite density

Basics of Conductivity

Ohm's Law

$$\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

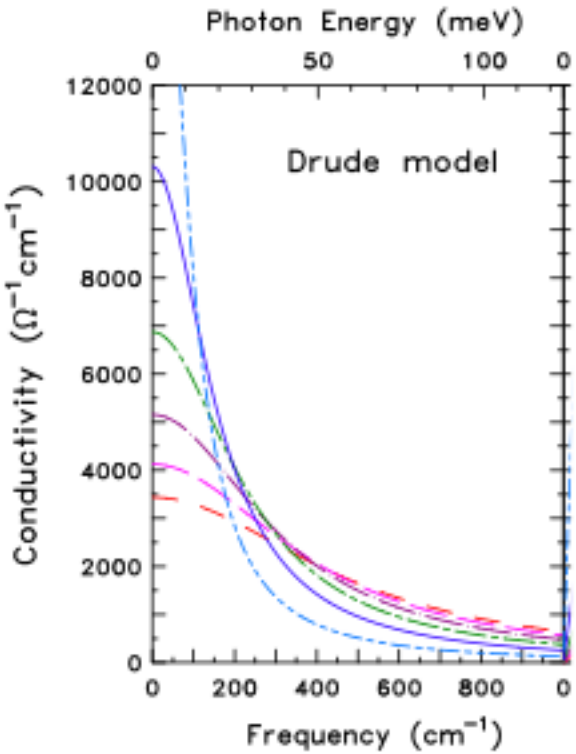
$$\vec{E} = \vec{E}(\omega)e^{-i\omega t}$$
$$\vec{j} = \vec{j}(\omega)e^{-i\omega t}$$

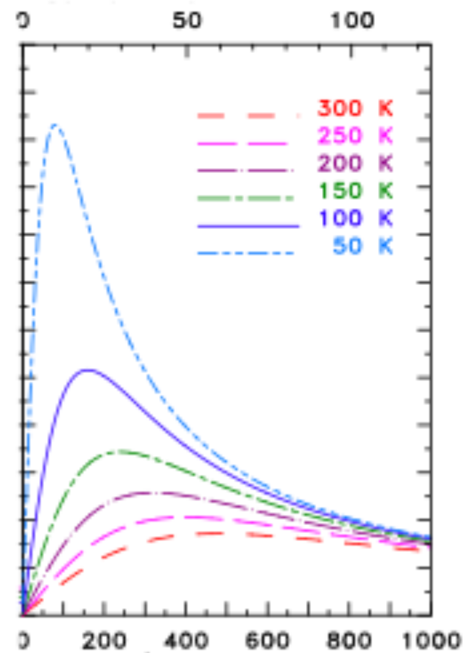
Drude Model

$$m\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = q\vec{E}$$

$$\vec{j} = nq\vec{v} \quad \Longrightarrow \quad \boxed{\sigma(\omega) = \left(\frac{nq^2\tau}{m}\right)\frac{1}{1 - i\omega\tau}}$$

Drude Model



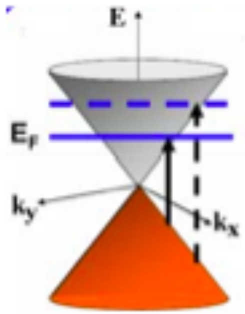


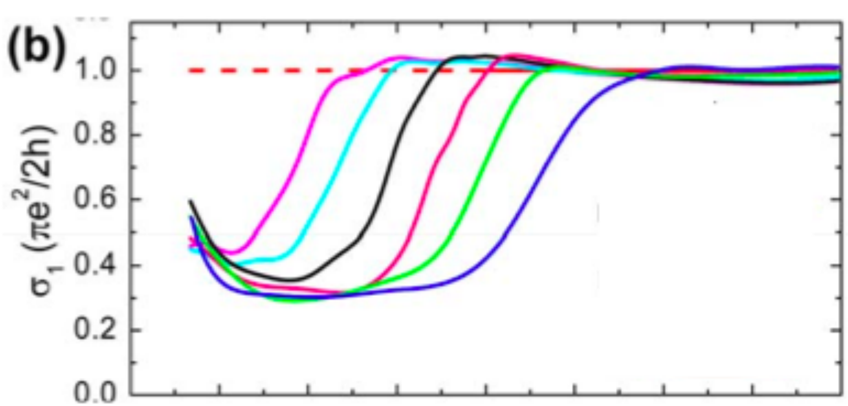
$$\mathrm{Re}(\sigma) = \frac{\sigma_0}{1+\omega^2\tau^2}$$

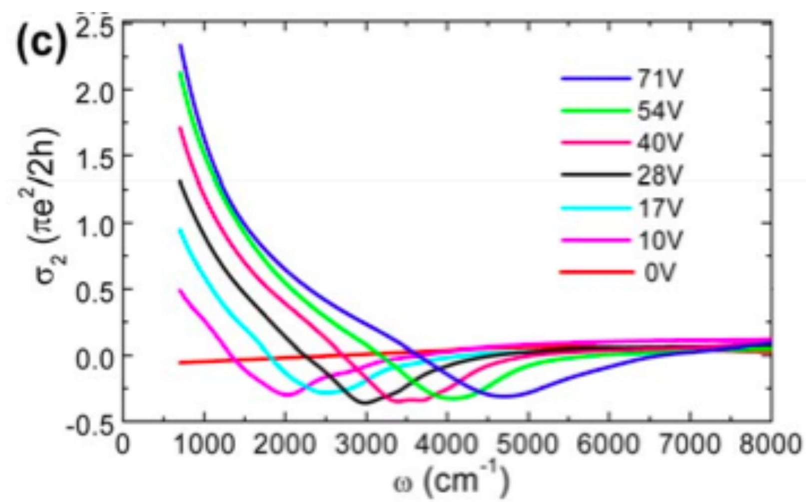
$$\mathrm{Im}(\sigma) = \frac{\sigma_0\omega\tau}{1+\omega^2\tau^2}$$

Graphene

Li et al. (2008)

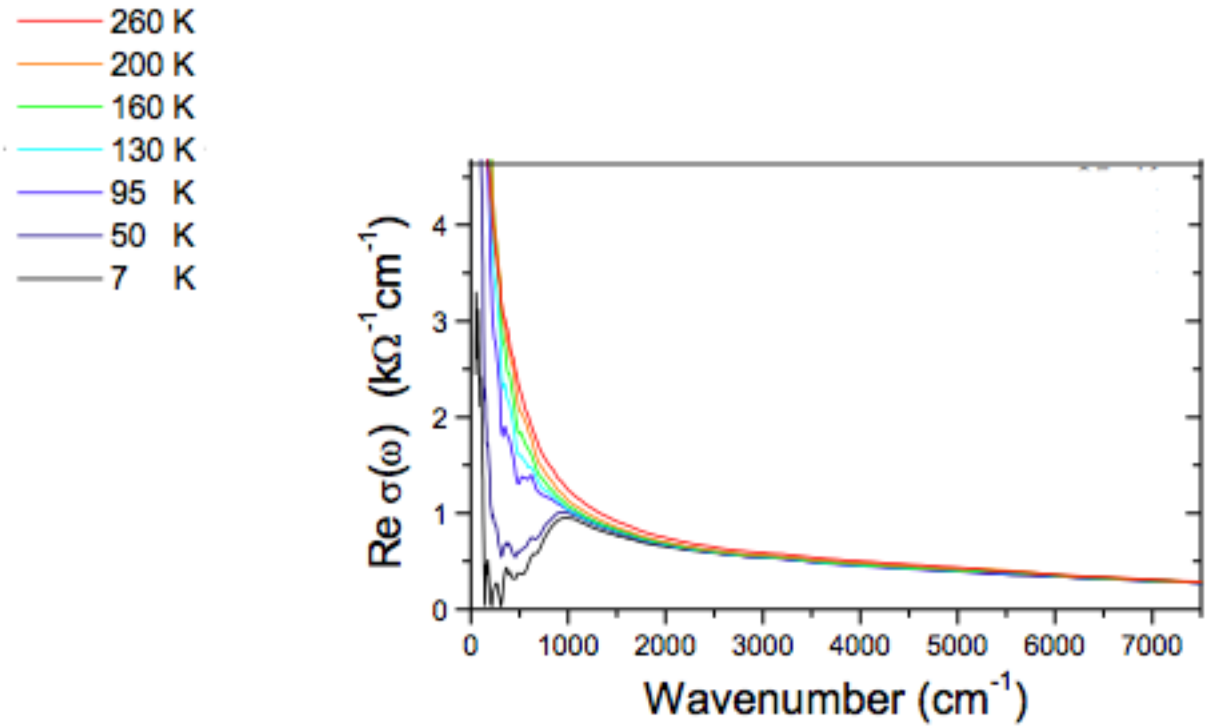






Cuprates

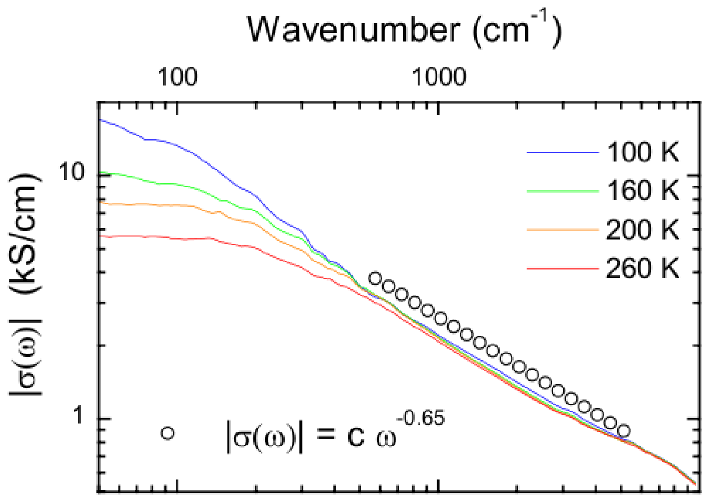
van der Marel et al.

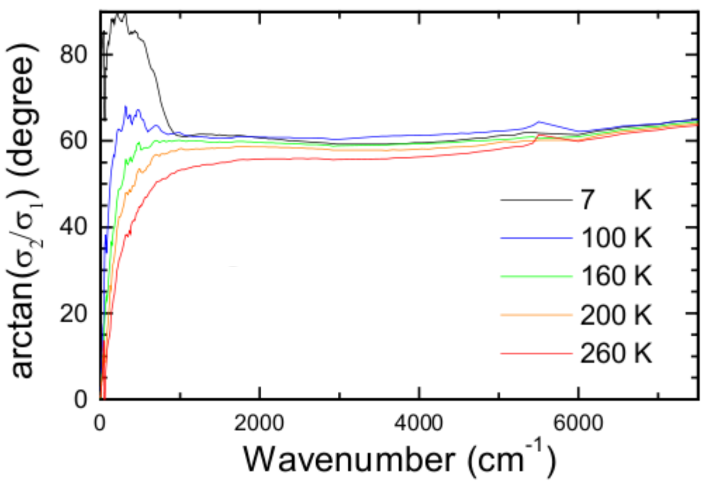


$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$

Cuprates

van der Marel et al.

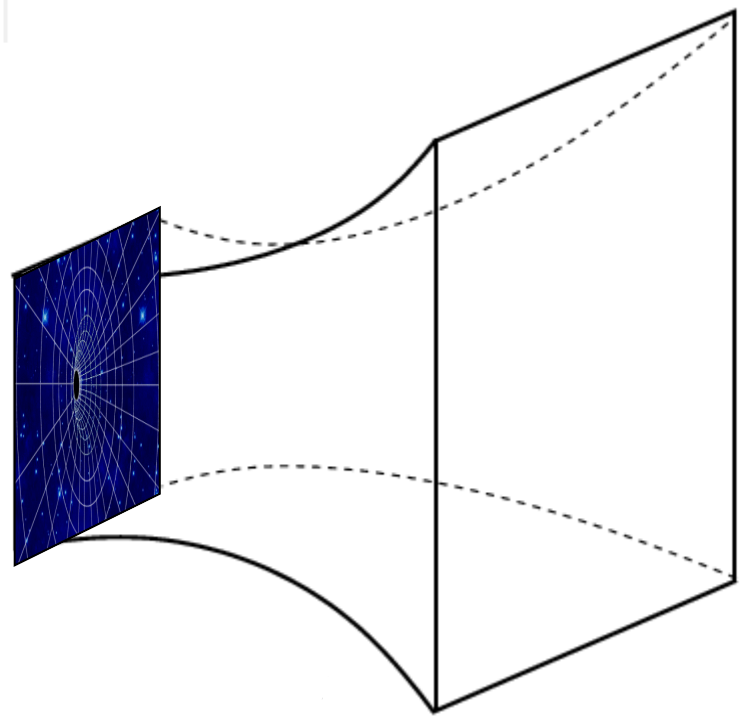




$$|\sigma(\omega)| \sim \frac{1}{\omega^{2/3}}$$

Back to Holography

Computing the Conductivity



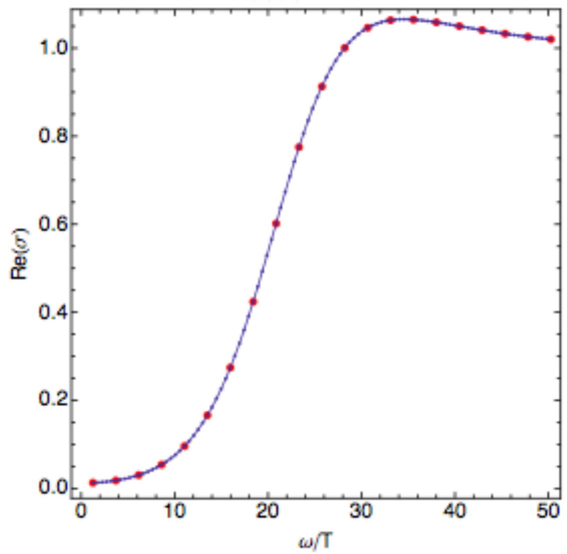
Boundary field theory

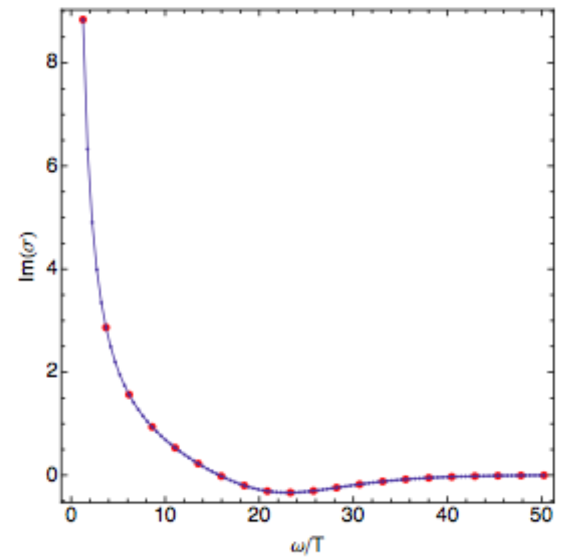
$z = 0$

$$A_x = \frac{E}{\omega} + J_x z + \dots$$

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$

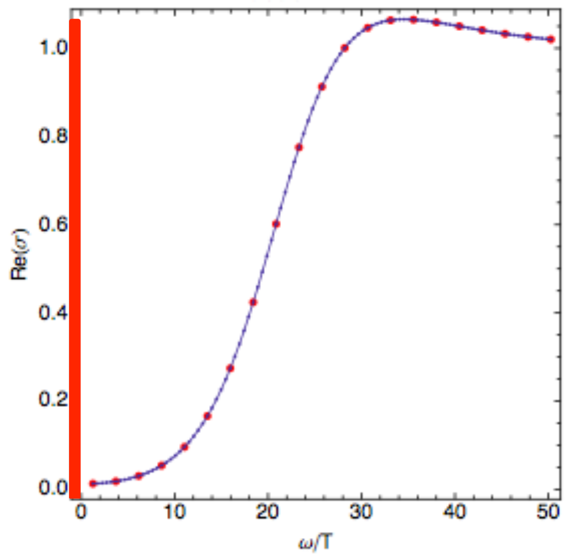


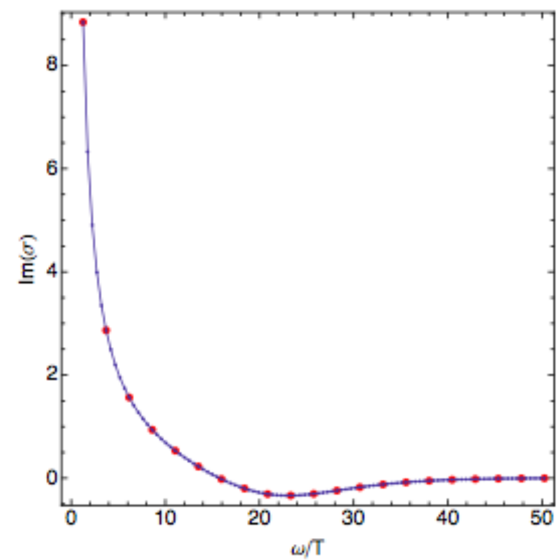


Herzog, Kovtun, Sachdev and Son; Hartnoll

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$





$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

$$\mathrm{Im}(\sigma) \to \frac{K}{\pi\omega}$$

Herzog, Kovtun, Sachdev and Son; Hartnoll

Resolving the Delta-Function

$$\mathrm{Re}\,\sigma(\omega) \sim K\,\delta(\omega)$$

Due to:

- Finite density of charge carriers
- Translational invariance

Two options to resolve

1. Dilute charge carriers
2. Break translational invariance

Option 1: Diluting Charge Carries

Probe Branes:

DC Resistivity: $\rho \sim T^{2/z}$

Optical Conductivity: $\sigma(\omega) \rightarrow \begin{cases} (i\omega)^{-1} & z < 2 \\ (i\omega)^{-2/z} & z > 2 \end{cases}$

Karch and O'Bannon;
Hartnoll, Silverstein, Polchinski, Tong

Breaking Translational Invariance

How to Build a Lattice

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \mu \mathcal{Q} + \phi_0(x,y)\mathcal{O}$$

Choices:

- Ionic lattice: $\mathcal{O} = \mathcal{Q}$
- Scalar lattice: introduce a neutral bulk scalar. $\mathcal{O} \longleftrightarrow \Phi$

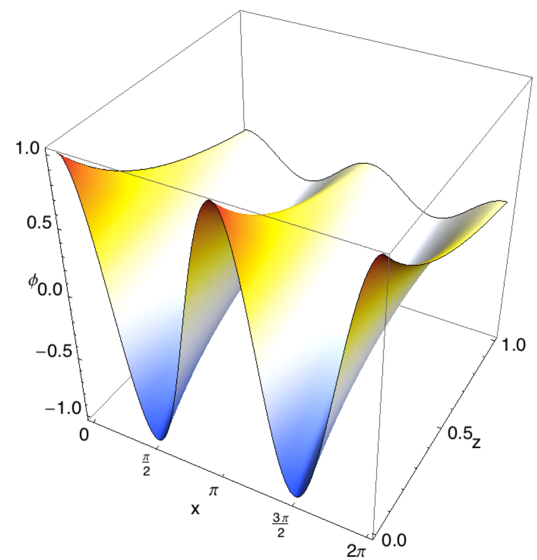
  Pick $m_\Phi^2 L^2 = -1 \Longrightarrow$ relevant operator

Source: $\phi_0 = A\cos(k_L x)$

The Lattice

Solve Einstein equations subject to lattice boundary conditions: $A_0(z,x)$, $\Phi(x,z)$ and

$$ds^2 = \frac{L^2}{z^2}\Big[-g_{tt}(z,x)dt^2 + g_{zz}(z,x)dz^2 + g_{xx}(z,x)(dx + a(z,x)dz)^2 + g_{yy}(z,x)dy^2\Big]$$

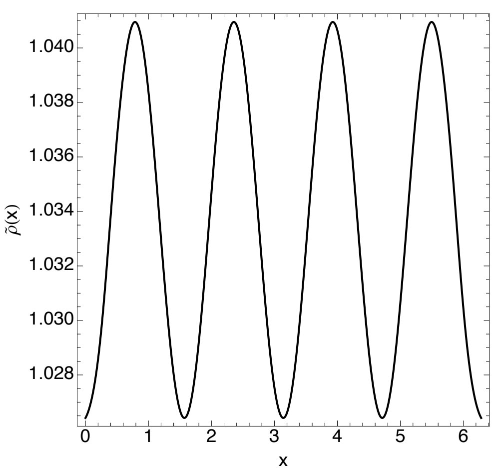


Note: $\mathcal{O}$ relevant; $e^{ik_L x}\mathcal{O}$ irrelevant

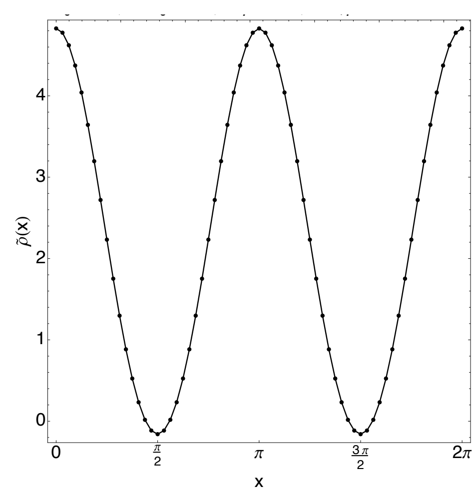
parameters
$T,\ \mu,\ k_L,\ A$

Charge Density

Scalar Lattice



Ionic Lattice

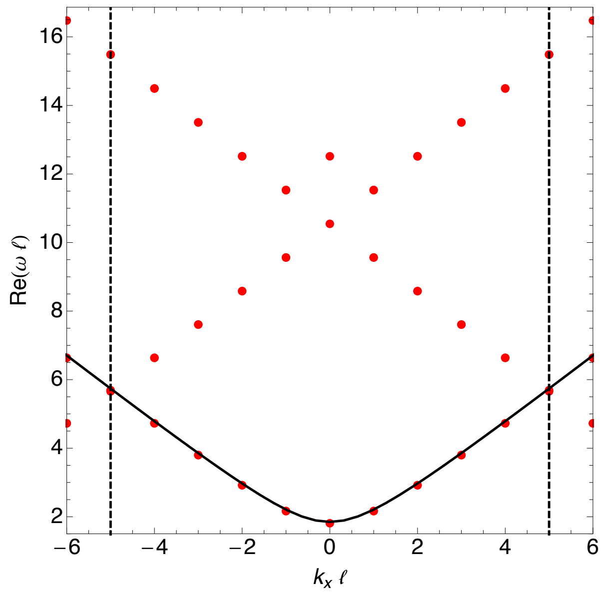


$$\text{Charge Density} \sim k_L^2$$

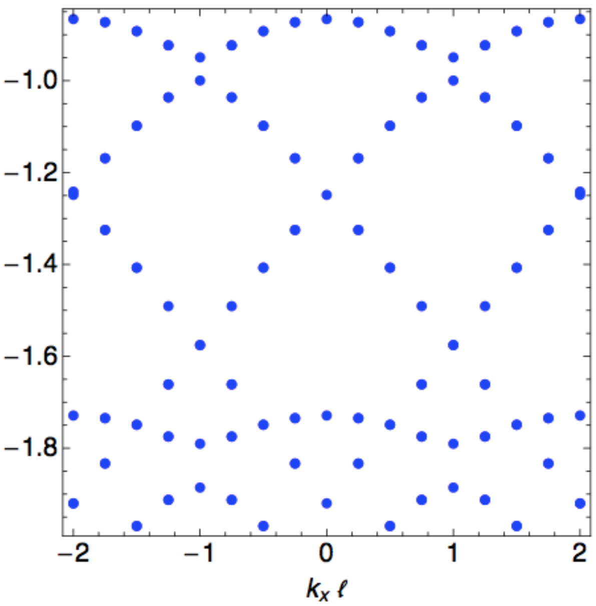
Band Structure

Add a probe scalar field in the lattice background

Scalar Lattice

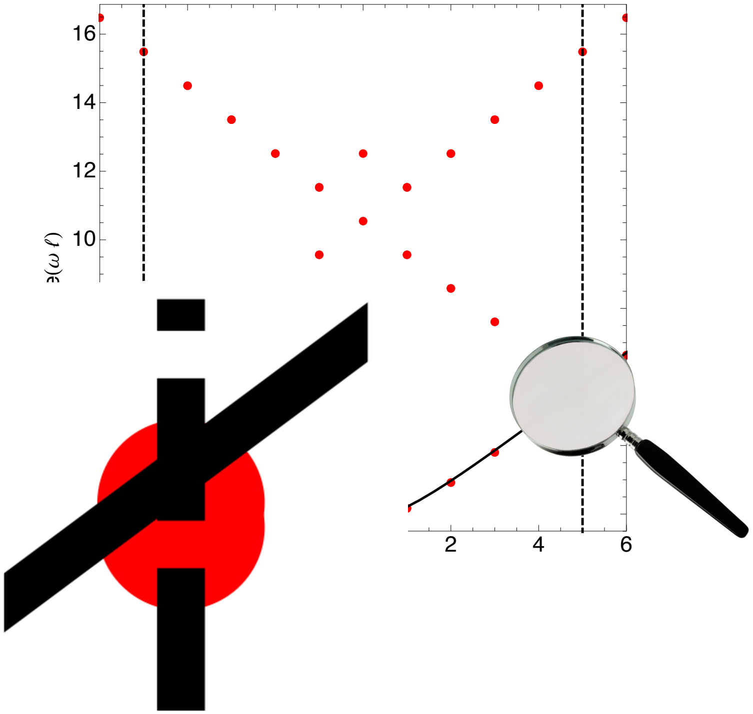


Ionic Lattice

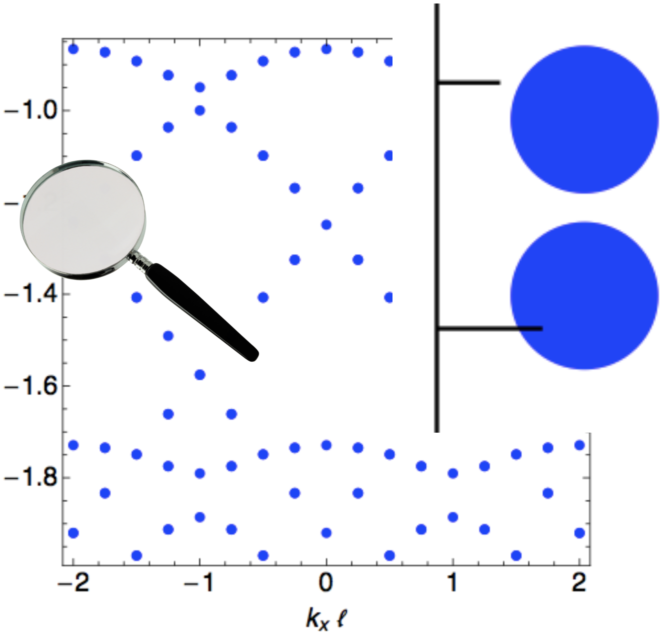


Band Structure

Scalar Lattice



Ionic Lattice



Band gaps small

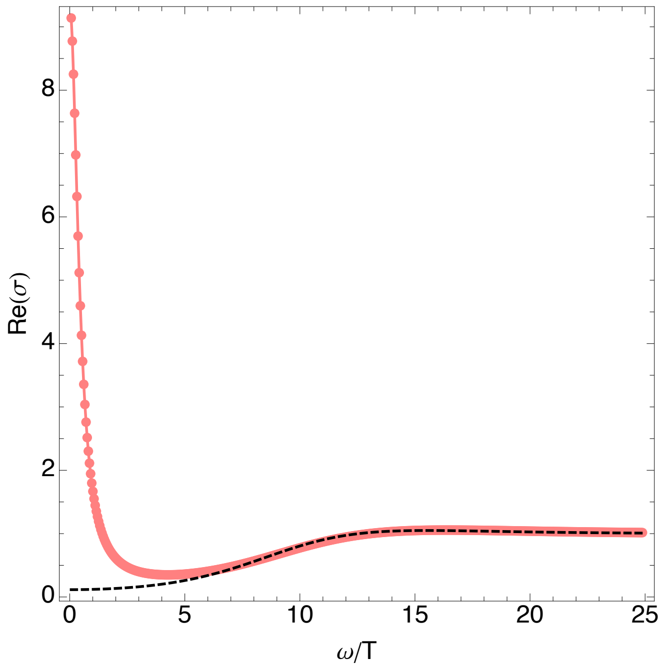
Perturbing the Lattice

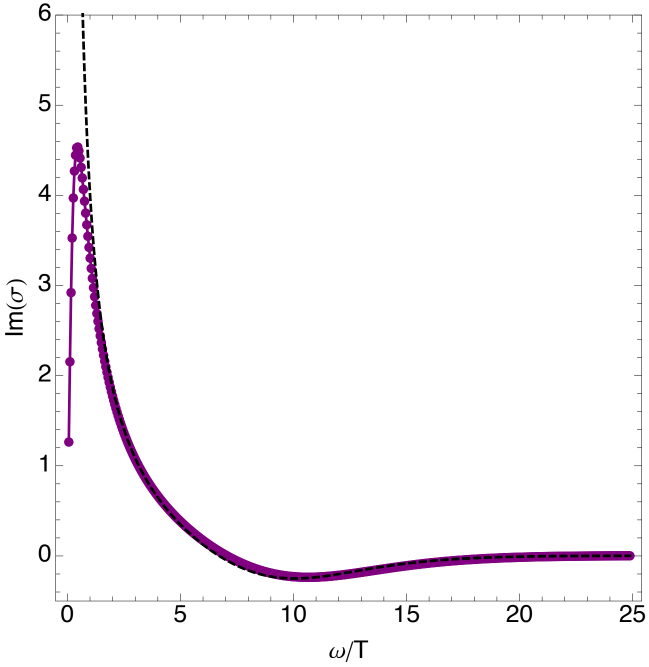
$$\delta g_{tt},\ \delta g_{tz},\ \delta g_{tx},\ \delta g_{zz},\ \delta g_{zx},\ \delta g_{xx},\ \delta g_{yy}$$

$$\delta A_t,\ \delta A_z,\ \delta A_x,\ \delta\Phi$$

Optical Conductivity

$$j(\omega) = \sigma(\omega) E(\omega)$$

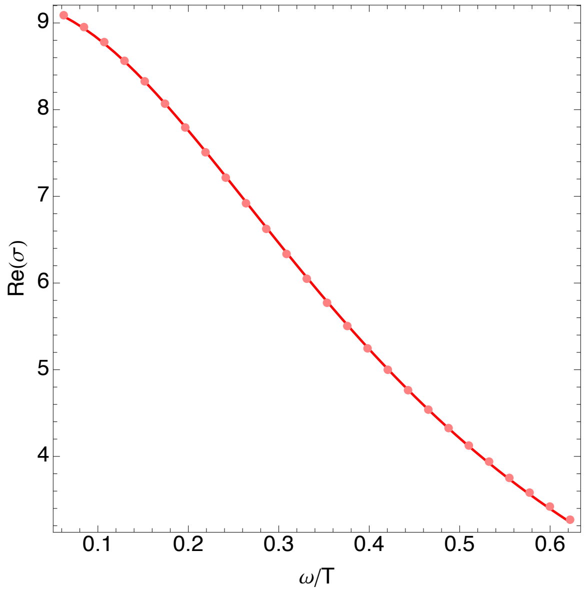


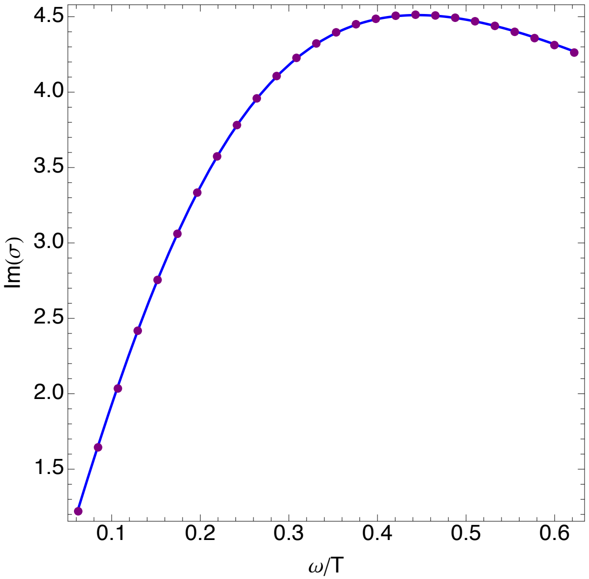


$\mu = 1.4 \qquad T = 0.115\mu \qquad k_L = 2 \qquad A = 1.5$

Low Frequency Behaviour

$$\omega \lesssim T$$

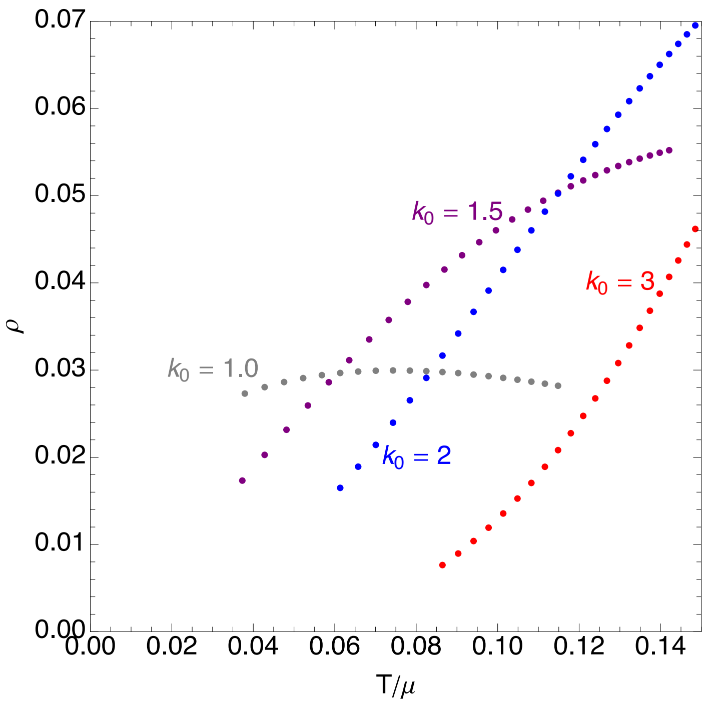




$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

Note: No quasiparticles

DC Resistivity



$$\rho = \frac{1}{K\tau}$$

Nearly all temperature dependence in τ

DC Resistivity

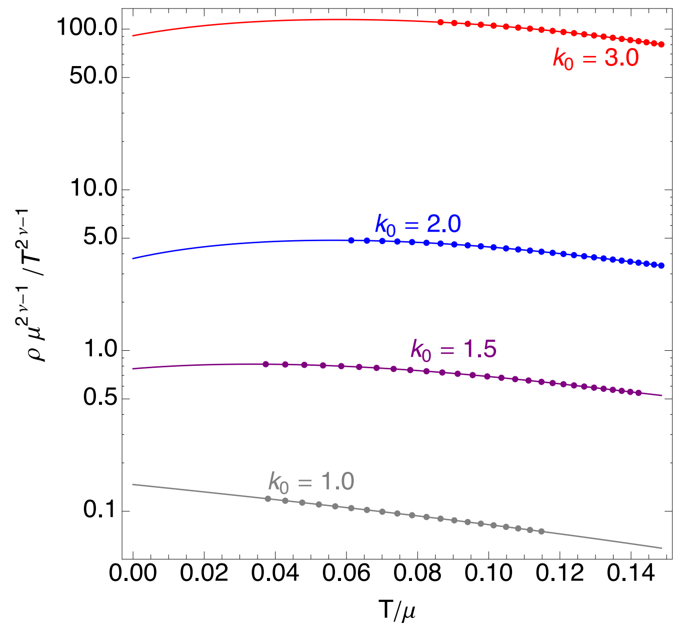
Low energy excitations around black hole governed by *locally critical* theory

- Field Theory: $z \to \infty$
- Geometry: $AdS_2 \times \mathbf{R}^2$

Hartnoll and Hofman: $\rho \sim T^{2\nu-1}$

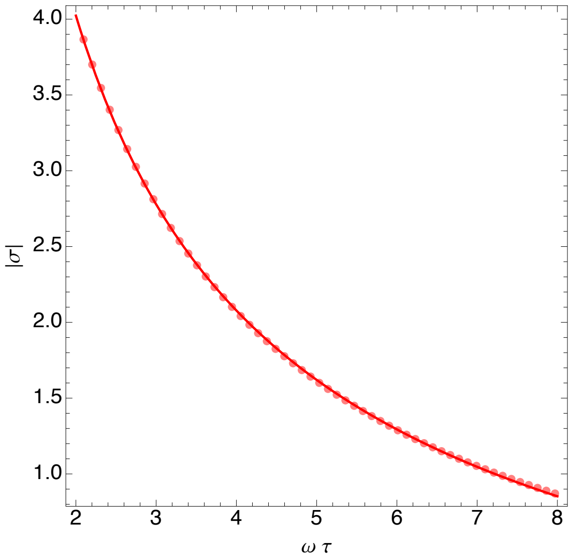
$$\nu = \frac{1}{2}\sqrt{5 + 2(k/\mu)^2 - 4\sqrt{1 + (k/\mu)^2}}$$

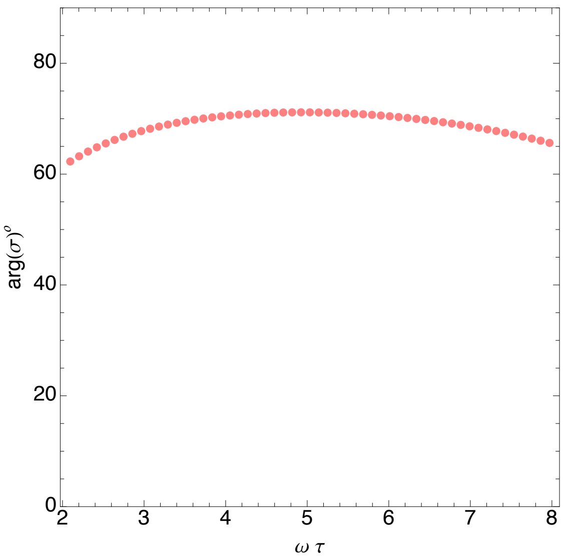
DC Resistivity



Mid-Frequency Behaviour

$$\omega \gtrsim T$$
$$2 < \omega\tau < 8$$



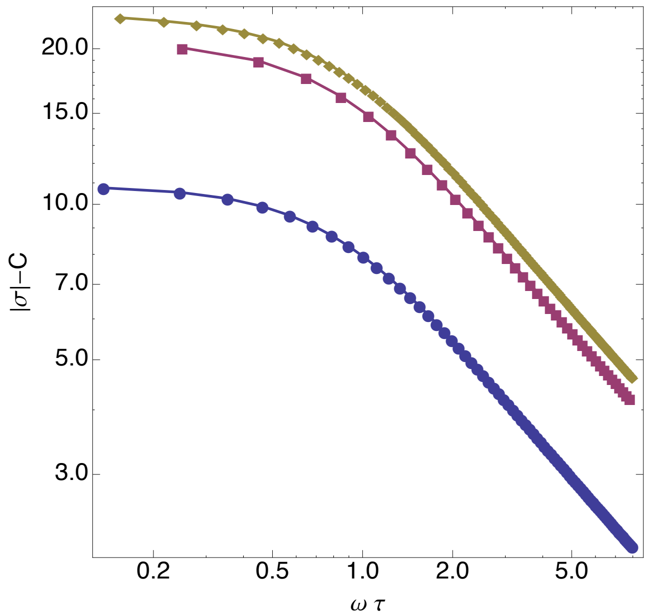


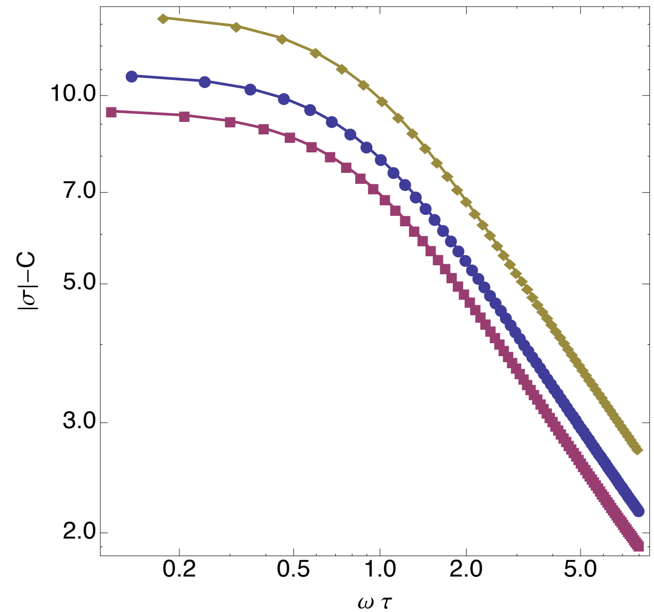
$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

Not arising from near horizon geometry alone

Robust Power-Law

Log-log plots



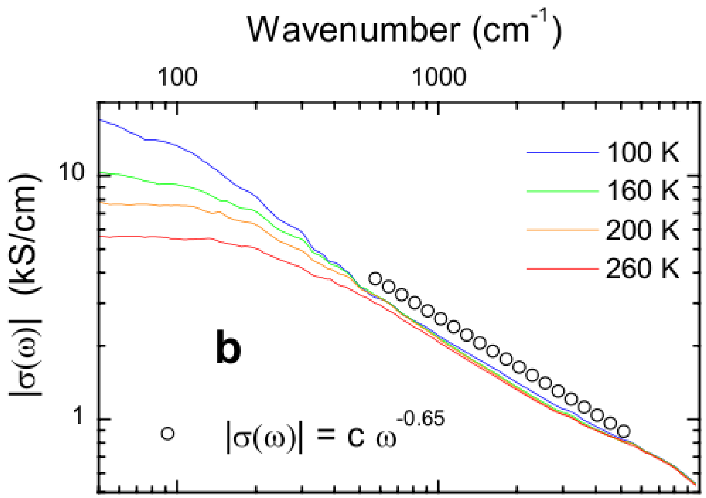


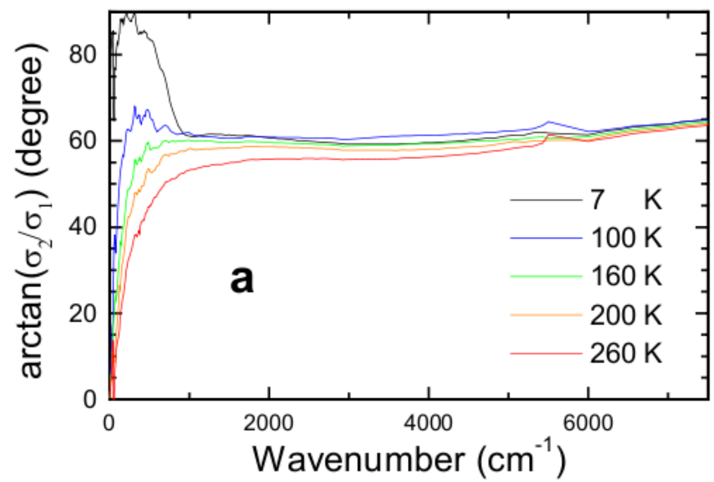
$k_L = 3,\ k_L = 1,\ k_L = 2$

$T = 0.98\mu,\ T = 0.115\mu,\ T = 0.13\mu$

Comparison to Cuprates

van der Marel et al.

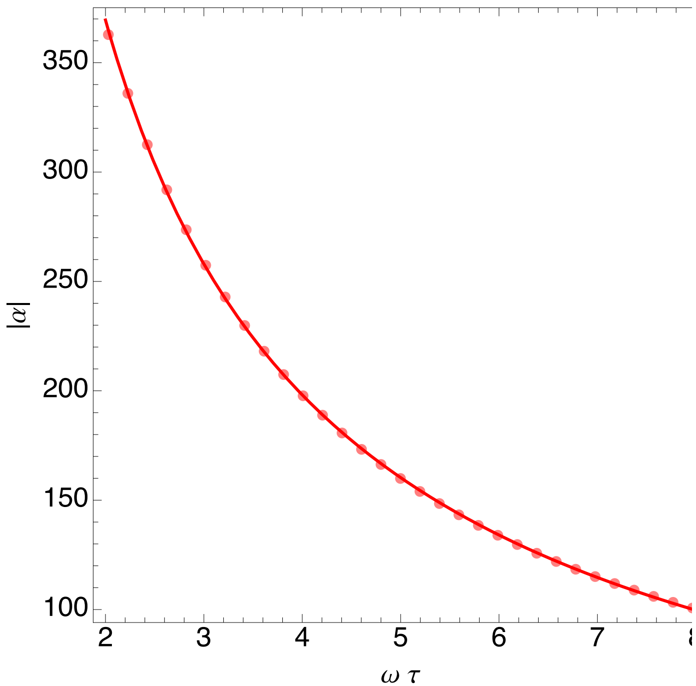




$$|\sigma(\omega)| \sim \frac{1}{\omega^{2/3}}$$

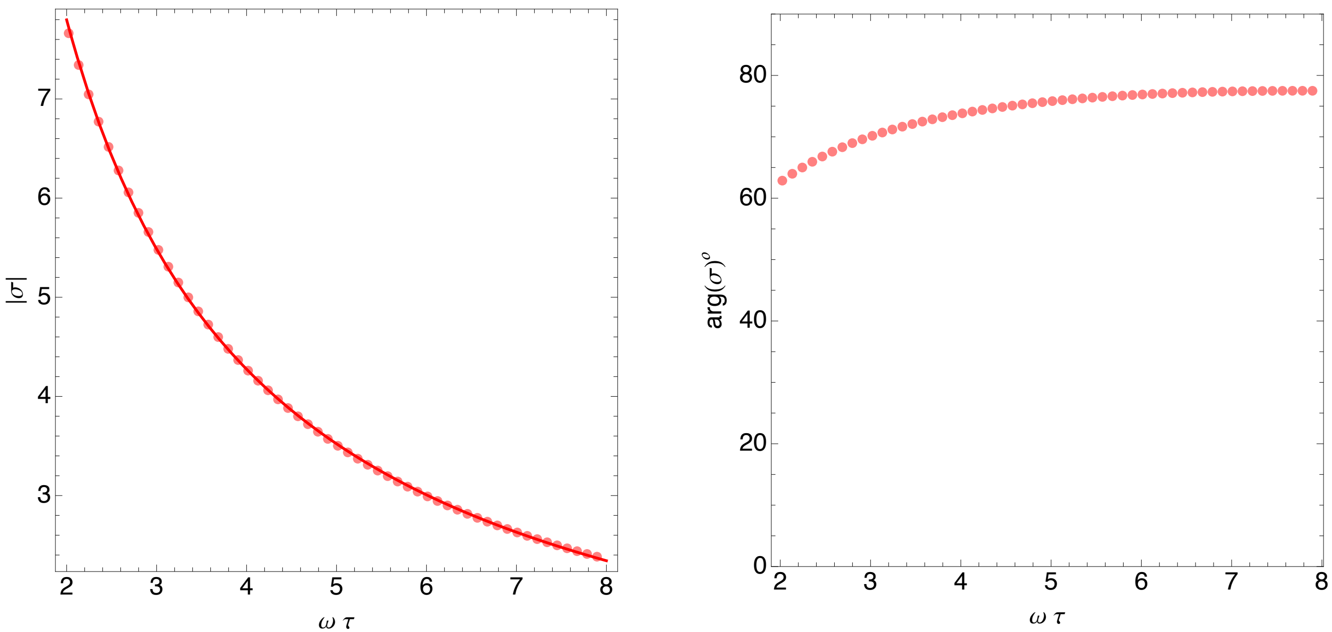
More Evidence for Scaling

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & \kappa T \end{pmatrix} \begin{pmatrix} R \\ -\nabla T / T \end{pmatrix}$$



$$\alpha(\omega) \sim \frac{B'}{\omega^{4/5}} + C'$$

Conductivity in AdS_5



$$|\sigma(\omega)| \sim \frac{1}{\omega^{0.867}}$$

Summary

- Clear evidence for lattice induced, mid-infrared, scaling regime
- Robust agreement with cuprates.
- Why?!