Holographic superfluids and the dynamics of symmetry breaking

Jerome Gauntlett

- Part I (1/4)

  Equilibrium AdS/CFT - overview of different holographic phases

  Aristomenis Donos, Christiana Pantelidou

- Part II (3/4)

  Non-equilibrium AdS/CFT - quench of a holographic superfluid

  Joe Bhaseen, Ben Simons, Julian Sonner, Toby Wiseman

Part I: Equilibrium AdS/CMT

- One focus: use holography to analyse the equilibrium behaviour of CFTs when held at finite temperature and charge density and/or in a uniform magnetic field and try to make contact with different phases that are seen in condensed matter
- Approach:
  - Construct all AdS black hole solutions with the relevant asymptotic behaviour
  - Calculate the free energies and deduce the phase diagram
- Hard!

Questions:

- What type of phases are possible?
- What kind of zero temperature ground states are possible?
- Do we find interesting new emergent scaling behaviour in the far IR? eg Lifshitz, Schrodinger, ..., something new??
- Transport?

Top-down solutions of D=10/11 supergravity preferable

AdS/CMT has certainly led to new insights into string/M-theory - including rich new classes of black hole solutions

Superfluid Phases

- Key ingredient for superfluidity: charged bulk fields that can spontaneously break a global U(1) symmetry

- s-wave superfluids have $l = 0$ order parameter
  Use charged bulk scalar fields. [Gubser][Hartnoll,Herzog,Horowitz] [Gauntlett,Sonner,Wiseman][Gubser,Herzog,Pufu,Tesileanu]

- p-wave superfluids have $l = 1$ order parameter
  Seen in eg He_3, heavy fermions, organics, Sr_2RuO_4

  In D=4,5 use SU(2) gauge fields. Take the background to be charged with respect to $U(1) \subset SU(2)$ and then spontaneously break the U(1) [Gubser]

  In D=5 can use a charged self-dual two-form [Aprile,Franco,Rodriguez,Russo]

- These black hole solutions have been found by solving ODEs

- The above examples have been found for CFTs at finite chemical potential with respect to the global U(1) symmetry. The black holes are electrically charged

- Holographic superconductivity can also occur for CFTs in a magnetic field.

  - Examples with bulk charged scalars [Almuhairi,Polchinksi][Donos,Gauntlett,Pantelidou]

  - Examples with bulk charged vectors [Ammon,Erdmenger,Kerner,Strydom][Almuhairi,Polchinksi] [Donos,Gauntlett,Pantelidou] - connections with condensation of rho mesons in QCD? [Chernodub]

  - Both occur for N=4 d=4 SYM and N=8 d=3 SYM and there is an interesting interconnection with supersymmetry...

  - Inferred using a linearised analysis - to construct back reacted black holes need to solve PDEs

Spatially Modulated Phases

- In condensed matter there is a variety of phases that are spatially modulated, spontaneously breaking translation invariance.
- For example: charge density waves and spin density waves.
- The modulation is fixed by an order parameter associated with non-zero momentum.
- Spatially modulated superconducting phases are also possible. FFLO phase is a variation of BCS. Perhaps seen in some heavy fermions (eg $CeCoIn_5$) and some organic superconductors

- Spatially modulated black holes are possible
- d=3 or d=4 CFTs at finite chemical potential with respect to U(1) and we find examples of spatial modulated phases [Nakamura,Ooguri,Park][Donos,Gauntlett] [Domokos,Harvey][Bergman,Jokela,Lifschytz,Lippert]
- Also find spatial modulation when d=3 or d=4 CFT is placed in a magnetic field [Donos,Gauntlett,Pantelidou]
- Examples have spatially modulated currents. Can also have [Donos,Gauntlett,Pantelidou]
    - Charge density waves
    - Spatially modulated superfluids. Specifically p-wave with a helical order
    - Top-down examples in D=10,11 supergravity

- Spatially modulated phases are not exotic. Typical?
- The spatially modulated phases above have been inferred using a linearised perturbative analysis. Generically to go beyond this one needs to solve PDEs.....many interesting questions here
- An interesting exception[Donos,JPG]: back reacted black holes that describe d=3+1 p-wave superfluids with a helical structure. The D=5 gravity model has a metric, a gauge-field and a charged two-form
- The solutions are spatially homogeneous with a helical structure. More precisely there is a Bianchi VII_0 symmetry and we constructed black holes by solving ODEs!
- At zero temperature the black holes become domain wall solutions and interpolate between AdS5 in the UV and a new scaling solution in the IR with helical symmetry (c.f. [Iizuka,Kachru,....])

Superconductor Week

Commercialization · Markets · Products
Business Developments · R&D · Cryogenics

June 30, 2012 Vol. 26, No. 10

Superconductor Week

Commercialization · Markets · Products
Business Developments · R&D · Cryogenics

June 30, 2012 Vol. 26, No. 10

Part 2: Non equilibrium Dynamics

- Studying the far from equilibrium dynamics of any system is a very challenging problem
- Receiving much interest not least because of new experiments e.g. in the context of cold atoms
- Can we use AdS/CFT to obtain new insights?
- Basic idea is very simple: analyse time dependent black hole solutions.
- Technically challenging because it requires solving non-linear PDEs.
  [.....][Chesler,Yaffe][Murata][Bizon,Rostworowski][Garfinkle,Pando Zayas] [Bantilan,Pretorius,Gubser][Buchel,Lehner,Myers][Heller,Janik,Witaszczyk][.....]

- Quantum quench
- Start with a state of initial Hamiltonian that is abruptly changed e.g.

$$H = H_0 + g(t)H \qquad\qquad g(t) = g\theta(t)$$

How does the sate evolve?

- a) Nature of thermalisation? Relaxation times?
- b) Universal behaviour near critical points?

- Here we will study a quantum quench of superfluids. Analyse in the context of AdS/CFT but discover some universal features that apply more broadly

Quantum quench of a BCS superconductor

- Abruptly switch on pairing interactions in a BCS setting [Barankov,Levitov]
- Approximations: collisionless, no thermal dissipation, no vortex production

BCS Hamiltonian

$$\mathcal{H} = \sum_{p,\sigma} \epsilon_p a^\dagger_{p,\sigma} a_{p\sigma} - \frac{\lambda(t)}{2} \sum_{q,p} a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} a_{-q\downarrow} a_{q\uparrow}$$

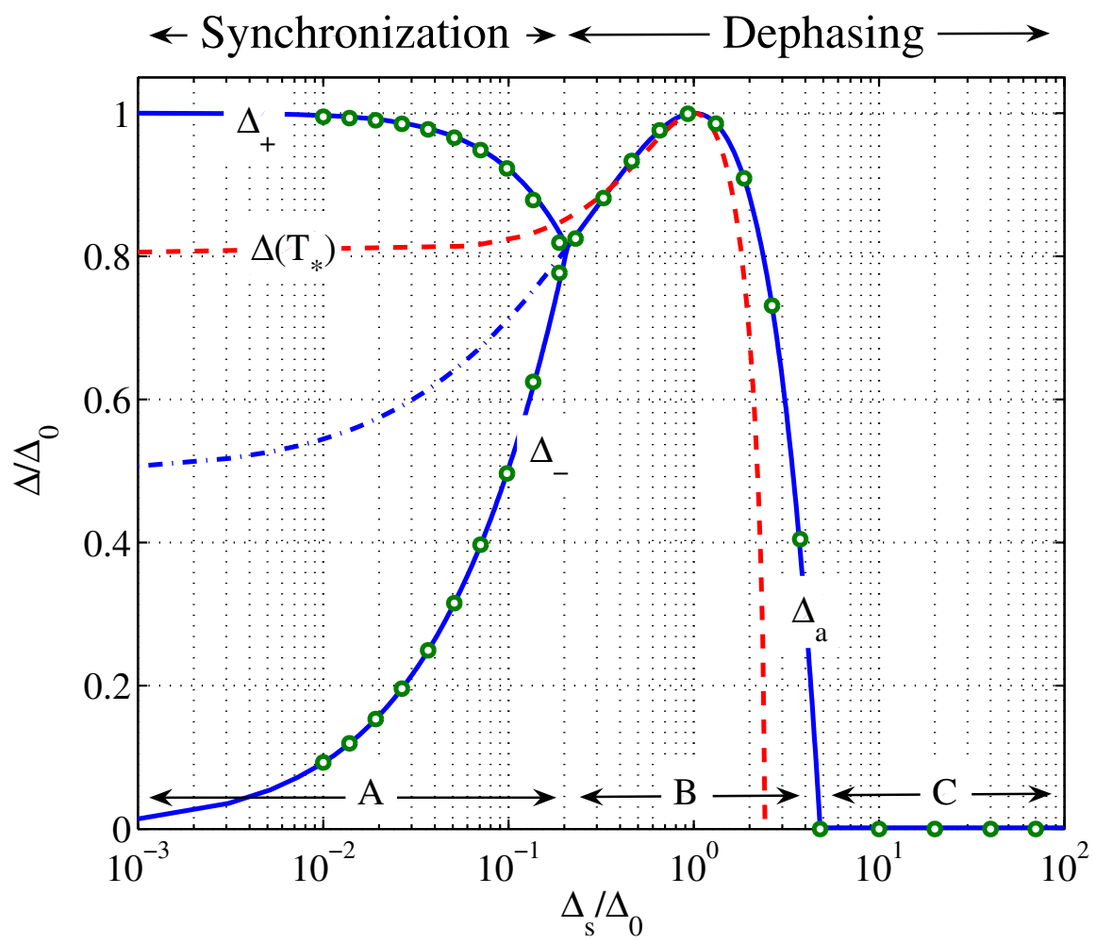
$$\lambda(t) = \lambda_s, \quad t < 0$$

$$\lambda(t) = \lambda, \quad t \geq 0$$

B-L showed this is an integrable system

- B-L Dynamical phase diagram

Final pairing gap



Strength of quench

- Region A: persistent oscillations in a final superfluid state
- Region B: power-law decay to final superfluid state
- Region C: power-law decay to unbroken phase final state

- What happens if we relax the assumptions and consider the role of collisions, thermal damping, non-BCS, strong coupling ...
- What survives?
- Explore using AdS/CFT

THE MODEL [Hartnoll,Herzog,Horowitz]

$$S = \int d^4x \left(R + 6 - F^2 - |D\psi|^2 - m^2|\psi|^2\right)$$

$$D\psi = d\psi - iqA\psi$$

AdS4 vacuum is dual to a d=2+1 CFT with a global U(1) symmetry

Order parameter for superfluid $\psi \leftrightarrow \mathcal{O}$

For simplicity we choose $q = 2 \qquad m^2 = -2 \qquad \Delta(\mathcal{O}) = 2$

Unbroken phase black holes

- The electrically charged AdS-RN black hole

$$ds^2 = -g dt^2 + g^{-1} dr^2 + r^2 (dx^2 + dy^2)$$

$$A = \phi dt \qquad\qquad \psi = 0$$

$$g = r^2 - (r^2 + \mu^2)\frac{r_+}{r} + \frac{r_+^2 \mu^2}{r^2}$$

$$\phi = \mu(1 - \frac{r_+}{r})$$

- Describes CFT at chemical potential μ and temperature T

Superfluid phase black holes

- Spatially homogeneous and isotropic

$$ds^2 = -ge^{-\beta}dt^2 + g^{-1}dr^2 + r^2(dx^2 + dy^2)$$

$$A = \phi dt \qquad \psi = \psi(r) \in \mathbb{R}$$

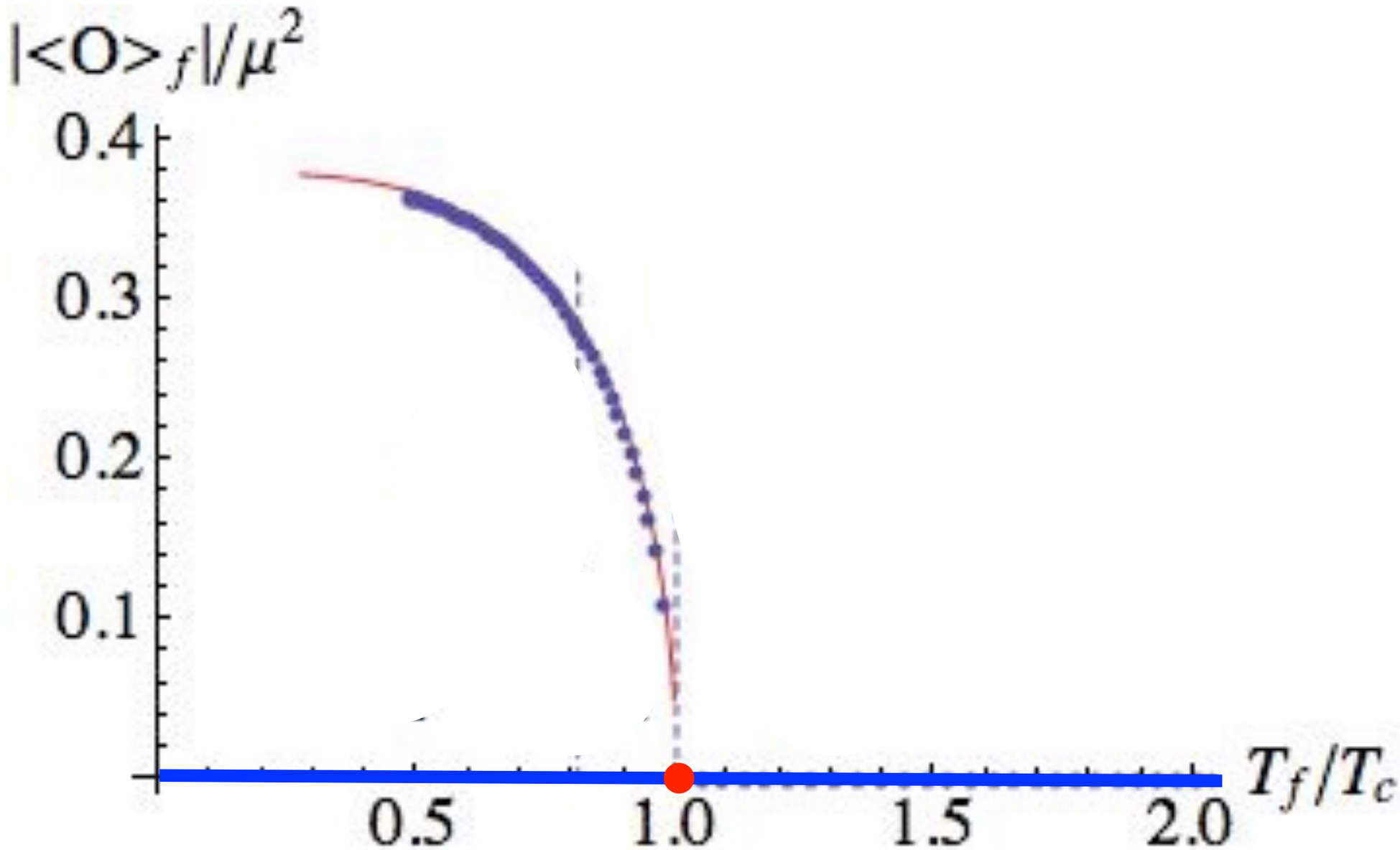
$$g = r^2 + \ldots \qquad \beta = 0 + \ldots \qquad \phi = \mu - \frac{q}{r} + \ldots$$

$$\psi = \frac{\psi_1}{r} + \frac{\psi_2}{r^2} + \ldots$$

$\psi_1 \leftrightarrow$ adding a source to CFT $\qquad \psi_2 \leftrightarrow \langle \mathcal{O} \rangle$

Superfluid black holes have $\psi_1 = 0$

Superfluid phase transition



Note solutions for this bottom up model are singular at T= 0
(The top-down examples [JPG,Sonner,Wiseman] have an emergent AdS4 at T=0)

The dynamical quench from the superfluid phase

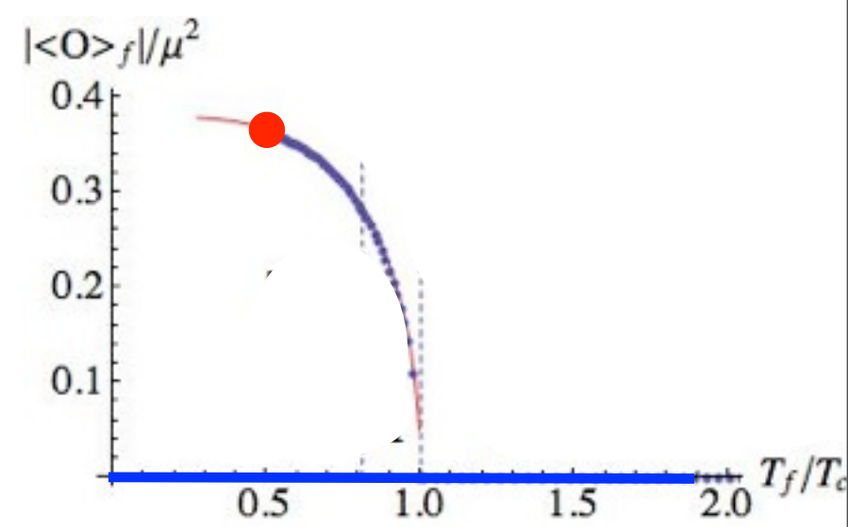
No source
$t = \infty$

Final black hole solution

Gaussian pulse of
field dual to $\mathcal{O}$

No source
$t = -\infty$

Initial superfluid black hole
solution $T = 0.5T_c$



AdS4 boundary

Consider homogeneous and isotropic quenches

Use ingoing Eddington Finklestein coordinates

$$ds^2 = z^{-2}\left[-F\,dv^2 - 2\,dv dz + S^2(dx_1^2 + dx_2^2)\right]$$

$$F = F(v,z) \qquad S = S(v,z)$$

$$A = A_v(v,z)dv \qquad \psi = \psi(v,z)$$

Asymptotic AdS4 boundary is at $z = 0$

[Kinoshita, Murata, Tanahashi]

- 5 real variables
- 8 PDEs: 5 evolution equations and 3 constraint equations

- Asymptotic expansion

$$\psi(v,z) = z\psi_1(v) + z^2\psi_2(v) + \dots$$

$$A_v(v,z) = \mu(v) - z\rho(v) + \dots$$

$$F(v,z) = 1 - \tfrac{1}{2}|\psi_1|^2 z^2 + \tilde{F}(v)z^3 + \dots$$

$$S(v,z) = 1 - \tfrac{1}{4}|\psi_1|^2 z^2 + \tilde{S}(v)z^3 \dots$$

- Apply a sharply peaked Gaussian pulse with strength δ

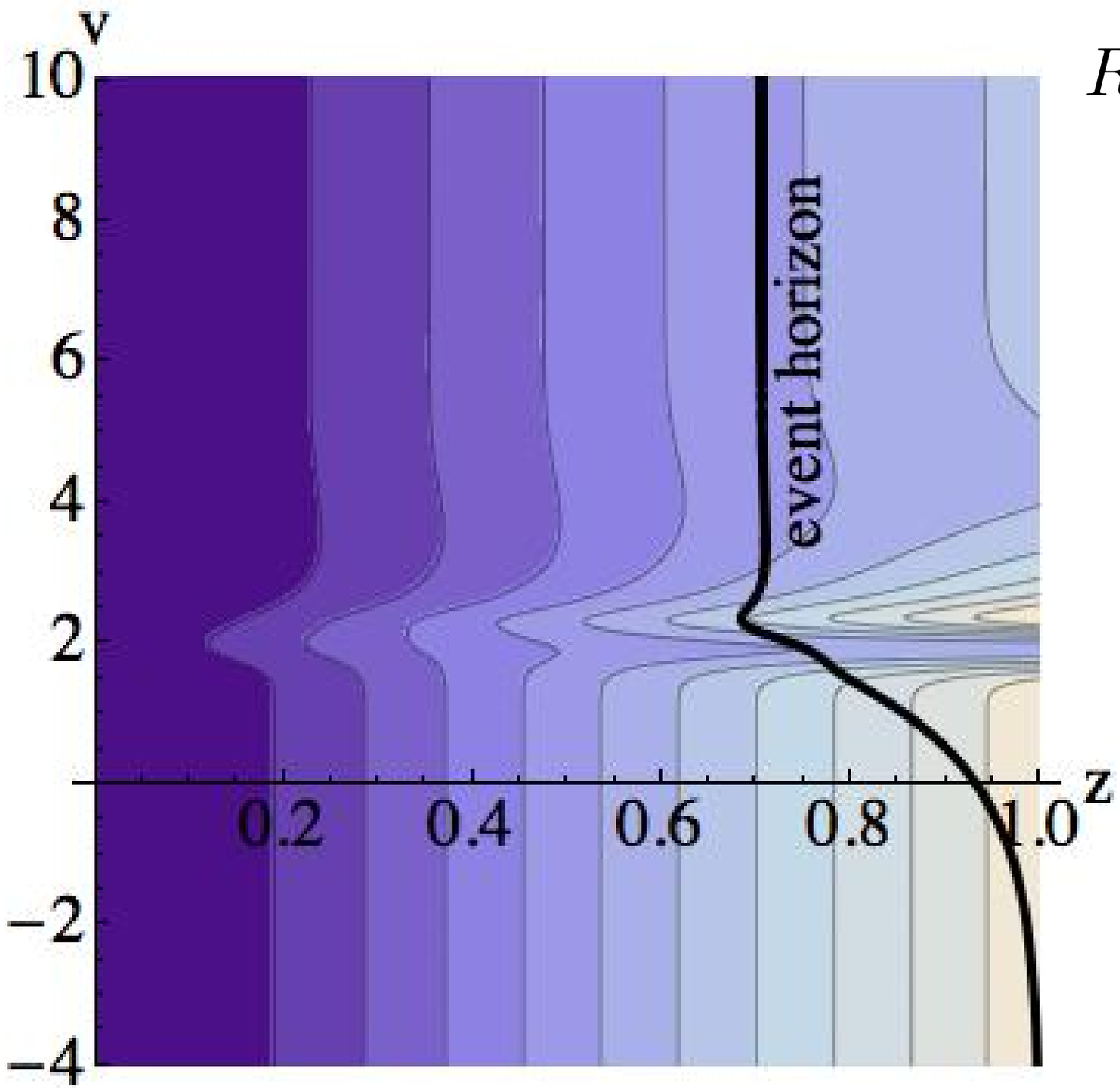
$$\psi_1(v) = \delta e^{-10v^2}$$

- Subtlety in holographic renormalisation

$$\langle \mathcal{O} \rangle \sim \psi_2 - \mu\psi_1 \qquad\qquad \langle J_t \rangle \sim \rho + \partial_v \mu$$

- Quench satisfies: $\langle J_t \rangle|_{initial} = \langle J_t \rangle|_{final}$

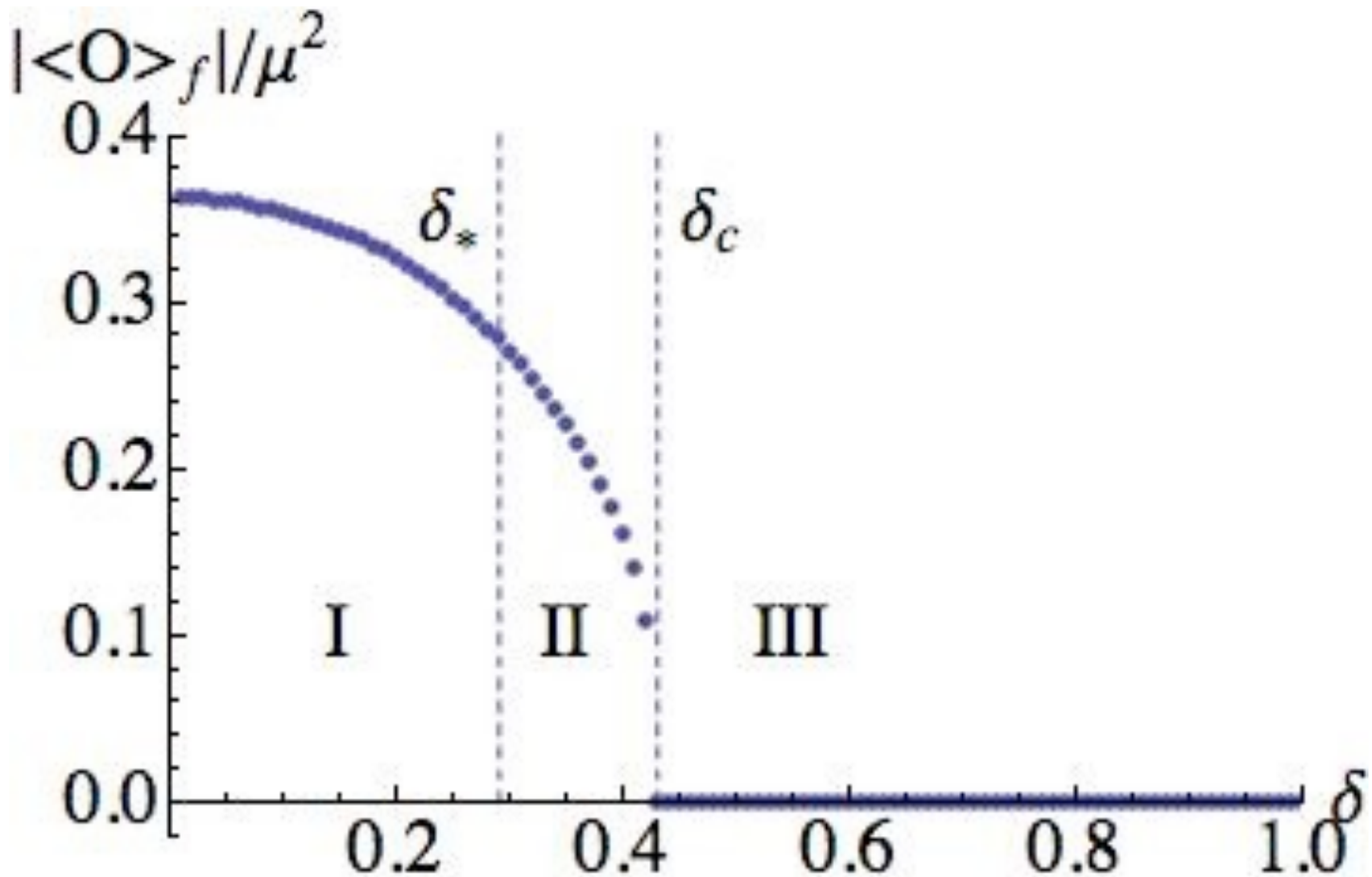
Results



$Re(\psi)$

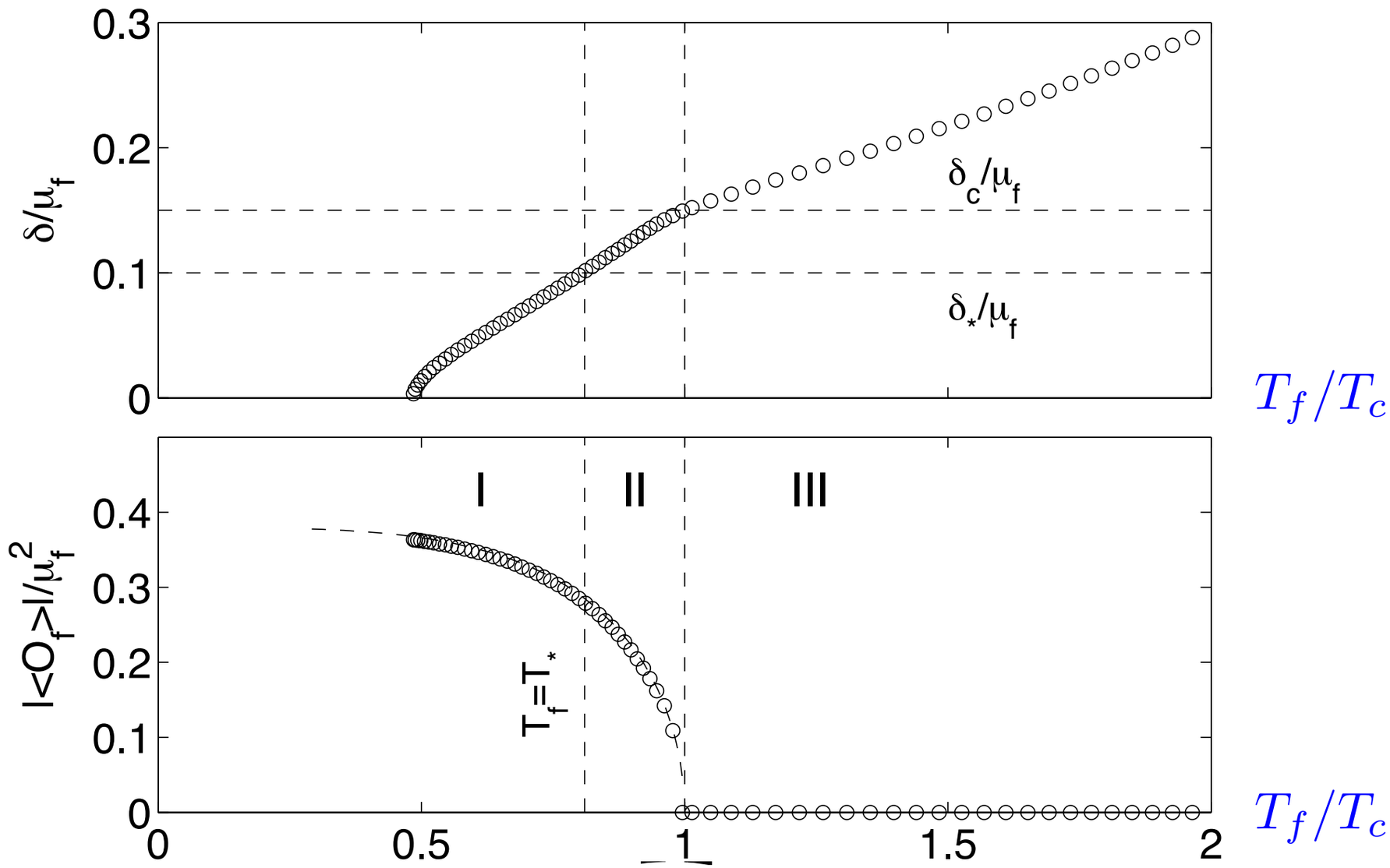
At late times it settles down to an equilibrium black hole solution

Dynamical phase diagram



Strength of quench

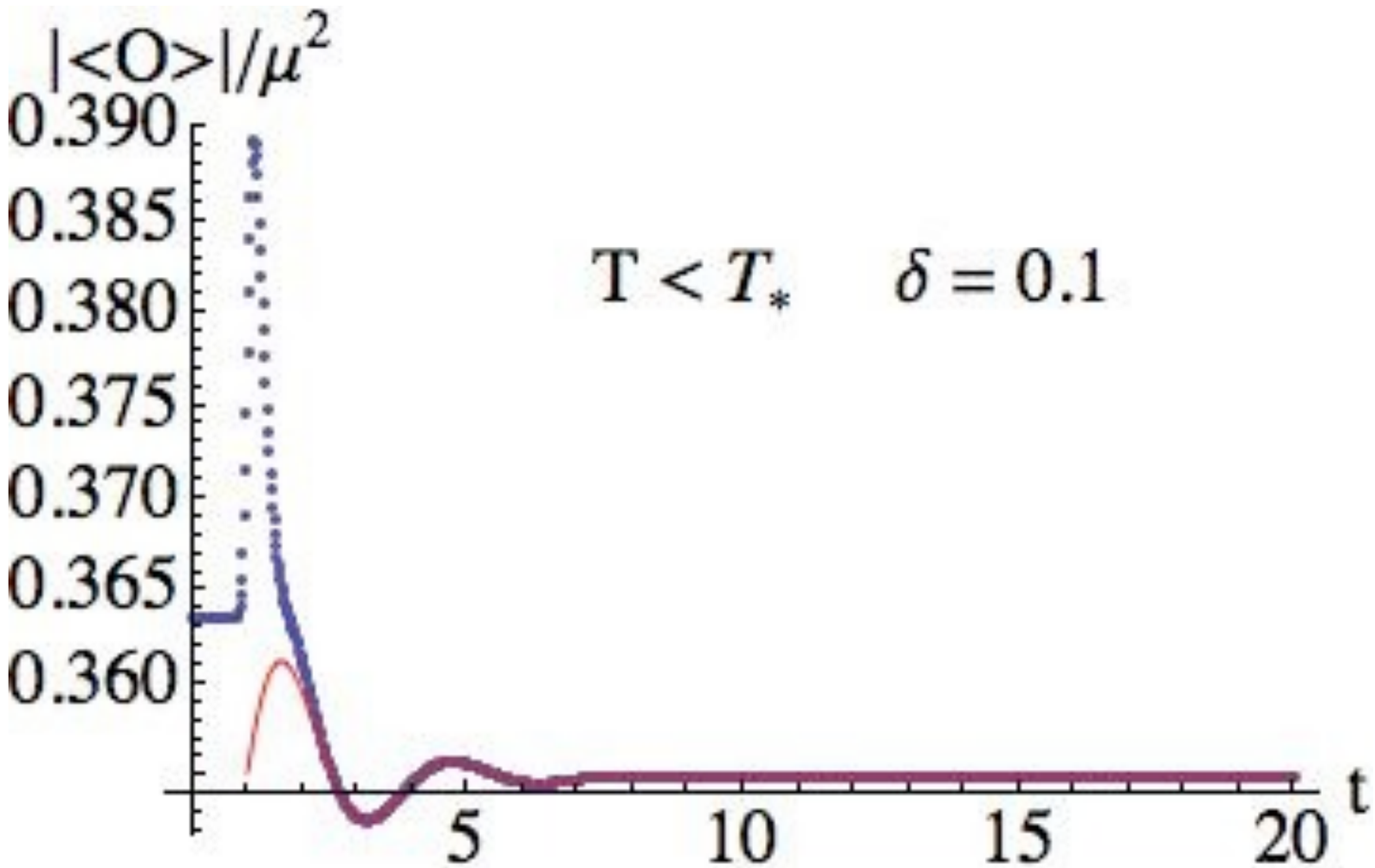
The quench adds energy and heats the superfluid



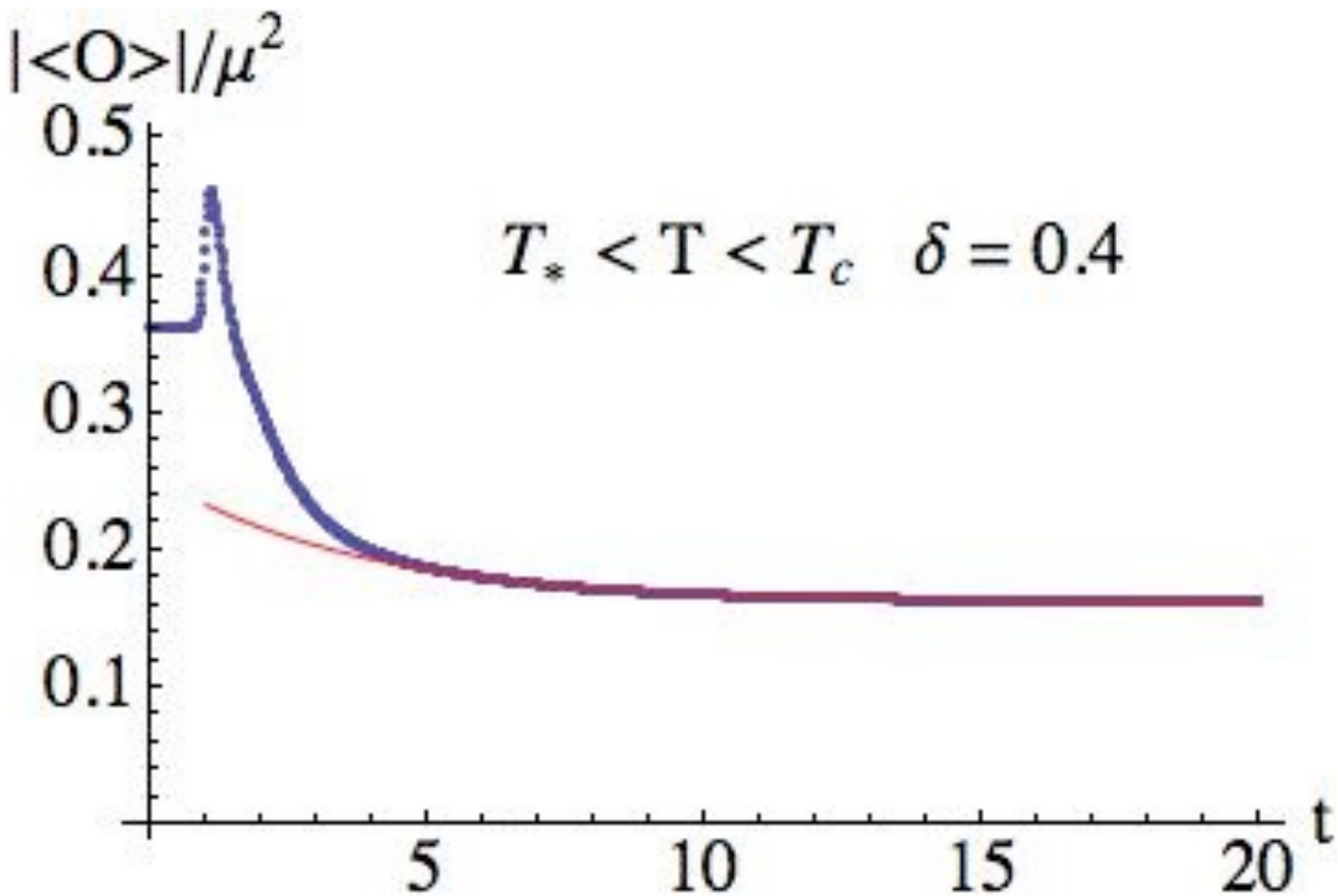
The final black hole is indeed an equilibrium black hole

Further analysis reveals that T_* corresponding to the quench δ_* is an emergent dynamical temperature scale

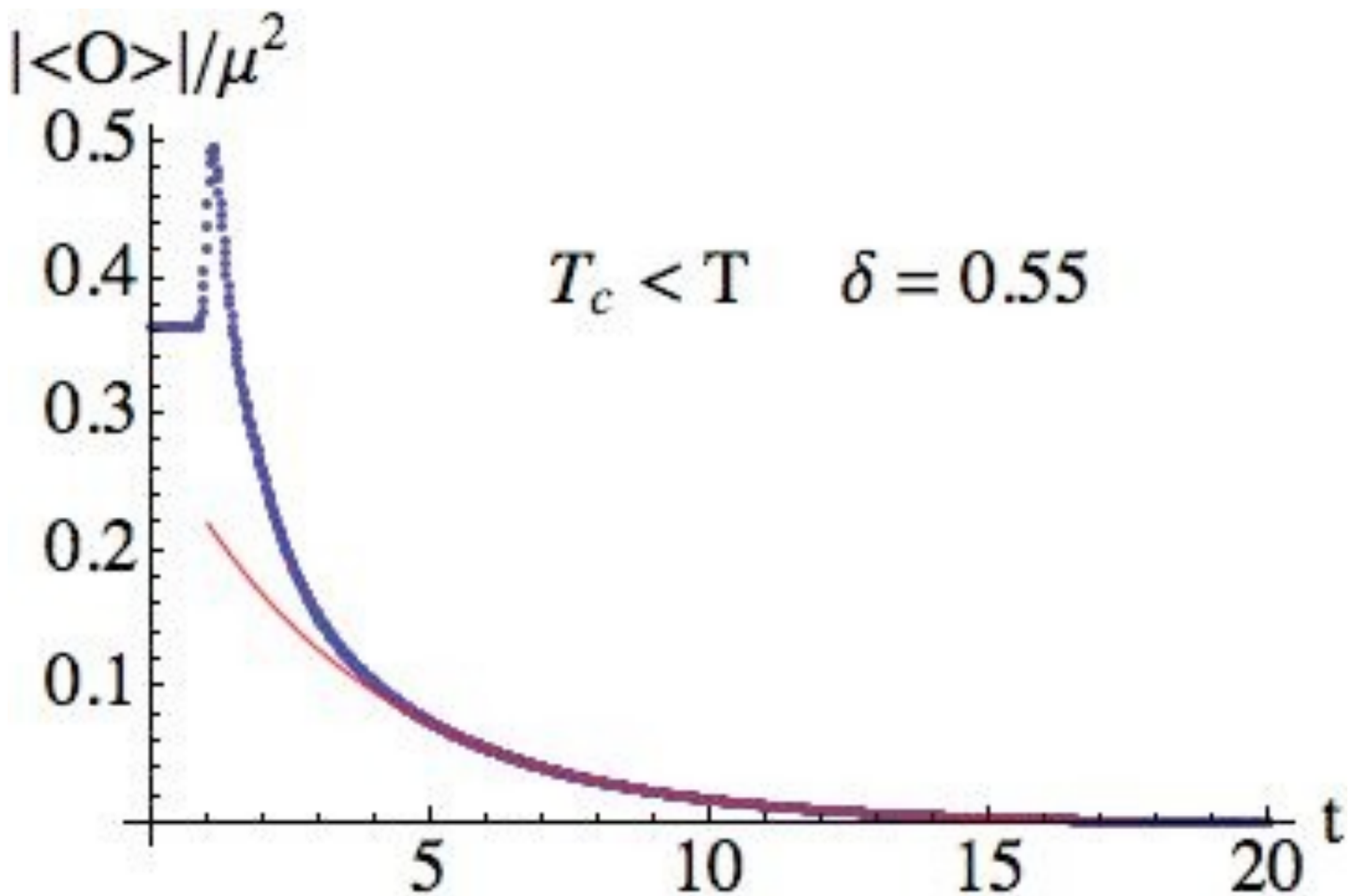
Region I: Small quenches give rise to damped oscillations to a final superfluid black hole



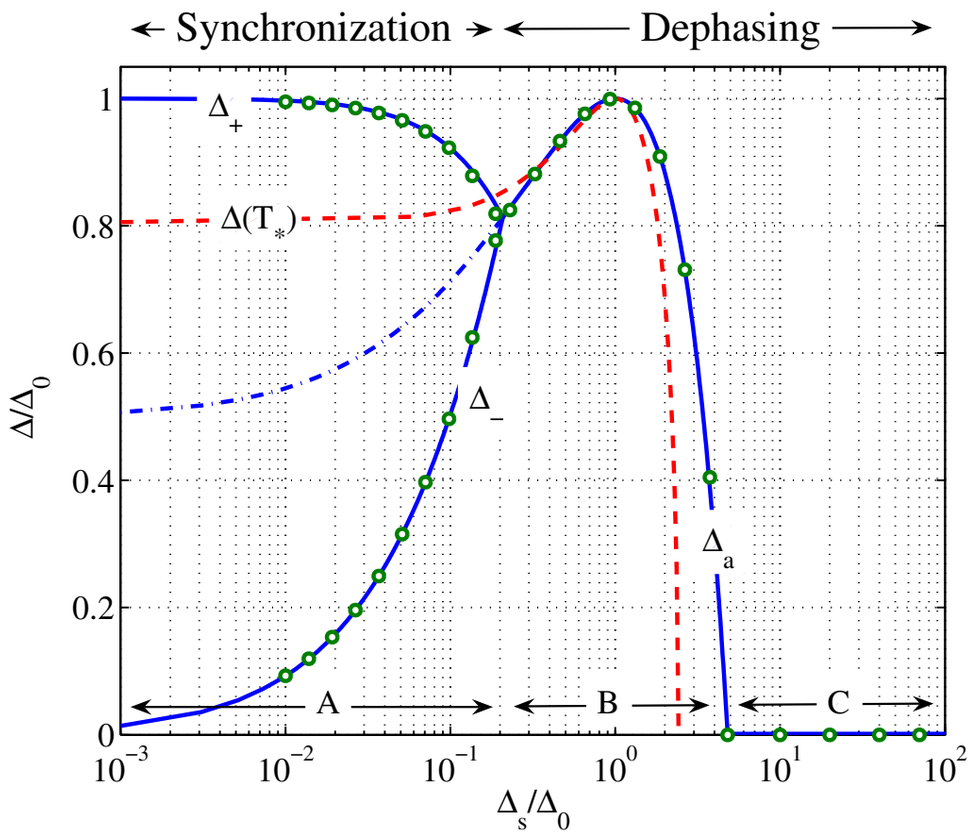
Region II: Larger quenches give rise to a decayed approach to a final superfluid black hole

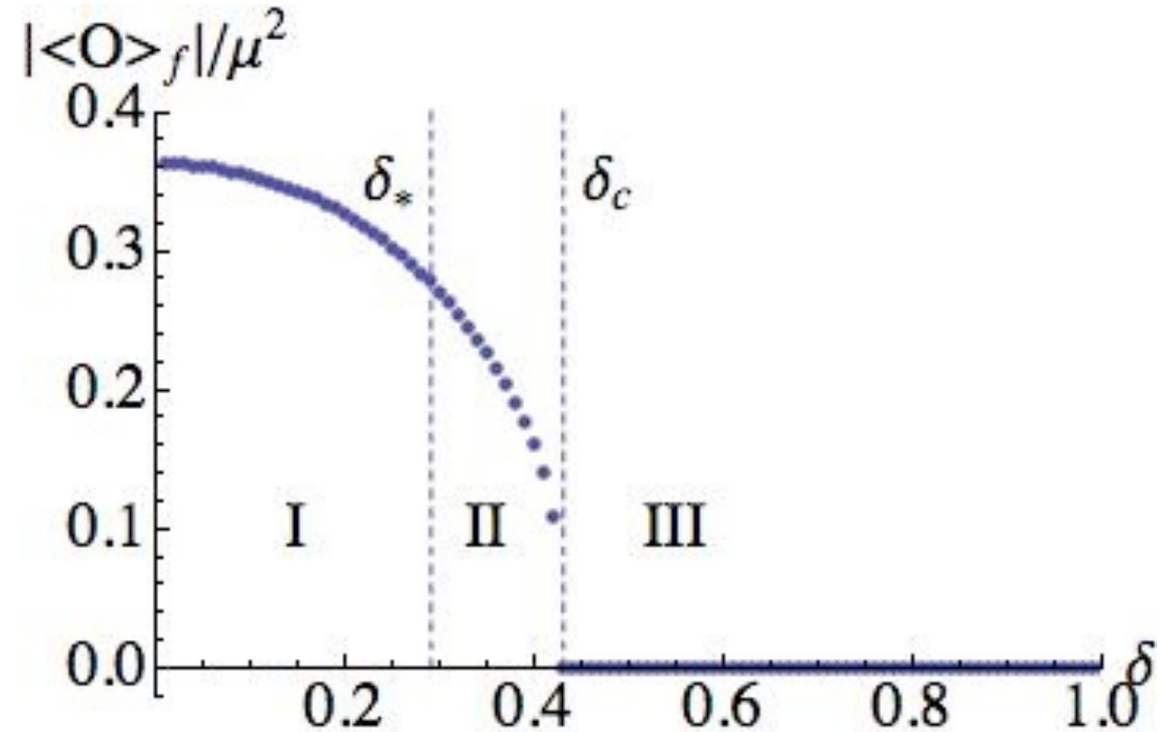


Region III: Larger quenches still give rise to a decayed approach to the unbroken phase



$T_c < \mathrm{T} \quad \delta = 0.55$





- The three regions of B-L survive the inclusion of collisions, thermal affects and also in a strongly coupled set up
- Region I: Persistent oscillations in B-L replaced with exponentially damped oscillations
- Region II: power-law damped oscillations in B-L replaced with decay
- δ_* is an analogue of the B-L dephasing transition

Quasi normal modes

- The late time behaviour should be governed by linear response theory, which is determined by the quasi-normal modes of the black hole
- Provides excellent check of numerics and also leads to key insight into where the emergent temperature T_* comes from
- Recall that the QNMs are linearise perturbations with ingoing boundary conditions at the black hole event horizon and are normalisable at the AdS boundary
- They are functions of complex ω . For stable black holes they lie in the lower half plane
- The late time dynamics should be governed by the dominant quasi-normal modes i.e. the QNMs that are closest to the real axis

- For the homogeneous and isotropic quenches we only consider the QNMs at zero momentum
- Consider perturbations about the equilibrium black holes

$$\begin{aligned} \psi(v,z) &= \psi_0(z) + \delta\psi(v,z) \\ g_{ab}(v,z) &= g_{ab,0}(z) + \delta g_{ab}(v,z) \\ A(v,z) &= A_0(z) + \delta A(v,z) \end{aligned}$$

- Only consider sector involving $\delta\psi$
- Diffeomorphisms and U(1) gauge symmetry dealt with by defining gauge invariant variables and we consider

$$\delta\Phi_I(v,z) = e^{-i\omega v}\Phi_I^{\omega}(z)$$

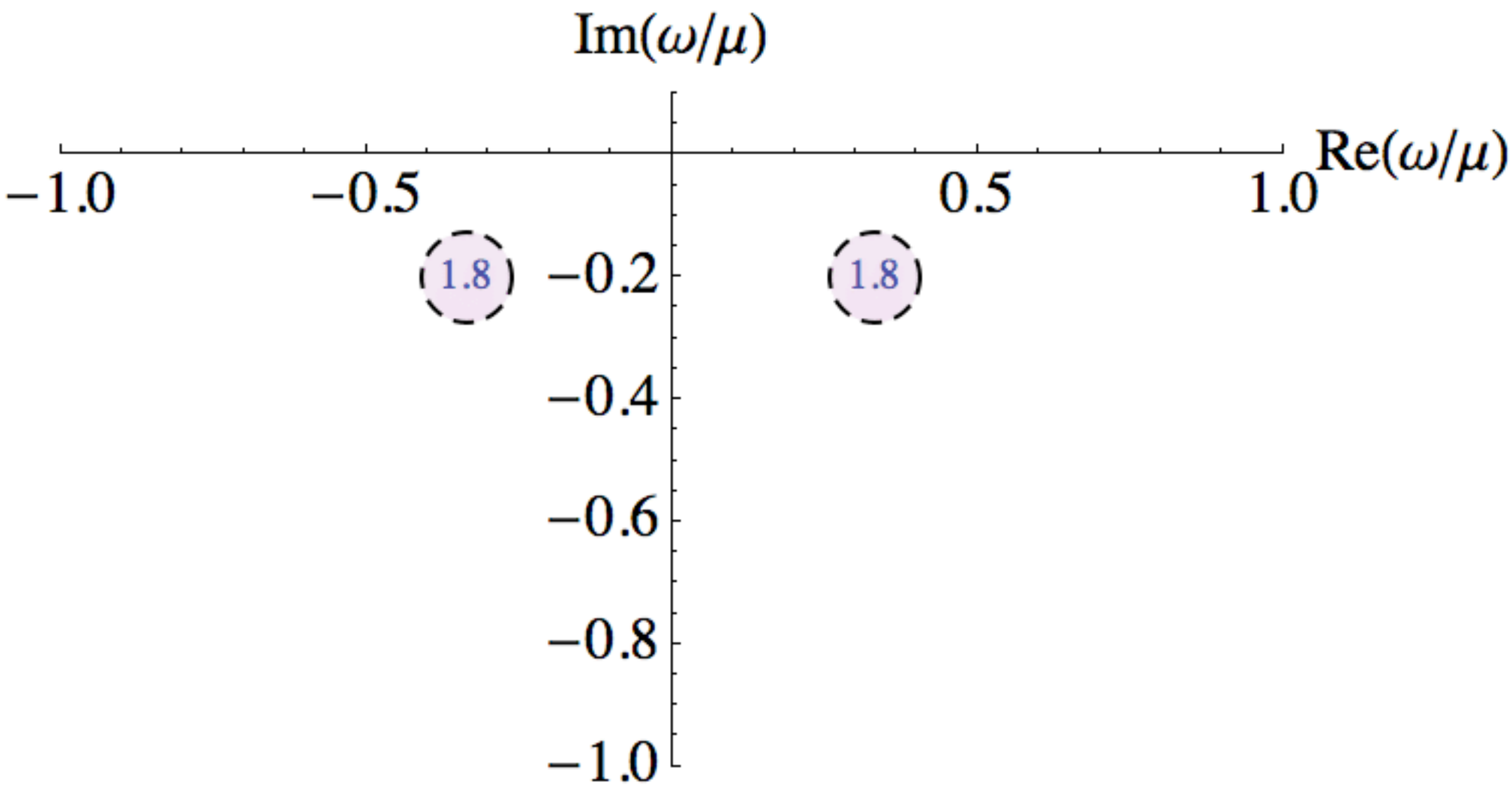
with $\omega \in \mathbb{C}$

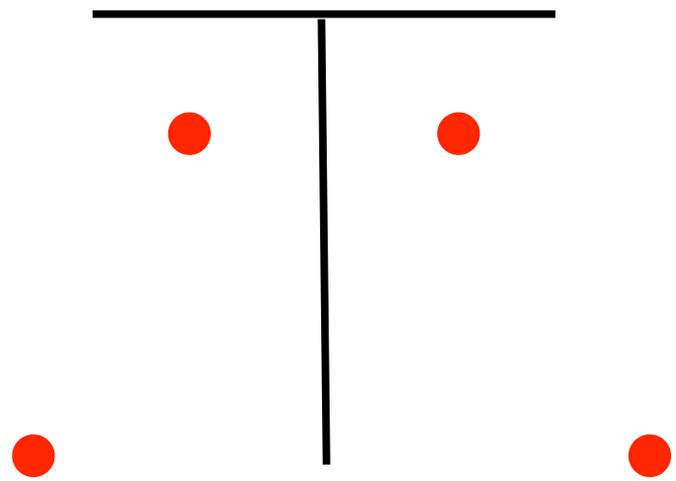
The QNM pole-dance

Start with QNMs at $T > T_c$ and follow motion as we decrease T

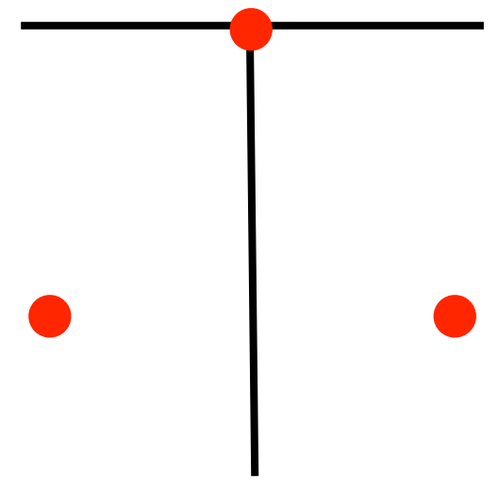
The QNM pole-dance

Start with QNMs at $T > T_c$ and follow motion as we decrease T

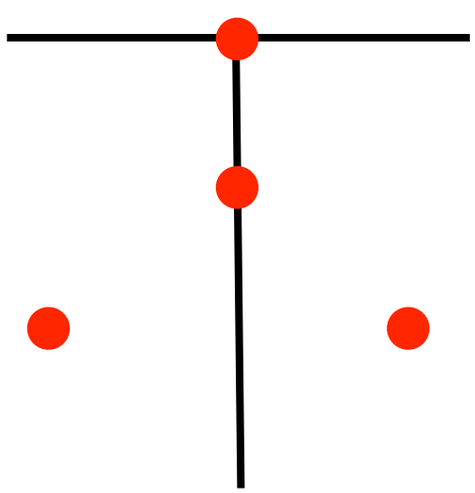




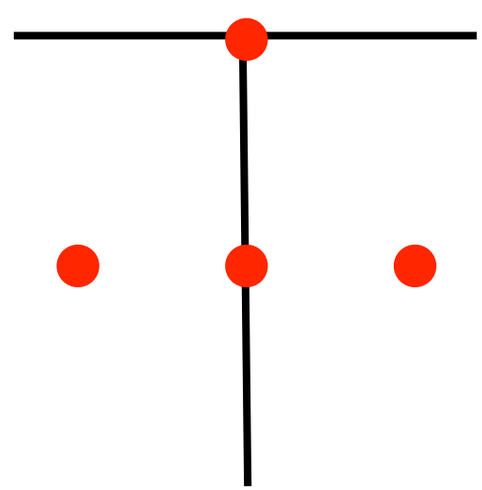
$T > T_c$



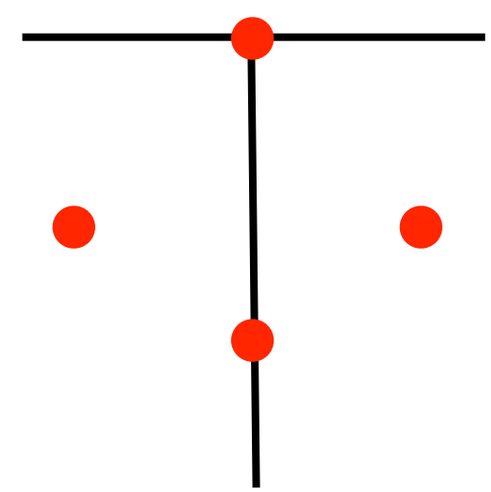
$T = T_c$



$T_* < T < T_c$



$T = T_*$



$T < T_*$

- Notice symmetry under $\omega \to -\omega^*$ .This comes from time-reversal invariance of the system
- The QNM at $\omega = 0$ is the Goldstone mode. The one sailing down the imaginary axis is the amplitude mode
- The late time dynamics is determined by the dominant QNMs - the ones that lie closest to the real axis
- Roughly, expect the real part of the QNMs are associated with oscillations and the imaginary part with decay
- More precisely we have

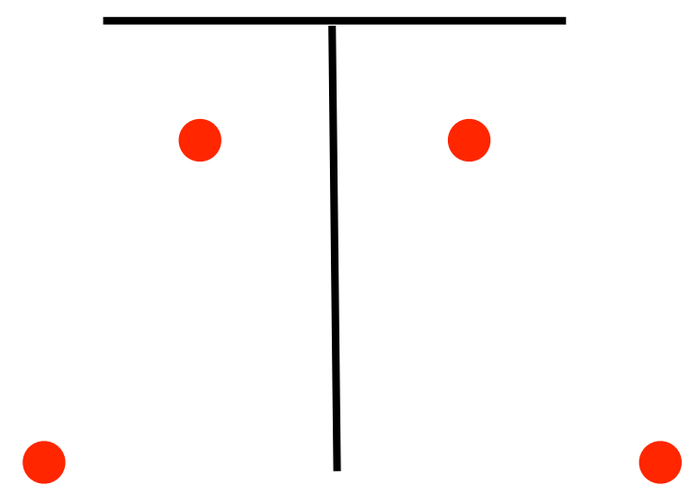
$$|\langle \mathcal{O}(t) \rangle| = |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|$$

where ω corresponds to the dominant QNMs

Region III: $T > T_c$ decayed approach to a final unbroken phase black hole

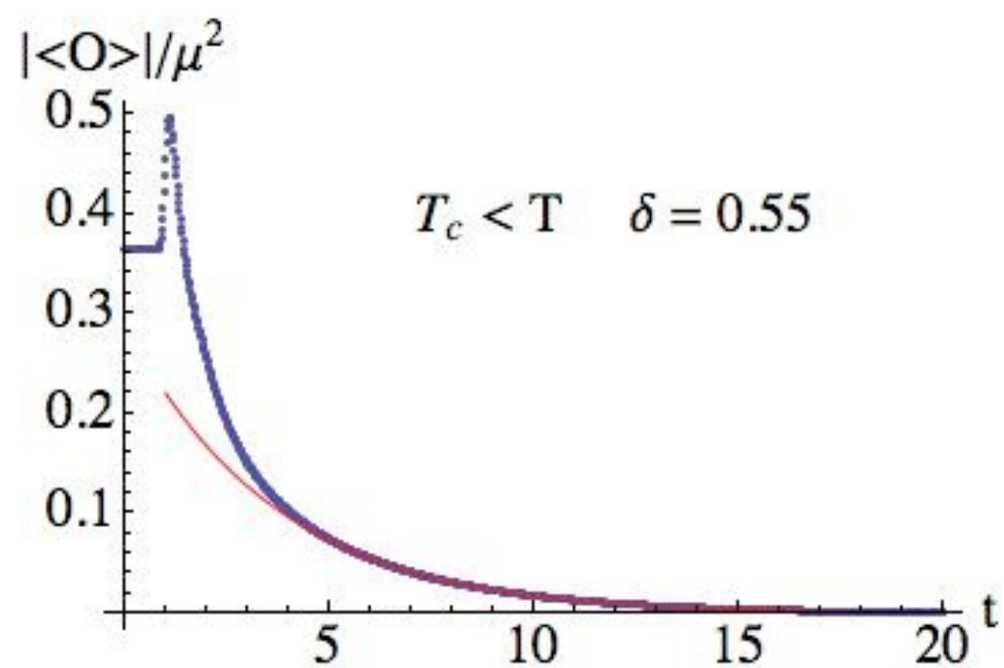
- The dominant QNMs have

$$Re(\omega) \neq 0 \quad Im(\omega) \neq 0$$



- But $\langle \mathcal{O} \rangle_f = 0$ so

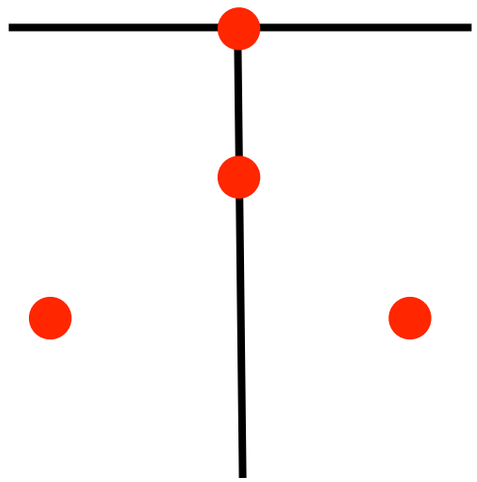
$$\begin{aligned} |\langle \mathcal{O}(t) \rangle| &= |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}| \\ &= |\mathcal{A}| e^{Im(\omega)t} \end{aligned}$$



Region II: $T_* < T < T_c$ decayed approach to a final superfluid phase black hole

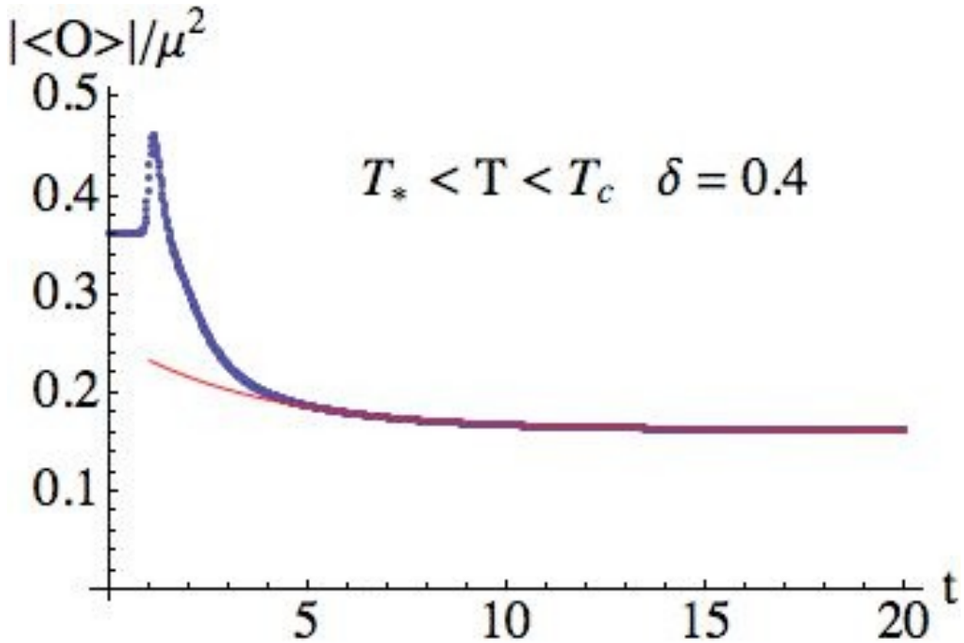
- The dominant QNMs have

$$Re(\omega) = 0 \quad Im(\omega) \neq 0$$



- Now $\langle \mathcal{O} \rangle_f \neq 0$ so

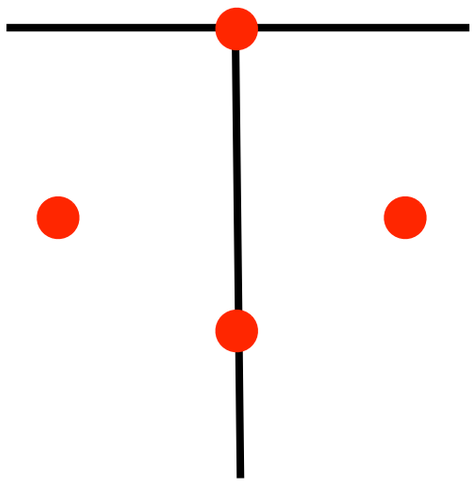
$$|\langle \mathcal{O}(t) \rangle|^2 = |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|^2$$
$$= |\langle \mathcal{O} \rangle_f|^2 + |\mathcal{A}|^2 e^{2Im(\omega)t}$$
$$+ 2e^{Im(\omega)t} Re[\langle \mathcal{O} \rangle_f A^*]$$



Region I: $T < T_*$ damped oscillations approaching a final superfluid phase black hole

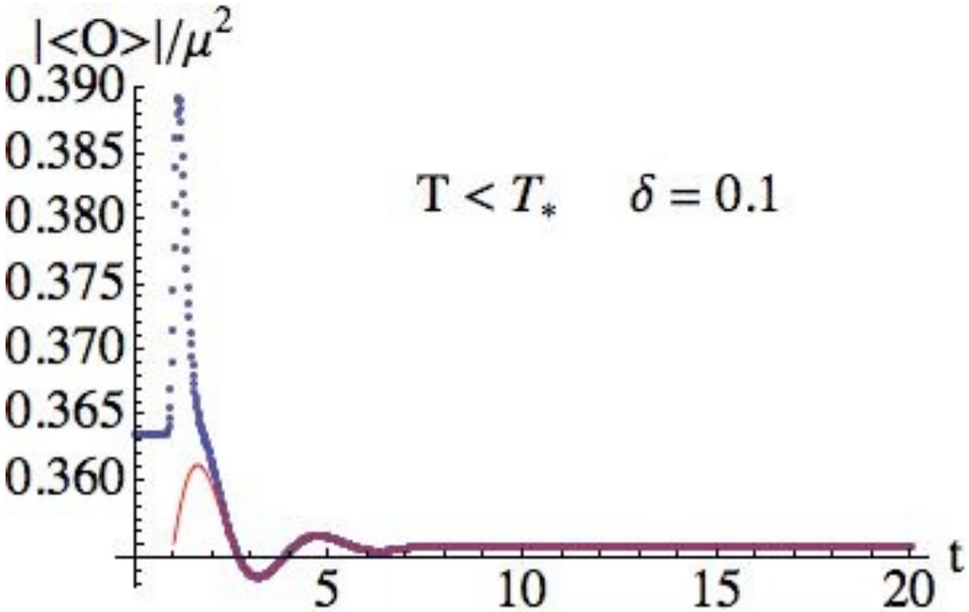
- The dominant QNMs have

$$Re(\omega) \neq 0 \qquad Im(\omega) \neq 0$$



- Now $\langle \mathcal{O} \rangle_f \neq 0$ so

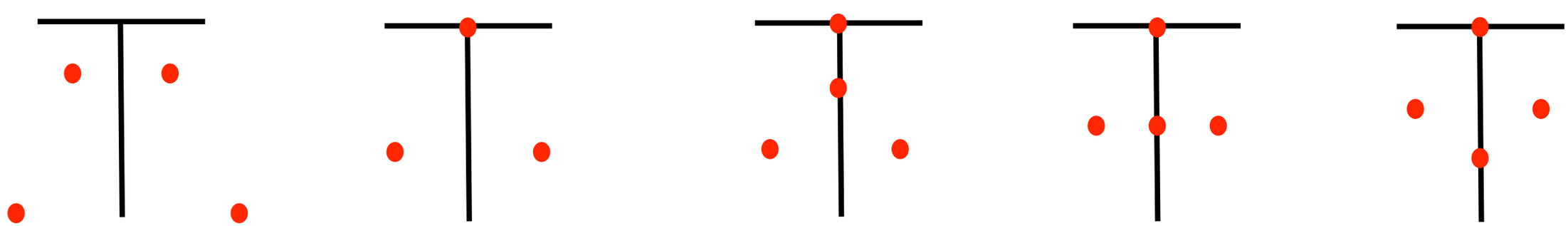
$$\begin{aligned} |\langle \mathcal{O}(t) \rangle|^2 &= |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|^2 \\ &= |\langle \mathcal{O} \rangle_f|^2 + |\mathcal{A}|^2 e^{2Im(\omega)t} \\ &+ 2e^{Im(\omega)t}\Big(Re[\langle \mathcal{O} \rangle_f A^*] \cos[Re(\omega)t] \\ &\qquad - Im[\langle \mathcal{O} \rangle_f A^*] \sin[Re(\omega)t] \Big) \end{aligned}$$



Universality

- The main features, including the new dynamical temperature scale T_* in the superfluid phase are captured by the QNMs
- Expect similar results for other holographic superfluids both in d=2+1 but also in 3+1
- The results also have significance for non-holographic systems!
- Recall that the location of QNMs correspond to the location of the poles of the retarded Green's function for the operator $\mathcal{O}$ in the dual CFT
- Thus ANY system that has poles in the retarded Greens function as we have here will give rise to the same late time linear response under a quench

- Key point: the pole structure below is the generic structure we expect for a time-reversal invariant system that breaks a continuous symmetry
- The precise value of T_* will depend on the details of the system - in principle could be zero.
- The phenomenon should also be seen if a local symmetry is broken
- Can this been seen in experiment? eg Cold atom experiments



$T > T_c$ $\quad$ $T = T_c$ $\quad$ $T_* < T < T_c$ $\quad$ $T = T_*$ $\quad$ $T < T_*$

Final Comments

- We have used AdS/CFT to obtain the far from equilibrium dynamics of a strongly coupled superfluid under a quantum quench
- Determined the dynamical phase diagram and explained how its late-time features are determined by the structure of QNMs
- Top down model of [JPG,Sonner,Wiseman]. This model captures infinite class of CFTs and can quench from arbitrary low temperature. Work in progress.
- A universal picture has emerged which covers holographic and non-holographic systems that have time-reversal invariance and assuming spatial homogeneity and isotropy.
- Can we calculate T_* in a weakly coupled theory?
- Can the phenomenology be verified experimentally?

- Gauge-choice for μ

$$Im(\psi_2 - D\psi_1) = 0 \qquad\qquad D = \partial_v - 2i\mu$$

- Evolution equations imply

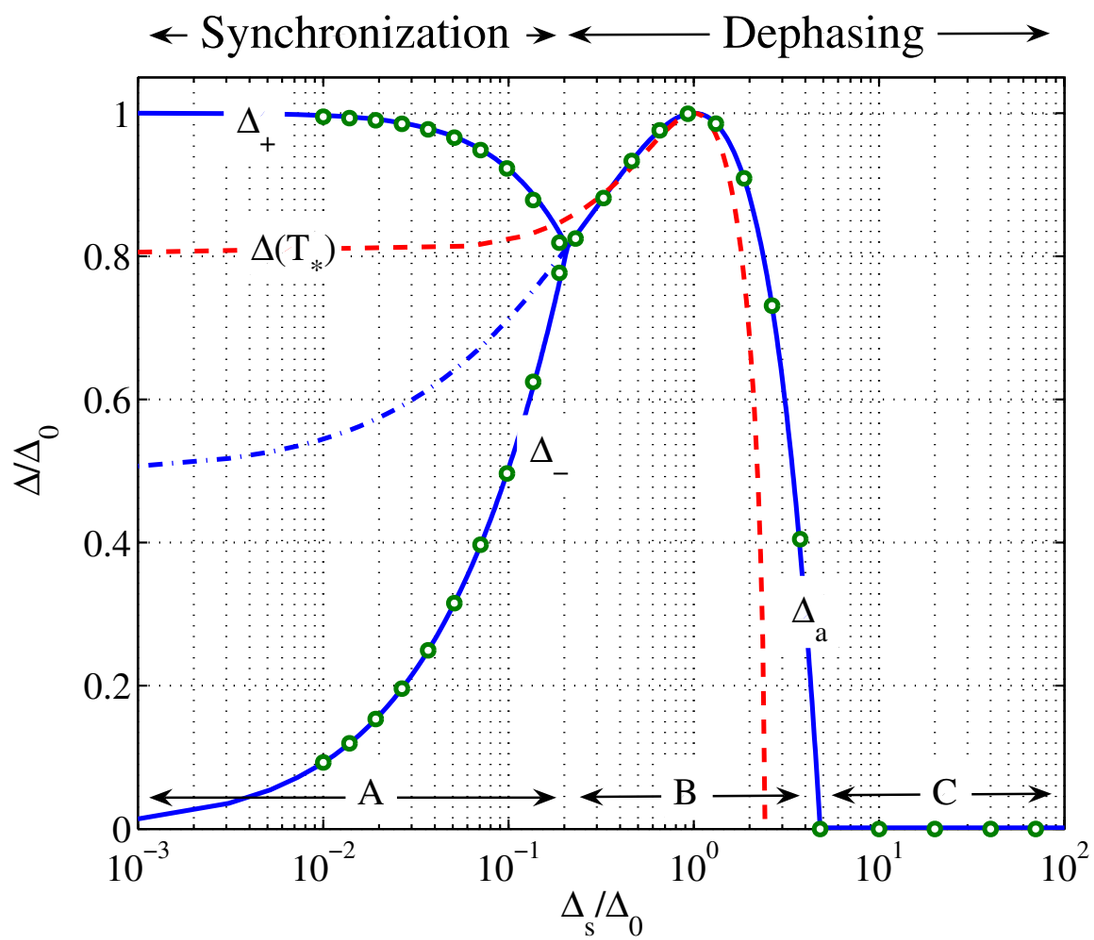
$$\dot{\rho} = -4Im[\psi_1^*(\psi_2 - D\psi_1)]$$

- Our quench $\psi_1(v) = \delta e^{-10v^2}$ is real

- Conclude that quench conserves charge

- B-L Dynamical phase diagram

Final pairing gap



Strength of quench

- Region A: persistent oscillations - final superfluid state
- Region B: power-law decay to final superfluid state
- Region C: power-law decay to unbroken phase final state

Quantum quench of a BCS superconductor

- Abruptly switch on pairing interactions in a BCS setting [Barankov,Levitov]
- Approximations: collisionless, no thermal dissipation, no vortex production

BCS Hamiltonian

$$\mathcal{H} = \sum_{p,\sigma} \epsilon_p a^\dagger_{p,\sigma} a_{p\sigma} - \frac{\lambda(t)}{2} \sum_{q,p} a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} a_{-q\downarrow} a_{q\uparrow}$$

$$\lambda(t) = \lambda_s, \quad t < 0$$
$$\lambda(t) = \lambda, \quad t \geq 0$$

BCS Ground state

$$|\Psi(t)\rangle = \prod_p \left[u_p(t) + v_p(t) a^\dagger_{p\uparrow} a^\dagger_{-p\downarrow} \right] |0\rangle$$

Pairing amplitude

$$\Delta(t) = \lambda(t) \sum_p u_p(t) v^*_p(t)$$

Initial pairing Δ_s

Eq'm pairing Δ_0