Minimal Model Holography

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Gravity Theories and their Avatars
Heraklion, Crete
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based mainly on

MRG, R. Gopakumar, arXiv:1011.2986
MRG, R. Gopakumar, T. Hartman & S. Raju, arXiv:1106.1897
MRG, R. Gopakumar, arXiv:1205.2472

AdS/CFT correspondence

Recall that in the AdS/CFT correspondence the relation between the parameters of the two theories is

$$\left(\frac{R}{l_{\mathrm{Pl}}}\right)^4 = N \qquad g_{\mathrm{string}} = g_{\mathrm{YM}}^2 \qquad \left(\frac{R}{l_{\mathrm{s}}}\right)^4 = g_{\mathrm{YM}}^2 N = \lambda$$

AdS radius in Planck units

AdS radius in string units

't Hooft parameter

Strong weak duality

For example, in the large N limit of gauge theory at large ‘t Hooft coupling

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g^2_{\rm YM} \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g^2_{\rm YM} N = \lambda$$

large small large

Supergravity (point particle) approximation is good for AdS description.

AdS/CFT duality

This is interesting since it gives insights into strongly coupled gauge theories using supergravity methods, e.g.

- anomalous dimensions in N=4 SYM
  [Minahan,Zarembo,Beisert,Staudacher,Janik,...]
- structural insights into amplitudes
  [Witten,Cazacho,Arkani-Hamed,Alday,Maldacena, Korchemsky,Drummond,Sokatchev,...]
- quark gluon plasma
  [Liu,Rajagopal,Wiedemann,Gubser,...]
- quantum critical systems
  [Polichastro,Son,Starinets,Hartnoll,Herzog,Horowitz, Kachru,Sachdev,Kiritsis,...]
- holographic QCD
  [Karch,Katz,Kruczenski,Mateos,Myers,Erdmenger,Kirsch, Sakai,Sugimoto,Kiritsis,...]
- ...

Conceptual understanding

However, at present, we are far from a conceptual understanding of why the duality works, and what ingredients are crucial for it, e.g. whether it requires

supersymmetry
integrability
...

This is obviously an important question since in many applications these features are absent.

Weakly coupled gauge theory

In order to make progress in this direction analyse another corner of AdS/CFT: consider case where gauge theory is weakly coupled

$$\left(\frac{R}{l_{\rm Pl}}\right)^4 = N \qquad g_{\rm string} = g_{\rm YM}^2 \qquad \left(\frac{R}{l_{\rm s}}\right)^4 = g_{\rm YM}^2 N = \lambda$$

large

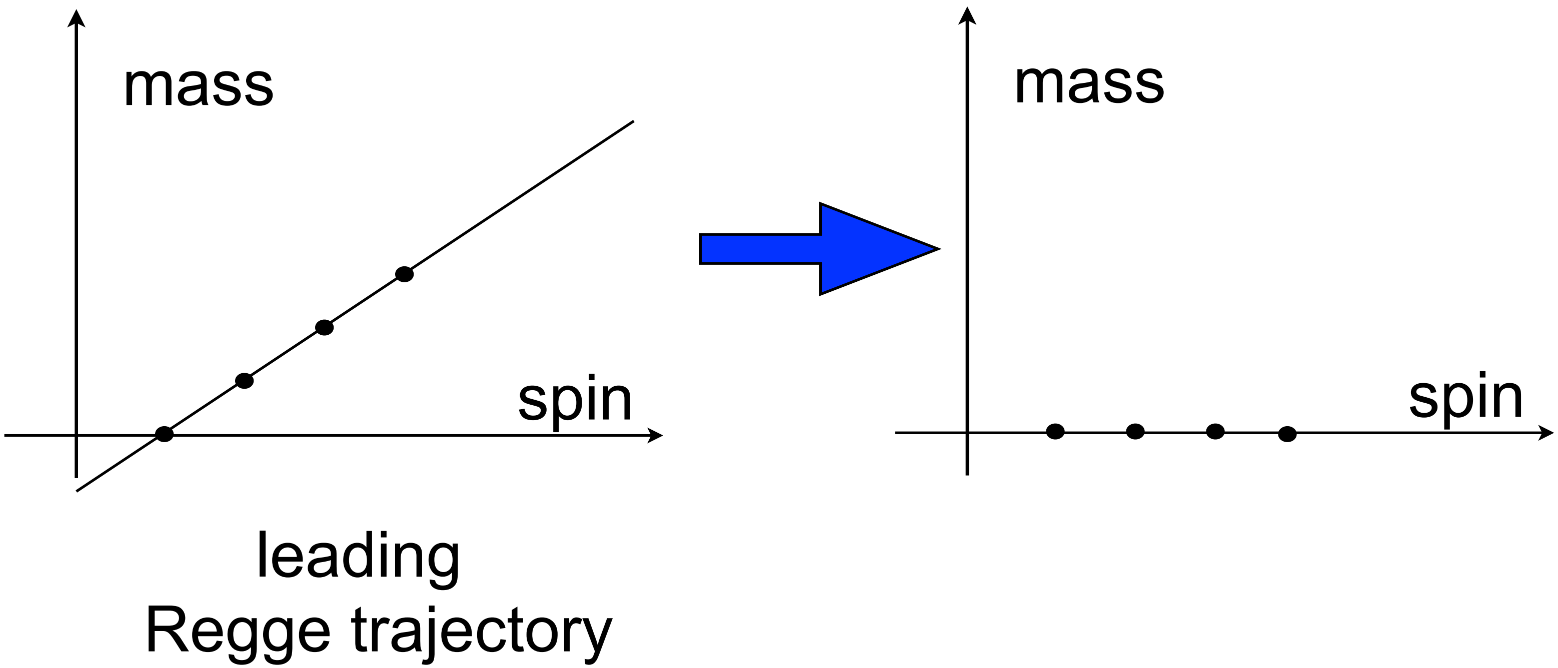
small

[Sundborg], [Witten], [Sezgin,Sundell]

$l_{\rm s} \to \infty$ `tensionless strings'

Tensionless limit

In tensionless limit all string excitations become massless:



Higher spin theory

Resulting theory has an infinite number of massless higher spin fields, which generate a very large gauge symmetry:

maximally unbroken phase of string theory

Idea: try to understand AdS/CFT correspondence starting from this highly symmetric theory!

see also [Sagnotti, et.al.], [Jevicki et.al.], [Douglas et.al.], ...

Higher spin theories

Higher spin (HS) theories have a long history.

- Fronsdal (1978): free HS theory in flat space with gauge symmetry

$$\delta\varphi_{\mu_1\cdots\mu_s} = \partial_{\mu_1}\epsilon_{\mu_2\cdots\mu_s} + \text{symmetrisations}$$

doubly traceless

traceless

Generalisation to AdS straightforward: $\partial \rightarrow \nabla$

Higher spin theories

- Fradkin & Vasiliev (1987): interacting HS theory on AdS (or dS) background.

  - involves infinitely many higher spin fields

  - cosmological constant allows for higher derivative interactions

(Evades various no-go theorems a la Coleman-Mandula.)

Higher spin currents

Note that large symmetry is mirrored on the field theory side since free field theory has conserved (traceless) higher spin currents

$$J_{\mu_1 \cdots \mu_s} = \phi^i \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^i + \cdots$$

other arrangement of derivatives

s=2: stress-energy tensor --- dual to graviton under AdS/CFT correspondence

higher s currents --- dual to higher spin fields on AdS

Disclaimer

Take the above considerations as general motivation to understand dualities between

HS theory on AdS $\longleftrightarrow$ weakly coupled gauge theory

Disclaimer: At present do not understand how these dualities fit into the stringy AdS/CFT correspondence, see however [Giombi et.al.], [Chang et.al.] .

Higher spin duality

Concrete proposal for HS - CFT duality

[Klebanov-Polyakov]
[Sezgin-Sundell]

higher spin theory on AdS4	$\longleftrightarrow$	3d O(N) vector model in large N limit

Actually different versions, depending on whether vector model fields are bosons or fermions and on whether one considers free or interacting fixed point.

N=1 susy generalisation: [Leigh, Petkou]

Checks of the proposal

During the last few years impressive checks of the duality have been performed, in particular

3-point functions of HS fields on AdS4

have been matched to

3-point functions of HS currents in O(N) model to leading order in 1/N. [Giombi & Yin]

Recently, generalisations to a family of parity-violating theories have also been proposed. [Giombi et.al.], [Aharony et.al.]

AdS3 / CFT2

Here: describe 3d/2d CFT version of this duality.

Lower dimensional version interesting

- 2d CFTs well understood
- Higher spin theories simpler in 3d

Also, 3d conformal field theories with unbroken higher spin symmetry and finite number of d.o.f. (finite N) are necessarily free, but this is not the case in 2d.

[Maldacena,Zhiboedov]

3d proposal

The 3d/2d proposal takes the form

[MRG,Gopakumar]

AdS3:
higher spin theory
with a complex
scalar of mass M

$\longleftrightarrow$

2d CFT:
$\mathcal{W}_{N,k}$ minimal models
in large N ‘t Hooft limit
with coupling λ

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

Scalars

In original version of conjecture there were two scalars.

Given our more detailed understanding of the symmetries (see below), it now seems that one of the scalars should be rather thought of as a non-perturbative state.

cf. also [Chang, Yin]

[This new point of view resolves also some puzzles regarding the structure of the correlation functions.]

[Papadodimas, Raju]
[Chang, Yin]

Outline

In the rest of the talk I want to explain the proposal in more detail and indicate which consistency checks have been performed.

- The HS theory in 3d
- Matching the symmetries
- The spectrum
- Conclusions

The HS theory on AdS3

The AdS3 HS theory can be described very simply.

Recall that pure gravity in AdS3: Chern-Simons theory based on

$$sl(2,\mathbb{R})$$

[Achucarro & Townsend]
[Witten]

Higher spin description: replace

$$sl(2,\mathbb{R}) \to \mathrm{hs}[\lambda]$$

[Vasiliev]

Higher spin algebra

The higher spin algebra $\mathrm{hs}[\lambda]$ is an infinite dimensional Lie algebra that can be thought of as

$$\mathrm{hs}[\lambda] \equiv sl(\lambda, \mathbb{R})$$

[Bordemann et.al.]
[Bergshoeff et.al.]
[Pope, Romans, Shen]
[Fradkin, Linetsky]

since

$$\mathrm{hs}[\lambda]\Big|_{\lambda=N} \cong sl(N, \mathbb{R})$$ for integer N.

Higher spin algebra

More explicitly this algebra can be defined as follows: consider the associative algebra

[Bordemann et.al.]
[Bergshoeff et.al.]
[Pope, Romans, Shen]

$$B[\lambda] = \frac{U(sl(2))}{C_2 - \frac{1}{4}(\lambda^2 - 1)\mathbf{1}}$$

On this vector space then define Lie algebra with Lie brackets given by commutators; as vector space

$$B[\lambda] = \mathrm{hs}[\lambda] \oplus \mathbb{C} \ .$$

Higher spin algebra

Generators of $\mathrm{hs}[\lambda]$:

$$V_n^s \;\; \text{with} \; |n| < s \;, \;\; s = 2, 3, \ldots$$

\`wedge algebra'

Commutation relations can be easily determined explicitly, e.g.

$$[V_2^3, V_1^3] = 2\, V_3^4 \qquad [V_2^3, V_0^3] = 4\, V_2^4$$

$$[V_2^3, V_{-1}^3] = 6\, V_1^4 + \tfrac{1}{5}(4 - \lambda^2)\, V_1^2 \qquad [V_2^3, V_{-2}^3] = 8\, V_0^4 + \tfrac{4}{5}(4 - \lambda^2)\, V_0^2 \; .$$

Asymptotic symmetries

For these higher spin theories asymptotic symmetry algebra can be determined following Brown & Henneaux, leading to classical

$$\mathcal{W}_\infty[\lambda] \ \text{algebra}$$

[Henneaux & Rey]
[Campoleoni et al]
[MRG, Hartman]

Extends algebra `beyond the wedge':

pure gravity: $sl(2,\mathbb{R}) \quad \rightarrow \quad \text{Virasoro}$

higher spin: $\text{hs}[\lambda] \quad \rightarrow \quad \mathcal{W}_\infty[\lambda]$

[Figueroa-O'Farrill et.al.]

W-algebra

Resulting algebra generated by W^s_n, but now there is no restriction on n any longer.

In the generic case, the resulting W-algebra is non-linear, and hence does not contain $\mathrm{hs}[\lambda]$ as a subalgebra.

[MRG,Hartman]

Exception: $\lambda = 1$ for which $\mathcal{W}_\infty[1] \cong \mathcal{W}_\infty$

linear W-algebra of Pope, Romans & Shen

Henneaux-Rey: analysis for $\lambda = \frac{1}{2}$.

Campoleoni et.al.: analysis for sl(N) and formal large N limit.

Dual CFT

By the usual arguments, dual CFT should therefore have

$$\mathcal{W}_\infty[\lambda] \text{ symmetry.}$$

Basic idea:

$$\mathcal{W}_\infty[\lambda] = \lim_{N\to\infty} \mathcal{W}_{N,k} \qquad \text{with} \qquad \lambda = \frac{N}{N+k} \ .$$

‘t Hooft limit of 2d CFT!

The minimal models

The minimal model CFTs are the cosets

$$\mathcal{W}_{N,k}: \quad \frac{su(N)_k \oplus su(N)_1}{su(N)_{k+1}}$$

e.g. Ising model (N=2, k=1)
tricritical Ising (N=2, k=2)
3-state Potts (N=3,k=1),..

with central charge

$$c_N(k) = (N-1)\Big[1 - \frac{N(N+1)}{(N+k)(N+k+1)}\Big] .$$

General N: higher spin analogue of Virasoro minimal models. [Spin fields of spin s=2,3,..,N.]

Relation of symmetries

On the face of it, the two symmetries

$$\mathcal{W}_\infty[\lambda] \qquad \text{vs} \qquad \lim_{N,k\to\infty} \mathcal{W}_{N,k}$$

appear to be quite different. However, the asymptotic symmetry analysis only determines the classical symmetry algebra, i.e. the commutative Poisson algebra.

In order to understand above relation, we need to understand the quantum version of this algebra.

Quantum symmetry

The full structure of the quantum algebra can actually be determined completely. [MRG, Gopakumar]

There are two steps to this argument. To illustrate them consider an example. Naive quantisation of classical algebra leads to

$$[W^3_m, W^3_n] = 2(m-n)W^4_{m+n} + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n}$$
$$+\frac{8N_3}{c}(m-n)\,(LL)_{m+n} + \frac{N_3 c}{144}m(m^2-1)(m^2-4)\delta_{m,-n}$$

spin-3 field

non-linear term

Jacobi identity

$$[W^3_m, W^3_n] = 2(m-n)W^4_{m+n} + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n} + \frac{8N_3}{c}(m-n)(LL)_{m+n} + \frac{N_3 c}{144}m(m^2-1)(m^2-4)\delta_{m,-n}$$

spin-3 field

non-linear term

Jacobi identity

$$[W_m^3, W_n^3] = 2(m-n)W_{m+n}^4 + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n}$$
$$+ \frac{8N_3}{c}(m-n)\,(LL)_{m+n} + \frac{N_3 c}{144}m(m^2-1)(m^2-4)\delta_{m,-n}$$

Jacobi identity determines quantum correction

$$[W_m^3, W_n^3] = 2(m-n)W_{m+n}^4 + \frac{N_3}{12}(m-n)(2m^2+2n^2-mn-8)L_{m+n}$$
$$+ \frac{8N_3}{c+\frac{22}{5}}(m-n)\,\Lambda_{m+n}^{(4)} + \frac{N_3 c}{144}m(m^2-1)(m^2-4)\delta_{m,-n}$$

where

$$\Lambda_n^{(4)} = \sum_p :L_{n-p}L_p: + \tfrac{1}{5}x_n L_n$$

Similar considerations apply for the other commutators.

Structure constants

The second step concerns structure constants. The fields can be rescaled so that

$$W^{(3)} \cdot W^{(3)} \sim \frac{c}{3} \cdot \mathbf{1} + 2 \cdot L + \frac{32}{(5c+22)} \cdot \Lambda^{(4)} + 4 \cdot W^{(4)}$$

but then coupling constant

$$W^{(3)} \cdot W^{(4)} \sim C_{33}^{4} \cdot W^{(3)} + \cdots$$

characterises algebra. Classical analysis determines

[MRG, Hartman]

$$\left(C_{33}^{4}\right)^2 = \frac{64}{5} \frac{\lambda^2 - 9}{\lambda^2 - 4} + \mathcal{O}\left(\tfrac{1}{c}\right) .$$

Structure constants

Classical analysis determines

$$\left(C_{33}^{4}\right)^2 = \frac{64}{5}\,\frac{\lambda^2-9}{\lambda^2-4} + \mathcal{O}\!\left(\tfrac{1}{c}\right) \, .$$

Structure constants

Classical analysis determines

$$(C_{33}^{4})^2 = \frac{64}{5}\,\frac{\lambda^2-9}{\lambda^2-4} + \mathcal{O}(\tfrac{1}{c})\ .$$

Requirement that representation theory agrees for $\lambda = N$ with $\mathcal{W}_N$:

$$(C_{33}^{4})^2 = \frac{64\,(c+2)\,(\lambda-3)\,\big(c(\lambda+3)+2(4\lambda+3)(\lambda-1)\big)}{(5c+22)\,(\lambda-2)\,\big(c(\lambda+2)+(3\lambda+2)(\lambda-1)\big)}\ .$$

[Note: $\mathrm{hs}[\lambda]\Big|_{\lambda=N} \cong sl(N,\mathbb{R})$ implies $\mathcal{W}_\infty[\lambda]|_{\lambda=N} = \mathcal{W}_N$.]

Higher Structure Constants

Similarly, higher structure constants can be determined [Blumenhagen, et.al.] [Hornfeck]

$$C_{33}^4 C_{44}^4 = \frac{48\big(c^2(\lambda^2-19)+3c(6\lambda^3-25\lambda^2+15)+2(\lambda-1)(6\lambda^2-41\lambda-41)\big)}{(\lambda-2)(5c+22)\,\big(c(\lambda+2)+(3\lambda+2)(\lambda-1)\big)}$$

$$(C_{34}^5)^2 = \frac{25(5c+22)(\lambda-4)\big(c(\lambda+4)+3(5\lambda+4)(\lambda-1)\big)}{(7c+114)(\lambda-2)\,\big(c(\lambda+2)+(3\lambda+2)(\lambda-1)\big)}$$

$$\begin{aligned} C_{45}^5 = {} & \frac{15}{8(\lambda-3)(c+2)(114+7c)\big(c(\mu+3)+2(4\lambda+3)(\lambda-1)\big)}\, C_{33}^4 \\ & \times \Big[c^3(3\lambda^2-97)+c^2(94\lambda^3-467\lambda^2-483)+c(856\lambda^3-5192\lambda^2+4120) \\ & \quad +216\lambda^3-6972\lambda^2+6756\Big]\,. \end{aligned}$$

Higher Structure Constants

Actually, can rewrite all of them more simply as

[MRG, Gopakumar]

$$C_{44}^{4} = \frac{9(c+3)}{4(c+2)}\,\gamma - \frac{96(c+10)}{(5c+22)}\,\gamma^{-1}$$

$$(C_{34}^{5})^2 = \frac{75(c+7)(5c+22)}{16(c+2)(7c+114)}\,\gamma^2 - 25$$

$$C_{45}^{5} = \frac{15\,(17c+126)(c+7)}{8\,(7c+114)(c+2)}\,\gamma - 240\,\frac{(c+10)}{(5c+22)}\,\gamma^{-1}$$

where $$\gamma^2 \equiv \left(C_{33}^{4}\right)^2$$

These structure constants (and probably all) are actually determined in terms of γ^2 by Jacobi identity.

[Candu, MRG, Kelm, Vollenweider, to appear]

Quantum algebra

Thus full quantum algebra characterised by two free parameters

[MRG, Gopakumar]

$$\gamma^2 \quad \text{and} \quad c \ .$$

But

$$(C_{33}^4)^2 \equiv \gamma^2 = \frac{64(c+2)(\lambda-3)\big(c(\lambda+3)+2(4\lambda+3)(\lambda-1)\big)}{(5c+22)(\lambda-2)\big(c(\lambda+2)+(3\lambda+2)(\lambda-1)\big)} \ .$$

Thus there are three roots that lead to the same algebra:

$$\mathcal{W}_\infty[\lambda_1] \cong \mathcal{W}_\infty[\lambda_2] \cong \mathcal{W}_\infty[\lambda_3] \qquad \text{at fixed } c$$

\`Triality'

Triality

In particular,

$$\mathcal{W}_\infty[N] \cong \mathcal{W}_\infty[\tfrac{N}{N+k}] \cong \mathcal{W}_\infty[-\tfrac{N}{N+k+1}] \quad \text{at } c = c_{N,k}$$

minimal model

asymptotic symmetry algebra of hs theory

This is even true at finite N and k, not just in the 't Hooft limit!

This triality generalises level-rank duality of coset models of [Kuniba, Nakanishi, Suzuki] and [Altschuler, Bauer, Saleur].

Higher spin symmetry

Since $\mathcal{W}_\infty[\lambda]$ algebra is non-linear, it does not contain higher spin algebra $\mathrm{hs}[\lambda]$ as a subalgebra at finite c:

Finite c: hs-symmetry is `broken', but non-linear deformation remains true symmetry.

cf. [Maldacena, Zhiboedov]

NB. Exception for $\lambda = 1$ --- free theory.

Symmetries

So the symmetries suggest that we should have

HS on AdS3	$\longleftrightarrow$	2d CFT with
=CS with		$\mathcal{W}_\infty[\lambda]$
$\mathrm{hs}[\lambda]$		symmetry
		=
		minimal models

Semiclassical limit: take c large --- ‘t Hooft limit!

Spectrum

Higher spin fields themselves correspond only to the vacuum representation of the W-algebra!

To see this, calculate partition function of massless spin s field on thermal AdS3

[MRG, Gopakumar, Saha]

$$Z^{(s)} = \prod_{n=s}^{\infty} \frac{1}{|1-q^n|^2} \,.$$

$$q = \exp(-\frac{1}{k_{\rm B}T})$$

[Generalisation of Giombi, Maloney & Yin calculation to higher spin, using techniques developed in David, MRG, Gopakumar.]

1-loop partition function

The complete higher spin theory therefore contributes

$$Z_{\mathrm{hs}} = \prod_{s=2}^{\infty} \prod_{n=s}^{\infty} \frac{1}{|1-q^n|^2} \, .$$

MacMahon function!

This reproduces precisely contribution of CFT vacuum representation in ‘t Hooft limit

--- not a consistent CFT by itself.....

Representations

Indeed, the full CFT also has the representations labelled by

$$(\rho, \mu; \nu)$$

$$\begin{array}{cccc} \text{rep of} & \mathfrak{su}(N)_k & \mathfrak{su}(N)_1 & \mathfrak{su}(N)_{k+1} \end{array}$$

Compatibility constraint: $\rho + \mu - \nu \in \Lambda_R(\mathfrak{su}(N))$

fixes μ uniquely: label representations by $(\rho; \nu)$.

Simple representations

Simplest reps that generate all W-algebra reps upon fusion: (f;0) and (0;f) (& conjugates).

't Hooft limit: $h(\mathrm{f};0) = \frac{1}{2}(1+\lambda)$ $\quad h(0;\mathrm{f}) = \frac{1}{2}(1-\lambda)$

semiclassical: $h(\mathrm{f};0) = \frac{1}{2}(1-N)$ $\quad h(0;\mathrm{f}) = -\frac{c}{2N^2}$

dual to perturbative scalar

non-perturbative

Proposal

Contribution from all representations of the form (*;0) is accounted for by adding to the hs theory a complex scalar field of the mass

[MRG,Gopakumar]

$$-1 \leq M^2 \leq 0 \quad \text{with} \quad M^2 = -(1-\lambda^2)\ .$$

[Compatible with hs symmetry since hs theory has massive scalar multiplet with this mass.]

[Vasiliev]

Corresponding conformal dimension then

$$M^2 = \Delta(\Delta - 2) \ \Rightarrow \ \Delta = 1 + \lambda\ .$$

Quantisation

Note that for masses in above window, there are two quantisations

[Klebanov & Witten]

$$\Delta_{\pm} = 1 \pm \lambda \ .$$

The scalar is quantised in the `usual' (+) quantisation.

[At least formally, the other primitive CFT representation (0;f) seems to correspond to a scalar with the (-) quantisation --- see original version of proposal.]

Checks of proposal

Main evidence from 1-loop calculation:

Contribution of single real scalar to thermal partition function is

[Giombi, Maloney & Yin]

$$Z^{(1)}_{\text{scalar}} = \prod_{l=0,l'=0}^{\infty} \frac{1}{(1-q^{h+l}\bar{q}^{h+l'})} \ ,$$

where

$$h = \frac{1}{2}\Delta = \frac{1}{2}(1+\lambda) \ .$$

Total 1-loop partition function

The total perturbative 1-loop partition function of our AdS theory is then:

$$Z^{(1)}_{\text{pert}} = \prod_{s=2}^{\infty} \prod_{n=s}^{\infty} \frac{1}{|1-q^n|^2} \times \prod_{l,l'=0}^{\infty} \frac{1}{(1-q^{h+l}\bar{q}^{h+l'})^2}$$

higher spin fields

scalar fields

Total 1-loop partition function

The total perturbative 1-loop partition function of our AdS theory is then:

$$Z^{(1)}_{\text{pert}} = \prod_{s=2}^{\infty} \prod_{n=s}^{\infty} \frac{1}{|1 - q^n|^2} \times \prod_{l,l'=0}^{\infty} \frac{1}{(1 - q^{h+l} \bar{q}^{h+l'})^2}$$

$\mathcal{W}_N$ modes

$$(\mathrm{f};0)^{\otimes r_1} \otimes (\bar{\mathrm{f}};0)^{\otimes r_2}$$

Total 1-loop partition function

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$\mathcal{W}_N$ modes

$$(\mathrm{f};0)^{\otimes r_1}\otimes(\bar{\mathrm{f}};0)^{\otimes r_2}$$

We have shown analytically that this agrees exactly with CFT partition function of (*;0) representations in 't Hooft limit!

[MRG,Gopakumar]
[MRG,Gopakumar,Hartman,Raju]

Lowest orders

For example, for single scalar first non-trivial terms (including higher spin mode contributions) are

$$
\begin{aligned}
Z^{(1)} &= q^h \bar{q}^h \Big(1 + q + 2q^2 + 4q^3 + \cdots\Big)\Big(1 + \bar{q} + 2\bar{q}^2 + 4\bar{q}^3 + \cdots\Big) \\
&\quad + q^{2h}\bar{q}^{2h}\Big(1 + q + 3q^2 + \cdots\Big)\Big(1 + \bar{q} + 3\bar{q}^2 + \cdots\Big) \\
&\quad + q^{2h+1}\bar{q}^{2h+1}\Big(1 + q + \cdots\Big)\Big(1 + \bar{q} + \cdots\Big) + \cdots \ .
\end{aligned}
$$

This is of the form

$$
Z^{(1)} = \chi_{h_1}(q)\chi_{h_1}(\bar{q}) + \chi_{h_2}(q)\chi_{h_2}(\bar{q}) + \chi_{h_3}(q)\chi_{h_3}(\bar{q}) + \cdots
$$

characters of $\mathcal{W}_N$ reps

Lowest orders

$$
\begin{aligned}
Z^{(1)} \quad = \quad & q^h \bar{q}^h \Big(1 + q + 2q^2 + 4q^3 + \cdots\Big)\Big(1 + \bar{q} + 2\bar{q}^2 + 4\bar{q}^3 + \cdots\Big) \\
& + q^{2h}\bar{q}^{2h}\Big(1 + q + 3q^2 + \cdots\Big)\Big(1 + \bar{q} + 3\bar{q}^2 + \cdots\Big) \\
& + q^{2h+1}\bar{q}^{2h+1}\Big(1 + q + \cdots\Big)\Big(1 + \bar{q} + \cdots\Big) + \cdots \ .
\end{aligned}
$$

with

$$
\begin{aligned}
\chi_{h_1}(q) \quad &= q^h \big(1 + q + 2q^2 + 4q^3 + \cdots\big) && = \chi(\mathrm{f}; 0) \\
\chi_{h_2}(q) \quad &= q^{2h} \big(1 + q + 3q^2 + \cdots\big) && = \chi([0, 1, 0^{N-3}]; 0) \\
\chi_{h_3}(q) \quad &= q^{2h+1} \big(1 + q + \cdots\big) && = \chi([2, 0^{N-2}]; 0) \ .
\end{aligned}
$$

calculated from first
principles in CFT!

Non-perturbative states

The remaining states, i.e. those of the form

$$(*;\nu) \qquad \text{with } \nu \neq 0$$

seem to correspond to conical defect solutions (possibly dressed with perturbative excitations).

[Castro, Gopakumar, Gutperle, Raeymaekers]
[MRG, Gopakumar]

Generalisations

Various generalisations of the duality have also been proposed and tested, in particular

- supersymmetric version

  [Creutzig, Hikida, Ronne]
  [Candu, MRG]
  [Henneaux,Gomez,Park,Rey]
  [Hanaki,Peng]
  [Ahn]

- orthogonal (instead of unitary) groups

  [Ahn], [MRG, Vollenweider]

Correlation functions

A comparison between 3-point functions has also been performed (as for AdS4), and again perfect agreement has been found to leading order in 1/N.

[Chang, Yin]

With the above interpretation, only the scalar (f;0) has to behave as a perturbative state, and the 1/N corrections in the mixed correlators do not pose any problems any longer.

[Papadodimas, Raju]

Classical solutions

Another very interesting development concerns the classical solutions of the HS theory.

[Gutperle, Kraus, et.al.]

Very interesting lessons (that are maybe applicable more generally): because of large HS gauge symmetry, usual GR tensors are not gauge invariant any longer!

Characterisation of regular classical solutions is therefore subtle!

Black holes

However CS description allows for HS gauge invariant formulation. Using this point of view, black hole solutions for these theories have been constructed.

[Gutperle,Kraus,et.al.]

Their entropy can be matched to dual CFT description.

[Kraus,Perlmutter]
[MRG,Hartman,Jin]

Conclusions

Given strong evidence for duality between

AdS3:
higher spin theory
with a complex
scalar of mass M

$\longleftrightarrow$

2d CFT:
$\mathcal{W}_{N,k}$ minimal models
in large N ‘t Hooft limit
with coupling λ

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

Conclusions

- The duality is non-supersymmetric.
- It allows for detailed precision tests: spectrum, correlation functions, etc.
- Can shed maybe interesting light on conceptual aspects of quantum gravity.

Main challenges

- Prove the duality in the ‘t Hooft limit. [Main challenge: understand scalars from CS point of view.]

- Understand phase structure of partition function.

- Reproduce calculable quantum corrections of CFT from Higher Spin Quantum Gravity on AdS3.

- Embed hs theories on AdS3 into string theory, e.g. into the D1-D5 system.

cf [Kiritsis, Niarchos]