Universality or non-universality in applications of gauge/gravity duality

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Based on joint work with
M. Ammon, V. Grass, M. Kaminski, P. Kerner, S. Müller, A. O'Bannon, J. Shock, M. Strydom, H. Zeller

Motivation:

Gauge/gravity duality: New tools for strongly coupled systems

Famous result: Shear viscosity/Entropy density

Kovtun, Son, Starinets

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

From `Planckian time' $\tau_P = \dfrac{\hbar}{k_B T}$, Universal result

This talk:
Deviations from this result at leading order in λ and N

Search for similar result in condensed matter context

Holographic proof of universality relies on space-time isotropy

Key ingredient for changes to the
universal result: Spacetime anisotropy

Rotational invariance broken

Holographic p-wave superfluids/superconductors

Nematic phase (Condensate breaks rotational symmetry)

New holographic quantum critical points

Outline

- Holographic superconductors
- Transport coefficients in anisotropic systems
- Universality in AdS/CMT
- Condensates at finite magnetic field

Reminder: Holographic Superfluids/Superconductors

- Holographic Superconductors from charged scalar in Einstein-Maxwell gravity
  (Gubser; Hartnoll, Herzog, Horowitz)

- p-wave superconductor
  current dual to gauge field condensing
  (Gubser, Pufu)
  SU(2) Einstein-Yang-Mills model

s-wave superconductor:

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{ab}F_{ab} - V(|\psi|) - |\nabla\psi - iqA\psi|^2$$

Operator $\mathcal{O}$ dual to scalar ψ condensing

Herzog, Hartnoll, Horowitz 2008

p-wave superconductor:

$$S = \frac{1}{2\kappa^2}\int d^4x \left[R - \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{6}{L^2}\right]$$

Current J^1_x dual to gauge field component A^{1x} condensing

Gubser, Pufu 2008

P-wave superconductor from probe branes

Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864

- A holographic superconductor with field theory in 3+1 dimensions for which
- the dual field theory is explicitly known
- there is a qualitative ten-dimensional string theory picture of condensation

This is achieved in the context of
adding flavour to gauge/gravity duality

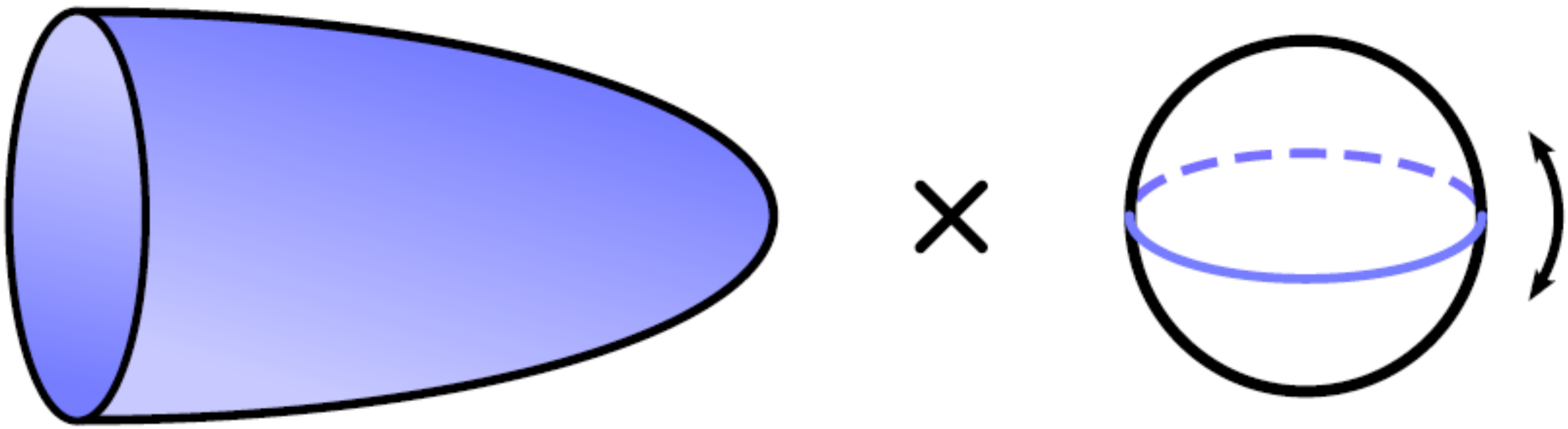
Brane probes added on gravity side $\Rightarrow$
fundamental d.o.f. in the dual field theory (quarks)

Additional D-branes within $AdS_5 \times S^5$ or deformed version thereof

Quarks within Gauge/Gravity Duality

Adding D7-Brane Probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		



Probe brane fluctuations $\Rightarrow$ Masses of mesons ($\bar{\psi}\psi$ bound states)

On gravity side:
Probe brane fluctuations described by Dirac-Born-Infeld action

$$S_{\mathrm{DBI}} = -\,T_{D7} \int \mathrm{d}^8\xi \,\mathrm{Str}\sqrt{|\det(G + 2\pi\alpha' F)|}$$

On field theory side: Lagrangian explicitly known

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4} + \mathcal{L}({\psi_q}^i, {\phi_q}^i)$$

Turn on finite temperature and isospin chemical potential:

Finite temperature: Embed D7 brane in black hole background

Isospin chemical potential: Probe of two coincident D7 branes

Additional symmetry $U(2) = SU(2)_I \times U(1)_B$

$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots\,, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots$$

Condensate $\langle J_3 \rangle, \quad J_3 = \bar{\psi}_d \gamma_3 \psi_u + \ bosons$

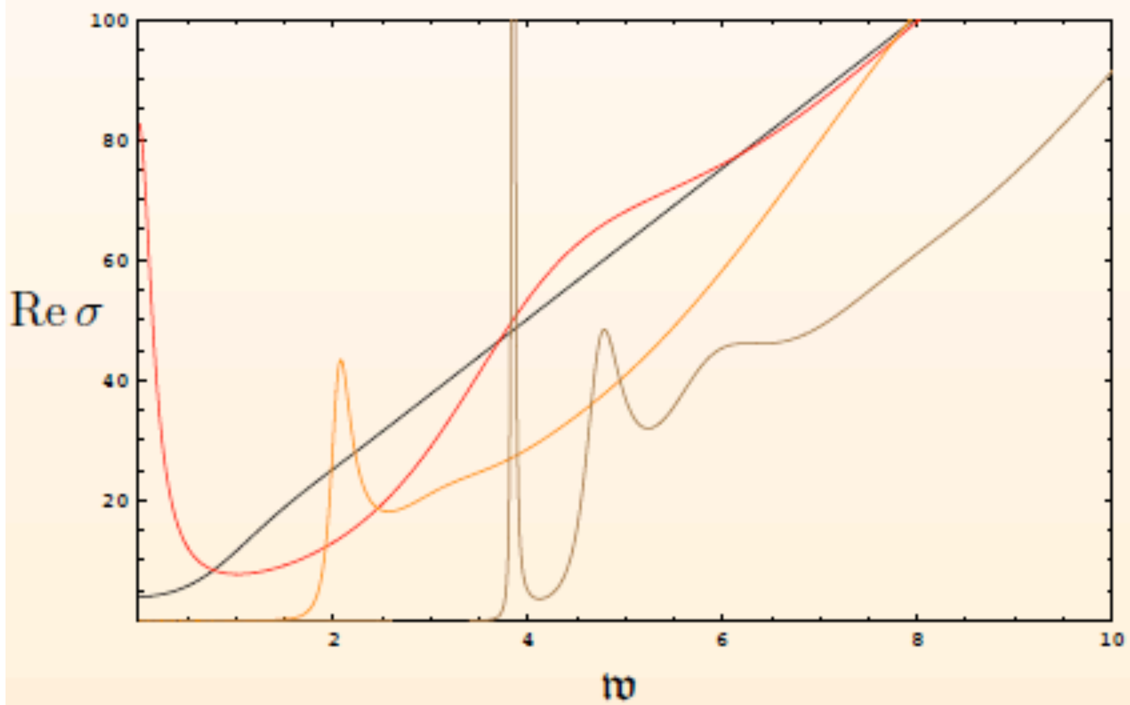
rho meson condensation in Sakai-Sugimoto:
Aharony, Peeters, Sonnenschein, Zamaklar '07

Calculate correlators from fluctuations

Conductivity

Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3



Ammon, J.E., Kaminski, Kerner '08

$\mathfrak{w} = \omega/(2\pi T)$

T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

(Vanishing quark mass)

Interpretation: Frictionless motion of mesons through plasma

Effective 5d model ➡ anisotropic shear viscosity

Bottom-up: Including the backreaction

Ammon, J.E., Graß, Kerner, O'Bannon 0912.3515

Einstein-Yang-Mills-Theory with $SU(2)$ gauge group

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R - \Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right]$$

$$\alpha = \frac{\kappa_5}{\hat{g}}$$

$\alpha^2 \propto$ number of charged d.o.f./all d.o.f.

In presence of $SU(2)$ chemical potential, same condensation process as before

Hairy black hole solution

- metric ansatz

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + \frac{r^2}{f(r)^4}\mathrm{d}x^2 + r^2 f(r)^2\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)$$

with

$$N(r) = -\frac{2m(r)}{r^2} + r^2$$

AdS boundary $r = r_{\mathrm{bdy}} \to \infty$ & black hole horizon $r = r_h$

- gauge field ansatz

$$A = \phi(r)\tau^3\mathrm{d}t + w(r)\tau^1\mathrm{d}x$$

Field Theory	$\Leftrightarrow$	Gravity
chemical potential μ $SU(2) \to U(1)_3$		$A_t^3 = \phi(r) \neq 0$ $SU(2) \to U(1)_3$
$\langle \mathcal{J}_1^x \rangle \neq 0$ $U(1)_3 \to \mathbb{Z}_2,\ SO(3) \to SO(2)$		$A_x^1 = w(r) \neq 0$ $U(1)_3 \to \mathbb{Z}_2,\ SO(3) \to SO(2)$

- $w(r_{\text{bdy}}) = 0 \Rightarrow$ SSB $U(1)_3 \to \mathbb{Z}_2$ & $SO(3) \to SO(2)$

$\Rightarrow$ holographic p-wave superfluid with backreaction

- 5 fields: $\{\phi(r), w(r), \sigma(r), f(r), m(r)\}$

Variation of on-shell action at AdS boundary gives

- energy-momentum tensor

$$\langle \mathcal{T}_{\mu\mu} \rangle \propto T^4 \cdot \mathrm{Func}(m_0^b, f_2^b), \text{ with: } \langle \mathcal{T}_{yy} \rangle = \langle \mathcal{T}_{zz} \rangle \neq \langle \mathcal{T}_{xx} \rangle$$
$$\langle \mathcal{T}_{\mu\nu} \rangle = 0 \text{ for } \mu \neq \nu$$

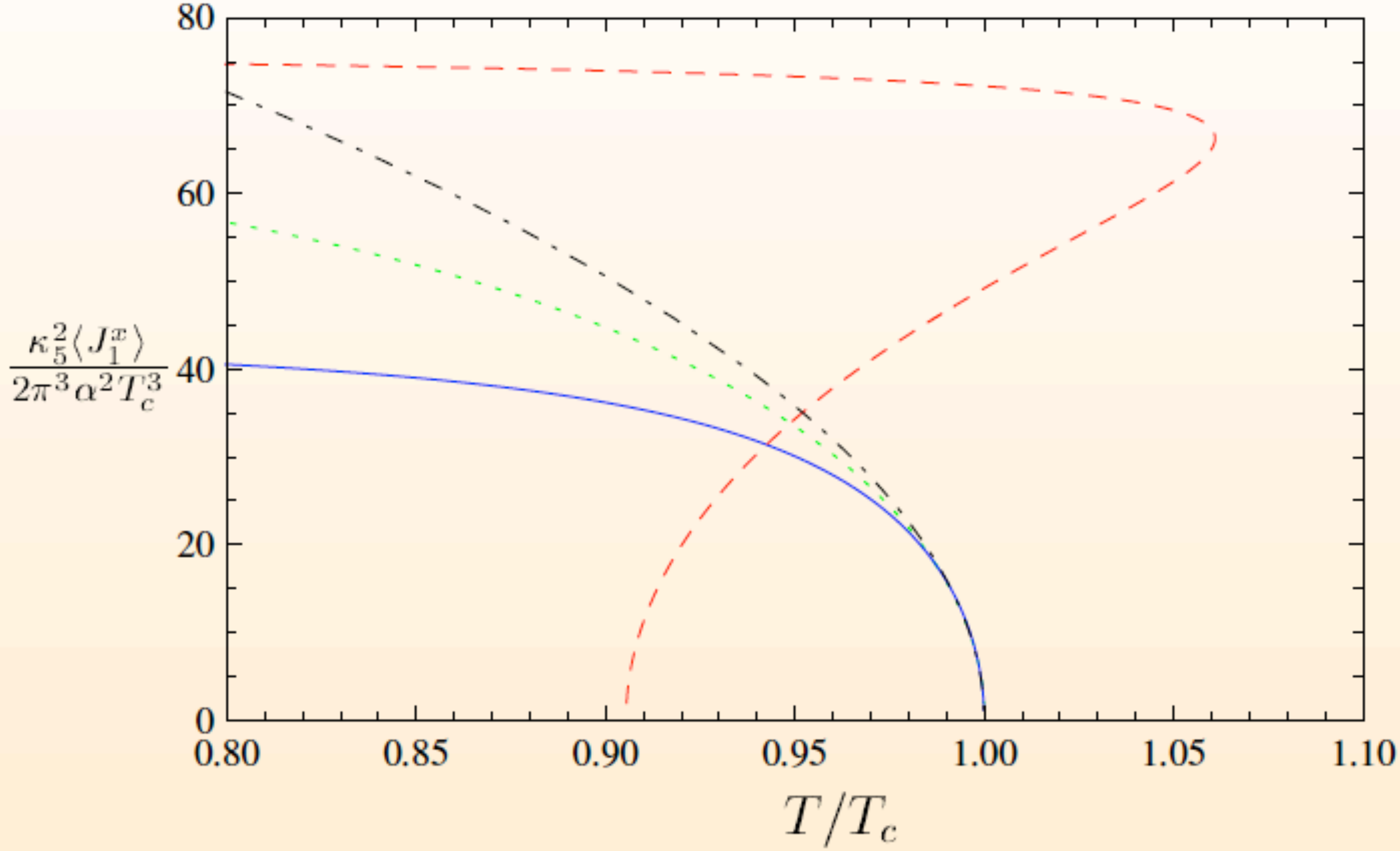
- density current

$$\langle \mathcal{J}_3^t \rangle \propto T^3 \phi_1^b$$

- condensate

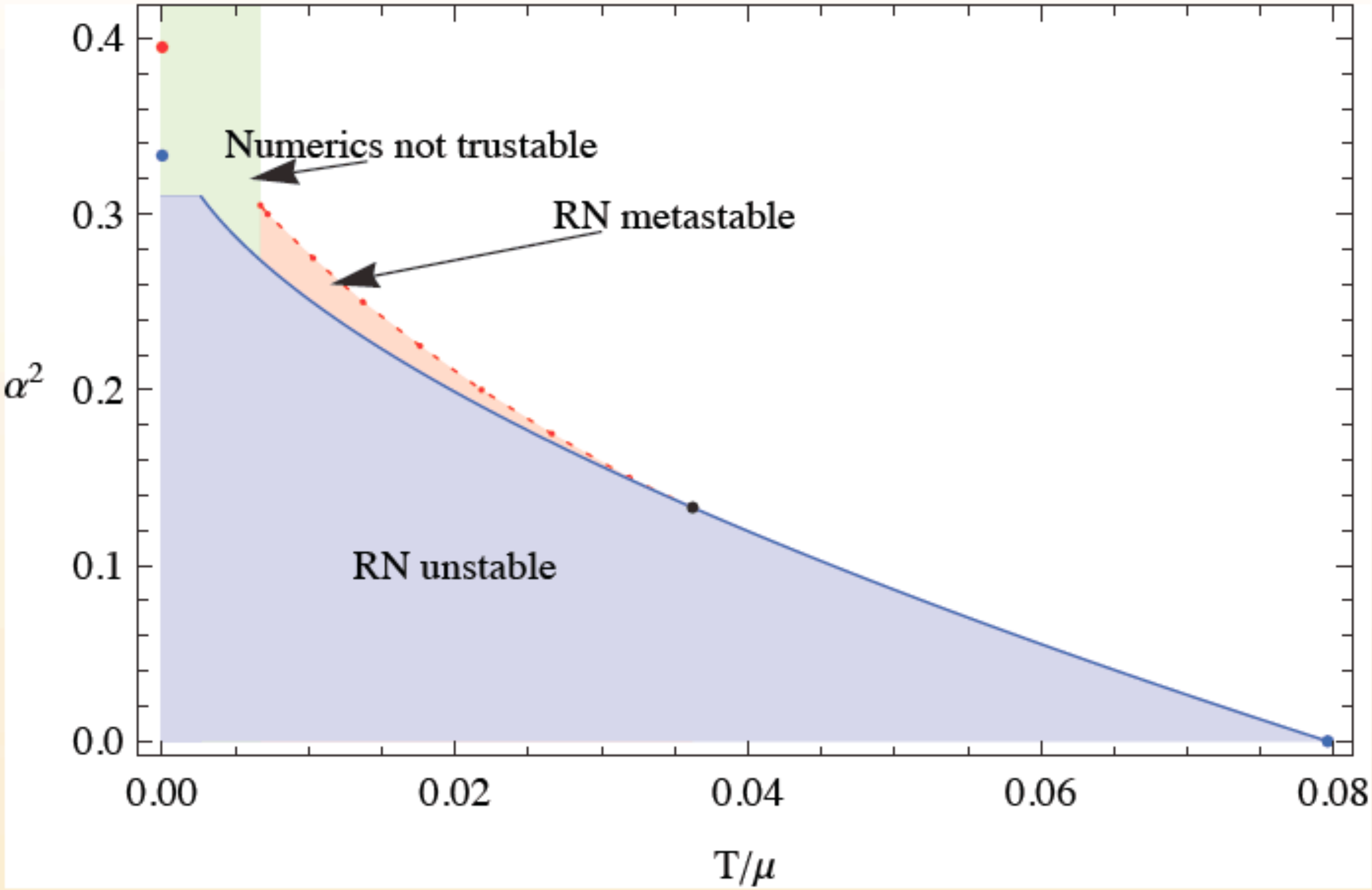
$$\langle \mathcal{J}_1^x \rangle \propto T^3 w_1^b$$

Phase transition



Phase transition becomes first order above α_{crit}

Phase diagram



Fluctuations about equilibrium

small perturbations:

- metric $\hat{g}_{MN} = g_{MN}(r) + h_{MN}(x^\mu, r)$
- gauge field $\hat{A}^a_M = A^a_M(r) + a^a_M(x^\mu, r)$
- x^μ-spacetime translational invariance still unbroken

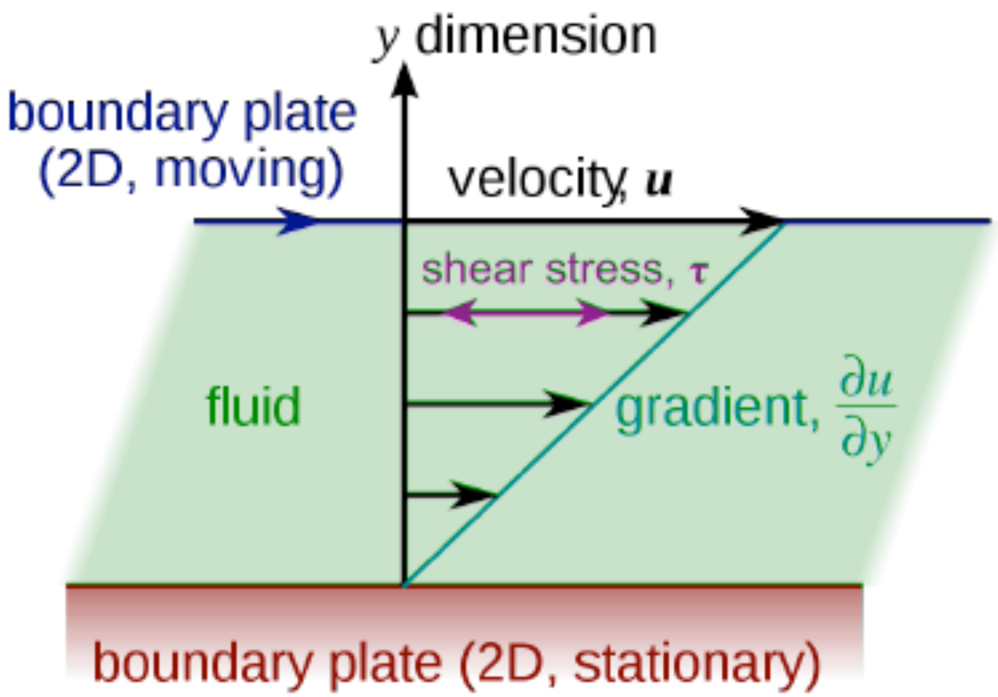
$\Rightarrow$ Fourier decomposition of fluctuations possible:

$$h_{MN}(x^\mu, r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i}k_\mu x^\mu} h_{MN}(k^\mu, r)$$
$$a^a_M(x^\mu, r) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{\mathrm{i}k_\mu x^\mu} a^a_M(k^\mu, r)$$

- $SO(2)$ symmetry $\Rightarrow$ two distinct momenta needed: $k_{\|}$ and $k_{\perp}$

Anisotropic shear viscosity

- viscosity tensor $\eta_{\alpha\beta\gamma\delta}$
- anisotropic systems:
  21 components
- isotropic systems:
  1 shear viscosity
- transversely isotropic systems:
  2 shear viscosities



Holographic calculation: J.E., Kerner, Zeller 1011.5912; 1110.0007

Classification of Fluctuations

- set $k_\perp = 0$

$\Rightarrow$ classification under $SO(2)$ rotational symmetry around x-axis possible:

	dynamical fields	constraints	# physical modes
helicity 2	$h_{yz}, h_{yy} - h_{zz}$	none	2
helicity 1	$h_{ty}, h_{xy}; a^a_y$	h_{yr}	4
	$h_{tz}, h_{xz}; a^a_z$	h_{zr}	4
helicity 0	$h_{tt}, h_{xx}, h_{yy} + h_{zz}, h_{xt};$ a^a_t, a^a_x	$h_{tr}, h_{xr}, h_{rr}; a^a_r$	4

gauge choice $h_{Mr} = 0$ and $a^a_r = 0 \Rightarrow$ 14 physical modes

Transport coefficients from Green functions

One non-trivial helicity 2 mode ➡

gives well-known result $\eta/s = 1/4\pi$

Helicity 1 modes:

- in $\vec{k} \to 0$ limit additional symmetry:
  $\mathbb{Z}_2$: $x \to -x$, $w \to -w$

$\Rightarrow$ helicity 1 modes decouple in 2 blocks:

even parity: $\{\Psi_t = g^{yy} h_{t\perp}, a^3_\perp, h_{r\perp}\}$

odd parity: $\{\Psi_x = g^{yy} h_{x\perp}, a^1_\perp, a^2_\perp\}$ $\Rightarrow$ 3 independent fields: $\Psi_x, a^1_\perp, a^2_\perp$

$\Rightarrow$ Green's function: 3×3 matrix

Linear response

- choose basis: $a_\perp^\pm = a_\perp^1 \pm \mathrm{i} a_\perp^2$

$\Rightarrow$ transform in fundamental repr. of unbroken $U(1)_3$

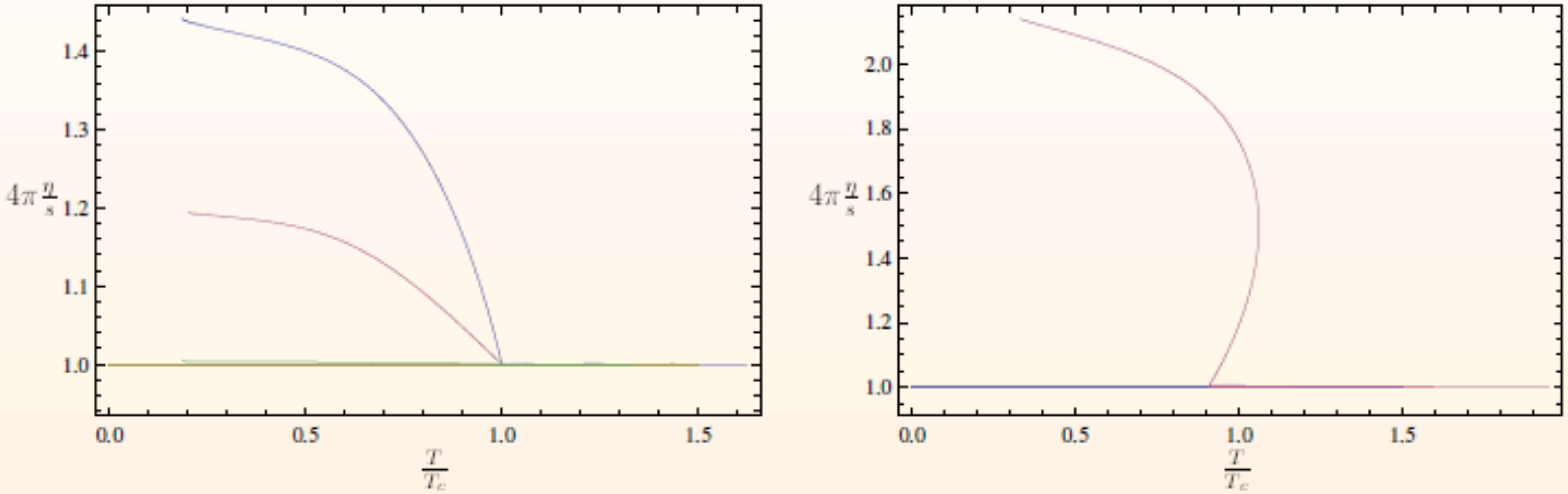
- field theory:

$$\begin{pmatrix} \langle J_+^\perp \rangle \\ \langle J_-^\perp \rangle \\ \langle T^{x\perp} \rangle \end{pmatrix} = \begin{pmatrix} G_{+,+}^{\perp,\perp} & G_{+,-}^{\perp,\perp} & G_+^{\perp\, x\perp} \\ G_{-,+}^{\perp,\perp} & G_{-,-}^{\perp,\perp} & G_-^{\perp\, x\perp} \\ G^{x\perp}{}_+^{\perp} & G^{x\perp}{}_-^{\perp} & -\langle T_{xx} \rangle - \mathrm{i}\omega\eta_{x\perp} \end{pmatrix} \begin{pmatrix} a_\perp^+ \\ a_\perp^- \\ h_{x\perp} \end{pmatrix}$$

- with

$$\eta_{x\perp} = -\lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\left(G^{x\perp,x\perp} \right)$$

Anisotropic shear viscosity



$\eta_{yz}/s = 1/4\pi$; η_{xy}/s dependent on T and on α

Critical behaviour: $1 - 4\pi\frac{\eta_{xy}}{s} \propto \left(1 - \frac{T}{T_c}\right)^{\beta}$ with $\beta = 1.00 \pm 3\%$, α-independent

Non-universal behaviour at leading order in λ and N

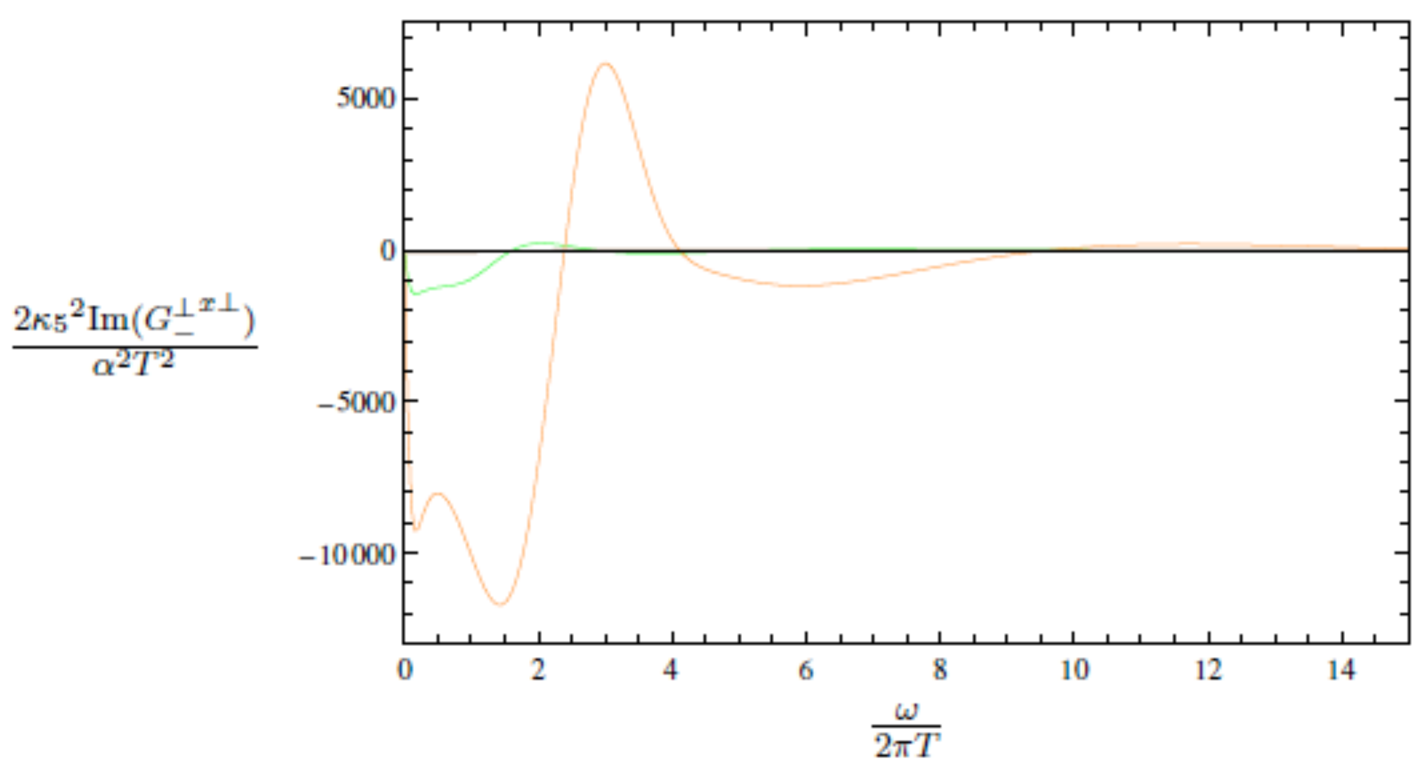
Critical exponent confirmed analytically in Basu, Oh 1109.4592

Flexoelectric Effect

Nematic crystals:
A strain introduces spontaneous electrical polarization

In our case:

A strain $h_{x\perp}$ introduces an inhomogeneity in the current $\mathcal{J}_1^x$ which introduces a current $\mathcal{J}_\pm^\perp$



J.E., Kerner, Zeller
1110.0007

Is there a similar universal result as $\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$ within condensed matter applications of holography?

Candidate: Homes' Law

Homes' Law

Homes' Law $\rho_s = C\sigma(T_c)T_c$

Shown to hold experimentally to great accuracy (Homes et al, Nature 2004)

Zaanen (Nature, 2004): $\tau(T_c) = \frac{\hbar}{k_B T_c}$

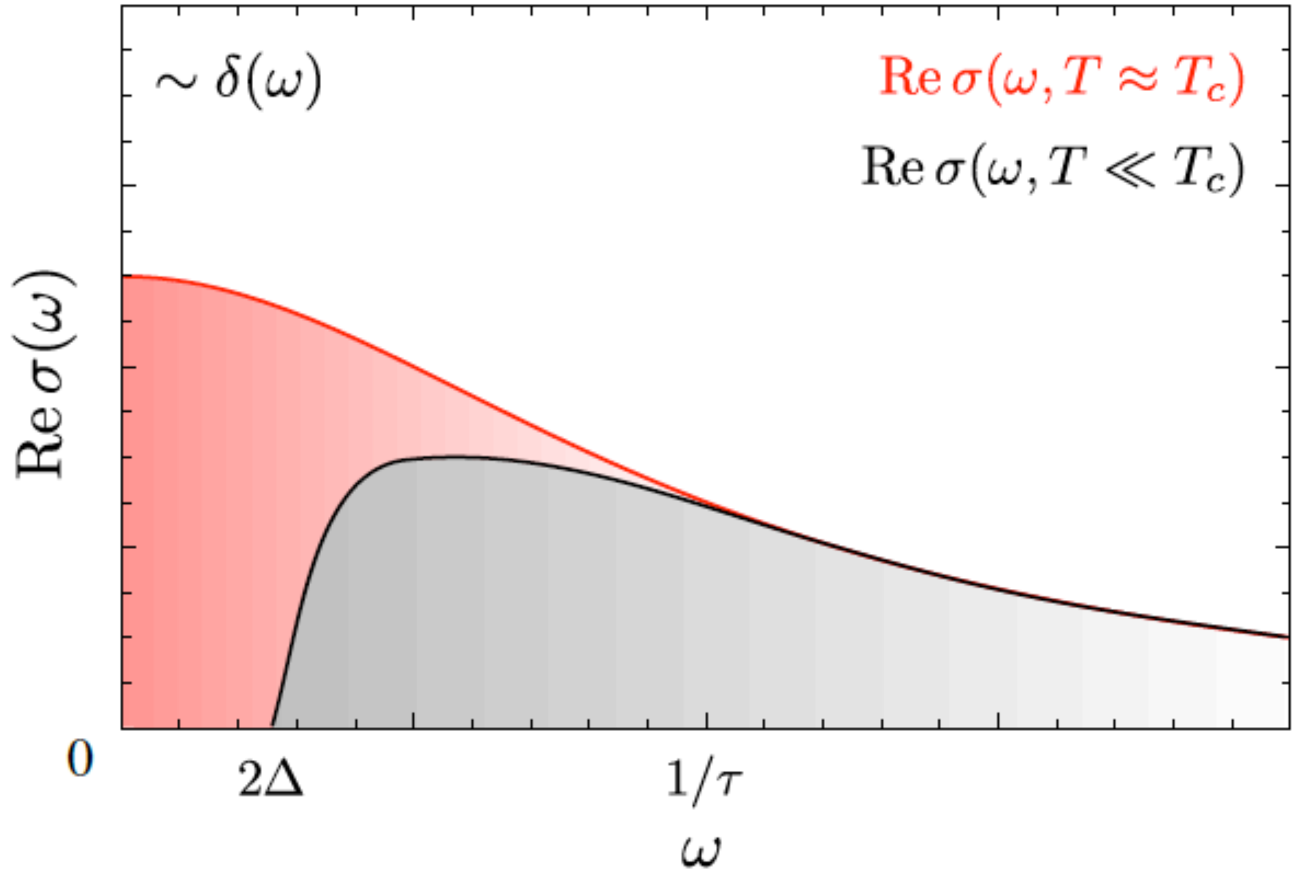
Planckian dissipation: Shortest possible dissipation timescale

Holographic version:

Preliminary results in J.E., Kerner, Müller 1206.5305

Not possible to calculate superconducting density ρ_s holographically

$$\sigma(\omega) = \rho_s \delta(\omega)$$



Idea: Rewrite Homes' Law using sum rules

Homes' Law

Homes' Law: $\rho_s = C\sigma(T_c)T_c$

Sum rule: ${\omega_P}^2(T=0) = {\omega_P}^2(T=T_c)$

$\rho_s \propto \omega_P(T=0)$

Drude law: $\sigma = \frac{ne^2\tau}{m}, \quad {\omega_P}^2 = \frac{4\pi ne^2}{m}$

$\Rightarrow \quad 4\pi\sigma(T_c) = {\omega_P}^2(T_c)\tau(T_c)$

$\Rightarrow$ Homes' Law equivalent to

$$\tau(T_c)T_c = const$$

Homes' Law

Assume diffusion can be used to determine the timescale

$\Rightarrow D(T_c)T_c = const$

Holography in the probe limit without backreaction (Einstein-Maxwell theory):

$$D = \frac{1}{4\pi}\frac{d}{d-2}\frac{1}{T}$$

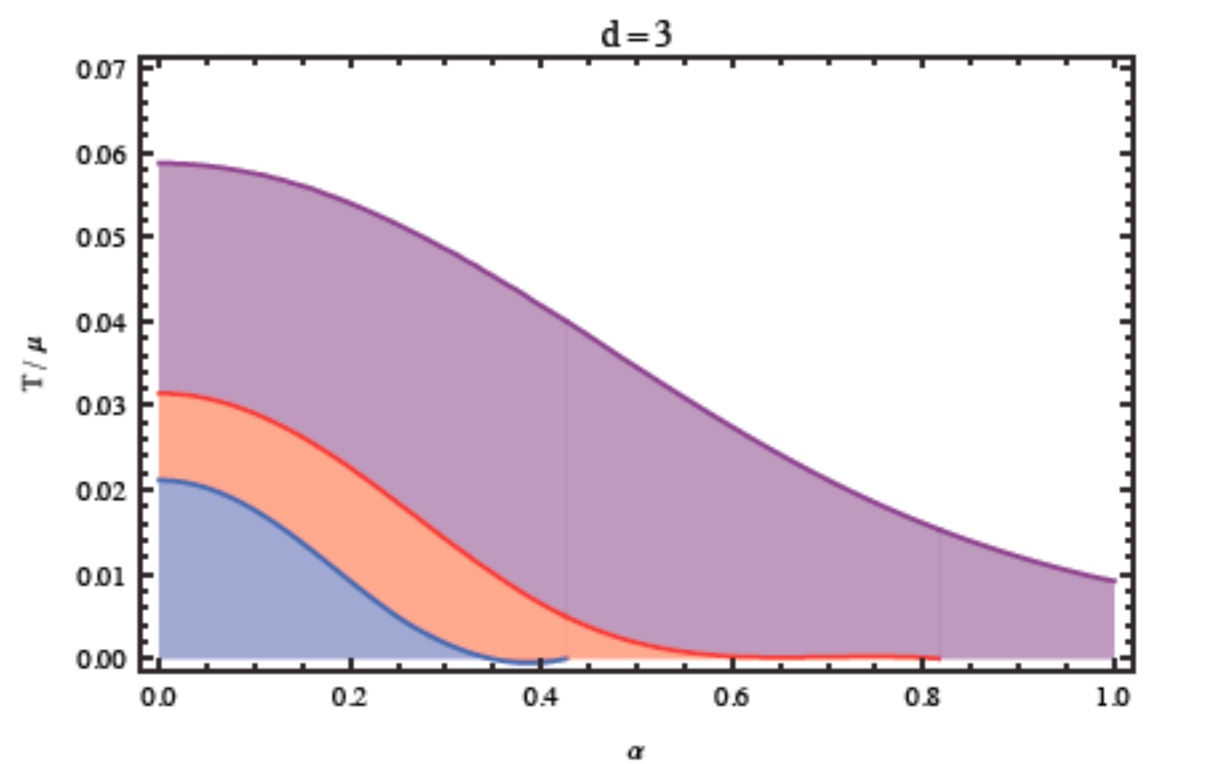
Including the backreaction we expect $D = \frac{1}{4\pi}\frac{d}{d-2}\frac{1}{T}f(\frac{T}{\mu})$

Including the backreaction

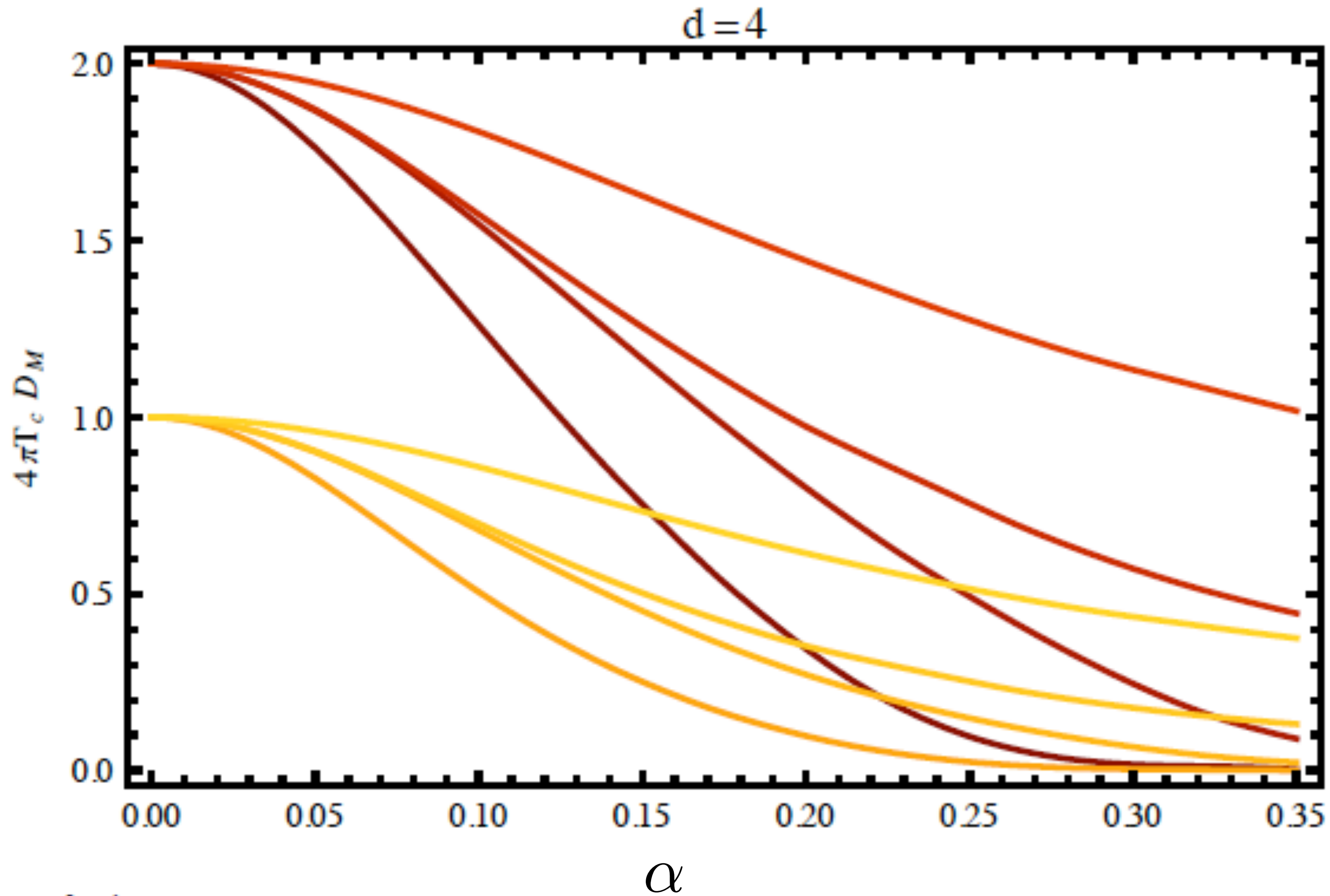
$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^{d+1}x\,\sqrt{-g}\left[R - 2\Lambda - \frac{2\kappa^2}{e^2}\left(\frac{1}{4}F_{ab}F^{ab} - |\nabla\Phi - \mathrm{i}A\Phi|^2 - V(|\Phi|)\right)\right]$$

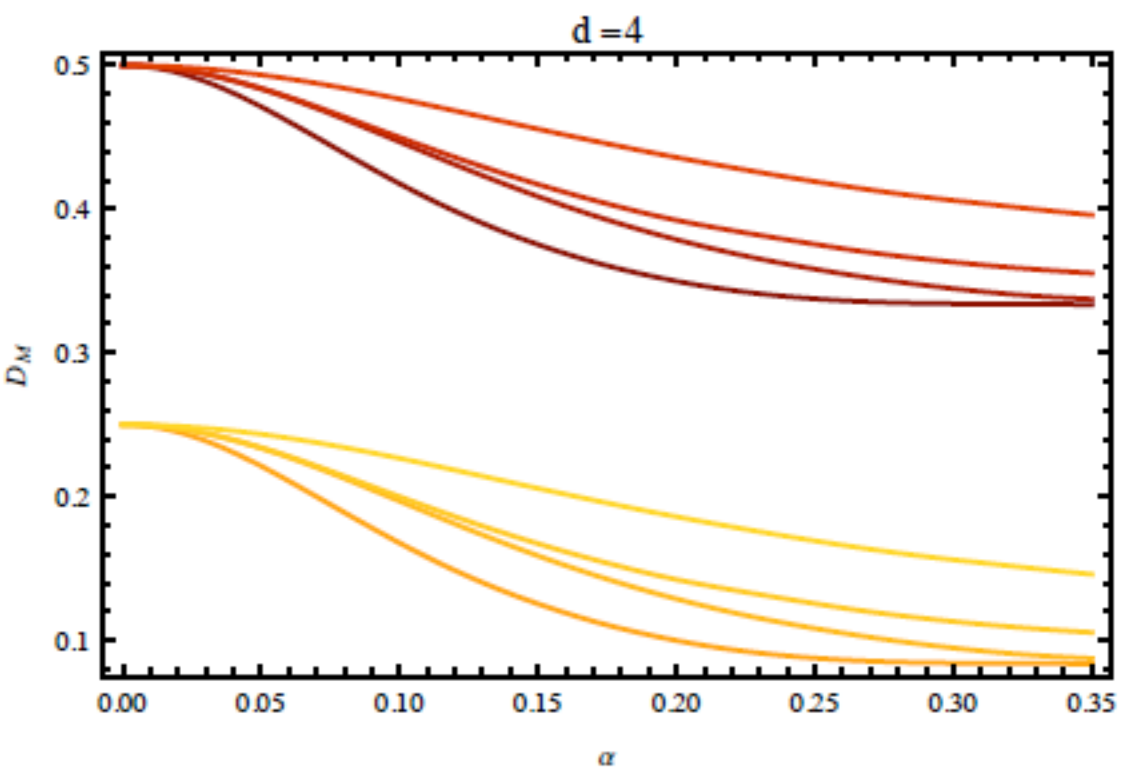
Backreaction parameter $\alpha^2 L^2 = \frac{\kappa^2}{e^2}$

Phase diagram



R charge and momentum diffusion times T_c vs. α





Reasons for decrease

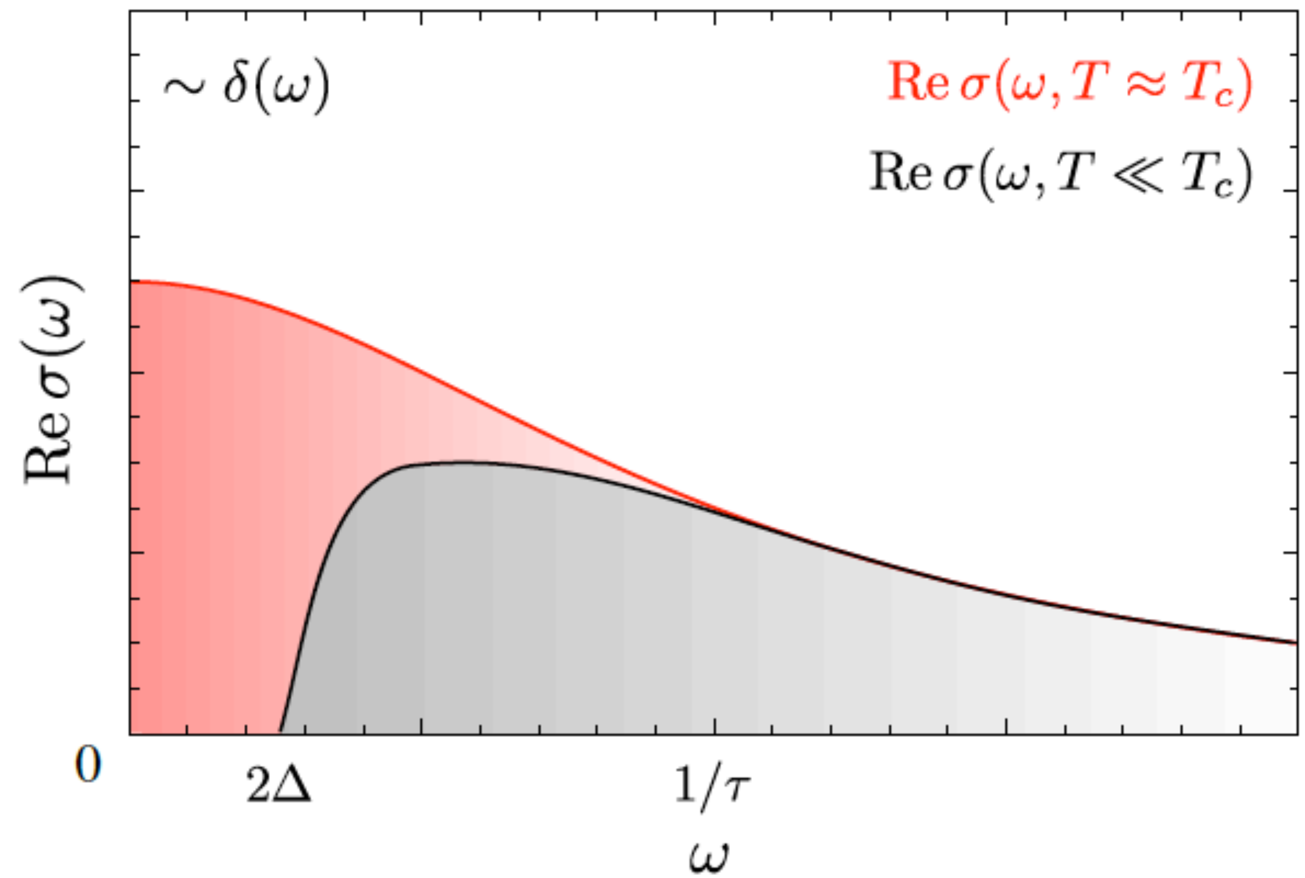
Sum rule $$\frac{\omega_{\mathrm{P}}^2}{8} = \int_0^\infty \mathrm{d}\omega\, \mathrm{Re}\,\sigma(\omega)$$

$$N_{\mathrm{n}} = \int_0^\infty \mathrm{d}\omega\, \mathrm{Re}\,\sigma(\omega)\Big|_{T>T_c} = \frac{\omega_{\mathrm{Pn}}^2}{8},$$

$$N_{\mathrm{s}} = \int_{0+}^\infty \mathrm{d}\omega\, \mathrm{Re}\,\sigma(\omega)\Big|_{T<T_c}$$

$$\rho_{\mathrm{s}} \equiv \omega_{\mathrm{Ps}}^2 = 8\,(N_{\mathrm{n}} - N_{\mathrm{s}}) = \omega_{\mathrm{Pn}}^2 - 8N_{\mathrm{s}}$$

$$1 - \frac{8N_{\mathrm{s}}}{\omega_{\mathrm{P}n}^2} = \frac{C}{4\pi}\tau_c T_c$$



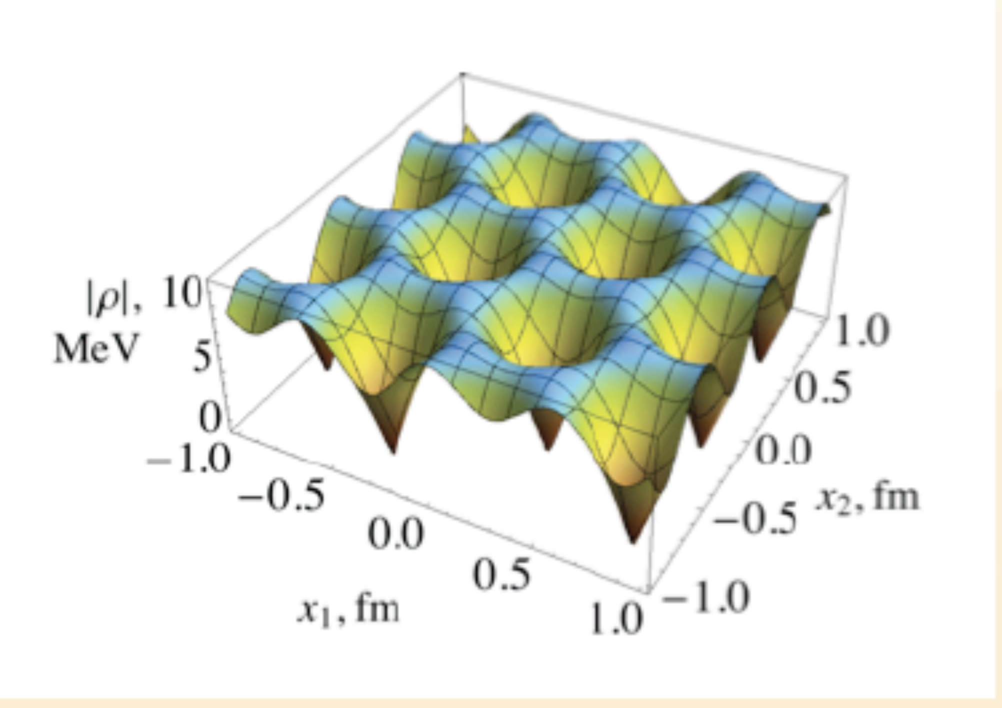
Decrease may originate from pseudogap states whose number increases with backreaction

External electromagnetic fields

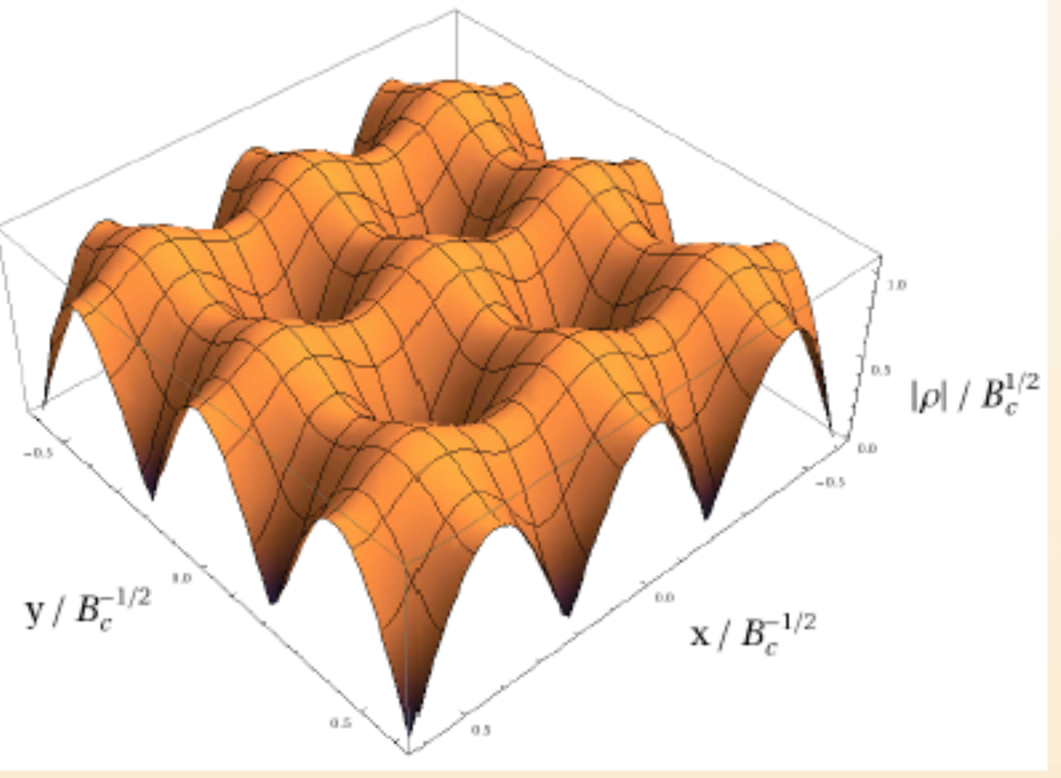
A magnetic field leads to

ρ meson condensation and superconductivity in the QCD vacuum

Effective field theory:
(Chernodub)



Gauge/gravity duality
magnetic field in black hole supergravity background
(J.E., Kerner, Strydom PLB 2011)

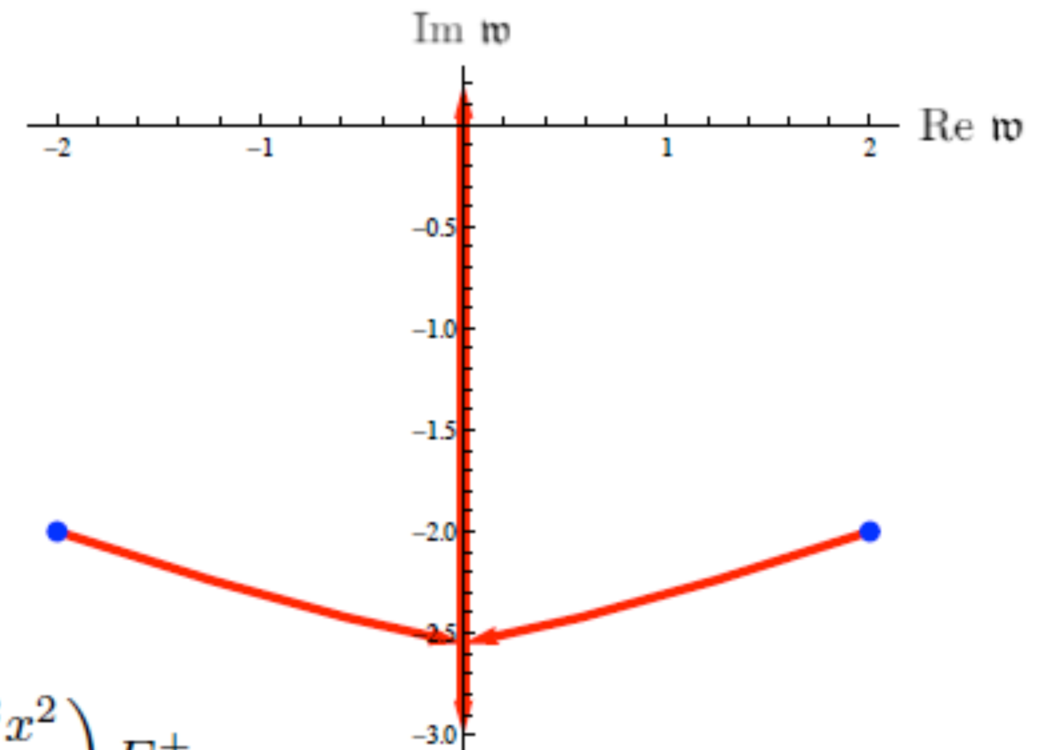


Condensation in magnetic field

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{2\kappa_5^2} (R - 2\Lambda) - \frac{1}{4\hat{g}^2} F^a_{\mu\nu} F^{a\mu\nu} \right] + S_{\mathrm{bdy}}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu$$

$$A^3_y = xB$$

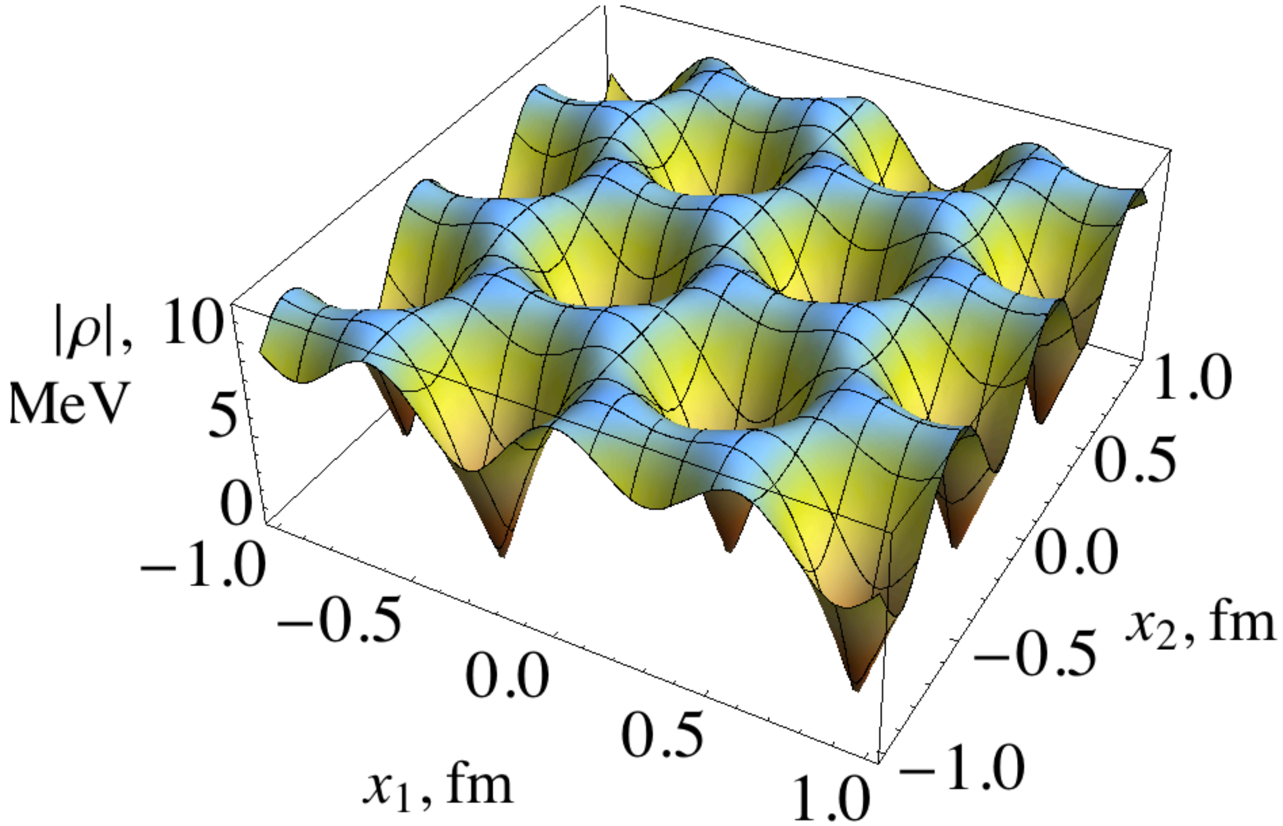


Fluctuations

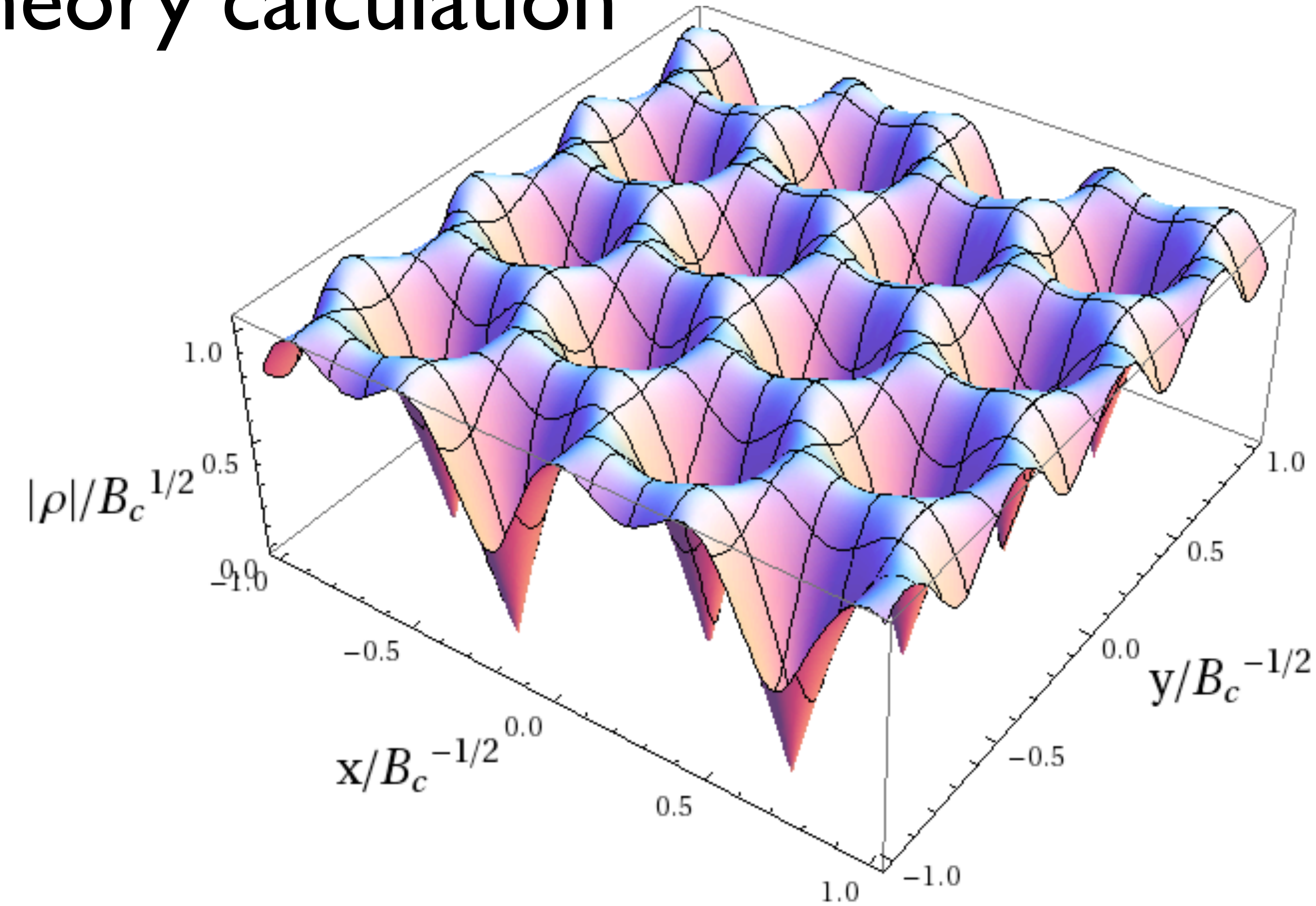
$$0 = \partial_u^2 E_x^+ + \frac{1}{f} \partial_x^2 E_x^+ + \left(\frac{f'}{f} - \frac{1}{u} \right) \partial_u E_x^+ - \frac{2}{xf} \partial_x E_x^+ + \left(\frac{\omega^2}{f^2} - \frac{B^2 x^2}{f} \right) E_x^+$$

cf. Chernodub;
Callebaut, Dudas, Verschelde;
Donos, Gauntlett, Pantelidou

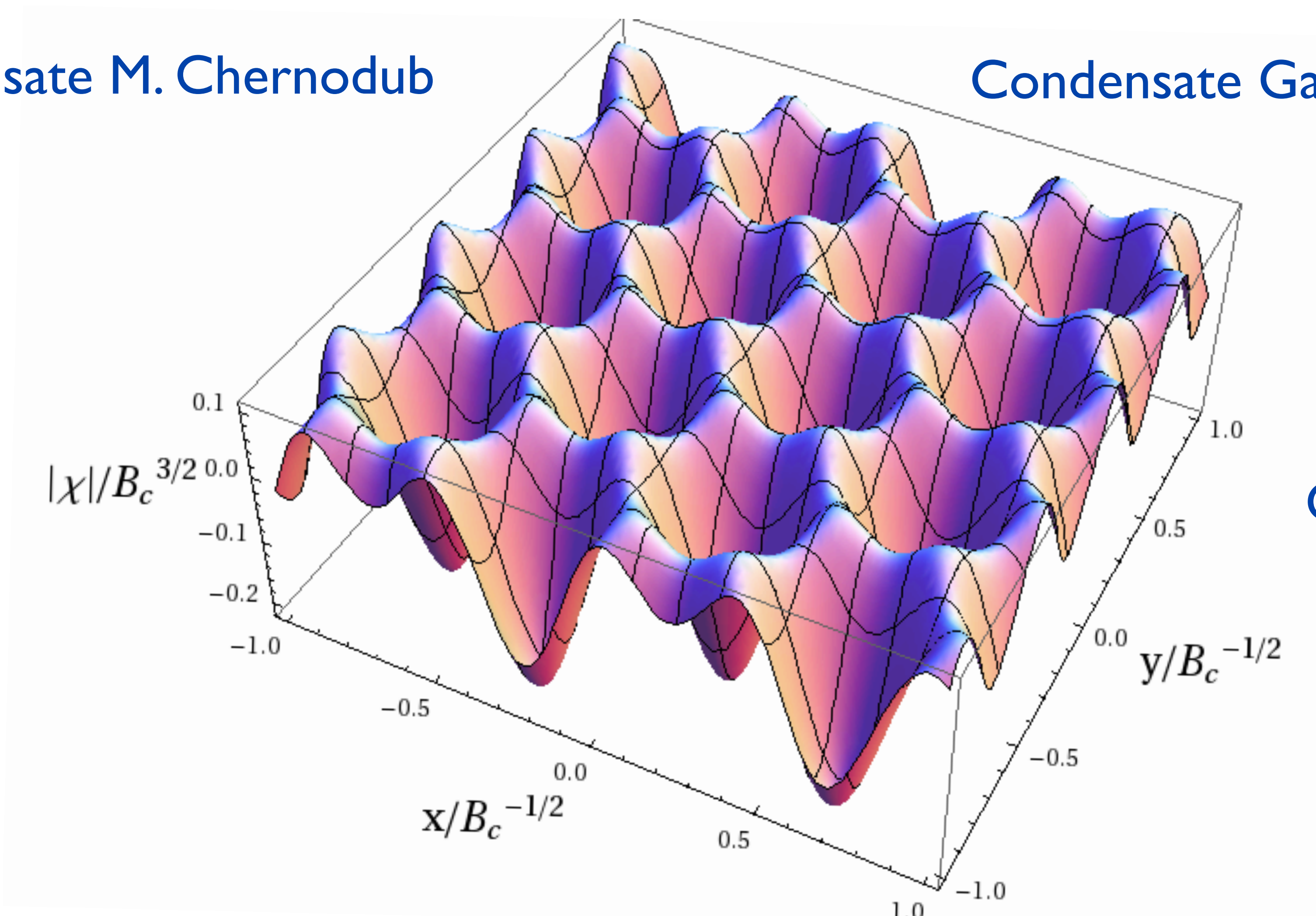
Comparison to field theory calculation



Condensate M. Chernodub



Condensate Gauge/Gravity Duality



Magnetization Gauge/Gravity Duality

J.E., J. Shock, M. Strydom, to appear

Conclusions

- D3/D7 with finite isospin: Holographic p-wave superconductor with known dual field theory
- Add backreaction in bottom-up model
- Anisotropic shear viscosity:
  Non-universal contribution at leading order in N and λ
- Flexoelectric effect
- Progress towards holographic Homes' law
- Condensation at finite magnetic field