Universal incoherent metallic transport

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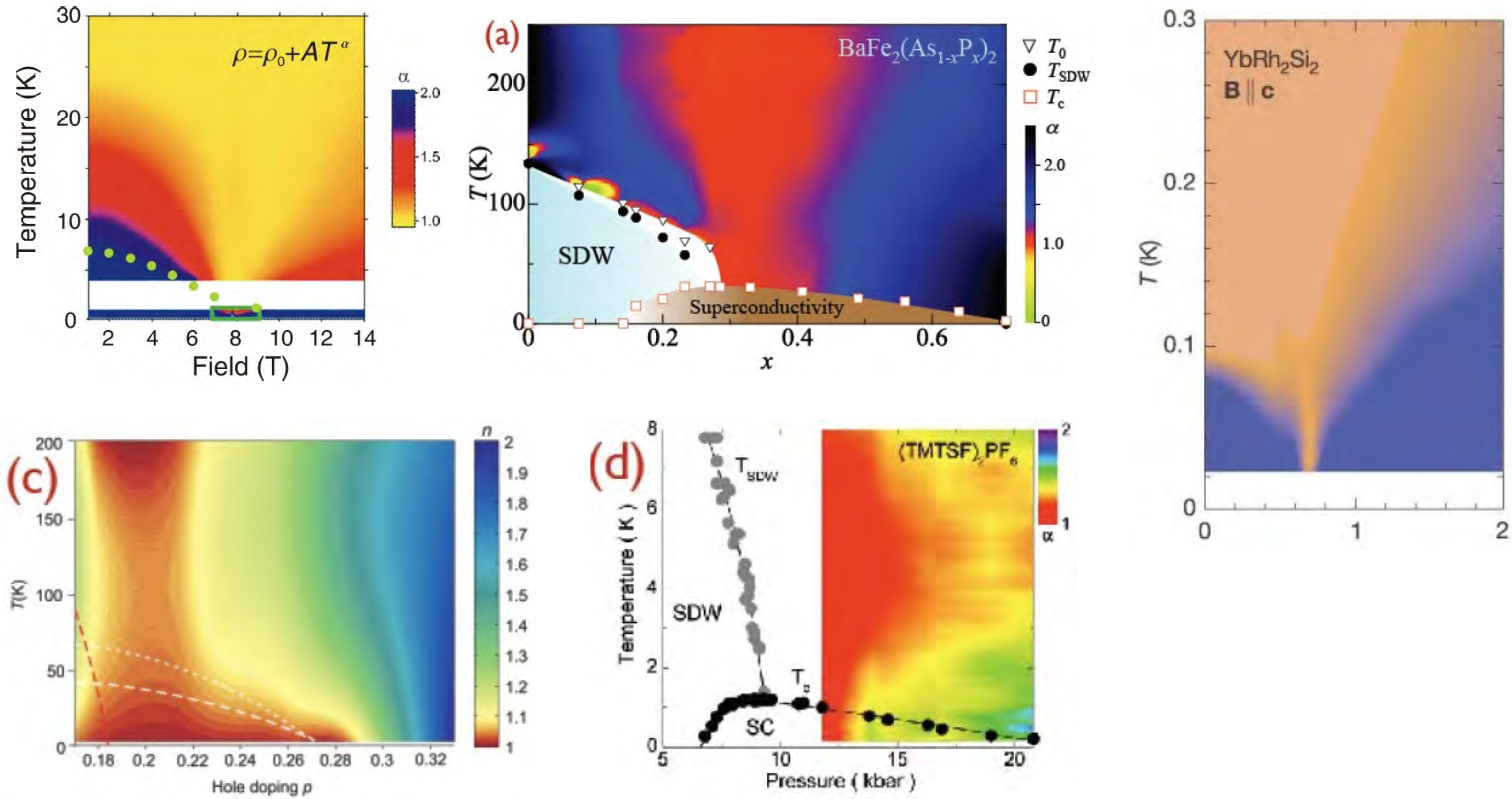
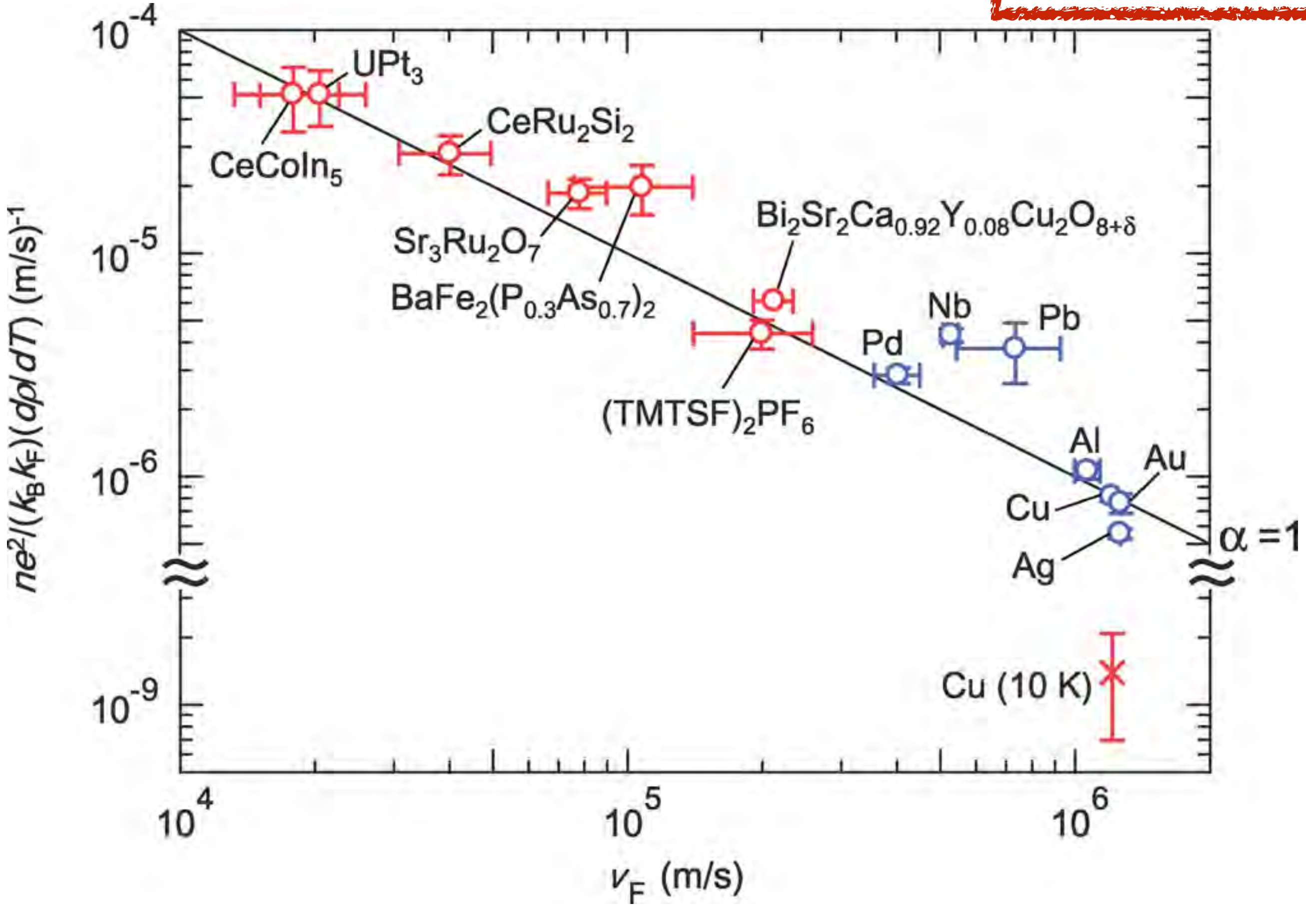
Ubiquity of T-linear resistivity



(after Sachdev-Keimer ’11)

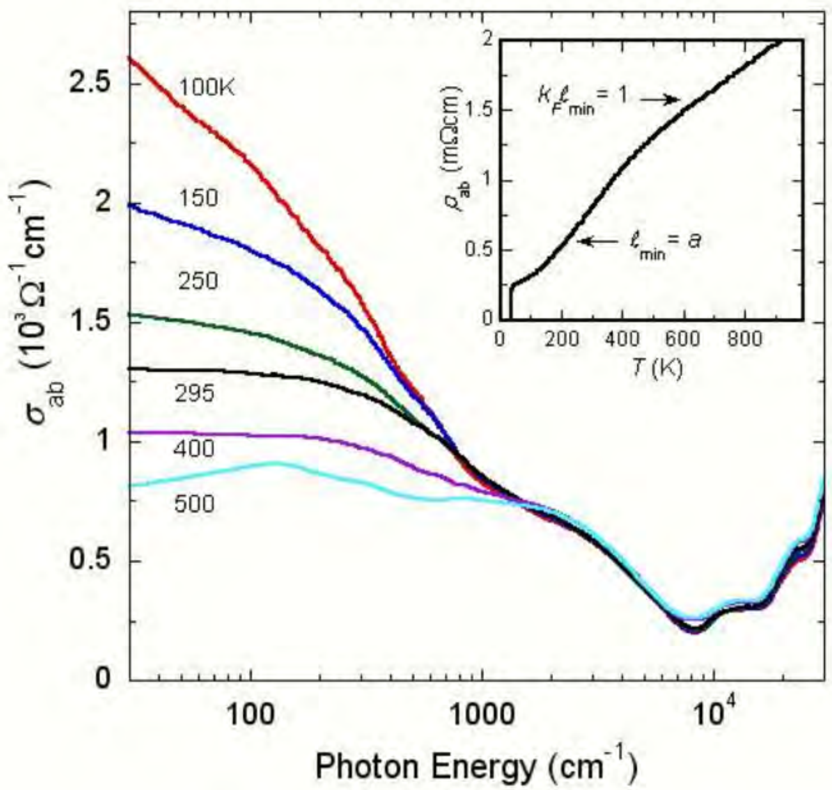
Universal timescale

- **Write:** $\sigma = \dfrac{ne^2\tau}{m}$ $\left(n = \dfrac{k_F^2}{2\pi d}\right)$
- **Measure: σ (from resistivity), k_F,m (quantum oscillations).**
- **Extract τ. Find:** $\boxed{\tau = \alpha \dfrac{\hbar}{k_B T}}$ $(\alpha \approx 1 - 2)$

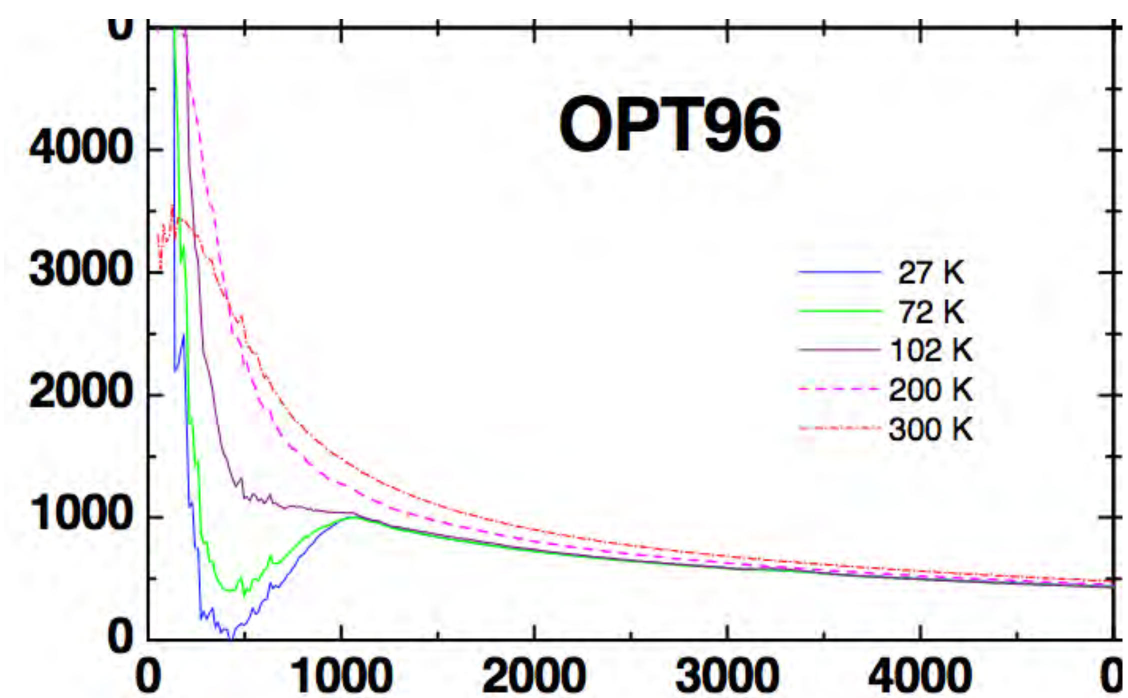


(Bruin et al. '13)

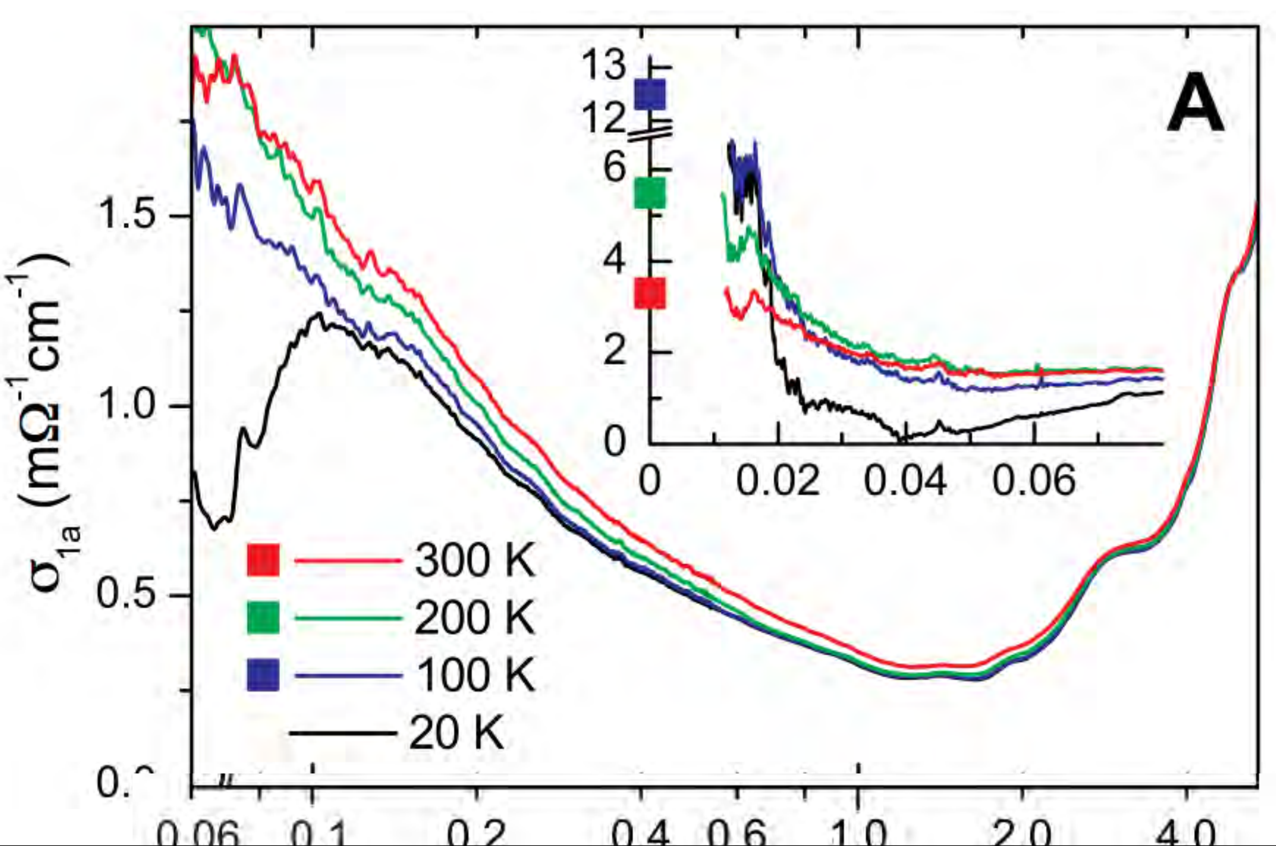
$$\frac{1}{\tau} \sim \frac{k_B T}{\hbar}$$



- **LSCO, Takenaka et al. '03**

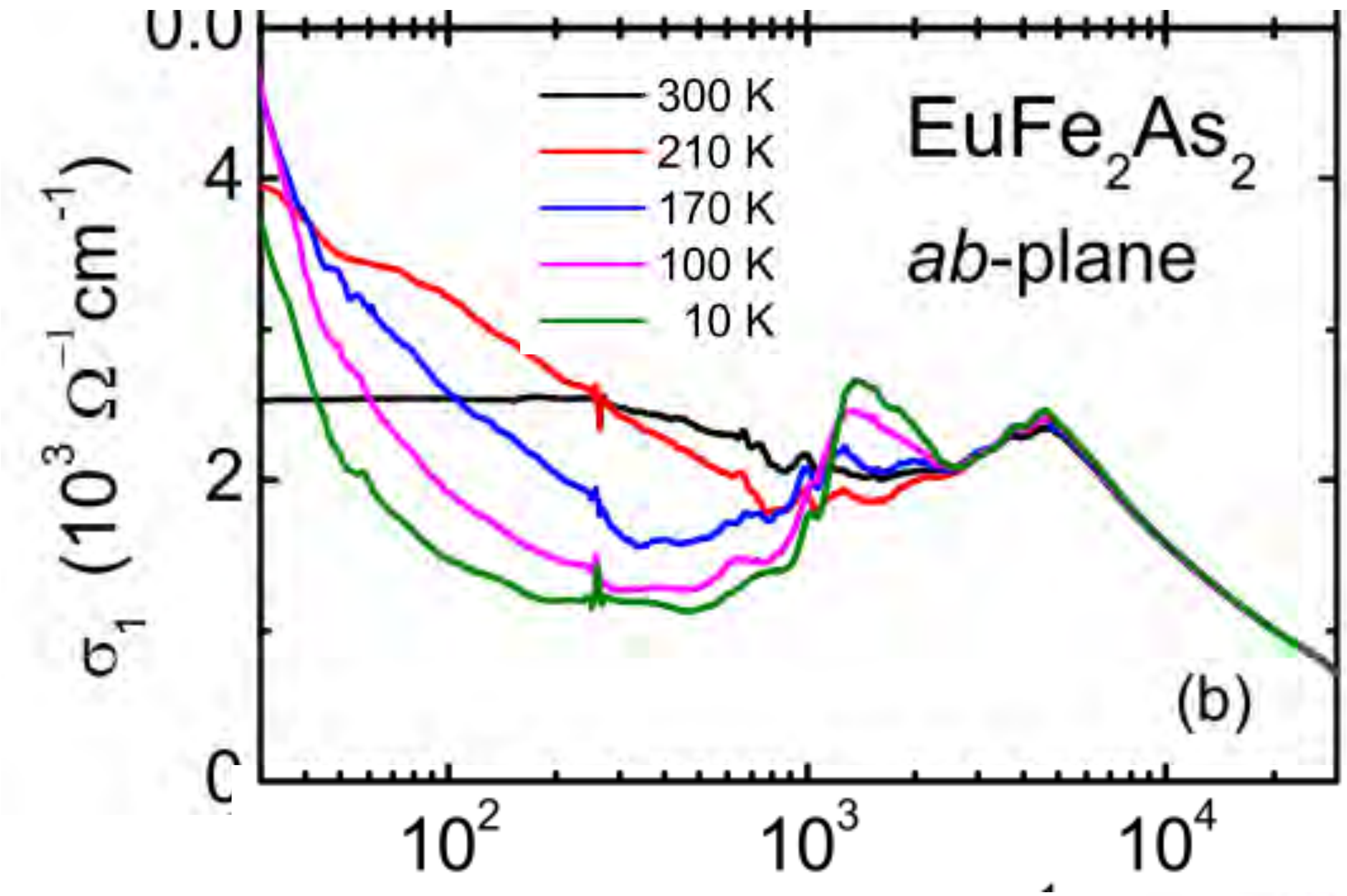


- **BSCCO, Hwang et al. '07**

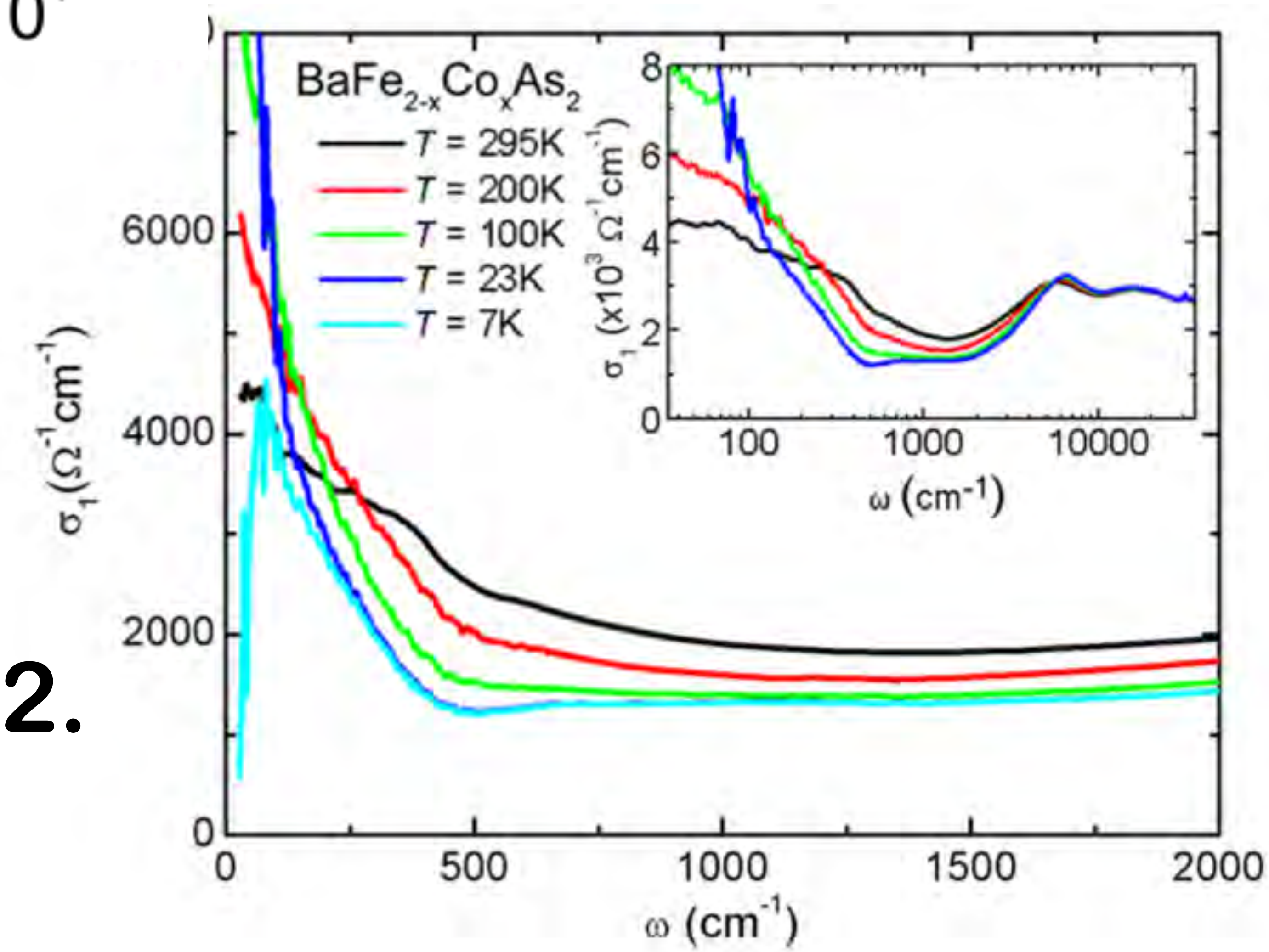


- **YBCO, Boris et al. '04**

$$\frac{1}{\tau} \sim \frac{k_B T}{\hbar}$$



- **Wu et al. '09.**



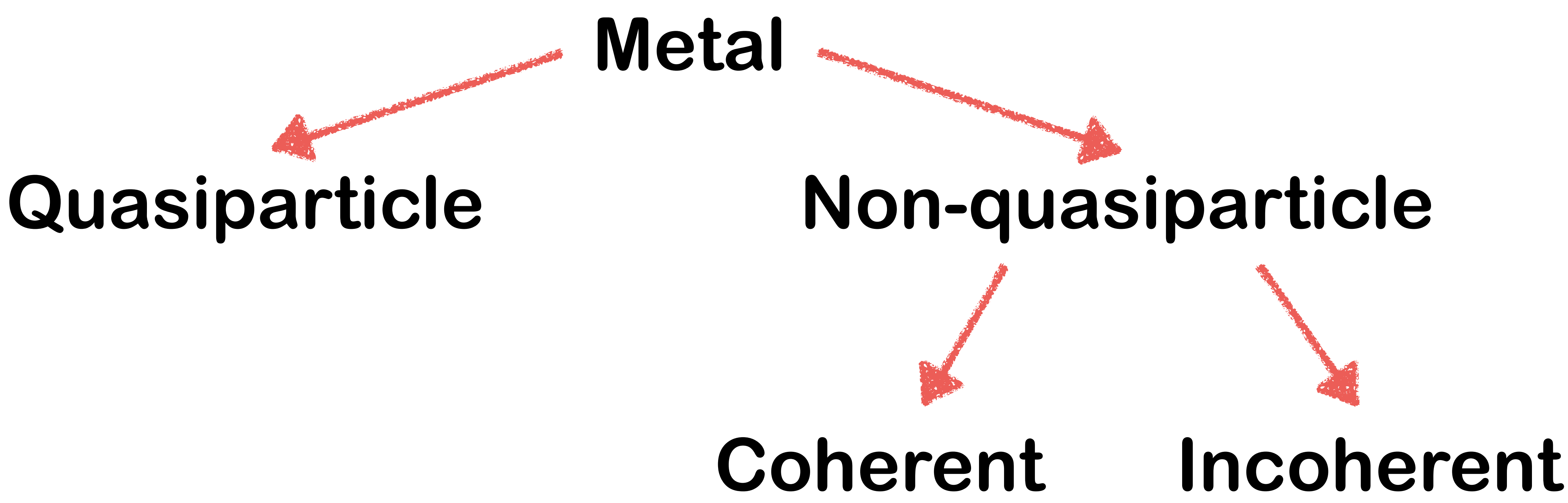
- **Schafgans et al. '12.**

Mission statement

- Perhaps these systems show similar behavior because they are close to saturating a universal bound?
- Longstanding idea that quantum criticality might saturate a timescale bound (Sachdev).
- Bolstered by Kovtun-Son-Starinets viscosity bound (cf. Zaanen, Bruin et al.).
- This talk: quasi-concrete proposal for such a bound in the context of metals.

Classification of metals by transport

- The resistivity of a metal is determined by the longest lived excitations that carry charge (or heat).



Quasiparticle transport

- **Longest lived excitations: δn_k.**
- **Study with Boltzmann equation.**

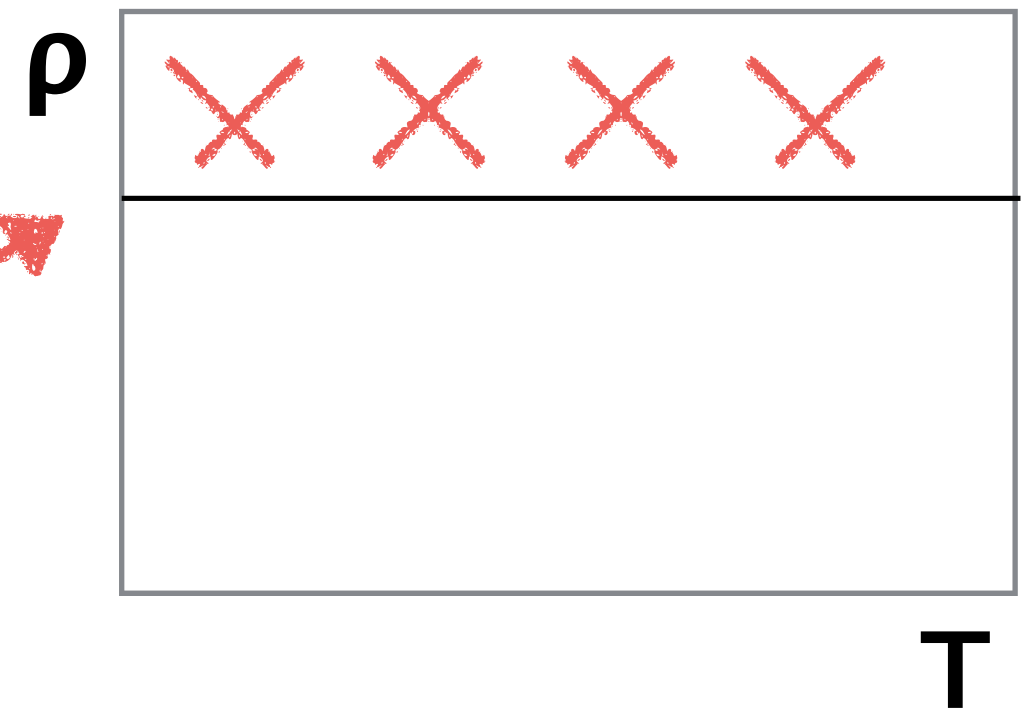
$$\sigma = \frac{ne^2\tau}{m}$$

$$\sim (k_F \ell)\, k_F^{d-2} \frac{e^2}{\hbar}$$

$$\gtrsim k_F^{d-2} \frac{e^2}{\hbar}$$

mean free path

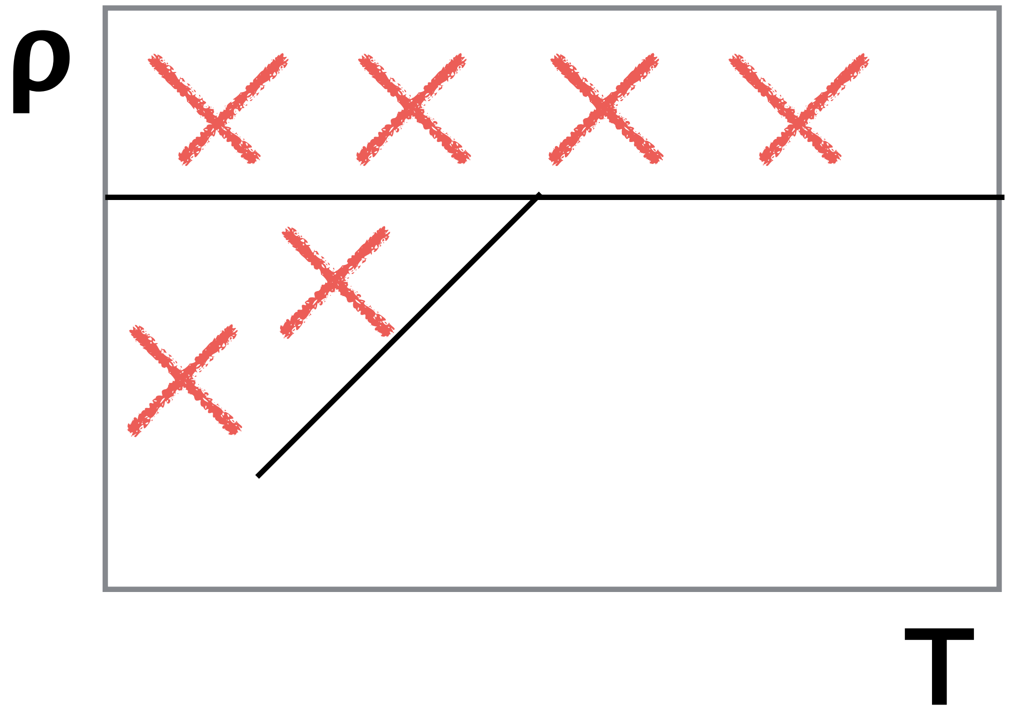
'MIR' bound



Quasiparticle transport

- **Lifetime instead of mean free path:**

$$\sigma = \frac{ne^2\tau}{m}$$
$$\sim \frac{\tau E_F}{\hbar} k_F^{d-2} \frac{e^2}{\hbar}$$
$$\gtrsim \frac{E_F}{k_B T} k_F^{d-2} \frac{e^2}{\hbar}$$



if qp have energy ~ $k_B T$
⇒ uncertainty principle: $(k_B T)\tau \gtrsim \hbar$

(requires sufficient inelastic scattering)

Einstein vs. Drude

- Drude: conductivity as momentum relaxation.

$$\sigma = \frac{ne^2}{m}\tau_{\text{mom.}}$$

- Einstein: conductivity as diffusion of charge.

$$\sigma = \chi D_{\text{charge}}$$

- Conventional quasiparticle metal: typically

$$\tau_{\text{mom.}} \sim \tau_{\text{qp.}} \sim \tau_{\text{charge}}$$

- Without quasiparticles, (apparently) different approaches to conductivity.

Coherent metals

- **Only momentum P is long-lived:**

$$\langle P(t)\rangle \sim e^{-\Gamma t}, \qquad \Gamma \ll k_B T$$

(momentum-conserving interactions strong, umklapp+disorder weak)

- **Then:**

$$\sigma = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{1}{-i\omega + \Gamma}$$

Narrow Drude peak

(e.g. Jung-Rosch '07, Hartnoll-Hofman '12, Mahajan-Barkeshli-Hartnoll- '13).

Incoherent metals

- Nothing is long-lived that overlaps with the current J.
- Longest lived quantities are total energy and charge. Relate to currents:

$$\frac{\partial n_A}{\partial t} + \nabla \cdot j_A = 0\,, \qquad (n_A = \{\epsilon, \rho\})$$

- Implies diffusion and Einstein relation:

$$\frac{\partial n_A}{\partial t} = D_{AB} \nabla^2 n_B \qquad \sigma_{AB} = D_{AC} \chi_{CB}$$

No 'Drude' peak but structure at $\omega \sim T$ generic

Incoherent metals

- **Conductivities:**

$$j_A = -\sigma_{AB} \nabla \mu_B \,, \qquad (\mu_A = \{T, \mu\})$$

- **Susceptibilities:**

$$\nabla n_A = \chi_{AB} \nabla \mu_B$$

- **Diffusion rates:**

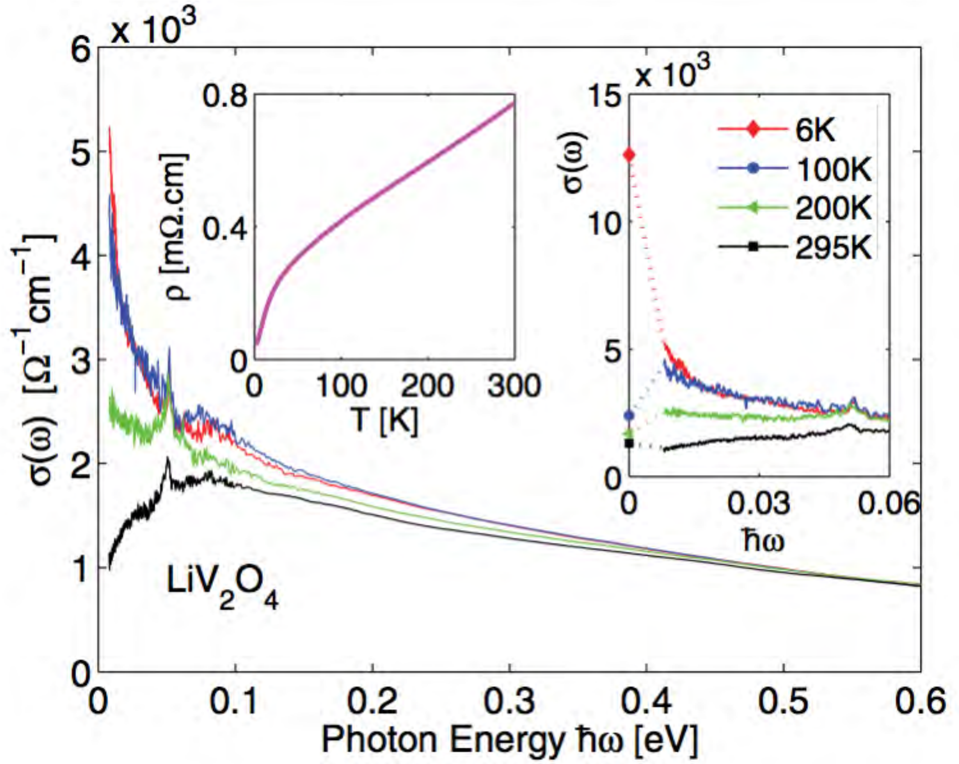
$$D_+ D_- = \frac{\sigma}{\chi} \frac{\kappa}{c} \,,$$

$$D_+ + D_- = \frac{\sigma}{\chi} + \frac{\kappa}{c} + \frac{T(\xi\sigma - \chi\alpha)^2}{c\chi^2\sigma} \,.$$

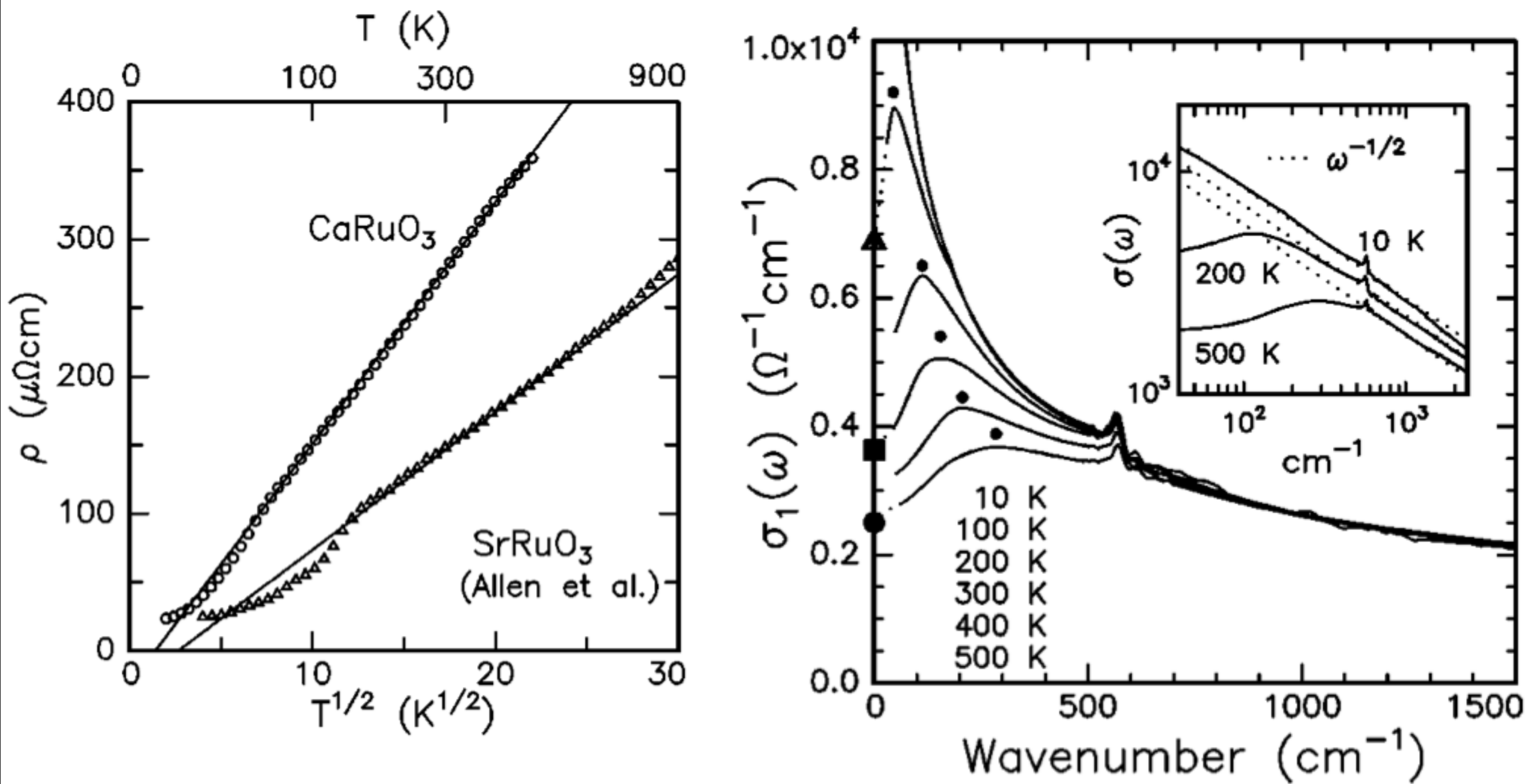
Optical conductivities

- Materials with high temperature T-linear resistivity regimes <u>all</u> show the onset of incoherence ...

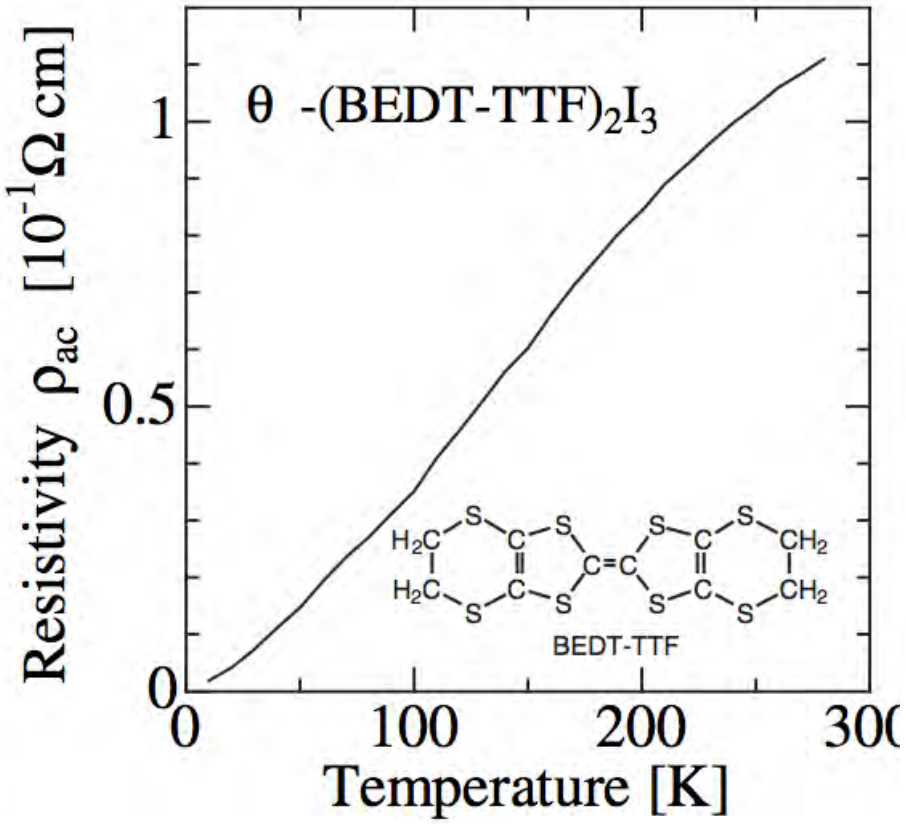
(in low T regimes, with Drude peaks, coherent metal approach likely applicable)

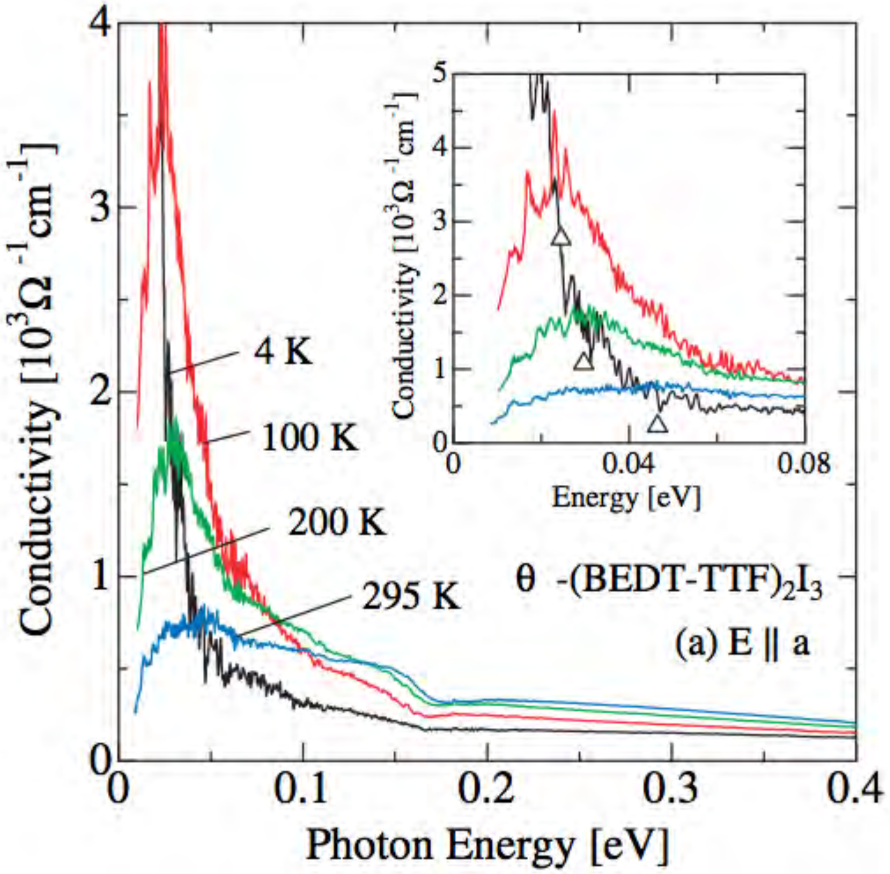


- **Jonsson et al. '07.**

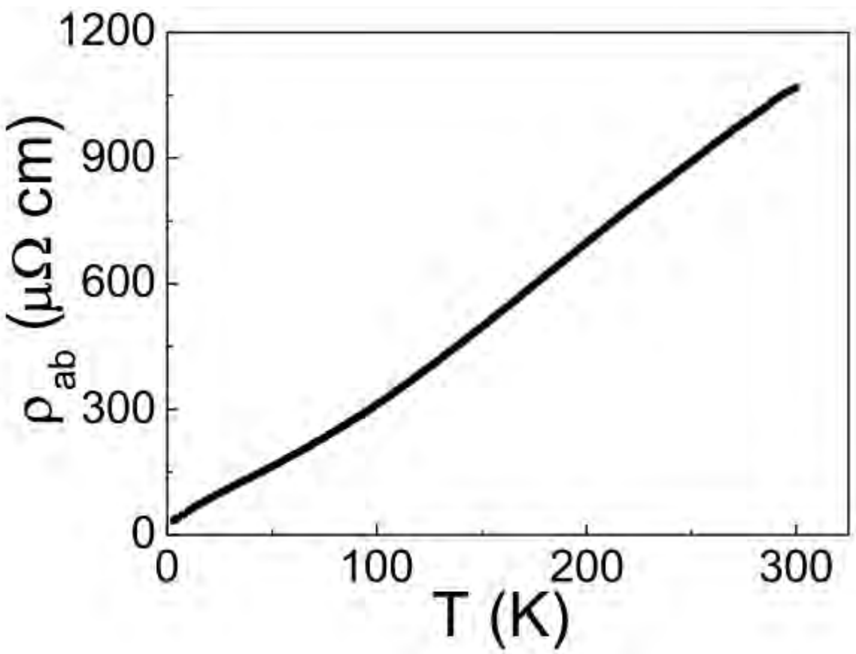


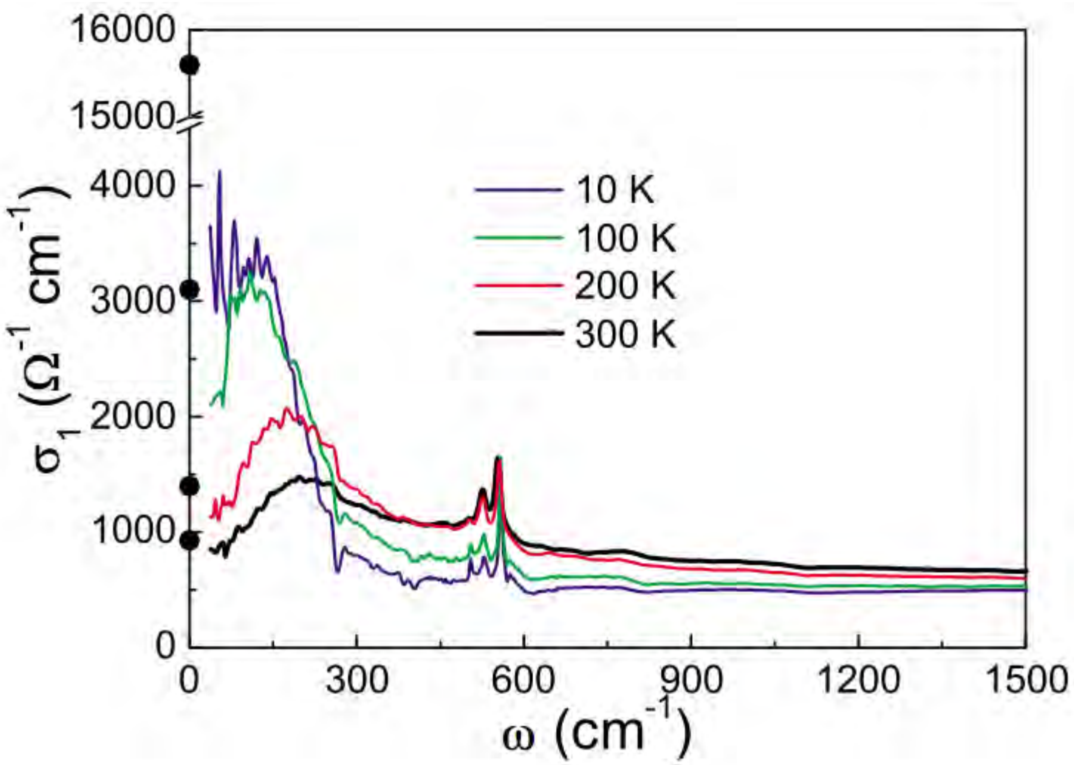
- **Lee et al. '02.**





- **Takenaka et al. ’05.**





- **Wang et al. ’04.**

$Na_{0.7}CoO_2$

Incoherent metals

- Dropping 'thermoelectric' terms:

$$\sigma = \chi D_+ \,,$$
$$\kappa = c D_- \,.$$

- Unlike momentum relaxation, diffusion is a process that is intrinsic to the system.

- Might the D's be fundamentally bounded? e.g. with quasiparticles:

$$D \sim v_F^2 \tau \gtrsim \frac{v_F^2 \hbar}{k_B T}$$

Aside on screening

- **In an actual metal, Coulomb interactions instantaneously screen fluctuations in charge.**

$$\chi_{\rho\rho}(\omega,k) = \frac{k^2 D\chi}{i\omega - Dk^2} \qquad \sigma^L(\omega,k) = \frac{-i\omega\chi_{\rho\rho}(\omega,k)}{k^2 - \chi_{\rho\rho}(\omega,k)}$$

- **Charge does not diffuse.**

$$\sigma^L(\omega,k) = \frac{-i\omega D\chi}{i\omega - D(k^2+\chi)}$$

- **However, the Einstein relation still holds:**

$$\sigma_{\text{d.c.}} = \frac{1}{i\pi}\int_{-\infty}^{\infty} \frac{\operatorname{Im}\sigma^L(\omega,k)}{\omega} d\omega = D\chi$$

Universal bounds?

- The KSS bound can be stated as a bound on momentum diffusion:

$$D_{\rm mom.} \gtrsim \frac{\hbar c^2}{k_B T}$$

- They proposed that this bound continued to hold in the absence of quasiparticles.
- The qp bound in metals suggests: $c \to v_F$

$$D_{\pm} \gtrsim \frac{\hbar v_F^2}{k_B T}$$

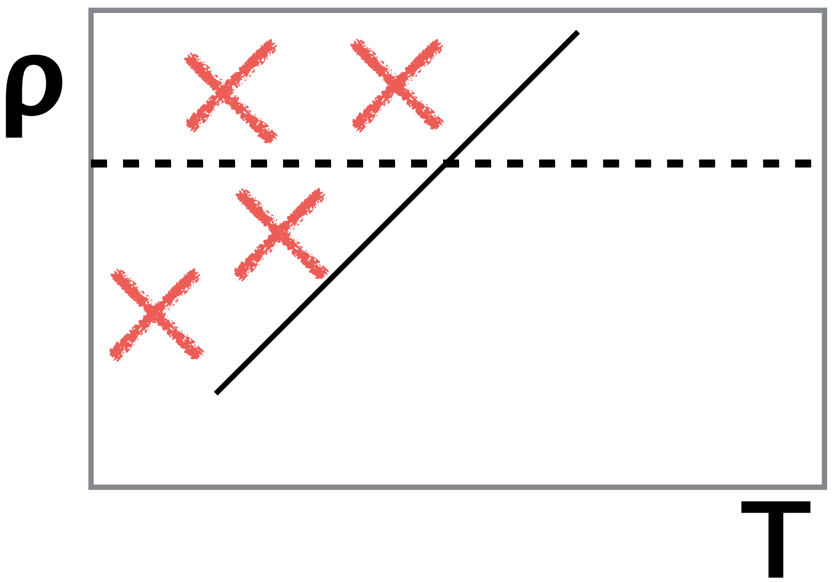
Resistivity

- A system approximately saturating the bound will have:

$$\rho \sim \frac{1}{\chi D} \sim \frac{\hbar}{k_F^{d-2} E_F} \frac{k_B T}{e^2}\,, \qquad (\text{with } \; \chi \sim e^2 k_F^d / E_F)$$

- Linear resistivity. If analyzed à la Bruin et al. would give the measured: $\tau \sim \hbar/(k_B T)$

- Can cross MIR bound:



Incoherence vs. phonons

- Electron-phonon-type scattering above a 'Debye' scale mimics many features of incoherent transport.

- However:
(i) e-ph scattering cannot cross MIR bound.
(ii) Above Debye scale, elastic scattering, and hence the Wiedemann-Franz law:

$$\frac{\kappa}{\sigma T} = \frac{\pi^2}{3}\frac{k_B^2}{e^2}$$

- In an incoherent metal: $\frac{\kappa}{\sigma T} = \frac{cD_+}{T\chi D_-} \sim \frac{c}{T\chi}$

The importance of D

- **Direct measurements of the diffusion constants can distinguish different scenarios and potentially falsify the bound.**

- **Eg. I am proposing:** $\chi \sim 1\,, \quad D \sim \frac{1}{T}$

- **Ultra high T expansion (e.g. DMFT) gives:**

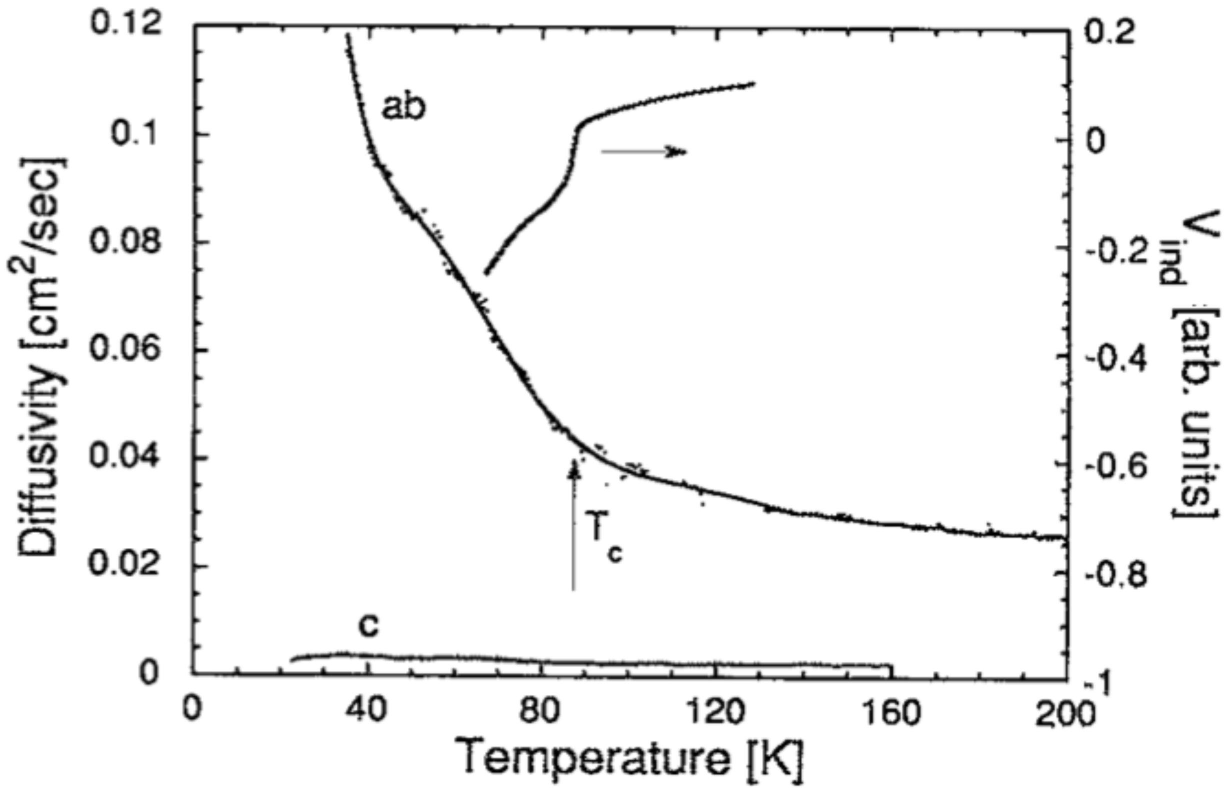
$$\chi \sim \frac{1}{T}\,, \quad D \sim 1$$

- **DMFT has also argued for:**

$$\chi \sim T\,, \quad D \sim \frac{1}{T^2}$$

The importance of D

- An old measurement of thermal diffusivity in BSCCO exists:



- Wu et al. '93. BSCCO
- Compatible with bounds once phonons subtracted.

Summary

- Proposed that incoherent metallic transport is subject to a diffusivity bound.
- This may explain the ubiquity of T-linear resistivity.
- Materials can cross the MIR bound while saturating the diffusivity bound.
- Known T-linear materials are incoherent.

Looking forward

- **Experimental counterexamples? (cf. low spin diffusivity measured in cold atomic Fermi gases).**

Universal spin dynamics in two-dimensional Fermi gases

Marco Koschorreck[1]*, Daniel Pertot[1], Enrico Vogt[1] and Michael Köhl[1,2]*

Harnessing spins as information carriers has emerged as an elegant extension to the transport of electrical charges[1]. The coherence of such spin transport in spintronic circuits is determined by the lifetime of spin excitations and by spin diffusion. Fermionic quantum gases allow the study of spin transport from first principles because interactions can be precisely tailored and the dynamics is on directly observable timescales[2-12]. In particular, at unitarity, spin transport is dictated by diffusion and the spin diffusivity is expected to reach a universal, quantum-limited value on the order of the reduced Planck constant $\hbar$ divided by the mass m. Here, we study a two-dimensional Fermi gas after a quench into a metastable, transversely polarized state. Using the spin-echo technique[13], for strong interactions, we measure the lowest transverse spin diffusion constant[14,15] so far 6.3(8) x 10^{-3} $\hbar/m$.

where λ_{dB} is the de Broglie wavelength of the col
In the degenerate regime (of any dimensionality)
wavelength λ_{dB} is of the order of $1/k_F$ and hence the
is $l_{mfp} = 1/(n\sigma) \approx 1/k_F$, where k_F is the Fermi wave
and $n \sim k_F^D$ is the density. Hence, the spin diffusion
by $\hbar/m$. This quantum limit can also be viewed as a
certainty principle by noticing that the mean-free
is limited by the mean interparticle spacing[11]. Th
argument, however, hides much of the rich underl
particular, it cannot explain the Leggett–Rice effect[1
ence between longitudinal and transverse spin dif
the transition to weak interactions where the phy
cause the system evolves from collision-dominated
The lowest spin diffusion constant for longitudin
has been measured to be $\mathcal{D}_{\parallel} = 6.3\hbar/m$ in three-di

Looking forward

- Controlled models of incoherent metals?
  e.g. (i) disordered fixed points.
  (ii) emergent particle-hole symmetry.
  (iii) holographic models of incoherence

- Some holographic geometries with strong momentum relaxation are known. Very interesting to probe the diffusivities and optical conductivities in these systems.