Towards a Field Theory over Tensor Network States

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Chris Hooley[2]
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Tanja Duric
Vid Stojevic

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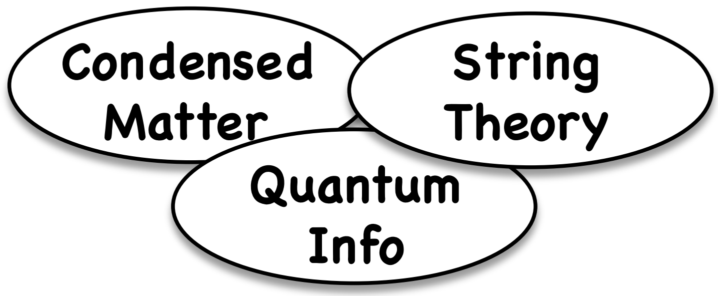
Background:

- Convergence of ideas



- Suggestive links

  Holography vs hierarchical tensor networks

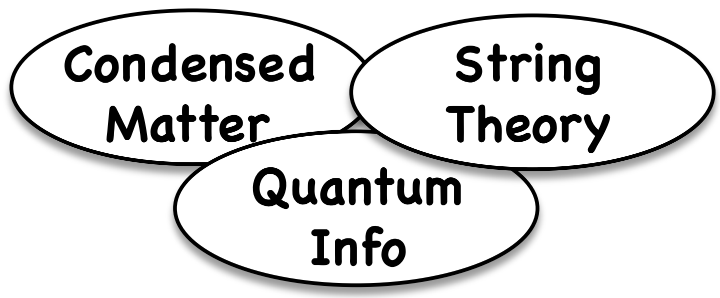
  Is Holography an entanglement ansatz?

  Can we capture new strongly-correlated phases?

- Ideas formulated in very different language

Background:

- Convergence of ideas



- Suggestive links

  Holography vs hierarchical tensor networks

  Is Holography an entanglement ansatz?

Need to develop a common Language
Import insights of tensor networks to field theory

Outline:

- Background

  Variational States in Condensed Matter
  Tensor Networks
  RG and Hierarchical Tensor Networks

- Towards a Field Theory Over Tensor Networks

  Goal and Key steps
  Formulating the Field Theory
  Interpretations and Applications
  Extensions to Higher Dimensions
  Extension to Critical Systems

- Conclusions

I Background:

Variational States in Condensed Matter

Tensor Networks and Strong Correlation

Tensor Networks and AdS/CFT

Background

Variational States in Condensed Matter

New State Observed — Guess a Wavefunction — Find a toy Hamiltonian — Field Theory

The Dream!
New emergent state predicted

Examples:

FQHE: Expt[DPvK]->Exact Diag->Wavefunction[Laughlin]->Hamiltonian [Haldane]
->Composite Fermion Picture[Jain/Read]->Field Theory[Lopez/Fradkin]

BCS: Experiment -> Toy Hamiltonian -> Guess wavefunction -> Field Theory

Exceptions:

Haldane Conjecture -> Field Theory -> Expt
Topological Insulator -> Toy Hamiltonian -> wavefunction -> experiment
Bethe Ansatz: variational wavefunction -> tremendous power in 1d
Conformal Field Theory, Renormalisation Group… etc. etc.

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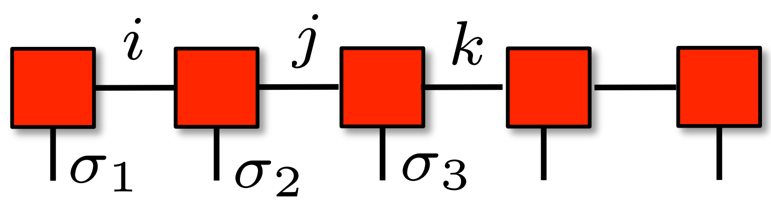
Background

Tensor Networks

- Class of variational wavefunctions
- Embody insights about entanglement structure
- Describe groundstates of local Hamiltonians efficiently
- Exact for some model H (AKLT, Majumdar-Ghosh, etc)

- Matrix Product States (1d tensor network)

$$|\phi\rangle = \sum_{\{\sigma\}} A_i^{\sigma_1} A_{ij}^{\sigma_2} A_{jk}^{\sigma_3} A_{kl}^{\sigma_4} \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$



- MPS is a restricted sum of product states
- MPS dense on Hilbert space

Background

Tensor Networks

- Class of variational wavefunctions
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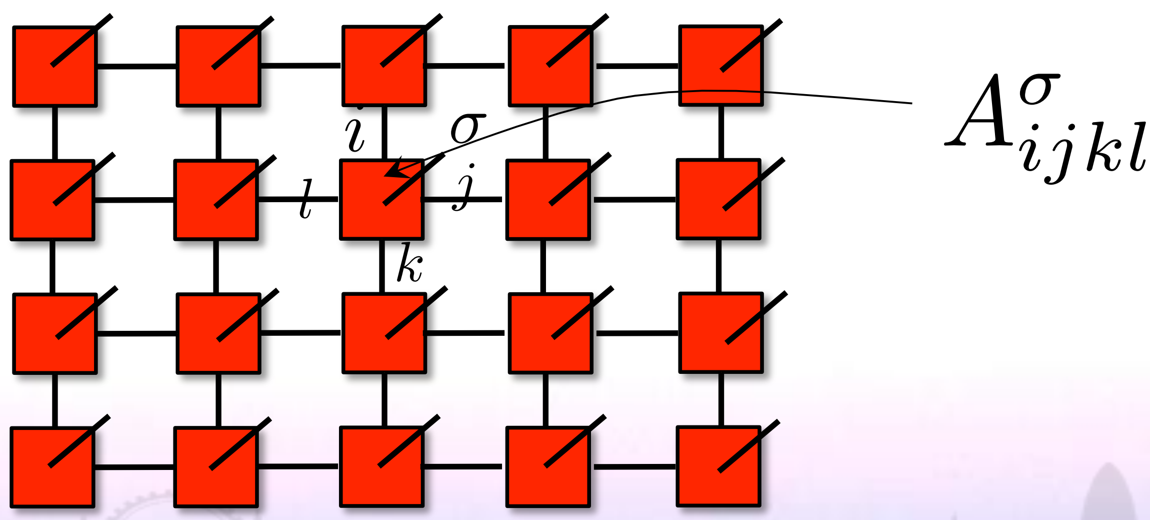
- Describe groundstates of local Hamiltonians efficiently
- Exact for some model H (AKLT, Majumdar-Ghosh, etc)

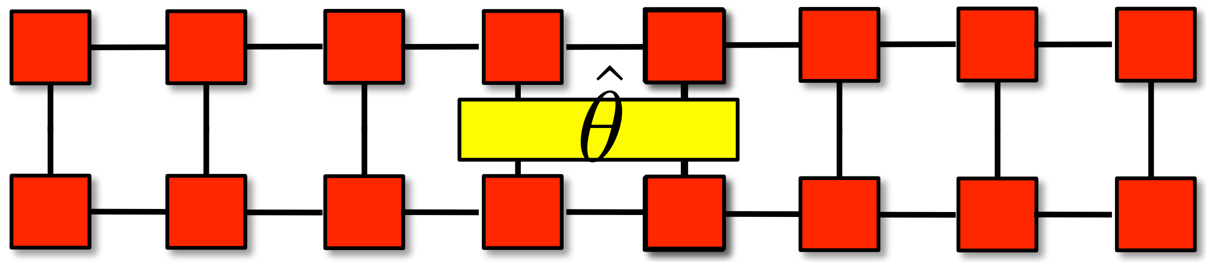
- PEPS (projected entangled pair state - 2d tensor network)



Background

Contracting Tensor Networks

$$\langle\psi|\hat{\theta}|\psi\rangle =$$



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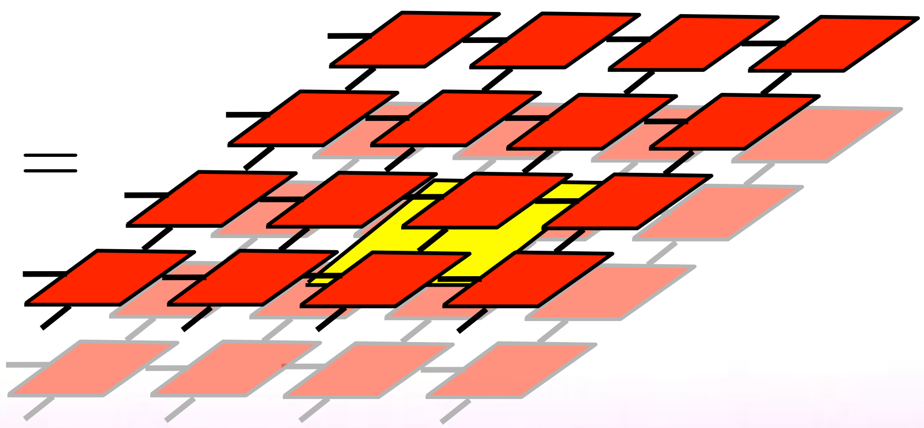

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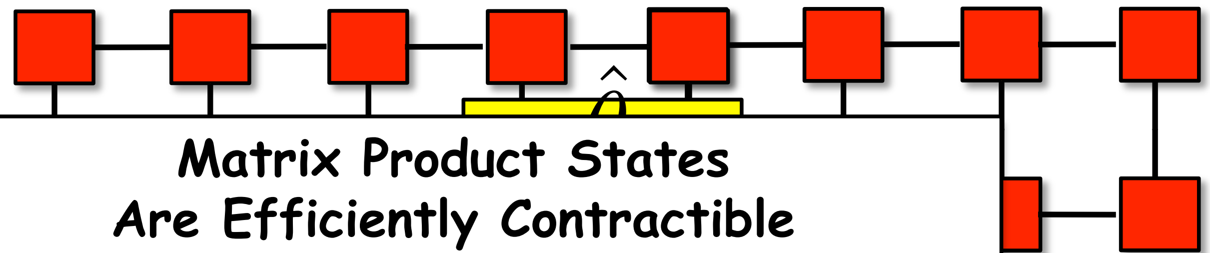

Matrix Product States
Are Efficiently Contractible
t~Poly[N]

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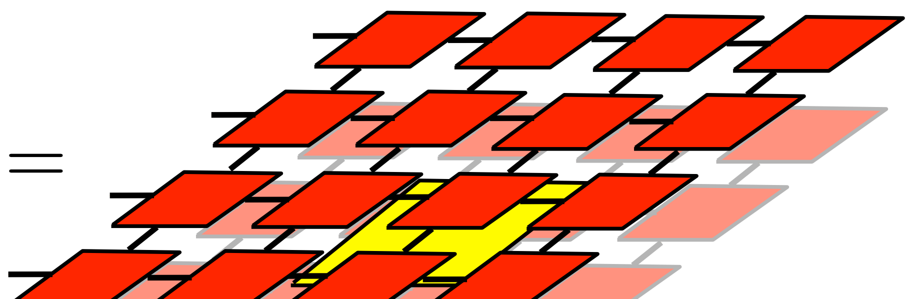


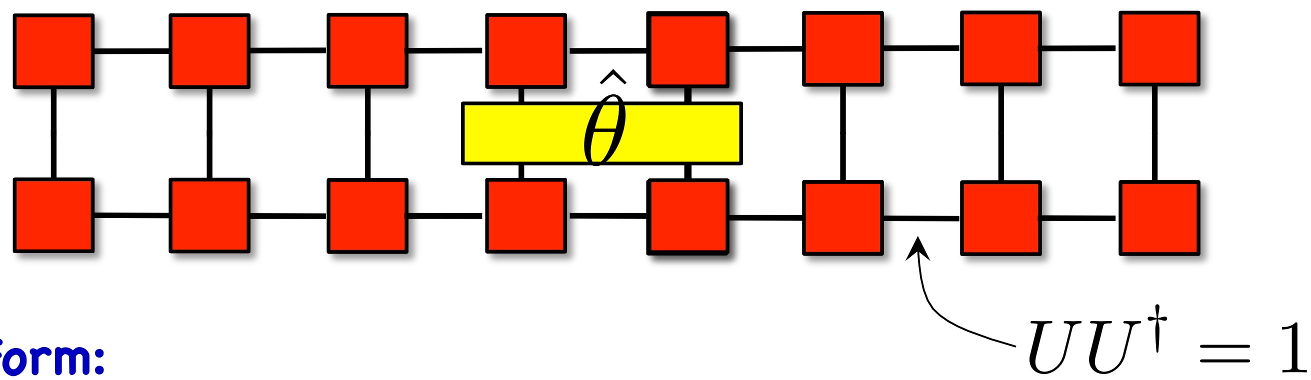
Tensor Network States in D>1
Are NOT Efficiently Contractible
t~Exp[N]

Background

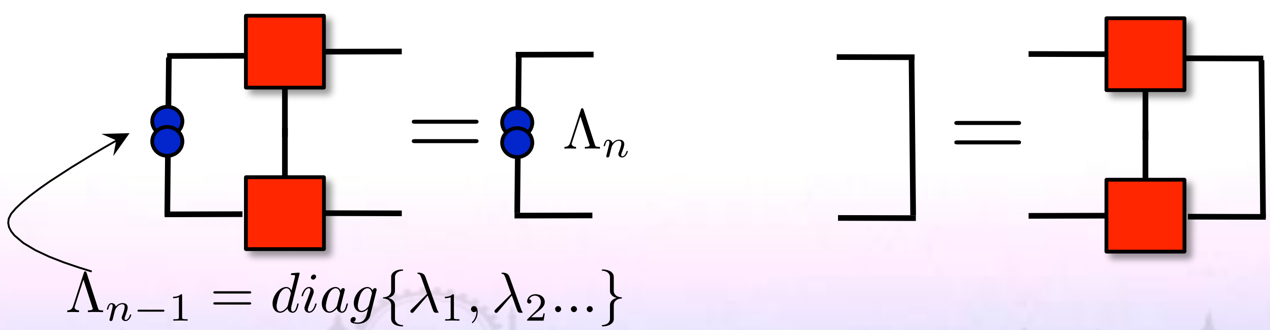
Tensor Networks – Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
- **A.** Not in general. In 1d can always gauge fix to make it so.

$$\langle\psi|\hat{\theta}|\psi\rangle =$$



Canonical Form:



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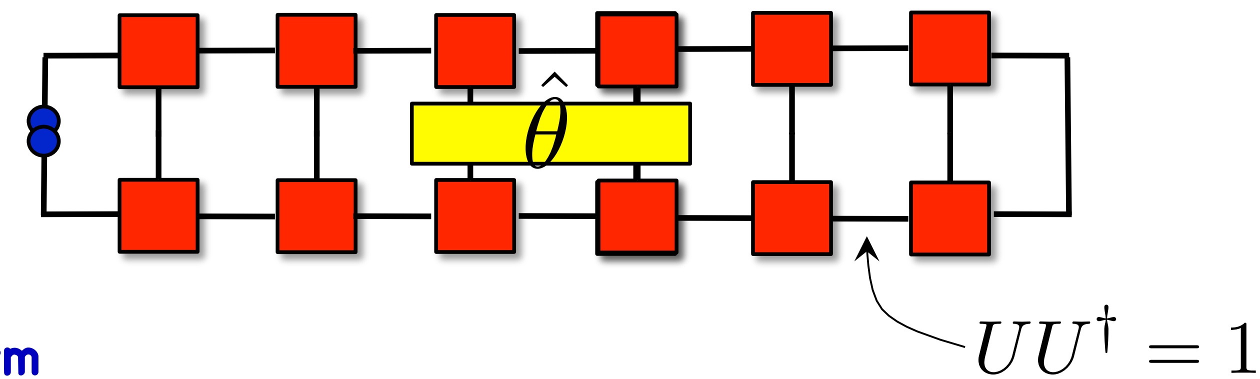

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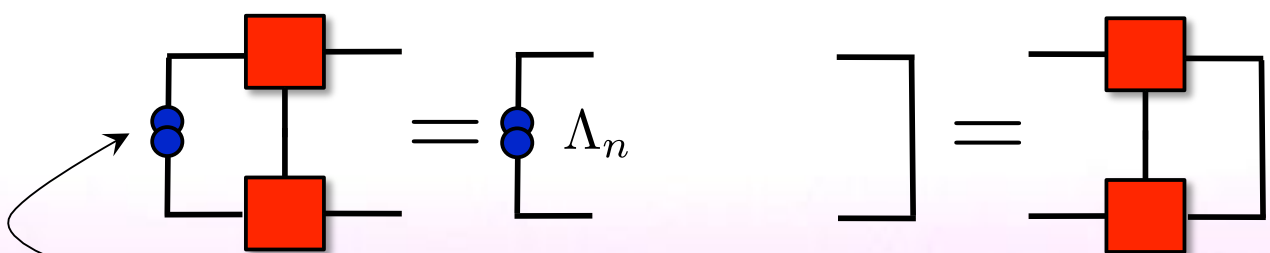

$$\Lambda_{n-1} = diag\{\lambda_1, \lambda_2 ...\}$$

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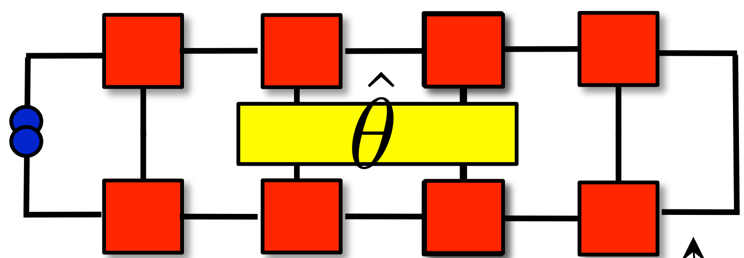
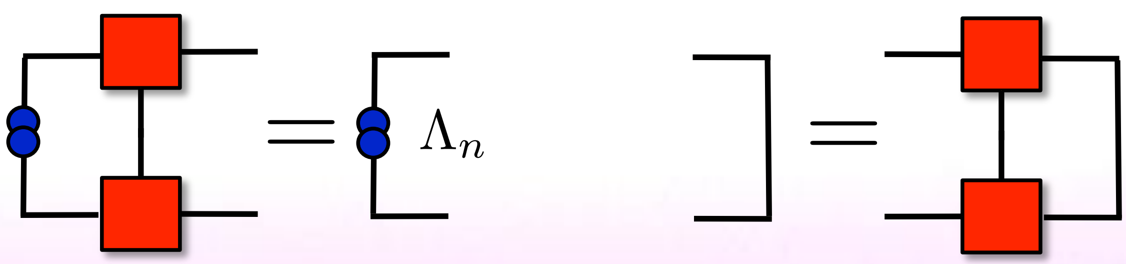


$UU^\dagger = 1$

Canonical Form



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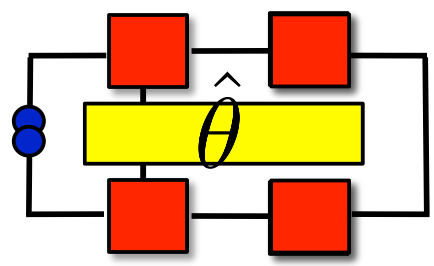

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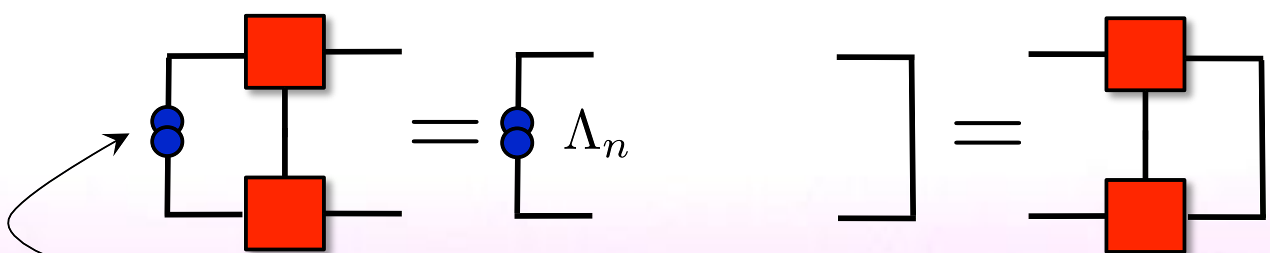

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- **Q.** Is the expectation of local operators local in terms of As?
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Canonical Form:

- In canonical form, MPS is a Schmidt decomposition $|\phi\rangle = \sum_\alpha \lambda_\alpha |\phi_L^\alpha\rangle|\phi_R^\alpha\rangle$ of each bond [Vidal,PRL91,147902,(2003)]

$$|\phi\rangle = \sum_{\{\sigma\}} A_i^{\sigma_1} A_{ij}^{\sigma_2} A_{jk}^{\sigma_3} A_{kl}^{\sigma_4} \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$

vs

$$|\phi\rangle = \sum_{\{\sigma\}} \Gamma_i^{\sigma_1} \lambda_i^1 \Gamma_{ij}^{\sigma_2} \lambda_i^2 \Gamma_{jk}^{\sigma_3} \lambda_i^3 \Gamma_{kl}^{\sigma_4} \lambda_i^4 \ldots |\sigma_1, \sigma_2, \sigma_3, \sigma_4, \ldots\rangle$$

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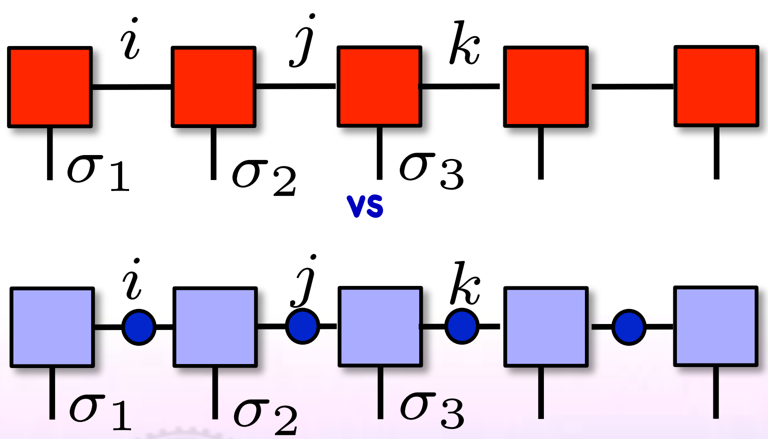

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Tensor Networks – Locality and Gauge Fixing

- **Q.** Is the expectation of local operators local in terms of As?
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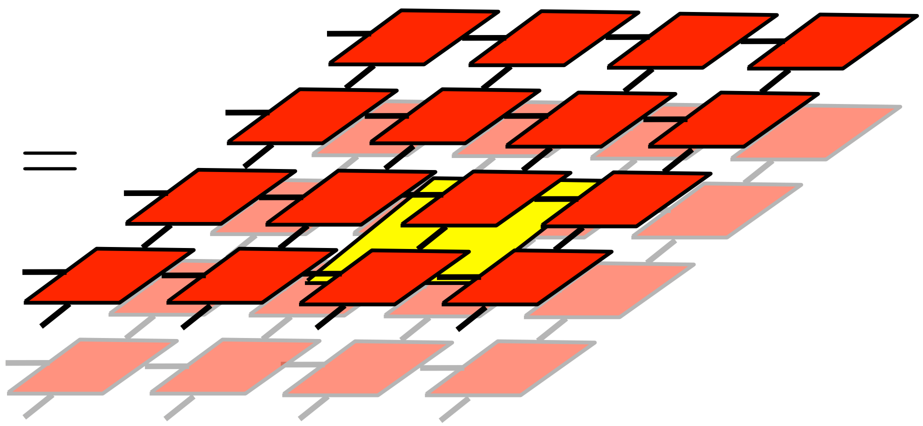

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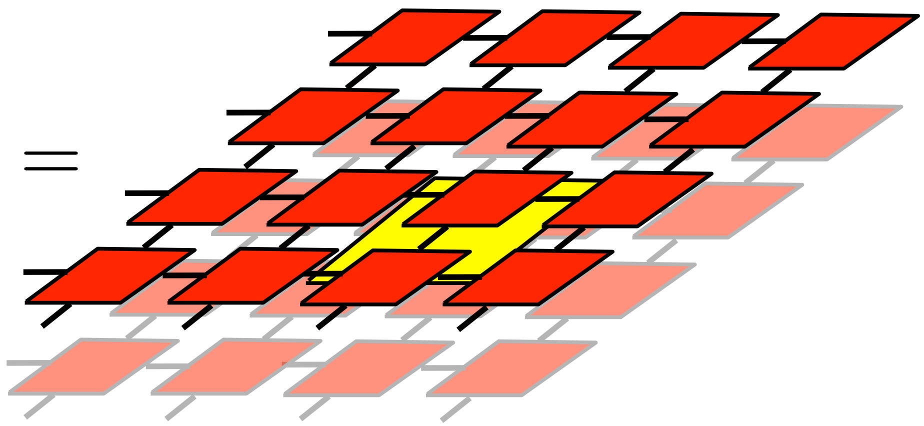

- Physically, expect entanglement to decay, so should be local
- Obey area laws by construction
- Numerically, various approximation schemes seem to work
- Similar to maximally localised Wanier orbitals?

[Mazari et al Rev Mod Phys (2012)]

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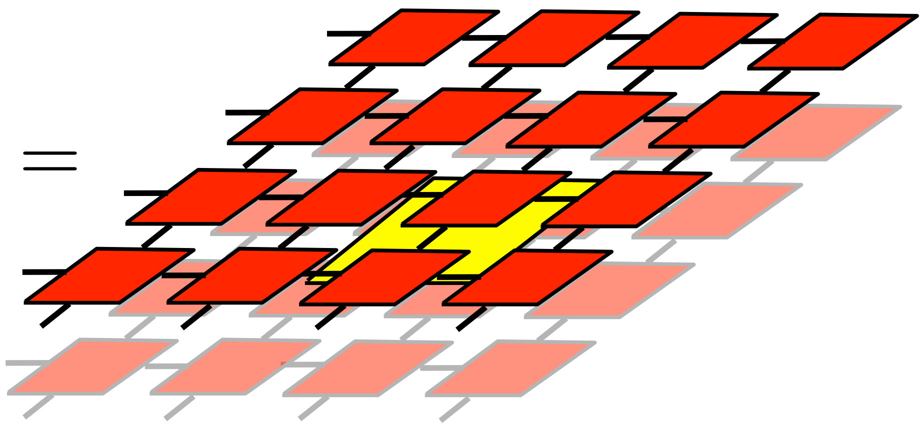
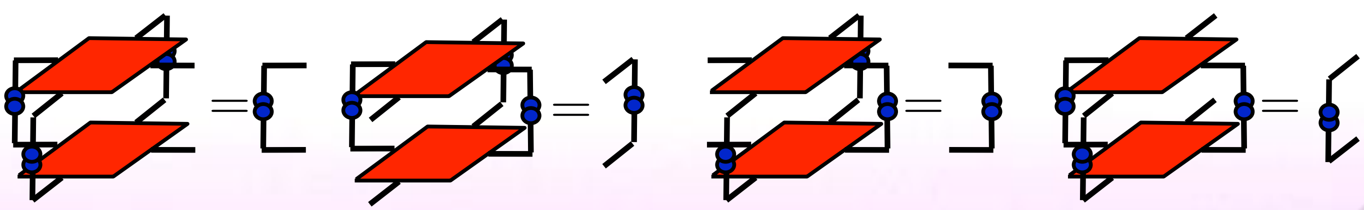


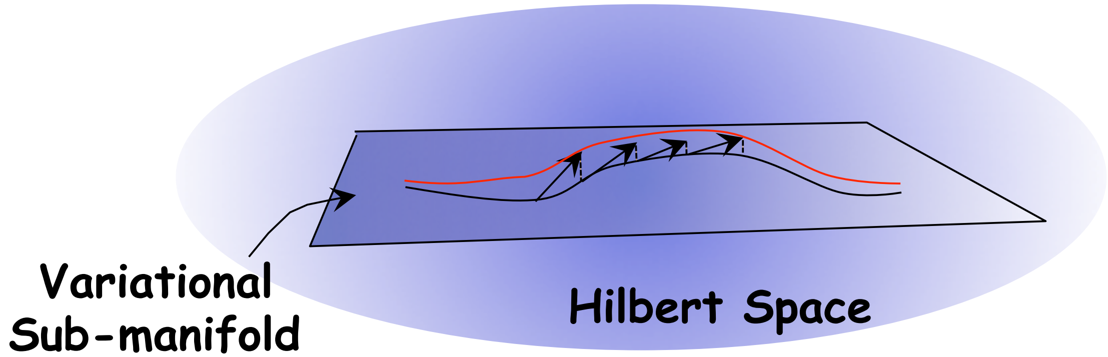
Quasi-Canonical Form:



Background

Time dependence of Tensor Network States

- Fixed bond order – restricted sub-manifold of Hilbert space
- Higher bond order – higher dimension sub-manifold



Time Dependent Variational Principle:

- Bond-order grows under Hamiltonian evolution
- Continually Project back to fixed bond order

$$\langle \partial_{\bar{A}_i} \psi | \partial_{A_j} \psi \rangle \dot{A}_j = i \langle \partial_{\bar{A}_i} \psi | \hat{\mathcal{H}} | \psi \rangle$$

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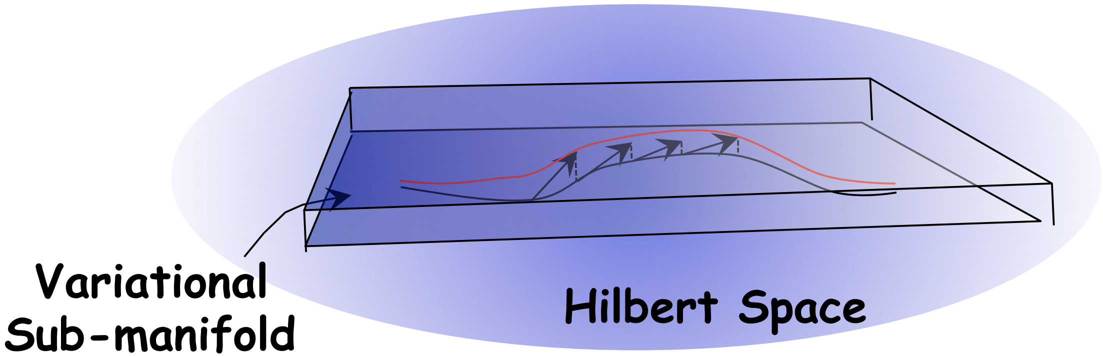

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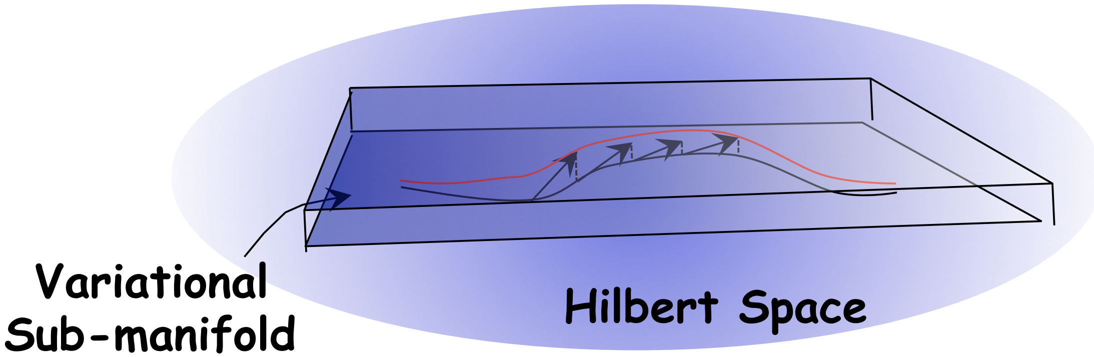
Background

Variational Manifold vs Classical Phase space

$$\langle \partial_{\bar{A}_i} \psi | \partial_{A_j} \psi \rangle \dot{A}_j = i \langle \partial_{\bar{A}_i} \psi | \hat{\mathcal{H}} | \psi \rangle$$



TDVP is a semi-classical equation of motion
Variational manifold is a semi-classical phase space
Tensors are semi-classical collective coordinates

I Background:

Variational States in Condensed Matter
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Tensor Networks and AdS/CFT

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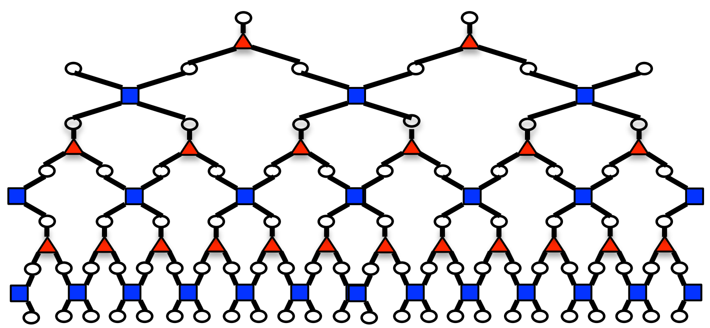
RG and Hierarchical Tensor Networks

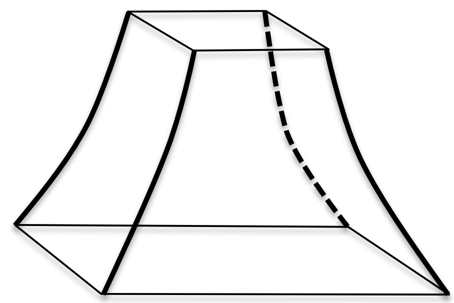
- Critical systems don't obey area laws
- Exponentially large bond order required
- Scaling suggests a more efficient way to encode

MERA (multi-scale entanglement renormalisation ansatz)

[Vidal Phys Rev Lett 101, 110501 (2008)]



vs



Similarity to AdS [Swingle Phys Rev D86, 065007 (2012); ArXiv1209.3304]

- Extends 1d-2d
- Extra dimension - entanglement RG scale
- Entanglement minimal surface [Ryu, Takayanagi PRL 96, 181602 (2006)]
- Finite T -> finite extent

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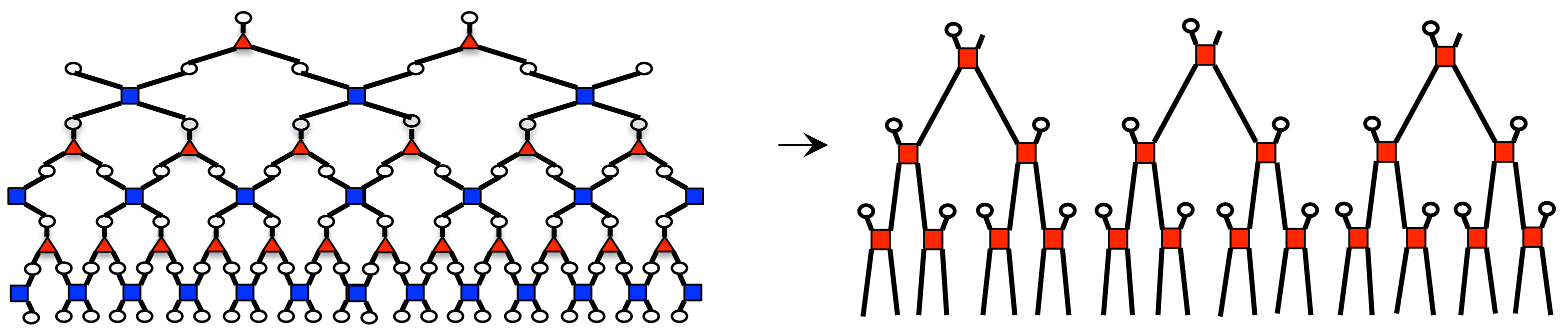
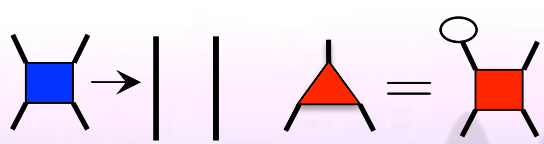


Exact Holographic mapping [Xiao-Liang Qi arXiv:1309.6282]

- Unitary transformation to disk
- Wavelet transform on Cayley tree
- Residual entanglement -> metric



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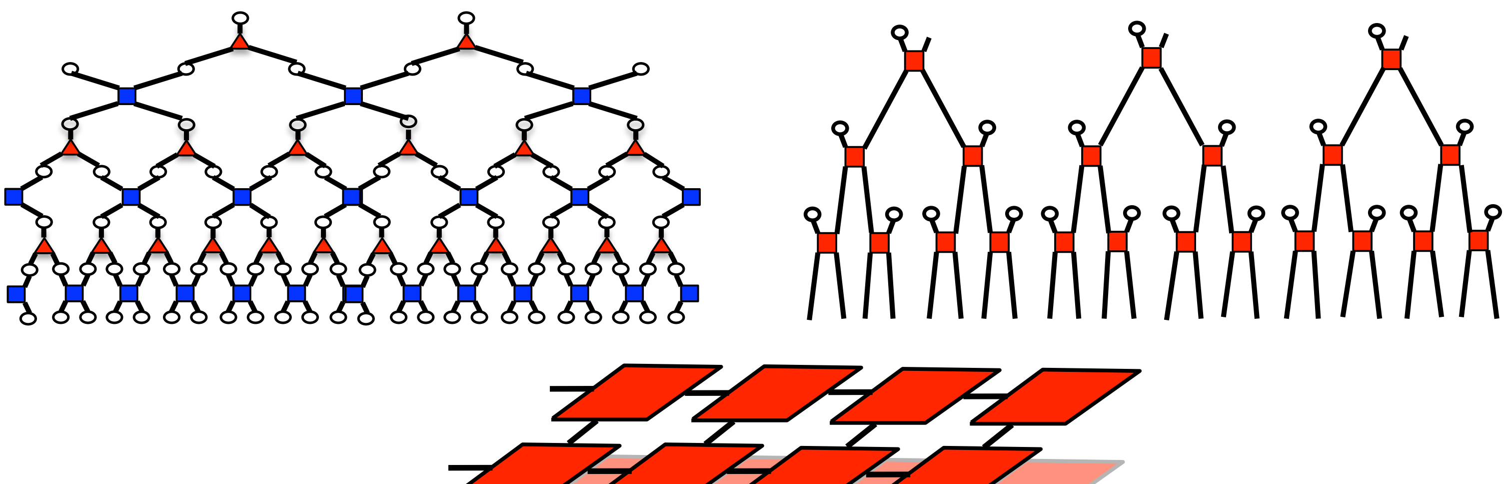
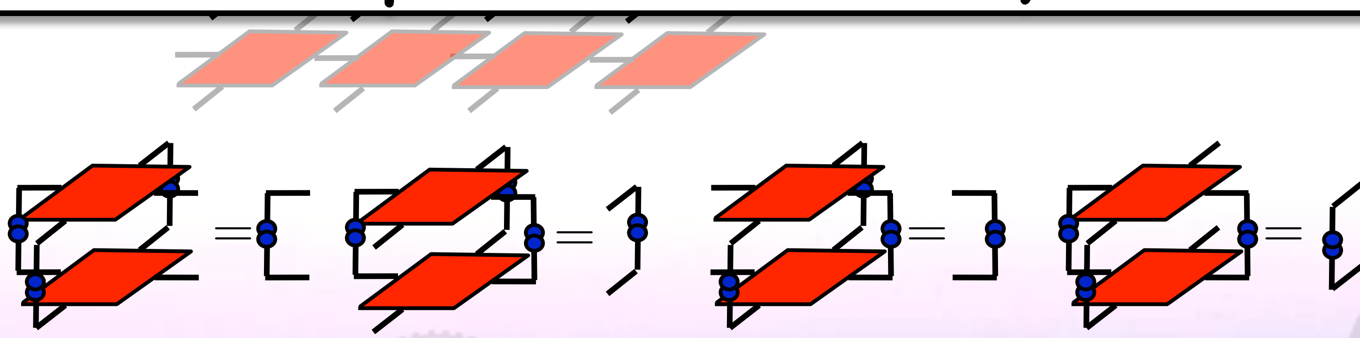


Tensor Networks harbour deep insights
Import to field theory



II Towards a Field Theory over Tensor Network States:

Goal and Key Steps

Formulating the Field Theory
Interpretation and Applications
Extensions to Higher Dimensions
and Critical Systems

Towards a Field Theory over MPS

Goal: Import insights from tensor networks into a Field theory over tensor network states

$$\mathcal{Z} = Tr\, e^{-\beta\mathcal{H}} = \int [DA] e^{-\mathcal{S}[A]}$$

MPS data

Advantages

- Saddle points=> TDVP (time-dependent variational principle)
- Fluctuations - expansion about saddle or increase bond order
- Field theory treatment of gauge freedoms in MPS?
- Extension to higher dimensions?
- Various potential applications…

Towards a Field Theory over MPS

Goal: Import insights from tensor networks into a Field theory over tensor network states

$$\mathcal{Z} = Tr\ e^{-\beta\mathcal{H}}$$

$$\mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}\ \mathbb{1}$$

$$\mathbb{1} = \int D\psi |\psi\rangle\langle\psi|$$

$$= \int D\psi e^{\int d\tau \left[\langle\psi|\partial_\tau \psi\rangle - \langle\psi|\hat{\mathcal{H}}|\psi\rangle\right]}$$

Key Steps

- Saddle points=> TDVP (time-dependent variational principle)
- Insert resolutions of identity over over-complete set
- Usually $|\psi\rangle$ product states
- Can we do the same with matrix product states?

Towards a Field Theory over MPS

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$$\mathbb{1} = \int DA |A\rangle\langle A|$$

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Key Steps

- Saddle po … rinciple)
- Insert res … t
- Usually $|\psi$ …
- Can we de …

Q. What is the Measure?

Q. What is the Berry Phase?

Q. Is the theory local?

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Gauge Fixing, Locality and General Parameterization

Local Field Theory => Gauge Fix to Canonical Form

- Canonical Equations

$$\sum_\sigma A^{\sigma\dagger}\Lambda_{n-1}A^\sigma = \Lambda_n \qquad \sum_\sigma A^\sigma A^{\sigma\dagger} = 1$$

- General Parameterization

$$A^\sigma = N^\sigma U^\sigma, \quad \underbrace{\sum_\sigma U^{\sigma\dagger}N^{\sigma\dagger}\Lambda_{n-1}N^\sigma U^\sigma = \Lambda_n}_{\text{Residual canonical equations}}$$

Diagonal matrix of spin coherent state spinors

SU(N)/DU(N)

Gauge Fixing, Locality and General Parameterization

Local Field Theory => Gauge Fix to Canonical Form

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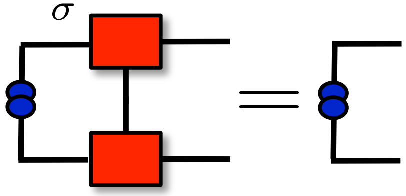

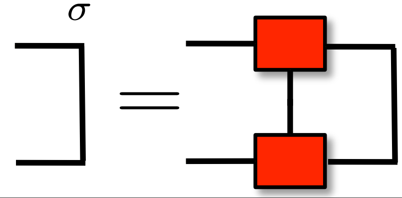

- **Bond Order 2 Parameterization**

$$\begin{aligned} N^\sigma &= \begin{pmatrix} n_1^\sigma & 0 \\ 0 & n_2^\sigma \end{pmatrix} \\ U^\sigma &= \cos\theta^\sigma/2 + i\boldsymbol{\tau}.\mathbf{u}\sin\theta^\sigma/2 \\ \mathbf{u} &= (\cos\phi, \sin\phi) \end{aligned}$$

$$\begin{aligned} d\Lambda_n &= \sum_\sigma \Gamma^\sigma \cos\theta_\sigma \\ 0 &= \sum_\sigma \Gamma^\sigma \sin\theta_\sigma \end{aligned}$$

$$\begin{aligned} \Lambda_n &= (\mathbb{1} + \tau_z d\Lambda_n)/2 \\ 2\Gamma_n &= [d\Lambda_{n-1}(1+\sigma\bar{n}^z) + \sigma\Delta n^z/2] \end{aligned}$$

Gauge Fixing, Locality and General Parameterization

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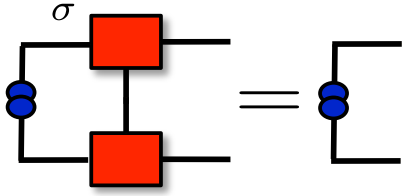
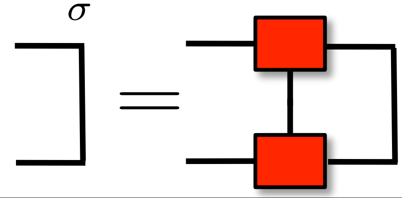




- General Bond Order – split into SU(2) subgroups

$$N^\sigma = \mathrm{diag}(n_1^\sigma, n_2^\sigma, \ldots)$$

$$U^\sigma = \prod_l e^{i\theta^\sigma \mathbf{v}_l^\sigma . \tau/2}$$

$$d\Lambda_n^i = \sum_\sigma \Gamma^{\sigma,j}\left(\overrightarrow{\prod_l} e^{\theta^\sigma \mathbf{v}_l^\sigma . \mathbf{f}}\right)_{j,i}$$

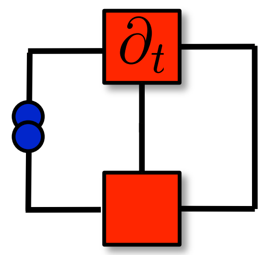
$$\Gamma^{\sigma,j} = Tr\left[\tau_j \Lambda_{n-1} N^{\sigma\dagger} N^\sigma\right] \qquad [\tau_i, \tau_j] = 2i f_{ijk}\tau_k \qquad \Lambda_n = \frac{1 + \sum'_{\beta_d} d\Lambda_n^{\beta_d}\tau_{\beta_d}}{2}$$

Berry Phase

- **Contribution from nth site in chain**

$$\langle\psi|\partial_t\psi\rangle_n = \sum_\sigma Tr\left[A_n^{\sigma\,\dagger}\Lambda_{n-1}\partial_t A_n^\sigma\right]$$

$$=$$



$$= \sum_\sigma Tr\left[U_n^{\sigma\,\dagger}N_n^{\sigma\,\dagger}\Lambda_{n-1}\partial_t\left(N_n^\sigma U_n^\sigma\right)\right]$$

$$= \sum_\sigma Tr\left[\Lambda_{n-1}N_n^{\sigma\,\dagger}\partial_t N_n^\sigma\right] + \sum_\sigma Tr\left[U_n^{\sigma\,\dagger}\left(\Lambda_{n-1}N_n^{\sigma\,\dagger}N_n^\sigma\right)\partial_t U_n^\sigma\right]$$

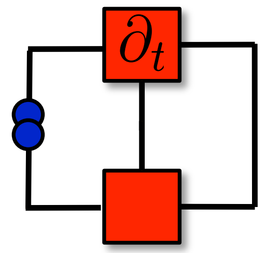
Towards a Field Theory over MPS

Berry Phase

- **Contribution from n^th site in chain**

$$\langle\psi|\partial_t\psi\rangle_n = \sum_\sigma Tr\left[A_n^{\sigma\dagger}\Lambda_{n-1}\partial_t A_n^\sigma\right]$$

$$=$$



Bond Order 2

$$= \sum_n \left[\sum_{\alpha=1}^{\chi}\Lambda_{n-1}^\alpha\langle\mathbf{n}^\alpha|\partial_t\mathbf{n}^\alpha\rangle + \partial_t\phi(d\Lambda_n - d\Lambda_{n-1})\right]$$

General

$$= \sum_n \left[\sum_{\alpha=1}^{\chi}\Lambda_{n-1}^\alpha\langle\mathbf{n}^\alpha|\partial_t\mathbf{n}^\alpha\rangle + \sum_{i=1}^{\chi(\chi-1)}\partial_t\phi_i u_\gamma^i(d\Lambda_n^\gamma - d\Lambda_{n-1}^\gamma)\right]$$

γ **Labels diag generators**
i **Labels SU(2) subgroups**

$$\Lambda_n = \frac{1}{2}\left(\mathbb{1} + \sum_{i=1}^{\chi-1}\tau_i d\Lambda_n^i\right) \qquad u_i^\gamma = \frac{1}{2}Tr[\tau_\gamma^z, \tau_i]$$

Gauge Fixing

- **Canonical constraints**

$$d\Lambda_n = \sum_\sigma \Gamma^\sigma \cos\theta_\sigma \ , \qquad 0 = \sum_\sigma \Gamma^\sigma \sin\theta_\sigma$$

Iteratively defines $d\Lambda_n$, (together with def'n of Γ^σ)

Fixes $\theta^\downarrow$, given $\theta^\uparrow$

$$d\Lambda_n \equiv d\Lambda_n(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\}) \quad \theta^\downarrow \equiv \theta^\downarrow(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\})$$

- **Gauge fixing**

$$\mathbb{1} = \int D\Lambda D\theta^\downarrow \delta[\theta^\downarrow - \theta^\downarrow(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\})]\delta[d\Lambda_n - d\Lambda_n(\{\mathbf{n}_1, \mathbf{n}_2, \theta^\uparrow\})]$$

$$D\mathbf{n}D\chi D\theta^\sigma D\psi \Longleftrightarrow D\mathbf{n}D\chi D\theta^\uparrow D\psi \Longleftrightarrow D\mathbf{n}D\chi D\Lambda D\psi$$

Interesting Special Cases

- **Maximally Entangled States** $\Lambda \propto \mathbb{1} \quad \Rightarrow \quad A^\sigma = N^\sigma U$

$$\langle\psi|\partial_t\psi\rangle = \frac{1}{\chi}\sum_n \sum_{\alpha=1}^{\chi} \langle \mathbf{n}^\alpha|\partial_t \mathbf{n}^\alpha\rangle$$

$$\langle\psi|\hat{\mathcal{H}}|\psi\rangle = \langle\frac{1}{2}\sum_n J\sigma_n.\sigma_{n+1}\rangle = \frac{1}{2}\sum_n J|U_{\alpha,\beta}|^2 \mathbf{n}_n^\alpha \mathbf{n}_{n+1}^\beta$$

– **Spatially Uniform**

$$\langle\psi|\partial_t\psi\rangle \quad = \quad \sum_n \sum_{\alpha=1}^{\chi} \Lambda^\alpha \langle \mathbf{n}^\alpha|\partial_t \mathbf{n}^\alpha\rangle$$

- In both Cases, Effectively χ replicas of system
- glued together with $SU(\chi)$ field
- No intrinsic dynamics for U
  - Behaves as fancy Lagrange multiplier (maximally entangled)
  - Inherited through those of N^σ (spatially uniform case)

LCN UCL

II Towards a Field Theory over Tensor Network States:

Goal and Key Steps

Formulating the Field Theory

Interpretation and Applications

Extensions to Higher Dimensions and Critical Systems

Towards a Field Theory over MPS

Instantons in MPS Field Theory

- Disconnected configurations for product states
- Smooth field/tensor for MPS

- Instantons at χ_0 -> semi-classical configurations at $\chi > \chi_0$

Two Ways to Include Q Fluctuations in Field Theory

- i. Expand about semi-classical saddle point
- ii. Increase bond-order of field integral.

- Complementary
  - May use simultaneously
  - Accommodate different effects
  - If dominant effects captured by ii. + scaling => AdS?

Towards a Field Theory over MPS

Instantons in MPS Field Theory

- Disconnected configurations for product states
- Smooth field/tensor for MPS

- Instantons at χ_0 -> semi-classical configurations at $\chi > \chi_0$

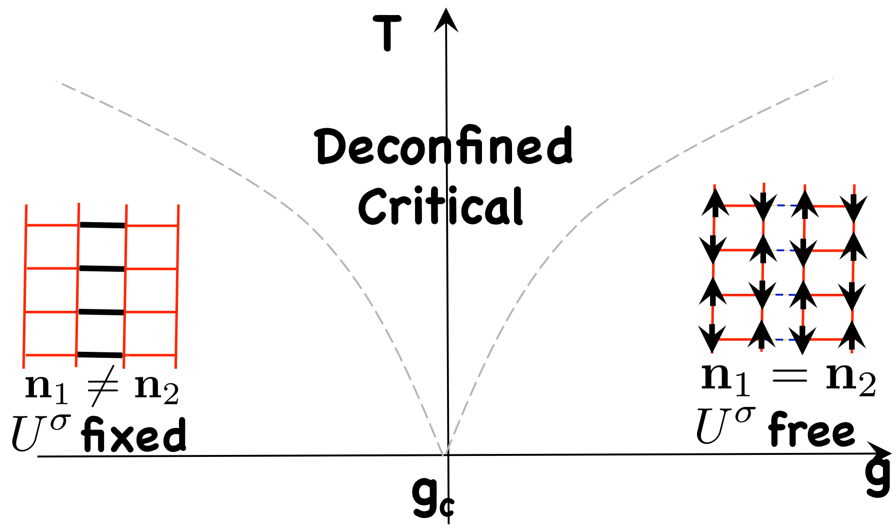
Two Ways to Include Q Fluctuations in Field Theory

- i. Expand about semi-classical saddle point
- ii. Increase bond-order of field integral.

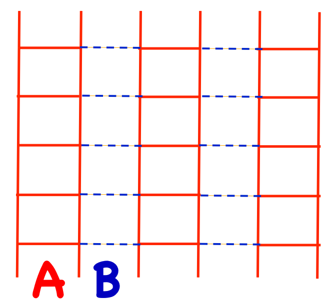
Q. Is holography an ansatz for the entanglement structure of the dominant saddle point?

Towards a Field Theory over MPS

Deconfined Criticality



$$\mathcal{H} = g \sum_{\langle ij \rangle}^{A} J \hat{\sigma}_i . \hat{\sigma}_j + \sum_{\langle ij \rangle}^{B} J \hat{\sigma}_i . \hat{\sigma}_j$$



- [Senthil et al, Science 303, 1490 (2004)]
- Critical theory not described by order parameter fluctuations
- Gauge fields/Lagrange multipliers determine critical behaviour
- MPS states may characterize both sides at low bond order
- Certain MPS degrees of freedom soften at transition
- => MPS field theory dual to gauge theory?
- => Ginzburg-Landau description of transition in terms of MPS fields

Towards a Field Theory over MPS

Potential Applications

Extended Truncated Wigner Approximation

- Propagates density matrix using saddle-point approx to Keldysh

$$\hat{\rho}(t) = \int D\psi_+\psi_-\hat{\rho}(0)e^{-i\mathcal{S}_+ + i\mathcal{S}_-}$$

- All entanglement contained in $\hat{\rho}(0) = \sum_\alpha \lambda_\alpha |\psi_\alpha\rangle\langle\psi_\alpha|$
- Propagate $|\psi_\alpha\rangle$ independently
- Perturbative corrections increase Schmidt rank [Polkovnikov PRA68, 053604 (2003)]

Alternative

- Propagate with saddle points of MPS Keldysh
  i. Decompose $\hat{\rho}(0)$ over product states – propagate with MPS saddle point
  =>Allows entanglement to grow to some limit
  ii. Decompose $\hat{\rho}(0)$ over MPS – propagate with MPS saddle point
  =>Allows restructuring of entanglement

Q. Late time hydrodynamics of eigenstate thermalization?

Potential Applications

Fluctuation corrections to MPS time evolution

- Two ways to include fluctuations about saddle point
- Re-summing the effects of fluctuations (usual field theory)
- Increasing bond order
- Re-summing fluctuations may improve low bond-order saddle point and fidelity of time evolution over low bond order states

Q. Fluctuation Corrections to MPS?

Many body localization

- Product state field theory – saddle points describe low energy
- Many body localization is a dynamical phase transition through spectrum
- MPS field theory may permit description of dynamics higher in the spectrum.

Q. Field Theory of mid-spectrum states?

II Towards a Field Theory over Tensor Network States:

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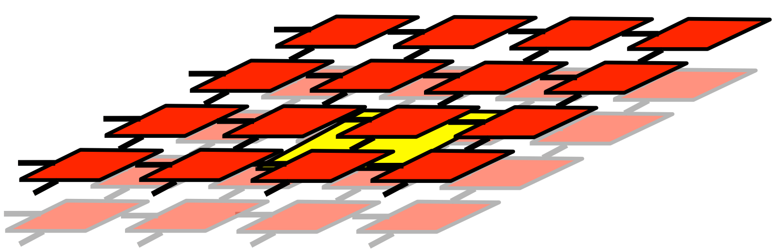
Extension to Higher Dimensions

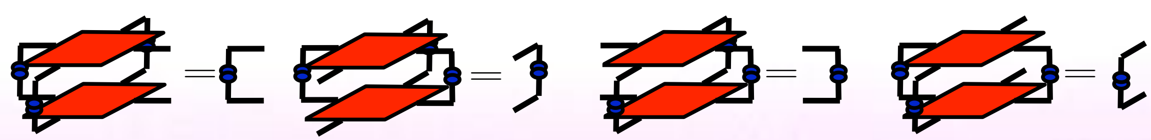
Fundamental difficulty…

- No canonical form in higher dimensions
- Action not local in terms of A
- Decay of entanglement suggests approximately true
- Quasi canonical form
- Connection to multi-band Wannier functions
- [Marzari et al RevModPhys84, 1419 (2012)]?

$$\langle \psi | \hat{\theta} | \psi \rangle =$$



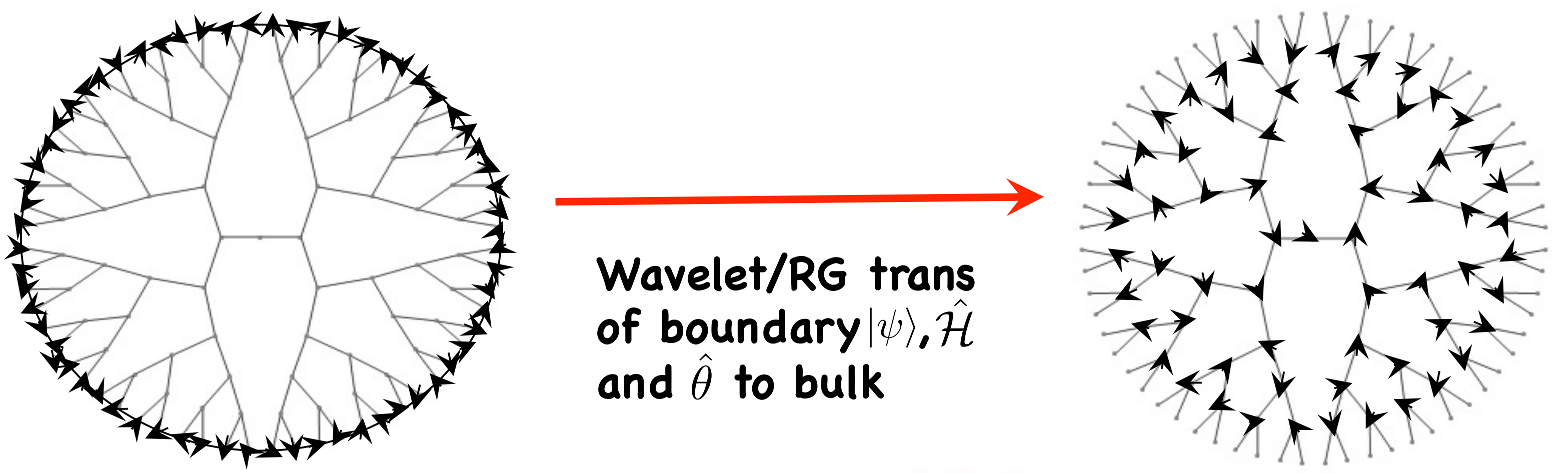
Quasi-Canonical Form:



Towards a Field Theory over MPS

Extensions to Critical Systems

- Various RG Schemes
- MERA (multiscale entanglement renormalization ansatz) [Vidal, PRL99, 220405(2007)]
- TRG (tensor RG) [Verstrate et al, Adv. Phys 57, 143 (2008)]
- SRG (second RG) [Xie et al PRL103, 16069 (2009)]
- HOTRG (higher order TRG) [Xie et al PRB86, 045139 (2012)]
- Exact Holographic Mapping [Xiao-Liang Qi[ArXiv:1309.6282]



- Relation to AdS/CFT [Swingle, Phys. Rev. D 86, 065007 (2012),ArXiv:1209.3304]
- Applying RG to field theory over MPS ->[S.-S. Lee,NPB 832,56 (2010); 851,143(2011)]

Conclusions

So Far

- Field Theory over Tensor Network States

MPS data

$$\mathcal{Z} = Tr\, e^{-\beta \mathcal{H}} = \int [DA] e^{-\mathcal{S}[A]}$$

- Imports insights about entanglement to field theory
- Managed for 1d – exploring potential applications

Future

- RG-> describing both renormalization of fluctuations and entanglement structure?
- Extension to higher dimensions and critical theories?
- Links to AdS/CFT?