Holography and condensed matter experiments: a status report.

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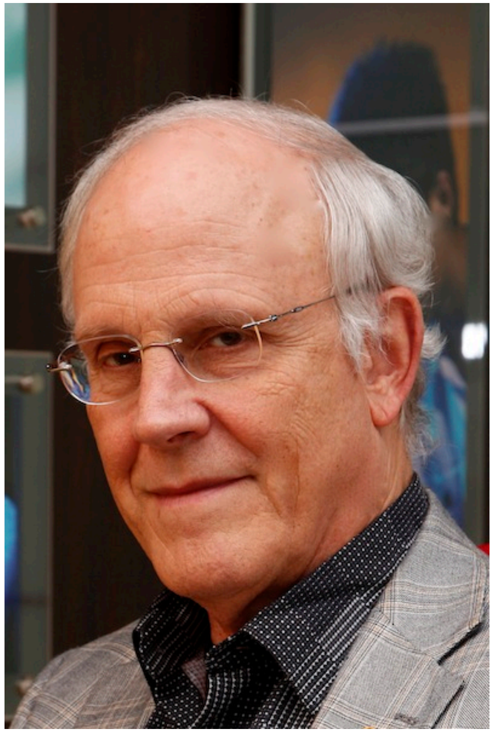
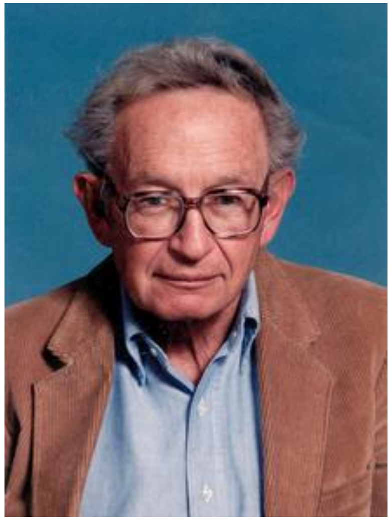
My dear friend David …



Jan, AdS/CMT needs a killer App !

Condensed Matter theory: the first law.



Phil Anderson: "The only meaning of theory is *to demonstrate possibility*. It is *about learning to think differently, be it on basis of flawed solutions to wrong problems*."
Case in point: "Anderson-Morel" for p-wave superconductivity.

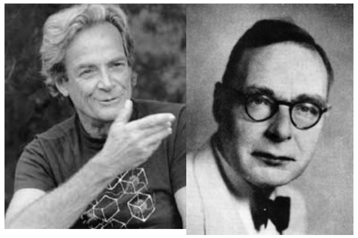
Quantum field theory = Statistical physics.

$$Z = \sum_{configs.} e^{-\frac{E_{config}}{k_B T}}$$

Path integral mapping

"Thermal QFT", Wick rotate:

$$t \rightarrow i\tau$$



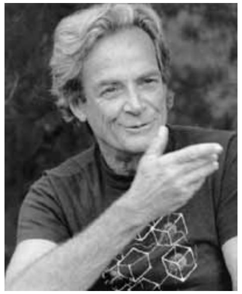
$$Z_\hbar = \sum_{worldhistories} e^{-\frac{S_{history}}{\hbar}}$$

But generically: the quantum partition function is not probabilistic: "sign problem", no EQUATIONS that work!

$$Z_\hbar = \sum_{worldhistories} (-1)^{history} e^{-\frac{S_{history}}{\hbar}}$$

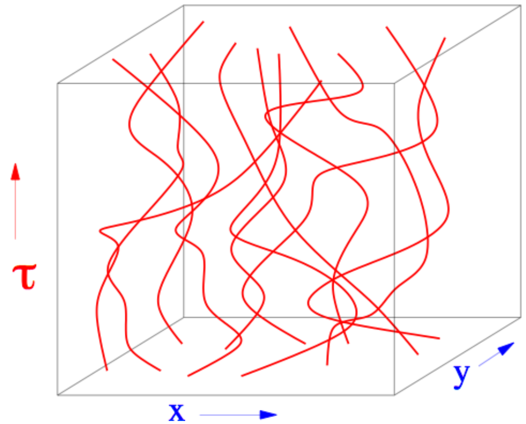
Fermions at a finite density: the sign problem.

Imaginary time first quantized path-integral formulation



$$\mathcal{Z} = \mathrm{Tr}\exp(-\beta\hat{\mathcal{H}}) = \int \mathrm{d}\mathbf{R}\rho(\mathbf{R},\mathbf{R};\beta)$$

$$\mathbf{R} = (\mathbf{r}_1,\ldots,\mathbf{r}_N) \in \mathbb{R}^{Nd}$$



$$\rho_{B/F}(\mathbf{R},\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\rho_D(\mathbf{R},\mathcal{P}\mathbf{R};\beta) = \frac{1}{N!}\sum_{\mathcal{P}}(\pm 1)^{\mathcal{P}}\int_{\mathbf{R}\to\mathcal{P}\mathbf{R}}\mathcal{D}\mathbf{R}(\tau)\exp\left\{-\frac{1}{\hbar}\int_0^{\hbar/T}\mathrm{d}\tau\left(\frac{m}{2}\dot{\mathbf{R}}^2(\tau)+V(\mathbf{R}(\tau))\right)\right\}$$

Boltzmannons or Bosons:

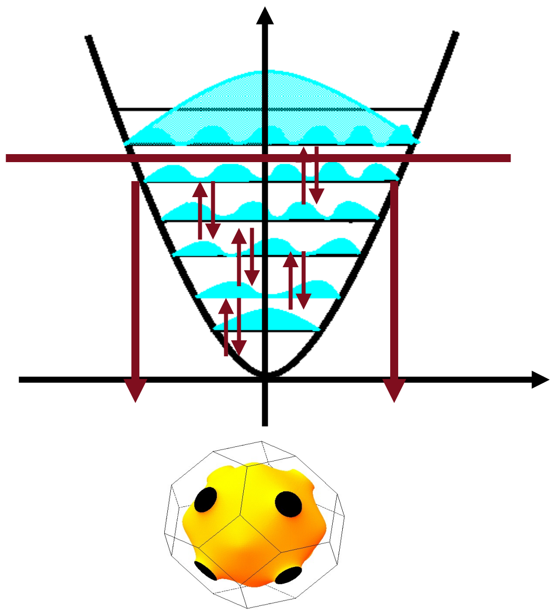
- integrand non-negative
- probability of equivalent classical system: (crosslinked) ringpolymers

Fermions:

- negative Boltzmann weights
- non probablistic: NP-hard problem (Troyer, Wiese)!!!

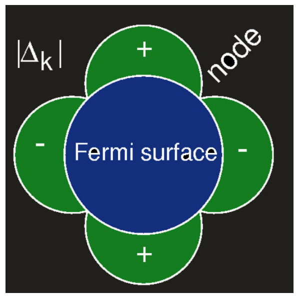
Fermions: the tiny repertoire …

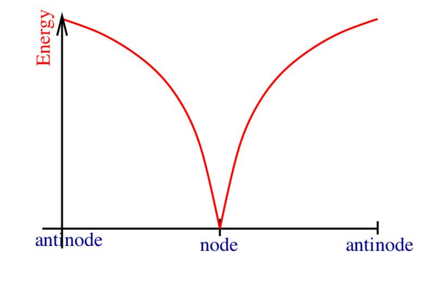
Fermiology



BCS superconductivity

$$\Psi_{BCS} = \Pi_k \left(u_k + v_k c^+_{k\uparrow} c^+_{-k\downarrow} \right) |vac.\rangle$$



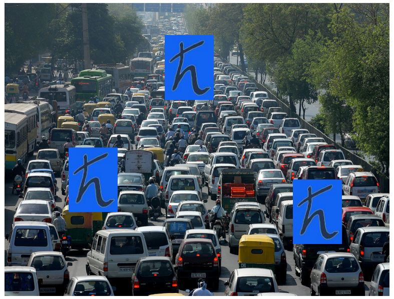


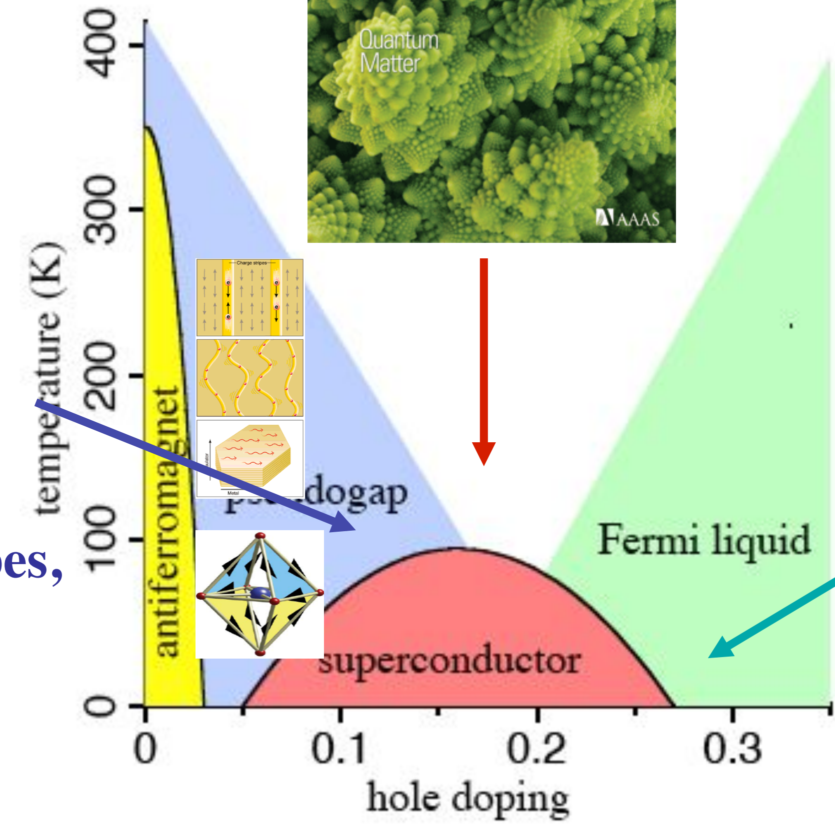
Phase diagram high Tc superconductors

The clash: the quantum critical metal

… which is good for superconductivity!

The quantized traffic jam

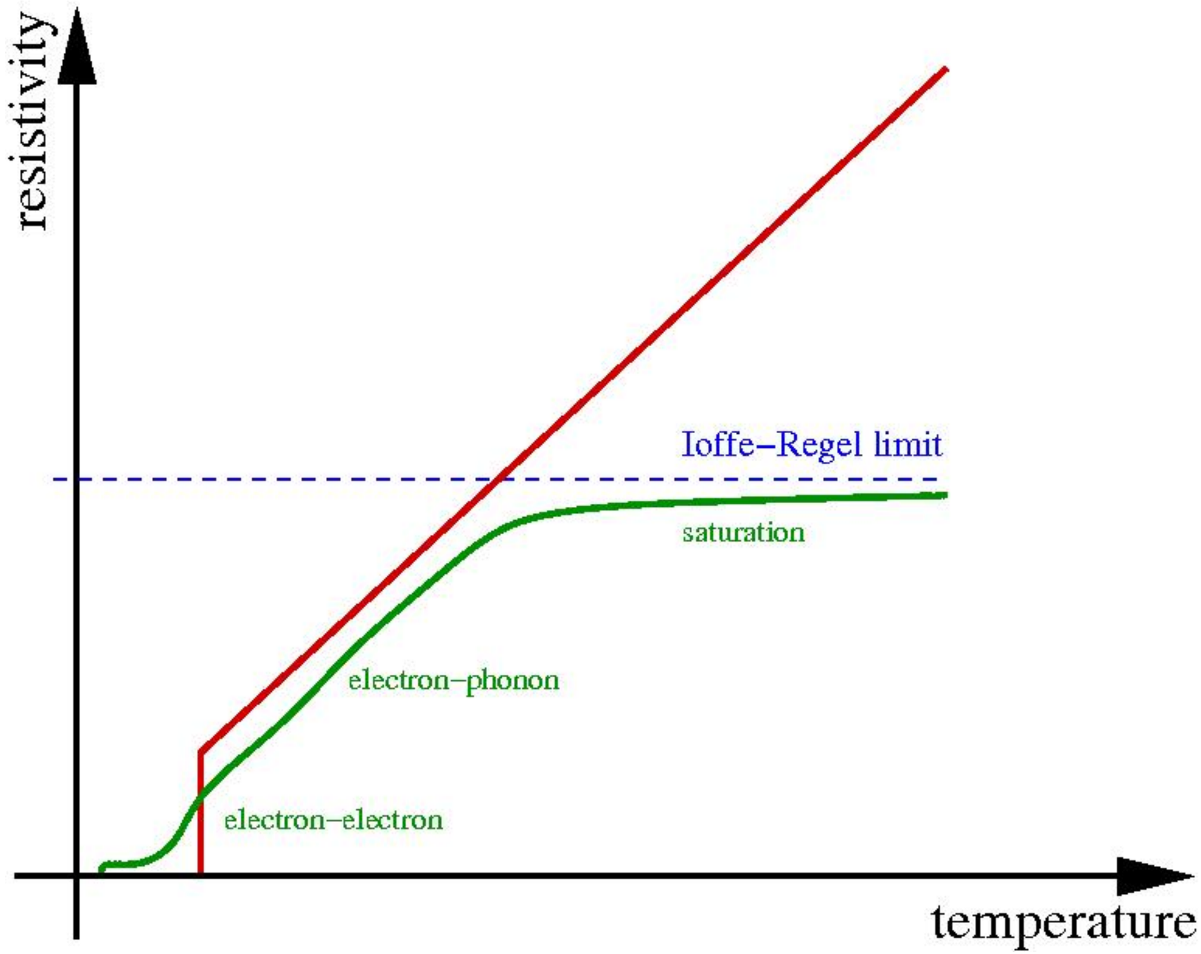




The quantum fog (Fermi gas) returns

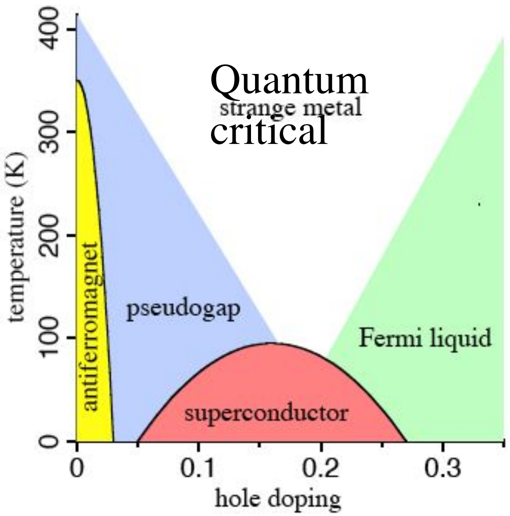
Exotic orders: stripes, orbital currents, nematics …

Divine resistivity

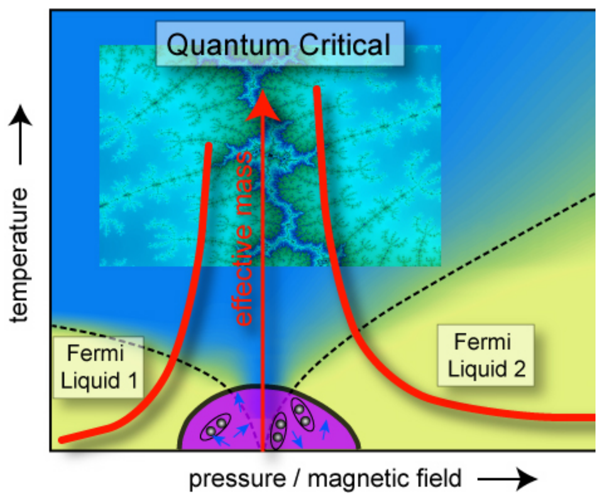


A universal phase diagram

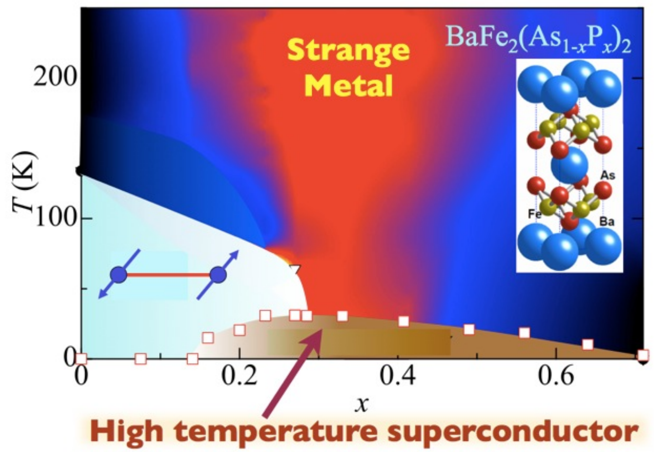
High Tc superconductors



Heavy fermions

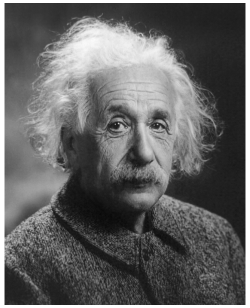


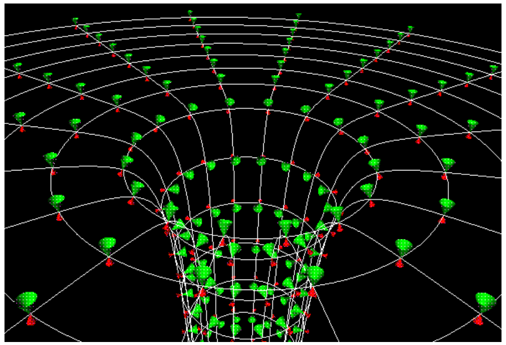
Iron superconductors (?)



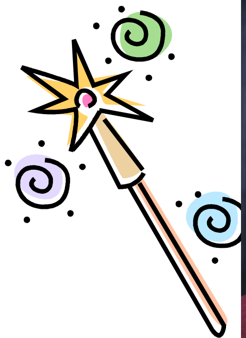
General relativity "=" quantum field theory

General relativity





'AdS/CFT'



Maldacena 1997

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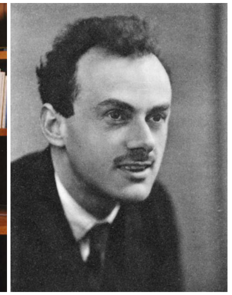
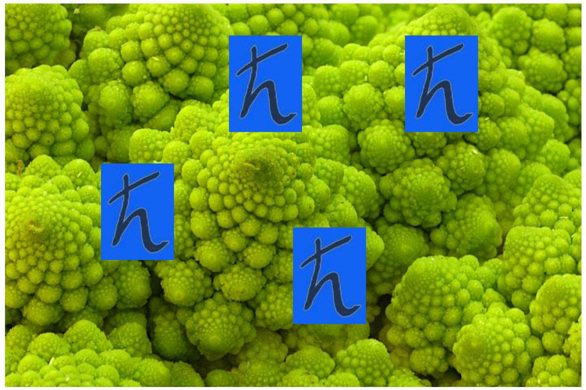
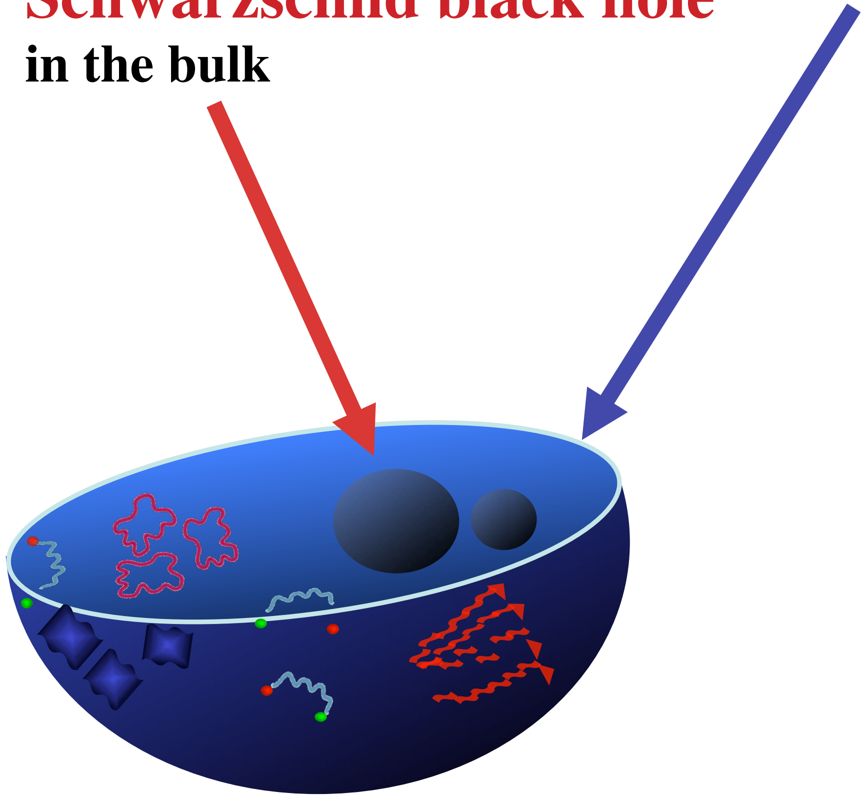
Quantum fields





The triumph: gravitational encoding of all thermal physics!

Schwarzschild black hole
in the bulk



Boundary: the emergence theories of finite temperature matter.

- **All of thermodynamics! Caveat: phase transitions are mean field (large N limit).**

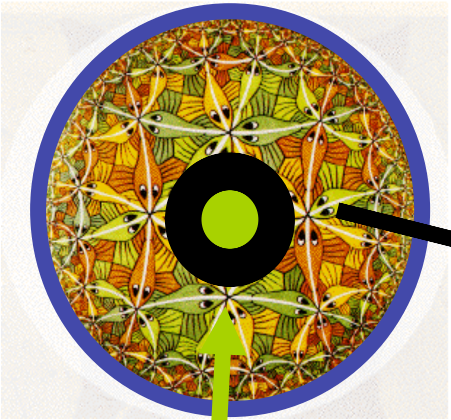
- **Precise encoding of Navier-Stokes hydrodynamics! Right now used to debug complicated hydrodynamics (e.g. superfluids).**

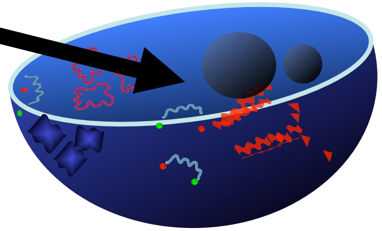
- **For special "Planckian dissipation" values of parameters (quantum criticality):**

$$\tau_{\hbar} = const.\frac{\hbar}{k_B T}, \quad const. = O(1)$$

The charged black hole encoding for finite density (2008 - ????)

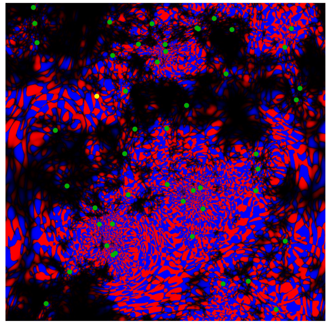
Anti de Sitter universe.



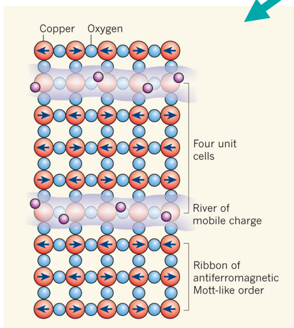


Charged black hole in the middle

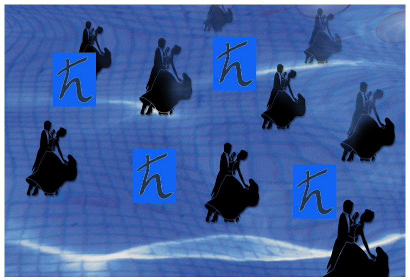
Finite density quantum matter:



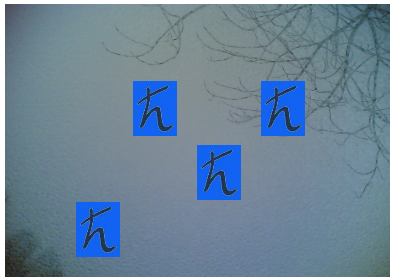
Holographic strange metals



Stripy pseudogap orders



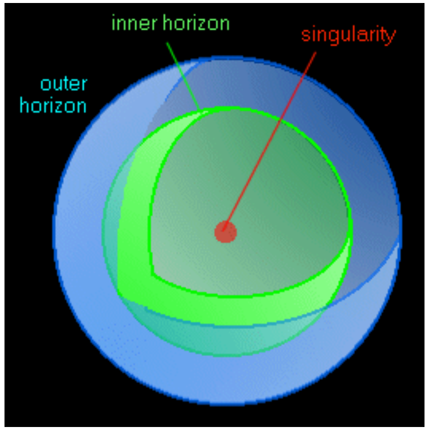
High Tc superconductors



Emergent Fermi liquids

The condensed matter text book: “stars” in the gravitation dual .

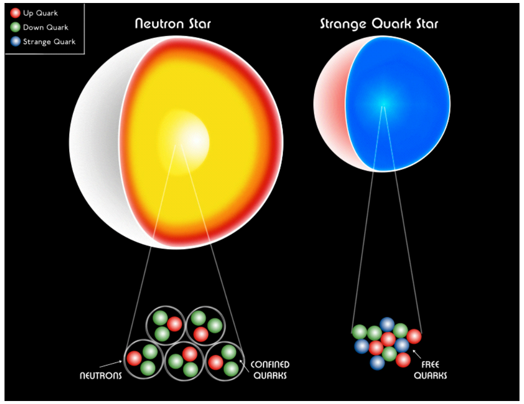
(Reissner-Nordstrom)
“Black hole like object”



“uncollapse”

Phase transition

“star like object”



“fractionalized”, “unstable”:
strange metal

“Cohesive state”:

Symmetry breaking:
superconductor, crystal
(“scalar hair”)

Fermi-liquid (“electron star”)

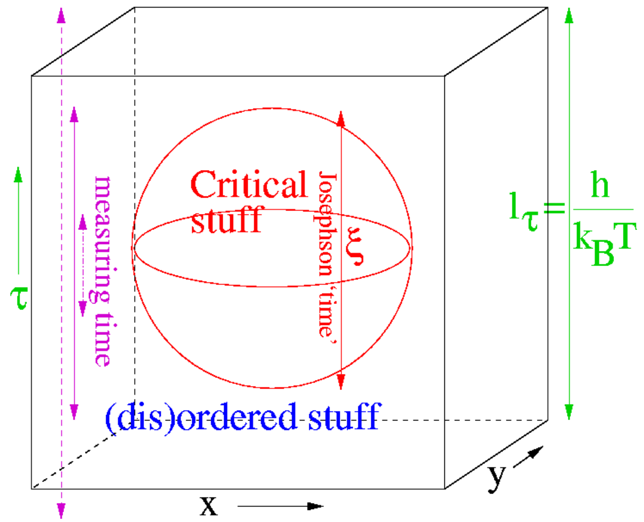

UV INDEPENDENCE

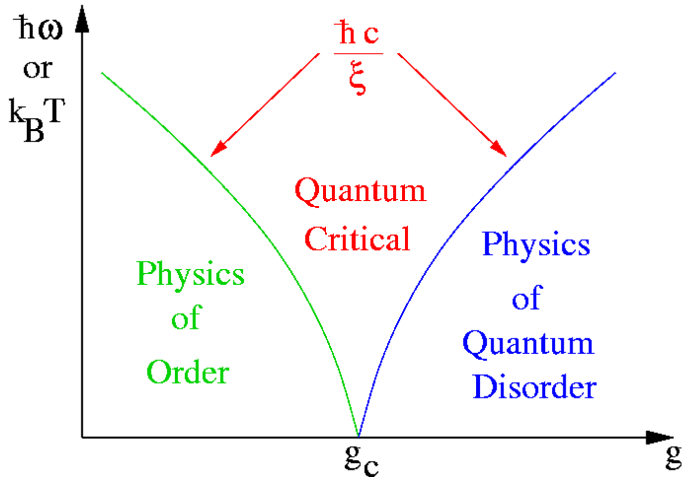
MATRIX LARGE N

Quantum criticality.

Sachdev's book "quantum phase transitions"

Scale invariance of the quantum dynamics (in space and time) is dynamically generated, as emergent phenomenon.



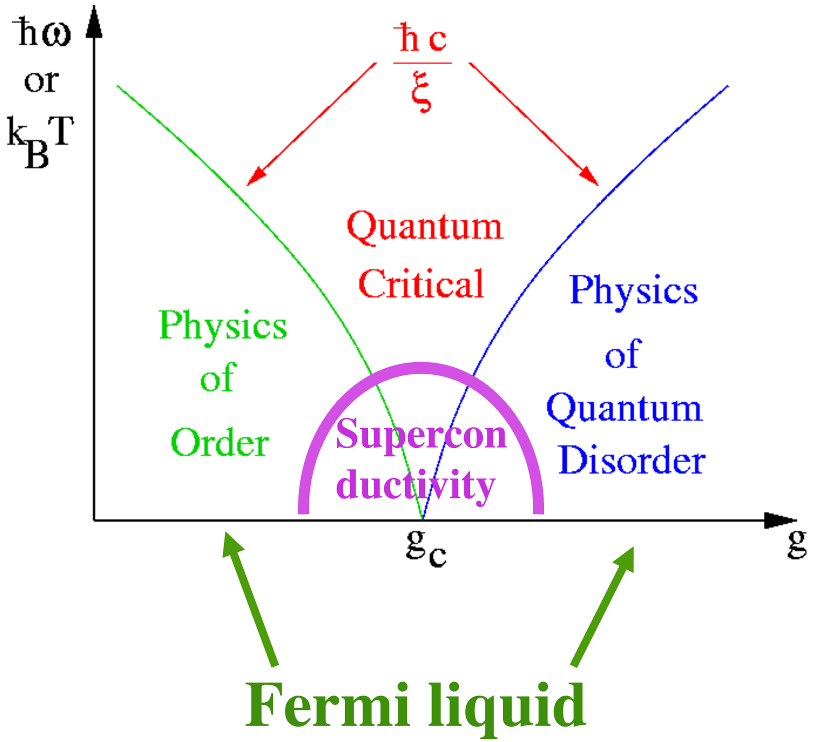


In the higher dimensional (bosonic) quantum field theories which are understood this only happens at isolated points in coupling constant space.

Hertz-Millis metallic "Quantum Critical Point"

Assertion: in the "UV" a *Fermi-liquid* is formed co-existing with an electronic *order parameter* (e.g. magnet) interacting via a Yukawa coupling.

The *order parameter* is subjected to *a bosonic quantum phase transition*: always isolated unstable fixed point (stat phys rule book).



Electrons: *Fermi-gas* = heat bath *damping bosonic critical fluctuations*.

Lingering singularities of QP *on the Fermi surface* due to critical bosons.

Likely resolved due to critical fluctuations acting as pairing glue: the "superconducting domes".

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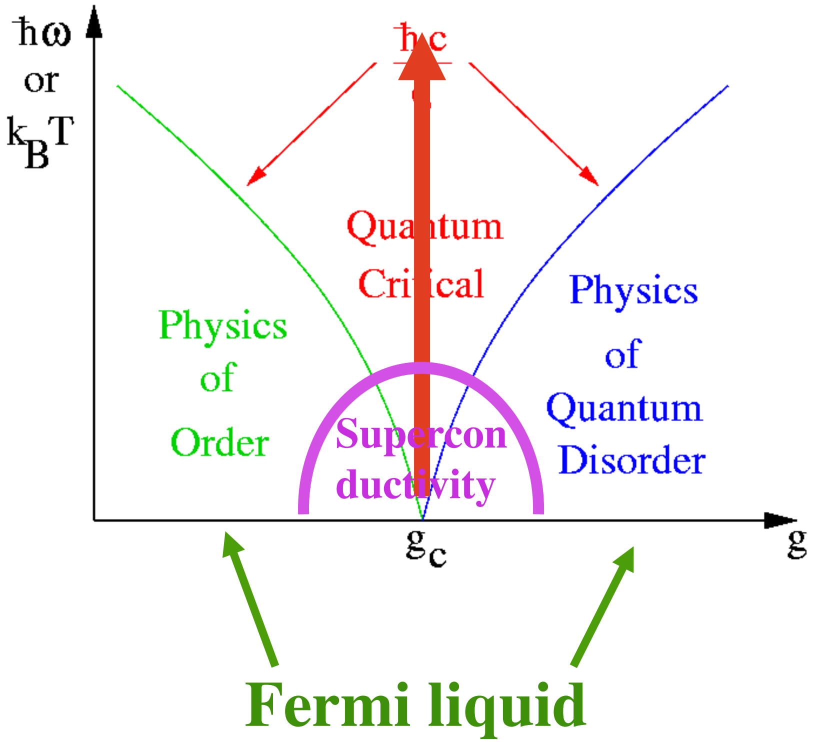

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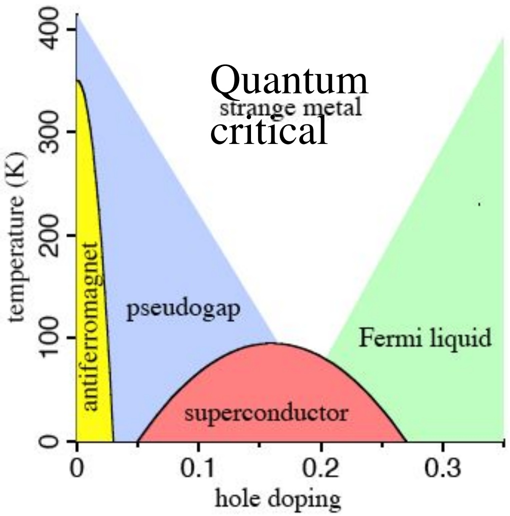
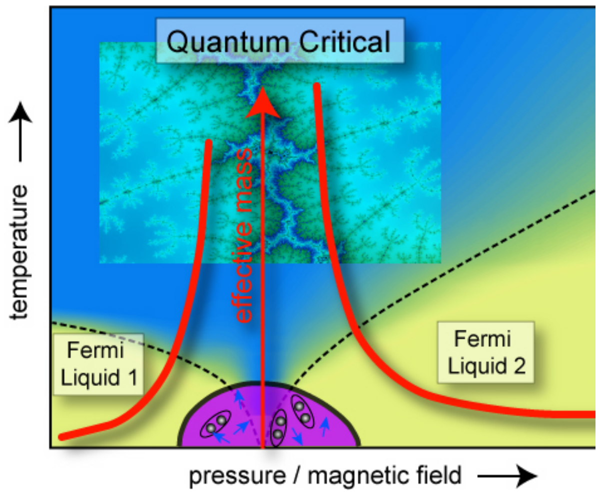
Metal is *perturbatively* shaken from *the below* (critical fluctuations in the IR).

A universal phase diagram

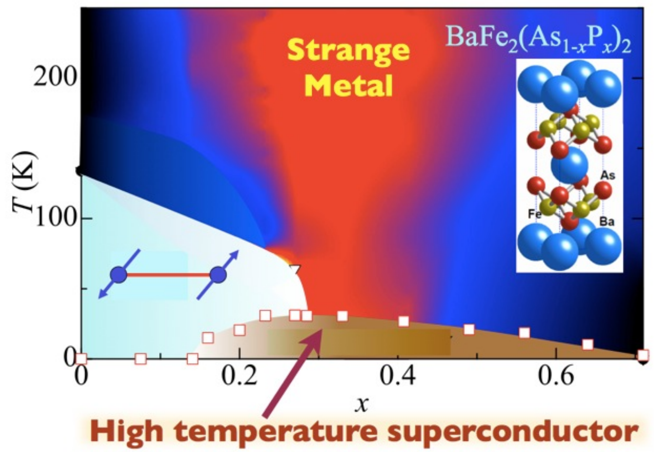
High Tc superconductors



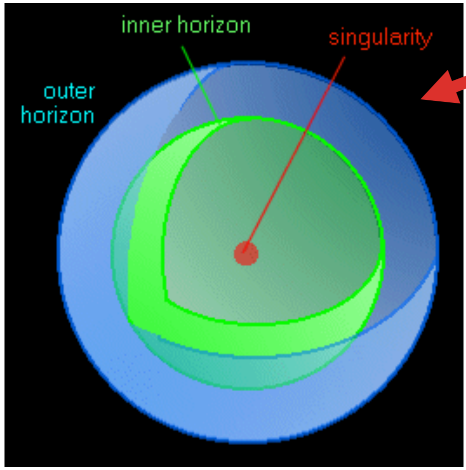
Heavy fermions



Iron superconductors (?)



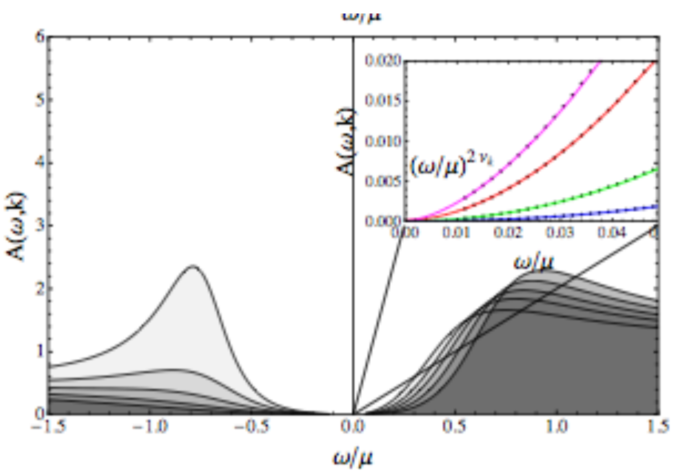
Finite density: the Reissner-Nordstrom strange metals (Liu et al.).



Near-horizon geometry of the extremal RN black hole:

- Space directions: flat, codes for simple Galilean invariance in the boundary.

- Time-radial(=scaling) direction: emergent AdS_2, codes for emergent temporal scale invariance!



Fermion spectral functions:

$$A(k,\omega) \propto G''_{AdS_2}(k,\omega) \propto \omega^{2\nu_k}$$

$$\nu_k = \frac{1}{\sqrt{6}}\sqrt{k^2 + \frac{1}{\xi^2}}$$

"Un-particle physics!"

"Scaling atlas" of holographic quantum critical phases.

Deep interior geometry sets the scaling behavior in the emergent deep infrared of the field theory. Uniqueness of GR solutions:

1. "Cap-off geometry" = confinement: conventional superconductors, Fermi liquids ….

2. Geometry survives: "hyperscaling violating geometries" (Einstein – Maxwell- Dilaton – Scalar fields –Fermions).

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\vartheta/(d-\theta)} dr^2 + dx_i^2 \right)$$

$$x_i \to \zeta x_i, \quad t \to \zeta^z t, \quad ds \to \zeta^{\theta/d} ds$$

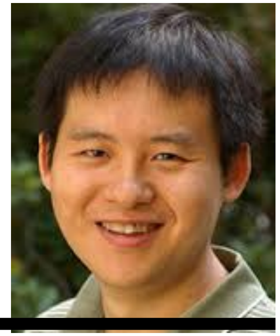
$$S \propto T^{(d-\theta)/z}$$

Quantum critical phases with unusual values of:

$z =$ **Dynamical critical exponent**

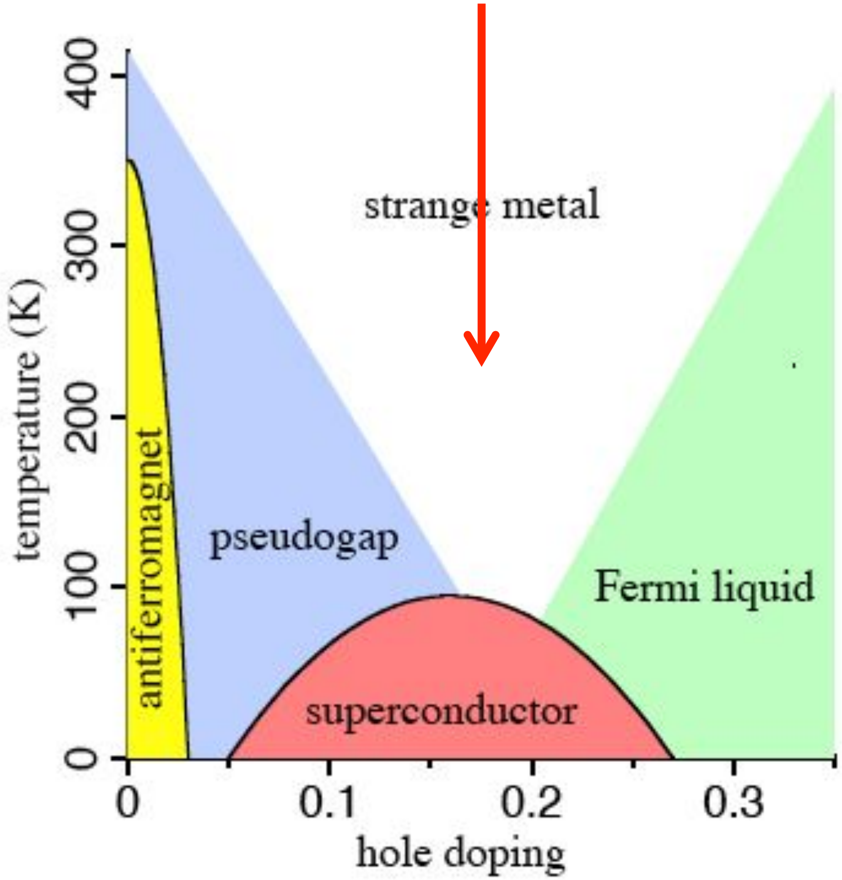
$\theta =$ **Hyperscaling violation exponent**

The unstable conformal metal of finite density holography.



Hong Liu

Conformal metal: *quantum critical* fermionic *phase.*

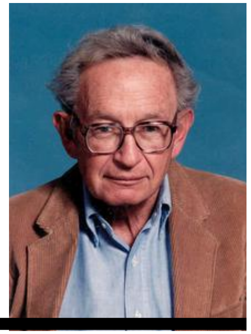


Characterised by *non-bosonic scaling properties*: large (infinite) z, hyperscaling violation, …

Intrinsically *very unstable*: "mother" of Fermi-liquids, superconductors, stripes, CDW's, loop currents, electronic nematics, …

The finite temperature state: governed by *Planckian dissipation*.

Intermediate temperature phase.



In recent years a fashion has grown up to ascribe great importance to "quantum critical points" at $T = 0$, at the boundary between the basins of attraction to the stable fixed points of ordered ground states. I argue that more physical significance in connecting microscopic interactions with observed phenomena lies in the common phenomenon of partially ordered "liquid" states at higher temperatures, unstable phases which define the relevant degrees of freedom and may order in many different ways as the temperature is further lowered.

Key Words: fixed point, liquid phases, partial ordering

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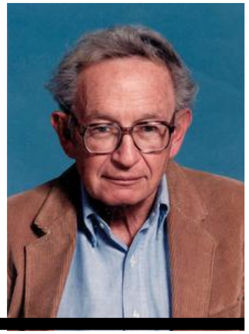
Intermediate temperature phase.



In our field of strongly-correlated electronic phenomena, the first fad of the 21^{st} century is the Quantum Critical Point.[1] (See Fig. 1.) This is defined as a point (along a line representing different values of some control parameter) where two ground states with different symmetries and different order parameters meet. The argument is that in the

In favour of the conformal metal ...

The resistivity is linear up to the melting point of the crystal: how can a puny deep IR critical boson get this done?

There appears to be a plethora of competing orders to come to an end simultaneously at optimal doping (stripes, CDW, loop currents, nematicity ..,)

At least in the "antinodal regime" quasiparticles are completely obliterated: the "incoherent backgrounds" look like critical branch-cuts …

Theoretical reasoning: it is impossible to form a UV Fermi-liquid in the presence of Mottness (=> Anderson).

But where is the smoking gun evidence after 30 years ??

Decoding the strange metals …

Challenges for the experimentalists:

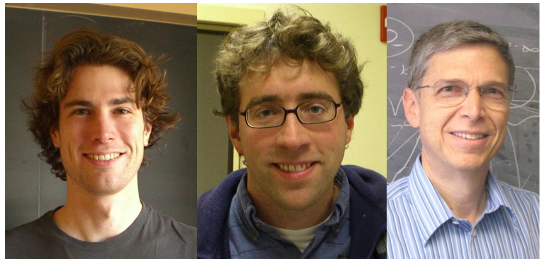
1. Measuring dynamical susceptibilities = *scaling properties* of propagators: the Goldman-Scalapino-Ferrell device for pairing.

2. Is the strange metal a "Planckian dissipator"? Linear resistivity versus the skin effect.

The Challenge for the holographists:

Pseudo-gap regime: *competing orders versus the fermion propagators*.

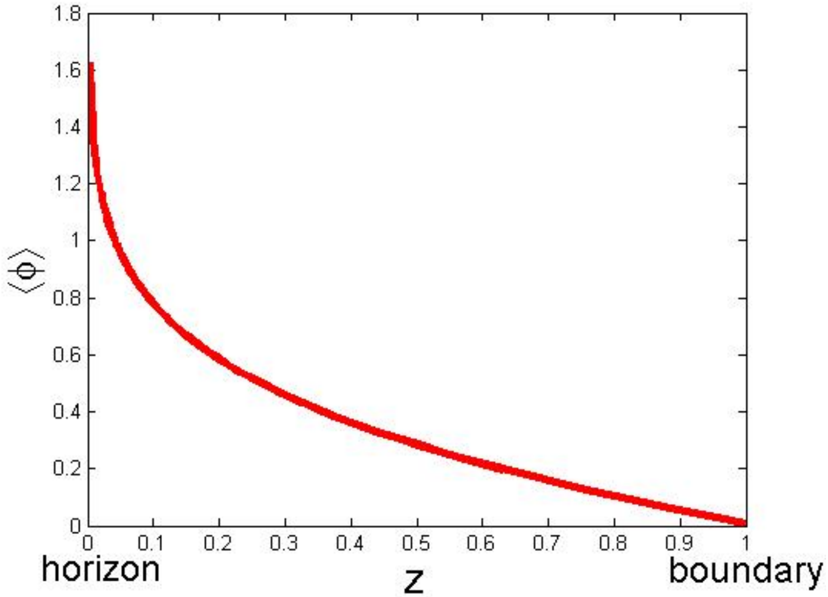
The Bose-Einstein Black hole hair

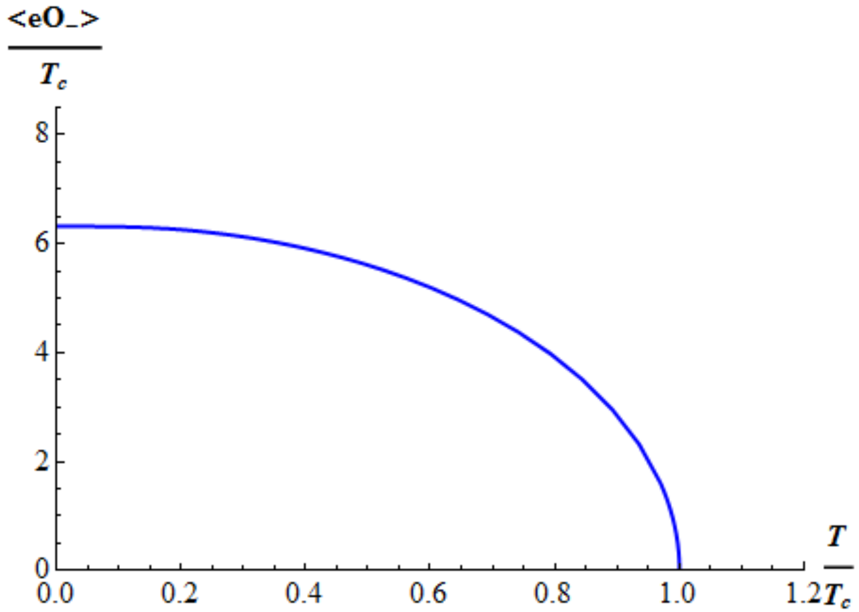


Hartnoll Herzog Horowitz

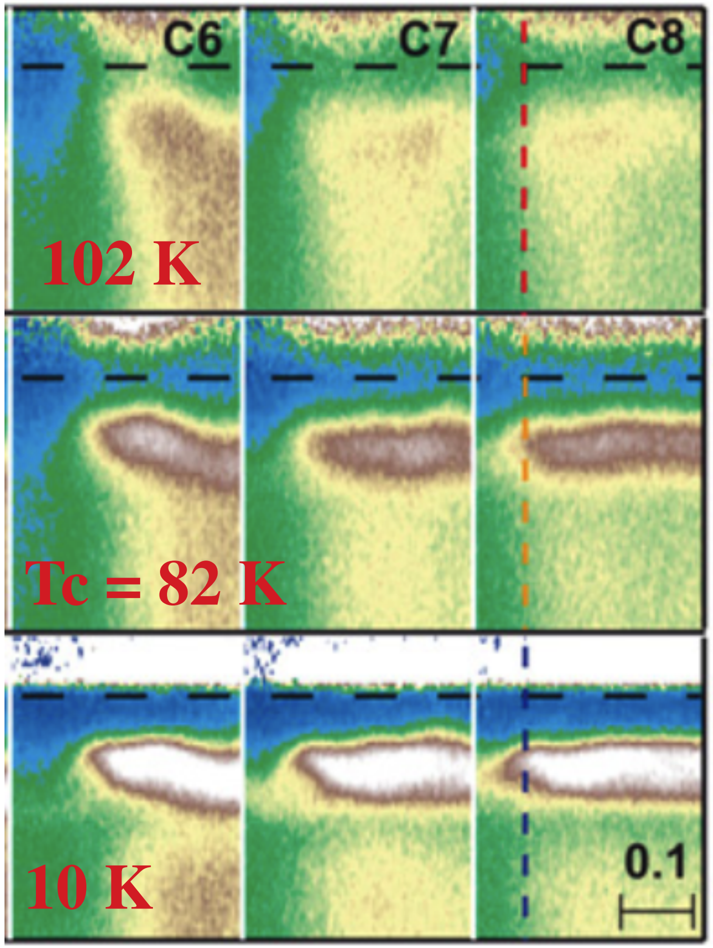
Scalar hair accumulates at the horizon

Mean field thermal transition.

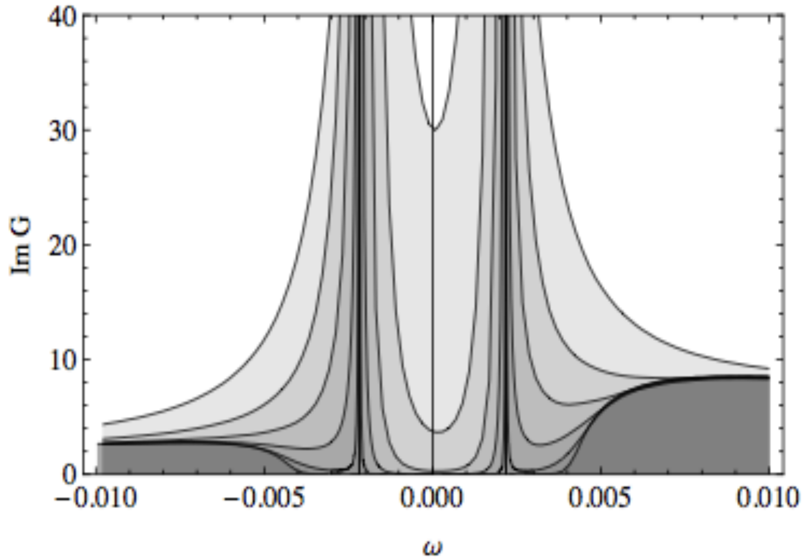




‘Pseudogap’ fermions in high Tc superconductors



Shen group, Nature 450, 81 (2007)

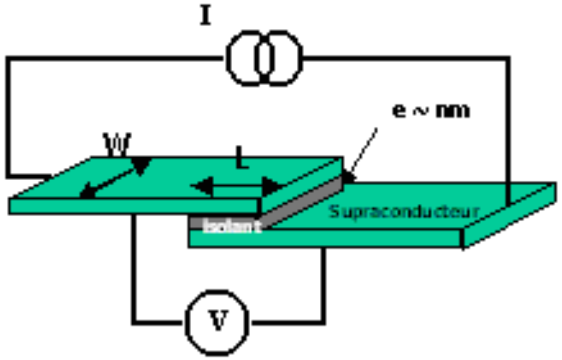


Gap stays open above Tc

But sharp quasiparticles disappear in incoherent ‘spectral smears’ in the metal

Observing holographic superconductivity

J.-H. She

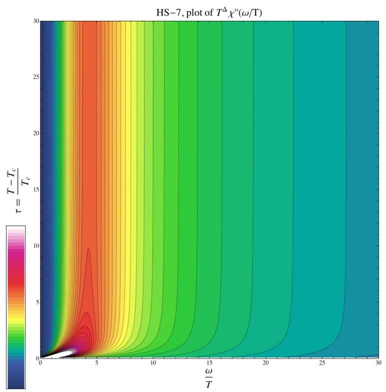


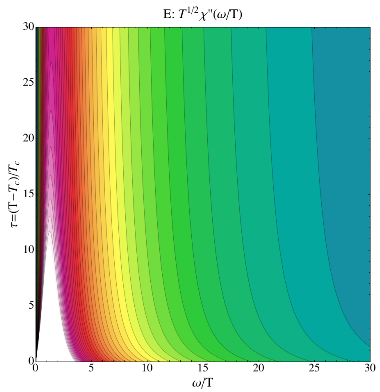
Measure the dynamical pair susceptibility in the strange metal using the "Goldman" Josephson junction (PRB 84, 144527).

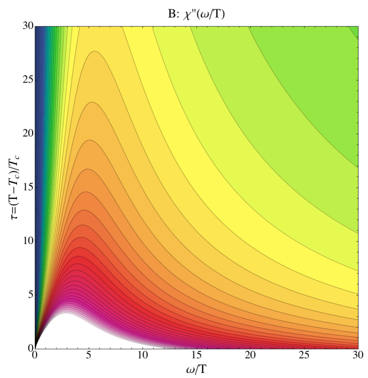
$T - T_c$

Holographic superconductors

Conventional ("critical glue")







$$T^{\Delta}\chi_p''\left(\frac{\hbar\omega}{k_B T}\right)$$

$$\hbar\omega/(k_B T)$$

Observing the pairing mechanism …

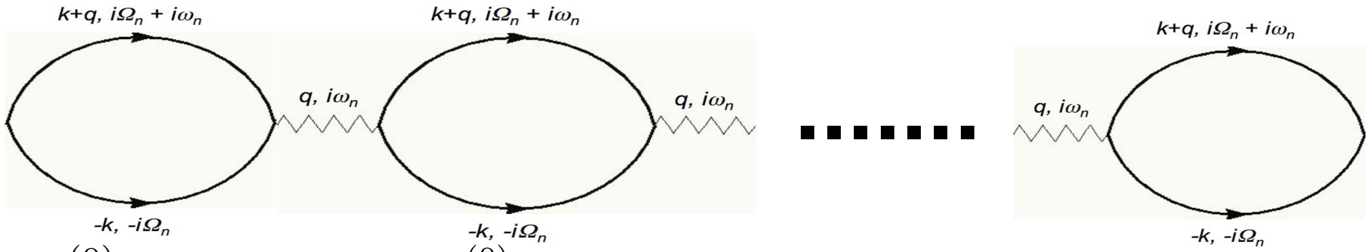
J.-H. She et al, Phys. Rev. B 84, 144527 (2011)

Claim: the maximal knowledge on the pairing mechanism is encoded in the temperature evolution of the normal state dynamical pair susceptibility:

$$\chi_p\left(q,\omega\right) = -i\int_0^{\infty} dt\, e^{i\omega t - 0^+ t}\left\langle\left[b^+(q,0), b(q,t)\right]\right\rangle$$

$$b^+\left(q,t\right) = \sum_k c^+_{k+q/2,\uparrow}(t)\, c^+_{-k+q/2,\downarrow}(t)$$

BCS and the pair susceptibility



$\chi^{(0)}_{\rm pair}(q,\omega)$ $\chi^{(0)}_{\rm pair}(q,\omega)$ $\chi^{(0)}_{\rm pair}(q,\omega)$

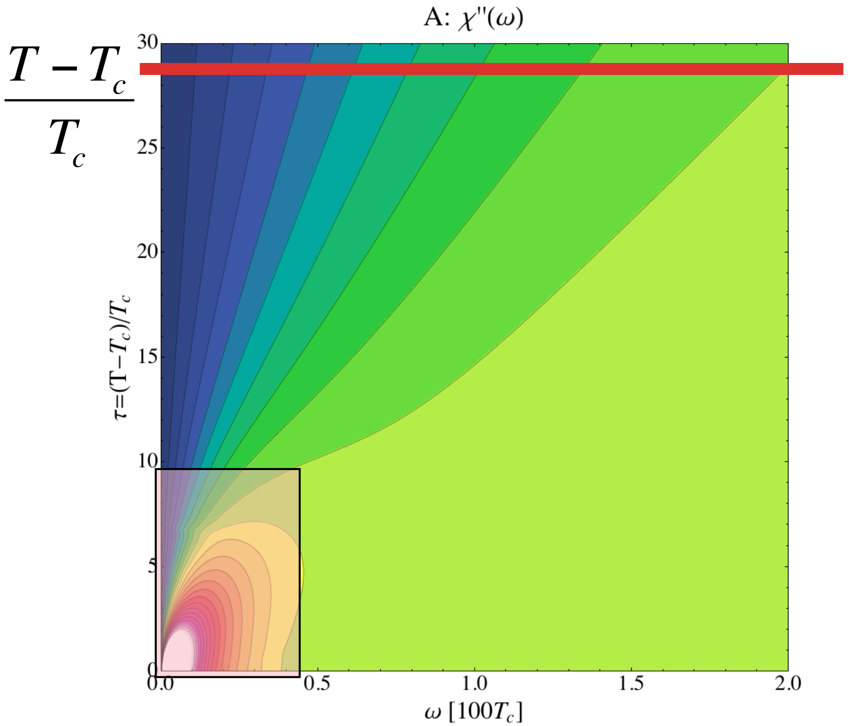
$$\chi_{\rm pair}(\omega) = \frac{\chi^{(0)}_{\rm pair}(\omega)}{1 - g\chi^{(0)}_{\rm pair}(\omega)} \rightarrow 1 - g{\rm Re}\chi^{(0)}_{\rm pair}(\omega = 0) = 0$$

$$\mathrm{Im}\,\chi^{(0)}_{pair}(\omega) = \frac{1}{2E_F}\tanh\left(\frac{\hbar\omega}{4k_BT}\right) \rightarrow {\rm Re}\chi(\omega = 0) = 2\int_0^{\omega_c} d\omega' \frac{{\rm Im}\chi(\omega')}{\omega'}$$

$$\Delta = 2\omega_B e^{-1/\lambda}$$

Imaginary part of the "regular" BCS pair susceptibility

$$\lambda(i\Omega) = \frac{g}{A}\frac{\omega_b^2}{\omega_b^2 + \Omega^2}$$



High temperature: the Fermi gas

$$\mathrm{Im}\,\chi^{(0)}_{pair}(\omega) = \frac{1}{2E_F}\tanh\left(\frac{\hbar\omega}{4k_B T}\right)$$

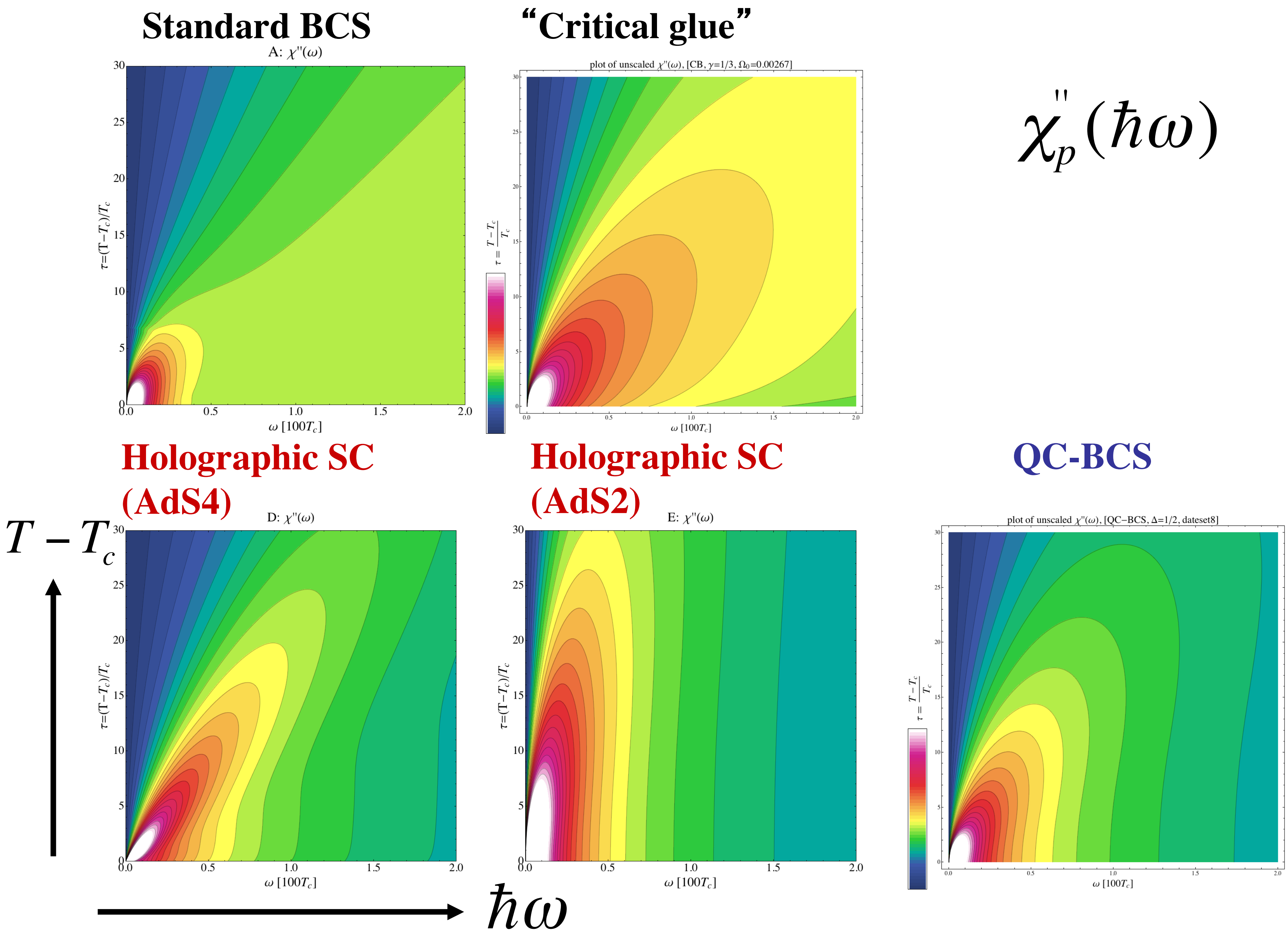
Close to Tc: "relaxational peak"

$$L = \frac{1}{\tau_r}\Psi\partial_t\Psi + |\nabla\Psi|^2 + \alpha_0(T - T_c)|\Psi|^2 + w|\Psi|^4 + \cdots$$

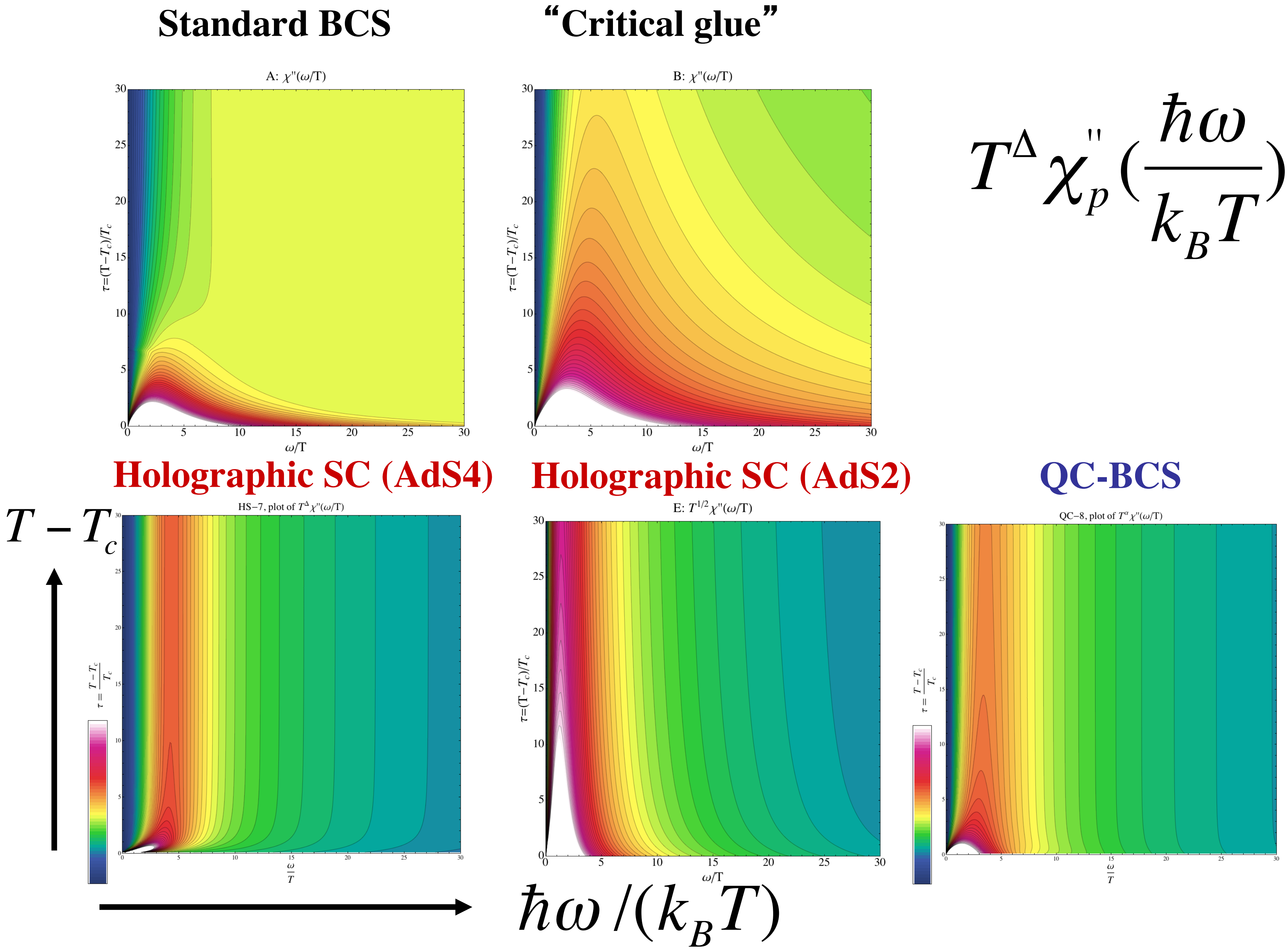
Assume mean-field thermal transition (true in all cases)

$$\chi_{pair}(\omega) = \frac{\chi'_{pair}(\omega = 0, T)}{1 - i\omega\tau_r}$$

$$\chi'_{pair}(\omega = 0, T) = 1/\left[\alpha_0(T - T_c)\right] \quad \tau_r = \frac{8}{\pi}\frac{\hbar}{k_B(T - T_c)}$$



Standard BCS
A: χ''(ω)
τ=(T−T_c)/T_c
ω [100T_c]
"Critical glue"
plot of unscaled χ''(ω), [CB, γ=1/3, Ω_0=0.00267]
τ = (T − T_c)/T_c
ω [100T_c]
χ''_p(ħω)
Holographic SC (AdS4)
D: χ''(ω)
τ=(T−T_c)/T_c
ω [100T_c]
Holographic SC (AdS2)
E: χ''(ω)
τ=(T−T_c)/T_c
ω [100T_c]
QC-BCS
plot of unscaled χ''(ω), [QC−BCS, Δ=1/2, dataset8]
τ = (T − T_c)/T_c
ω [100T_c]
T − T_c
ħω

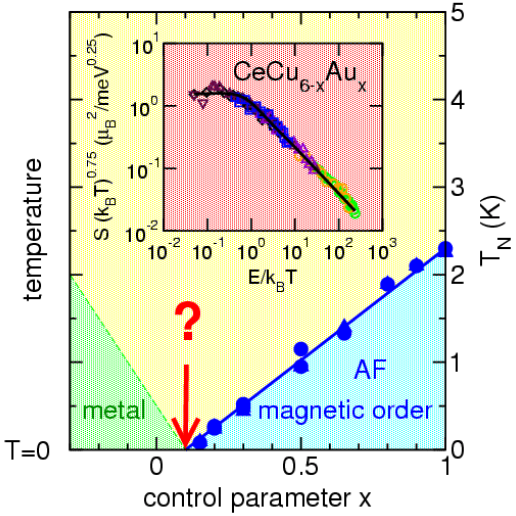


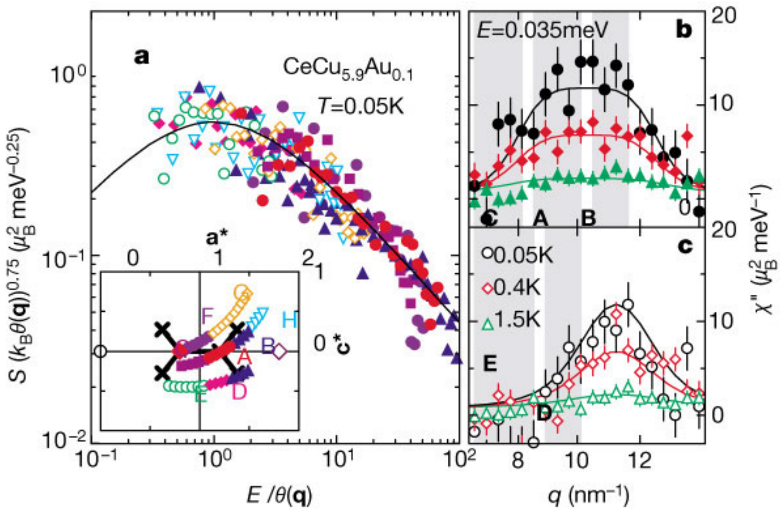
Standard BCS
"Critical glue"
A: χ''(ω/T)
B: χ''(ω/T)
τ=(T−Tc)/Tc
ω/T
$T^{\Delta}\chi''_p(\frac{\hbar\omega}{k_B T})$
Holographic SC (AdS4)
Holographic SC (AdS2)
QC-BCS
HS-7, plot of $T^{\Delta}\chi''(\omega/T)$
E: $T^{1/2}\chi''(\omega/T)$
QC-8, plot of $T^{\alpha}\chi''(\omega/T)$
$T - T_c$
$\hbar\omega/(k_B T)$

Local quantum criticality: heavy fermion magnetism

Aeppli et al.

Nature 407, 351 (2000)



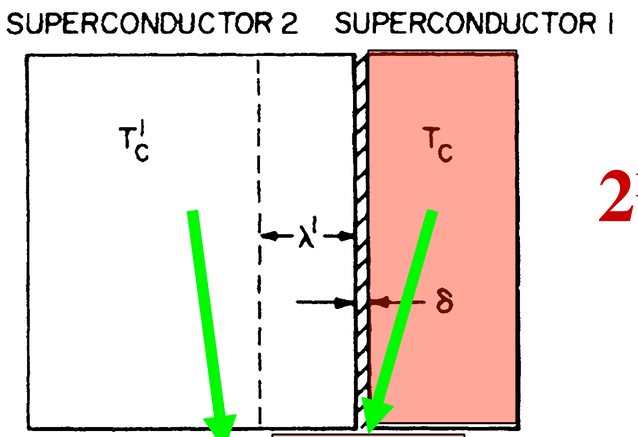


Magnetic fluctuations (neutrons) near the quantum critical point:

- **Space directions: smooth in momentum space.**
- **Time direction: “energy-temperature scaling”**

Most convincing evidence for local quantum criticality up to now!

Observing the origin of the pairing mechanism



$$\Delta c^{\dagger} c^{\dagger}$$

$$T_c' > T > T_c$$

2nd order Josephson effect

Ferrell Scalapino

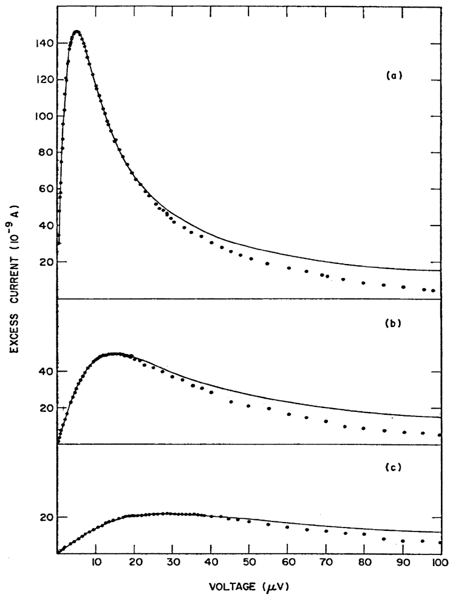
1969 1970

$$I_s(\mathbf{H}, V) \sim \frac{1}{R_N^2} \mathrm{Im}\chi_{\mathrm{pair}}(\mathbf{k}, \omega) \qquad \omega = 2eV$$

Proof of principle ….

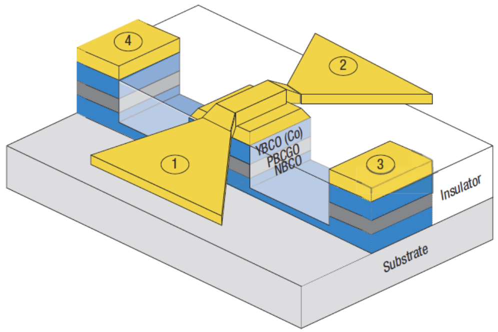
Goldman 1970's

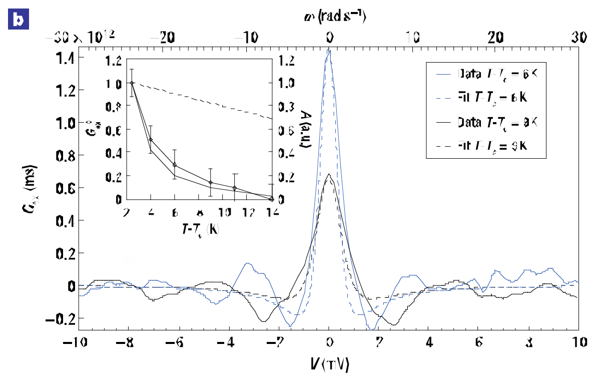
Al-Pb junction: "Relaxational peak" Al near the BCS transition



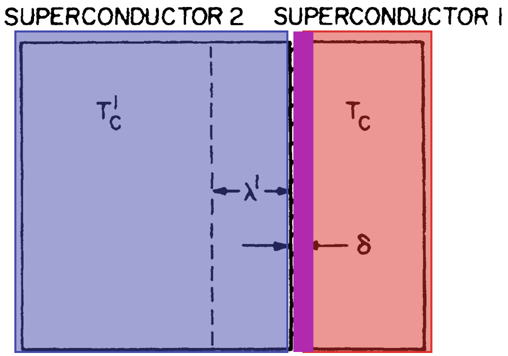
$$I_s(V) = \frac{4eA|C|^2}{dN_0\varepsilon} \frac{\omega/\Gamma_0}{1+(\omega/\Gamma_0)^2}$$

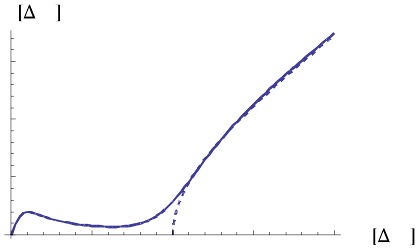
Recent: 60K-90K cuprate superconductors (Bergeal, Nature Physics 2008).





How to build the pairing telescope?





$$I_{tun}(V) = I_{qp}(V) + I_{pair}(V)$$

QC metal:

Need large dynamical range:

$$T, \omega \propto 10 - 100\, T_c$$

QC superconductor at ambient conditions with low Tc:

$CeIrIn_5$, $T_c = 0.4$K

Probe superconductor:

Cuprate ?

MgB_2?

Barrier is the challenge!

Decoding the strange metals …

Challenges for the experimentalists:

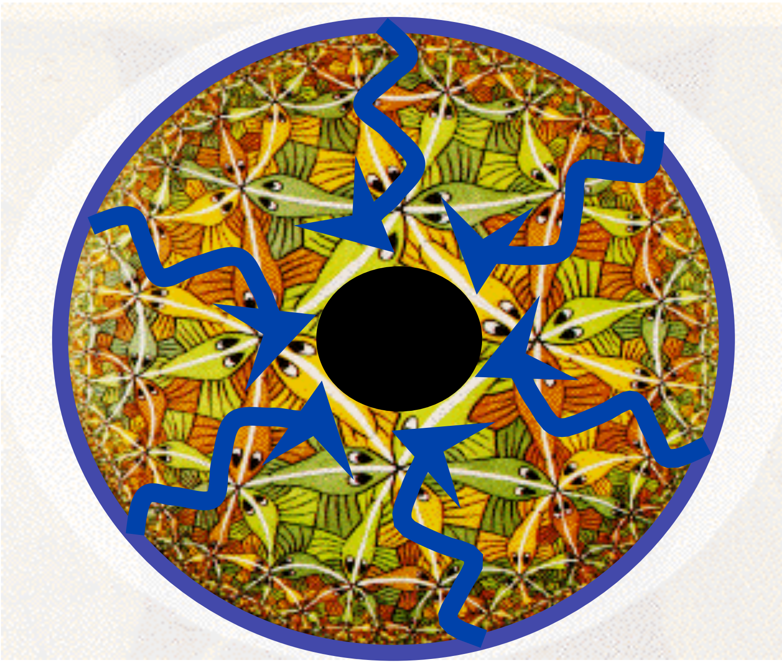
1. Measuring dynamical susceptibilities = *scaling properties* of propagators: the Goldman-Scalapino-Ferrell device for pairing.

2. Is the strange metal a "Planckian dissipator"? Linear resistivity versus the skin effect.

The Challenge for the holographists:

Pseudo-gap regime: *competing orders versus the fermion propagators*.

Dissipation = absorption of classical waves by Black hole!



Hartnoll-Son-Starinets (2002):

Viscosity: absorption cross section of gravitons by black hole

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G}$$

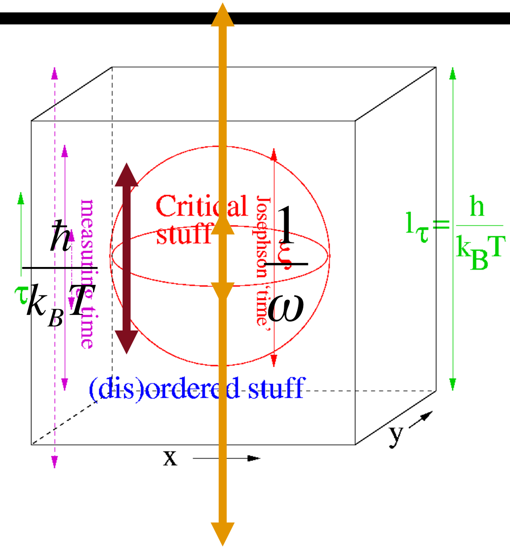
= area of horizon (GR theorems)

Entropy density s: Bekenstein-Hawking BH entropy = area of horizon

Universal viscosity-entropy ratio for CFT' s with gravitational dual limited in large N by:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Planckian dissipation …



Scaling form dynamical susceptibility:

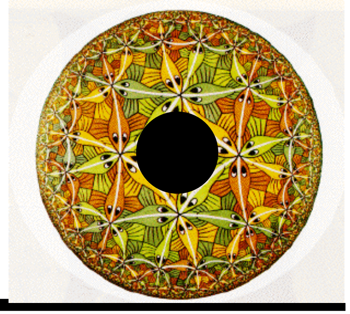
$$\chi(\omega) \propto \frac{1}{T^{2-\eta}} \Psi\left(\frac{\hbar\omega}{k_B T}\right)$$

Quantum critical regime $\quad k_B T >> \frac{\hbar c}{\xi}$

$$\hbar\omega >> k_B T : \quad \chi(\omega) \propto \frac{e^{i\frac{\pi}{2}(2-\eta)}}{|\omega|^{2-\eta}}$$

$$\hbar\omega << k_B T : \quad \chi(\omega) \propto \frac{1}{T^{2-\eta}} \frac{1}{1 - i\omega\tau_\hbar}$$

Planckian dissipation: $\quad \tau_\hbar = const. \frac{\hbar}{k_B T}, \quad const. = O(1)$

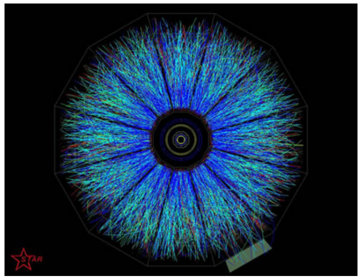


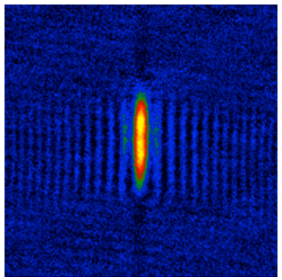
Planckian dissipation

Universal entropy production time in QC system: $$\tau = \tau_\hbar \approx \frac{\hbar}{k_B T}$$

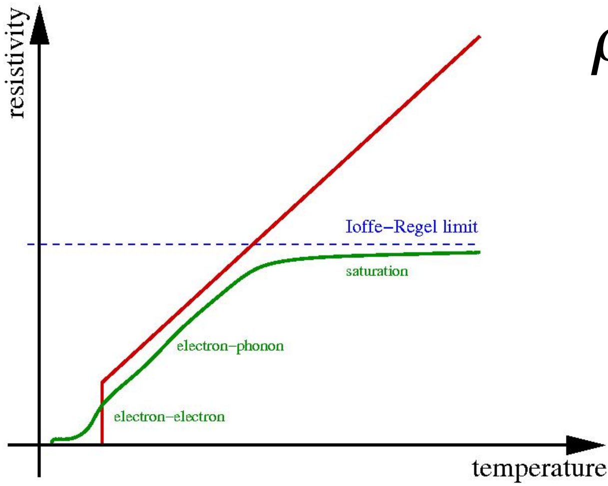
Observed in Quark gluon plasma (heavy ion colliders RIHC, LHC) and cold atom "unitary fermi gas":

$$\frac{\eta}{s} = T\tau_\hbar = \frac{1}{4\pi}\frac{\hbar}{k_B}$$





Since early 1990's recognized as responsible for strange metal properties, also linear resistivity high Tc metals ??:



$$\rho \propto \frac{1}{\tau_\hbar} \propto k_B T$$

Quantum matter.

"Macroscopic stuff that can quantum compute all by itself"

$$|\Psi\rangle = \sum_{configs} A_{configs} |configs\rangle$$

- Topological incompressible systems, no low energy excitations but the whole carries quantum information: fractional quantum Hall, top. Superconductors/insulators (Majorana's, theta vacuum, ..)

- Compressible systems: are the strange metals of this kind??

Strongly interacting fermions at finite density: the fermion signs as entanglement resource!

"Unparticle physics": macroscopic entangled matter.

Conjecture I: ***All strongly interacting quantum critical states are long ranged entangled. Support: dynamical critical scaling down in Euclidean signature, branch cut propagators.***

Conjecture II: ***Fermion signs are "stabilizers" of long range entangled vacuum states. If compressible these have (?) to form quantum critical phases.***

Conjecture III: ***this dense entanglement implies that such states are maximally efficient entropy generators = Planckian dissipation.***

Quasiparticle versus "Unparticle" transport.

Hydrodynamics is based on momentum conservation: how about the broken translational symmetry in metals?

Fermi liquid: the charge carriers are quantum-mechanical waves diffracting against the lattice while the Fermi-momentum is of order of the Umklapp momentum,

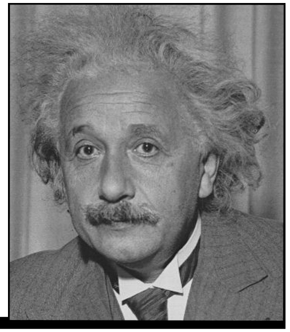
$$\frac{1}{\tau_{coll}} \simeq \frac{(k_B T)^2}{\hbar E_F} \qquad \frac{1}{\tau_K} = C \frac{1}{\tau_{coll}} \qquad \eta \simeq (n E_F) \tau_{coll} \sim \frac{1}{T^2}$$

Quantum critical metals:

- QC fluctuations are not waves, these do not diffract against the periodic lattice!

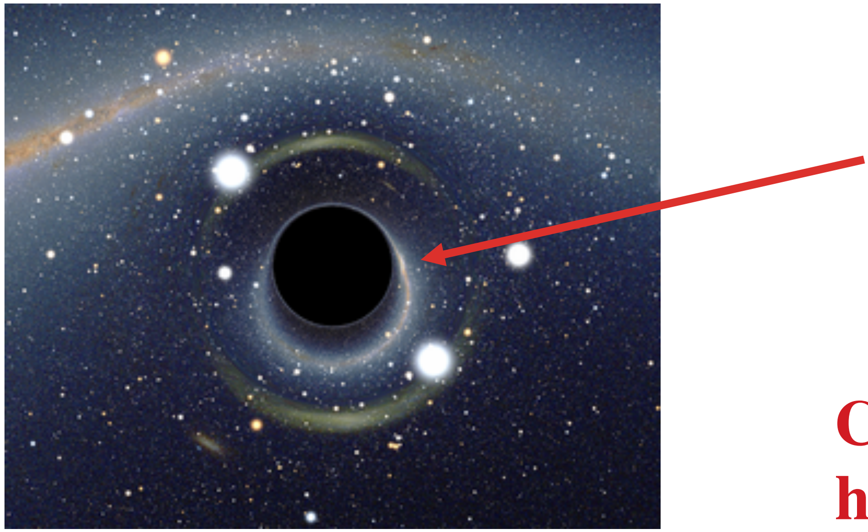
- "Planckian dissipation": extremely rapid equilibration, only after hydrodynamics is established momentum relaxes !?

Black holes with a corrugated horizon



Charged Black Hole: describes finite density strange metal .

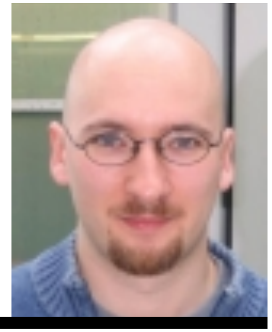
Breaking translational symmetry in the boundary:



Corrugate the black hole horizon ….

Not a favorite thing of general relativity -- hard work, still in progress!

Holographic quenched disorder.



David Vegh

Dictionary entry "number one":

Global translational invariance in the boundary (energy-momentum conservation)

<==>

General covariance in the bulk (Einstein theory)

Breaking of Galilean invariance in the boundary = elastic scattering (?)

<==>

Fix the (spatial) frame in the bulk = "Massive gravity"

$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\left(\mathcal{R} + \frac{6}{L^2} - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} + m^2\left(\alpha \mathrm{Tr}\left(\mathcal{K}\right) + \beta\left(\mathrm{Tr}\left(\mathcal{K}\right)^2 - \mathrm{Tr}\left(\mathcal{K}^2\right)\right)\right)\right)$$

$$\mathcal{K}^{\mu}_{\alpha}\mathcal{K}^{\alpha}_{\nu} = g^{\mu\alpha}f_{\alpha\nu}$$

Couple the metric g_{ab} to a fixed metric f_{xx}=f_{yy}=1

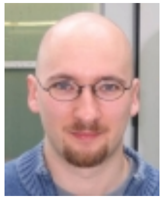
Holographic linear resistivity.

Davison, Schalm, JZ, arXiv:1311.2451

Richard Davison

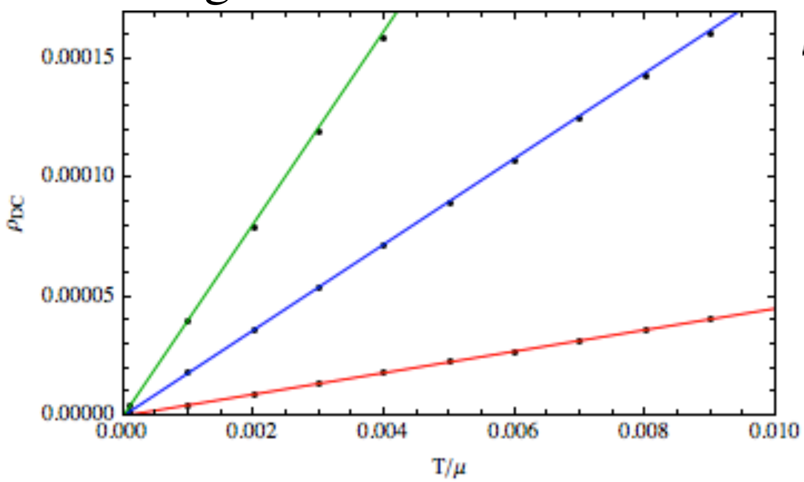
Steve Gubser

"Champion" strange metal: Einstein-Maxwell-dilaton (consistent truncation), local quantum critical, marginal Fermi-liquid (3+1D), susceptible to holo. superconductivity, healthy thermodynamics: unique ground state, Sommerfeld thermal entropy.



David Vegh

Breaking of Galilean invariance (finite conductivities) due to quenched disorder: "massive gravity" = fixing space-like diffeomorphisms in the bulk.



$$S = \frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{4}e^{\phi}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}\partial_\mu\phi\partial^\mu\phi + \frac{6}{L^2}\cosh\phi - \frac{1}{2}m^2\left(\mathrm{Tr}\left(\mathcal{K}\right)^2 - \mathrm{Tr}\left(\mathcal{K}^2\right)\right)\Big]$$

Explicit holographic construction explaining linear resistivity!

The secret of the linear resistivity …

Davison

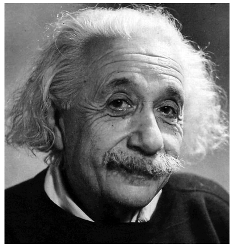
Hartnoll

Planckian dissipation = very rapid local equilibration: a hydrodynamical fluid is established before it realizes that momentum is non conserved due to the lattice potential (not true in Fermi-gas: Umklapp time is of order collision time).

Stokes

Resistivity in hydrodynamics

$$\rho(T) \propto \frac{1}{\tau_{rel}} = \frac{D}{l^2}$$



Einstein

Einstein relation:

$$D = \frac{\eta}{m_e n_e}$$

Sachdev Son

Planckian viscosity

$$\eta = A \frac{\hbar}{k_B} s$$

$$\rho(T) = \frac{1}{\omega_p^2 \tau_{rel}} = A \frac{\hbar}{\omega_p^2 l^2 m_e} \frac{S}{k_B}$$

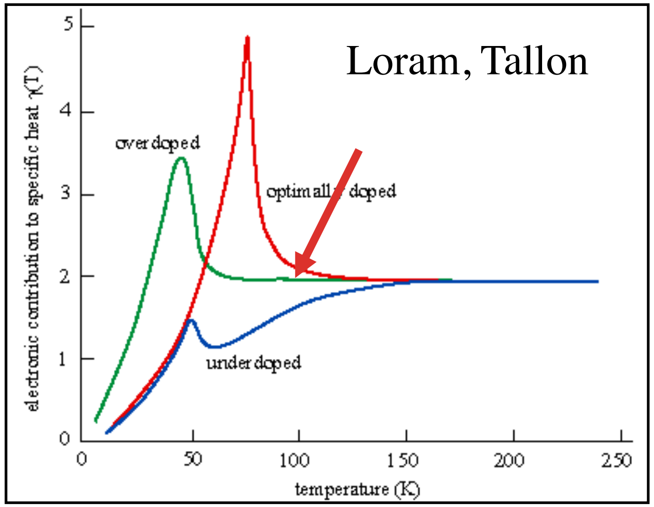
Caveat: only when z is infinite, otherwise l is running …

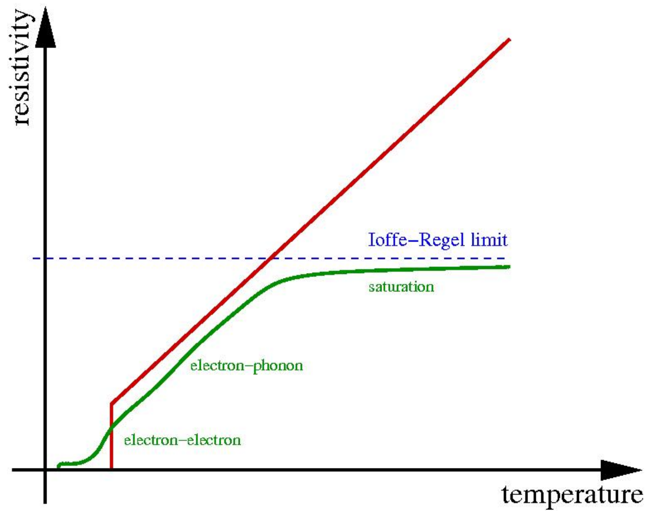
Entropy versus transport: optimal doping

Optimally doped

$$C = \gamma T \Rightarrow S = T / \mu$$

$$\rho \propto \frac{1}{\tau_{rel}} \propto S \propto T$$





Plugging in numbers: "mean-free path" $l \approx 10^{-8} m$

Quite dirty but no residual resistivity since the fluid becomes perfect at T =0 !

Magnitude of momentum relaxation.

According to the cuprate optical conductivity the momentum relaxation rate is:

$$\frac{1}{\tau_{\exp}} \approx \frac{k_B T}{\hbar}$$

According to "massive gravity", the RN strange metal has a momentum relaxation rate:

$$\frac{1}{\tau_{\exp}} = A\frac{\hbar}{l^2 m_e}\frac{S}{k_B} = A\frac{\hbar^2}{\mu l^2 m_e}\frac{k_B T}{\hbar} \qquad \textbf{assuming} \qquad \frac{S}{k_B} = \frac{k_B T}{\mu}$$

It follows for the microscopic mean free path:

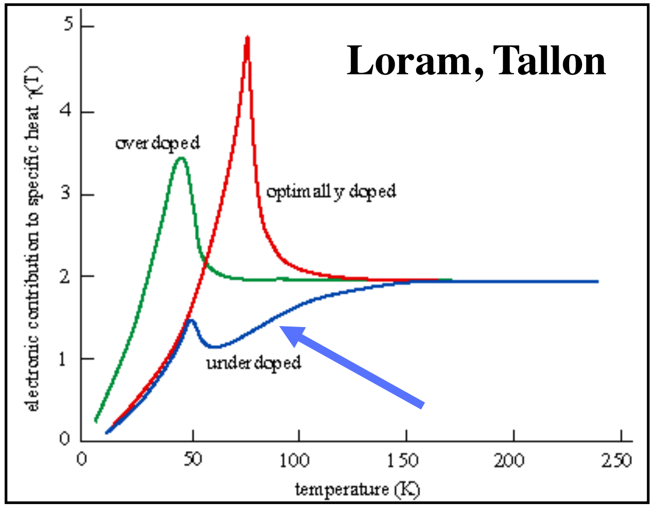
$$l = \hbar\sqrt{\frac{A}{\mu m_e}} \approx 10^{-9}\, m$$

"Fermi-liquid like" pseudogap transport

Greven vd Marel

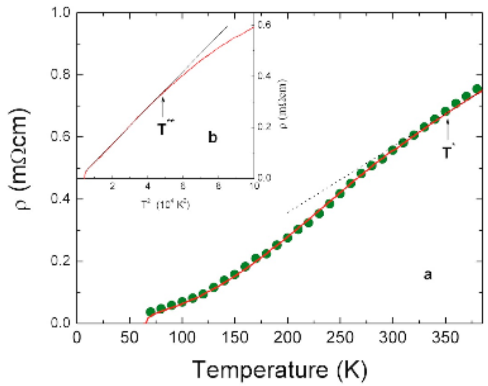
Pseudogap regime:

$$C \propto T^2 \Rightarrow S \propto T^2 (?)$$

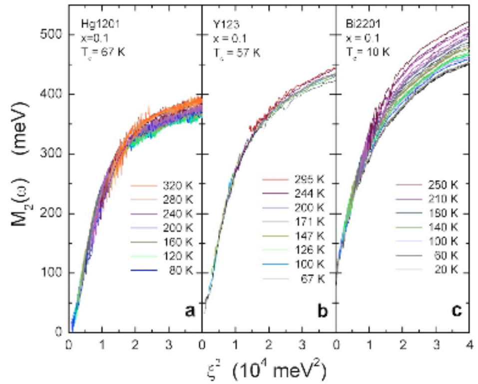


Massive gravity:

$$\rho \propto \frac{1}{\tau_{rel}} \propto S \propto T^2$$



DC resistivity



Energy/temperature scaling collapse

Holographic order parameter is not a potential!

Order parameter

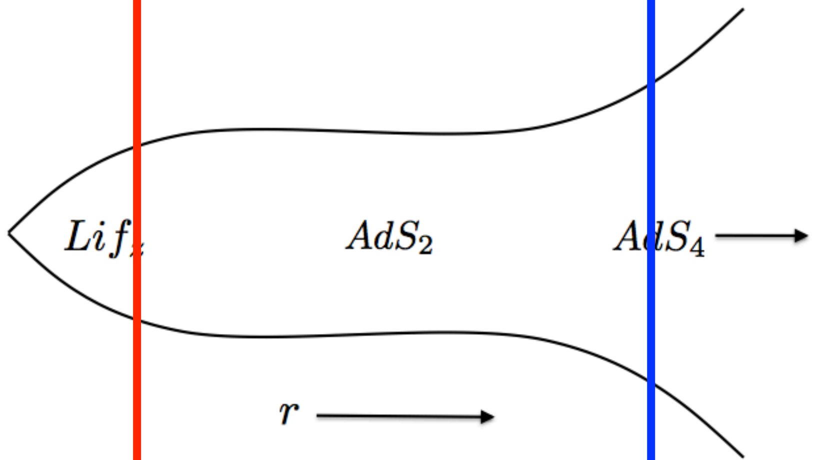
Chemical potential

Scaling geometry in the deep interior of the star!

Strange metal scaling geometry.

Pristine AdS

Typically "Lifshitz": characterised by z.



Modified quantum critical phase "stabilized" by order parameter.

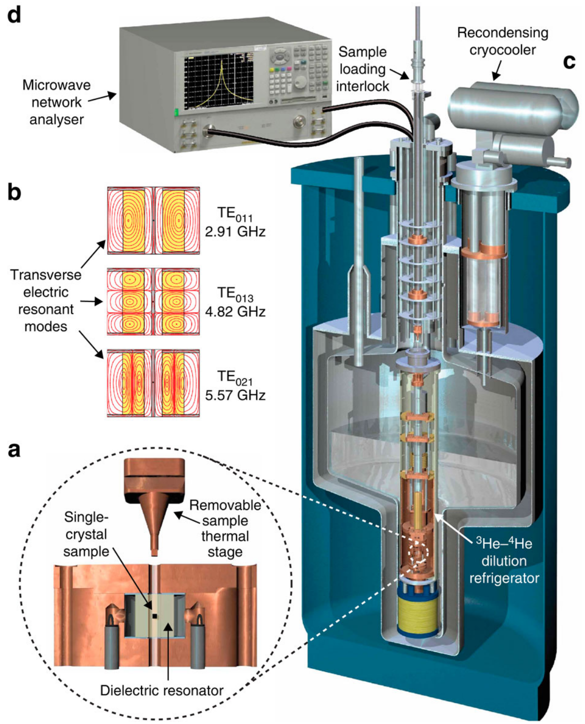
Strange metal quantum critical phase

UV Conformal fields

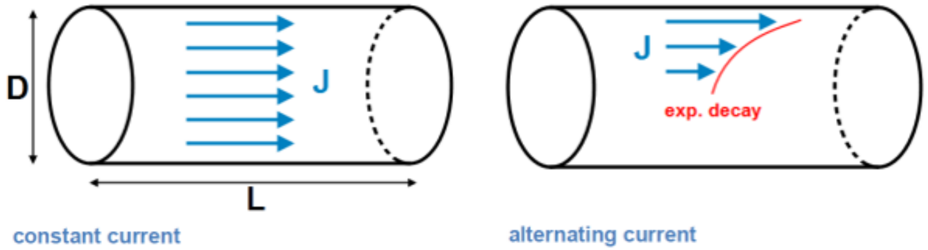
Skin effect in metals …

Measure impedance of sample in microwave cavity:

$$Z(\omega) = Z'(\omega) + iZ''(\omega)$$



AC currents penetrate over a skin depth $\delta \simeq Z'(\omega)$



Fermi-liquid at high temperature: "classical" skin effect

$$\delta \simeq \sqrt{\frac{\rho(T)}{\omega}}$$

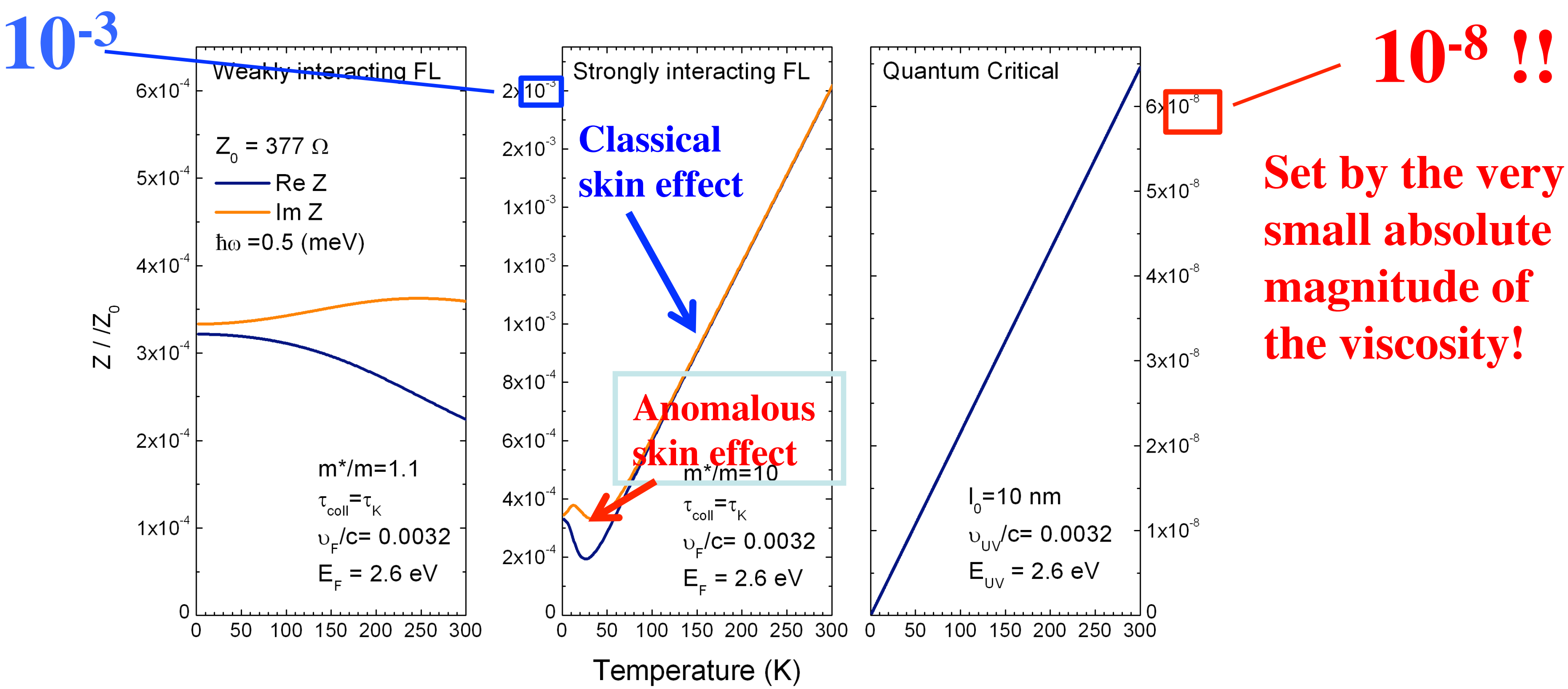
Collision-less regime at low temperature: "anomalous" skin effect

$$\delta \simeq \frac{1}{\omega^{1/3}}$$

Skin effect and viscosity…

Forcella, JZ, Valentinis, van der Marel, arXiv:1406.1356.

Reformulation skin effect in magneto-hydrodynamical language: set by propagation of transversal sound, sensitive to viscosity!



Decoding the strange metals …

Challenges for the experimentalists:

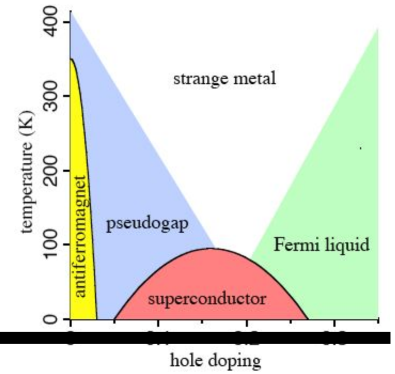
1. Measuring dynamical susceptibilities = *scaling properties* of propagators: the Goldman-Scalapino-Ferrell device for pairing.

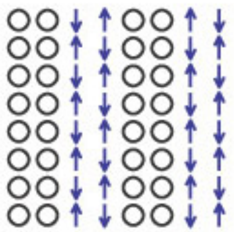
2. Is the strange metal a "Planckian dissipator"? Linear resistivity versus the skin effect.

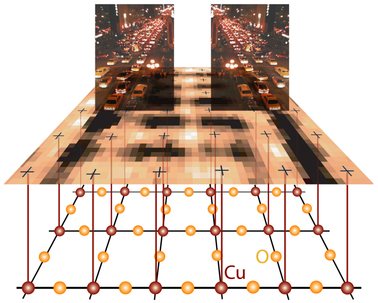
***The* Challenge for the holographists:**

Pseudo-gap regime: *competing orders versus the fermion propagators*.

Pseudogap: orderly but confusingly competing!

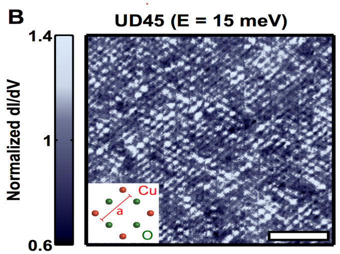




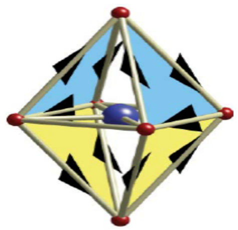


"Classic" stripes: only static in the bulk of special 214's (=> insulating, 1/8 commensuration), associated with pseudogap energy. Might freeze out in large magn. fields.

Indications for static "stripy" nematic order.



"Charge density wave" (123, 2212): static but weakly coupled, different wavevectors than stripes, small energy scale ("nodal regime").



Spontaneous loop currents: break time reversal, strong signal in neutron scattering, no relation to fermionic response !?

Strong evidence for fluctuating superconductivity.

Hartree-Fock …

Assume classical symmetry breaking state = product vacuum:

Particle-hole channel (crystal, magnet, currents…):

$$c^\dagger c c^\dagger c \simeq O c^\dagger c + h.c. - \frac{1}{2}|O|^2 + \delta O \delta O^*$$

The order parameter turns into a potential diffracting the quasiparticle waves: the "gap function"

$$\Delta = VO = V\langle c^\dagger c \rangle$$

To be perturbatively dressed by the order parameter fluctuations:

$$\delta O = c^\dagger c - O$$

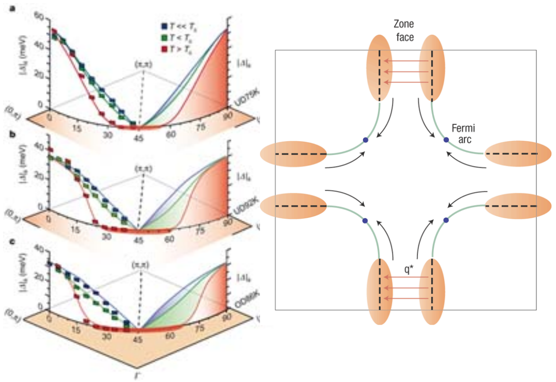
The fermion spectra going their own strange way ..

Nodal-antinodal dichotomy:

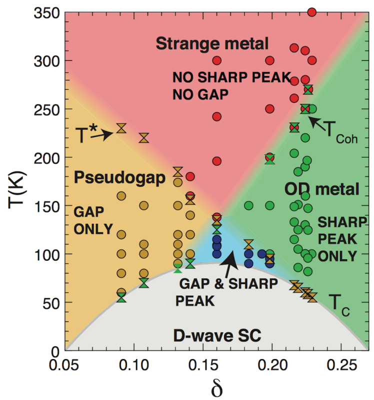
Nodal region: “marginal Fermi-liquid like” on “Fermi-arcs” in strange metal, cohere in BCS bologoliubons in SC.

Antinodes: incoherent backgrounds + pseudogap unrelated to order, in SC state QP’s with pole strength set by superfluid density.

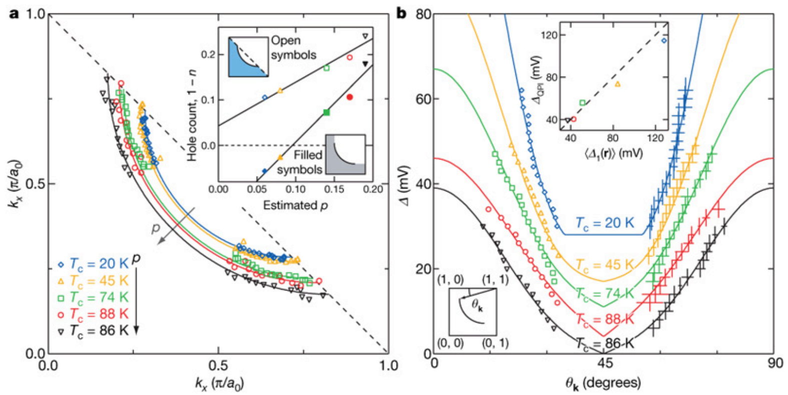
ARPES (Z.X. Shen)



ARPES (Campuzano)



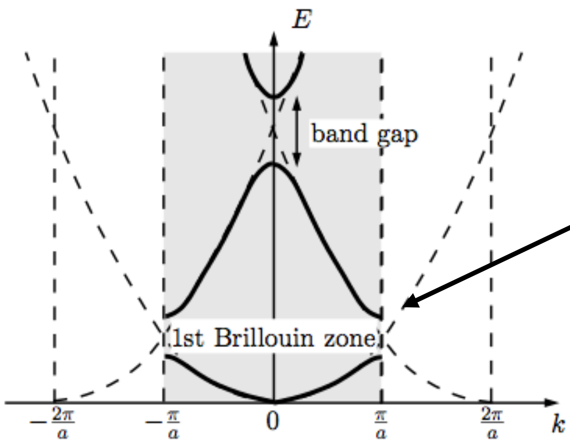
STS (Seamus Davis)



The AdS2 metal and weak potential.

Liu, Schalm, Sun, JZ, arXiv:1205.5227

Electrons are waves diffracting against periodic potentials causing band gaps. Explanation of insulators vs metals (Wilson 1931):



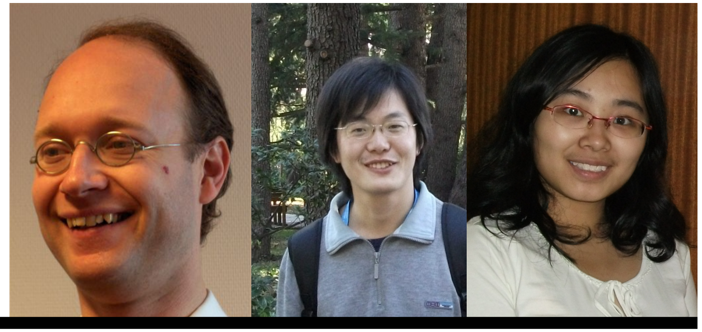
$$\psi_{1k}(r) = u_k e^{ikr} - v_k e^{i(k+K)r}$$

$$k = \frac{K}{2} : |\psi_{1k}(r)|^2 = (1 - \cos(Kr))/2$$

How does a static periodic potential affect the AdS2 metal ?

Answer: it is not waves but energy scaling!!

Set up RN + static potential.



Schalm Liu Sun

Background geometry: extremal RN black hole with a small space periodical gauge potential added encoding for the weak static potential.

$$ds^2 = -r^2 f(r)\, dt^2 + \frac{dr^2}{r^2 f(r)} + r^2(dx^2+dy^2), \quad A_t = \left[\mu_0 + \mu_1(x)\right]\left(1-\frac{1}{r}\right),$$

$$f(r) = 1 - \frac{1+Q^2}{r^3} + \frac{Q^2}{r^4}, \quad \mu_0 = 2Q, \quad \mu_1(x) = 2\varepsilon \cos\frac{x}{a}, \text{ with } \varepsilon << \mu_0$$

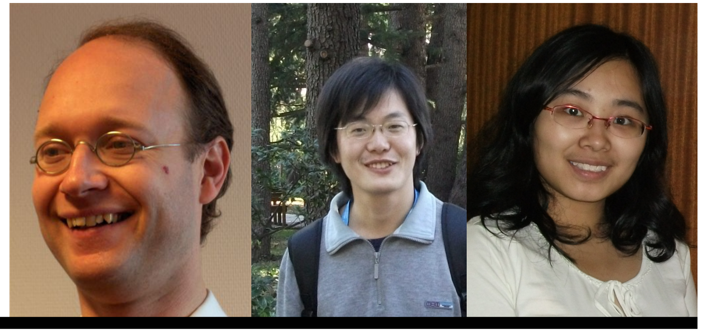
Strategy: study the lattice effect on the spectral function of the probe fermion.

Bulk Dirac equation: $(\not{D} - m)\Phi = 0, \quad m \Leftrightarrow \Delta = m + \frac{3}{2}$ **UV scaling dimension**

Bloch expansion: $$\Phi_\alpha = \int \frac{d\omega dk_x dk_y}{2\pi} \sum_{\ell \in Z} \Phi_\alpha^{(\ell)}(r,\omega,k_x,k_y) e^{-i\omega t + i\left[(k_x+\ell K)x + k_y y\right]}, \quad \alpha = 1,2$$

Umklapp momentum: $K = \frac{1}{a}$ $\quad k_x \in (-K/2, K/2)$, $\ell \in Z$ denotes the ℓ - th Brillouin zone

Domain wall fermions diffract ….

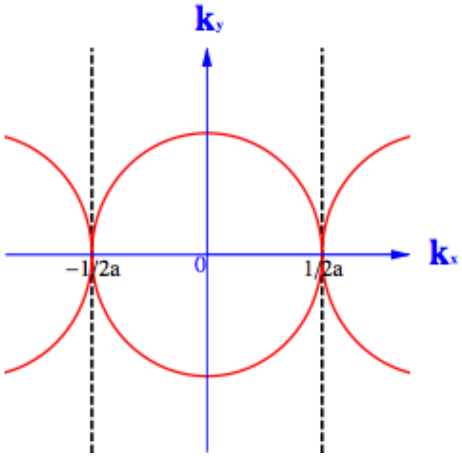


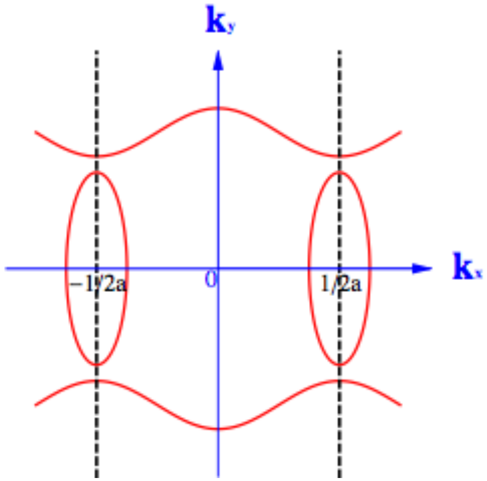
Schalm   Liu   Sun

In the quasi-Fermi-surface regime the "domain wall fermions" scatter just as helical Dirac fermions (e.g., surface states 3D topological insulators)

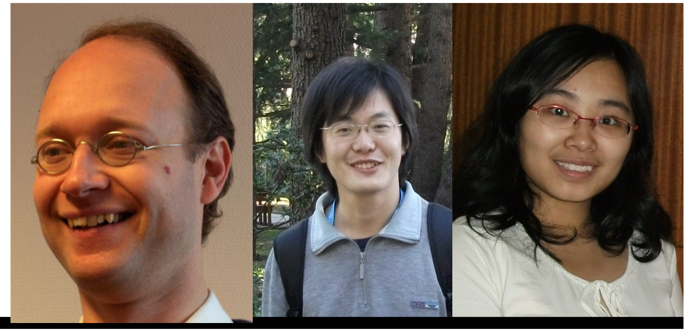
$$G(k,\omega) \propto \frac{\left(\omega - v_F\left(k-k_F\right)\right)}{\left(\omega - v_F\left(k-k_F\right)\right)^{\wedge}2 - \Delta^2(k) + \left(\omega - v_F(k-k_F)\right)\Sigma_{AdS_2}(k,\omega)}$$

$$\Delta^2 \approx \varepsilon^2\left(1 - \frac{1}{\sqrt{1+\left(2ak_y^2\right)}}\right)$$





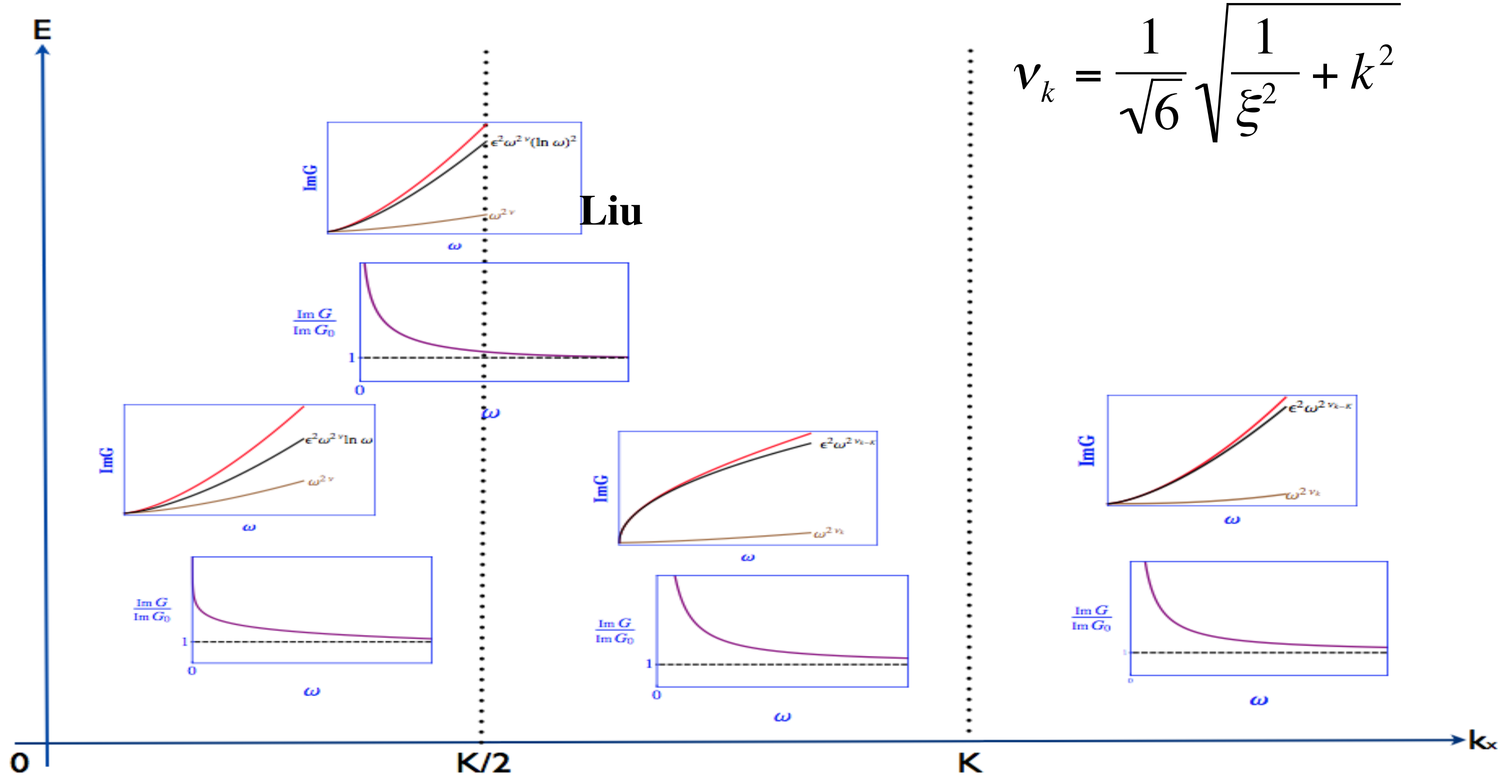
But the AdS2 states modify energy scaling!



Schalm Liu Sun

$$G_{IR}\left(\omega,\vec{k}\right) = \alpha_{\vec{k}} G^0_{IR}\left(\omega,\vec{k}\right) + G^1_{IR}\left(\omega,\vec{k}\right)$$
$$= \alpha_{\vec{k}}\,\omega^{2\nu_{\vec{k}}} + \beta^{(-)}_{\vec{k}}\,\omega^{2\nu_{\vec{k}-\vec{K}}} + \beta^{(0)}_{\vec{k}}\,\omega^{2\nu_{\vec{k}}}\ln\omega + \beta^{(+)}_{\vec{k}}\,\omega^{2\nu_{\vec{k}+\vec{K}}} + \ldots$$

$$\nu_k = \frac{1}{\sqrt{6}}\sqrt{\frac{1}{\xi^2} + k^2}$$



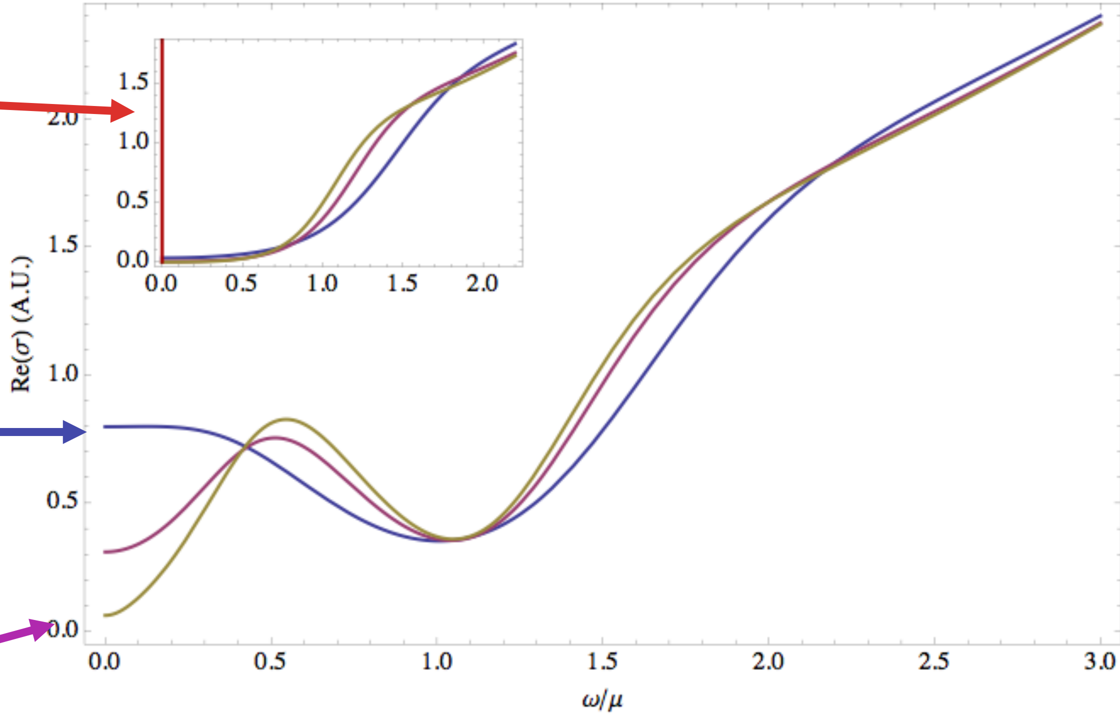
The higher dimensional quantum smectic.


Donos **Hartnol l**

“ab-plane” optical conductivity:
perfect Drude metal.

“c-axis” optical conductivity:

turns from “bad metal”



to “pseudogap-insulator” when potential increases:

$$\sigma(\omega) \propto \omega^{4/3} \quad at\ T = 0$$

The moral of this story ..

Holography is a source of *metaphors* based on general physics principle, to learn to think differently about quantum matter.
Very powerful in suggesting unusual experimental questions: *the ball is in the experimentalist's court!*

Is strange metal transport "Planckian dissipation" hydrodynamics?

1. **Correlate resistivity and entropy.**
2. **Anomalous-anomalous skin effect.**
3. **Violation Wiedemann-Franz law (Hartnoll at al., arXiv:1304.4248)).**

See also Hartnoll, arXiv:1405.3651; Lucas, Sachdev, Schalm, arXiv:1401.7933

Scaling properties of the strange metals: need better info on dynamical response functions!

Dynamical pair susceptibility, but also spin – (neutrons), charge- (RIXS ?), susceptibilities, …

The moral of this story ..

To impress the condensed matter side, the challenge is for the holographist's

to shed light on the physics of the pseudogap phase!

Strong periodic potentials, inhomogeneous spaces, backreact and allow for holographic orders:

Can this explain the "*disconnect*" between fermion propagators and order? Is the *pseudogap like the "relevant potential"* of Bianchi type VII, can this eventually explain *the nodal-antinodal dichotomy*?

Empty.

Empty.