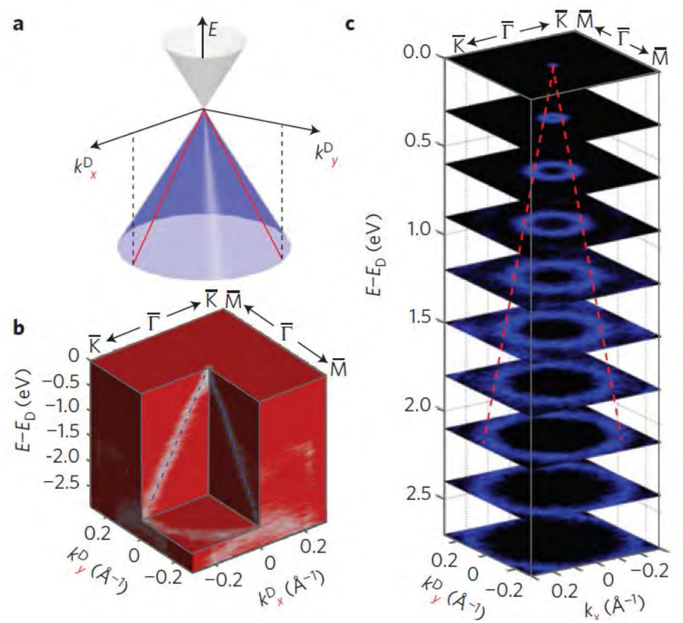
# Holographic Interaction Effects on Transport in Dirac Semimetals





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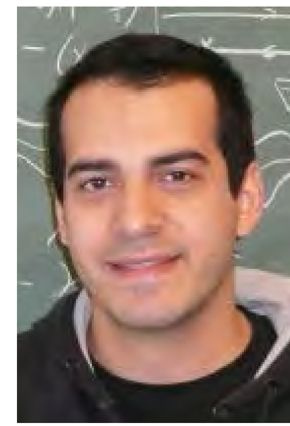

Chen *et al.*, Nature Mat. (2014).


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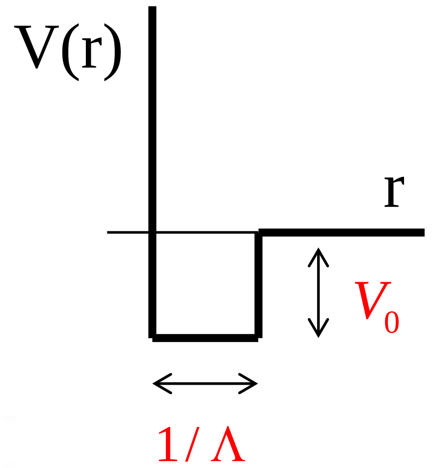
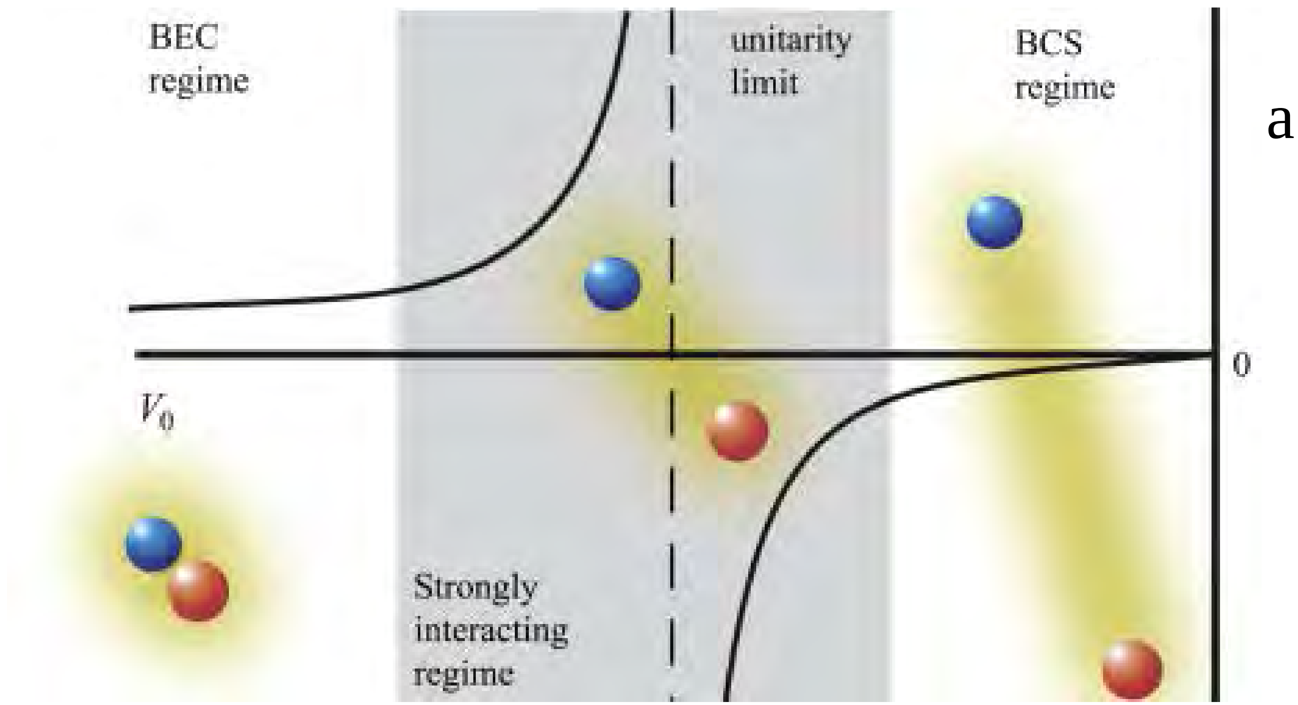
# Long-Term Goal

# BCS-BEC Crossover



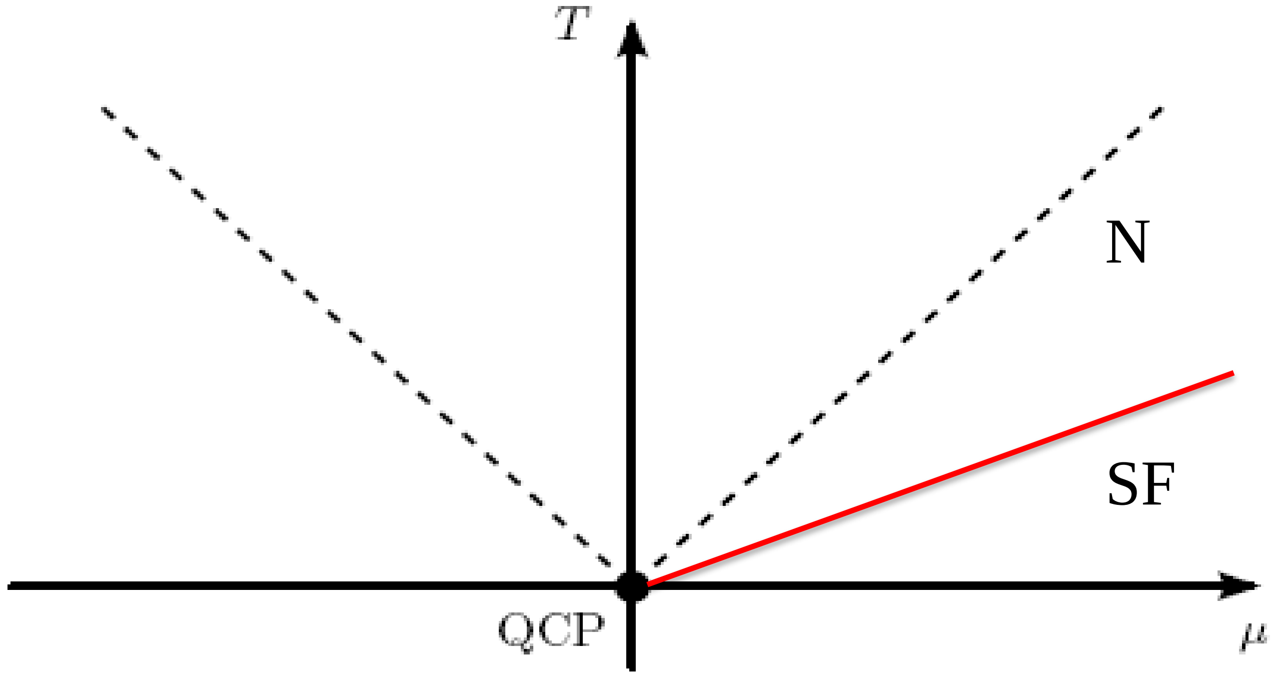

- Now fully explored with ultracold atoms:




$$a = \frac{mV_0}{4\pi\hbar^2}\,\frac{1}{1 + mV_0\Lambda / 2\pi^2\hbar^2}$$

# Unitary Fermi Gas

- Phase diagram:



# Conformal Field Theory

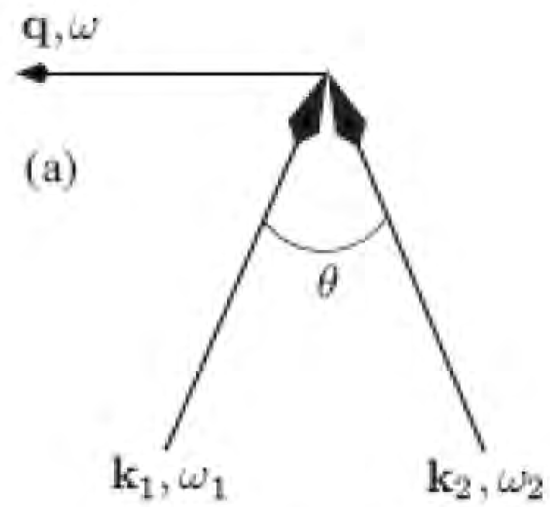
- We do not have Lorentz invariance (z=1):

$$\vec{x} \to \ell \vec{x}, \;\; t \to \ell t$$

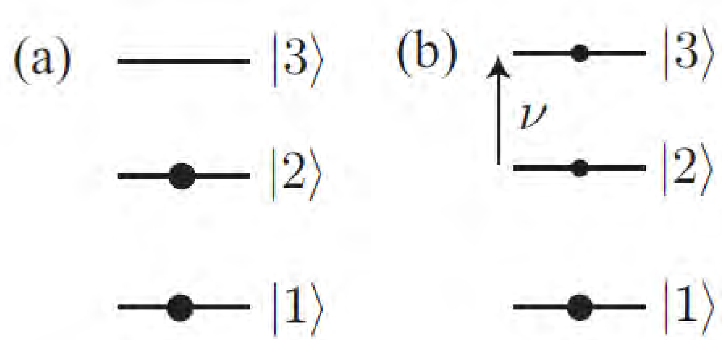
- but instead Galilean invariance (z=2):

$$\boxed{\vec{x} \to \ell \vec{x}, \;\; t \to \ell^2 t}$$

# Measurements




- Bragg scattering (Conductivity):
- RF Spectroscopy (ARPES):




Sum rule:

$$\int d\omega\, \rho(\vec{k},\omega) = 1$$

$$\rho = -\frac{1}{\pi}\,\mathrm{Im}(G_R)$$

# Bottom-Up
# (Semi-)Holography

# AdS/CFT (I)

- Properties of CFT are encoded in the geometry of a 'bulk' (d+1)-dimensional spacetime. We consider always d=4, $\mu = 0$!

$$ds^2 = \frac{dr^2}{r^2 V^2(r)} - V^2(r)\, r^{2z} dt^2 + r^2 d\vec{x}^{\,2} \,,$$

$$V^2(r) = 1 - \left(\frac{r_h}{r}\right)^{d+z-1}$$

$$T = \frac{d+z-1}{4\pi} (r_h)^z$$

# AdS/CFT (II)

- Conductivity:

$$\sigma \propto \frac{e^2 T^{1/z}}{g_5^2}$$

Kovtun & Ritz, PRD (2008);
Pang, JHEP (2010).

- Fermion Green's function (z=1):

$$G_R(k) \propto \frac{1}{k^{2M+1}} k_\mu \sigma^\mu$$

Liu *et al.*, PRD (2011);
Cubrovic *et al.*, Science (2009);
Gürsoy *et al.*, JHEP (2012).

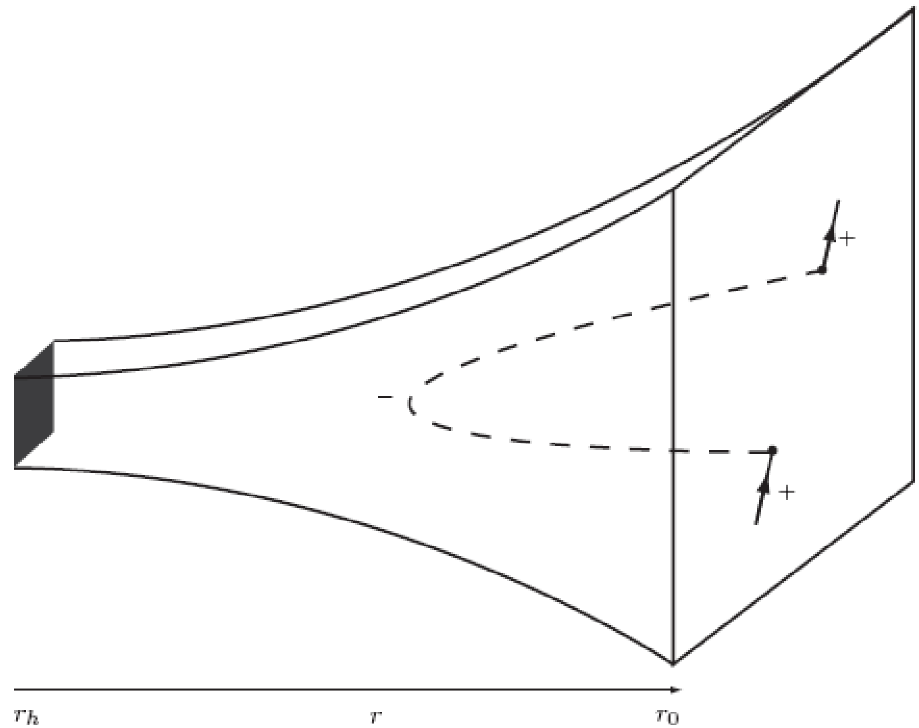
$$\sigma^\mu = (1, \vec{\sigma})$$

- Conductivity does not depend on the bulk fermion mass M and divergent sum rule.

$$G_R(k) = \frac{1}{G_0^{-1}(k) - \Sigma(\mathrm{k})} \underset{g \to \infty}{\square} -\frac{1}{\Sigma(k)}$$

# AdS/CFT (III)

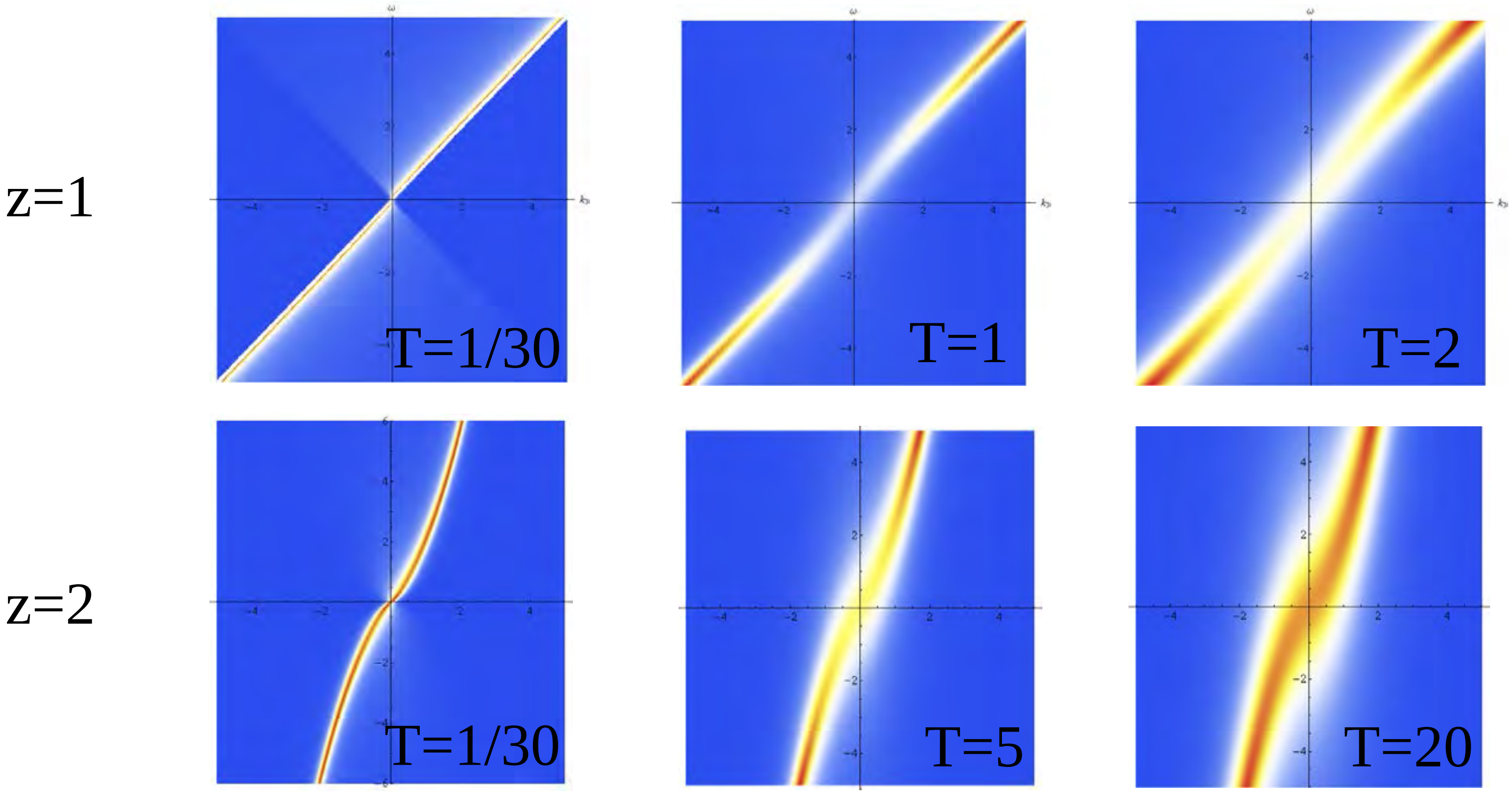
- Chiral fermions live on the UV slice $r=r_0$ and have appropriate double-scaling limit $r_0 \to \infty$ (-z/2<M<z/2):




$$G_R(\vec{k},\omega) = -\left(\omega - \frac{1}{\lambda}\vec{\sigma}\cdot\vec{k}\,k^{z-1} - \Sigma(p)\right)^{-1}$$

# AdS/CFT (IV)

- Spectral functions obey sum rule (M=1/4; $\lambda = 1$ ):



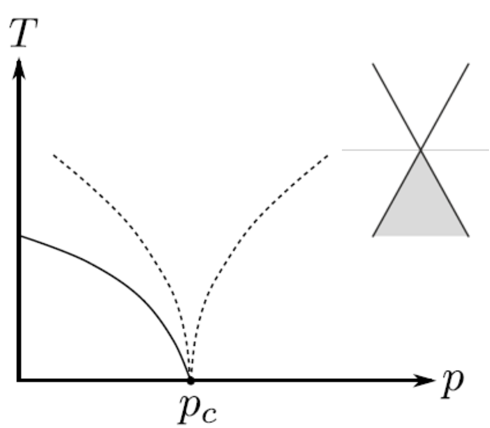
# Three-Dimensional Dirac Semimetal

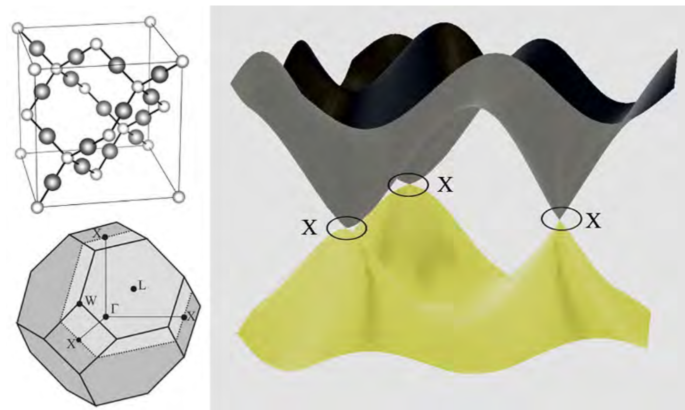
# Interacting Dirac Semimetal (I)

- Two species of Dirac fermions in the bulk with masses M and -M, respectively, give selfenergy (z=1; $\mu = 0$ ; T=0):

$$\Sigma(k) = g_M \gamma^0 \gamma^\mu k_\mu k^{2M-1}$$

- Condensed-matter interpretation:

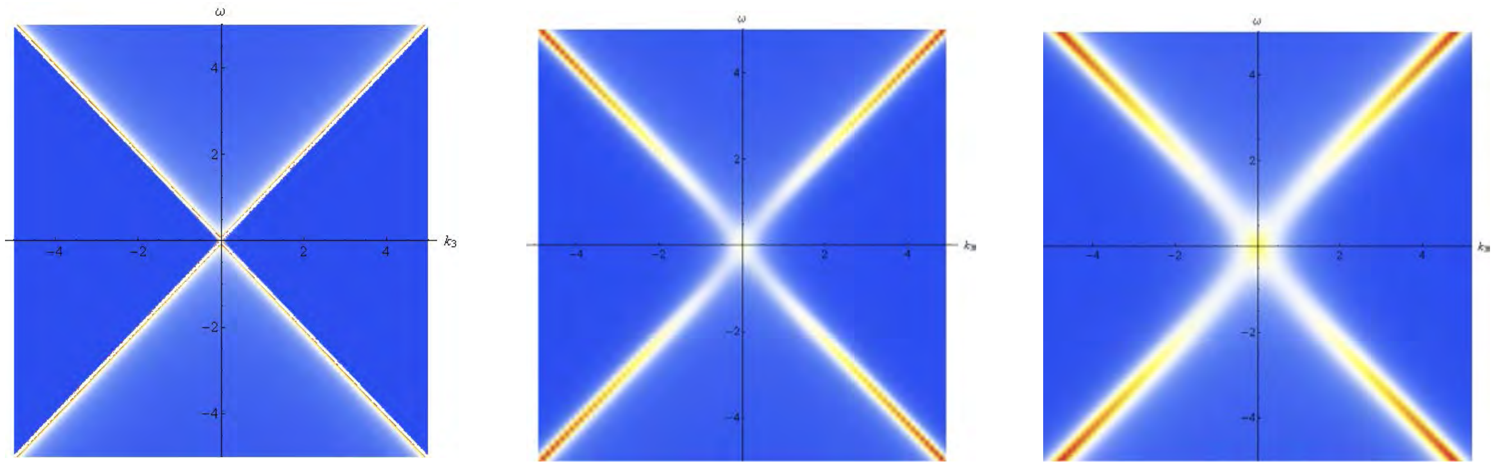



$BiO_2$

# Interacting Dirac Semimetal (II)

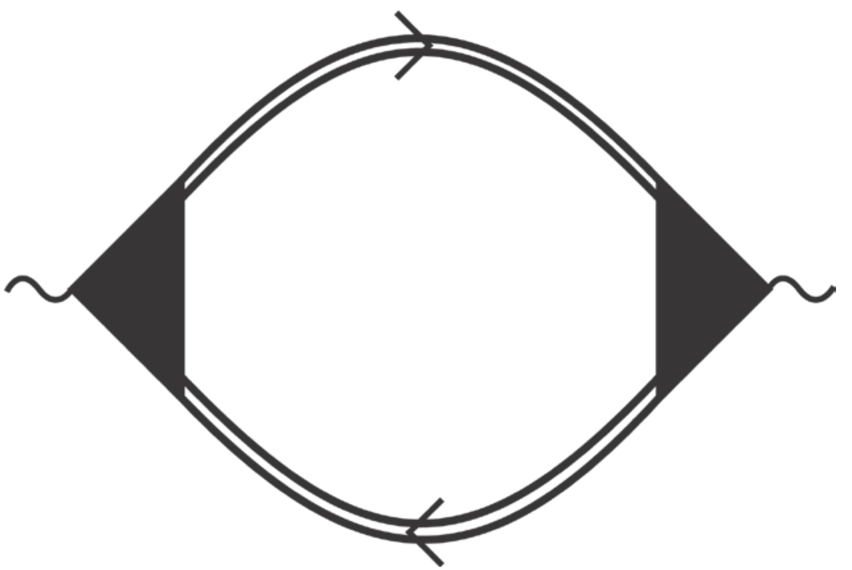
- Spectral functions (M=1/4; T=1/30, 2/3, 1):

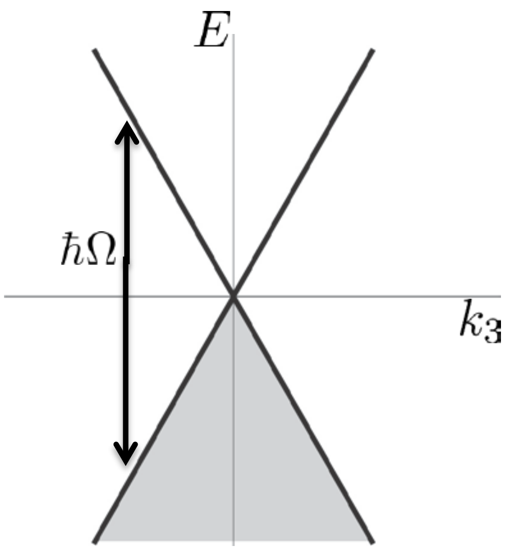


# Interacting Dirac Semimetal (III)

- Optical conductivity:

$$\langle j_i(\Omega)\rangle = \sigma_{ij}(\Omega)\, E_j(\Omega)$$



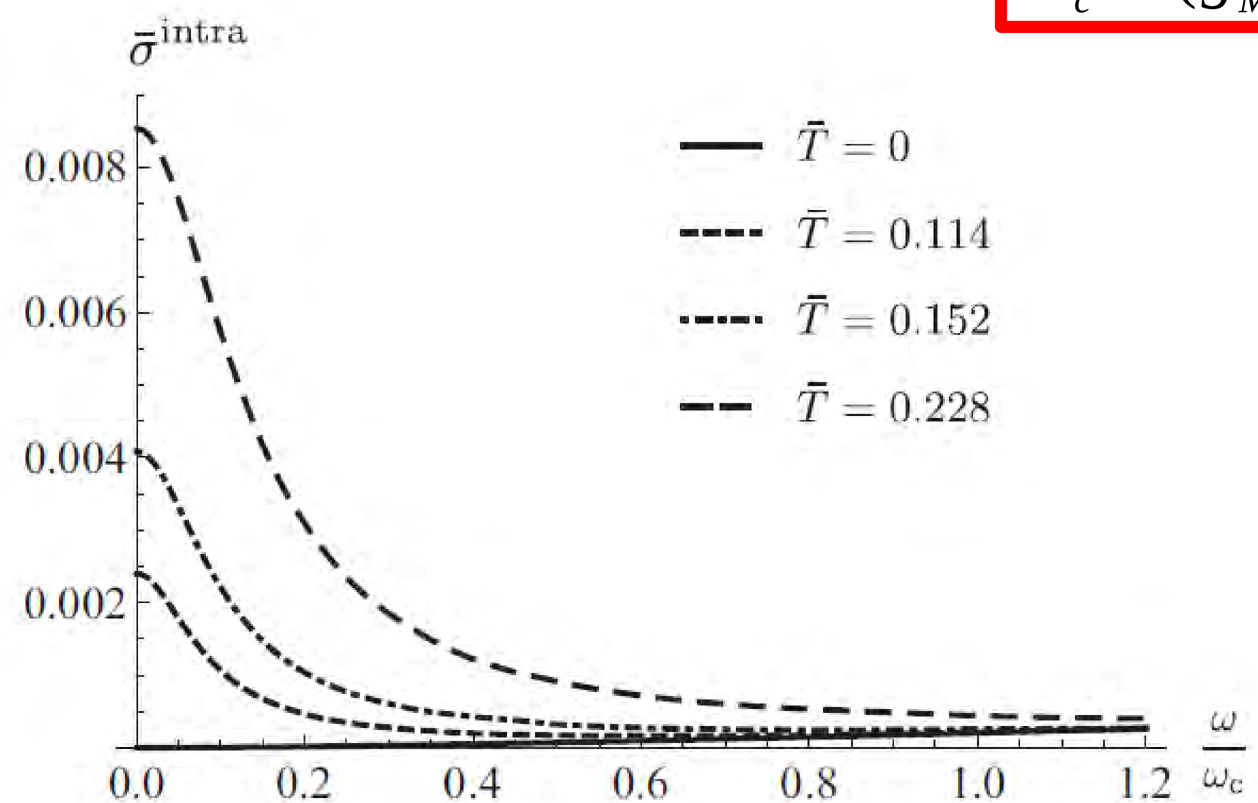



$$\sigma_0(\Omega) = \frac{e^2\,|\,\Omega\,|}{12\pi\hbar c}$$

# Interacting Dirac Semimetal (IV)

- Optical conductivity neglecting vertex corrections (M=1/4):

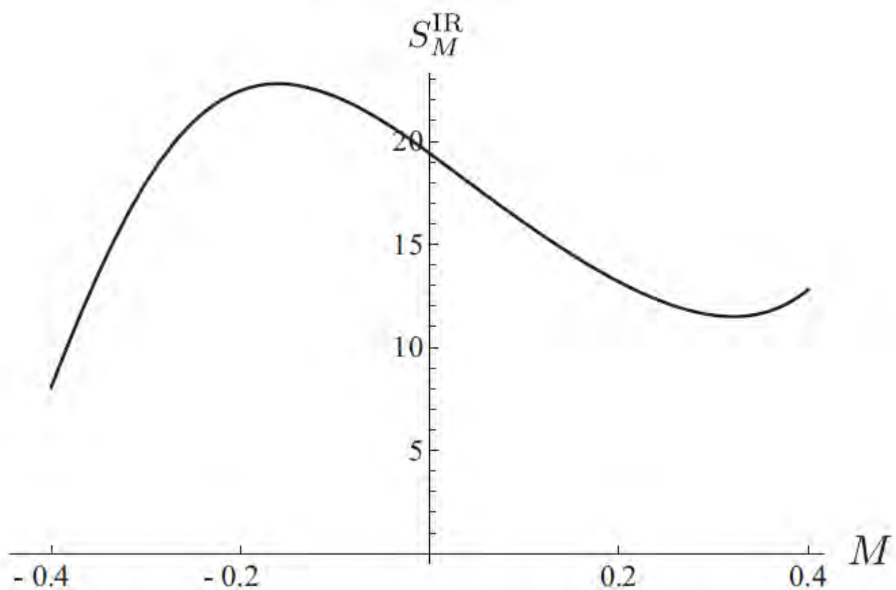
$$\omega_c \propto (g_M)^{1/(1-2M)}$$



# Interacting Dirac Semimetal (V)

- Conductivity:

$$\sigma = \lim_{\hbar\omega\beta\to 0} \mathrm{Re}\,\sigma_{xx}(\mathbf{0},\omega) \simeq \frac{e^2\omega_c}{12\pi\hbar c}\left(\frac{k_B T}{\hbar\omega_c}\right)^{3-4M} S_M^{\mathrm{IR}},$$




$$\sigma \underset{M\to 1/2}{\square} \frac{e^2 k_B T}{\pi\hbar^2 c}$$

# Interpretation of Results

# Fock model (I)

- Consider: $$Z[A] = \int d[\bar{\psi}]d[\psi] \exp\left[\frac{i}{\hbar}\Big(S_0[\bar{\psi},\psi;A] + S_\Delta[\bar{\psi},\psi]\Big)\right],$$
- with

$$S_0[\bar{\psi},\psi;A] = \int d^4x \sum_{i=1}^{N} \bar{\psi}_i(x)\Big(-i\hbar \not{D}\Big)\psi_i(x)$$

$$S_\Delta[\bar{\psi},\psi] = \int d^4x \int d^4x' \sum_{i,i'=1}^{N} \frac{\hbar g}{2N} \Delta(x-x')\bar{\psi}_i(x)\psi_{i'}(x)\bar{\psi}_{i'}(x')\psi_i(x')$$

$$\Delta(x-x') = \int \frac{d^4k}{(2\pi)^4} \frac{1}{-k^{2-\eta}} e^{ik\cdot x}.$$

# Fock model (II)

- Hubbard-Stratonovich (auxiliary-field) transformation:

$$1 = \int d[\rho] \exp\left[\frac{i}{\hbar} \sum_{i,i'=1}^{N} (\rho + \frac{\hbar g}{N} \psi_i \bar{\psi}_i || \frac{N\Delta}{2\hbar g} || \rho + \frac{\hbar g}{N} \psi_{i'} \bar{\psi}_{i'})\right].$$

- leads to

$$Z[A] = \int d[\rho] \exp\left[N \operatorname{Tr} \ln\Big(-G_\psi^{-1}[A,\rho]\Big) + N(\rho || \frac{i}{\hbar} \frac{\Delta}{2\hbar g} || \rho)\right].$$

$$G_\psi^{-1}[\rho, A](x, x') = -i\delta^4(x - x')(\partial\!\!\!/ - \frac{ie}{\hbar} A\!\!\!/) - \frac{1}{\hbar}\rho(x, x')\Delta(x - x')$$

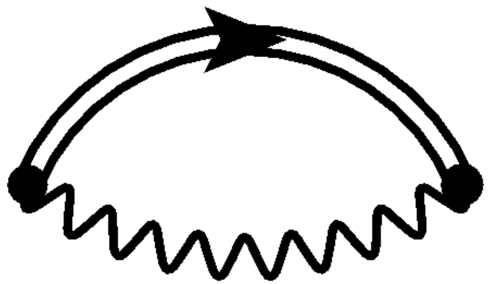
# Fock model (III)

- Large-N limit:

$$\langle \rho \rangle = \frac{g\hbar}{i} G_\psi[\langle \rho \rangle]$$

- thus:

$$G_\psi^{-1}(k) = \not{k} + ig \int \frac{d^4 q}{(2\pi)^4} \Delta(q) G_\psi(k-q)$$



# Fock model (IV)

- In the strong-coupling limit $g \to \infty$ : $G_\psi(k) = -1/\Sigma_\psi(k).$
  and we obtain

$$\Sigma_\psi(k) = \frac{\sqrt{g}}{h(\eta)} \not{k} k^{\frac{\eta}{2}}.$$

$$\frac{1}{h^2(\eta)} = \frac{\Gamma(1-\frac{\eta}{4})}{(4\pi)^2\Gamma(1+\frac{\eta}{4})}\left[\frac{\Gamma(1+\frac{\eta}{2})\Gamma(-\frac{\eta}{4})}{\Gamma(1-\frac{\eta}{2})\Gamma(2+\frac{\eta}{4})} - \frac{4+\eta}{2\eta}\frac{\Gamma(-1-\frac{\eta}{4})\Gamma(2+\frac{\eta}{2})}{\Gamma(3+\frac{\eta}{4})\Gamma(-\frac{\eta}{2})}\right].$$

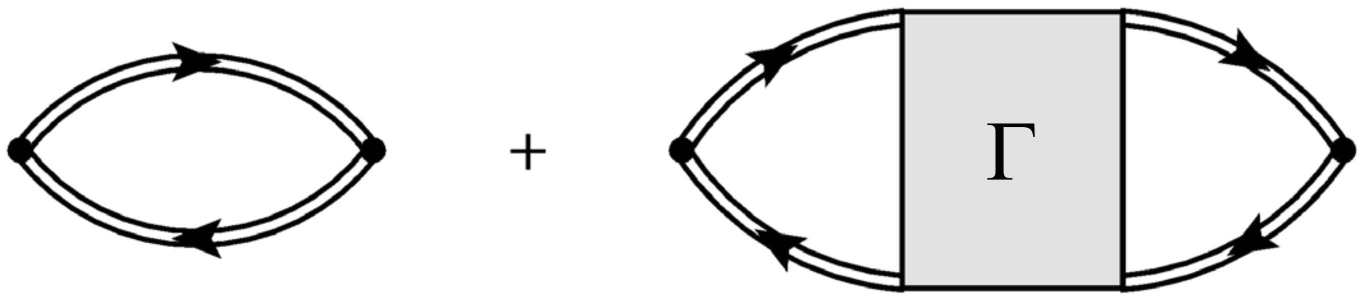
- So

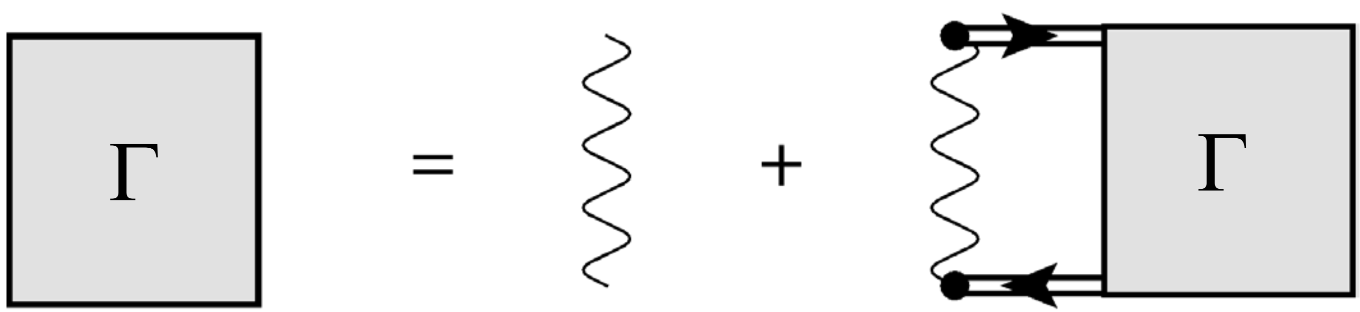
$$\eta = 4M - 2$$

$$\sqrt{g} = h(4M-2) g_M$$

# Fock model (V)

- Optical conductivity in large-N limit:

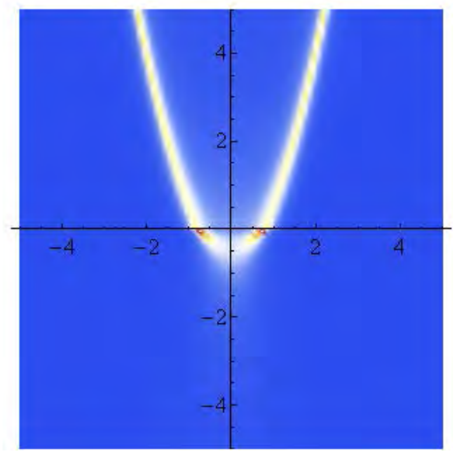


# Outlook

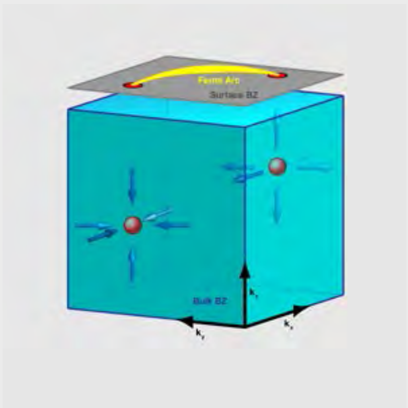
# Conclusions and Outlook (I)

- Towards ultracold fermions by using massive Dirac fermions on the boundary and a charged black brane.




- Ultracold bosons at unitarity.

- Weyl semimetals.



# Conclusions and Outlook (II)

- There are some interesting connections between spin-imbalanced fermions at unitarity and quark-gluon plasmas: