QFT, STRING THEORY and CONDENSED MATTER PHYSICS
KOLYMVARI, GREECE
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Towards collisions of inhomogeneous shockwaves in AdS

(Based on Arxiv:1407.5628)

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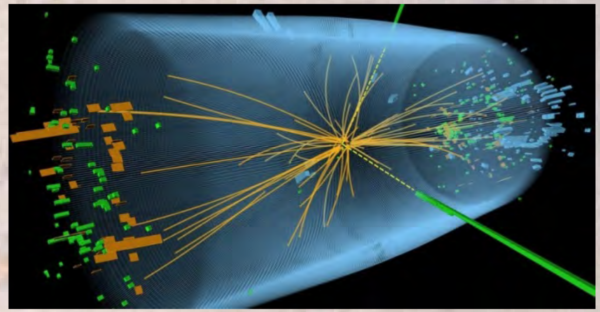
Heavy Ion Collisions

RHIC: Au-Au ($Z = 79$)

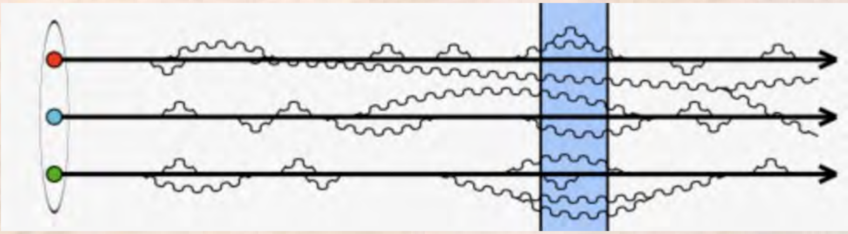
LHC: Pb-Pb ($Z = 82$) $\longrightarrow$ 1.36 TeV per nucleon
Lorentz factor > 1000

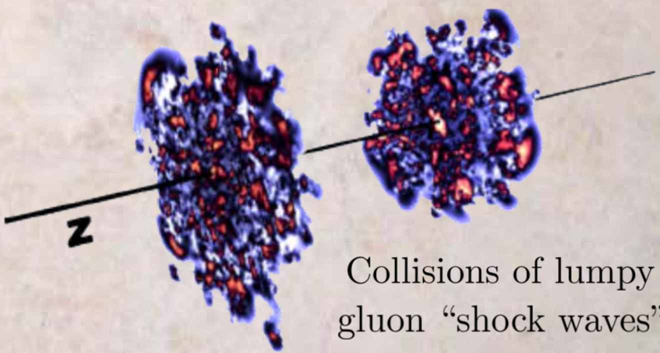


Result: formation of a *Quark Gluon Plasma*

- Thermal fluid, new state of matter
- Temperature $\sim 170 - 230$ MeV
- Lasts for $\tau \sim 10$ fm/c

Idealized dual gravitational description:
Stable AdS **black hole** at same Temp.





Collisions of lumpy gluon "shock waves"

⋆ Long timescales: gluon fluctuations are short-lived

⋆ Strong interaction timescales:
gluon fluctuations in quark background are dilated ⇨ Dynamics dominated by gluons

Out-of-equilibrium holography

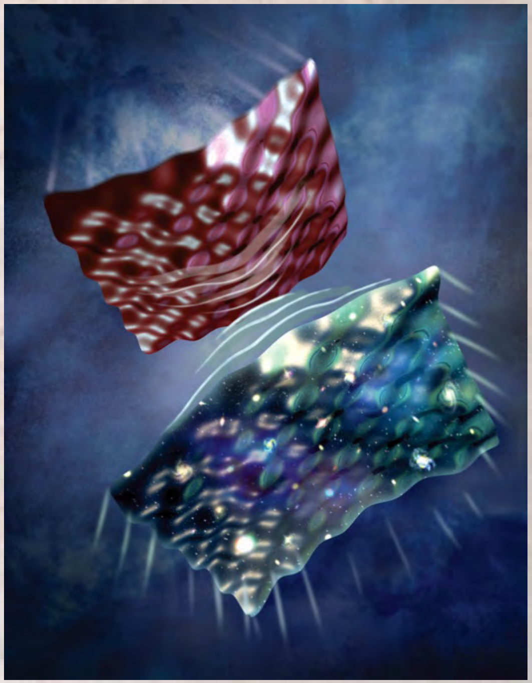
- Initial stage after the collision?

  → replace black hole by gravitational waves

<u>*Simplification:*</u>

Two planar sheets of finite thickness (and Gaussian profile), propagating toward each other at the speed of light.

Disclaimer:
$\mathcal{N} = 4$ SYM theory
in the limits
$g^2 N_c \to \infty$
$N_c \to \infty$



⋆ Gluon dynamics, no quarks ⇒ Pure GR, no strings

⋆ Lorentz contraction ⇒ Infinite planes

Output: Examine the evolution of the post-collision stress-energy tensor.

Albacete, Kovchegov, Taliotis '08

Outline

1. Ansatz and numerics abridged

2. Example: Completely homogeneous thermalization

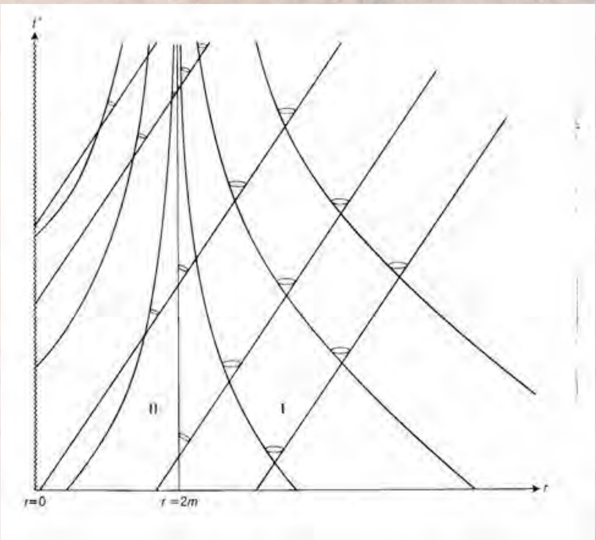
3. Review of holographic shockwave collisions

4. Transverse inhomogeneities

The characteristic formulation

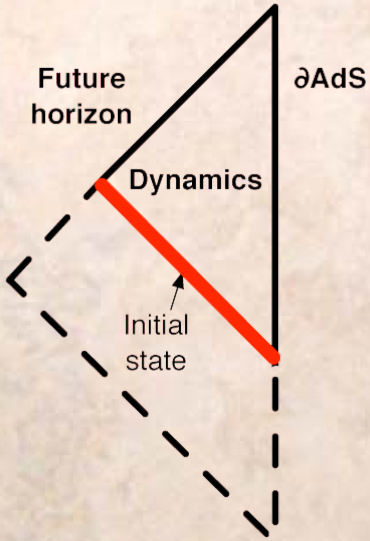
1) Eddington-Finkelstein coordinates with null holographic coord.

- ⋆ Fix part of diffeomorphism invariance
- ⋆ Caustics? Not a problem in AdS!



Bondi, Sachs '62

2) Determinant of spatial metric is a function.



3) Derivatives along outgoing null rays.

- ⋆ Fully covariant description
- ⋆ AdS feature: boundary reachable through geodesics

⇒ Most natural: start in null slice

The metric ansatz

Chesler, Yaffe '09

$$ds^2 = -A\,dt^2 + \beta\,dt\,dr + 2F_i\,dt\,dx^i + \Sigma^2 h_{ij}\,dx^i\,dx^j$$

r : Null coordinate
$r = \infty$: Boundary
t : time coordinate of boundary
$\det(h_{ij}) = 1$

Inv. under reparametrizations of r:
we fix $\beta = 2$
$\Rightarrow$ Residual gauge freedom: $r \to r + \xi(x^\mu)$

- Boundary conditions:

  Provide $A, F_i\,(t,\, r = \infty,\, x^i)$.

- Initial Condition:

  Provide $h_{ij}\,(t = 0,\, r,\, x^i)$ and EE fix the rest.

  No need for 1st time derivative.

- Solving nested linear ODEs:

$$\dot{f} = \left(\partial_t + \frac{1}{2} A\,\partial_r\right) f \quad \Longrightarrow$$

⋆ Solve for the $\dot{f}$ from EE.
⋆ EE_{ij} inv. under residual gauge, as is $\dot{f}$

$\Rightarrow$ The eqs. do not contain A.

A simple example:

Homogeneous thermalization

$$ds^2 = 2\,dt\,dr - A\,dt^2 + \Sigma^2\left(e^{-2B}dy^2 + e^{B}d\vec{x}_\perp^2\right)$$

Complete homogeneity in $x^i = (y, \vec{x}_\perp)$.
A, B, Σ functions of (t, r).

Heller, Mateos, van der Schee, Trancanelli '12

- Normalizable modes: $a_4(t)$, $b_4(t)$.
- Stress tensor:

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P_L(t) & 0 & 0 \\ 0 & 0 & P_T(t) & 0 \\ 0 & 0 & 0 & P_T(t) \end{pmatrix}$$

$$\epsilon = -\frac{3}{4}a_4$$

$$\Delta P(t) = 3b_4(t)$$

where

$$P_L(t) = \frac{\epsilon}{3} - \frac{2}{3}\Delta P(t), \quad P_T(t) = \frac{\epsilon}{3} + \frac{1}{3}\Delta P(t)$$

Resolution of homogeneous case

- Einstein Equations:

$$\Sigma\dot{\Sigma}' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0$$

$$2\Sigma\dot{B}' + 3\Sigma'\dot{B} + 3B'\dot{\Sigma} = 0$$

$$A'' + 3B'\dot{B} - 12\Sigma'\dot{\Sigma}/\Sigma^2 + 4 = 0$$

$$2\ddot{\Sigma} + \dot{B}^2\Sigma - A'\dot{\Sigma} = 0$$

$$2\Sigma'' + B'^2\Sigma = 0$$

Derivatives along ingoing/outgoing geodesics:

$$h' = \partial_r h, \quad \dot{h} = \partial_t h + \frac{1}{2}A\partial_r h$$

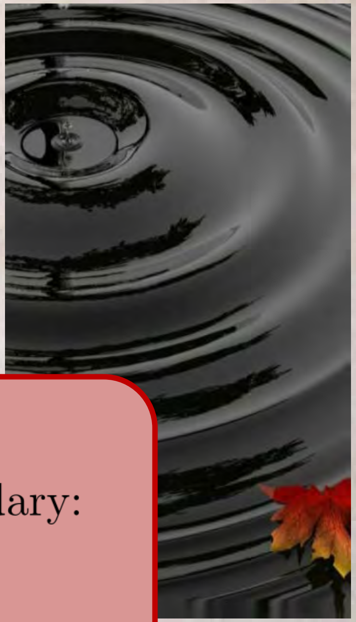
- Initial conditions: $B(t=0,r)$
  - ⋆ Procedure: $B \to \Sigma \to \dot{\Sigma} \to \dot{B} \to A \to \partial_t B$
- Boundary conditions: a_4

Constraint

To be imposed at the boundary:

$$\partial_t a_4 = 0$$

Conservation of energy



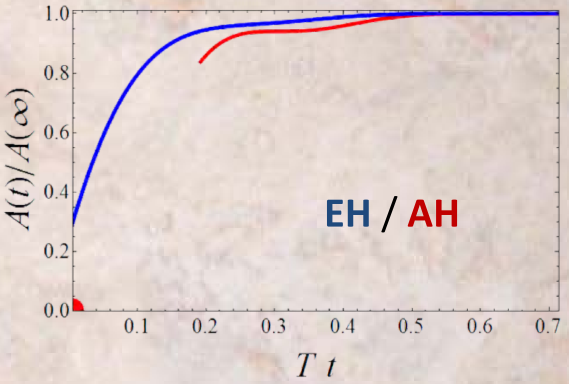
Horizons

- **Event Horizon**: Causal boundary of the black hole.
- **Apparent Horizon**: Surface where outgoing light rays are trapped.

Out of equilibrium, they do not coincide.

$$\partial_t r_{\mathrm{EH}}(t) - \frac{1}{2} A(t, r_{\mathrm{EH}}(t)) = 0$$

$$\dot{\Sigma}(t, r_{\mathrm{AH}}(t)) = 0$$



EH is teleological: Its eq needs to be supplemented with Final Condition: $r_{\mathrm{EH}}(t) \underset{t\to\infty}{\longrightarrow} \pi T$

If AH exists, it always lies inside EH.

Important because:

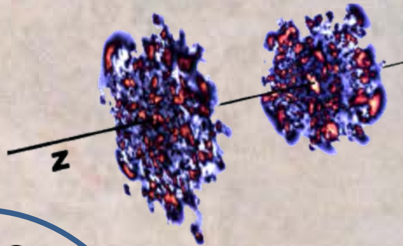
⋆ Their area is an easy-to-compute measure of entropy.
⋆ We need to fix their position for numerical evolution.

Collision of Shockwaves

$$ds^2 = 2\,dt\,dr - A\,dt^2 + 2F\,dt\,dy + \Sigma^2\left(e^{-2B}dy^2 + e^B d\vec{x}_\perp^{\,2}\right)$$

Homogeneity only in $\vec{x}_\perp$.
A, B, Σ, F functions of (t, r, y).

Chesler, Yaffe '11



- Normalizable modes: $a_4(t,y)$, $b_4(t,y)$, $f_4(t,y)$
- Stress tensor:

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \epsilon & s & 0 & 0 \\ s & P_L & 0 & 0 \\ 0 & 0 & P_T & 0 \\ 0 & 0 & 0 & P_T \end{pmatrix}$$

$$\epsilon = -\frac{3}{4}a_4$$

$$\Delta P = 3b_4$$

$$s = f_4$$

Resolution

⋆ Procedure: $B \to \Sigma \to F \to \dot{\Sigma} \to \dot{B} \to A \to \partial_t B$

⋆ *Constraints*: eqs. for $\ddot{S}$, $\dot{F}'$ $\Longrightarrow$ Boundary evolution:

$$\partial_t a_4 = -\frac{4}{3}\partial_y f_4$$

$$\partial_t f_4 = -\frac{1}{4}\partial_y a_4 - 2\partial_y b_4$$

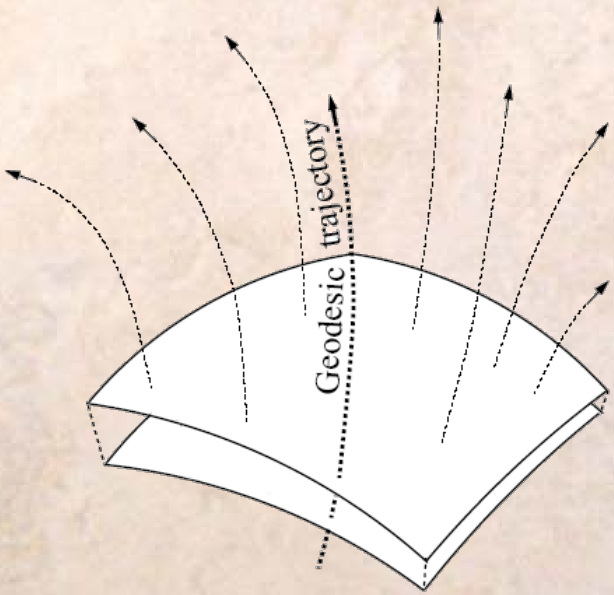
Initial condition

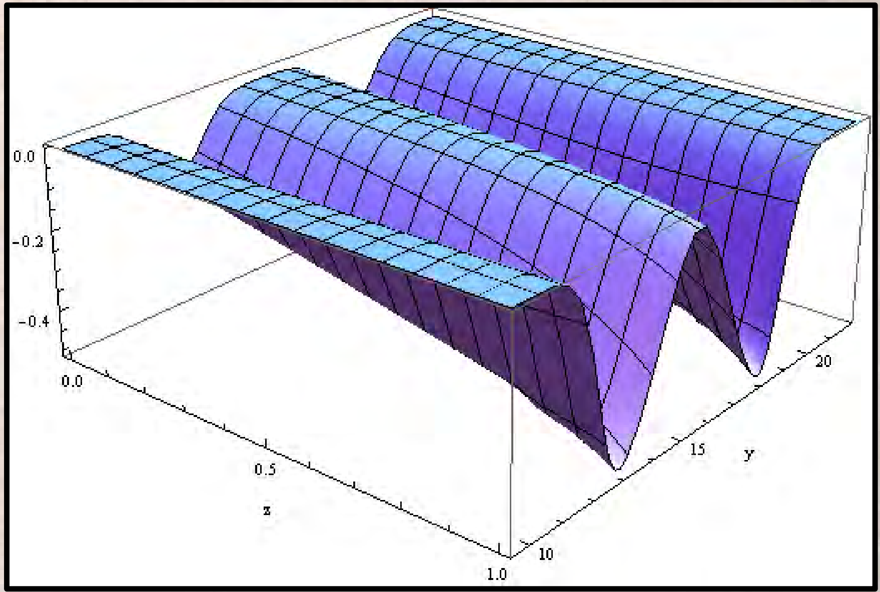
Single shock in Fefferman-Graham coord: Gaussian

$$ds^2 = \tilde{r}^2\left(-d\tilde{y}_+ d\tilde{y}_- + d\vec{x}_\perp^2\right) + \frac{1}{\tilde{r}^2}\left(d\tilde{r}^2 + h(\tilde{y}_\pm)d\tilde{y}_\pm^2\right) \quad \text{where} \quad \tilde{y}_\pm = \tilde{t} \pm \tilde{y}$$

then change into Eddington-Finkelstein coordinates, identify B:

$$g_{rr} = g_{rz} = 0,\ g_{rt} = 1 \qquad \longrightarrow \qquad B = \log\left(g_{x_\perp x_\perp}\right) - 2\log\Sigma, \quad \Sigma^6 r^4 = J^2|g|^2$$





Fixing $\begin{cases}\text{the residual gauge}\\ \text{the apparent horizon}\end{cases}$

Assume AH to lie at a fixed, constant $r = r_{\rm AH}$. Then,

$$dV = d\left(\int_{y_0}^{y} \Sigma^3(r,t,y)\,dy\right) = 0 \quad \text{along normal null rays}$$

($ds^2 = 0$ and extremize $dr - \frac{1}{2}A\,dt$)

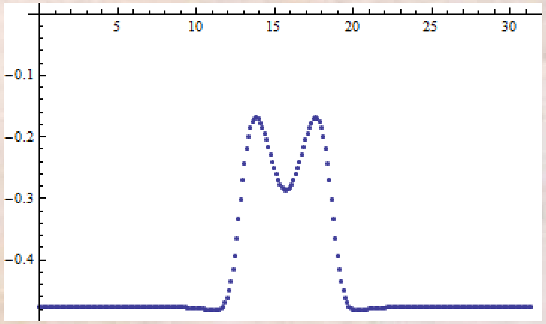
Result:
$$\boxed{3\Sigma^2\dot{\Sigma} - \partial_y\left(\Sigma F e^{2B}\right) + \frac{3}{2}e^{2B}F^2\Sigma' = 0 \quad \text{at} \quad r = r_{\rm AH}}$$

On the other hand: $ds^2 = -A\,dt^2 + 2\,dt\,dr + 2F_i\,dt\,dx^i + \Sigma^2 h_{ij}\,dx^i\,dx^j$

$\Rightarrow$ Residual gauge freedom: $r \to r + \xi(x^\mu)$.

If $r_{\rm AH}(t,y)$, modify:
$F \to F + \frac{\partial r_h}{\partial y}$

$\Rightarrow$ Choose $\xi(t,y)$ so that $r_{\rm AH} = 1$.



Facts and Motivation

- Linearizing the far-from-eq. state around the final state provides a *surprisingly accurate* description: always within 20%.

Heller, Mateos, van der Schee, Trancanelli '12

- Plasma thermalizes *very quickly*: hydrodynamics is applicable within a time $\tau_{iso} \sim 1/T$.

Chesler, Yaffe '11

- **Including radial flow**,
  - Fluctuations spread out rather quickly.
  - Stress tensor of the fluctuation, governed by hydrodynamics within 0.4 fm.

Van der Schee '13

Why transverse dynamics might matter

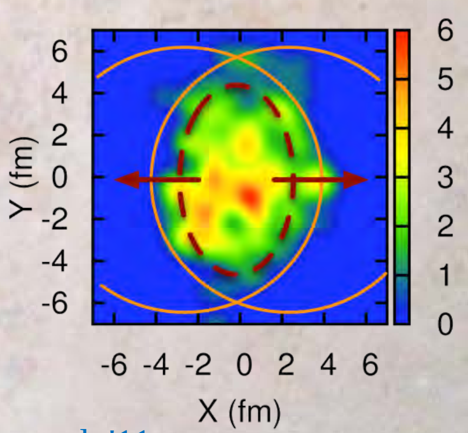
⋆ The experiments are not homogeneous at all.

⋆ Generalize the spectrum of QNM to non-zero momentum.

⋆ See if transverse expansion rate is faster or slower.

⋆ Make contact with elliptic flows, etc...

⋆ Since symmetry is not forced, we may see turbulence.



Sorensen et al. '11

Collision of inhomogeneous shockwaves

$$ds^2 = 2\,dt\,dr - A\,dt^2 + 2dt\,(F\,dy + G\,dx_1) +$$
$$\Sigma^2\left[e^{C-2B}\cosh D\,dy^2 + e^{B-C}\cosh D\,dx_1^2 + 2e^{-B/2}\sinh D\,dy\,dx_1 + e^B dx_2^2\right]$$

- ⋆ We keep the determinant given by $-\Sigma^6$.
- ⋆ No homogeneity.
- ⋆ A, B, Σ, F, C, D, G functions of (t, r, y, x_1).

Simplification:

$$h(t,r,y,x_1) \to h_0(t,r,y) + \epsilon\, e^{ikx_1}\,\delta h(t,r,y) \qquad k \in \mathbb{R}$$

where $C_0 = 0,\ D_0 = 0,\ G_0 = 0.$

- Input functions: $B_0,\ \delta B,\ \delta C,\ \delta D$

Free to choose ($\delta B,\ \delta C,\ \delta D$)

Shocks in EF coord. (B_0)

Collision of inhomogeneous shockwaves

- Stress tensor:

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix} \epsilon & s_y & s_x & 0 \\ s_y & P_y & \mathcal{T} & 0 \\ s_x & \mathcal{T} & P_{x_1} & 0 \\ 0 & 0 & 0 & P_{x_2} \end{pmatrix} \quad \text{where} \quad \begin{cases} \epsilon \sim a_4 \\ s_y \sim f_4 \\ s_x \sim g_4 \\ \mathcal{T}, \Delta P_i \sim b_4, c_4, d_4 \end{cases}$$

- Normalizable modes: $a_4(t,y)$, $b_4(t,y)$, $c_4(t,y)$, $d_4(t,y)$, $f_4(t,y)$, $g_4(t,y)$.

- Boundary evolution:

$$\partial_t a_4 = -\frac{4}{3}\left(\partial_y f_4 + \partial_{x_1} g_4\right)$$

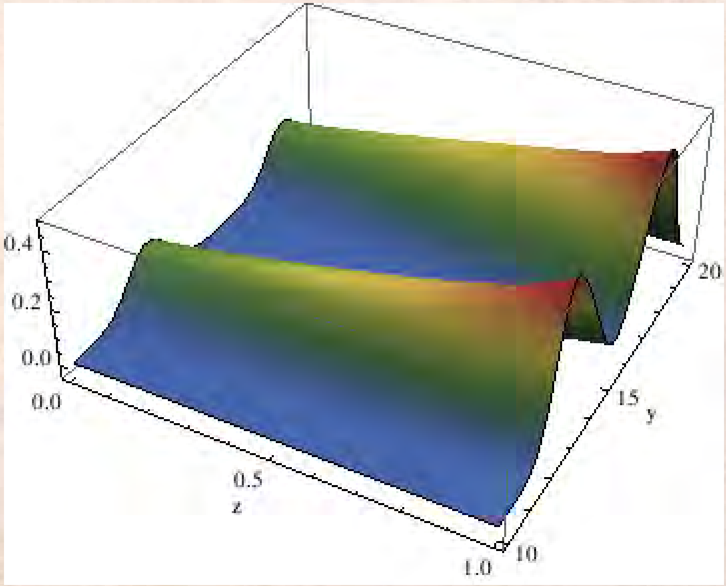
$$\partial_t f_4 = \partial_{x_1} d_4 - \frac{1}{4}\partial_y\left(a_4 + 8b_4 - 4c_4\right)$$

$$\partial_t g_4 = \partial_y d_4 - \frac{1}{4}\partial_{x_1}\left(a_4 + 4c_4 - 4b_4\right)$$

Animations

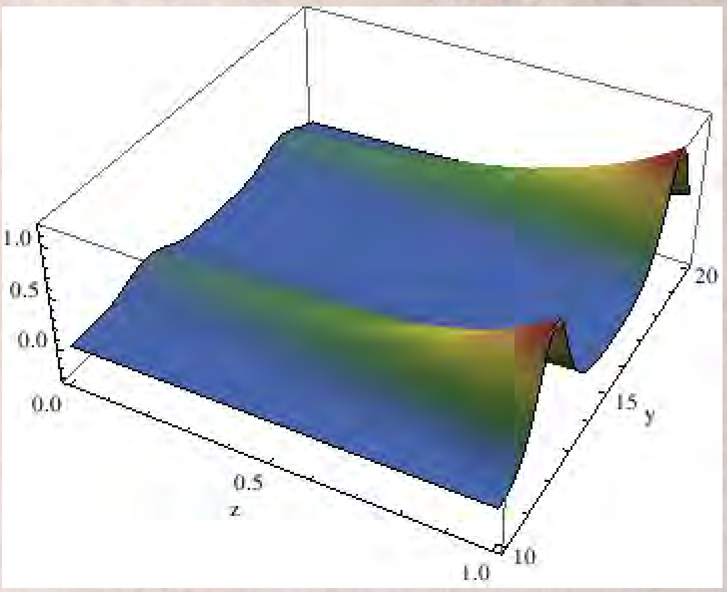
Dynamic background

$B(t, z, y)$



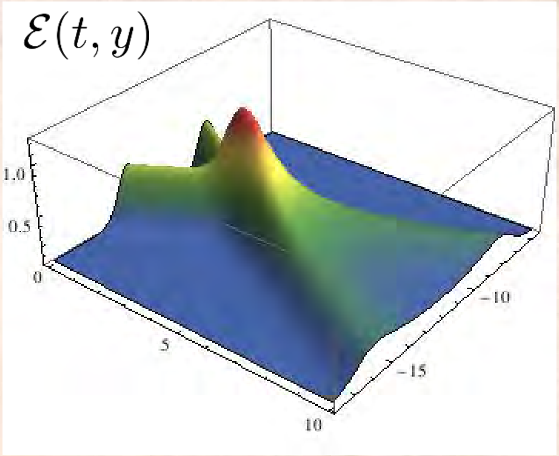
Perturbation

$\delta B(t, z, y)$

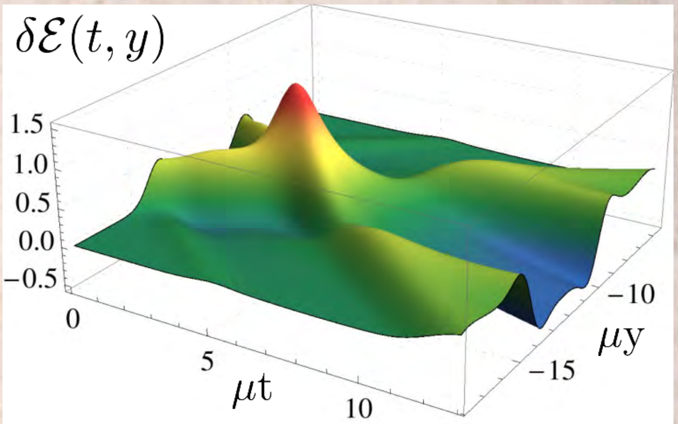


Results

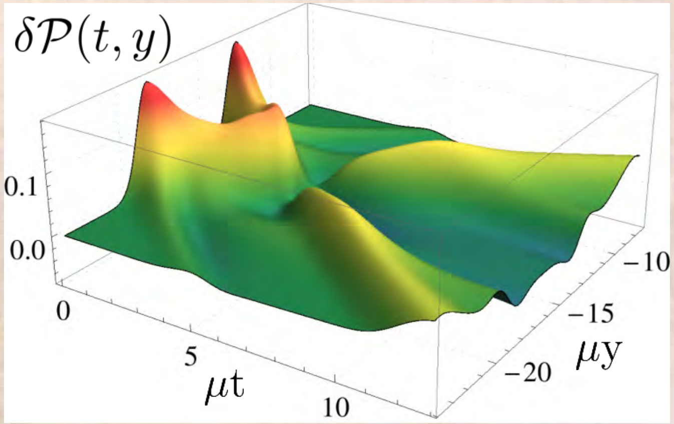
Contributions to the stress-energy tensor:



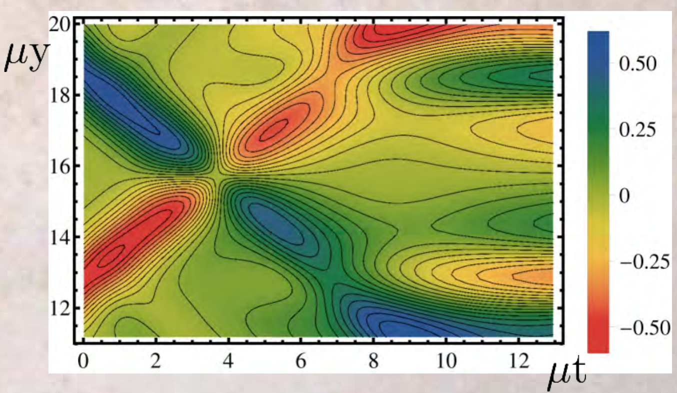
$$+\ \epsilon\ e^{ikx}$$



Inhom. on pressure anisotropy ($k = 0.2$):



Inhomogeneity on longitudinal energy flux, $\delta\mathcal{S}_y$ ($k = 0.5$):



Outlook:

- Does the evolution of the fluctuations obey hydrodynamics?
- Can we read off quasinormal modes with $k \neq 0$ of the BH?
- How much slower is the equilibration of fluctuations?
- Which modes are relevant/irrelevant depending on the value of k?

Thank you!

Technical Specs:

- 30 Chebychev points in z.
- 250 Fourier points in y.
- A timestep $dt = 0.0002$.
- Initial width 0.75, separation 2.6 (units of μ).
- Background energy density $\delta = 0.075$.

Convergence check: