A holographic Kondo model: RG flow and time dependence

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MAX-PLANCK-GESELLSCHAFT

Kondo effect

Kondo effect

Screening of a magnetic impurity by conduction electrons at low temperatures

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Metals: Fermi liquid + impurities: $\rho \sim \rho_0 + T^2$

In the presence of magnetic impurities: $\rho \sim -\log(T)$

Holographic Kondo Model: Motivation

- Original Kondo problem well-understood in field theory
- Open question: Magnetic impurity coupled to strongly coupled electron gas (Luttinger liquid)
- Here: Realization in gauge/gravity duality
- Describes RG flow, condensation process
- Possible extensions:
  Backreaction, Time dependence, Kondo lattices

Holographic Kondo Model

1. Kondo effect: Physics, CFT approach, large N
2. Holographic model: Similarities and differences
3. Backreaction, time dependence

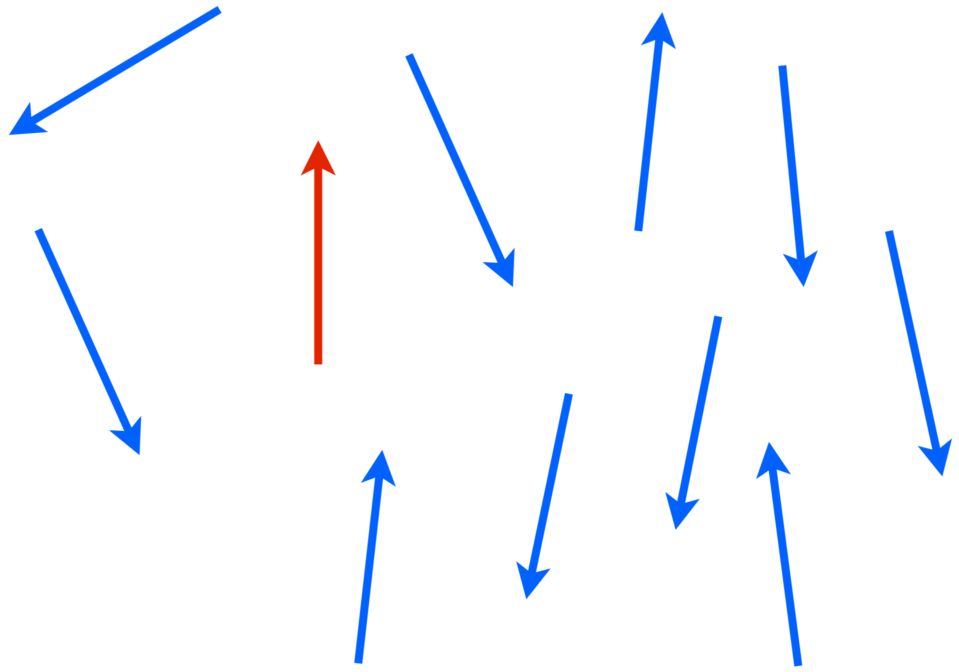
Based on joint work with

C. Hoyos (Tel Aviv Univ.), A. O'Bannon (DAMTP Cambridge),
J. Wu (NCTS Taiwan)

arXiv 1310.3271, published in JHEP

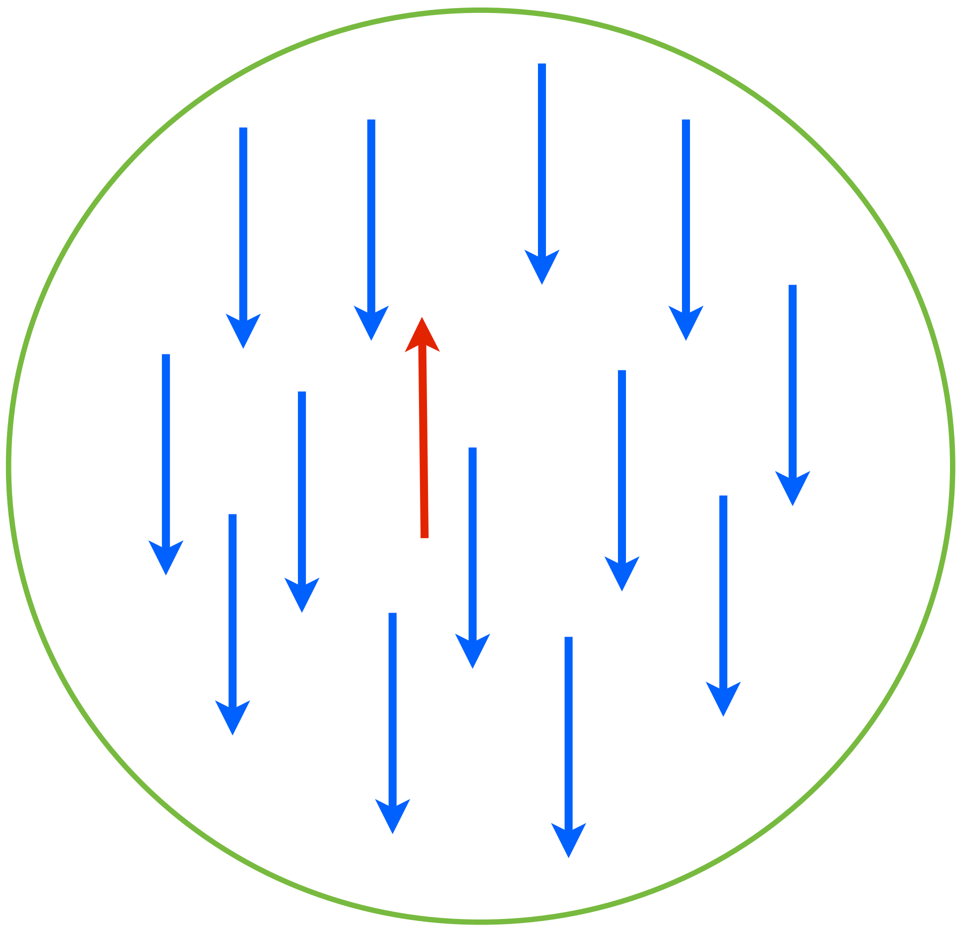
Physics of the Kondo effect

UV



Free electrons
+
impurity spin

IR



Impurity screened

Kondo model

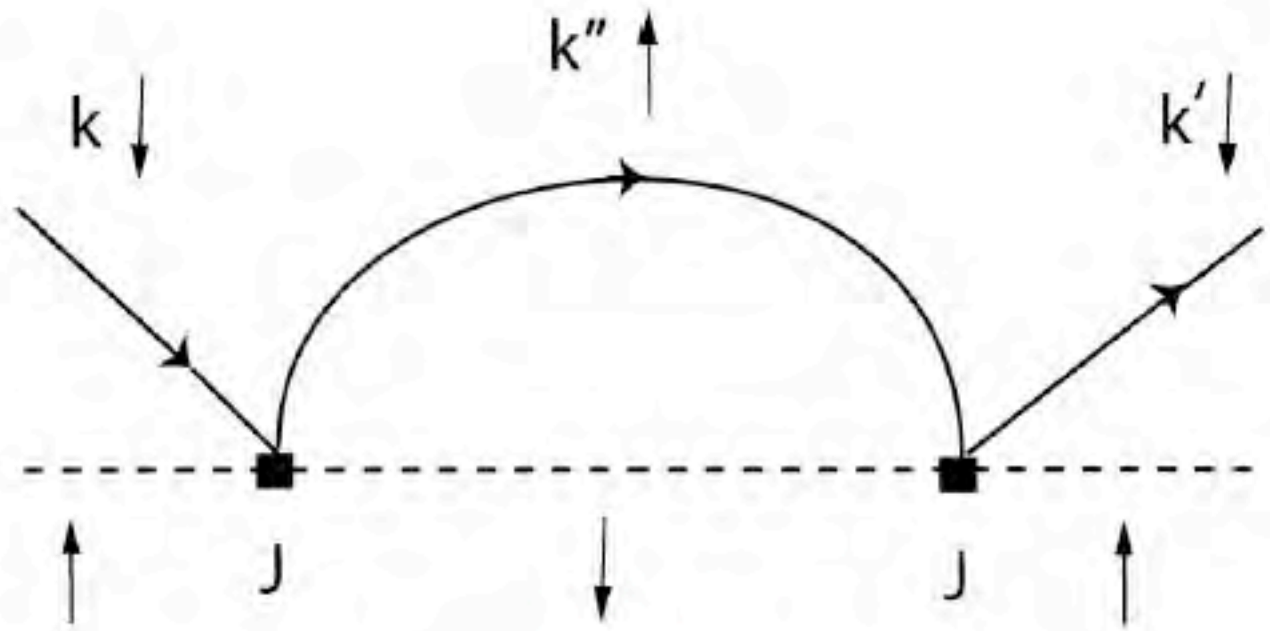
Kondo ’64, Affleck+Ludwig ‘90s

$$H = \frac{v_F}{2\pi}\psi_L^\dagger i\partial_x \psi_L + v_F \lambda_K \delta(x)\vec{S}\cdot\psi_L^\dagger \frac{1}{2}\vec{\tau}\psi_L$$

Free electrons

Local interaction
with magnetic impurity

Scattering with magnetic impurities



$$\rho \sim \rho_0 \left(1 + \kappa \log \frac{T}{|\epsilon - \epsilon_F|}\right)$$

Antiferromagnetic coupling $\kappa < 0$

Logarithmic behaviour at low temperatures

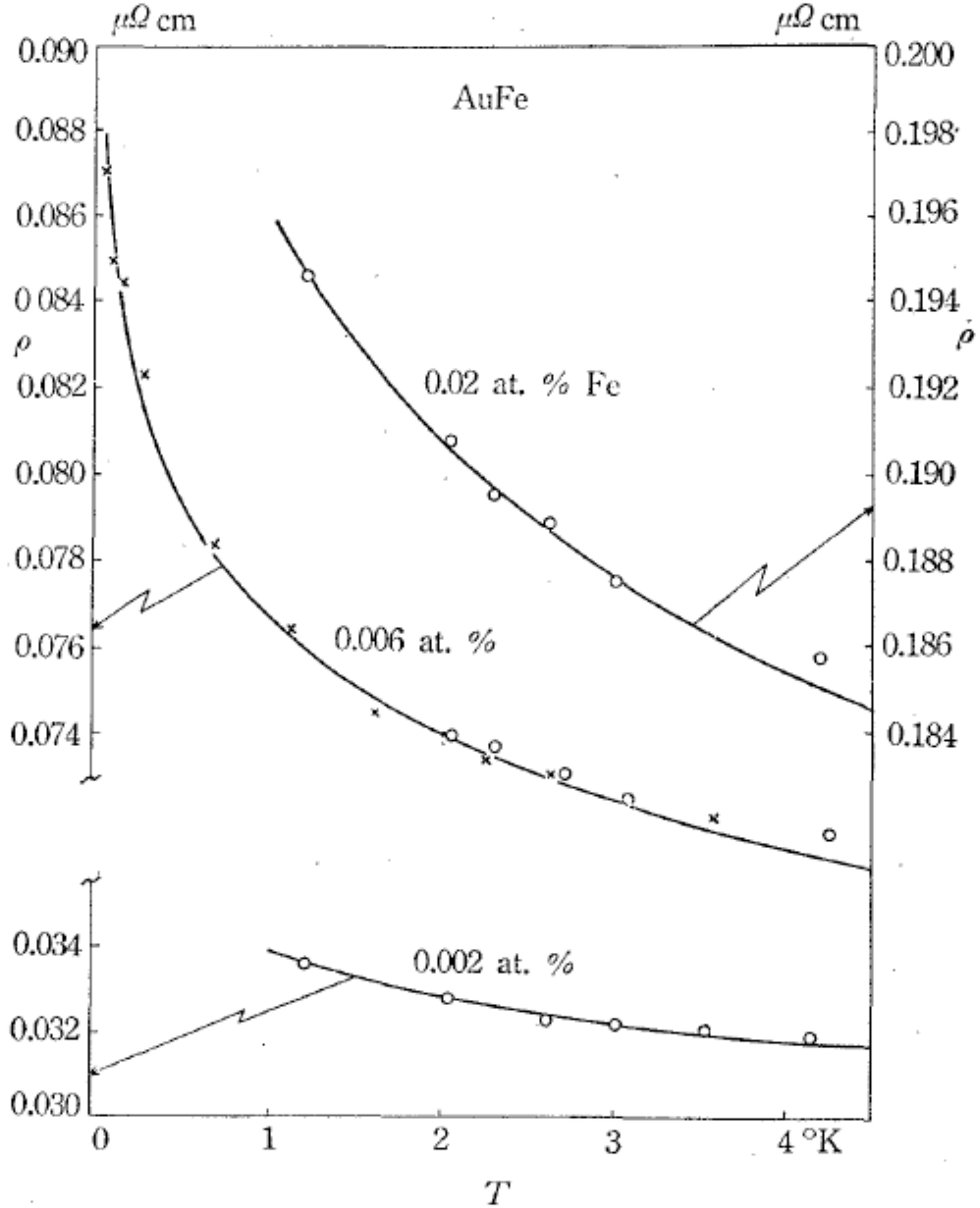


Fig. 1. Comparison of experimental and theoretical ρ–T curves for dilute AuFe alloys.

Jun Kondo:

- Progress of Theoretical Physics
- Volume 32, Issue 1
- Pp. 37-49

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IR theory is strongly coupled!

$$T_K \sim \Lambda_{\mathrm{QCD}}$$

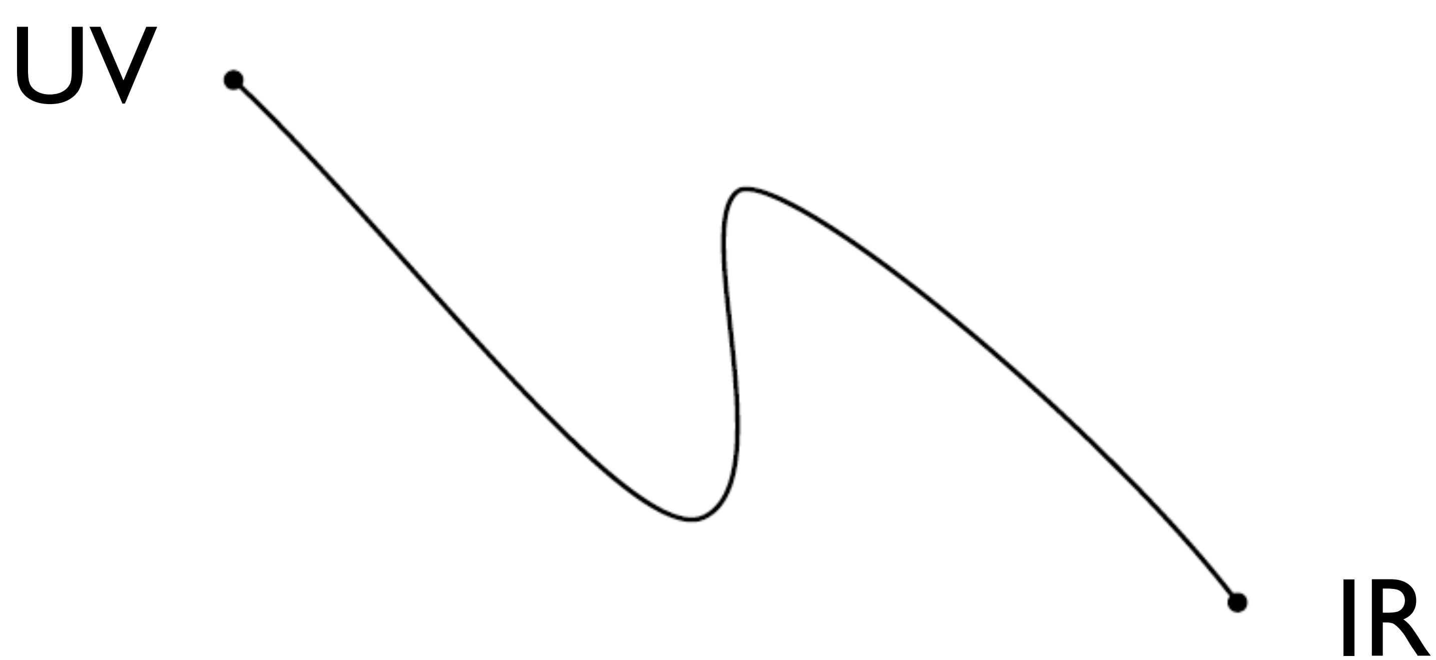
RG flow

The Kondo model was decisive in the development of the RG formalism. (Wilson)

Negative beta function: $\beta_\lambda \propto -\lambda^2 + \mathcal{O}(\lambda^3)$

UV fixed point: Free theory   Asymptotic freedom

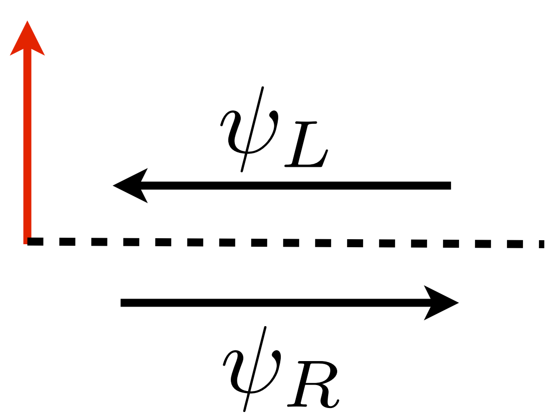
In some cases, the model flows to a strongly coupled IR fixed point.

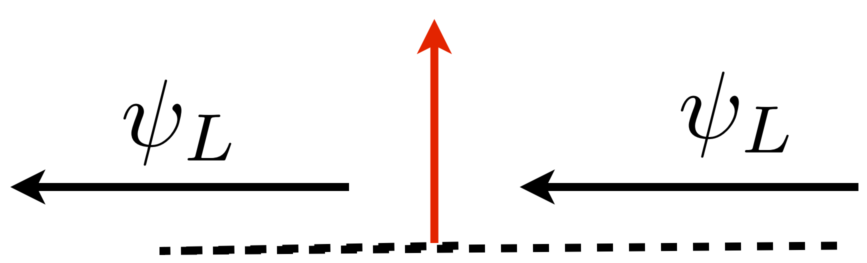


CFT approach

Affleck+Ludwig, 90's

- One-impurity problem: Spherical symmetry
- Reduces to 1+1-dimensional system with boundary
- Consider only left-movers





CFT approach

Compare CFT's at UV and IR fixed points

UV: Free fermions, boundary condition $\psi_- = \psi_+$

IR: Free fermions, boundary condition $\psi_- = -\psi_+$

Change in boundary condition induces change in spectrum

Generalizations

Generalizations

Spin $SU(N)$, k channels $SU(k)$, Charge $U(1)$

Kac-Moody algebra: $SU(N)_k \times SU(k)_N \times U(1)$

$$[J^a_n, J^b_m] = i f^{abc} J^c_{n+m} + k \frac{n}{2} \delta^{ab} \delta_{n,-m}$$

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Sugawara construction:
Separation of spin, channel and charge currents

$$H = \frac{1}{2\pi(N+k)} J^a J^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi N k} J^2 + \lambda_K \, \delta(x) \, \vec{S} \cdot \vec{J}$$

IR CFT

- Redefinition of spin current:

$$\mathcal{J}^a \equiv J^a + \pi(N+k)\lambda_K \delta(x) S^a$$

- Critical coupling

$$\lambda_K = \frac{2}{N+k}$$

- Hamiltonian:

$$H = \frac{1}{2\pi(N+k)}\mathcal{J}^a\mathcal{J}^a + \frac{1}{2\pi(k+N)}J^A J^A + \frac{1}{4\pi Nk}J^2$$

No impurity!

[Affleck & Ludwig '95]

- IR CFT = UV CFT with shifted spectrum
- IR spin representations = UV spin representations + impurity spin

Critical, under- and overscreening

Example of $SU(2)_k$:

- Underscreening: $2s_{\mathrm{imp}} > k$
  Fermi liquid + impurity of spin $|s_{\mathrm{imp}} - k/2|$

- Critical screening: $2s_{\mathrm{imp}} = k$ IR fixed point: k free left-movers

- Overscreening: $2s_{\mathrm{imp}} < k$

  Non-trivial IR fixed point: non-Fermi liquid behavior

Qualitatively similar for higher spin

Large N: Slave fermions

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Spin of impurity: Young tableau with Q boxes

$S^a = \chi^\dagger T^a \chi$ Totally antisymmetric representation

$$\chi^\dagger \chi = q, \quad Q = q/N$$

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$\chi^\dagger \chi = q, \;\; Q = q/N$

Kondo coupling as double-trace deformation:

$$\begin{aligned}
\lambda_K \, \delta(x) \, J^a S^a &= \lambda_K \, \delta(x) \, \left(\psi_L^\dagger T^a \psi_L\right) \left(\chi^\dagger T^a \chi\right) \\
&= \frac{1}{2} \lambda_K \, \delta(x) \left(\psi_L^\dagger \chi\right) \left(\chi^\dagger \psi_L\right) \\
&= \frac{1}{2} \lambda_K \, \delta(x) \, \mathcal{O} \mathcal{O}^\dagger
\end{aligned}$$

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&= \frac{1}{2} \lambda_K\, \delta(x) \left(\psi_L^\dagger \chi\right) \left(\chi^\dagger \psi_L\right) \\
&= \frac{1}{2} \lambda_K\, \delta(x)\, \mathcal{O}\mathcal{O}^\dagger
\end{aligned}$$

$\mathcal{O}$ $SU(N)$ singlet, charged under $U(N_f) \times SU(k) \times U(1)$

$[\mathcal{O}] = 1/2$

Large N: Slave fermions

Sachdev, Senthil, Voijta cond-mat/0209144

Kondo effect corresponds to condensation of $\mathcal{O} = \psi_L^\dagger \chi$

Mean field transition:

$$T > T_K, \quad \langle \mathcal{O} \rangle = 0, \quad SU(k) \times U(N_f) \times U(1)$$

$$T < T_K, \quad \langle \mathcal{O} \rangle \neq 0, \quad SU(k) \times U(N_f) \times U(1) \to U(1)_D$$

2. Holographic model

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Why holography?

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Gravity dual for well-understood RG flow

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Note however: Holographic model and standard Kondo model have significant differences

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Extensions: Time dependence, entanglement entropy

Impurities in string holographic models

Supersymmetric defects with localized fermions

- D5/D3 $AdS_2 \subset AdS_5$
  [Kachru, Karch, Yaida] [Harrison, Kachru, Torroba]
- M2/D2 in ABJM $AdS_2 \subset AdS_4$
  [Jensen, Kachru, Karch, Polchinski, Silverstein]
- D6 in ABJM $AdS_2 \subset AdS_4$, [Benincasa, Ramallo]
  with backreaction [Itsios, Sfetsos, Zoakos]
- $D(8-p)$ in Dp background $S^{7-p} \subset S^{8-p}$ [Benincasa, Ramallo]
  other sphere wrappings [Karaiskos, Sfetsos, Tsatis]
- Spectrum of Wilson loops
  [Mueck] [Faraggi, Pando Zayas] [Faraggi, Mueck, Pando Zayas]

New in our model:

- Model for entire RG flow
- Double-trace deformation
- Kondo temperature arises naturally
- Impurity screening
- Phase shift
- Power-law scalings at low T

Top-down probe brane model

- based on D7- and D5-probe branes in D3-brane background

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

Defect theory

Harvey and Royston 0709.1482, 0804.2854
Buchbinder, Gomis, Passerini 0710.5170

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

N_7 (1+1)-dimensional chiral fermions ψ_L

$$S_{3\text{-}7} = \int dx^+ dx^- \psi_L^\dagger \left(i\partial_- - A_-\right) \psi_L$$

Preserves 1/4 of SUSY

D5-brane probes

D5-branes: Impurity

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_5 D5	X				X	X	X	X	X	

Skenderis, Taylor hep-th/0204054

Camino, Paredes, Ramallo hep-th/0104082

Gomis and Passerini hep-th/0604007

(0+1)-dimensional fermions χ

$$\chi^\dagger \chi = q$$

Kondo interaction: Complex scalar

	0	1	2	3	4	5	6	7	8	9
N_5 D5	X				X	X	X	X	X	
N_7 D7	X	X			X	X	X	X	X	X

Dual operator: $\mathcal{O} \equiv \psi_L^\dagger \chi$

Kondo interaction: Complex scalar

	0	1	2	3	4	5	6	7	8	9
N_5 D5	X				X	X	X	X	X	
N_7 D7	X	X			X	X	X	X	X	X

Dual operator: $\mathcal{O} \equiv \psi_L^\dagger \chi$

TACHYON

$$m^2_{\text{tachyon}} = -\frac{1}{4\alpha'}$$

D5 becomes magnetic flux in the D7

Holography - Top-down model for Kondo

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N_c D3	•	•	•	•	–	–	–	–	–	–
N_7 D7	•	•	–	–	•	•	•	•	•	•
N_5 D5	•	–	–	–	•	•	•	•	•	–

- 3-7 strings = chiral fermions (current algebra)
- 3-5 strings = slave fermions
- 5-7 strings = bifundamental scalar (tachyon)

J.E., Hoyos, O'Bannon, Wu 1310.3271

Near-horizon limit

D3: $\mathrm{AdS}_5 \times S^5$

D7: $\mathrm{AdS}_3 \times S^5 \longrightarrow$ CS A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $\mathrm{AdS}_2 \times S^4 \longrightarrow \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar } \Phi \text{ dual to } \psi^\dagger \chi \end{cases}$

Bottom-up model

Action

$$S = S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi}\int \mathrm{tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right),$$

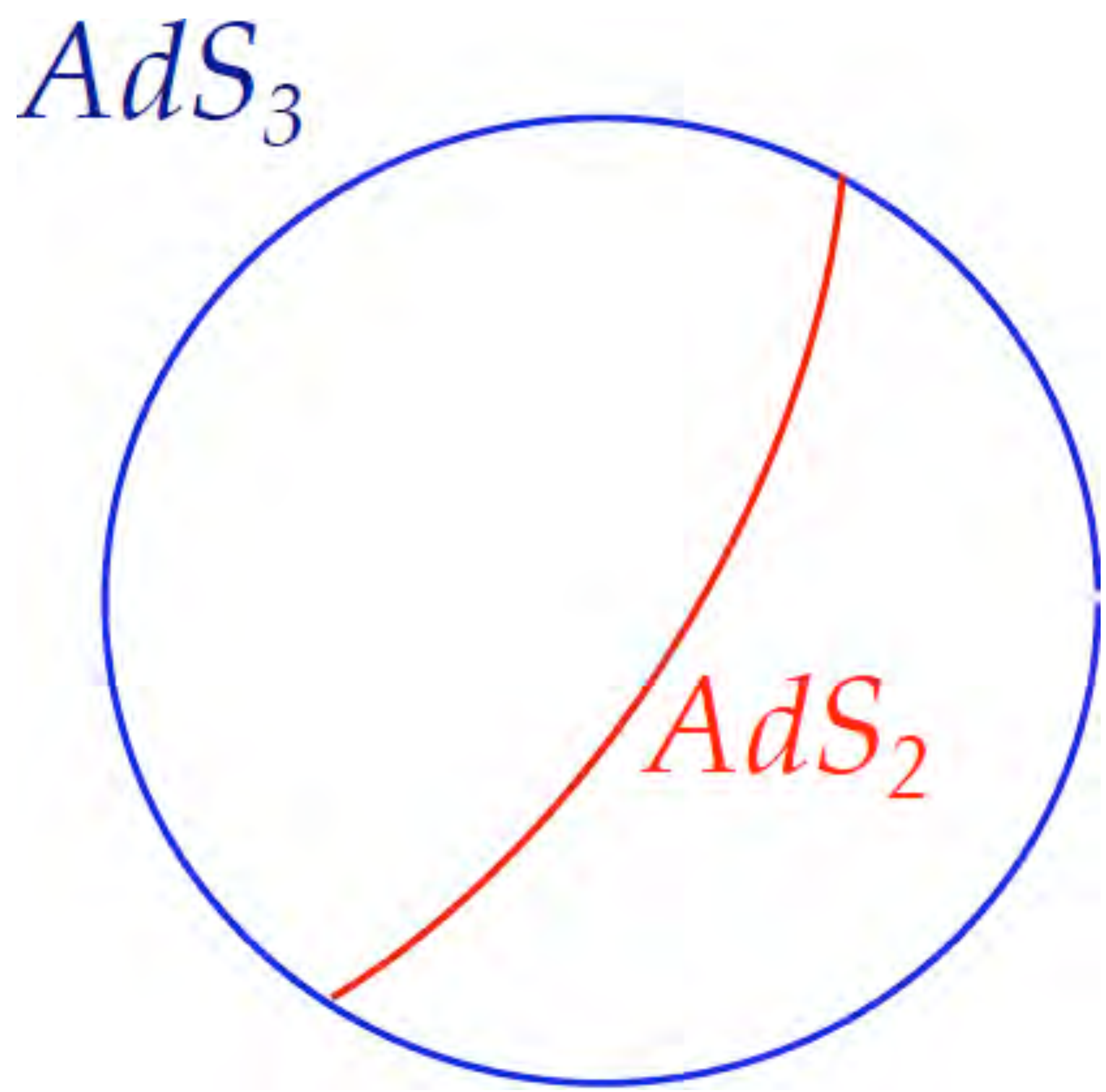
$$S_{AdS_2} = -N\int d^3x\,\delta(x)\sqrt{-g}\left[\frac{1}{4}\mathrm{tr} f^{mn} f_{mn} + g^{mn}\left(D_m\Phi\right)^\dagger D_n\Phi + V\left(\Phi^\dagger\Phi\right)\right]$$

BTZ black hole

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\,dt^2 + dx^2\right)$$

$$h(z) = 1 - z^2/z_H^2 \qquad\qquad T = 1/(2\pi z_H)$$

Defect space



Bottom-up model

Spin group SU(N) gauged

$U(k)_N$ Chern-Simons field dual to channel $SU(k)_N$, charge U(1) current

k=1

Defect Yang-Mills field encodes impurity spin representation, $a_t(z) = \frac{Q}{z} + \mu$

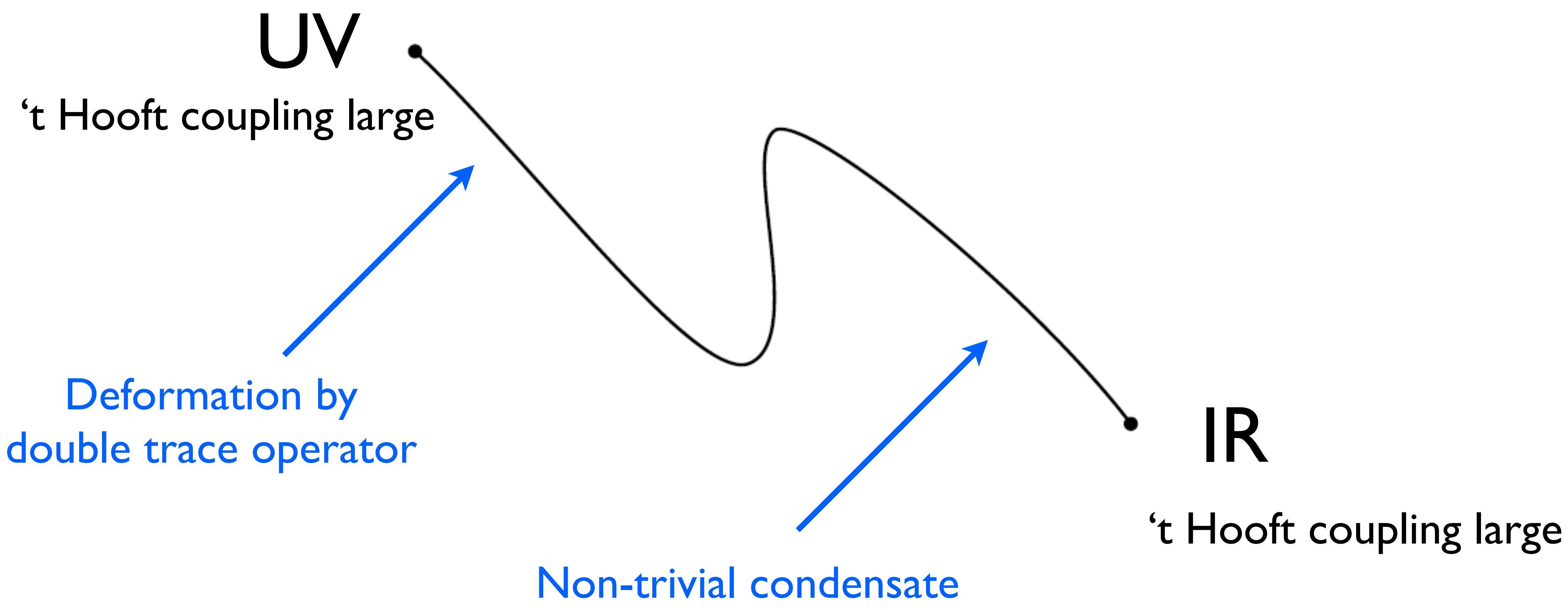
Φ complex scalar bifundamental under the two gauge fields, dual to $\mathcal{O} = \psi_L^\dagger \chi$

Potential $V(\Phi) = m^2 \Phi^\dagger \Phi$, mass at Breitenlohner-Freedman bound

Double-trace operator marginal

$$\Phi = z^{1/2}(\alpha \log(z) + \beta)$$

RG flow


UV
't Hooft coupling large
Deformation by
double trace operator
Non-trivial condensate
IR
't Hooft coupling large

Double trace deformation by $\mathcal{O}\mathcal{O}^\dagger$

$$\Phi = z^{1/2}(\alpha \log(z) + \beta)$$

Witten hep-th/0112258

$$\alpha = \kappa\beta$$

Renormalization:

$$\Phi = z^{1/2}\beta_0(\kappa_0 \log(\Lambda z) + 1) = z^{1/2}\beta(\kappa \log(\mu z) + 1)$$

Running coupling

$$\kappa = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{\mu}\right)}$$

Dynamical scale: $\Lambda_K = \Lambda e^{1/\kappa_0}$

Kondo coupling

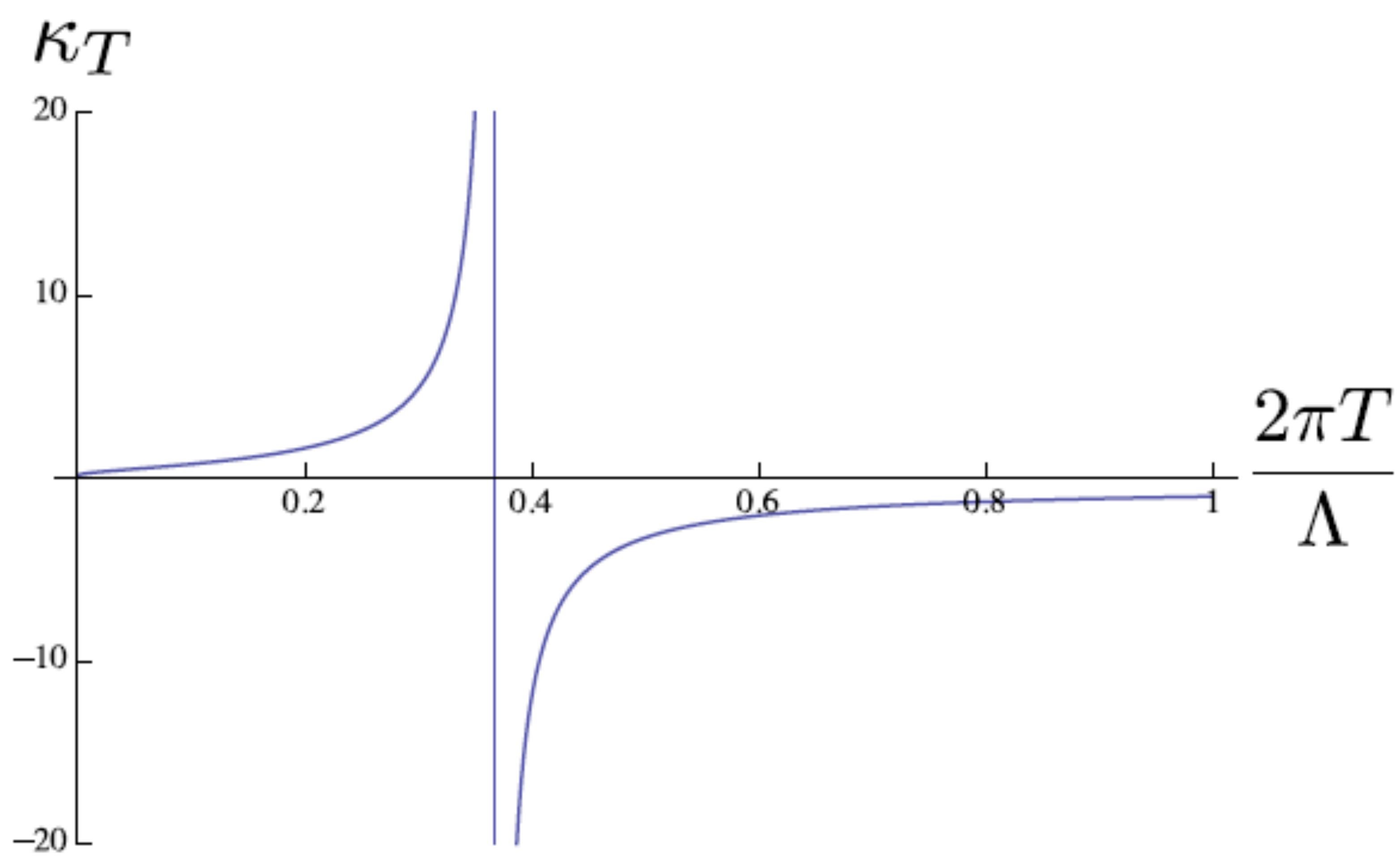
- Finite temperature solution:

$$\Phi = (z/z_H)^{1/2}\beta_T(\kappa_T \log(z/z_H) + 1) = \beta_0(\kappa_0 \log(\Lambda z) + 1)$$

- Temperature-dependent coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

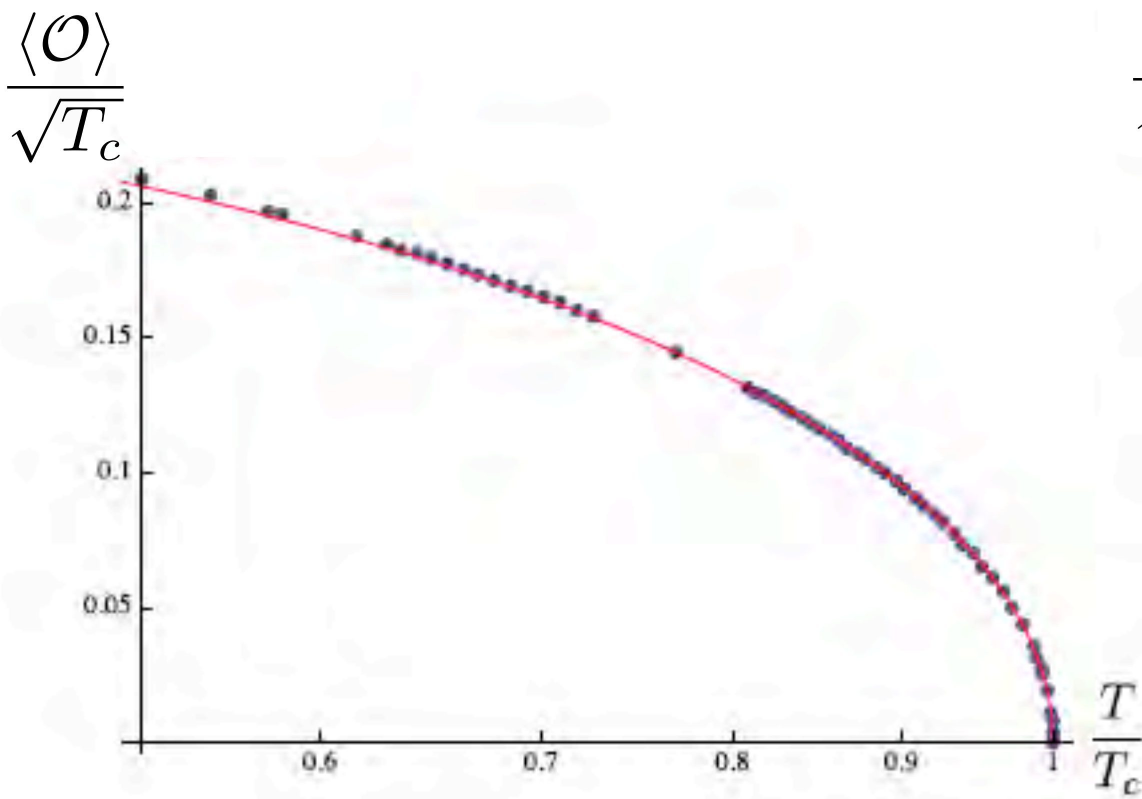
Scale generation

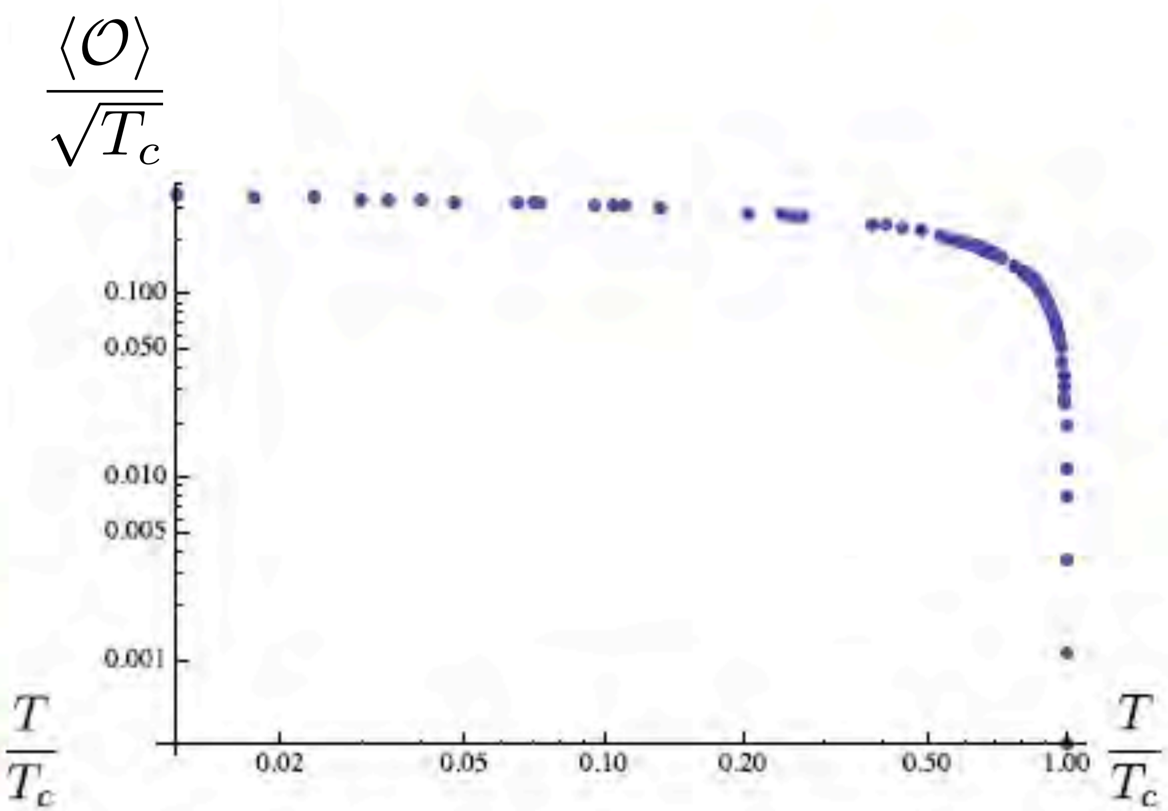


Divergence of Kondo coupling determines Kondo temperature

Below this temperature, scalar condenses

Condensate

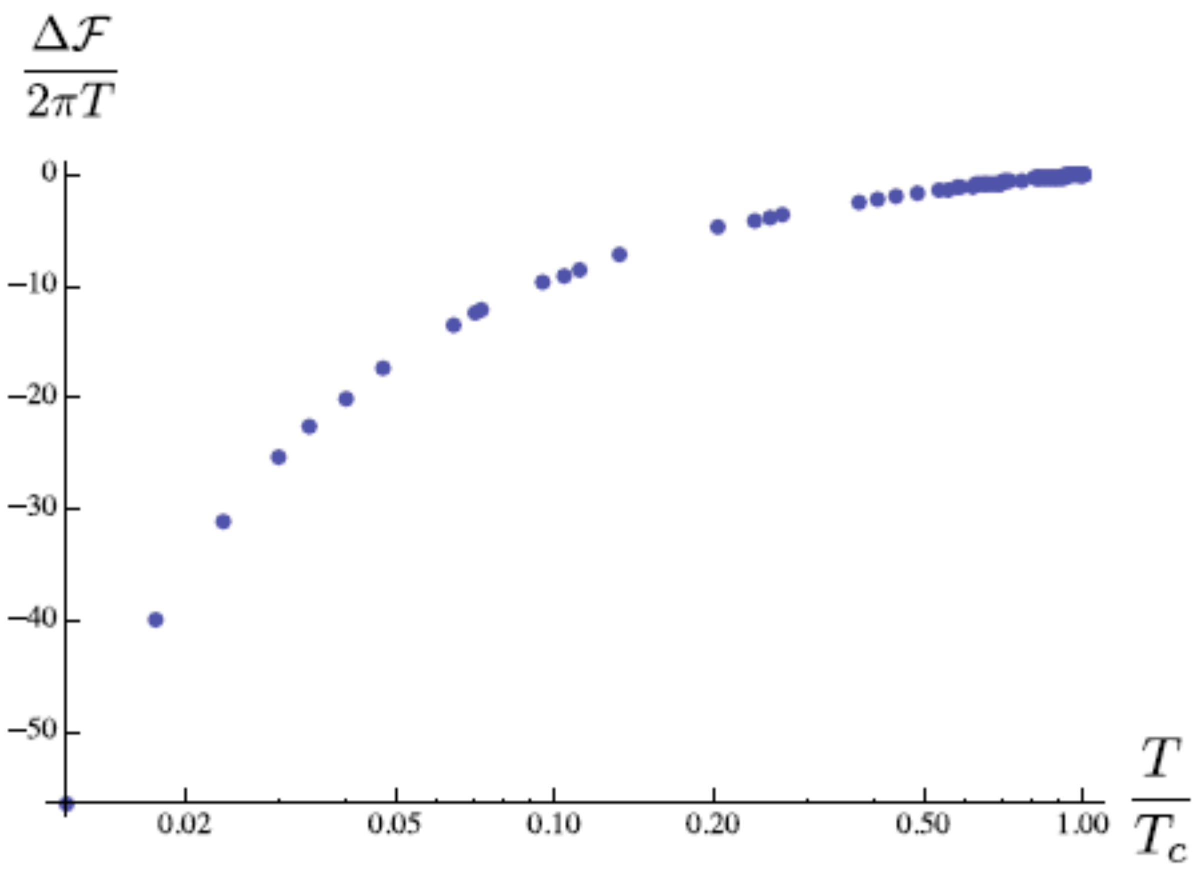




Mean field transition

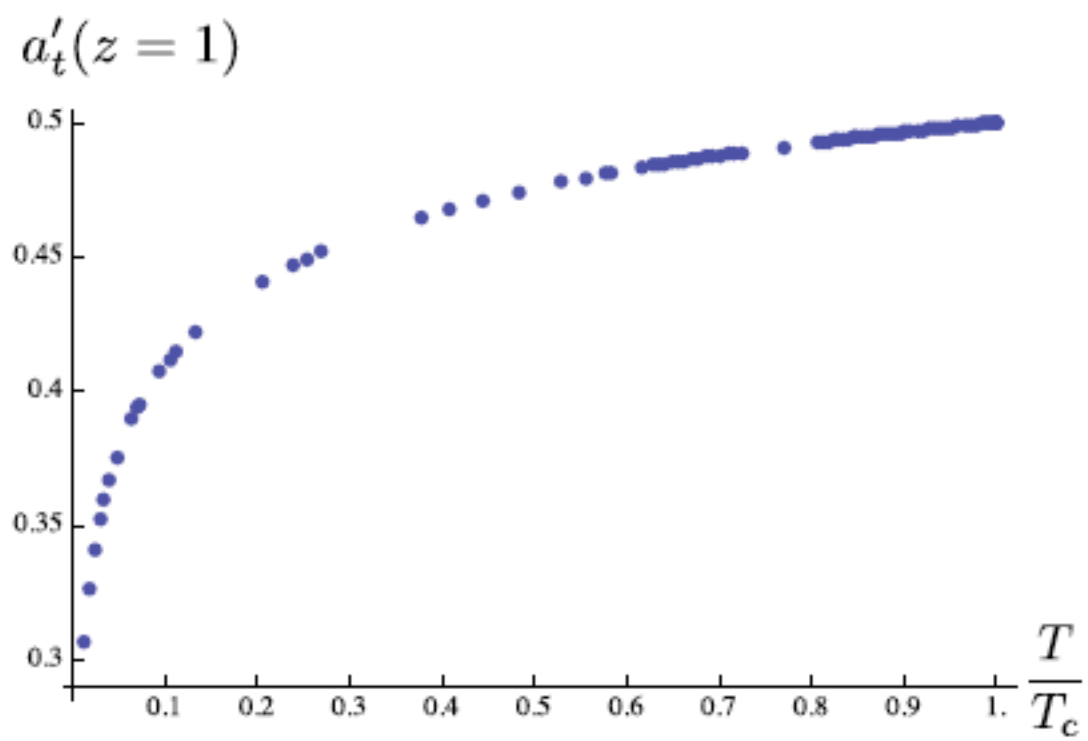
$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

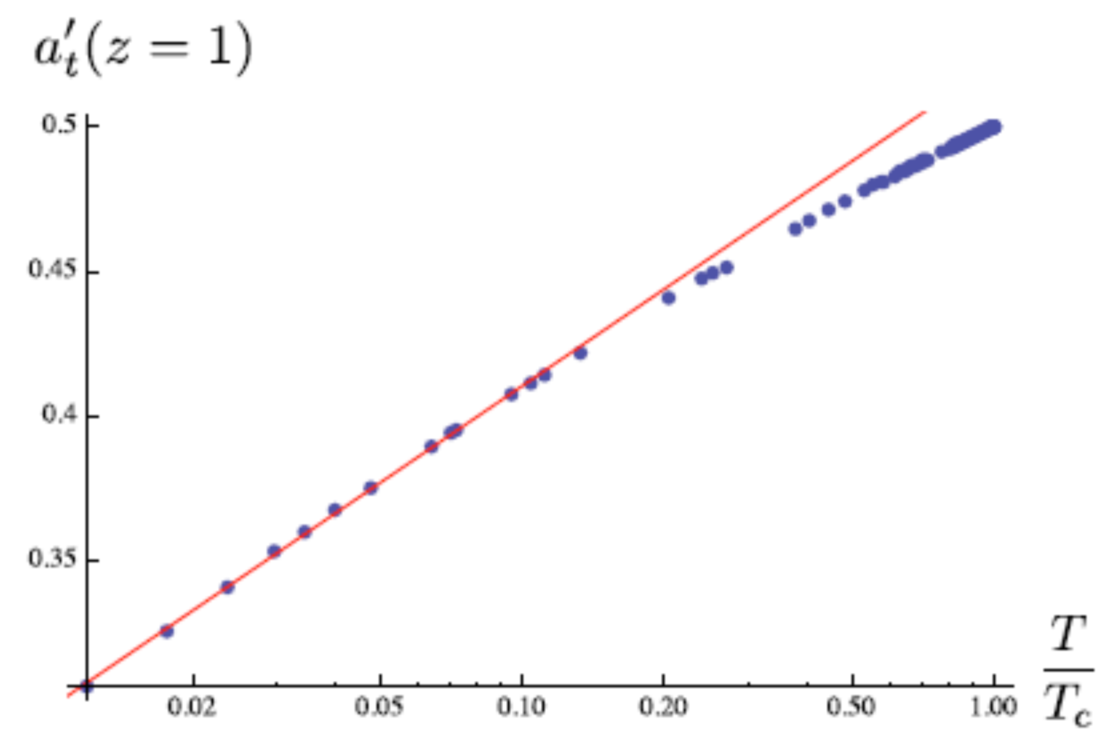
Free energy



Phase with scalar condensate more stable below critical temperature

Electric flux at horizon





$$\sqrt{-g} f^{tr}\Big|_{\partial AdS_2} = q = \chi^\dagger \chi$$

Impurity is screened

Phase shift

Below T_c , scalar transfers electric flux from 2d YM to 3d CS field

Wilson loop for 3d gauge field: $$W(z) \equiv \oint dx\, A_x(z)$$

Leads to phase shift e^{iW} for chiral fermions

Phase shift: Equations of motion

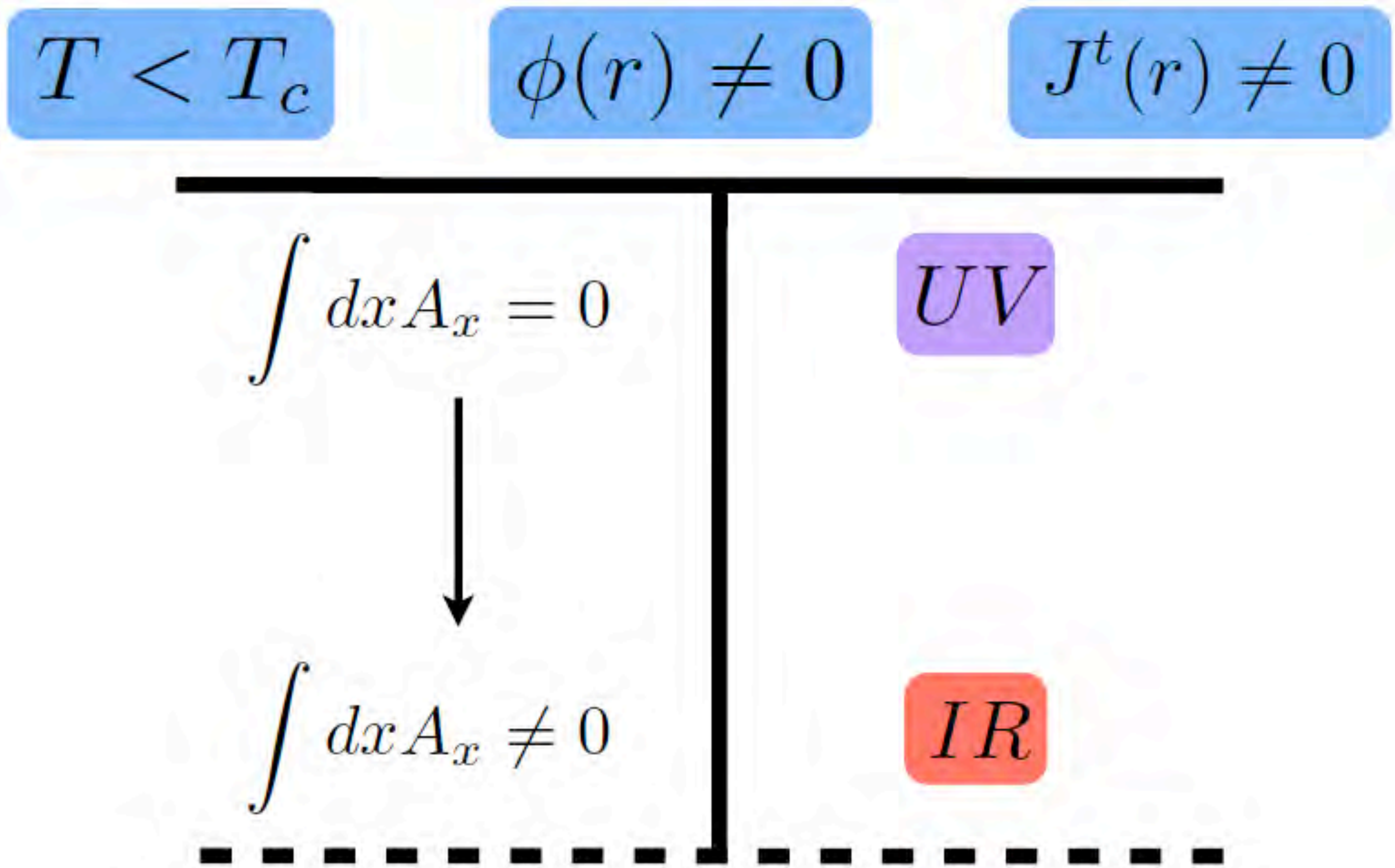
$$J^t(r) = -2\sqrt{-g}\, g^{tt} a_t \phi^2$$

$$\varepsilon^{trx} F_{rx} = -\frac{4\pi}{N}\delta(x) J^t(r)$$

$$\partial_r \left(\sqrt{-g}\, g^{rr} g^{tt} f_{rt}\right) = -J^t(r)$$

$T < T_c$ $\phi(r) \neq 0$ $J^t(r) \neq 0$

Phase Shift

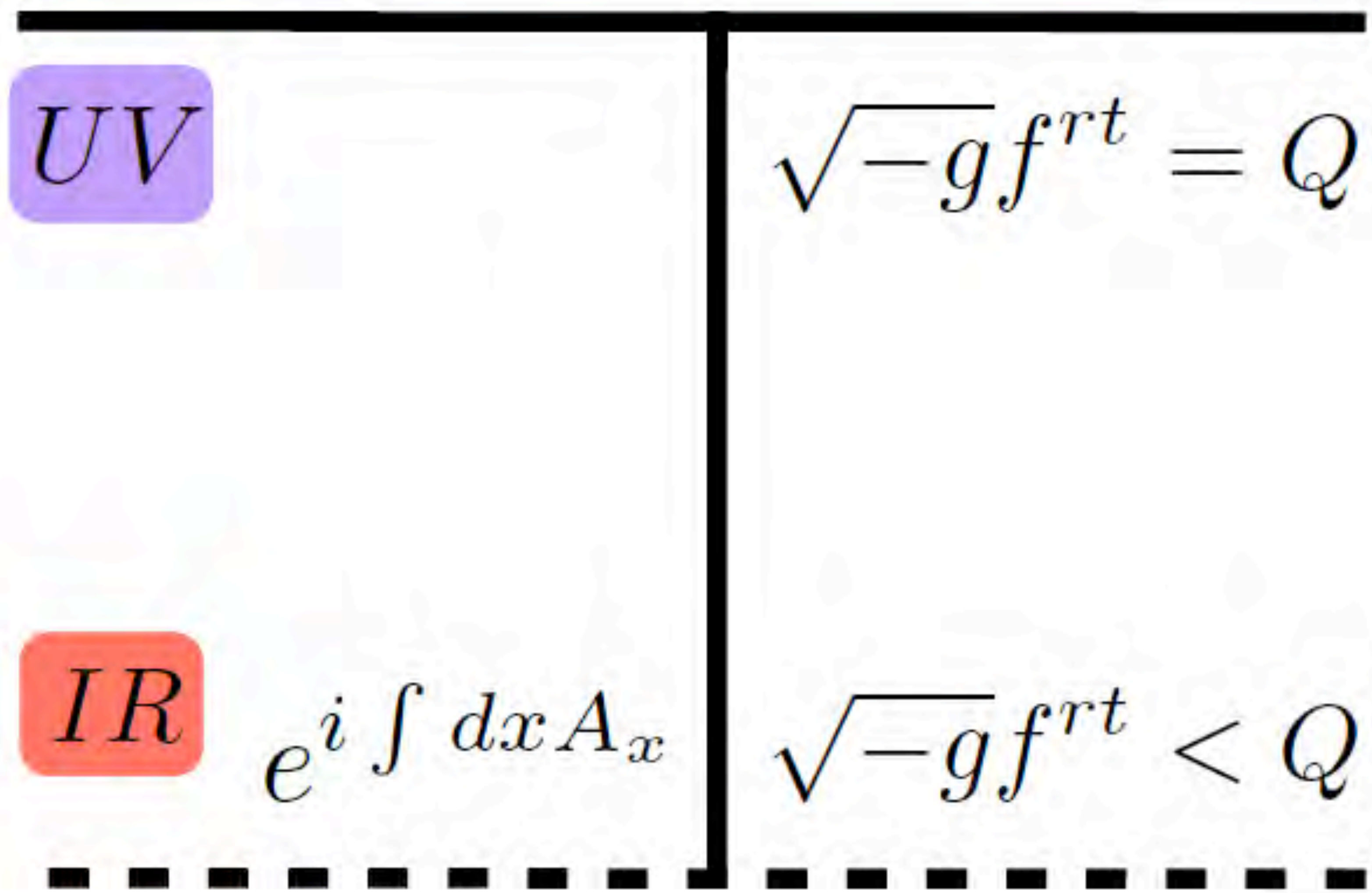


$$e^{i \int dx A_x}$$

Kraus and Larsen hep-th/0607138

Phase shift

$T < T_c$ $\qquad$ $\phi(r) \neq 0$ $\qquad$ $J^t(r) \neq 0$



Resistivity from leading irrelevant operator

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No log behaviour due to strong coupling

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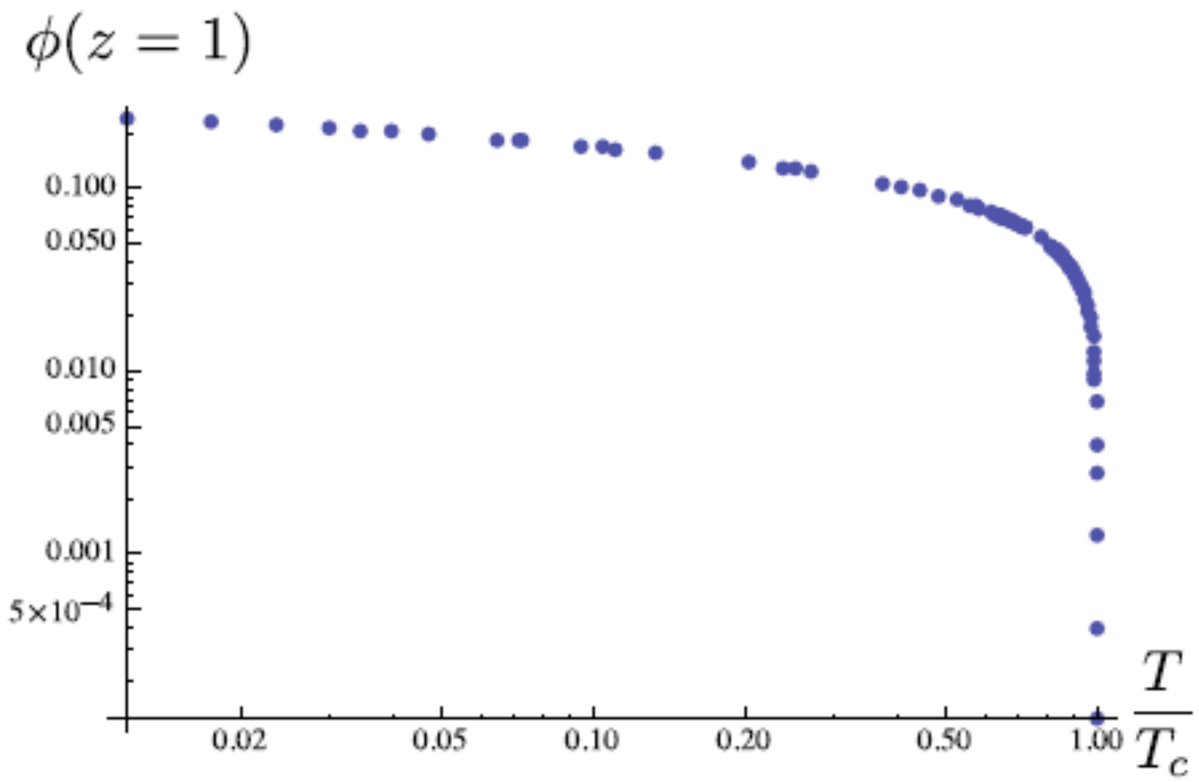
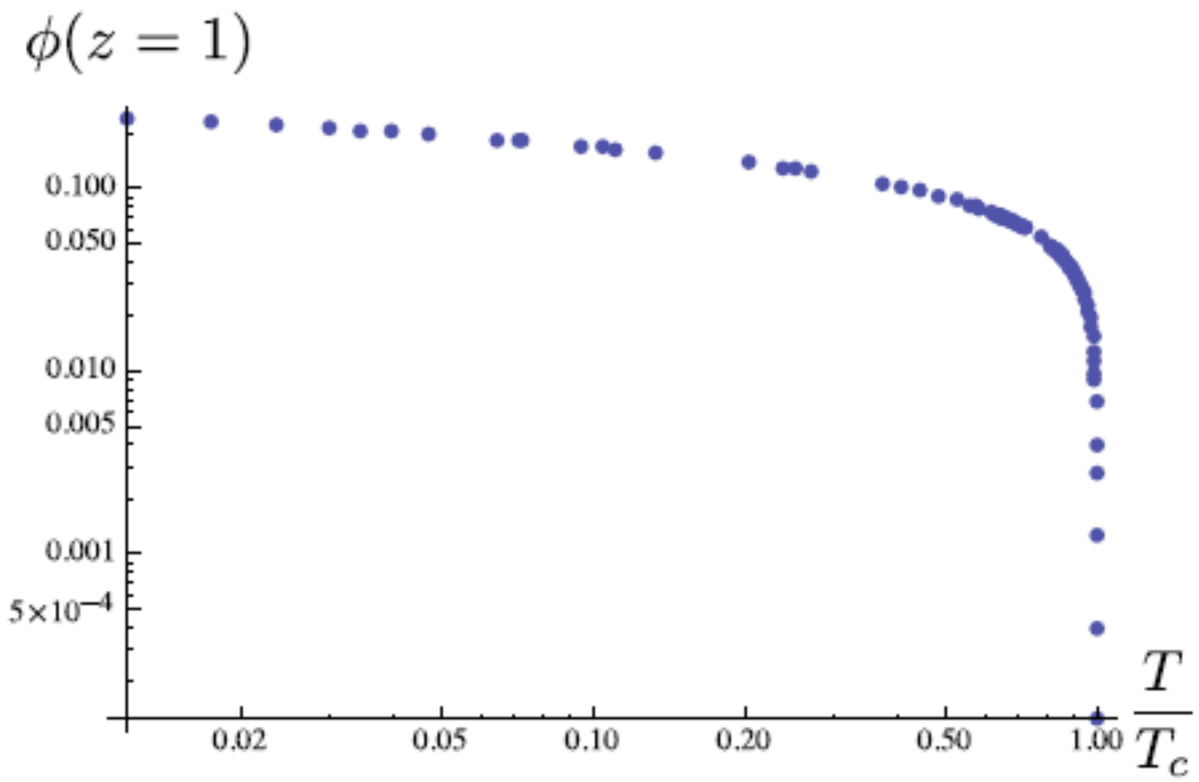


$$\phi_0(z=1) = \phi_\infty$$

Resistivity from leading irrelevant operator

No log behaviour due to strong coupling

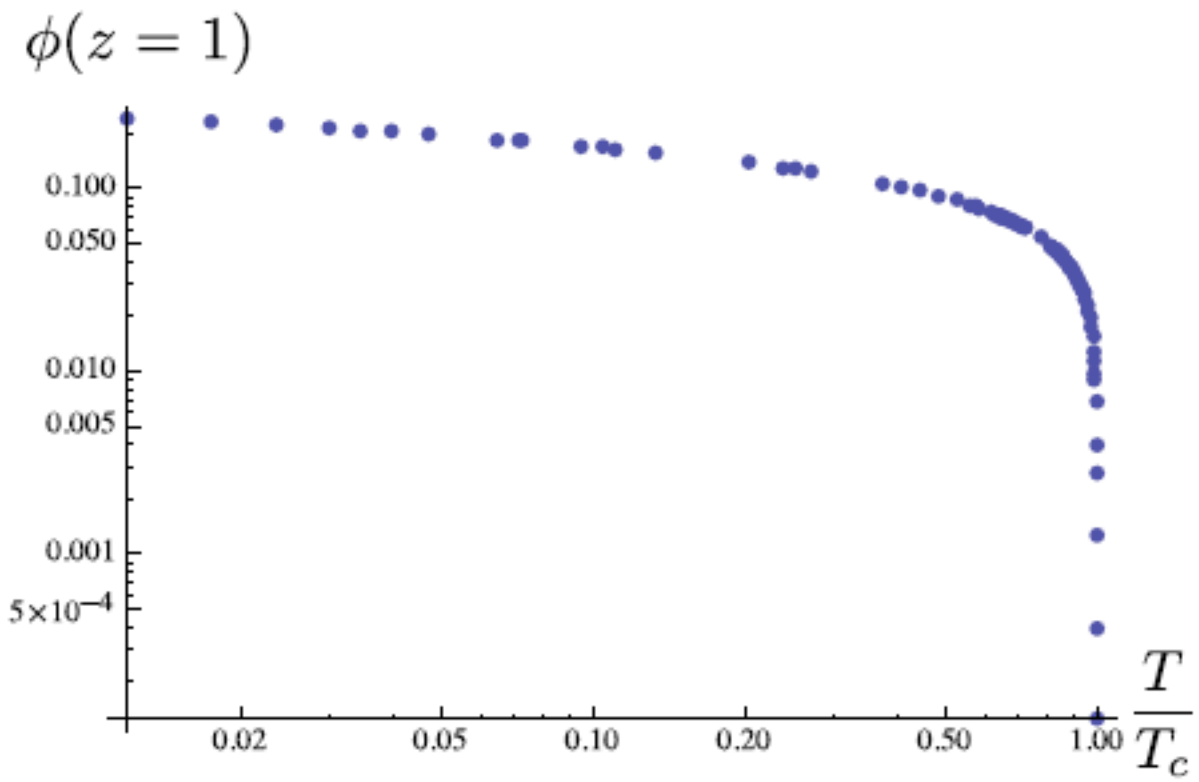


$$\phi_0(z=1) = \phi_\infty$$

IR fixed point stable: Flow near fixed point governed by operator dual to a_t

Resistivity from leading irrelevant operator

No log behaviour due to strong coupling



$$\phi_0(z=1)=\phi_\infty$$

IR fixed point stable: Flow near fixed point governed by operator dual to a_t

Dimension $$\Delta_+ = \frac{1}{2} + \sqrt{\frac{1}{4} + 2\phi_\infty^2}$$

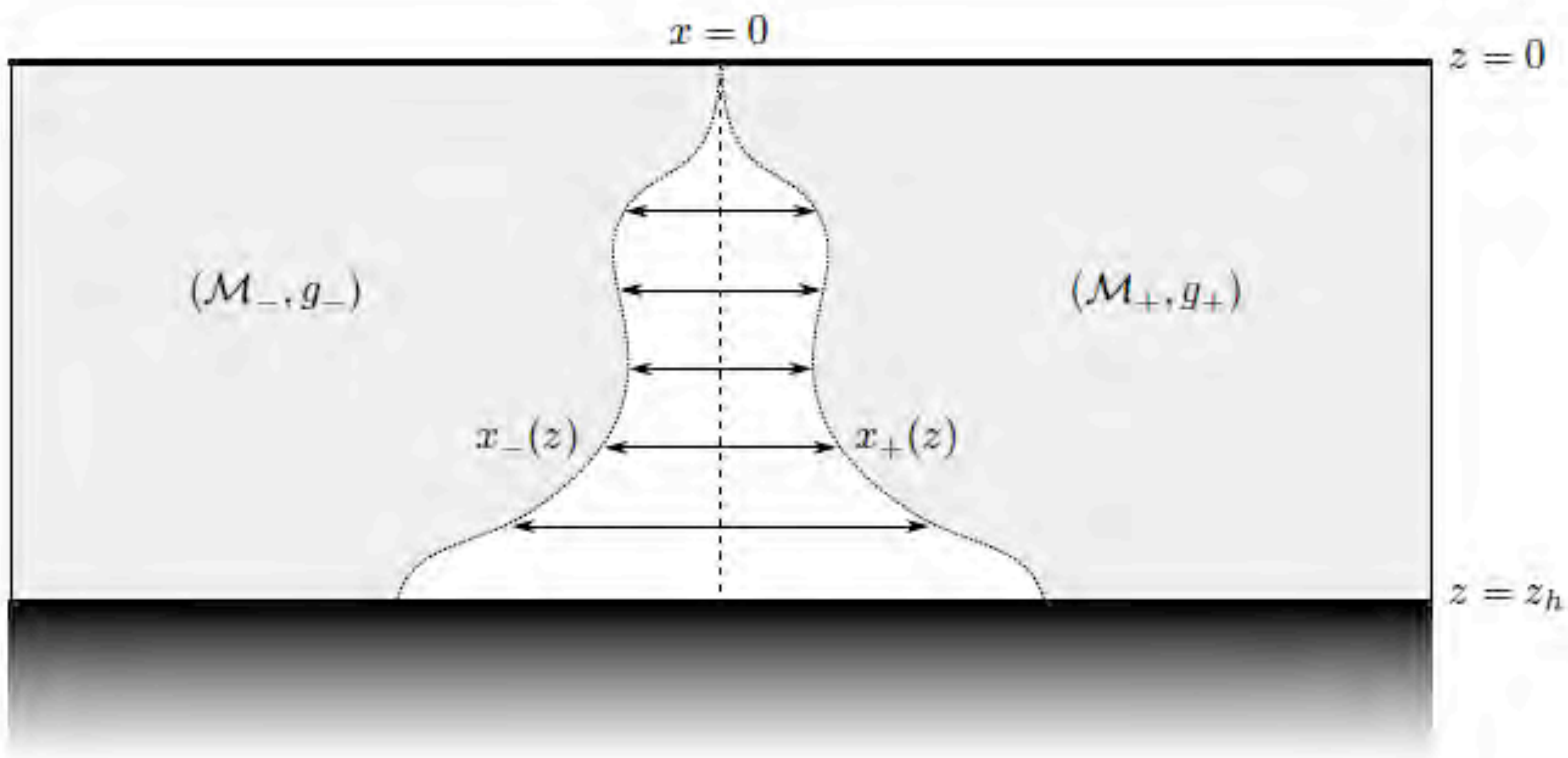
Resistivity from leading irrelevant operator

Entropy density $$s = s_0 + c_s\,\lambda_{\mathcal{O}}^2\,T^{-2+2\Delta_+}$$

Resistivity $$\rho = \rho_0 + c_+\,\lambda_{\mathcal{O}}^2\,T^{-1+2\Delta_+}$$

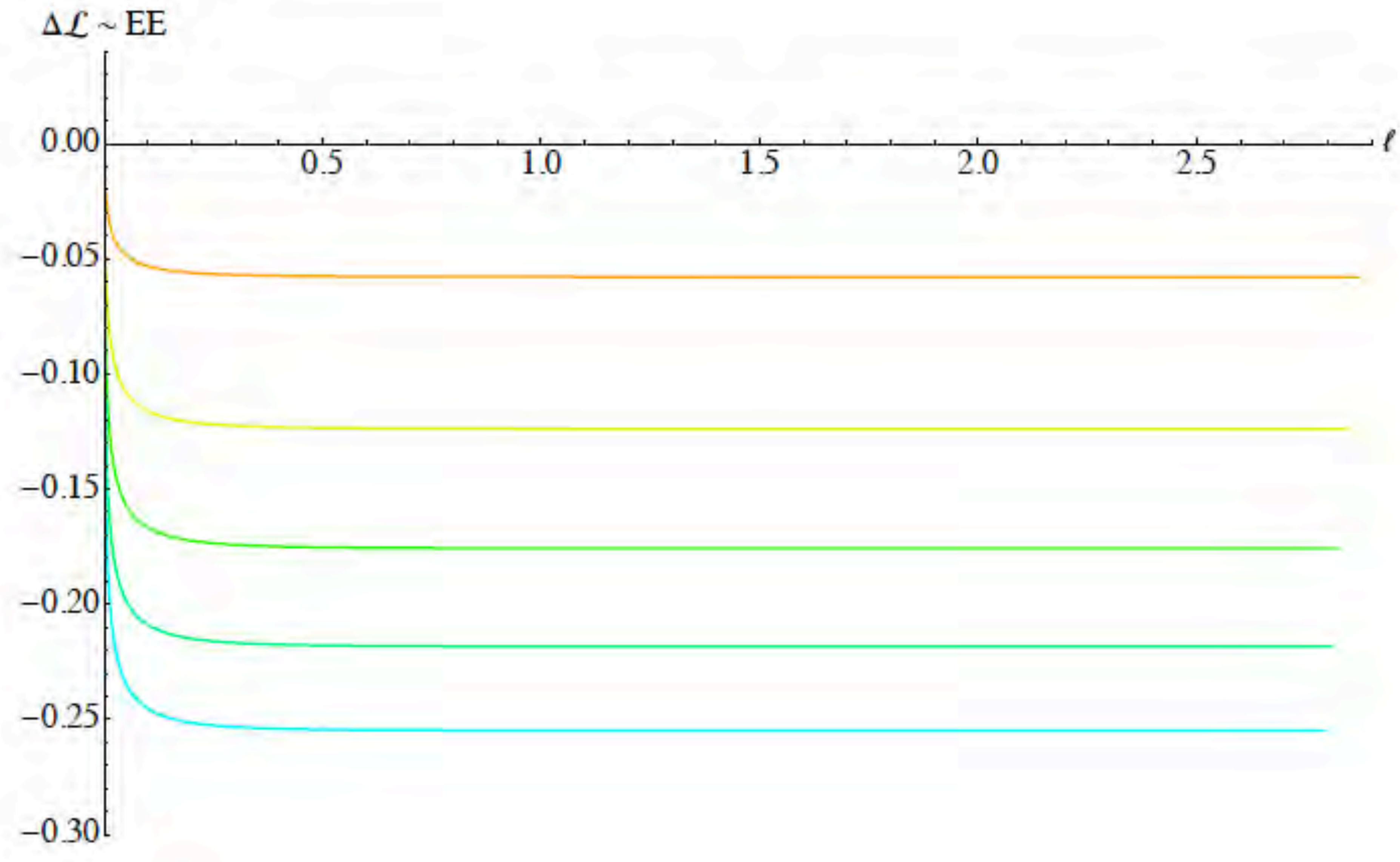
Including the backreaction

Flory, Newrzella, J.E., Hoyos, O'Bannon in progress



Impose Israel junction conditions

Defect Entanglement Entropy



Difference of EE with condensate (IR) minus EE without condensate (UV) as function of entangling region size for different temperatures

Time dependence

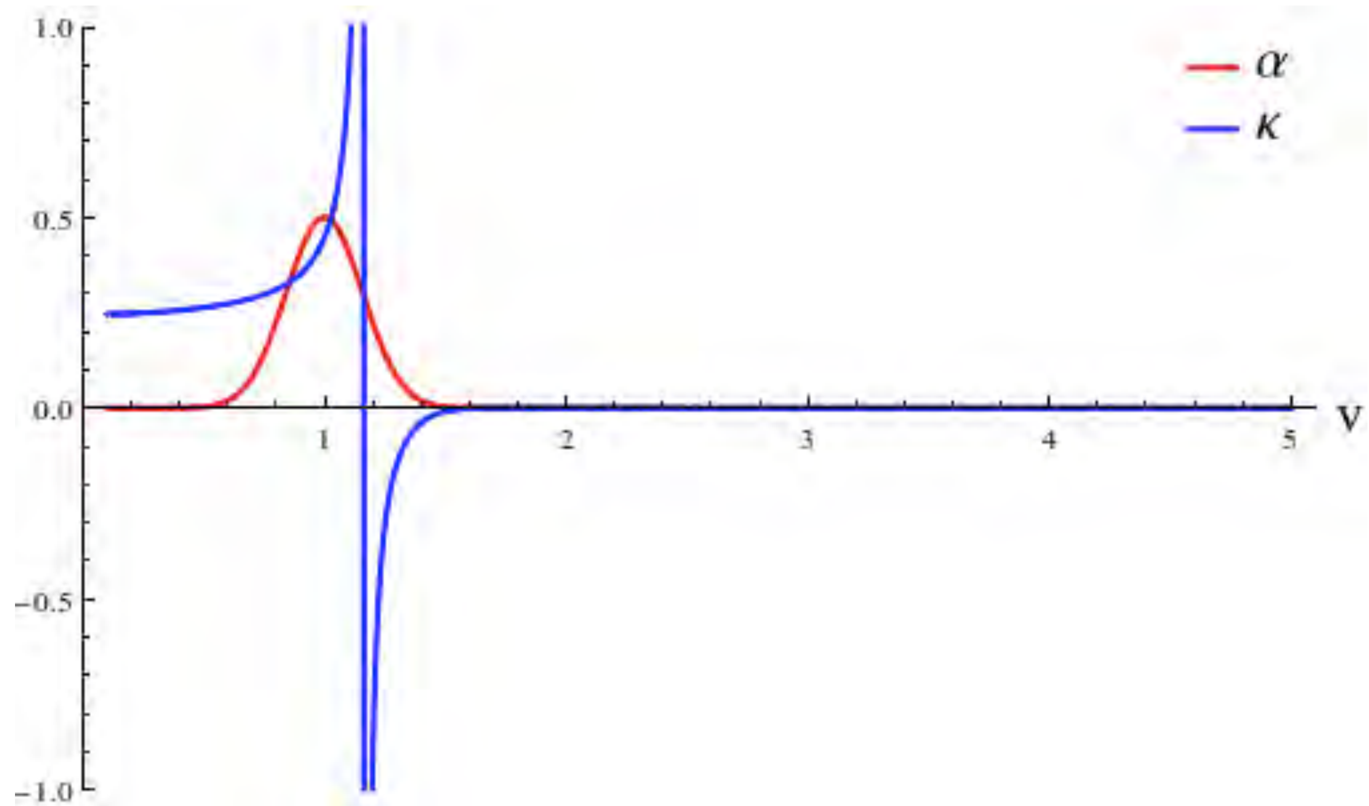
J.E., Flory, Newrzella, Strydom, Wu in progress

Look for time-dependent solutions of the equations of motion

$$A_x(z, x, t), a_t(z, t), a_z(z, t), \phi(z, t), \psi(z, t) \neq 0$$

modelling the evolution of the system
after turning on the Kondo interaction at $t = t_0$

T_c: Kondo coupling generating a Gaussian condensate pulse



J.E., Flory, Newrzella, Strydom, Wu (preliminary)

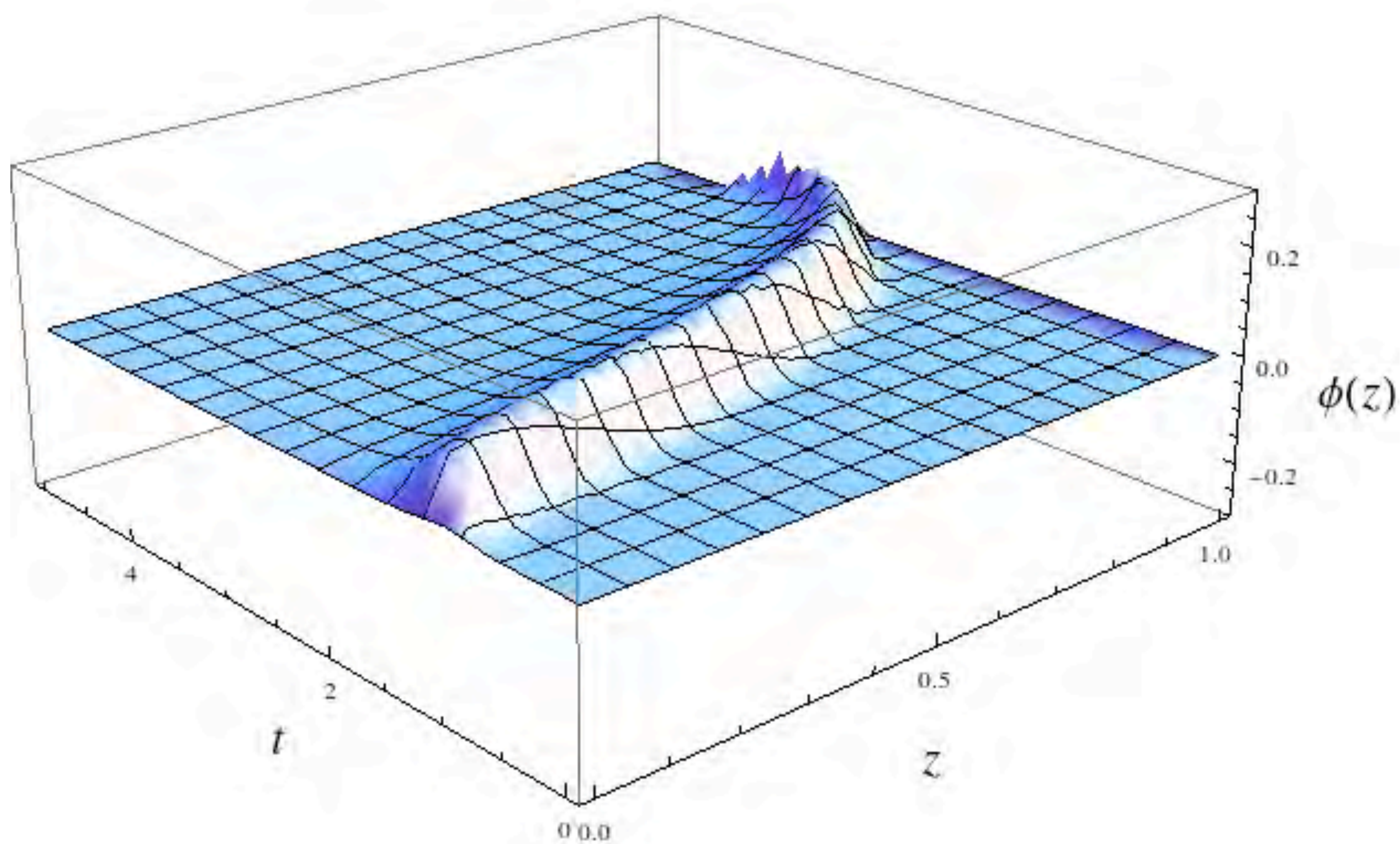
$$\phi(z) = \alpha\, z^{1/2} \ln(\Lambda z) + \beta\, z^{1/2} + \mathcal{O}\left(z^{3/2} \log(\Lambda z)\right)$$

Kondo coupling: κ

Condensate: $\alpha = \kappa\beta$

Scalar $\phi(z,t)$

Gaussian condensate pulse propagates to horizon
and falls into black hole

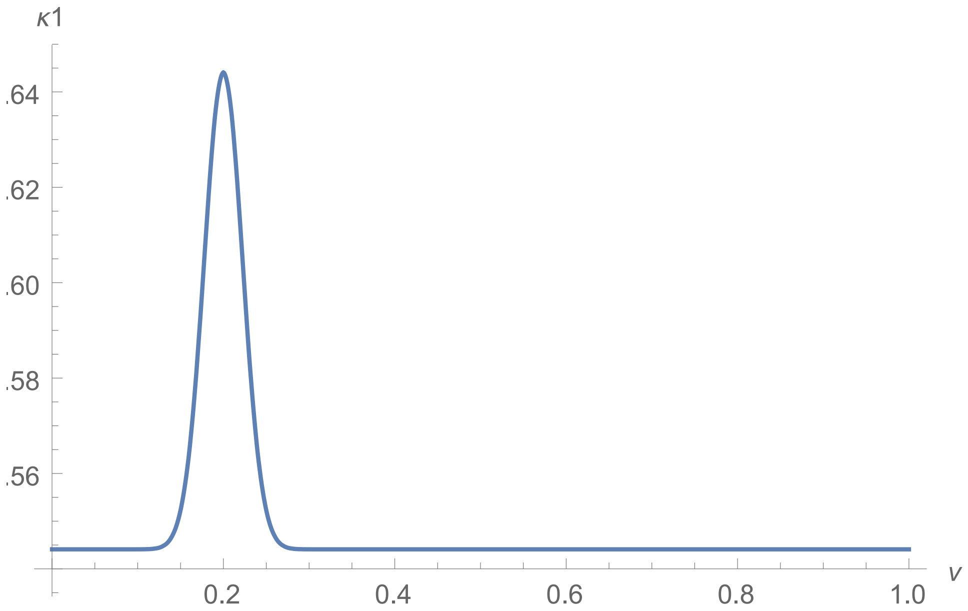


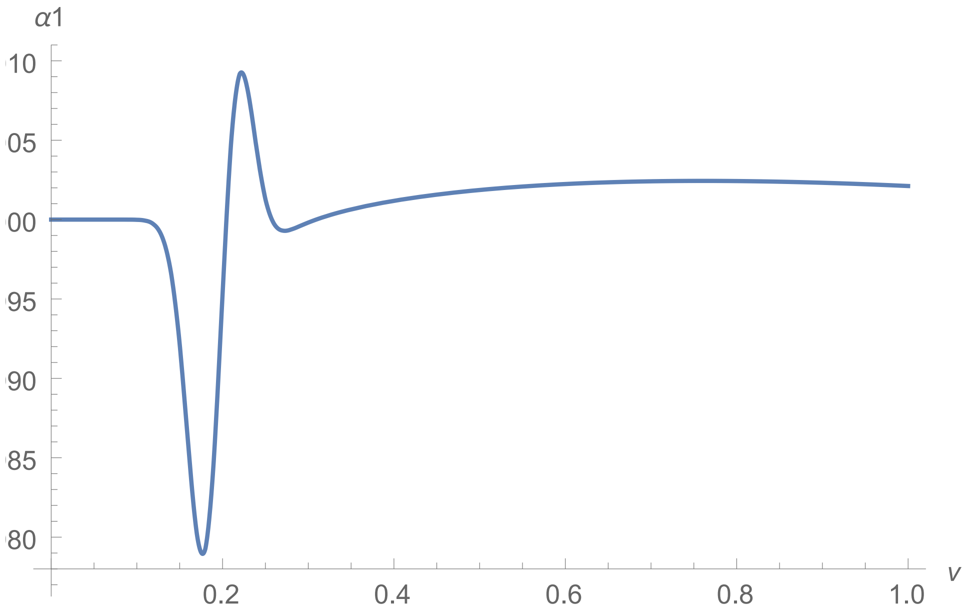
J.E., Flory, Newrzella, Strydom, Wu (preliminary)

Time dependence

Kondo coupling

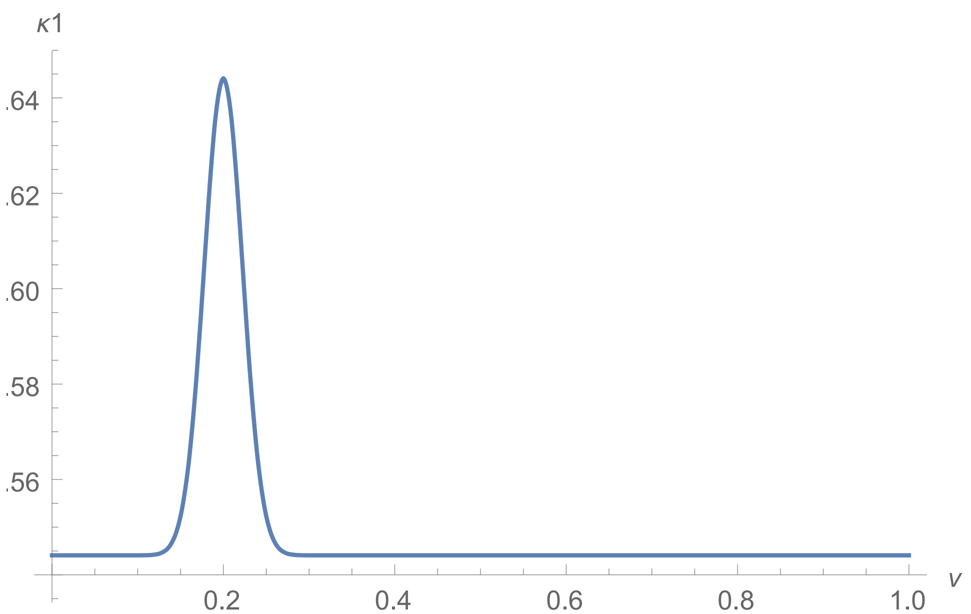
Condensate

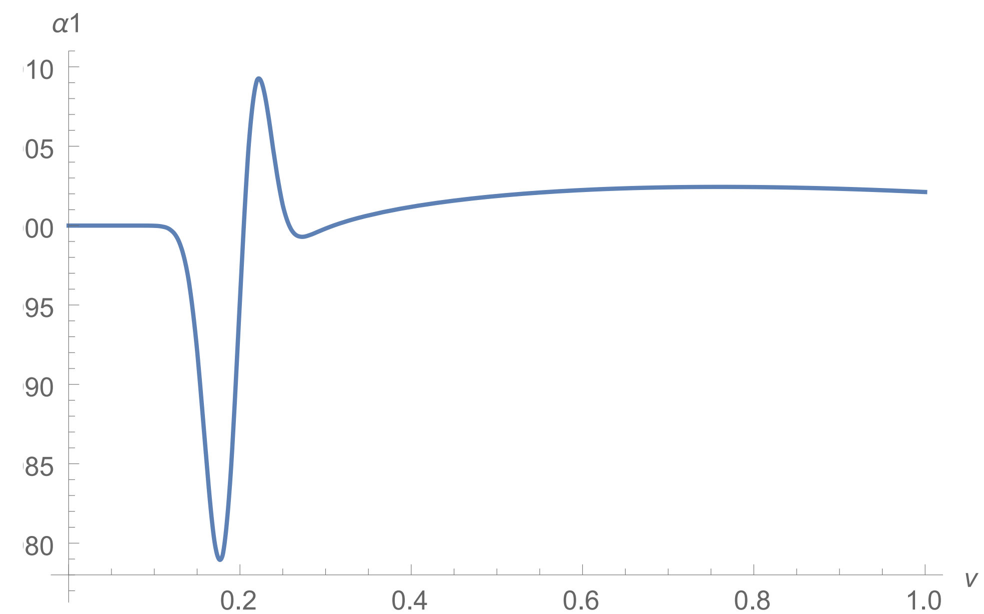


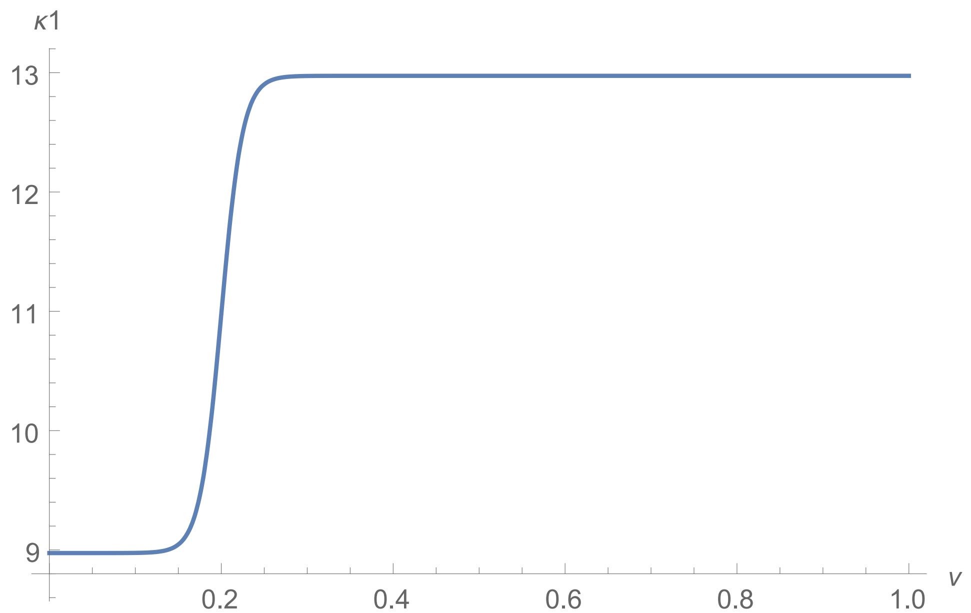


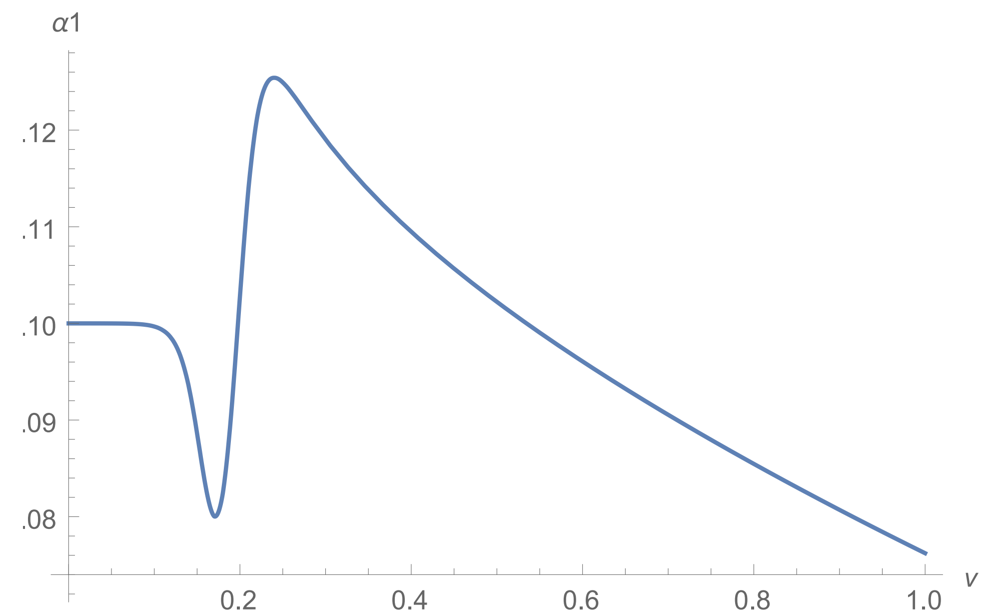
Time dependence

Kondo coupling → Condensate









Conclusion

- Kondo effect at large N: (0+1)-dimensional superfluid
- Holographic model: $S = S_{\rm CS} + S_{\rm AdS_2}$
- Two couplings: ‘t Hooft coupling (large), coupling of double-trace operator (runs)
- RG flow, screening, phase shift, power-law scaling
- Backreaction, time dependence and further extensions under investigation