Probing the structure of quantum phases of matter with holography

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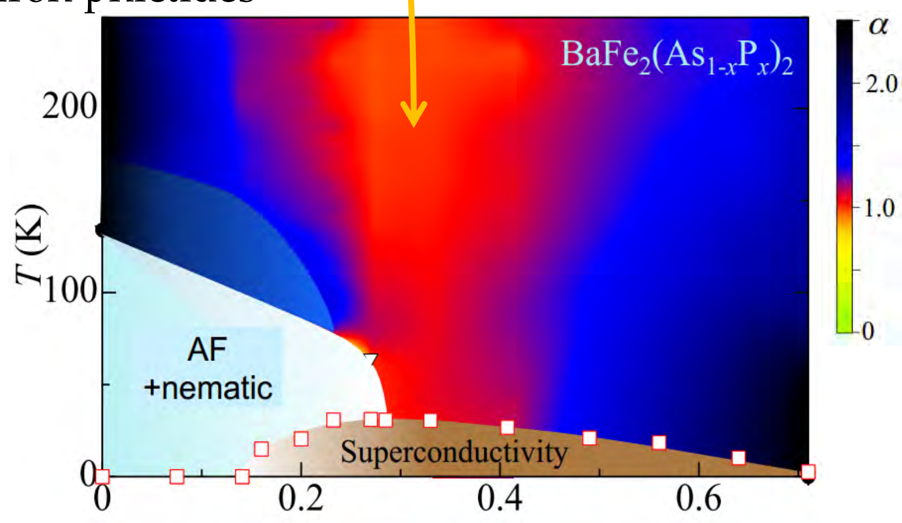
- Holographic applications to condensed matter
  - materials with unconventional scalings
  - new poorly understood phases of matter
  - systems with long range entanglement
  - …
- Common feature: d.o.f. are not weakly coupled
  (no notion of quasi-particles + Boltzmann/Landau theory does not apply)

natural setting for holography
→ new set of analytic tools

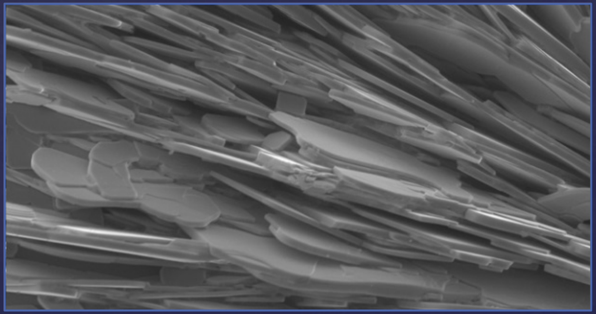
Iron pnictides



- On the GR side, from the dialogue between the two communities:
    - new classes of (black hole) solutions
    - new types of instabilities
    - non-relativistic, anisotropic geometries
    - new emergent scaling IR behavior
    - broken translations (`smectic' order) and/or rotations (`nematic' order)
    - …

- Underlying theme: ground states with reduced symmetries

- Recently many efforts to classify IR geometries (Iizuka, Kachru, Trivedi et al, Gouteraux + Kiritsis, …)



layered structure in cuprate superconductor

smectic order in a napoleon

My focus today:

- the vacuum structure of some of these scaling geometries (Lisfhitz scaling and hyperscaling violation)
- features and questions associated with the rich landscape of IR phases
- Potential applications to transport → RG flows with intermediate scaling regimes, visible in the conductivity

Lifshitz scaling and hyperscaling violation

- **Non-relativistic Lifshitz scaling**

Dynamical critical exponent z → anisotropy between space and time

$$\omega \sim k^z \qquad\qquad x \to \lambda x\,, \quad t \to \lambda^z t$$

Characterizes scaling of thermo quantities $\quad s(T) \sim T^{\frac{d}{z}}$

- **Hyperscaling violation θ** → anomalous scaling of free energy

→ critical excitations do not live in the naïve number of dimensions

$$s(T) \sim T^{\frac{d-\theta}{z}}$$

shifts effective dimensionality of the system $d_{eff} = d - \theta$

d_{eff}=1 of interest for compressible states and systems w/ Fermi surface (S_{ent}~ A log A) [Huijse/Sachdev/Swingle,Takayanagi et al] But FS not easily captured by holography.

How do we geometrize these scalings?

`Minimal′ model:

Exact solutions to simple EMD theory (either electric or magnetic field)

$$\mathcal{L}_{d+2} = R - 2(\partial\phi)^2 - e^{2\alpha\phi}F^2 - V_0 e^{-\eta\phi}$$

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z

$$ds^2_{d+2} = \left(-r^{-2z} dt^2 + \frac{dr^2 + d\vec{x}^2}{r^2}\right)$$

$$t \to \lambda^z t\,, \quad \vec{x} \to \lambda \vec{x}\,, \quad r \to \lambda r$$

$$\phi(r) \sim \log(r)$$

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z   θ

$$ds^2_{d+2} = \left(-r^{-2z}dt^2 + \frac{dr^2 + d\vec{x}^2}{r^2}\right) r^{2\theta/d}$$

$$t \to \lambda^z t\,, \quad \vec{x} \to \lambda\vec{x}\,, \quad r \to \lambda r \qquad ds \to \lambda^{\theta/d} ds$$

$$\phi(r) \sim \log(r)$$

no longer scale invariant

In general, **anomalous scaling of gauge field** ζ important to understand conductive properties [Gouteraux, Obers et al., Gouteraux/Kiritsis, Karch]

→ must assign separate scaling rules to matter fields to ensure invariance of EOMs

Natural question: IR endpoint of these scaling solutions?

Solutions are supported by a **running dilatonic scalar** **$\phi \sim \log r$**
→ not expected to be a good description of the geometry in the deep IR

$$\mathcal{L} = R - 2(\partial\phi)^2 - e^{2\alpha\phi}F^2 - V_0 e^{-\eta\phi}$$

Effective gauge coupling of the theory $g \equiv e^{-\alpha\phi}$ drives system to

strong coupling
(magnetic case)

Expect modifications to $g(\phi)$, e.g.

$$\frac{1}{g^2} \to \frac{1}{g^2} + \xi_1 + \xi_2 g^2 + \ldots$$

(toy model for QM corrections)

weak coupling
(electric case)

Expect higher derivative terms no longer negligible

(tree level terms comparable to F^4,…)

Also curvature + tidal singularities [Copsey/Mann, Horowitz/Way, Bao/Dong/Harrison/Silverstein]

IR completion of hyperscaling violation

[arXiv:1208.1752 – J. Bhattacharya, S.C. , A. Sinkovics]

See also Trivedi et al, 1208.2008

In the Lifshitz case, a toy model for QM corrections generates $AdS_2 \times R^2$ in deep IR [Harrison/Kachru/Wang 1202.6635]

Our starting point:

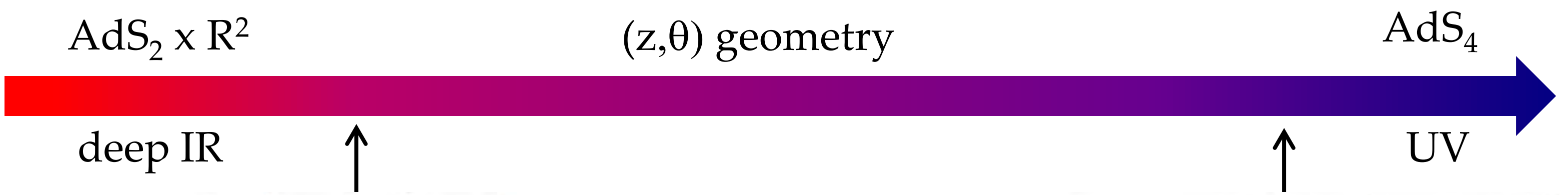
$$\mathcal{L} = R - 2(\partial\phi)^2 - f(\phi)F^2 - V(\phi)$$

$$f(\phi) = e^{2\alpha\phi}\,, \quad V(\phi) = -V_0 e^{-\eta\phi}$$

Explored **conditions for emergence of $AdS_2 \times R^2$** in deep IR :

- **generic IR modifications to $f(\phi)$ and $V(\phi)$** → whether of classical or \`quantum' origin (toy model of QM corrections as baby example)

We constructed **explicit zero temperature solutions which realize this RG flow**.

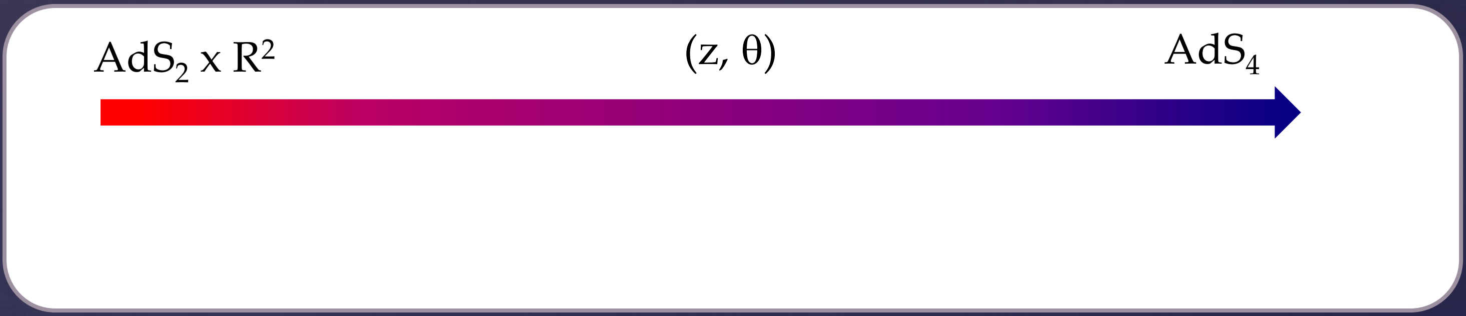


e.g. $\xi_2 g^4 + \xi_1 g^2 \simeq 1$ e.g. $V = -V_0(e^{-\eta\phi} + c_1 e^{\eta_1\phi})$

[arXiv:1208.1752]

Main Message:

- These scaling geometries should be thought of as intermediate solutions
- In many cases their ` naïve ' **IR completion is $AdS_2 \times R^2$**



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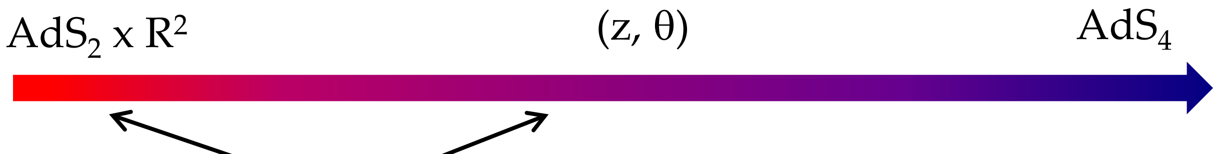

$$z,\theta \to \infty\,,\;\; \eta \equiv -\frac{z}{\theta} \;\; \text{fixed}$$

double scaling limit, Hartnoll/Shaghoulian

This picture has emerged in a number of setups:

- Dyonic charges [Trivedi et al, 1208.2008]
- Higher derivative and QM corrections provide stabilization mechanism → $AdS_2 \times R^2$ [Knodel/Liu, Peet et al, Cardoso/Haack et al,…]
- Various SUGRA truncations (sometimes with ` η-geometries’ in IR or mid-IR) [Donos/Gauntlett/Pantelidou, Kulaxizi/Parnachev/Schalm,…]

Spatially Modulated Instabilities of (z,θ) geometries
[S.C. arXiv:1310.3279, S.C. and A. Sinkovics arXiv:1212.4172]

- Well-known **extensive ground state entropy** of $AdS_2 \times R^2$ (violates 3rd law)
  Highly degenerate ground state – pathology or feature?
- New phases expected to emerge → **$AdS_2 \times R^2$ should not be typical ground state**

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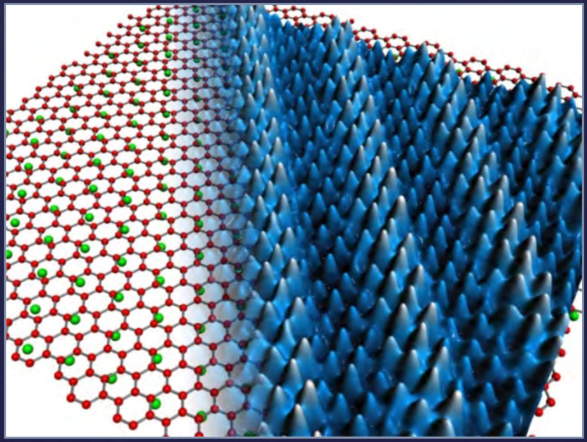
- $AdS_2 \times R^2$ suffers from **spatially modulated instabilities** in many setups [Nakamura/Ooguri/Park, Donos/Gauntlett/Pantelidou,…]

$$\delta g = h(r) \cos kx, \ldots$$



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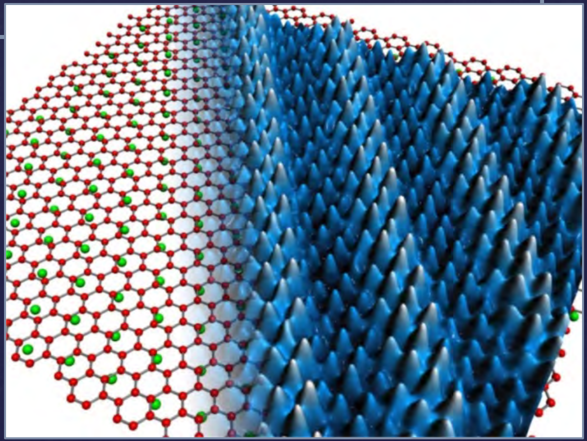
Our logic: use knowledge of instabilities of AdS_2 region to identify (z,θ) geometries which are **unstable to spatially modulated phases**

ubiquitous in CM systems
(smectics, spin/charge density waves…)



Spatially modulated instabilities

Magnetic case 1212.4172
Purely electric in 1310.3279

Simple EMD setup: $\mathcal{L} = R - V(\phi) - 2(\partial\phi)^2 - f(\phi)F_{\mu\nu}F^{\mu\nu}$

Strategy:

1) require $f(\phi)$ and $V(\phi)$ to support solutions with:

- $AdS_2 \times R^2$ in the deep IR
- an intermediate regime of (z, θ) scaling:

$$f(\phi) = e^{2\alpha\phi} + \ldots$$
$$V(\phi) = V_0 e^{-\eta\phi} + \ldots$$

corrections negligible in intermediate region

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2) turn on spatially modulated fluctuations of IR $AdS_2 \times R^2$ geometry

3) **identify conditions on $f(\phi_h)$ and $V(\phi_h)$ that imply instabilities**
(finite-k modes that are tachyonic and violate AdS_2 BF bound)

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4) **map to instability conditions for (z, θ)** and other parameters of the theory

To give you a flavor…

Spectrum of scaling dimensions (small k approximation)

$$\delta_{1,2,3,4} = \frac{1}{2} \pm \sqrt{\text{mess}}$$

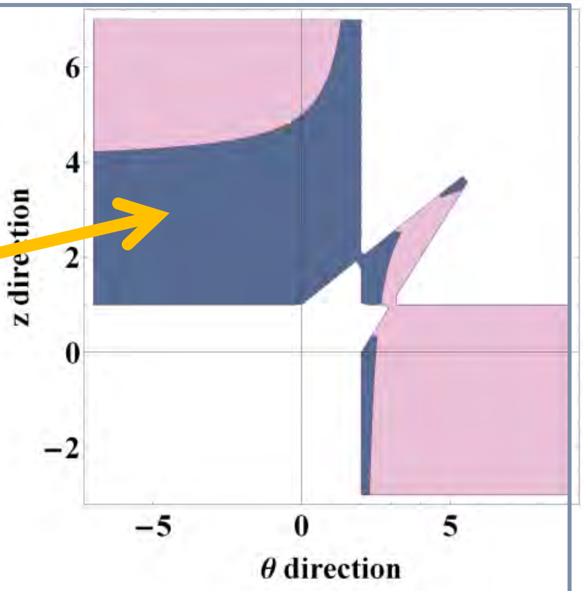
$$\text{mess} = [k=0 \text{ terms}] \pm k^2 \left[\frac{3}{2} \pm \frac{1}{2} \pm \frac{2}{8-V''_{eff}} \frac{f'^2}{f^2}\right]$$

Example from purely electric case [1310.3279]

$$f(\phi) = e^{\alpha\phi}\,, \qquad V = V_0 e^{-\eta\phi} + \mathcal{V}(\phi)$$

spatially modulated instabilities
(provided constraint is obeyed)

$$\frac{8}{\theta - 2z + 2} = L^2 \left(\mathcal{V}''(\phi_0) - \frac{\theta^2}{(\theta-2)(\theta-2z+2)} \mathcal{V}(\phi_0)\right)$$



Take home message:

- evidence for spatially modulated phases (`stripes’) as possible ground states of (certain) (z,θ) geometries

Similar instabilities found by analysing the (z,θ) geometries directly [Iizuka, Maeda 1301.5677]

Rich structure of IR phases

AdS_4

↓

intermediate
Lifshitz and hyperscaling
violating geometry

- spatially modulated geometries
- ? ?
- $AdS_2 \times R^2$
- $r^{\#} (AdS_2 \times R^2)$
- Lifshitz

An AdS_4 IR completion of (z,θ) geometries

[Work with J. Bhattacharya and B. Gouteraux, to appear]

Natural question: are there other possible ground states?

Emergent conformal symmetry in the IR?

Picture we are exploring:

deep IR → UV

AdS_4 — (z,θ) geometry — (different) AdS_4

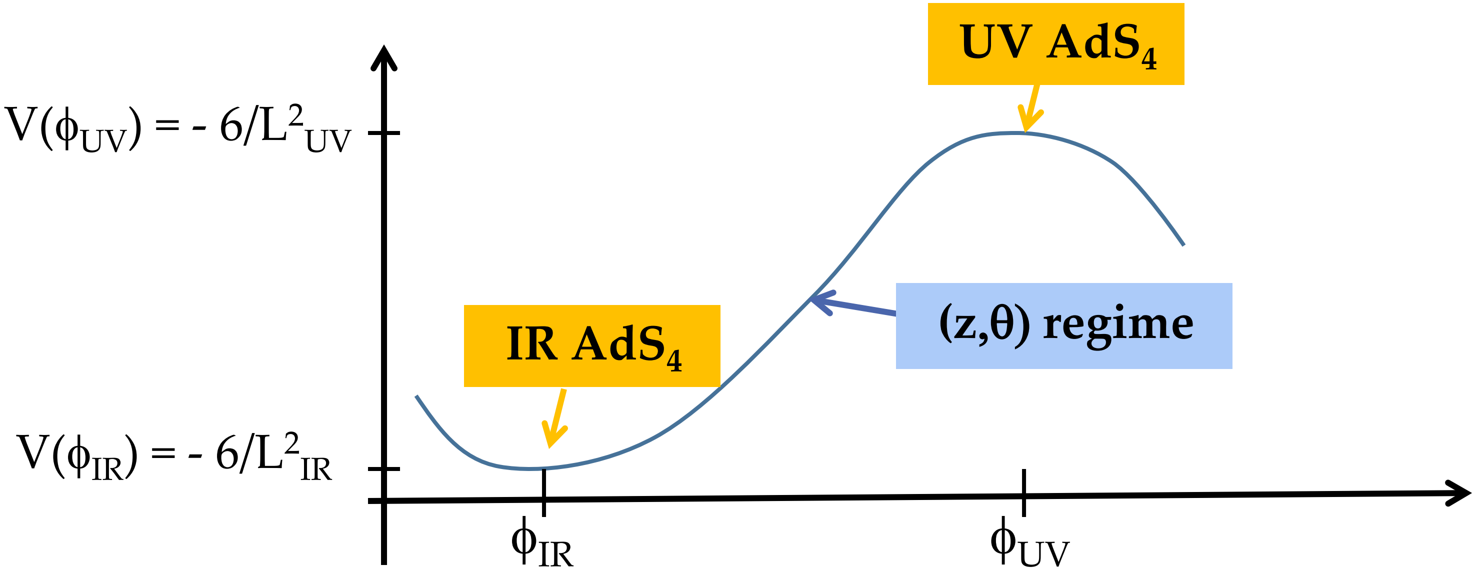
Analog of ground state of holographic superconductor but with intermediate hyperscaling violating regime

[Gubser/Rocha 0807.1737, Gubser /Nellore 0908.1972 … , Horowitz/Roberts 0908.3677]

How can we engineer this flow?

Our toy model:

$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}Z(\phi)\,F^2 - \frac{1}{2}W(\phi)\,A^2 - V(\phi)$$



For appropriate couplings, model could describe broken-symmetry phase of theory w/ U(1) symmetry and a charged complex scalar

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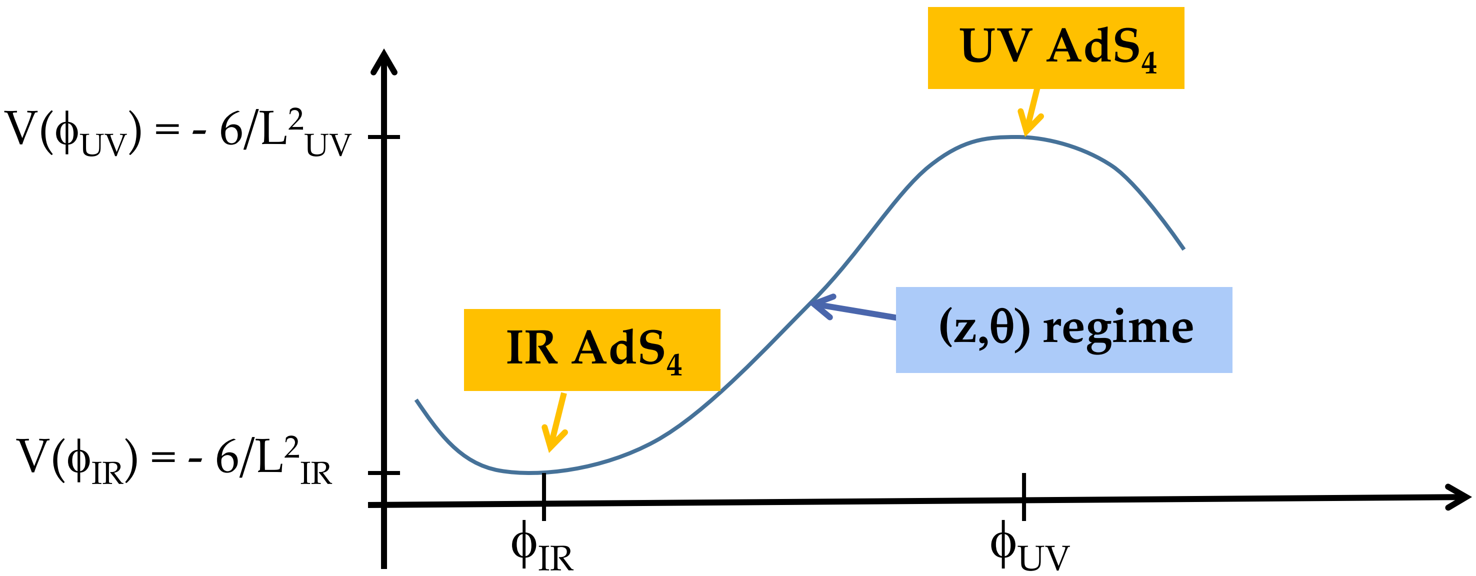
$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}Z(\phi)\,F^2 - \frac{1}{2}W(\phi)\,A^2 - V(\phi)$$

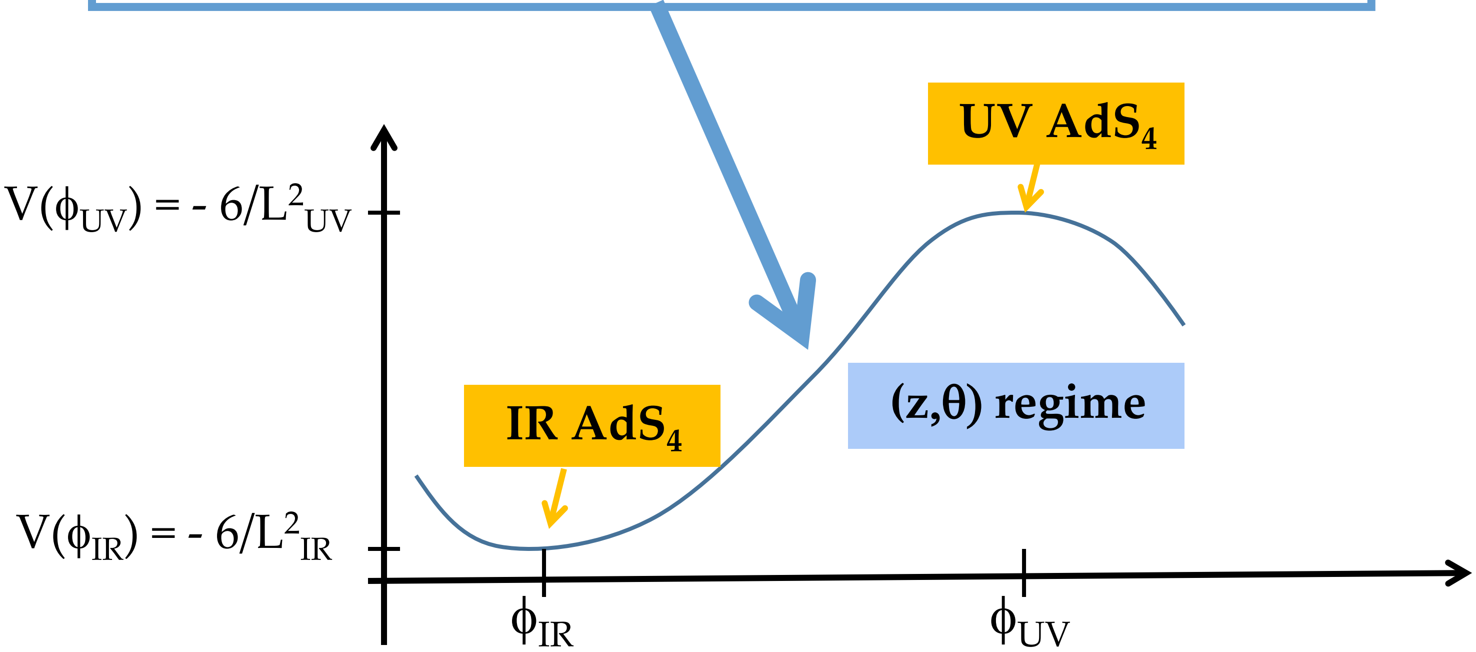
$$Z(\phi) = Z_0 e^{\,\alpha\phi}\,, \qquad W(\phi) = 4W_0 \sinh^2\left(\frac{\beta}{2}\phi\right)\,, \qquad V(\phi) = 2V_0 \cosh\delta\phi + 2V_1 \cosh\gamma\phi + V_3$$



- Massive gauge field needed to source IR AdS_4 (W must vanish in UV)
- Scalar potential engineered to get intermediate scaling

Intermediate scaling regime

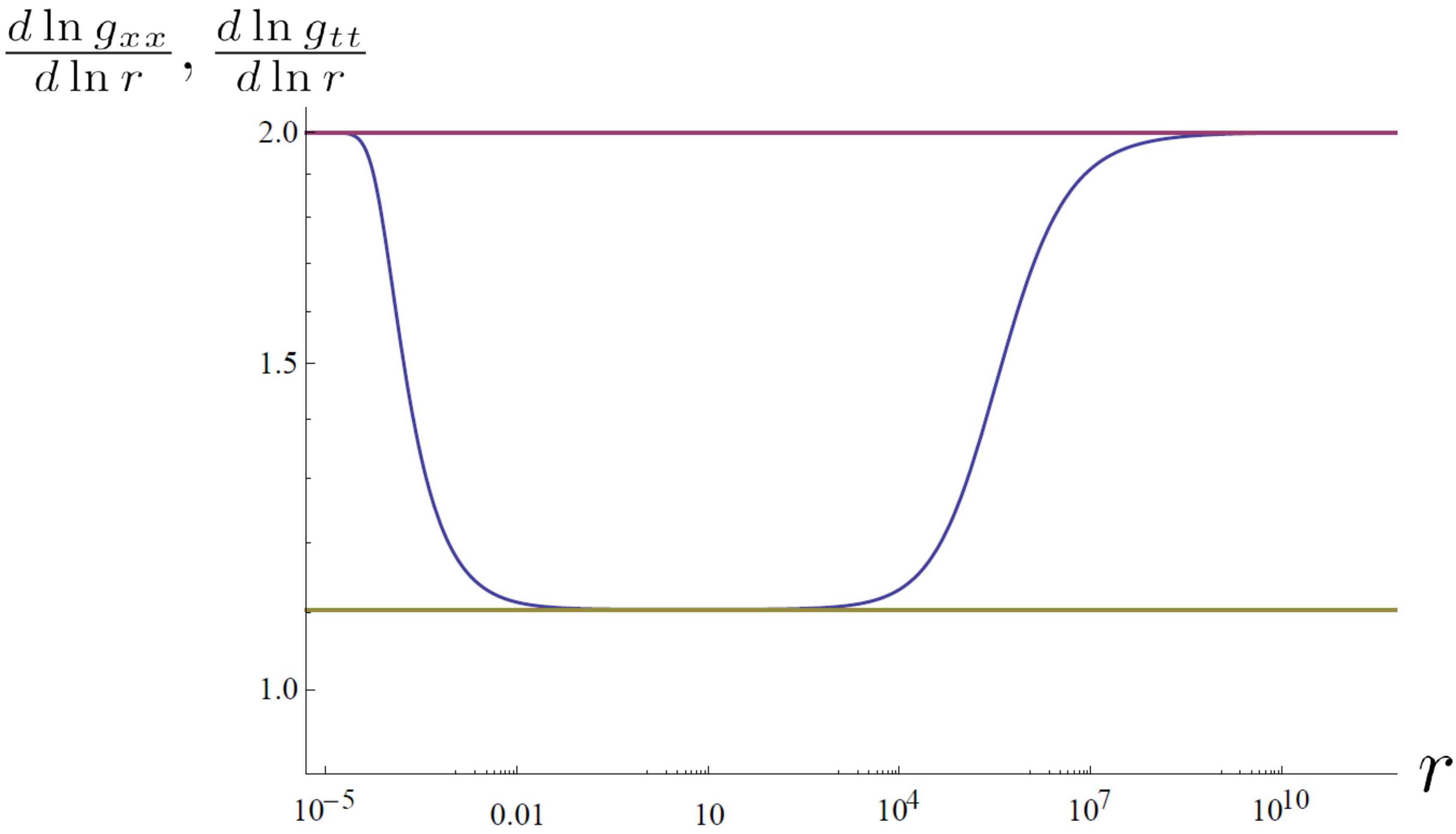
$$\mathcal{L}_{int} \sim R - \frac{1}{2}\partial\phi^2 - \frac{Z_0}{4}\, e^{\alpha\phi} F^2 - \frac{W_0}{2}\, e^{\beta\phi} A^2 - V_0 e^{-\eta\phi}$$



- Classes of (z,θ) solutions to `intermediate' model were found in e.g. Gouteraux + Kiritsis [arxiv:1212.2625] when $\beta = \alpha - \eta$

Background

$z = 1, \ \theta = -162/19$



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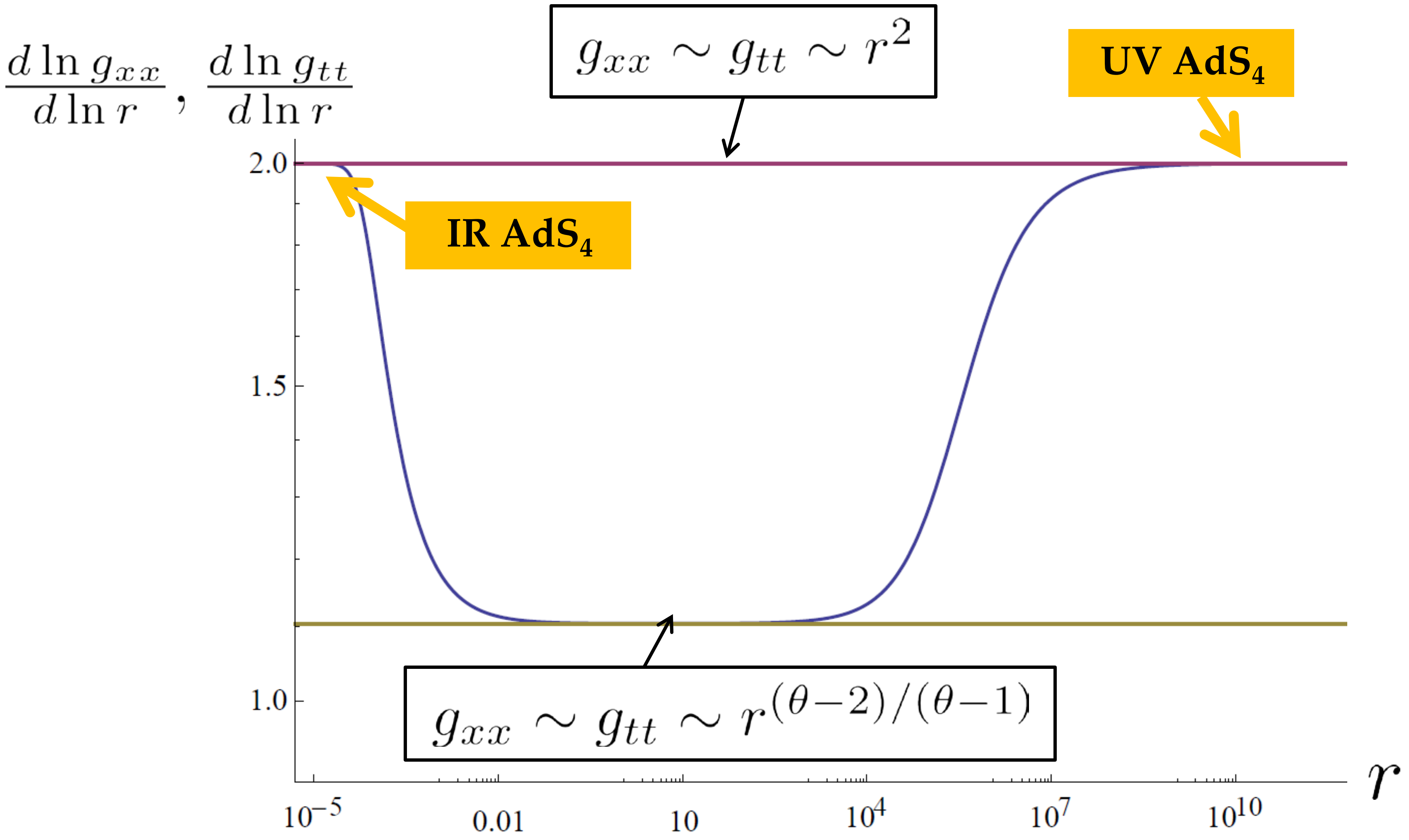
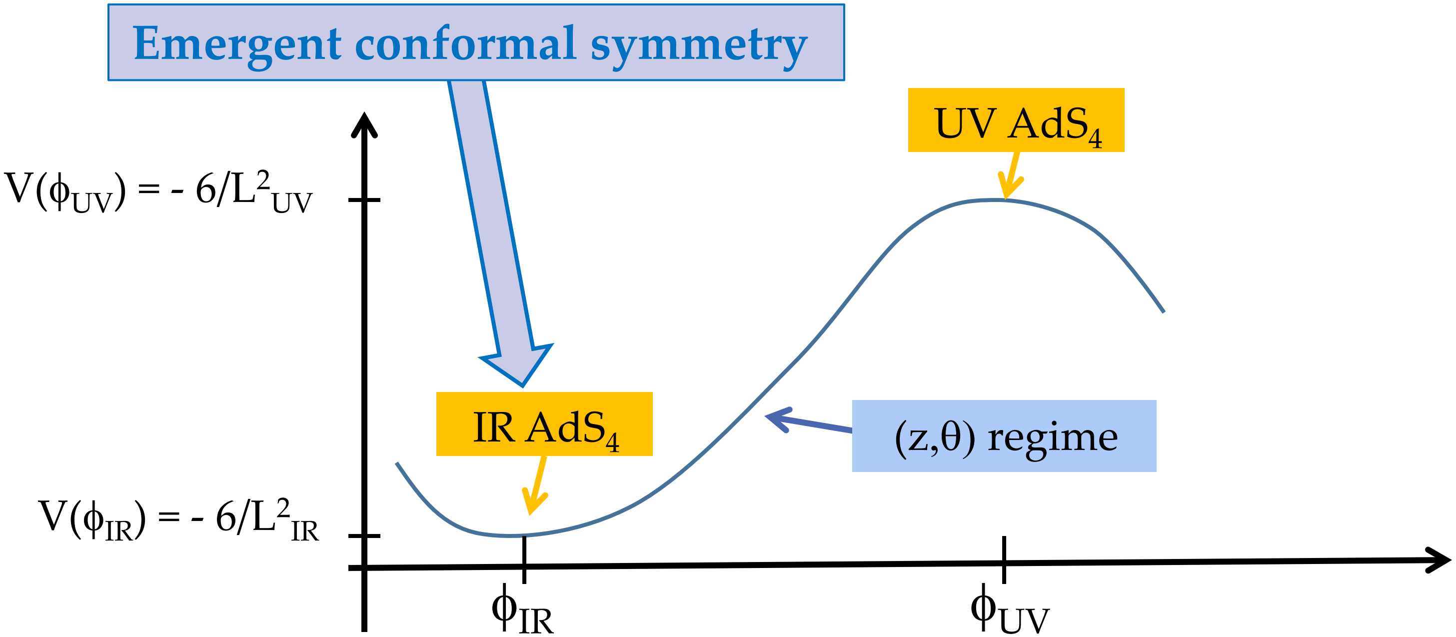


Features of our toy model



- **New ground state** for scaling solutions w/out extensive entropy
- Interplay between **different scalings at different energy scales**
- Applications to transport: **expect intermediate scaling regime**

[see also talk by Blaise Gouteraux]

AC Conductivity

$$\mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{Z(\phi)}{4}F^2 - \frac{1}{2}W(\phi)A^2 - V(\phi)$$

perturbations $\delta g_{tx}\,, \delta A_x$

The gauge field fluctuation $\delta A_x(t,r) = a_x(r)e^{-i\omega t}$ obeys:

$$\partial_r\left(Z\sqrt{\frac{D}{B}}\,\partial_r\, a_x\right) + \left[Z\sqrt{\frac{B}{D}}\,\omega^2 - \frac{Z^2}{\sqrt{BD}}(\partial_r A_t)^2 - W\sqrt{BD}\right] a_x = 0$$

Can be rewritten in **Schrodinger's form**:

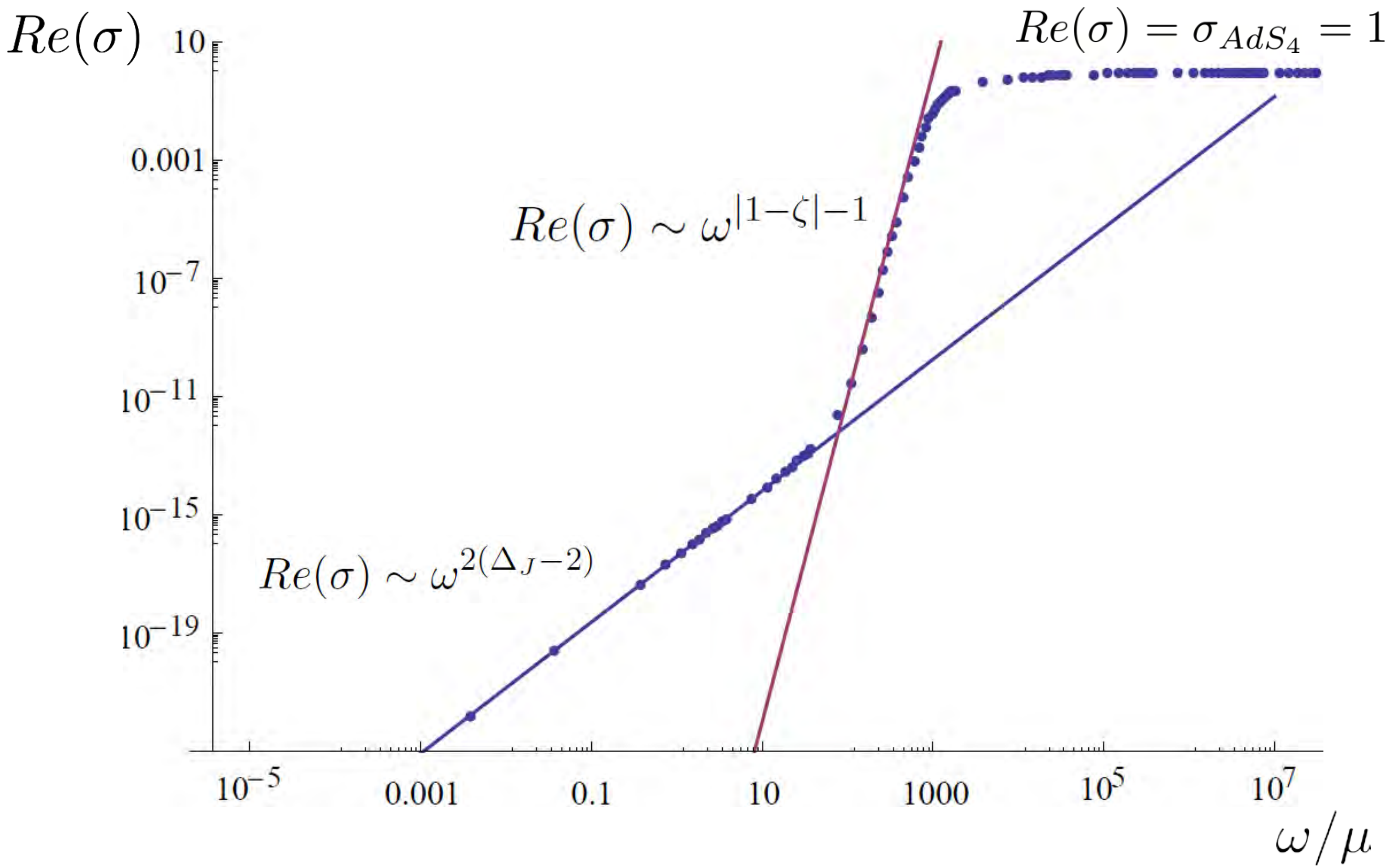
$$-a'' + \mathcal{V}\, a = \omega^2\, a$$

$$\mathcal{V} = \frac{Z\, A_t'^{\,2}}{g_{tt}} + \frac{W\, g_{tt}}{Z} - \frac{Z'^{\,2}}{4Z^2} + \frac{Z''}{2Z}$$

→ $\sigma(\omega)$ can be computed using the **method of matched asymptotics**

[Gubser + Rocha '08]

Preliminary plots – intermediate scaling regime



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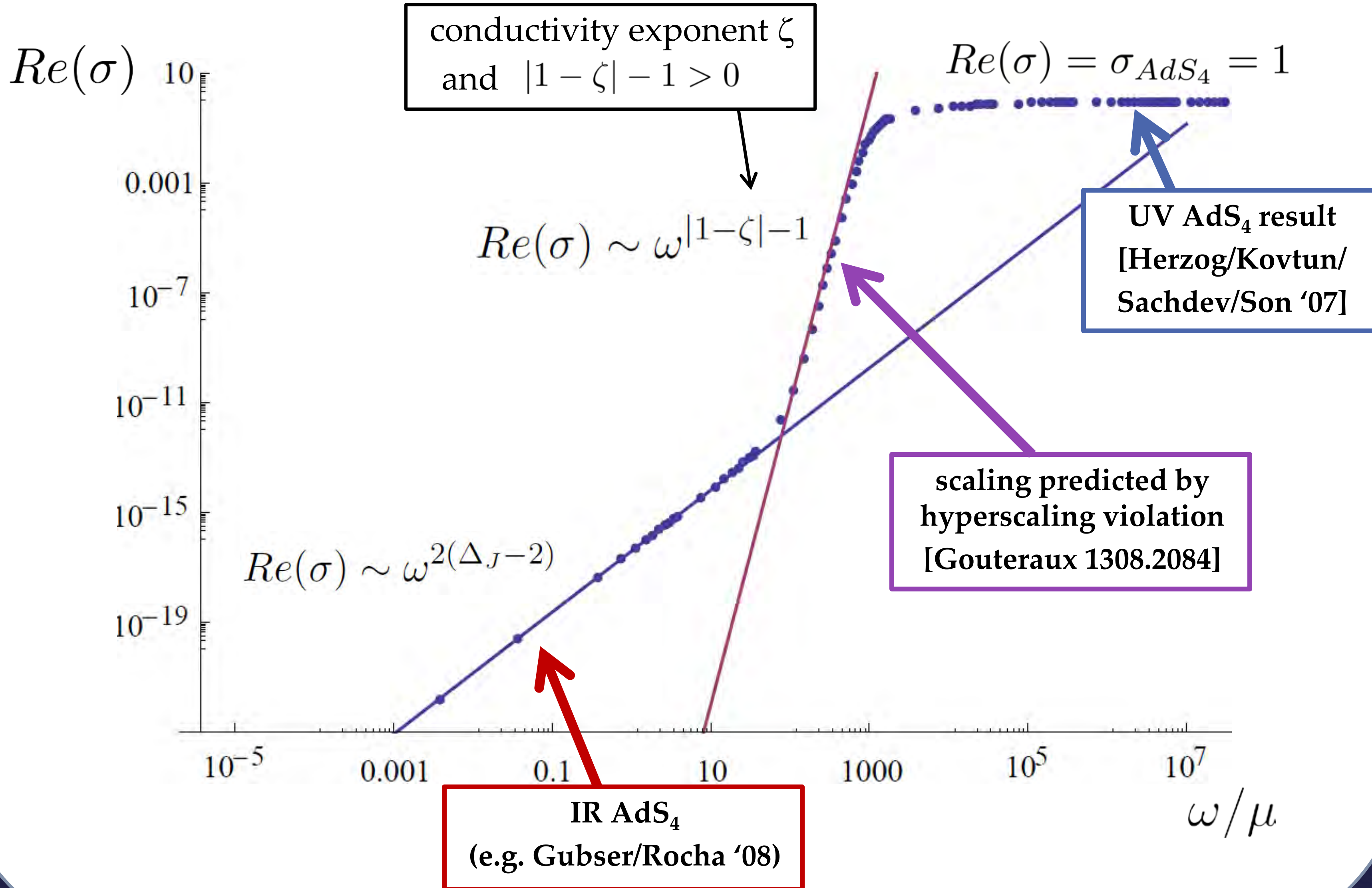
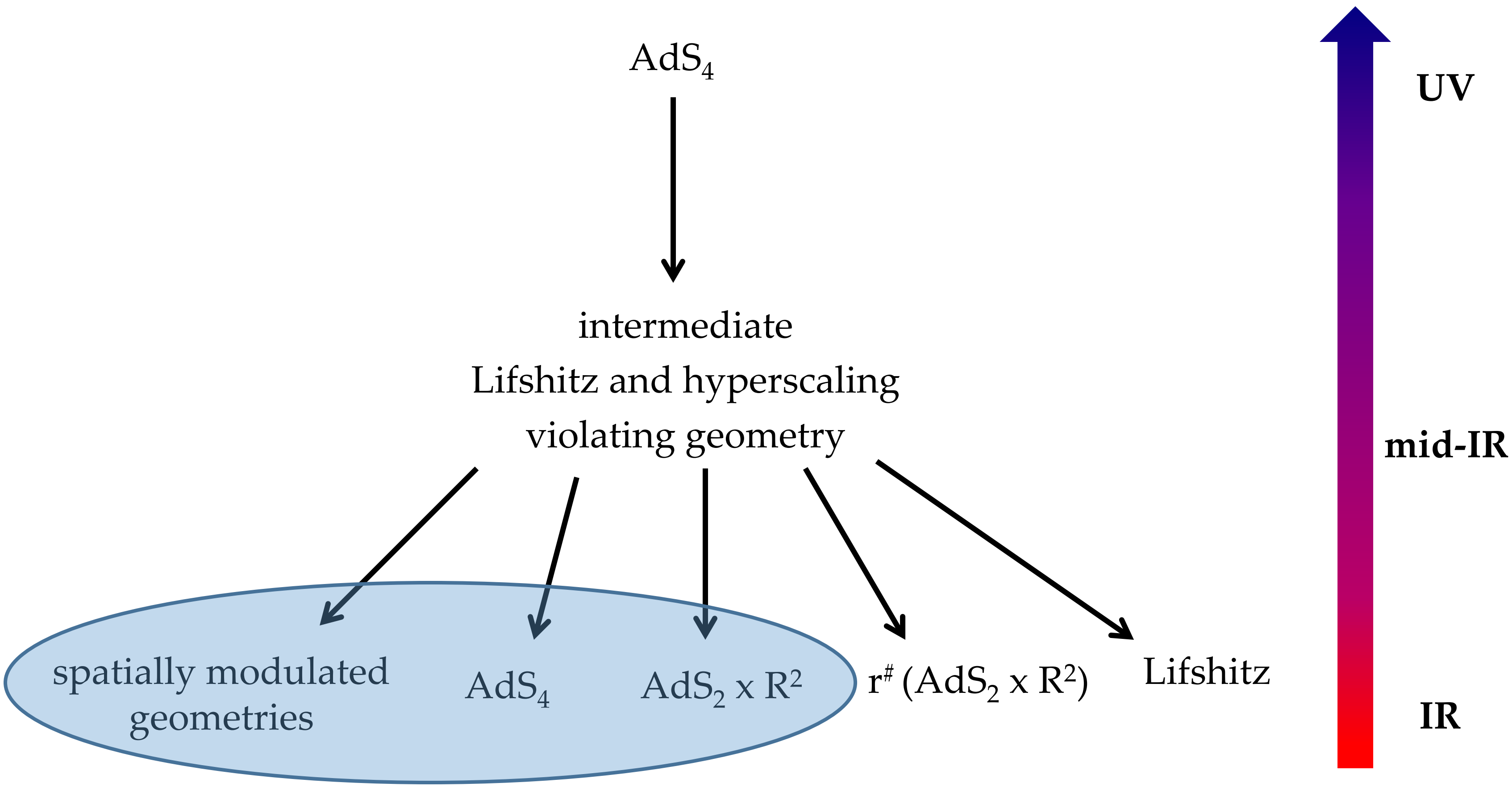


Rich structure of IR phases



To wrap up…

- The structure of phases from gravity is much richer than anticipated
- Interesting RG flows, emergent IR phases and intermediate scalings
- More lessons ahead as the dialogue between gravity and quantum field theories continues

Thank You