Strongly Coupled Anisotropic Fluids From Holography

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Special Thanks

Special Thanks To Elias!

Based On

- ``Strongly Coupled Anisotropic Fluids From Holography''
- Sachin Jain, Nilay Kundu, Kallol Sen, Aninda Sinha and S. P. T; arXive 1406.4874
- (92 pages!)

Some Related References

- Mateos and Trancenelli, arXive 1105.3472; arXive 1106.1632
- Rebhan and Steinedar; arXive 1110.6825
- Azenayagi, Li and Takayanagi, 0905.0688
- Erdminger, Kerner, Zeller, 1011.5912
- Donos and Gauntlett, JHEP 1404 (2014) 040, JHEP 1406 (2014) 007

Outline

- Motivation
- Brane Solution
- Viscosity
- Quasi Normal Modes and Stability
- Fluid Mechanics
- Conclusions

Motivation

- Holography: A useful tool to study strongly coupled field theories
- Qualitative insights, e.g., of transport properties have emerged.
- E.g bound of shear viscosity (Kovtun, Son, Starinets):

$$\eta/s \geq 1/(4\pi)$$

Motivation

- Most of the studies : Isotropic Phases
- Can something be learned from holography for anisotropic phases ?
- Anisotropic Phase: Equilibrium Configuration breaks rotational invariance
- Gravity description: Black Brane breaks rotational invariance

Motivation

- From the gravity perspective: Because such anisotropic black brane solutions exist, and are novel and not so well understood.
- From the Condensed matter perspective:
- i) Because many interesting states of matter are anisotropic. e.g., Graphene, Cuprates, spin density waves, spatially modulated phases etc.
- ii) Or perhaps can be emulated in Atom traps etc

Motivation

- Gravity side: Anisotropic Black Branes break rotational invariance.
- Are there even such solutions? This was not obvious, until a few years ago.
- (Rotation easy way to do it)
- Today, we know that Anisotropic Black brane solutions with planar horizons exist.

Motivation

- The developments leading to these anisotropic black brane solutions, and also hairy black brane solutions, are relatively recent.
- Intuition from condensed matter physics played an important role.

Motivation

- The no hair theorems for horizons in asymptotically flat space suggested that the rotationally symmetric Schwarzchild black brane (or its charged generalisation) were the only possible solutions.
- However, intuition from condensed matter physics suggested that anisotropic solutions, and more generally solutions with hair should exist.
- The intuition from condensed matter physics was right!
- Asymptotically AdS space is different from flat space. And anisotropic solutions, more generally hairy black branes do exist.

Motivation

- In fact a rich diversity of anisotropic black brane solutions are allowed (S. Cremonini's talk).
- Anisotropic homogeneous black brane solutions can be classified using the Bianchi classification developed earlier in cosmology.
- N. Iizuka, S. Kachru, N. Kundu, P. Narayan, N. Sircar, SPT, arXiv:1201.4861
- N. Iizuka, S. Kachru, N. Kundu, P. Narayan, N. Sircar, SPT, H. Wang, arXiv: 1212.1948

Motivation

- Special examples of anisotropic, homogeneous solutions are Lifshitz , Hyperscaling violation, etc. But many of the other 9 Bianchi classes can also be realised as black branes.
- (The Bianchi classification actually classifies all anisotropic homogeneous phases in nature. As such, should be of interest independent of gravity)

Motivation

- Since anisotropic phases have gravity duals the gravity description might teach us something interesting about such phases in strongly coupled systems.

Caveat

- **Anisotropic situations** can arise when the underlying phase is isotropic, due to appropriate initial conditions or boundary conditions. For example, in fluid mechanics. Or at collisions in RHIC or the LHC, since QCD at finite temperature does not break rotational invariance.

- However, in such situations the underlying equations of fluid mechanics are rotationally invariant.

- In contrast, in the cases of interest to us, the underlying equilibrium phase itself will be anisotropic and the equations of fluid mechanics which arise will then also be rotationally asymmetric.

Motivation

- Here we will consider a very simple system :

$$\int d^5x\sqrt{-g}[R - \Lambda - \frac{1}{2}(\nabla\phi)^2]$$

- Gravity with negative cosmological constant and a free scalar which we will call the dilation.
- Such a system arises as a consistent truncation in many gravity/string theory systems.
- (More on string compactifications later)

Basic Set Up

- The dual field theory is 3+1 dimensional. Lives in t,x,y,z, directions.
- Boundary conditions: linearly varying dilation on boundary of AdS space.

$$\phi = \rho z$$

- ρ has dimensions of mass ($(L)^{-1}$)
- (Apologies: $\hbar, k_B, c = 1$!)

Basic Set Up

$$AdS_5$$

$$ds^2 = \frac{r^2}{R^2}[-dt^2 + dx^2 + dy^2 + dz^2] + \frac{R^2}{r^2}dr^2$$

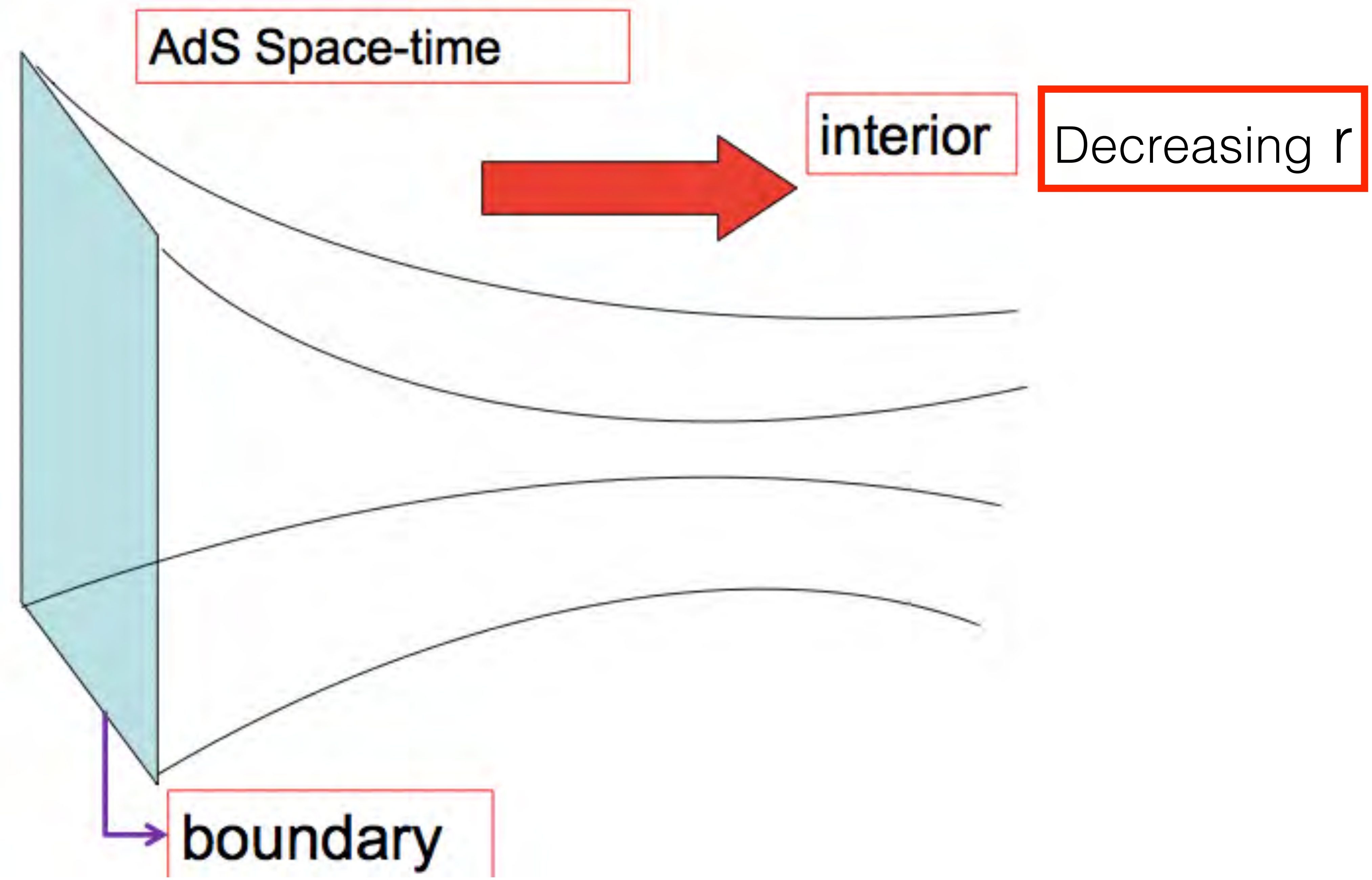
- Solution: homogeneous and isotropic.
- Boundary:

$$r \to \infty \qquad \phi = \rho z$$

- Resulting solution: Effect of dilation unimportant near the boundary. But gets more important for smaller values of r.
- ρ has dimensions of Mass.

Basic Set Up

- Thus the solution is asymptotically AdS, but deviates from AdS in the infrared, where the effects of the dilation stress tensor get significant.



AdS Space-time
interior
Decreasing r
boundary

Basic Set Up

- One important thing: Due to the linear dilation, this system breaks isotropy and also translational invariance.
- However, it turns out that the black brane solution preserves translation invariance, while breaking rotational symmetry.

Basic Set Up

- Thus, the equilibrium properties are those of a anisotropic phase with translational invariance intact.
- This happens on the gravity side because the stress tensor of the dilation depends on $\nabla\phi$ and not on ϕ .
- (See also J. Gauntlett's talk)

Basic Set Up

- Intuitively one should think of the field theory subjected to a driving force proportional to $\nabla\phi$, which is a constant.

Basic Set Up

- We will be interested in solutions at temperature T
- Dimensionless parameter: ρ/T
- $\rho/T << 1$: low anisotropy. Not so interesting
- The horizon appears before anisotropy in geometry can get significant.
- Temperature, T, cuts off the growth of anisotropy.

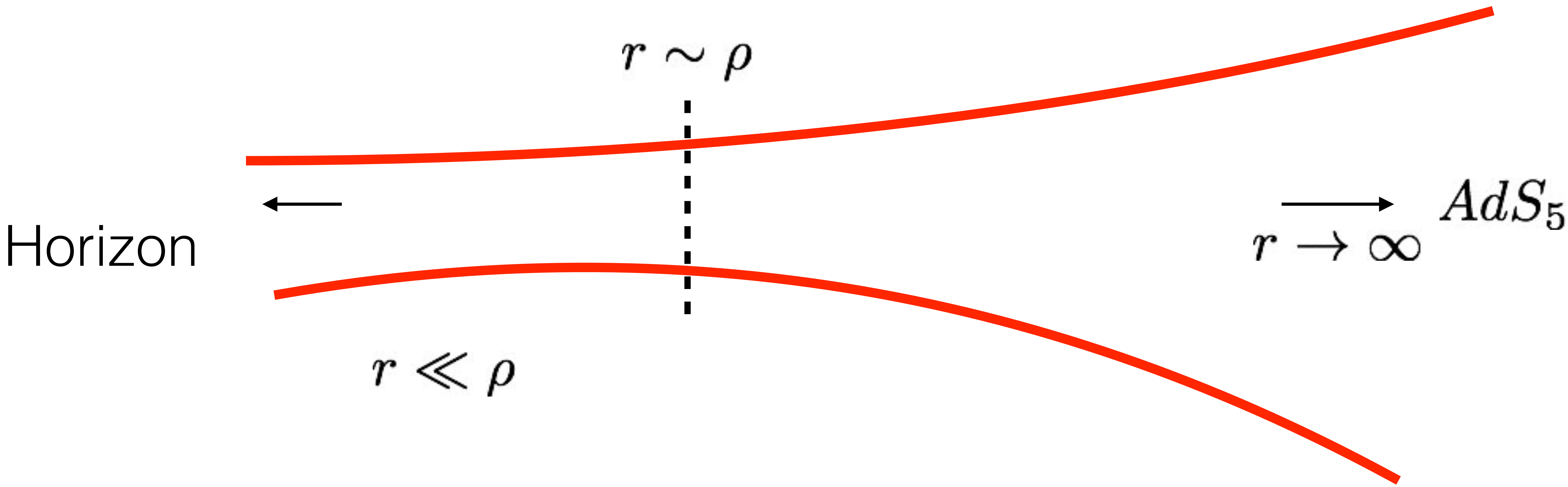
Basic Set Up

- We will be interested in solutions at temperature T
- Dimensionless parameter: ρ/T
- $\rho/T >> 1$: High anisotropy. More interesting. Mostly study this case.
- $\rho/T \rightarrow \infty$: Limit of extreme anisotropy. Can be obtained by taking the extremal limit, ρ fixed, $T \rightarrow 0$

The Extremal Black Brane Solution

$\phi = \rho z$

Dilaton is exactly linear for all r

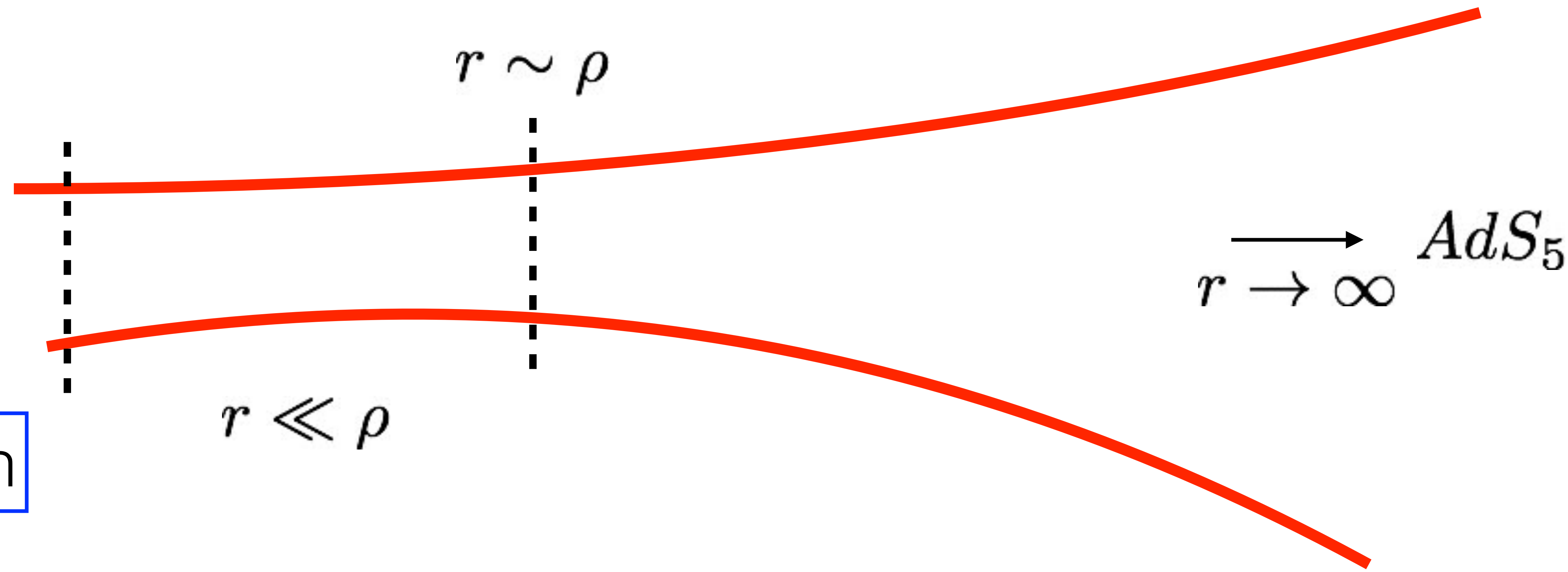


Near Horizon Geometry : $AdS_4 \times R$

The Near-Extremal Black Brane Solution

$\phi = \rho z$

Dilaton is exactly linear for all r



Horizon

Near Horizon Geometry: Black Brane in $AdS_4 \times R$

The Near-Extremal Black Brane

- The near horizon region is simple and can be obtained analytically.
- The full solution is obtained numerically.
- Symmetries of AdS_4 preserved along the full flow.

The Black Brane Solution

- The dilation perturbation is relevant in the IR and drives the system to a new fixed point.
- In the gravity description this corresponds to a new attractor geometry $AdS_4 \times R$
- The attractor geometry is an exact solution in its own right. Easy to study analytically.

Additional Motivation

- Study of Driven systems.
- With a slowly varying dilation and metric this was analysed earlier.
- S. Bhattacharyya, R. Loganayagam S. Minwalla, S. Nampuri, S. R. Wadia, S. P. Trivedi; 0806.006
- Here we have a fast varying dilation with a particular profile.

The Black Brane Solution

$$ds^2 = \frac{r^2}{R_4^2}(-dt^2 + dx^2 + dy^2) + \frac{R_4^2}{r^2}dr^2 + dz^2$$

$$R_4^2 = \frac{3}{4}R^2$$

- Near horizon region simple.
- Extra stress energy due to dilation prevents z direction from shrinking.
- Solution homogeneous but anisotropic.

$$SO(3) \to SO(2)$$

The Black Brane Solution

- Thermodynamics Near Extremality:
- Stable.
- Entropy density:

$$s = c_1 N^2 T^2 \rho$$

- Positive specific heat.

Viscosity

- Dissipation arises because of the black hole horizon.
- Things fall into a black brane/hole but cannot come out. This breaks time reversal invariance.
- Viscosity is related to the absorption of metric perturbations by the black brane horizon.

Viscosity

- In the isotropic case there are two independent components of the viscosity: Bulk and Shear viscosity. The Bulk viscosity vanishes in a CFT.

- The relevant metric perturbations have an absorption cross section which is proportional to the area.

- The entropy is also proportional to the area

$$S = \frac{A}{4G_N}$$

-

Viscosity

- As a result the ratio (s, the entropy density),

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- is universal in Einstein theory. Independent of details of black brane, as long as it is isotropic.

Viscosity (Anisotropic Case)

- Viscosity : In general is a tensor with 21 independent components.
- $$\eta_{ijkl} = \lim_{\omega \to 0} \frac{1}{\omega} Im G^R_{ijkl}(\omega)$$

$$G^R_{ijkl} = \int dt\, d\vec{x}\, e^{i\omega t} \theta(t) < T_{ij}(t) T_{kl}(0) >$$

Viscosity Anisotropic Case

- In our case since SO(2) is preserved, and there are 5 independent components.
- In terms of the metric :
- Spin 2 : 1 independent component
- Spin 1 : 1 independent component
- spin 0 : 3 independent components

Viscosity

- Spin 2 : $h_{xy}, (h_{xx} - h_{yy})/2$: $\eta_{||}$
- Spin 1 : h_{xz}, h_{yz} : $\eta_{\perp}$
- These viscosity coefficients can be calculated analytically.
- E.g., Membrane Paradigm, Liu and Iqbal, Phys.Rev. D79 (2009) 025023

Viscosity

- Close To Extremality

$$\frac{T}{\rho} \ll 1$$

$$\frac{\eta_{||}}{s} = \frac{1}{4\pi}$$

$$\frac{\eta_{\perp}}{s} = \frac{8\pi^2 T^2}{3\rho^2}$$

-

Thus, some components of viscosity become parametrically small.

Viscosity

- The low frequency response arises from the near horizon region. The 5 dim. metric gives rise to a 4 dim. metric, gauge field and scalar. These are absorbed differently by the black brane.

- This should be a general feature in anisotropic cases.

Stability

- For rotationally invariant case attempts to violate the KSS bound in a big way lead to pathologies.
- (Brigante, Liu, Myers, Shenker, Yaida; Myers and Sinha; etc)
- Therefore important to study the Quasi Normal modes of this system close to extremality

Stability

- One expects an instability, if it arises, to be localised near the horizon described by the $AdS_4 \times R$ geometry.
- Several Channels studied.
- No instability found. Poles lie in lower half plane.

$$\delta\phi \sim e^{-i\omega t}$$

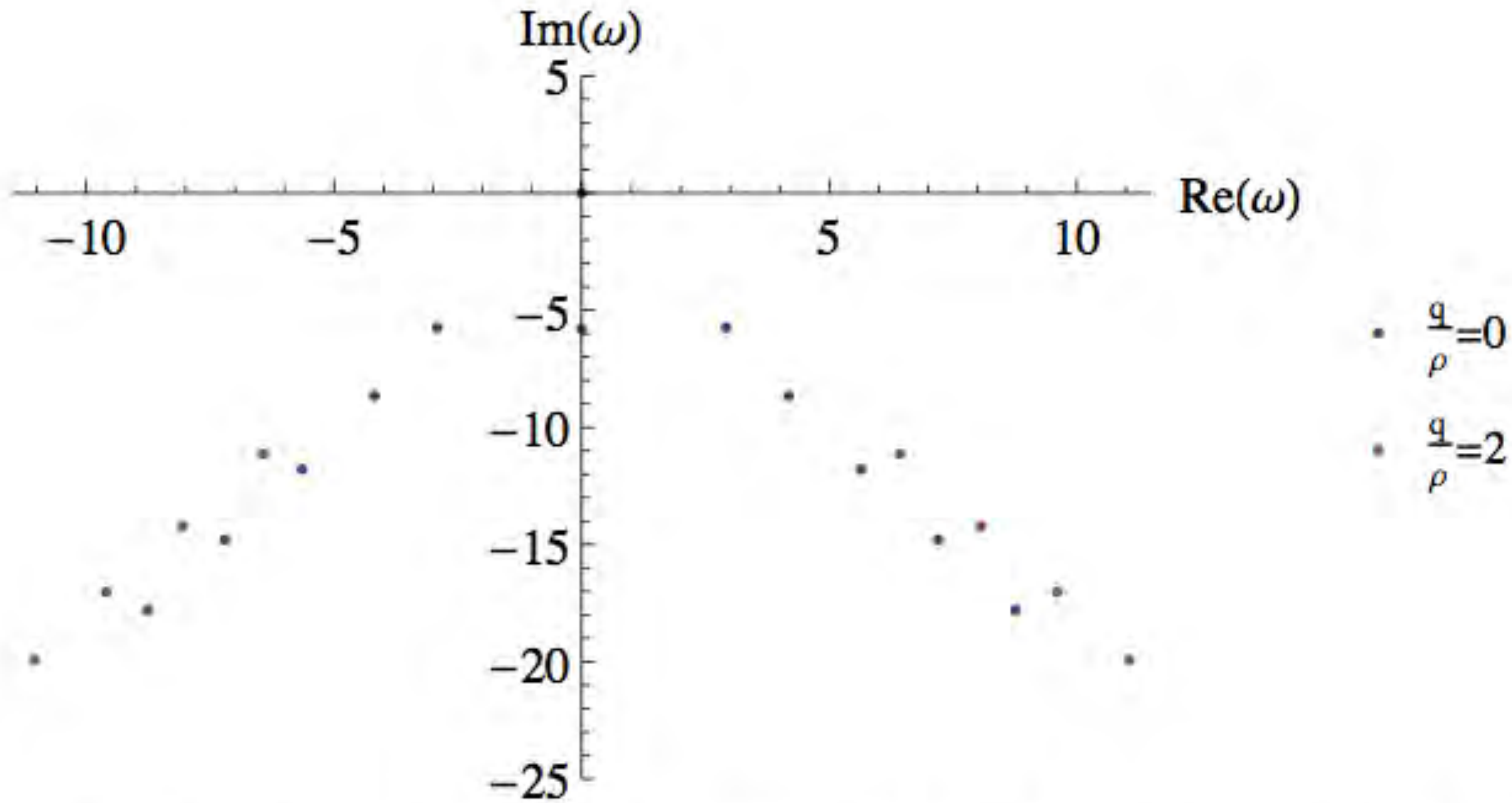


Figure 4: QNM plots for spin 1 mode with $\frac{q}{\rho} = 0, 2$ with $\vec{q}$ along z direction

More On Viscosity

- To summarise

- Different components of the viscosity behave very differently, when $T/\rho \ll 1$ and the anisotropy is big.

$$\frac{\eta_{||}}{s} = \frac{1}{4\pi}$$
$$\frac{\eta_{\perp}}{s} = \frac{8\pi^2 T^2}{3\rho^2}$$

- Some components become very small.

More On Viscosity

- This is different from what the weakly coupling picture would have suggested.

- If there are weakly interacting quasi-particles:

$$\frac{\eta}{s} \sim \frac{l_{mfp}}{\lambda_{dB}}$$

- Extrapolating to strong coupling $l_{mfp} \sim \lambda_{dB}$

$$\frac{\eta}{s} \sim O(1)$$

More On Viscosity

- For the anisotropic case the mean free path in different directions would be different. But still one expects l_{mfp} to be bounded by λ_{dB}

- Leading to the bound

$$\frac{\eta}{s} \geq\sim O(1)$$

- (for all components)

More On Viscosity

Comparison with hydrodynamics (to follow)

$$D = \frac{\eta}{\epsilon + P}$$

Einstein Relation

Component wise

$$D_{||} = \frac{\eta_{||}}{\epsilon + p} \qquad D_{\perp} = \frac{\eta_{\perp}}{\epsilon + p}$$

Diffusion Lengths

- ϵ energy density (above extremity)
- p : pressure
- s : entropy density

More On Viscosity

Comparison with hydrodynamics (to follow)

$$D = \frac{\eta}{\epsilon + P}$$

Einstein Relation

Component wise

Equation of state : $\epsilon + P \sim sT$

Leading to : $\frac{\eta}{s} \sim DT$

More On Viscosity

- This gives:

$$D_{||} = \frac{1}{3\pi T}$$
$$D_{\perp} = \frac{8\pi T}{3\rho^2}$$

- $D_{\perp}$ gets very small leading to the anomalously small value for $\eta_{\perp}$

More On Viscosity: Diffusion Lengths

- The diffusion lengths can be calculated directly from the gravity description also by looking at the dispersion relation for various hydrodynamic modes:

$$\omega = -iDq^2$$

- The results of course agree.

More On Viscosity

$$\frac{\eta}{s} \sim \frac{l_{mfp}}{\lambda_{dB}}$$

$$\frac{\eta}{s} \sim DT$$

$$l_{mfp} \to D \qquad \lambda_{dB} \to 1/T$$

- $$\frac{\eta}{s} \sim \frac{l_{mfp}}{\lambda_{dB}} \to DT$$

See also S. Hartnoll's

More On Viscosity

- So perhaps the weak coupling results can be extrapolated to strong coupling with the diffusion length playing the role of the mean free path.

$$l_{mfp} \to D \qquad \lambda_{dB} \to 1/T$$

- $$\frac{\eta}{s} \sim \frac{l_{mfp}}{\lambda_{dB}} \to DT$$

See S. Hartnoll's talk

Stability: Caveat

- Note here we are studying the 5 dim. theory of gravity and a massless scalar.

$$\int d^5x\sqrt{-g}[R - \Lambda - \frac{1}{2}(\nabla\phi)^2]$$

- Many string embeddings are known. All the ones we have looked at (S^5, orbifolds of S^5 , $T^{1,1}$) have instabilities.

Stability: Caveat

- It is unclear, at this moment, how serious this problem is.
- Either there are stable string embeddings, in the vast landscape of string vacua, with no instabilities
- Or there is a general field theory argument which rules out such a small value for the viscosity.
- (instabilities set in when $T \sim O(\rho)$)

Fluid Mechanics

- Effective field theory of hydrodynamic modes
- Valid at long distances.
- We will construct a theory valid for $L \gg 1/T, 1/\rho$

Fluid Mechanics

- Anisotropic Case:
- Extra Vector: $\xi_\mu = \partial_\mu \phi$ (constant in space, time)
- At zeroth order in the derivative expansion the constitutive relation:

$$T_{\mu\nu} = \epsilon u^\mu u^\nu + p(\eta^{\mu\nu} + u^\mu u^\nu) + f \xi^\mu \xi^\nu$$

- One more term in the anisotropic case.

Fluid Mechanics

- At the first derivative level: there are 10 terms in the constitutive relation.
- The coefficients, which include, $\eta_{||}, \eta_{\perp}$, are functions of T, ρ
- Some are not dissipative and can be calculated from equilibrium considerations.

Fluid Mechanics

- Since the system is subjected to a driving force, due to the varying dilation, momentum is not conserved.
- The Navier Stokes equations take the form

$$\partial_\mu T^{\mu\nu} = \partial^\nu \phi < O >$$

- $< O >$ expectation value of operator dual to dilation
- We have to therefore also write an analogue of the constitutive relation for $< O >$ expressing it in terms of T, ρ
- This introduces additional terms and coefficients. The coefficients of al these term scan be calculated from studying the Einstein and dilation equations in the bulk.

Fluid Mechanics

- Luckily most of these complications drop out in some simple situations.

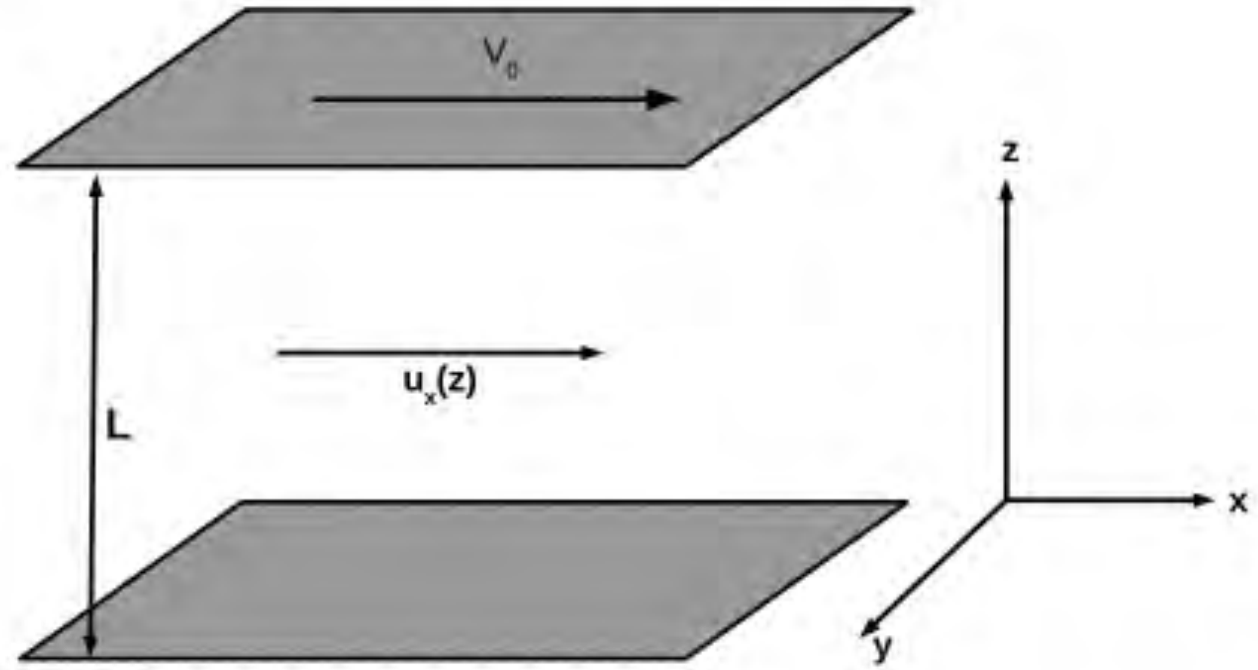


Figure 15: Fluid flow between two parallel plates separated in the z-direction by a distance L. The upper plate has velocity v_0 along the x-direction and the lower plate is at rest. The fluid generates a flow along x-direction with velocity $u_x(z)$.

Fluid Mechanics

- For a suitably orientation the force per unit area required to sustain the flow is

$$F/A = \frac{\eta_\perp v_0}{L}$$

- This becomes anomalously small at small T:

$$\eta_\perp \sim s\frac{T^2}{\rho^2}$$

Conclusions

- Anisotropic brane solutions are worth studying further. Both from the point of view of gravity, and also condensed matter physics.
- The simple system we have studied has already yielded some surprises.
- Some components of the viscosity become parametrically small, compared to the entropy density, when the anisotropy becomes large.

Conclusions

- One suspects that this parametric suppression happens in many other cases too.
- When an SO(2) symmetry survives, the spin 1 component of the metric, which is like a gauge field perturbation in the near horizon region, can be absorbed quite differently from the spin 2 component.

Conclusions

- It would be worth studying experimentally whether such behaviour is seen. Perhaps in cold atoms?
- Further studies on the gravitational side are bound to reveal more surprises !

Cautionary Note

- The brane solution in the gravity system we studied was stable (few quasi normal modes remain to be analysed).
- However, the string embeddings we studied all have instabilities due to additional modes. This is worrisome.
- Either we should be able to find compactifications in the landscape of string vacua without such instabilities.
- Or device a general argument in field theory which rules out such a small viscosity.

Thank You!