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Semiholographic non-Fermi liquids

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Holography gives us the possibility of exploring new possible (at least theoretically) phases of matter

It has confirmed ideas proposed before in the CM community but in a more controlled (though less realistic) setting

At finite density, relatively easy to obtain exotic states with attractive features also from a phenomenological point of view

Landau's theory of Fermi liquids is remarkably robust, accounting for the properties of most metals in terms of weakly interacting quasiparticles

$$C_v \sim T \qquad\qquad \rho \sim T^2$$

However many examples are now known of "strange metals" that do not fall into this class

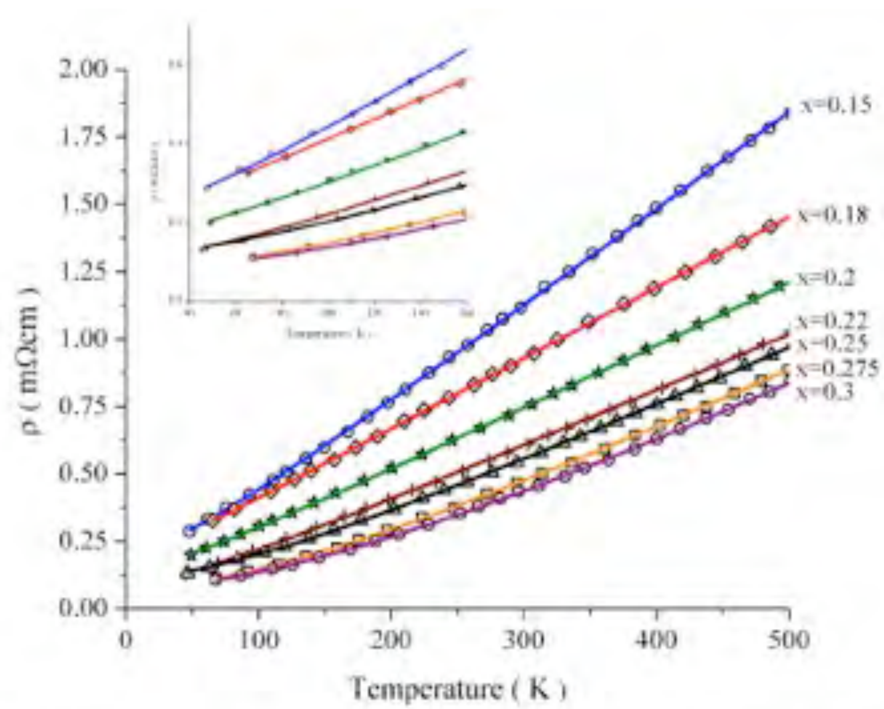
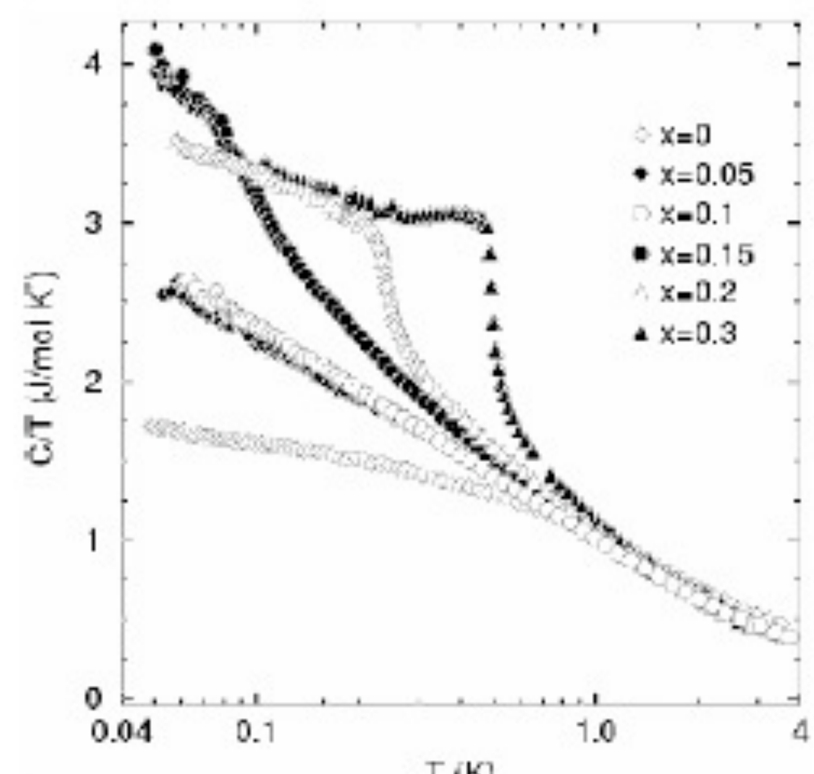
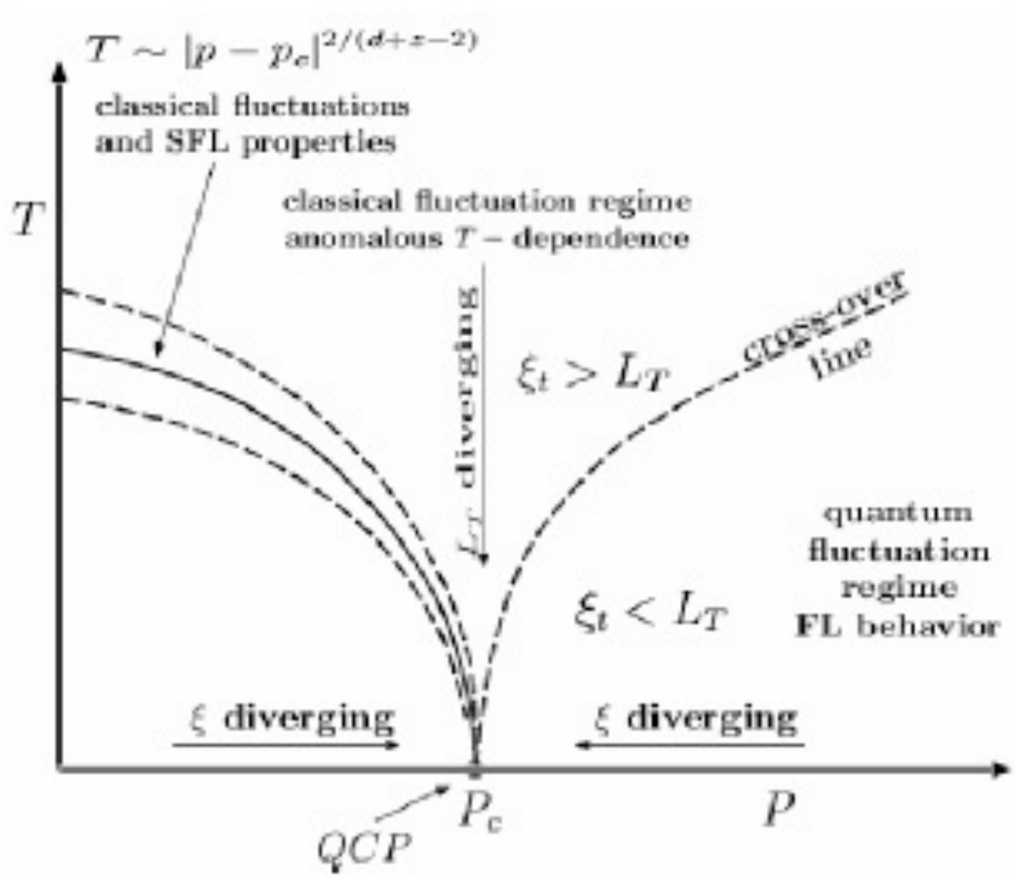


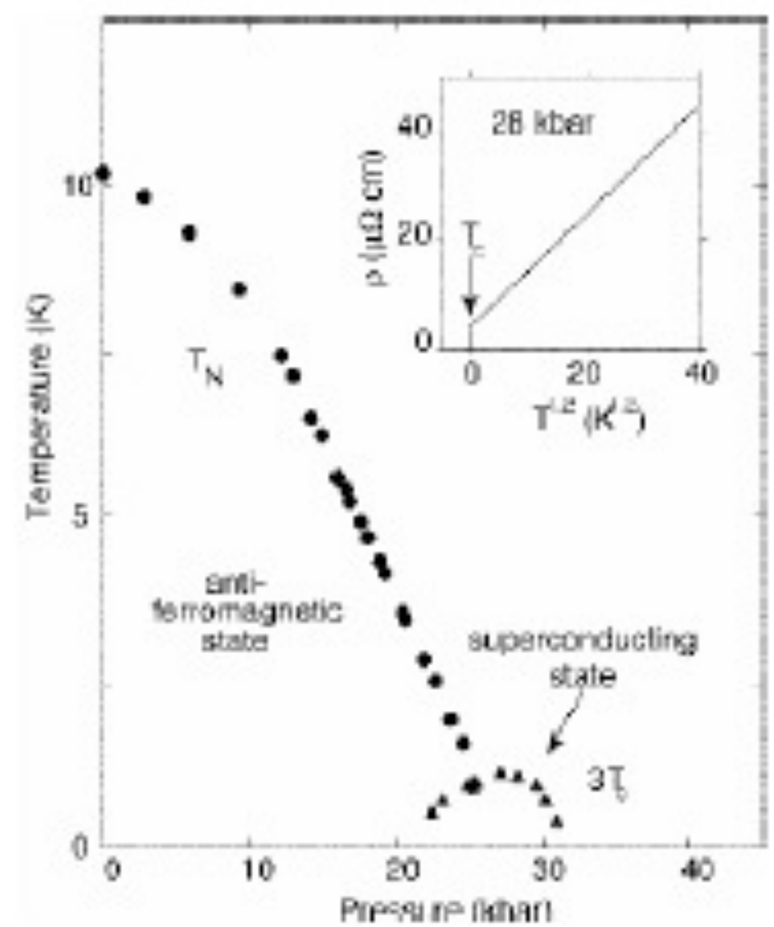
Figure 2. Comparison of the polycrystalline $La_{2-x}Sr_xCuO_4$ resistivity (data points) extracted from Ref. 18 with the "bottleneck" resistivity form of Eq. [10]. Inset slows the low temperature region in detail.



$$C_v \sim T \log T \qquad\qquad \rho \sim T$$

This behavior is often associated with the vicinity to a quantum critical point, and generically points to a short lifetime of excitations





Holography gives many ways of constructing metallic phases with abnormal properties [Sachdev, Hartnoll, Liu, McGreevy, Vegh, Charmousis, Kiritsis, Gouteraux, ...]

The gravitational solution does not show a Fermi surface, so the nature of the charge carriers is obscure

The Fermi surface is exhibited by models that include fermions populating the bulk and backreacting on the geometry but they have FL-like physics [Sachdev, Hartnoll, Tavanfar, Hofman,... Schalm, Zaanen,..., Nitti, Policastro, Vanel...]

[Liu, McGreevy, Vegh '09] showed that probe fermions in the background of an extremal AdS-RN black hole exhibit a Fermi surface in the spectral function

They can be understood as "domain-wall" fermions between the asymptotic AdS_4 and AdS_2 x R^2 regions

$$G_R(\omega, k) = \frac{A(\omega, k) + B(\omega, k)\mathcal{G}}{C(\omega, k) + D(\omega, k)\mathcal{G}} \qquad \mathcal{G} \propto \omega^{\nu(k)}$$

$$C(\omega = 0, k = k_*) = 0$$

ν, k_* functions of m, q

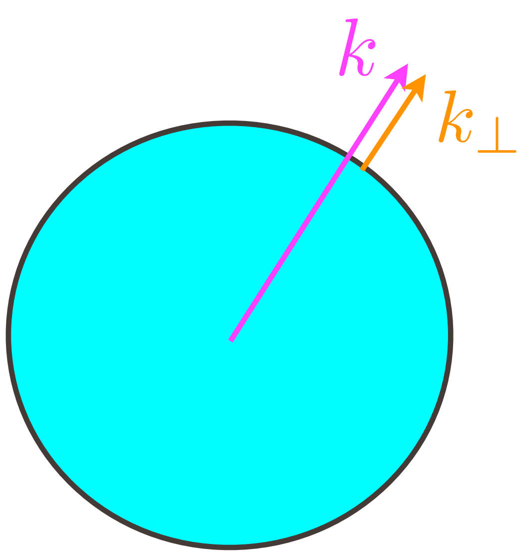
The Faulkner-Polchinski model [F-P, 10]

$$S = \int dt \left[\sum_k \left(\chi^\dagger_{\mathbf{k}} (i\partial_t - \epsilon_{\mathbf{k}} + \mu)\chi_{\mathbf{k}} + \sum_k (g_{\mathbf{k}} \chi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + c.c.\right) \right] + S_{\mathrm{CFT}}$$

Hybridization with fermion of dim $\Delta_\psi = \frac{\nu + 1}{2}$

Resummed propagator reproduces Liu et al.

$$G_{\chi\chi} = -\frac{a}{|g|^2 c\omega^\nu - v_F k_\perp}$$



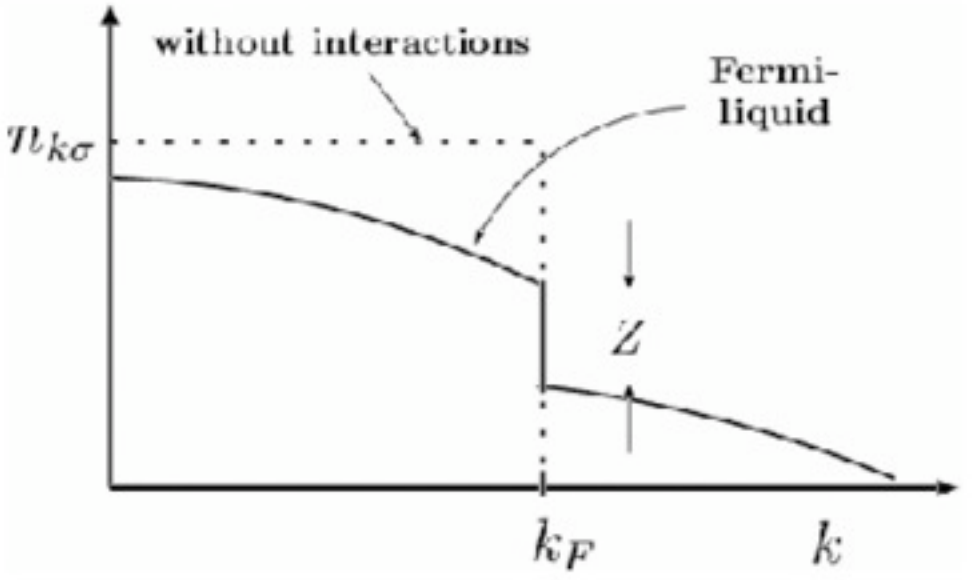
c, ν parameters of the model

Almost-solvable model of a non-Fermi liquid

Very broad quasiparticles

Zero residue at the Fermi surface

Lifshitz-like dispersion relation



[Varma, Nussinov, van Zaarlos, 01]

Similar models: marginal or singular FL (Varma)
hidden FL (Anderson)

We considered extensions of this model to extract "universal" predictions, i.e. that can be fitted with a finite number of parameters

Most studies focused on single-particle properties, that can be directly measured via ARPES etc.

We want to consider also multiparticle properties and collective excitations

Fermi liquid - reminder

Excitations adiabatically connected to free fermion state

$$\tilde{\varepsilon}(p,\sigma) = \varepsilon(p,\sigma) + \sum_{p',\sigma'} f(p,\sigma,p',\sigma')\delta n(p',\sigma')$$

↑ Landau parameters

$$f^{a,s}(\theta) = \sum_{l} F_l^{a,s} P_l(cos\theta)$$ for isotropic system

$$\frac{m^*}{m} = 1 + \frac{F_1^s}{3}$$

$$\chi = \frac{1}{1+F_0^a}\frac{\mu_B^2 k_F m^*}{\pi^2}$$

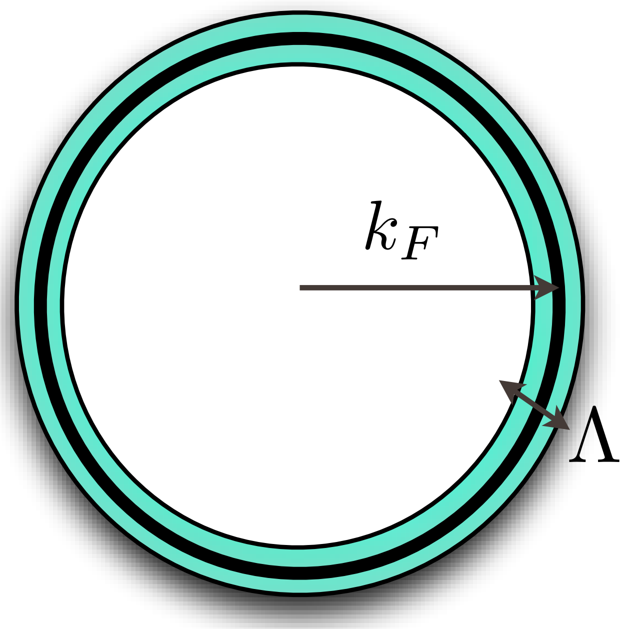
$$\kappa = \frac{1}{1+F_0^s}\frac{k_F m^*}{\pi^2\rho^2}$$

The interacting F-P model

$$
\begin{aligned}
S \quad &= N^2 S_{\mathrm{CFT}} \\
&+ \int dt \Bigg[\sum_k \Big(\chi^\dagger_{\mathbf{k}} (i\partial_t - \epsilon_{\mathbf{k}} + \mu) \chi_{\mathbf{k}} + N \sum_k (g_{\mathbf{k}} \chi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + c.c.\Big) \\
&+ N\eta \sum \chi^\dagger \chi \phi + N\tilde{g} \sum \chi^\dagger \chi \chi^\dagger \psi \\
&+ \sum (\lambda + V(\mathbf{q}))\, \chi^\dagger \chi\, \chi^\dagger \chi
\end{aligned}
$$

The N counting is chosen to reproduce the FP propagator at O(1) and suppress corrections to the vertex functions [cf. Stoof's talk]

Renormalization group [Polchinski, Sarkar]



$$\Lambda \to s\Lambda \qquad \mathbf{k} = \mathbf{k}_* + \mathbf{k}_\perp$$

$$[\mathbf{k}_*] = 0 \qquad [\mathbf{k}_\perp] = 1 \qquad [\chi] = -\ \tfrac{1}{2}$$

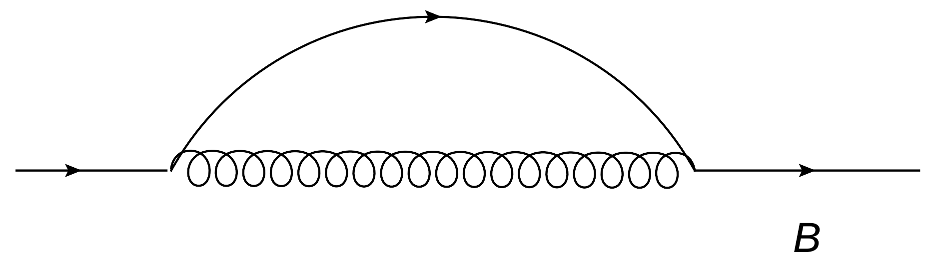
An interaction $\chi^n \Phi$ is relevant or marginal for

$$n \leq \frac{3-\nu}{2}$$

The RG argument needs refinement for special configurations (BCS instability)

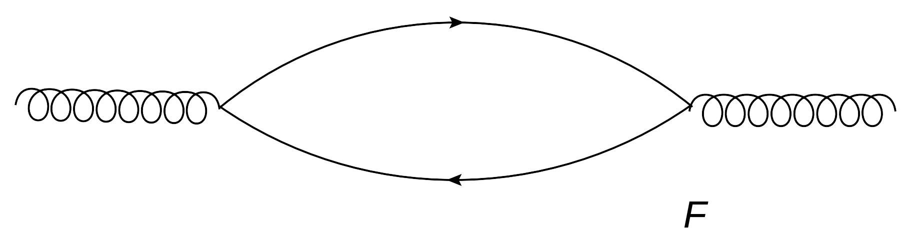
Self-energy corrections [Lee, 2009: breakdown of large N]

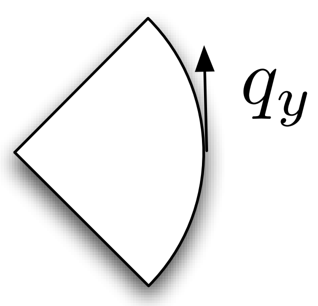
$$\Sigma(k) = $$



$$= \int G(k-k')\,\Pi(k')$$

$$\Pi(k) = $$





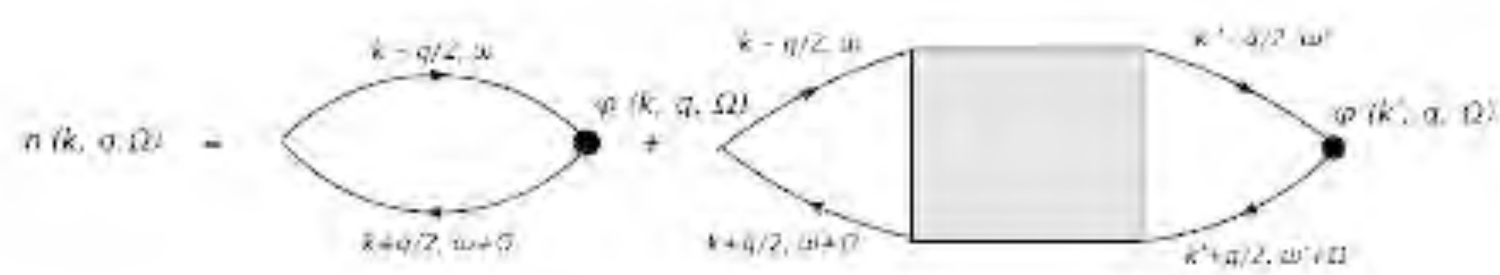
local patch

$$\Pi(\mathbf{q},\omega) = \begin{cases} 0\,, & 0 < \nu < 1/2 \\ \frac{|\omega|}{|q_y|}\,, & 1/2 < \nu < 0 \end{cases} \quad \text{as in FL}$$

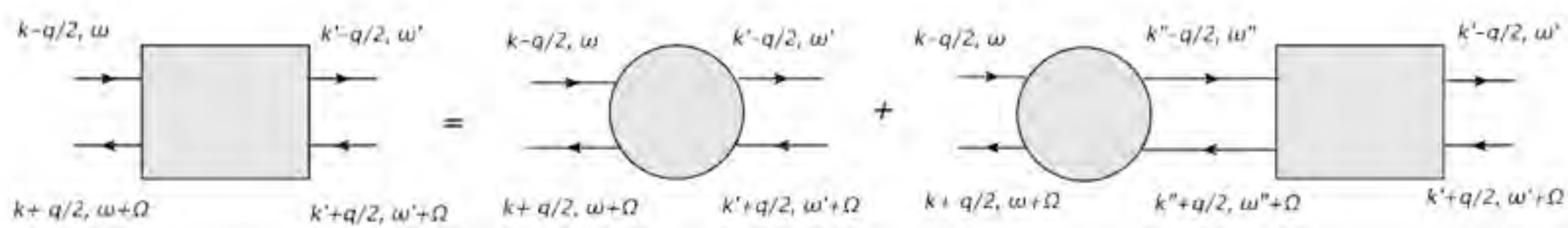
better behavior than in FL as $|\omega| << \omega^{\nu}$

Response function

$$\Delta\mathcal{L} = \int A_\mu(\chi^\dagger\gamma^\mu\chi + N^2 J^\mu)$$



$$R = G\,G(1 + \Gamma\,G\,G)$$



2PI part: $\Gamma = \Gamma_0 + \Gamma_0\, G\, G\, \Gamma$

IR leading behavior is determined by the singular part of the two propagator with momenta on the FS

$$G^{AB}(\mathbf{k}-\mathbf{q}/2,\omega)G^{CD}(\mathbf{k}+\mathbf{q}/2,\omega+\Omega) = (a^2\, Q^{ABCD} + g^{ABCD})(\mathbf{k},\omega,\mathbf{q},\Omega)$$

$$\Psi^A = \begin{pmatrix} \chi \\ \psi \end{pmatrix}$$

$$Q^{1111} = Q \sim \Theta(-k_\perp)\delta(\omega)k_\perp^{\frac{1}{\nu}-2}\frac{q\cos\theta}{q\cos\theta - \nu\Omega(\frac{|g|^2c}{v_F})^{\frac{1}{\nu}}k_\perp^{1-\frac{1}{\nu}}}\,.$$

compare with $Q_{FL} \sim \delta(k_\perp)\delta(\omega)\dfrac{q\cos\theta}{q\cos\theta - \Omega}$

The ω integral uses $\displaystyle\int \frac{d\omega}{2\pi}\frac{f(\omega)}{\omega^\nu - a} = \frac{i}{\nu}a^{\frac{1-\nu}{\nu}}f(a^{1/\nu})$

The analysis justifies a description in terms of Landau parameters

$$f(\mathbf{k}_*, \mathbf{k}'_*) = a^2\, \Gamma^{\chi\chi\chi\chi}(\mathbf{k}_*, \mathbf{k}'_*, \omega = \omega' = 0, \mathbf{q}, \Omega \to 0)$$

The response is encoded in the Landau-Silin equations

$$\delta n_k(q) = \int \mathrm{d}^2\, \mathrm{k}'\, \mathrm{h}(\mathbf{k}_*, \mathbf{k}'_*) \mathrm{Q}(\mathrm{k}', \mathrm{q}) \mathrm{E}_{\mathbf{k}'_*}(\mathrm{q})$$

$$E_{\mathbf{k}_*}(q) = \int \mathrm{d}^2\mathrm{k}'\, \mathrm{h}(\mathbf{k}_*, \mathbf{k}'_*) \mathrm{e}_{\mathbf{k}'_*}(\mathrm{q}) + \int \mathrm{d}^2\mathrm{k}'\, \mathrm{f}(\mathbf{k}_*, \mathbf{k}'_*) \mathrm{Q}(\mathrm{k}', \mathrm{q}) \mathrm{E}_{\mathbf{k}'_*}(\mathrm{q})$$

Zero sound corresponds to a solution of

$$fQ = 1$$

The equations can be solved explicitly for

$$f = \text{const} = F_0$$

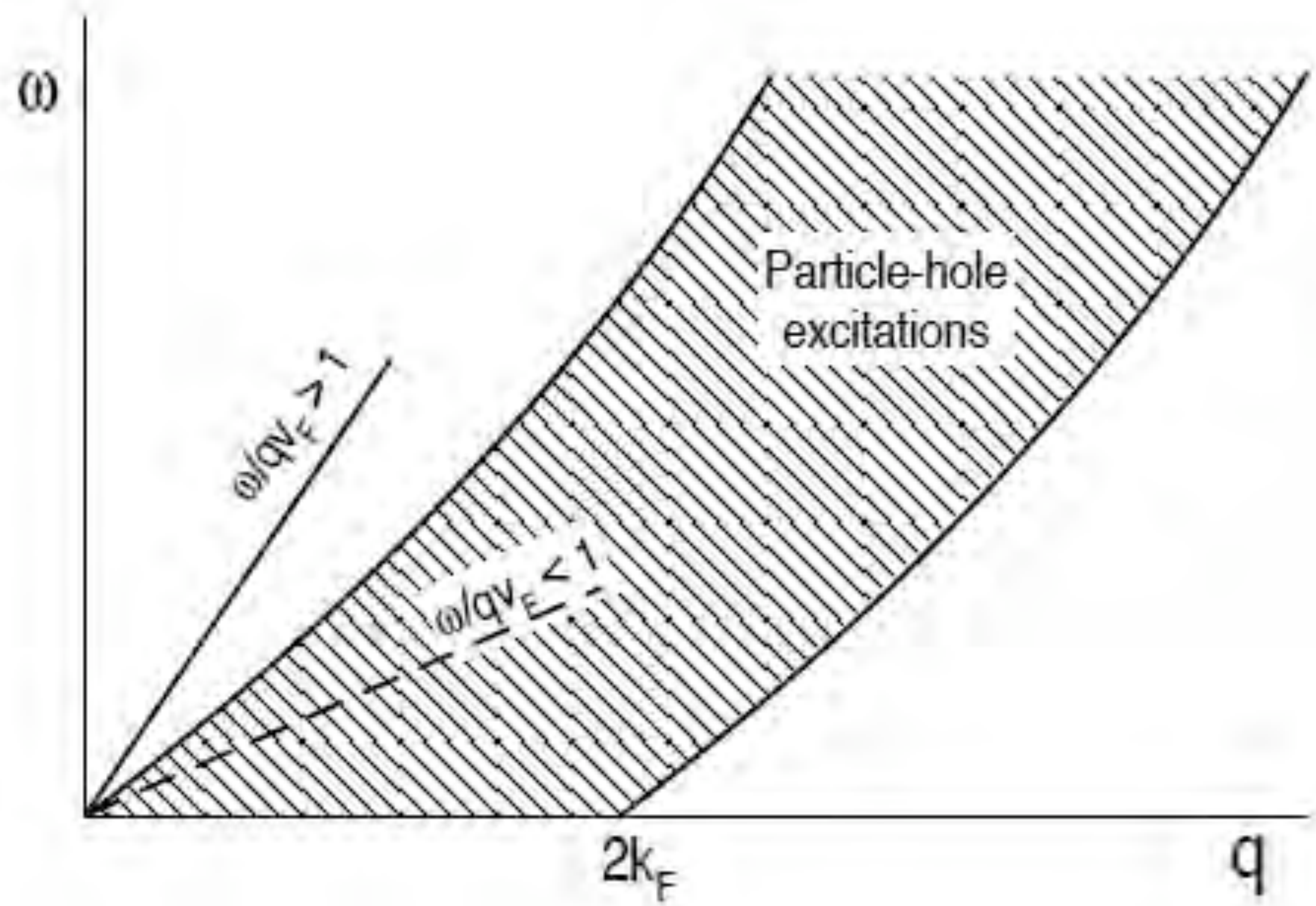
Zero-sound linear dispersion $\Omega = v_0\, q$ $\quad w = \frac{v_F}{|g|^2 c}$

$$\frac{\nu v_F^2}{F_0 w} = (w k_c)^{\frac{1-\nu}{\nu}} - \frac{\nu v_0}{w} \arcsin\left(\frac{w}{\nu v_0} (w k_c)^{\frac{1-\nu}{\nu}} \right)$$

at large F_0 $\quad$ $v_0 \sim c^{-3/2\nu} \sqrt{F_0}$

in FL $\quad \frac{1}{F_0} = \frac{v_0}{2 v_F} \log\left(\frac{v_0 + v_F}{v_0 - v_F} \right) - 1 \qquad v_0 \sim \sqrt{F_0/3}$

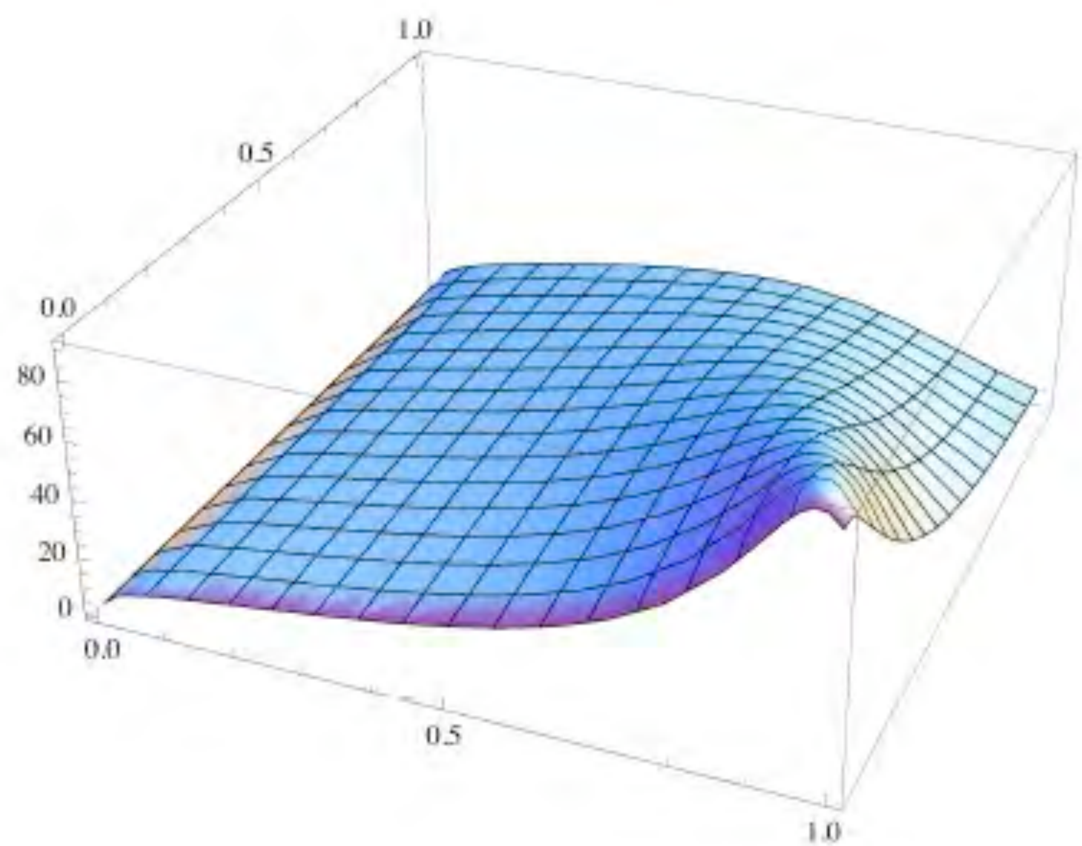
The FL zero sound can be a sharp resonance or can be embedded in the particle-hole continuum and be highly damped



Im(polarization function) integrated over frequency is peaked around

$$\Omega^{1/\nu} = \epsilon_p^{1/\nu} + \epsilon_h^{1/\nu}$$

consistent with contour integral argument, but how sharp is the peak?



$\nu = 0.7$

Analytical results for $\nu = \frac{1}{2}$

Finite temperature

$$G_R(\omega, \mathbf{k}) = \begin{cases} \frac{1}{\omega - v_F k_\perp + icT^\nu}, & \text{if } \omega << T \\ \frac{1}{\zeta_R \omega^\nu - v_F k_\perp + i\zeta_I \omega^\nu}, & \text{if } \omega >> T \end{cases}$$

Optical conductivity $\sigma(\omega) \sim \omega^{-\nu}$ $\omega >> T$

Entropy density $\frac{S}{V} \sim T^{1-\nu+\nu^2}$

The semiholographic approach offers a viable framework for phenomenological model building

The generalization of Landau theory can be developed and justified from first principles

Testable predictions depending on few parameters

Multiple directions for further generalizations, e.g. lattice, different IR sectors

Work in progress

Detailed study of the response function

Self-energy corrections

BCS instability

Open issues

The coupling g is dimensionful, is it a new scale different from E_F ?

Finite N breakdown [Lee, Mross et al.]

Dynamically determined scaling exponent

Backreaction on the geometry