Fermi surfaces and phase transitions in holographic mixed Bose-Fermi systems

Francesco Nitti

APC, U. Paris VII

Kolymbari
September 5th 2014

Work with G. Policastro, T. Vanel, 1307.4558, 1407.0410

Introduction

What is the $T = 0$ *ground state of a charged holographic system with several types of matter ?*

Introduction

Simplest $T = 0$ charged solution: extremal RN-AdS black hole

- The charge is hidden behind a horizon;
- Large ground state entropy;

Introduction

Simplest $T = 0$ charged solution: extremal RN-AdS black hole

- The charge is hidden behind a horizon;
- Large ground state entropy;

With bulk matter present, other solutions have lower free energy:

- Solutions with a bulk scalar condensate Horowitz, Roberts '09
    - Field theory side: spontaneous breaking of global $U(1)$ (ground state is a superfluid).
    - No charged horizon.

Introduction

Simplest $T = 0$ charged solution: extremal RN-AdS black hole

- The charge is hidden behind a horizon;
- Large ground state entropy;

With bulk matter present, other solutions have lower free energy:

- Solutions with a bulk scalar condensate Horowitz, Roberts '09
  - Field theory side: spontaneous breaking of global $U(1)$ (ground state is a superfluid).
  - No charged horizon.
- Fermionic matter in the bulk. Hartnoll, Tavanfar '10
  - In the fluid approximation, a bulk "Electron Star" solution: fermionic matter fills part of the geometry.
  - No horizon but IR Lifshitz scaling
  - Field theory side: large n. of Fermi surfaces associated to Fermion species coming from the states associated to boundary fermionic operators.

Introduction

What happens in the presence of both Fermionic and Bosonic charged matter ?

Introduction

What happens in the presence of both Fermionic and Bosonic charged matter ?

One can think of a bulk electron fluid interacting with a (possibly emergent) scalar condensate. This may for example arise as a composite of the fermions.

Introduction

What happens in the presence of both Fermionic and Bosonic charged matter ?

One can think of a bulk electron fluid interacting with a (possibly emergent) scalar condensate. This may for example arise as a composite of the fermions.

- Competition of Fermionic and Bosonic components, new quantum phase transitions.
- Deformation/destruction of Fermi surfaces in the presence of the condensate.

Introduction

What happens in the presence of both Fermionic and Bosonic charged matter ?

One can think of a bulk electron fluid interacting with a (possibly emergent) scalar condensate. This may for example arise as a composite of the fermions.

- Competition of Fermionic and Bosonic components, new quantum phase transitions.
- Deformation/destruction of Fermi surfaces in the presence of the condensate.

Lesson:

the ground state of charged holographic systems is as complex as it can be

Introduction

What happens in the presence of both Fermionic and Bosonic charged matter ?

One can think of a bulk electron fluid interacting with a (possibly emergent) scalar condensate. This may for example arise as a composite of the fermions.

- Competition of Fermionic and Bosonic components, new quantum phase transitions.
- Deformation/destruction of Fermi surfaces in the presence of the condensate.

Lesson:

the ground state of charged holographic systems is as complex as it can be

related work: Liu, Schalm, Sun, Zaanen, 1307.4572, 1404.0571

Setup

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)-\frac{1}{4e^2}F_{ab}F^{ab}\right]+S_{\text{matter}}$$

$U(1)$ gauge field $\Leftrightarrow$ Global $U(1)$ symmetry in the dual field theory.

Setup

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right] + S_{\text{matter}}$$

$U(1)$ gauge field $\Leftrightarrow$ Global $U(1)$ symmetry in the dual field theory.

Two types of bulk matter:

- A charged scalar field ψ

$$S_{\text{scalar}} = -\frac{1}{2} \int \mathrm{d}^4x \sqrt{-g} \left(|\partial_\mu \psi - iqA_\mu \psi|^2 + m_s^2 |\psi|^2 \right) \qquad -\frac{9}{4} < m_s^2 < 0$$

Setup

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab}F^{ab} \right] + S_{\text{matter}}$$

$U(1)$ gauge field $\Leftrightarrow$ Global $U(1)$ symmetry in the dual field theory.

Two types of bulk matter:

- A charged scalar field ψ

$$S_{\text{scalar}} = -\frac{1}{2} \int \mathrm{d}^4x \sqrt{-g} \left(|\partial_\mu \psi - iqA_\mu \psi|^2 + m_s^2 |\psi|^2 \right) \qquad -\frac{9}{4} < m_s^2 < 0$$

- A charged fermionic perfect fluid in local chemical equilibrium

$$T_{ab}^{\text{fluid}} = (\rho + p) u_a u_b + p g_{ab} \,, \qquad J^a_{\text{fluid}} = \sigma u^a$$

$$-p(r) = \rho(r) - \mu_l(r)\sigma(r), \quad \mu_l(r) = \text{local bulk chemical potential}$$

Solutions

Static homogeneous isotropic solutions:

$$\begin{aligned} \mathrm{d}s^2 &= \left[-f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \tfrac{1}{r^2}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)\right] , \\ A &= h(r)\mathrm{d}t \,, \quad \psi = \psi(r)\,, \quad u^a = (u^t(r), 0, 0, 0), \quad \rho_{fluid} = \rho(r). \end{aligned}$$

Solutions

Static homogeneous isotropic solutions:

$$\begin{aligned} \mathrm{d}s^2 &= \left[-f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \tfrac{1}{r^2}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)\right] , \\ A &= h(r)\mathrm{d}t , \quad \psi = \psi(r) , \quad u^a = (u^t(r), 0, 0, 0), \quad \rho_{fluid} = \rho(r). \end{aligned}$$

Close to the UV boundary: AdS asymptotics, no scalar source.

$$r \to 0 : \qquad f(r) \sim \frac{1}{r^2} , \quad h(r) \sim \mu - Qr , \quad \psi \sim \psi_+ r^{\Delta}$$

Solutions

Static homogeneous isotropic solutions:

$$\mathrm{d}s^2 = \left[-f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + \frac{1}{r^2}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right)\right] ,$$

$$A = h(r)\mathrm{d}t , \quad \psi = \psi(r) , \quad u^a = (u^t(r), 0, 0, 0), \quad \rho_{fluid} = \rho(r).$$

Close to the UV boundary: AdS asymptotics, no scalar source.

$$r \to 0 : \qquad f(r) \sim \frac{1}{r^2} , \quad h(r) \sim \mu - Qr , \quad \psi \sim \psi_+ r^{\Delta}$$

- (μ, Q, ψ_+): boundary theory parameters (chemical potential, total charge, scalar condensate).
- $\psi(r)$ obeys normalizable b.c. at $r = 0$.
  Field theory side: spontaneous breaking of global $U(1)$

Solutions

Static homogeneous isotropic solutions:

$$\begin{aligned} ds^2 &= \left[-f(r)dt^2 + g(r)dr^2 + \tfrac{1}{r^2}\left(dx^2 + dy^2\right)\right] , \\ A &= h(r)dt , \quad \psi = \psi(r) , \quad u^a = (u^t(r), 0, 0, 0), \quad \rho_{fluid} = \rho(r). \end{aligned}$$

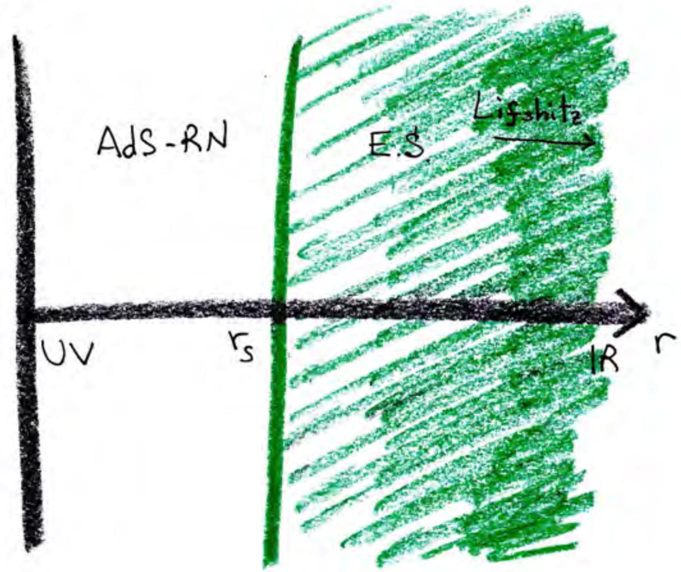
Close to the UV boundary: AdS asymptotics, no scalar source.

$$r \to 0 : \qquad f(r) \sim \frac{1}{r^2} , \quad h(r) \sim \mu - Qr , \quad \psi \sim \psi_+ r^\Delta$$

- (μ, Q, ψ_+): boundary theory parameters (chemical potential, total charge, scalar condensate).
- $\psi(r)$ obeys normalizable b.c. at $r = 0$.
  Field theory side: spontaneous breaking of global $U(1)$
- $\rho = \psi = 0$: solution is extremal RN black hole:

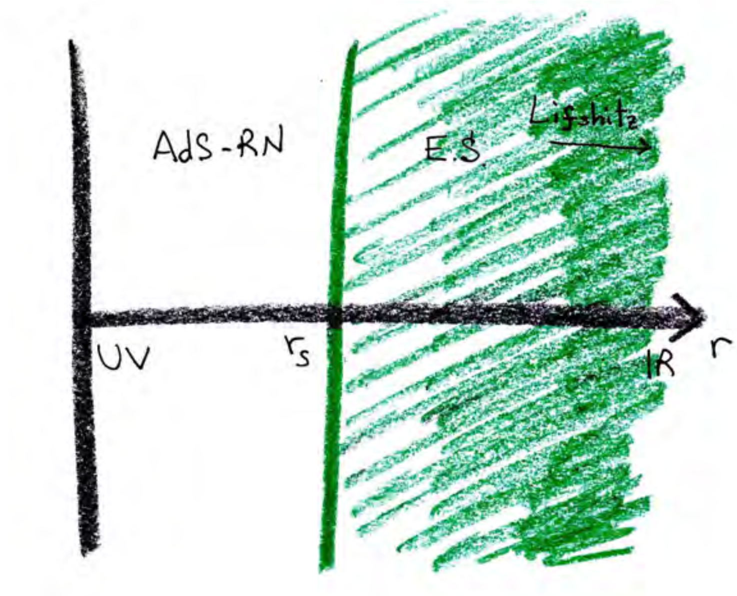
$$h = \mu - Qr, \qquad Q = \sqrt{6}/r_h^2, \quad \mu = \sqrt{6}/r_h,$$

The Electron Star Hartnoll, Tavanfar '09



$$\psi(r) = 0, \ \rho_{fluid}(r) \neq 0$$

The Electron Star Hartnoll, Tavanfar '09

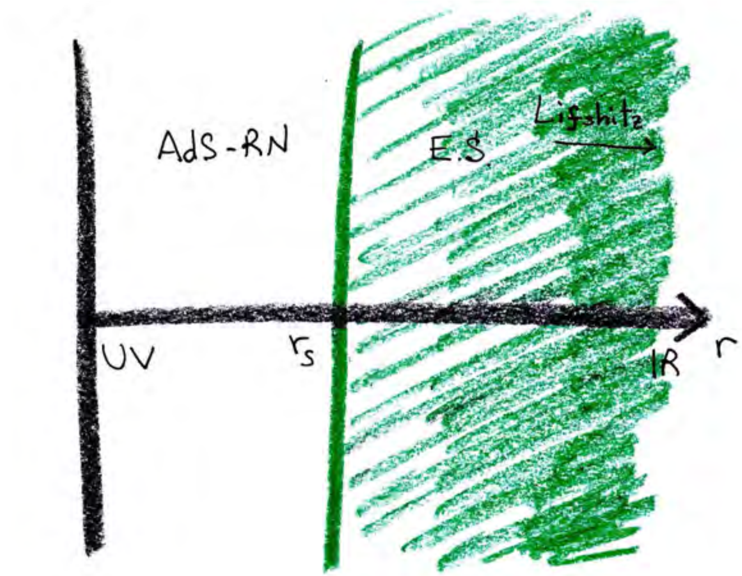


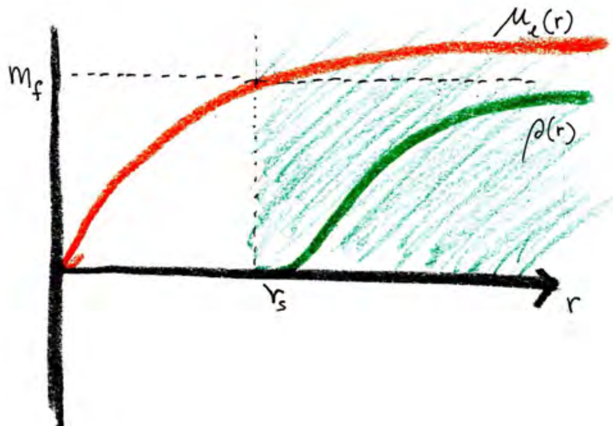
$$\psi(r) = 0, \;\; \rho_{fluid}(r) \neq 0$$

$$\rho(r) = \beta \int_{m_f}^{\mu_l(r)} \mathrm{d}\epsilon\, \epsilon^2 \sqrt{\epsilon^2 - m_f^2}$$

$$\mu_l(r) = \frac{h(r)}{\sqrt{f(r)}}, \quad 0 < m_f < 1$$

The Electron Star Hartnoll, Tavanfar '09



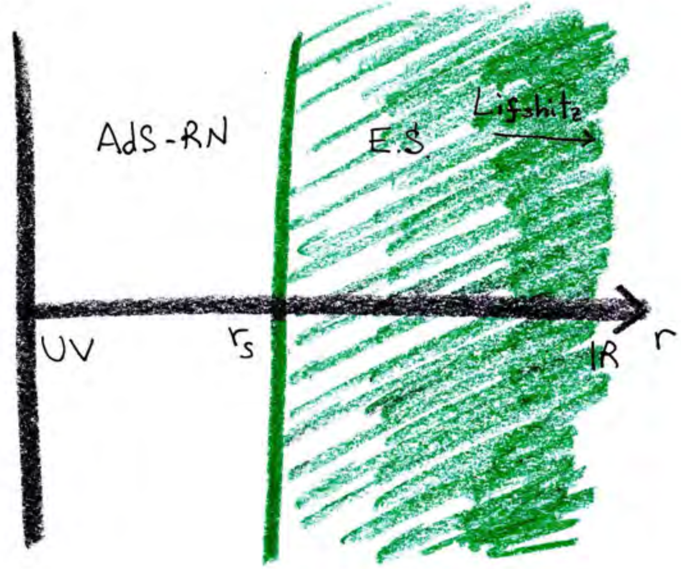


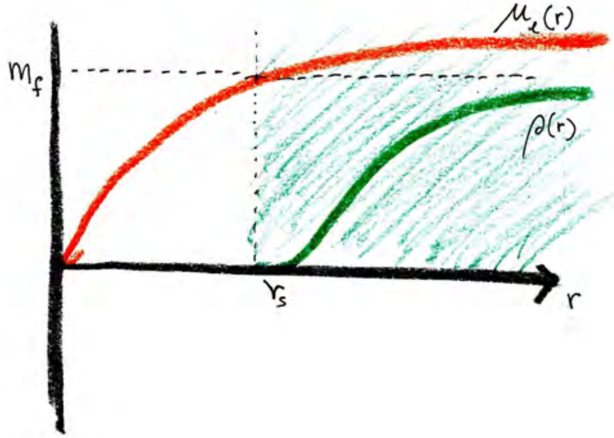
$$\psi(r) = 0, \;\; \rho_{fluid}(r) \neq 0$$

$$\rho(r) = \beta \int_{m_f}^{\mu_l(r)} d\epsilon \, \epsilon^2 \sqrt{\epsilon^2 - m_f^2}$$

$$\mu_l(r) = \frac{h(r)}{\sqrt{f(r)}}, \quad 0 < m_f < 1$$

The Electron Star Hartnoll, Tavanfar '09





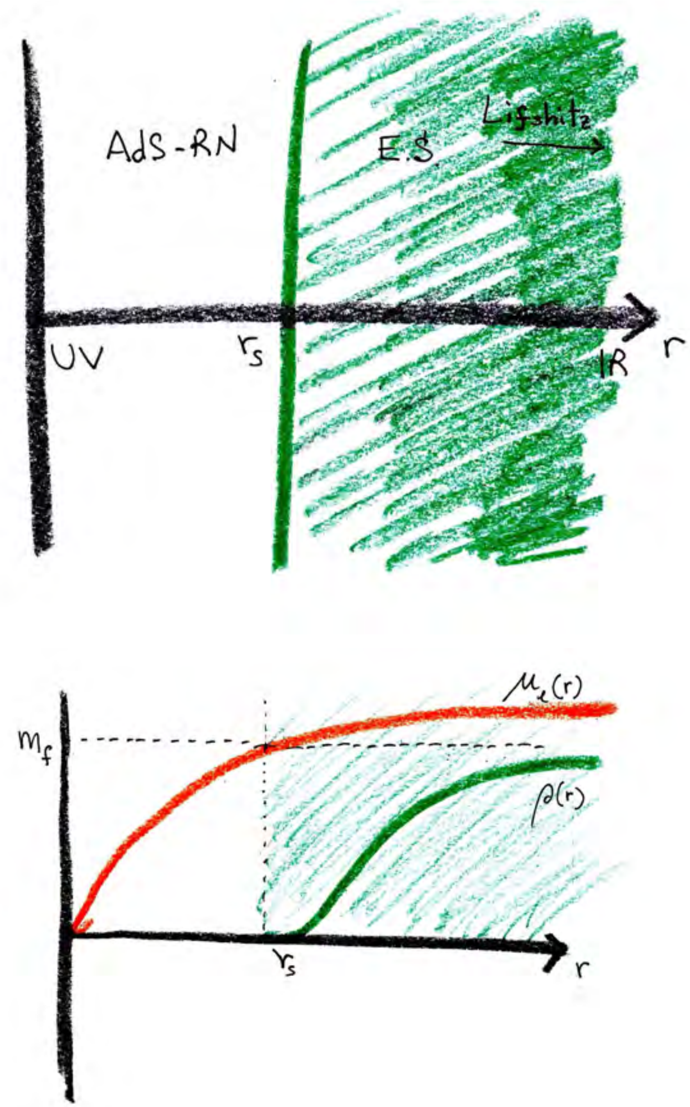
$$\psi(r) = 0, \;\; \rho_{fluid}(r) \neq 0$$

$$\rho(r) = \beta \int_{m_f}^{\mu_l(r)} \mathrm{d}\epsilon \, \epsilon^2 \sqrt{\epsilon^2 - m_f^2}$$

$$\mu_l(r) = \frac{h(r)}{\sqrt{f(r)}}, \quad 0 < m_f < 1$$

Need $e^2 \sim \ell_p/\ell_{AdS} \ll 1$

The Electron Star Hartnoll, Tavanfar '09



$$\psi(r) = 0, \;\; \rho_{fluid}(r) \neq 0$$

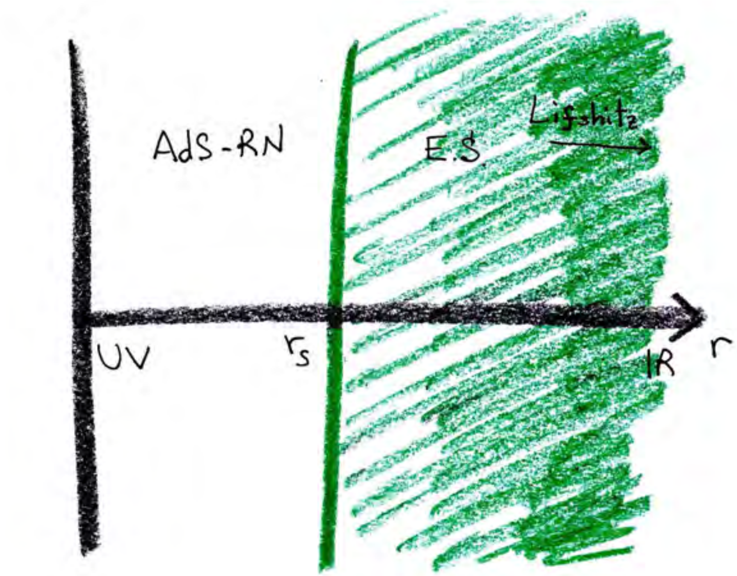
$$\rho(r) = \beta \int_{m_f}^{\mu_l(r)} d\epsilon \, \epsilon^2 \sqrt{\epsilon^2 - m_f^2}$$

$$\mu_l(r) = \frac{h(r)}{\sqrt{f(r)}}, \quad 0 < m_f < 1$$

Need $e^2 \sim \ell_p/\ell_{AdS} \ll 1$

$$r \to \infty : \quad h(r) \sim \frac{h_\infty}{r^z}, f(r) \sim \frac{1}{r^{2z}}$$

The Electron Star Hartnoll, Tavanfar ’09

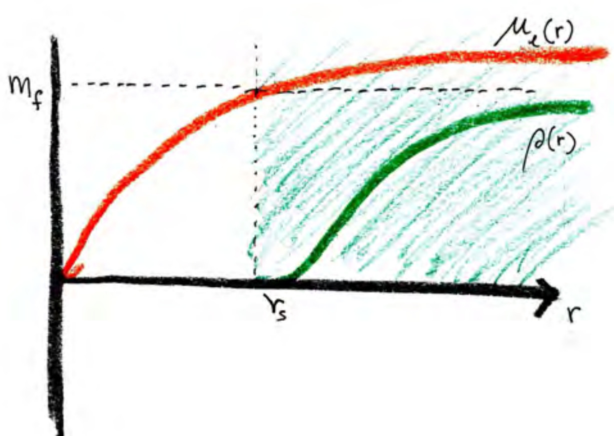


$$\psi(r) = 0,\ \rho_{fluid}(r) \neq 0$$

$$\rho(r) = \beta \int_{m_f}^{\mu_l(r)} \mathrm{d}\epsilon\, \epsilon^2 \sqrt{\epsilon^2 - m_f^2}$$

$$\mu_l(r) = \frac{h(r)}{\sqrt{f(r)}}, \quad 0 < m_f < 1$$

Need $e^2 \sim \ell_p/\ell_{AdS} \ll 1$



$$r \to \infty : \quad h(r) \sim \frac{h_\infty}{r^z}, f(r) \sim \frac{1}{r^{2z}}$$

For $m_f > 1$ the ground state is again the RN black hole (no star)

The $T = 0$ Holographic Superconductor

In the absence of fluid, the $T = 0$ ground state has $\psi(r) \neq 0$. The IR geometry has emergent $(2+1)d$ Poincaré invariance,

$$r \to \infty : \qquad f(r) \sim \frac{1}{r^2}, \quad h(r) \sim h_0 \frac{(\log r)^{1/2}}{r^\delta}, \quad \delta > 1.$$

The $T = 0$ Holographic Superconductor

In the absence of fluid, the $T = 0$ ground state has $\psi(r) \neq 0$. The IR geometry has emergent $(2+1)d$ Poincaré invariance,

$$r \to \infty : \qquad f(r) \sim \frac{1}{r^2}, \quad h(r) \sim h_0 \frac{(\log r)^{1/2}}{r^\delta}, \quad \delta > 1.$$

How is the Electron Star modified if we turn on the condensate?

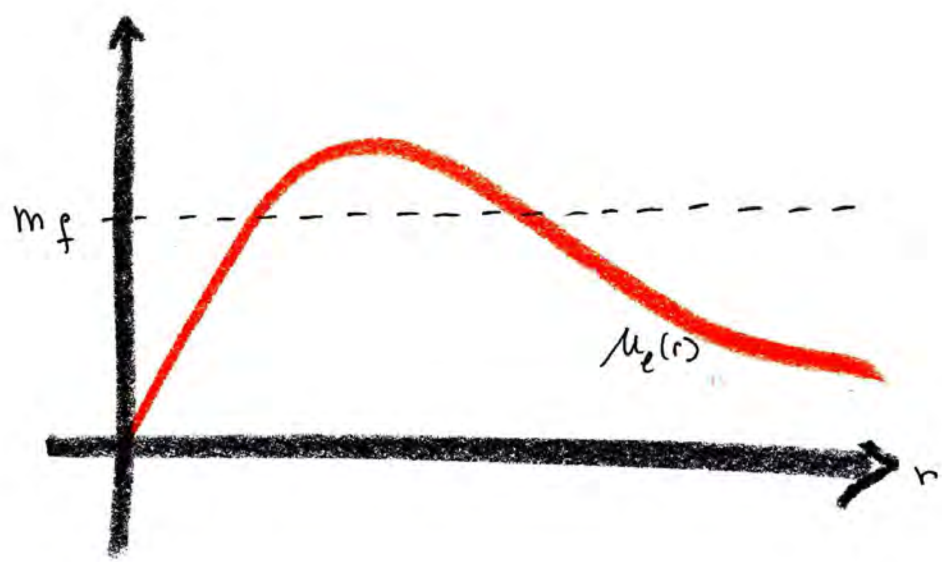
The $T = 0$ Holographic Superconductor

In the absence of fluid, the $T = 0$ ground state has $\psi(r) \neq 0$. The IR geometry has emergent $(2+1)d$ Poincaré invariance,

$$r \to \infty : \qquad f(r) \sim \frac{1}{r^2}, \quad h(r) \sim h_0 \frac{(\log r)^{1/2}}{r^\delta}, \quad \delta > 1.$$

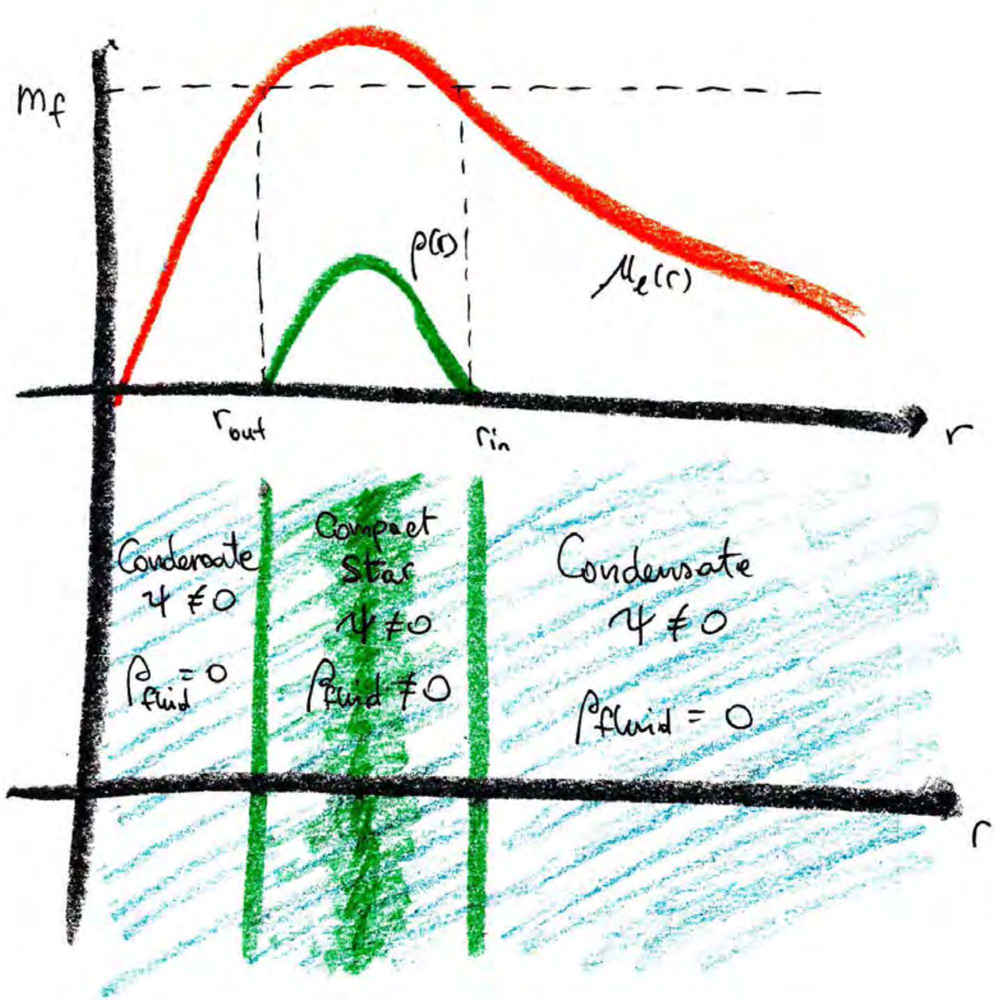
How is the Electron Star modified if we turn on the condensate?

Hint: $\mu_l = \frac{h}{\sqrt{f}} \to 0$ as both $r \to 0$ and $r \to \infty$



The Compact Star solution FN,Policastro,Vanel 1307.

Turning on $\rho(r)$ and $\psi(r)$ at the same time allows for new solutions:



A fluid layer sandwiched by the pure condensate solution.

Phase Transitions

For a given choice of the microscopic parameters (m_f, β, m_s, q), in any class of solutions all boundary quantities scale trivially with μ (the only scale on the boundary CFT at $T = 0$.)

$$\psi_+ \propto \mu^\Delta, \quad Q \propto \mu^2, \quad \ldots$$

Phase Transitions

For a given choice of the microscopic parameters (m_f, β, m_s, q), in any class of solutions all boundary quantities scale trivially with μ (the only scale on the boundary CFT at $T = 0$.)

$$\psi_+ \propto \mu^\Delta, \quad Q \propto \mu^2, \quad \dots$$

The Free energies of the various solutions will take the form:

$$\mathcal{F}_i = c_i(m_s, m_f, q)\mu^3$$

Phase Transitions

For a given choice of the microscopic parameters (m_f, β, m_s, q), in any class of solutions all boundary quantities scale trivially with μ (the only scale on the boundary CFT at $T = 0$.)

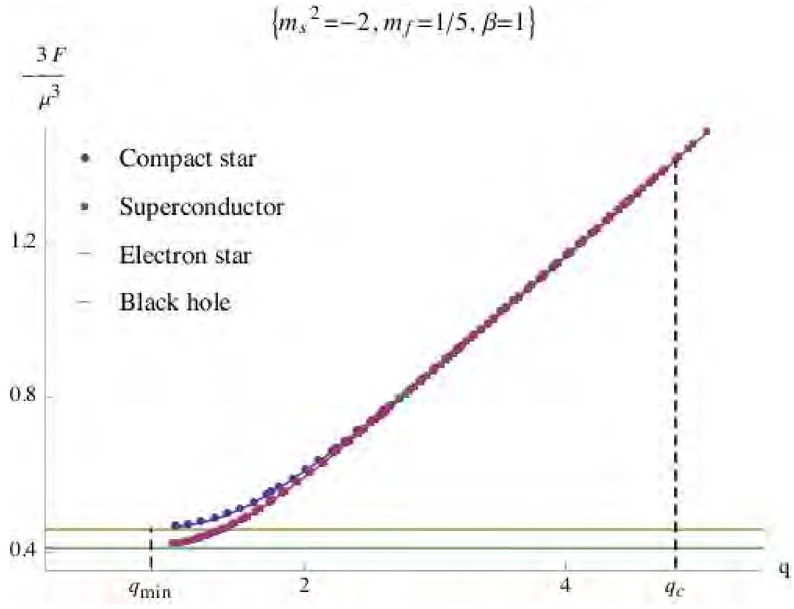
$$\psi_+ \propto \mu^\Delta, \quad Q \propto \mu^2, \quad \dots$$

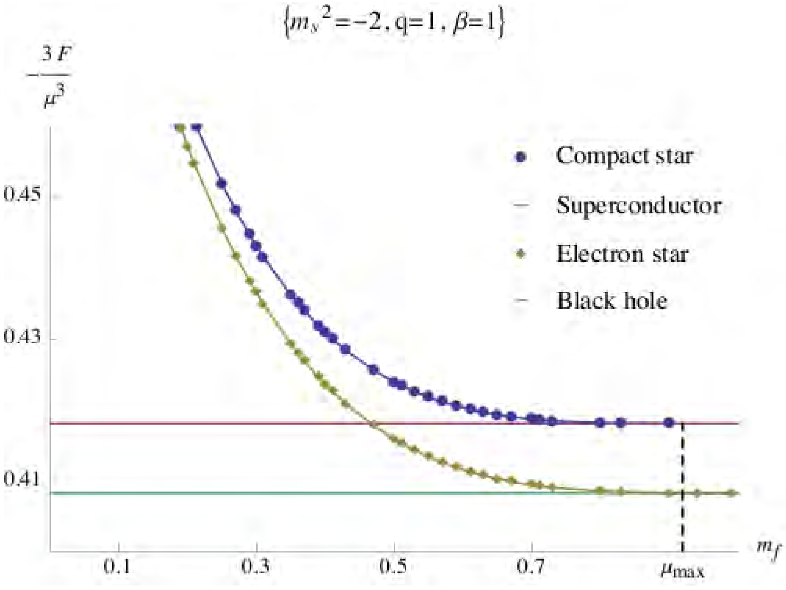
The Free energies of the various solutions will take the form:

$$\mathcal{F}_i = c_i(m_s, m_f, q)\mu^3$$

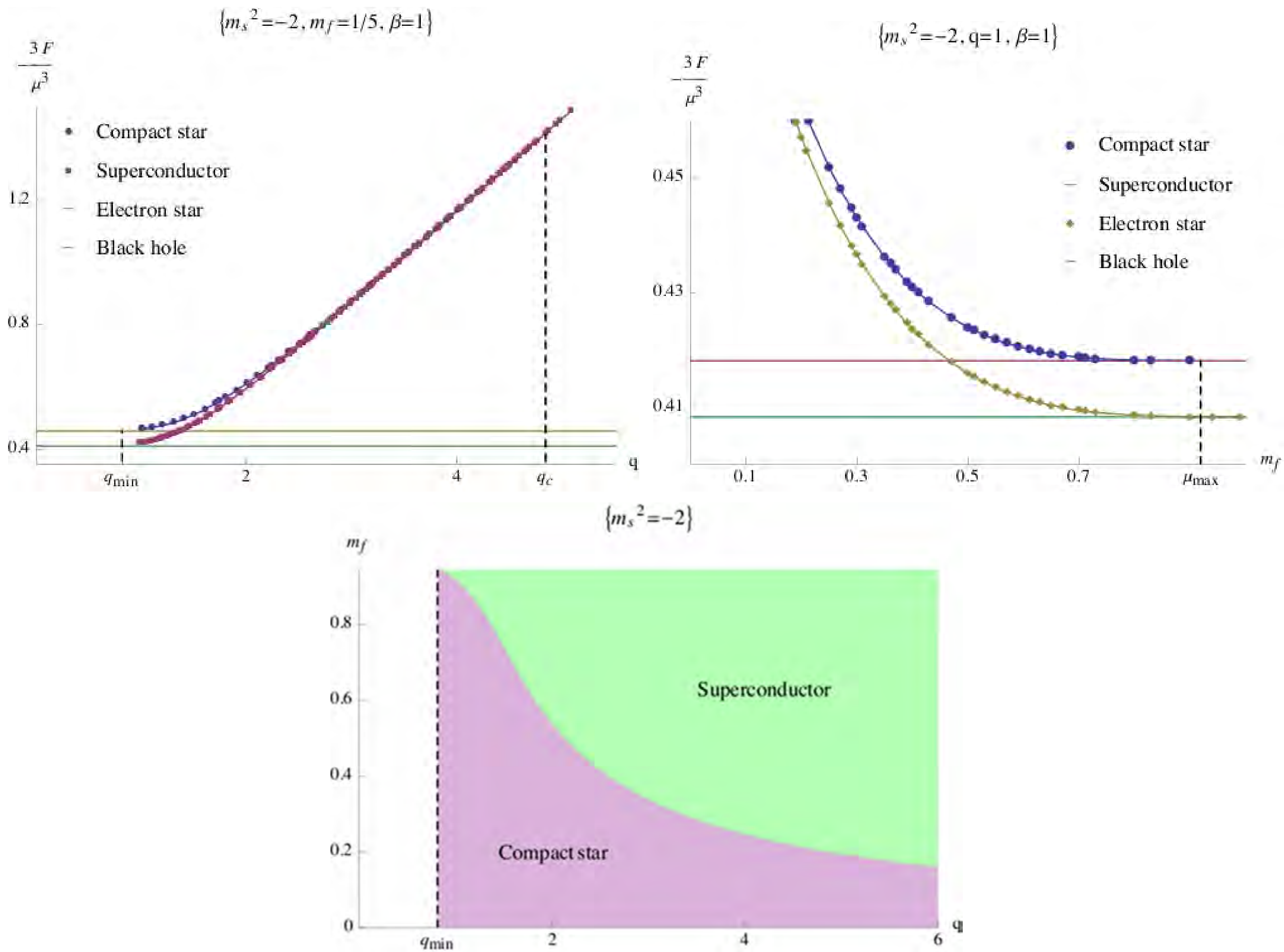
- The quantity $c_i(m_s, m_f, q)$ determines which solution has lowest free energy $\mathcal{F}_i$ at a given point in parameter space
- As we vary the control parameters (m_s, m_f, q) we can find quantum phase transitions between the various classes of solutions.

Phase Transitions





Phase Transitions



The compact Electron star dominates whenever it exists.

Adding current-current interaction

New interesting solutions arise if we add a direct current-current coupling:

$$\mathcal{L}_{int} = \eta J_a^{sc} J^a_{fluid} \qquad J_a^{sc} = -iq\left(\psi^* \mathcal{D}_a \psi - \psi(\mathcal{D}_a \psi)^*\right)$$

On the isotropic condensate background with $A = h(r)dt, \psi(r)$:

$$J_t = q^2 |\psi(r)|^2 h(r)$$

Adding current-current interaction

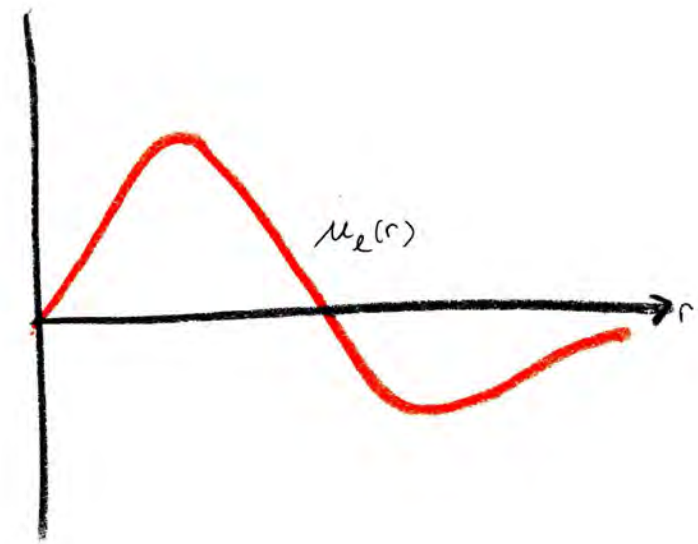
New interesting solutions arise if we add a direct current-current coupling:

$$\mathcal{L}_{int} = \eta J_a^{sc} J^a_{fluid} \qquad J_a^{sc} = -iq\left(\psi^* \mathcal{D}_a \psi - \psi(\mathcal{D}_a\psi)^*\right)$$

On the isotropic condensate background with $A = h(r)dt, \psi(r)$:

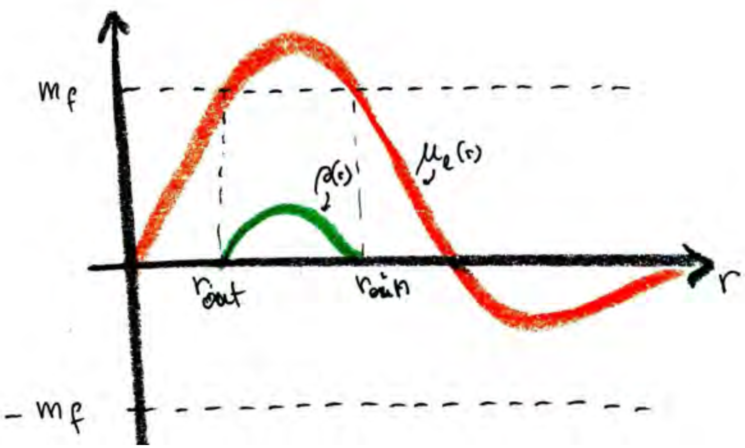
$$J_t = q^2|\psi(r)|^2 h(r)$$

$$\mu_l(r) = \frac{h(r)}{\sqrt{f(r)}}\left(1 - \eta q^2|\psi(r)|^2\right)$$



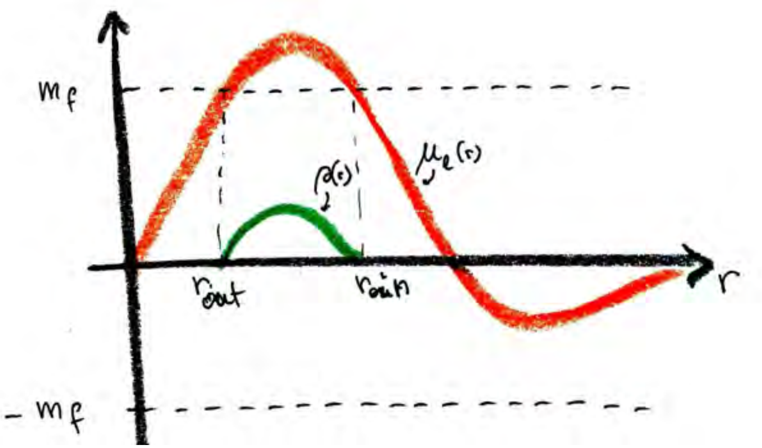
$\eta > 0$

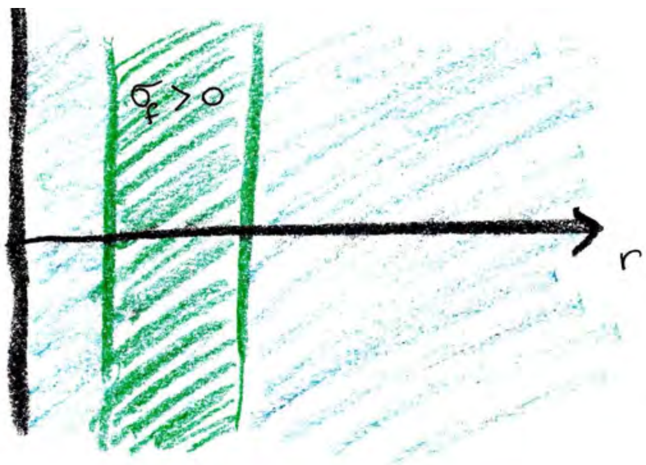
Compact Electron Stars



A positively charged fluid shell is allowed.

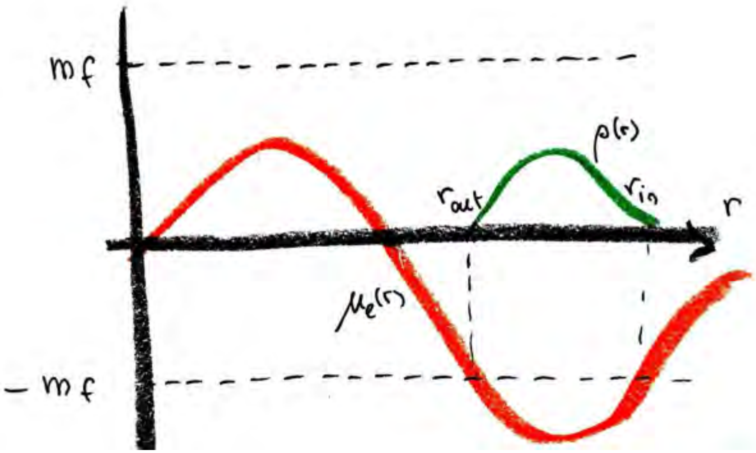
Compact Electron Stars





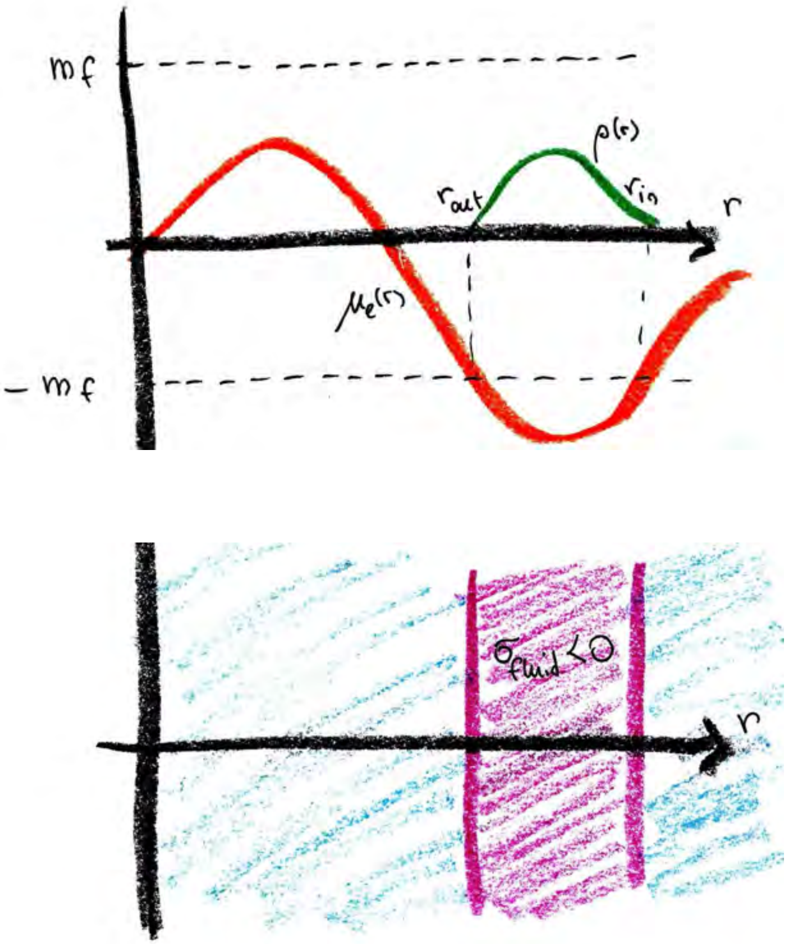
A positively charged fluid shell is allowed.

Compact Positron Stars



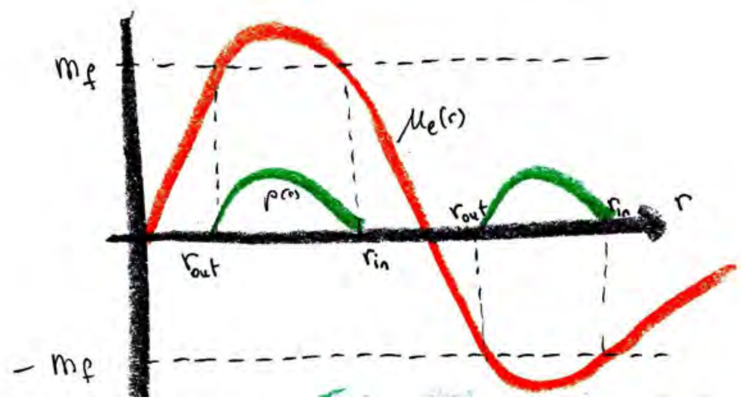
A negatively charged fluid shell is allowed.

Compact Positron Stars



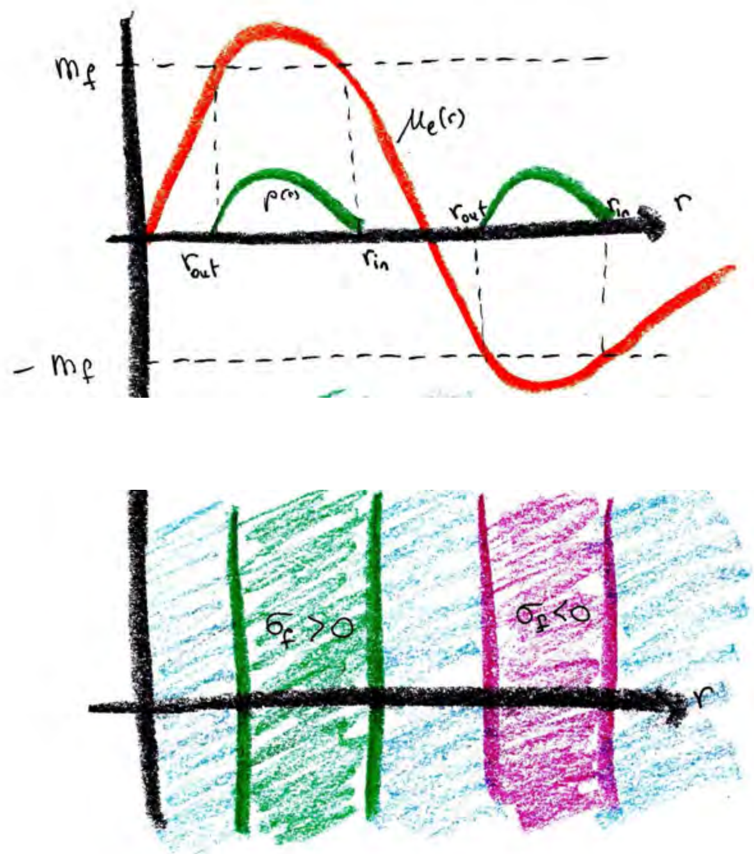
A negatively charged fluid shell is allowed.

Polarized solutions



Two fluid shells of opposite charge are allowed. The screening effect of the condensate keeps them apart.

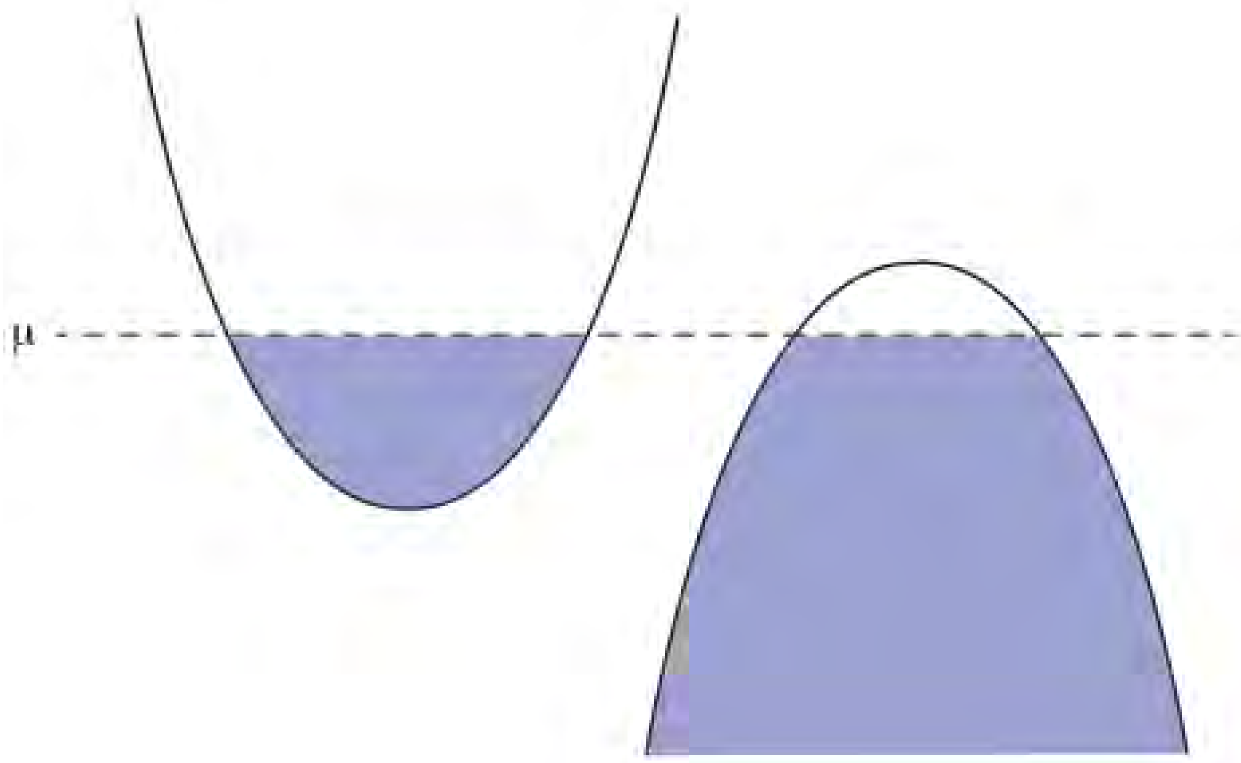
Polarized solutions



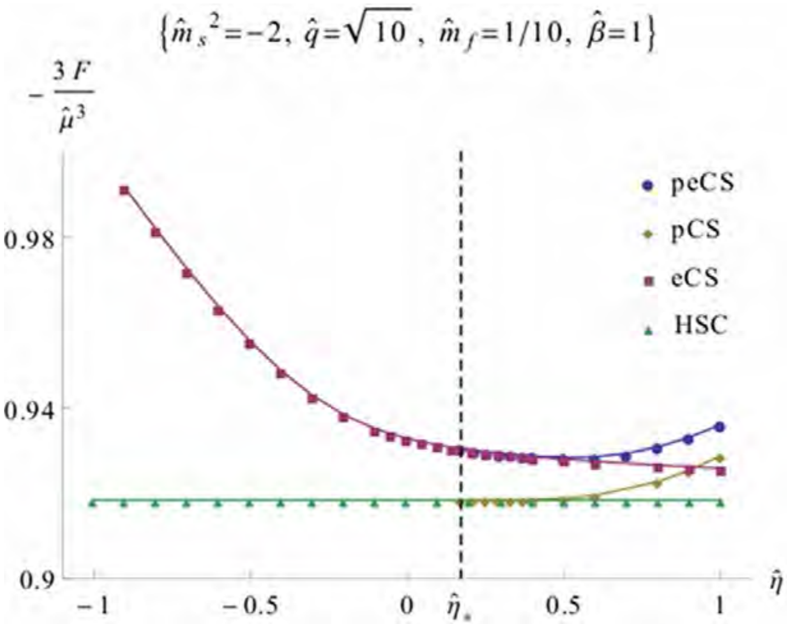
Two fluid shells of opposite charge are allowed. The screening effect of the condensate keeps them apart.

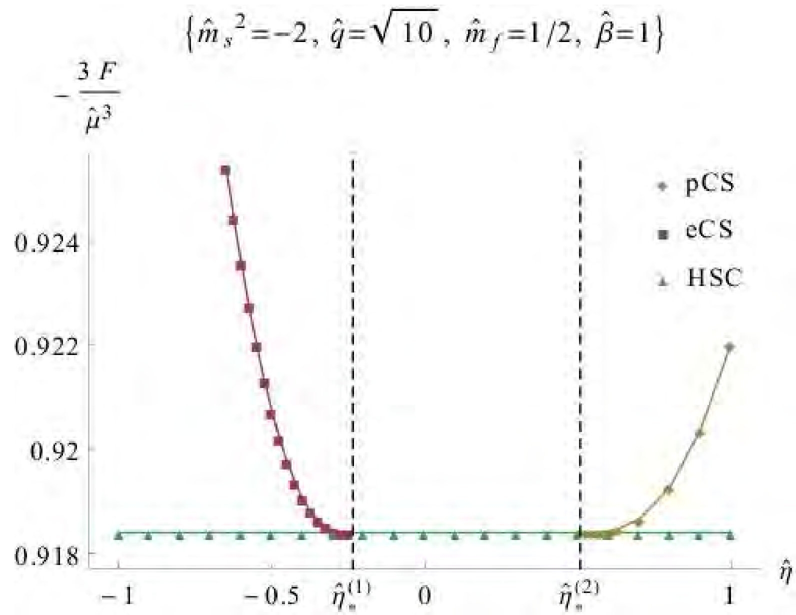
Field theory picture

Many Fermion species, such that zero energy level has a different offset for different flavors, so that a given chemical potential intersects the conductance band for some fermions and the valence band for others.

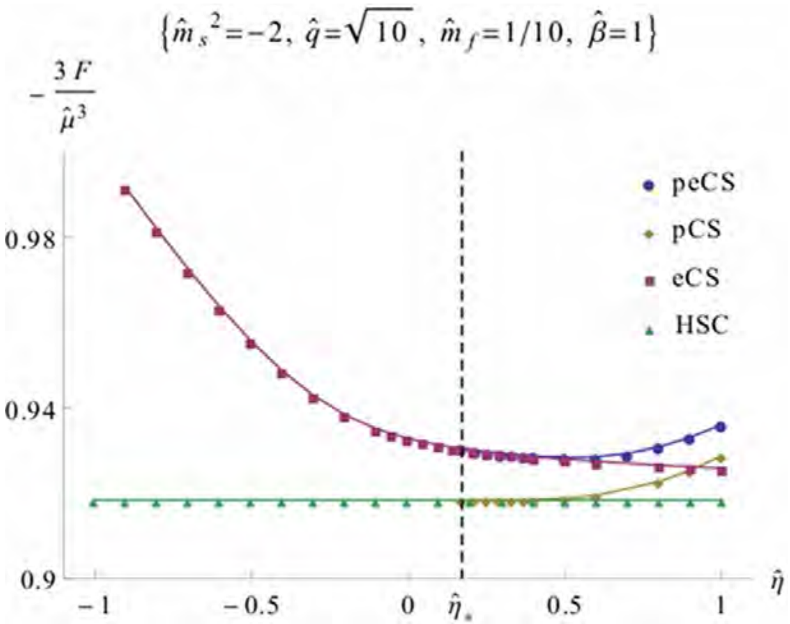


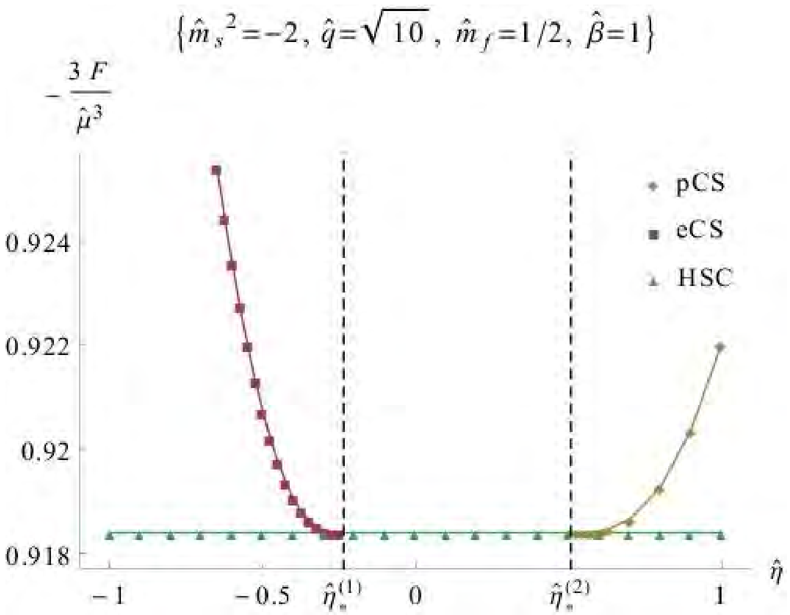
More phase Transitions

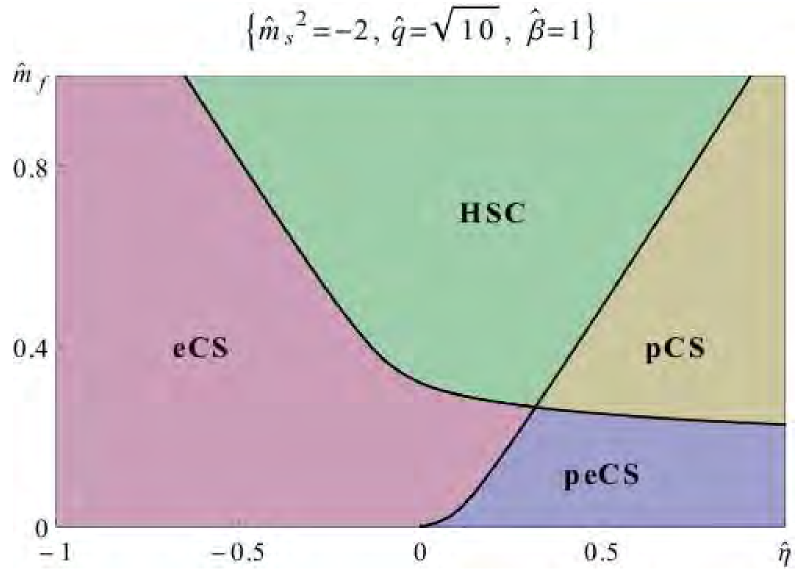




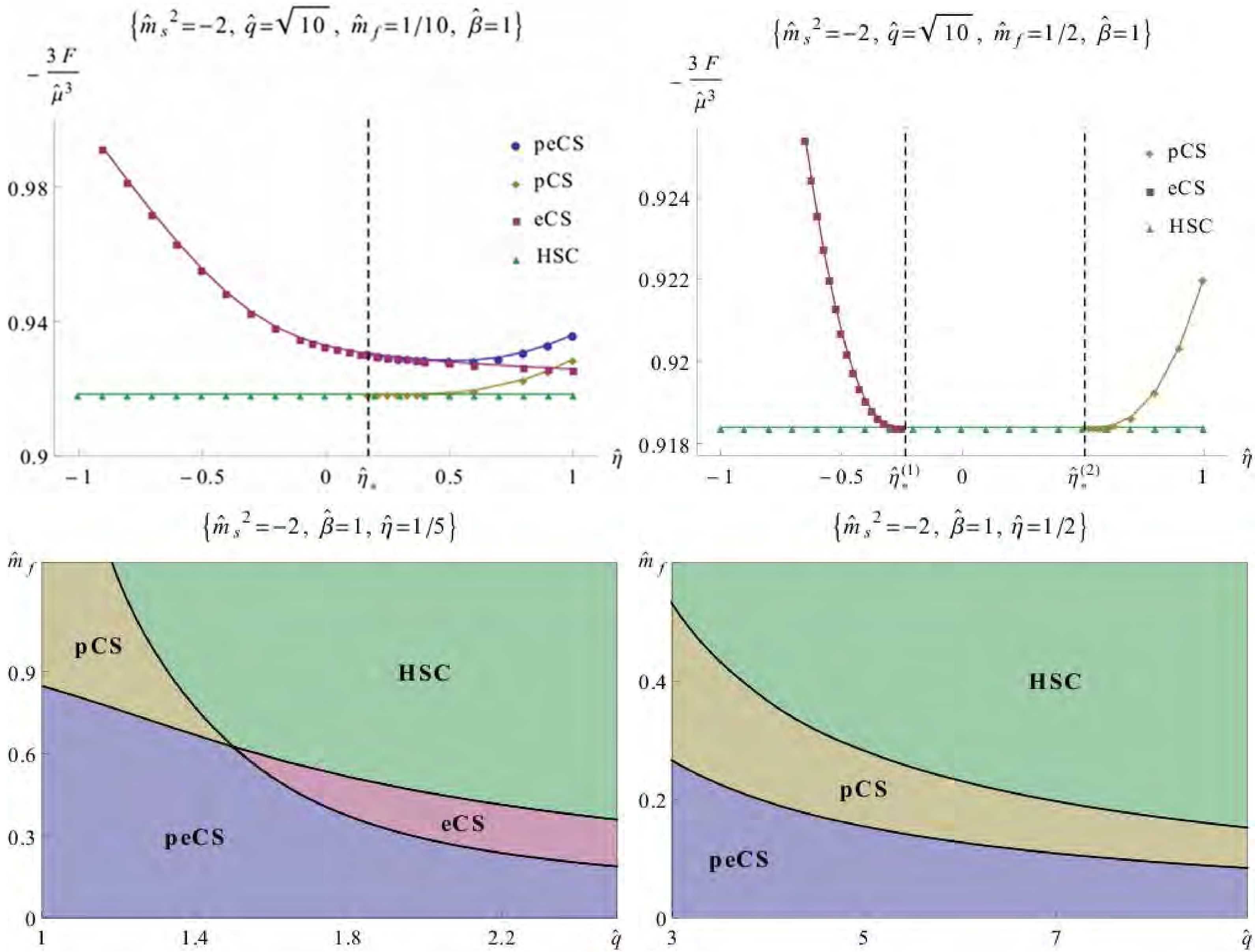
More phase Transitions







More phase Transitions



Low-energy spectrum and Fermi Surfaces

A Fermi surface on the Field theory side is signaled by a pole at zero frequency and finite momentum k_F in the two-point function $G(\omega, \vec{k})$ of a fermionic operator coupling to the bulk fermions.

Low-energy spectrum and Fermi Surfaces

A Fermi surface on the Field theory side is signaled by a pole at zero frequency and finite momentum k_F in the two-point function $G(\omega, \vec{k})$ of a fermionic operator coupling to the bulk fermions.

To compute $G(\omega, \vec{k})$ holographically, consider a probe bulk fermion χ on top of the homogeneous backgrounds (ES, CES, etc)

$$S_\chi = \int \mathrm{d}^4x\sqrt{-g}\left[-i\left(\bar{\chi}\Gamma^a\mathcal{D}_a\chi - m_f\bar{\chi}\chi\right) + \eta\bar{\chi}\Gamma_a\chi J^a_{sc}\right]$$

Pole in $G(0, k)$ $\Leftrightarrow$ normalizable solution with $\omega = 0$ and $k = k_F$ of the Dirac equation for the bulk spinor $\chi(r, \omega, \vec{k})$.

Fermi Surfaces: no condensate

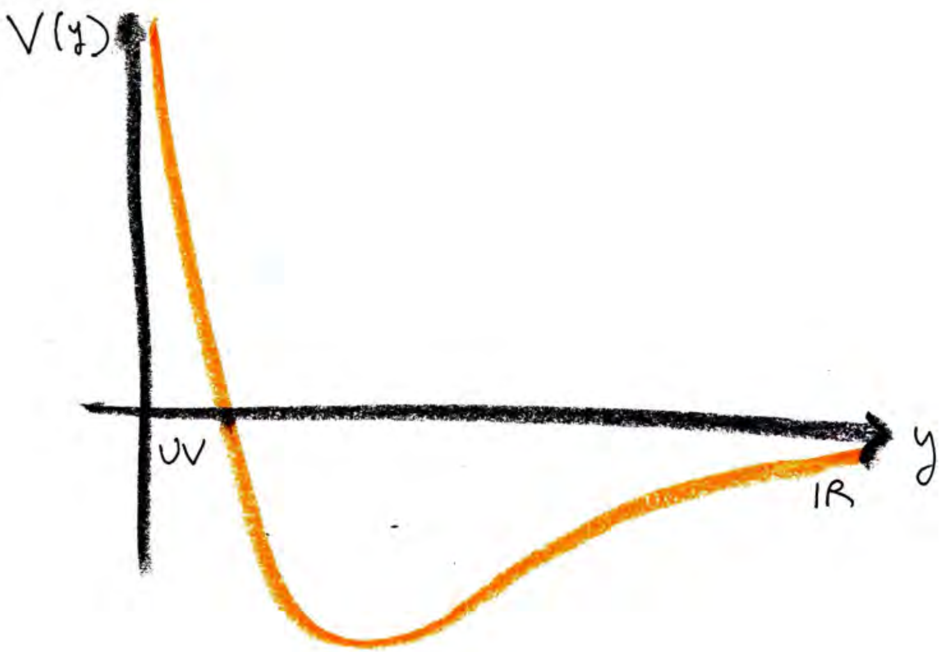
Recast Dirac's equation at $\omega = 0$ into a Schrodinger problem, with $|\vec{k}|^2$ appearing as energy:

$$-\partial_y^2\varphi + V(y)\varphi = -k^2\varphi$$

Fermi Surfaces: no condensate

Recast Dirac's equation at $\omega = 0$ into a Schrodinger problem, with $|\vec{k}|^2$ appearing as energy:

$$-\partial_y^2 \varphi + V(y)\varphi = -k^2 \varphi$$

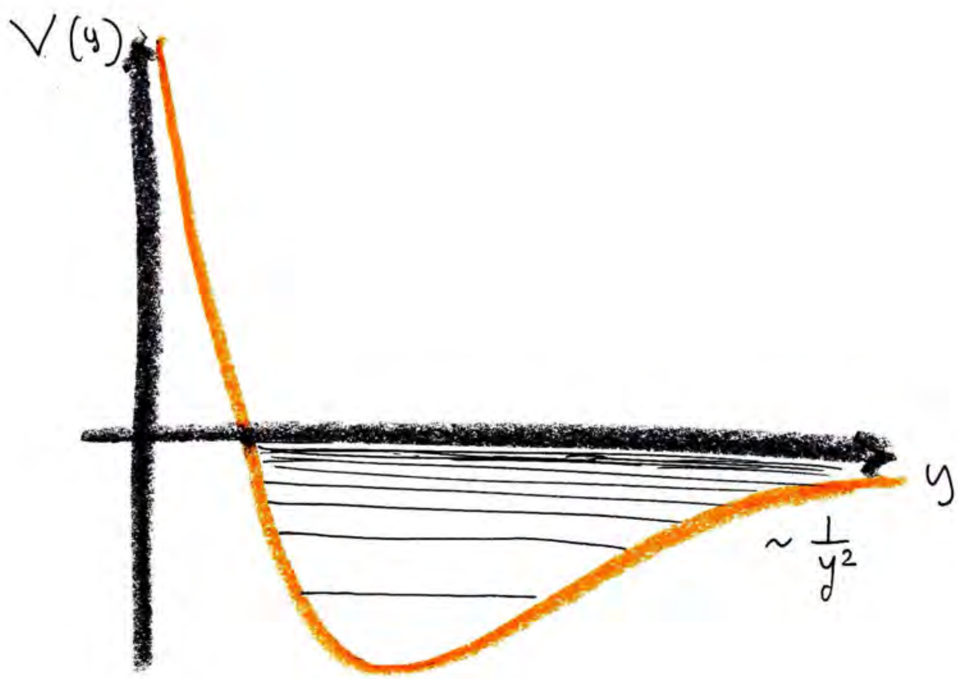


Electron Star $\psi = 0$ Hartonll,Hofman,Vegh '11

Fermi Surfaces: no condensate

Recast Dirac's equation at $\omega = 0$ into a Schrodinger problem, with $|\vec{k}|^2$ appearing as energy:

$$-\partial_y^2 \varphi + V(y)\varphi = -k^2 \varphi$$



States accumulate towards zero:

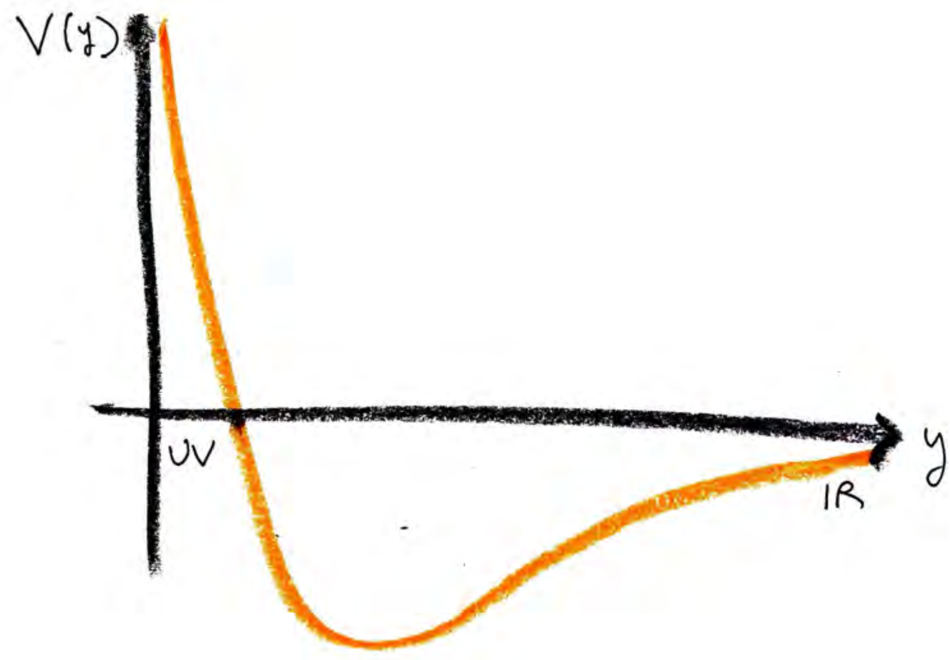
$$k_n \propto e^{-n}$$

$\Rightarrow$ an infinitenumber of Fermi surfaces

Electron Star $\psi = 0$ Hartonll,Hofman,Vegh '11

Fermi Surfaces: Compact stars

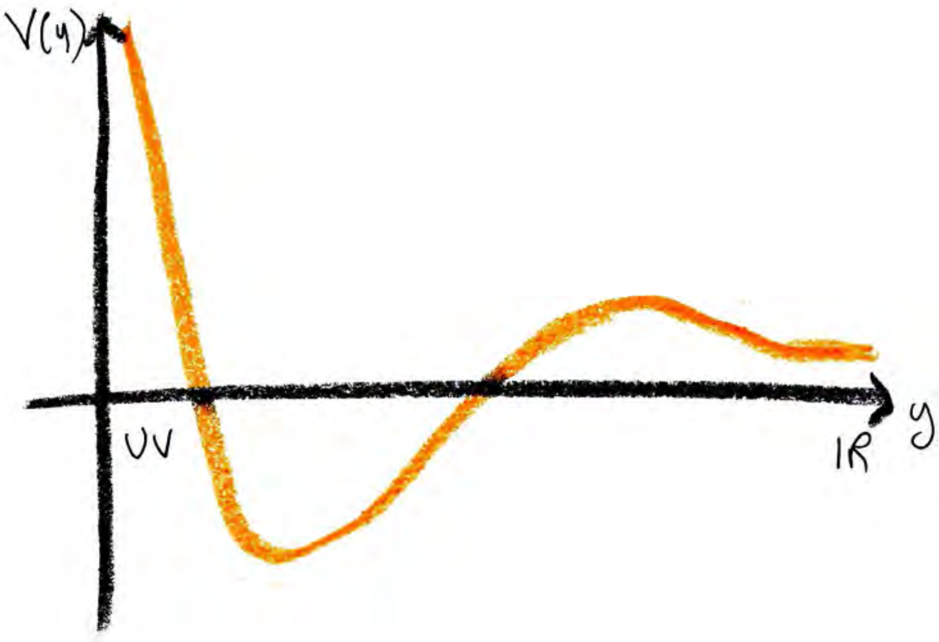
Turning on the scalar condensate changes the situation.



Electron star $\psi = 0$

Fermi Surfaces: Compact stars

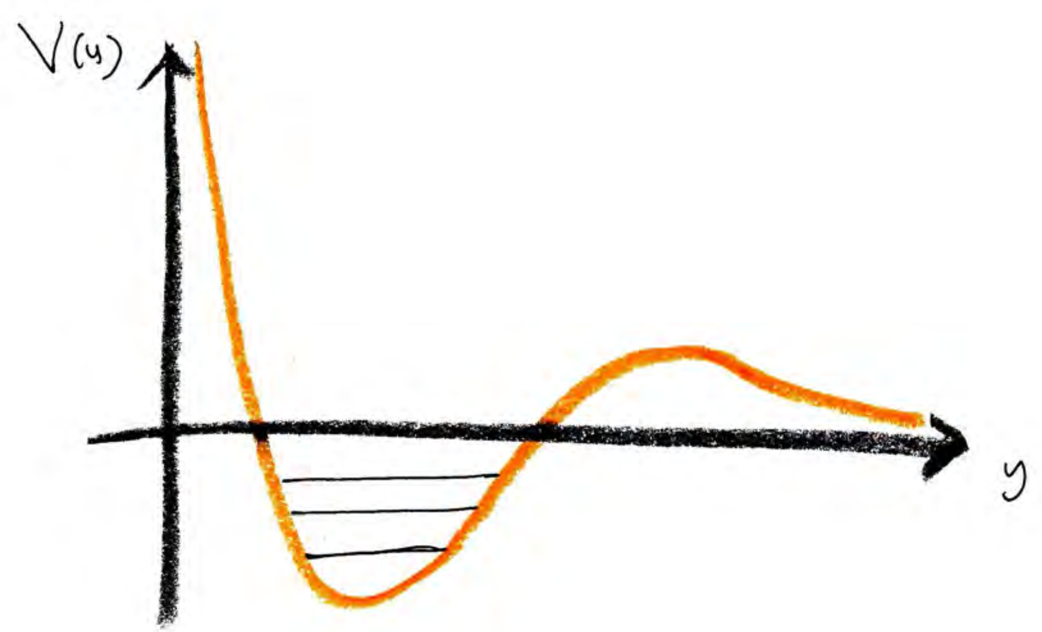
Turning on the scalar condensate changes the situation.



Compact stars $\psi \neq 0$

Fermi Surfaces: Compact stars

Turning on the scalar condensate changes the situation.



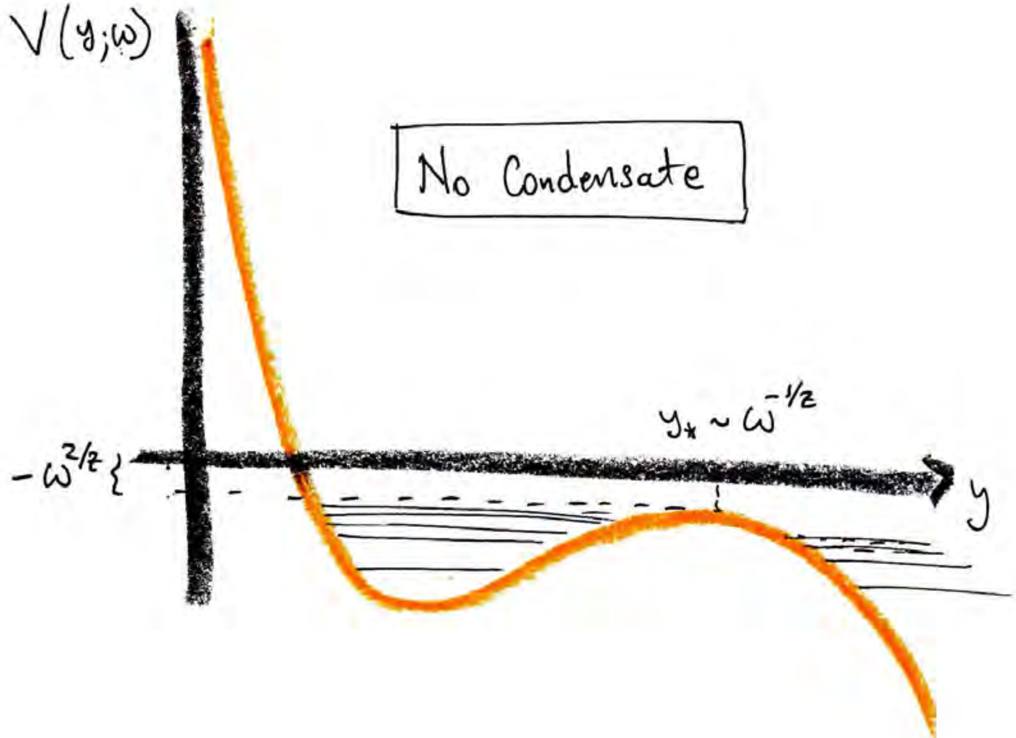
Finite number of Fermi surfaces with momenta $k_{max} > \ldots > k_{min}$

$\Rightarrow$ the condensate lifts most of the Fermi surfaces and leaves a finite number of them.

Conclusion

- Holographic systems with both bosonic and fermionic matter exhibit a rich structure, with various continous transitions between competing ground states.
- Among the solutions allowed, the ground state seems to always be the one with more ingredients present at the same time.
- A scalar condensate can gap (most of the) Fermi surfaces in an electron star.
- Investigate these systems further by computing conductivities.
- Examples in condensed matter systems ?

Finite ω



Finite ω