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MAX-PLANCK-GESELLSCHAFT

What are the prospects for finding hydrodynamic fluid behaviour in a real solid?

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Contents

1. Conditions under which hydrodynamic electron fluids might be found

2. Strong scattering: *T*-linear resistivity in a) Cuprates

   b) Quantum critical materials

   c) Elemental metals

3. Weak scattering: e.g. $PdCoO_2$

Relaxed definition of 'hydrodynamic' conditions

Least stringent condition: momentum of a fluid is relaxed much more slowly than its intrinsic rate of internal scattering.

Problem: in a solid with a lattice, momentum is relaxed *fast*.

Electron-impurity processes relax momentum

Normal electron-phonon processes relax momentum

Umklapp electron-phonon processes relax momentum

Umklapp electron-electron processes relax momentum

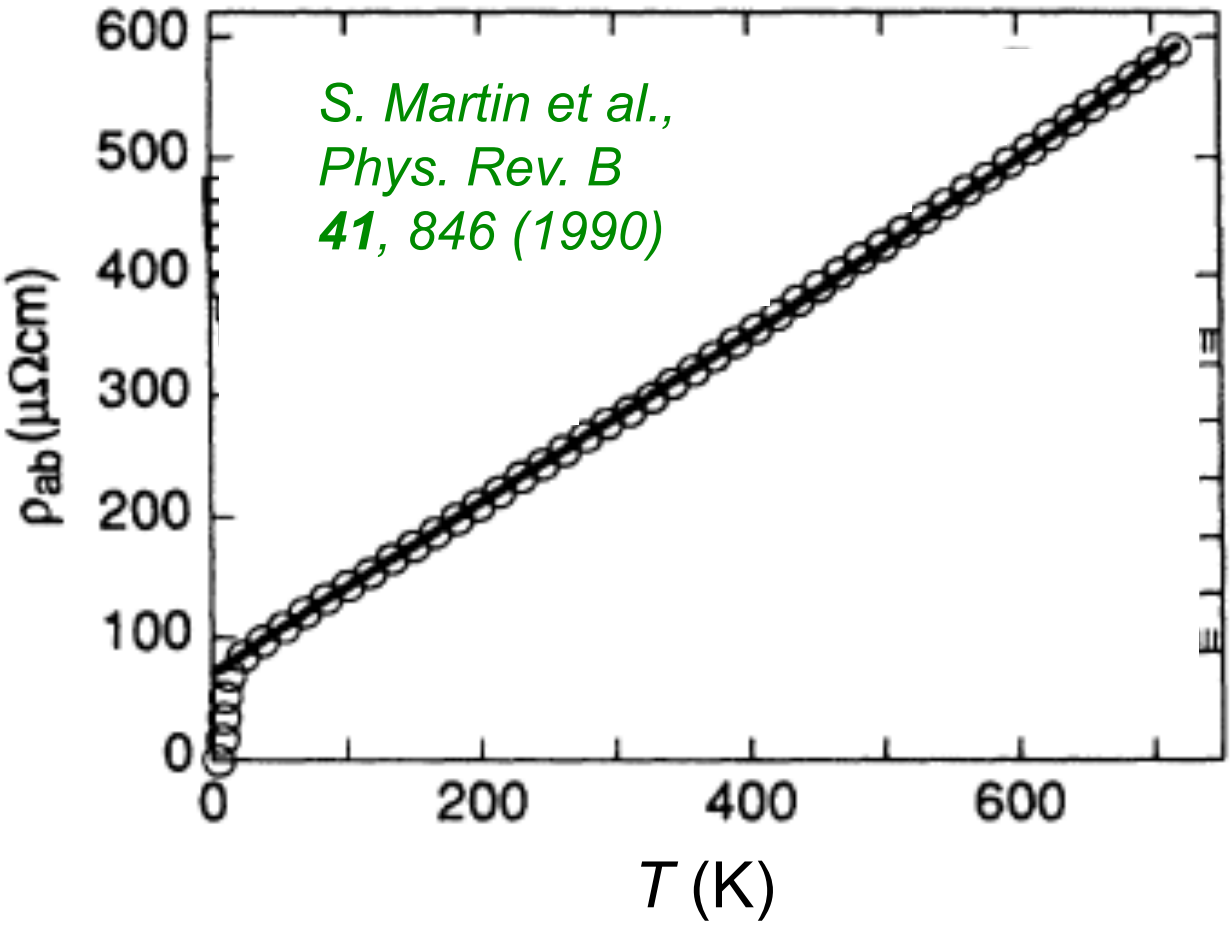
Only normal electron-electron processes conserve momentum.

Choice: a) Accept standard rates for the momentum-relaxing processes and crank up the momentum-conserving scattering OR

b) look for situations in which the momentum-relaxing scattering is suppressed.

NB: In terms of the classification used yesterday by Sean Hartnoll, I restrict consideration here to 'coherent' systems.

Choice a) extremely strong scattering: Cuprates 1980s



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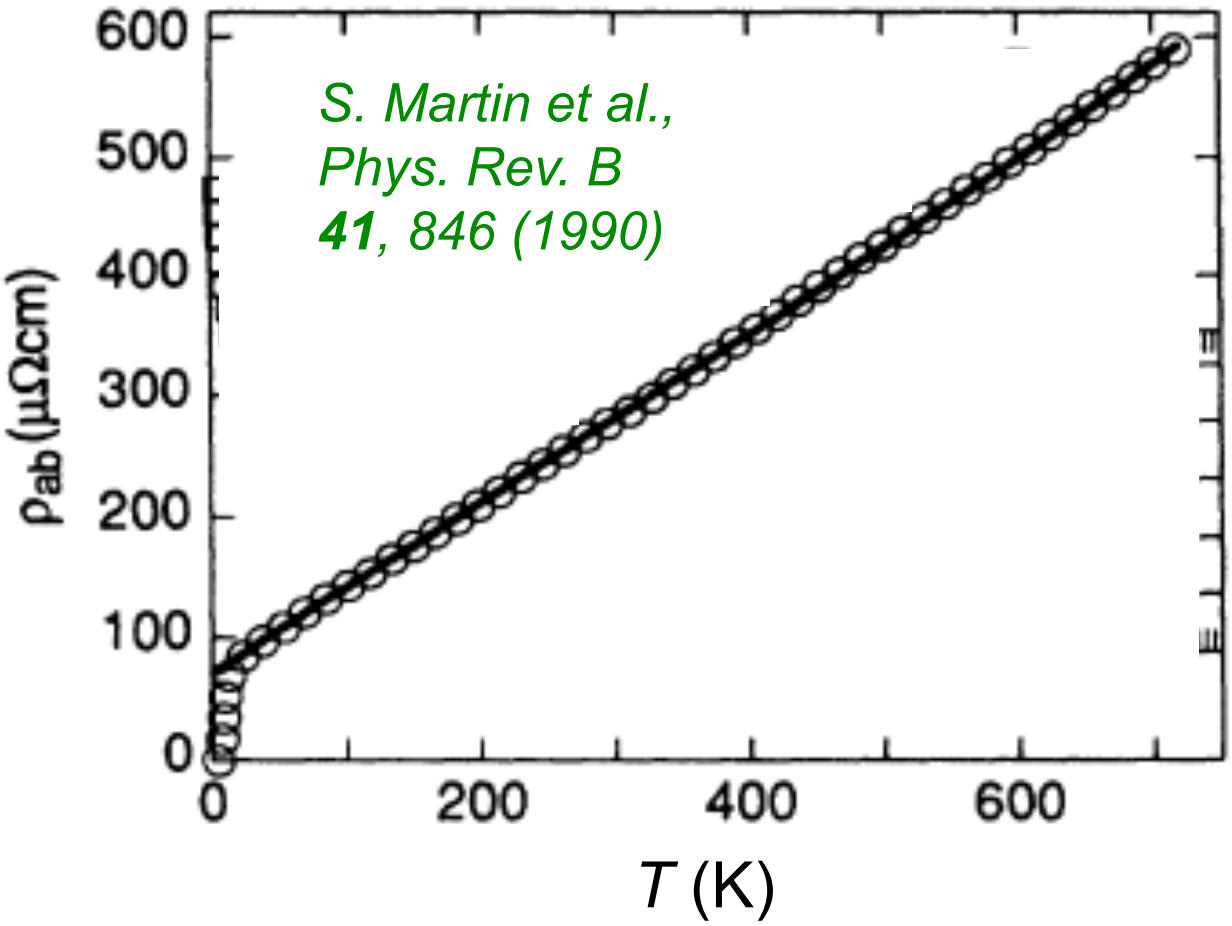


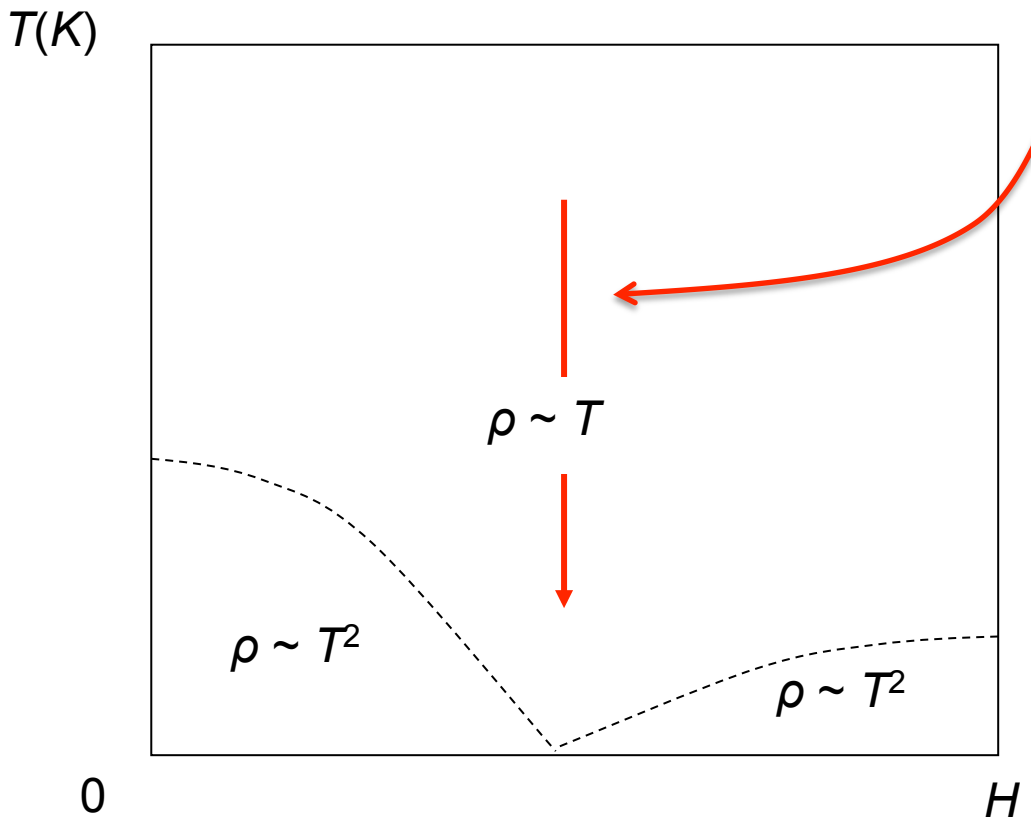
*J.W. Orenstein et al, Phys. Rev. B **42**, 6342 (1990)*
*M.A. Quijada et al, Phys. Rev. B **60**, 14917 (1999)*
*T. Valla et al., Science **285**, 2110 (1999)*
*D. van der Marel et al., Nature **425**, 271 (2003)*

Optical conductivity: linearity seen in d.c. transport is associated with a scattering rate of approximately $k_B/\hbar$ per kelvin. Same conclusions drawn later from photoemission linewidths.

Zaanen & colleagues: this means there is ultimately rapid 'Planckian dissipation' justifying a hydrodynamic treatment (JZ talk).

1990s onwards: tuned quantum critical systems

Sketch of metallic quantum criticality



T is the only energy scale, so a natural uncertainty-principle-based definition of a characteristic time is:

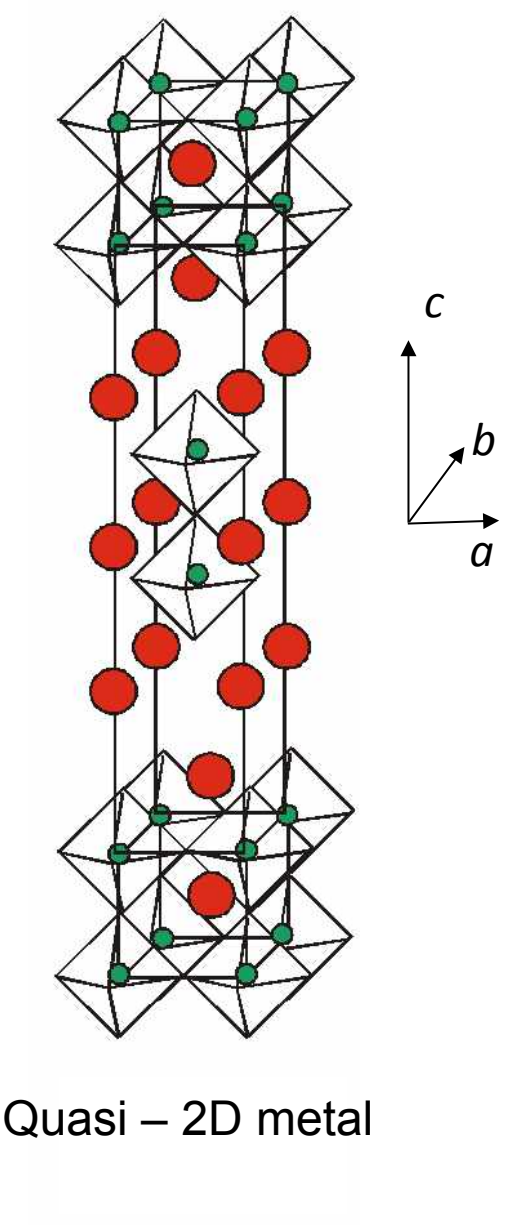
$$\tau \sim \hbar / k_B T$$

Equivalently, you can define a characteristic rate

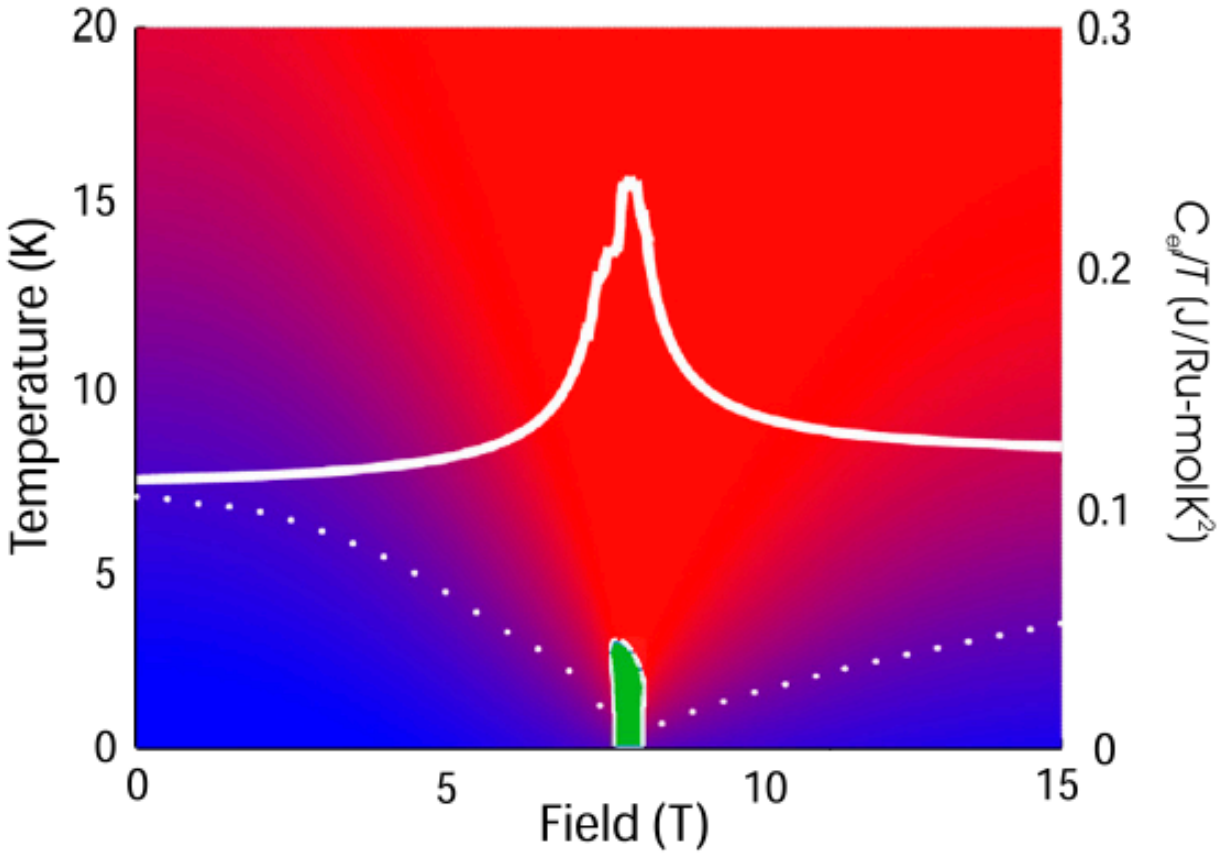
$$(\tau T)^{-1} = \alpha k_B / \hbar$$

where α is a dimensionless constant with no *a priori* justification for being of order 1 in a real material.

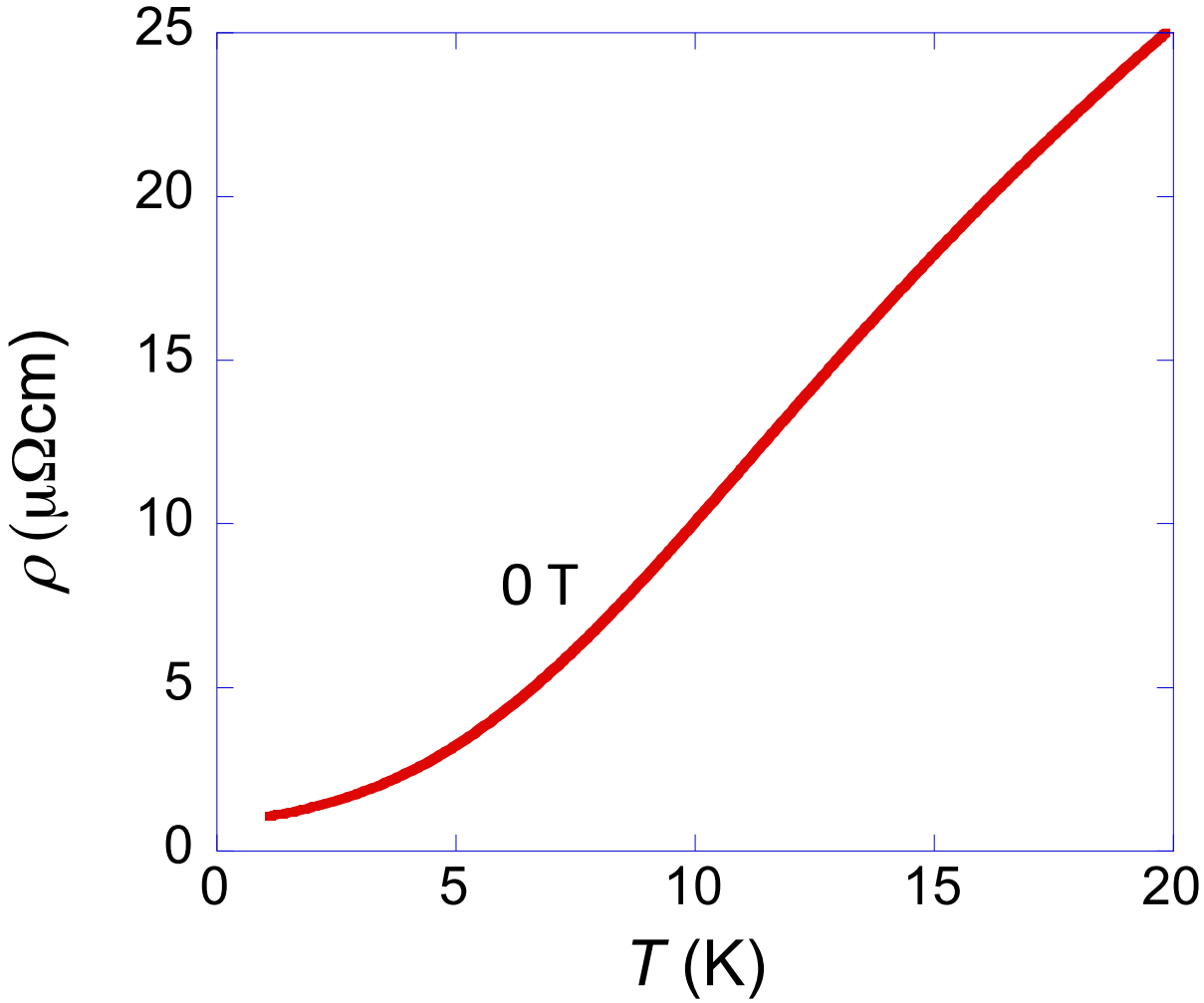
Evidence for quantum criticality in $Sr_3Ru_2O_7$



Quasi – 2D metal



Low temperature transport data from $Sr_3Ru_2O_7$ in an adiabatic demagnetisation cryostat



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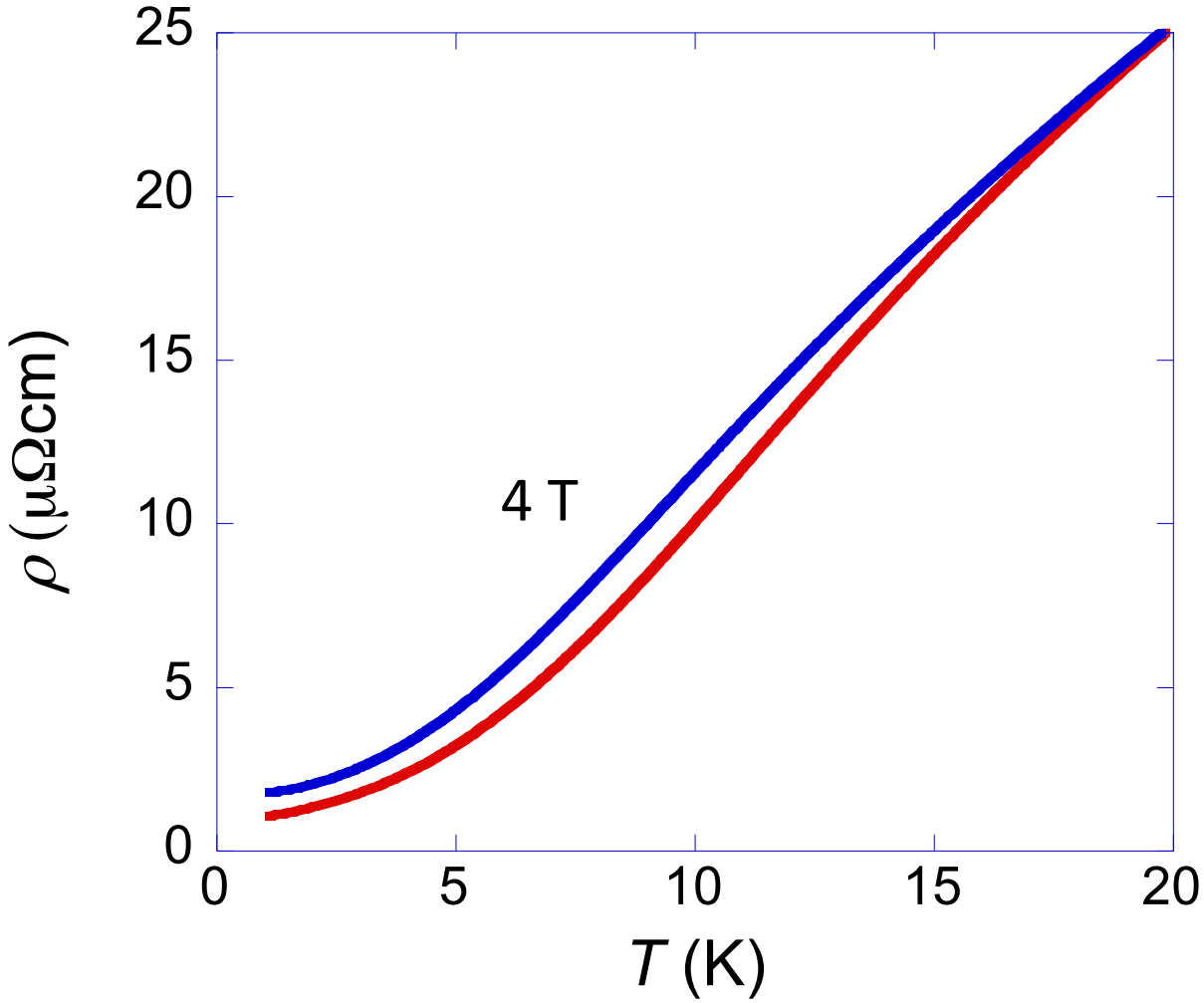

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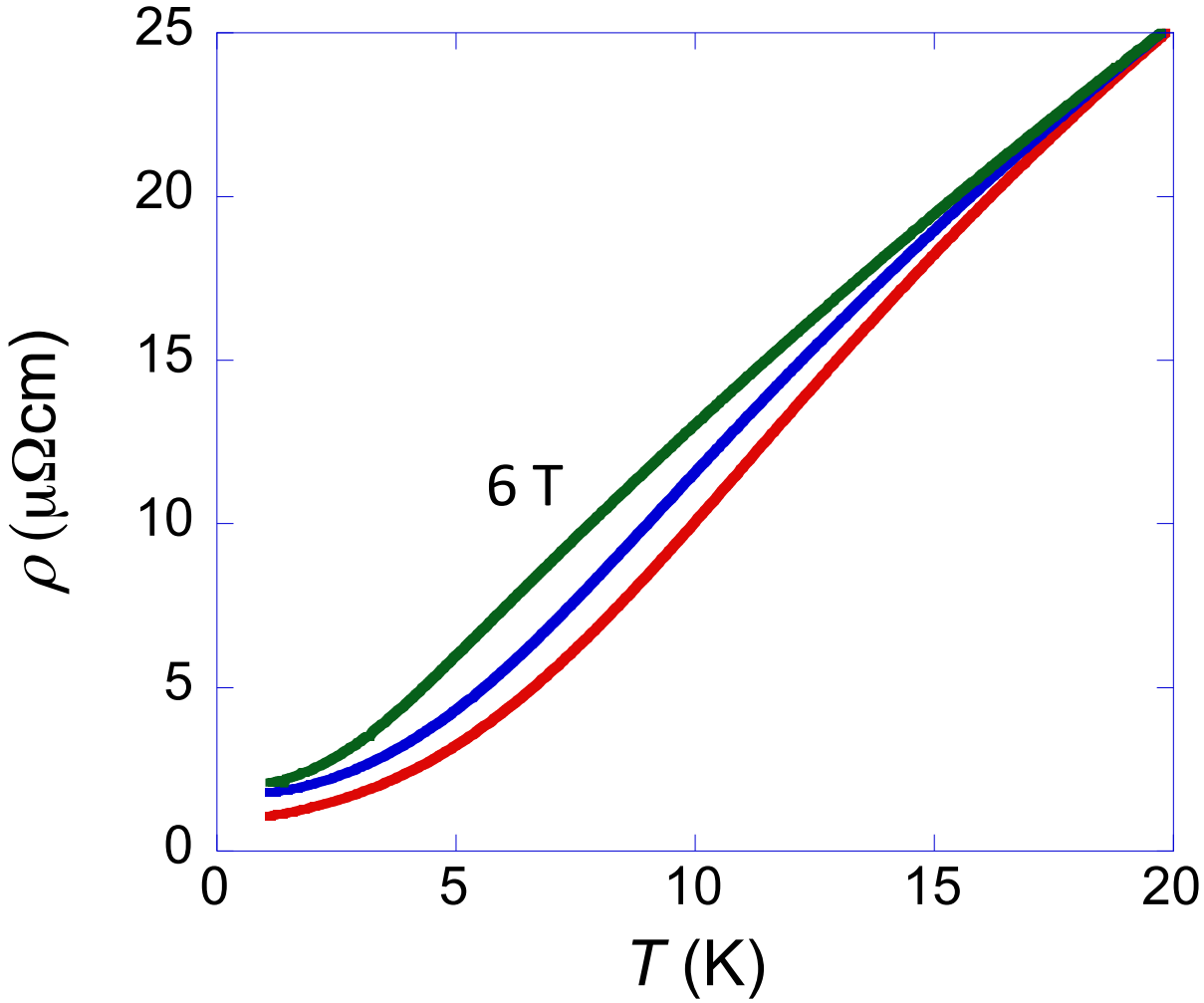

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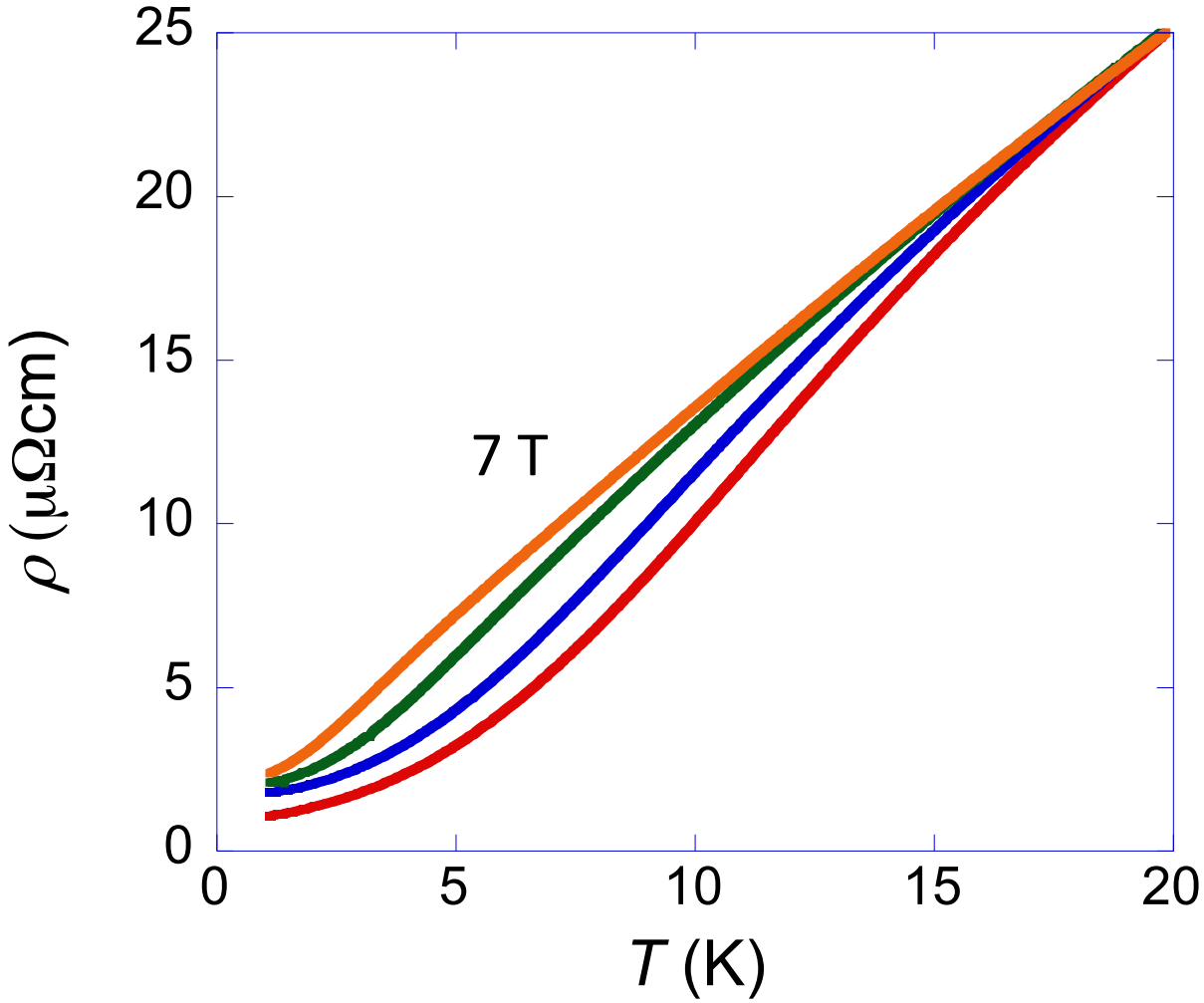

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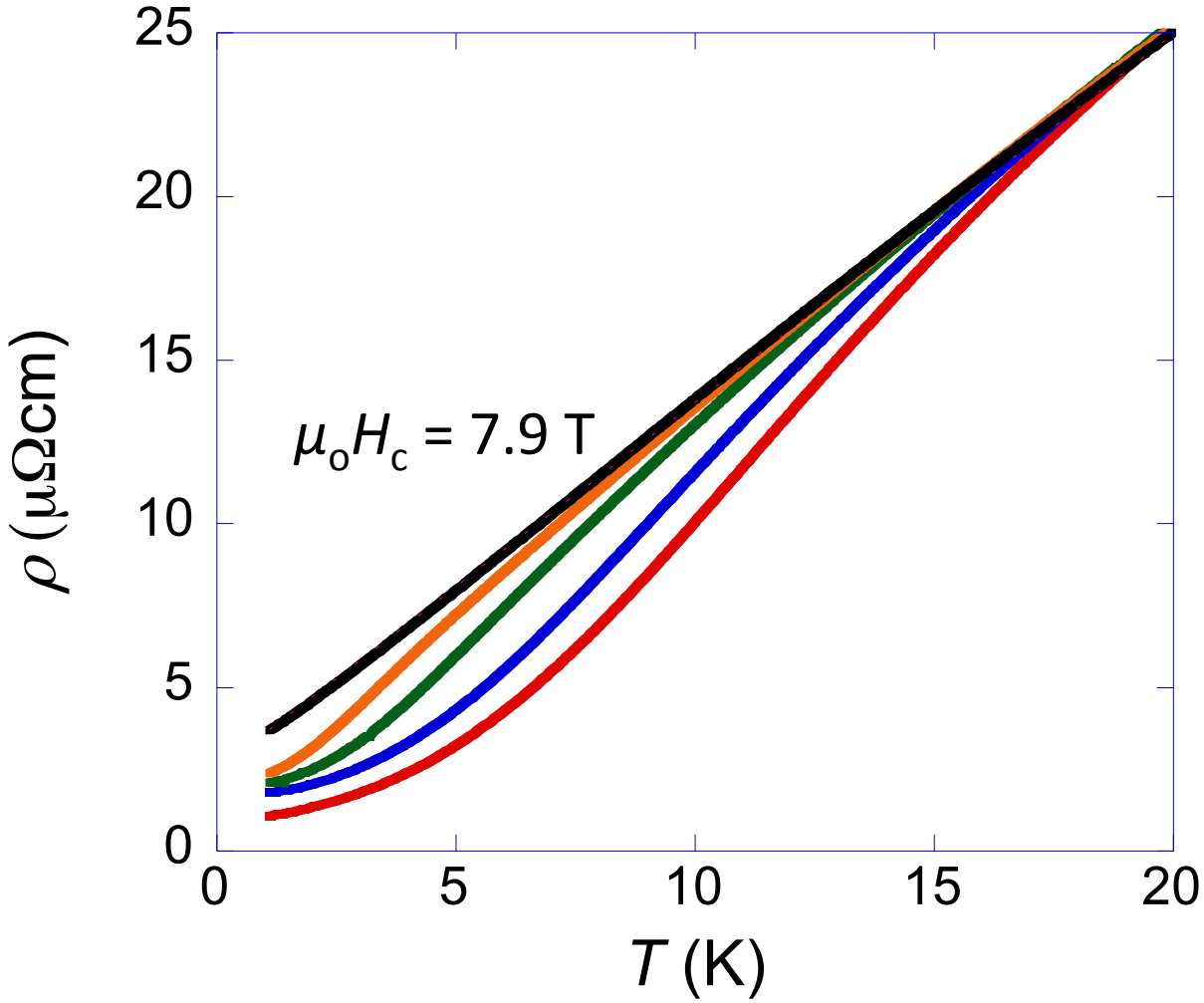
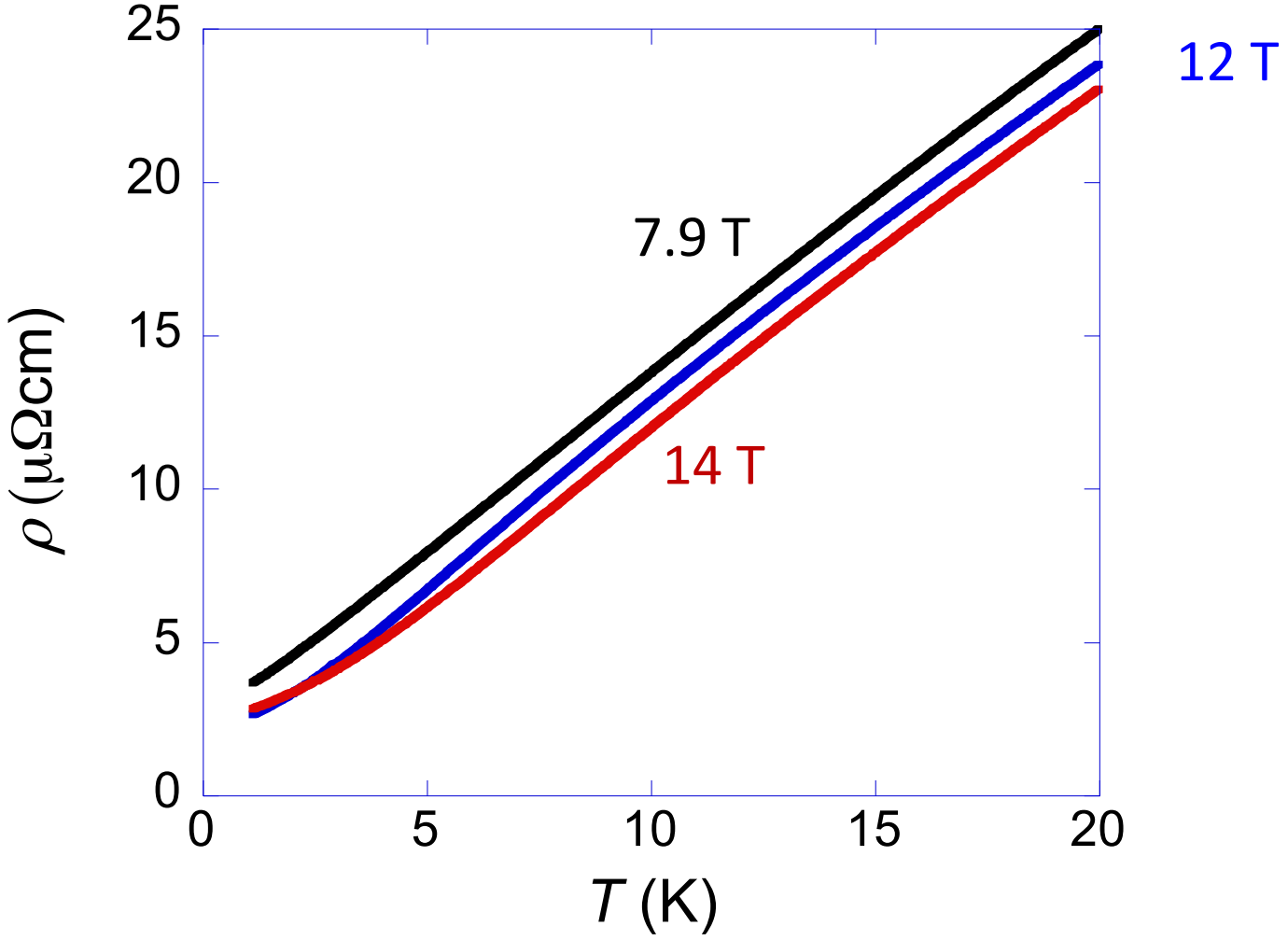


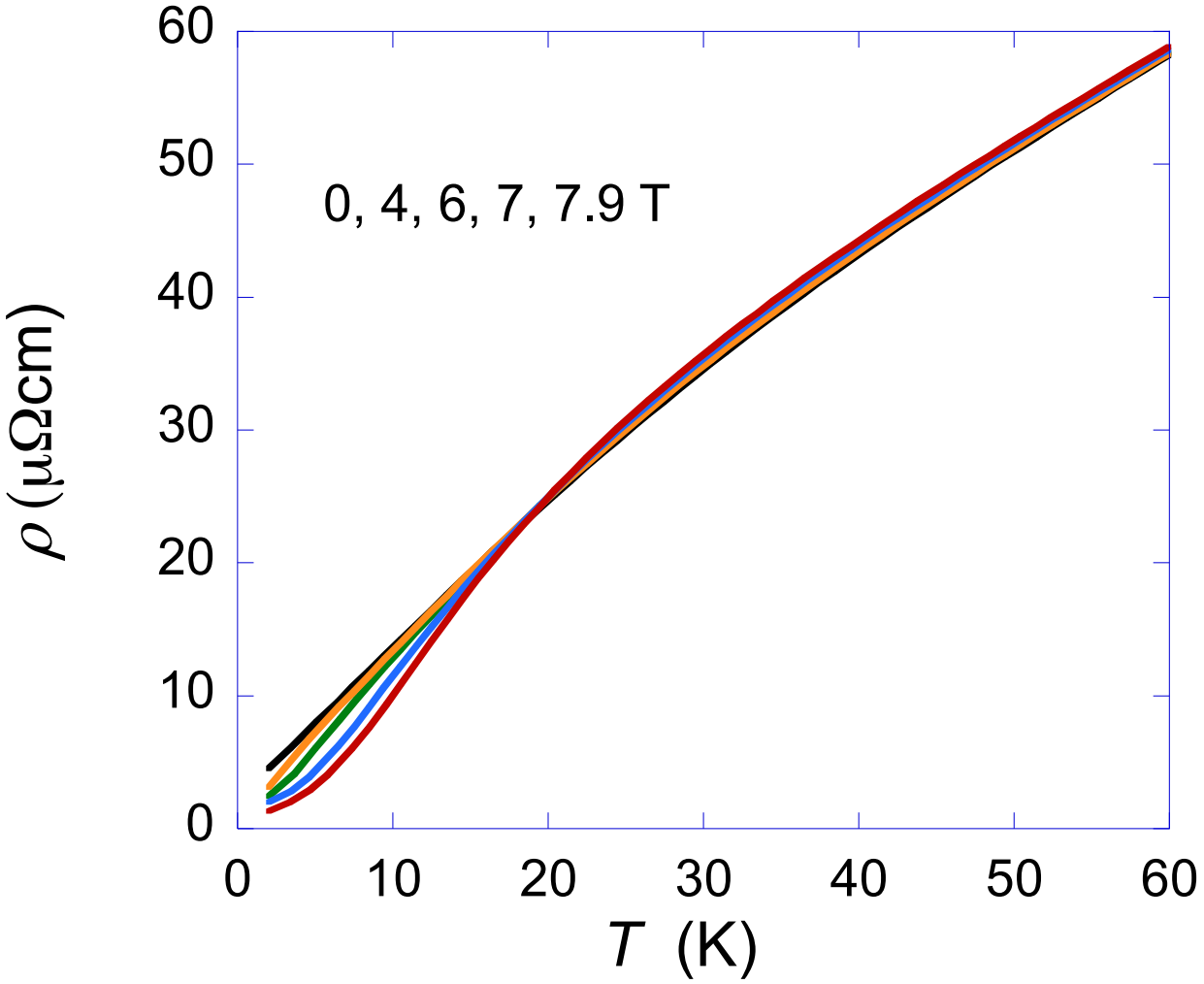
The linear resistivity at H_c is the upper bound for all data for $H < H_c$

Low temperature transport data from $Sr_3Ru_2O_7$ in an adiabatic demagnetisation cryostat



Qualitatively similar behaviour on the high-field side though the crossover temperatures are lower and the negative MR is more noticeable.

Extension of $Sr_3Ru_2O_7$ transport data to 60 K on low field side



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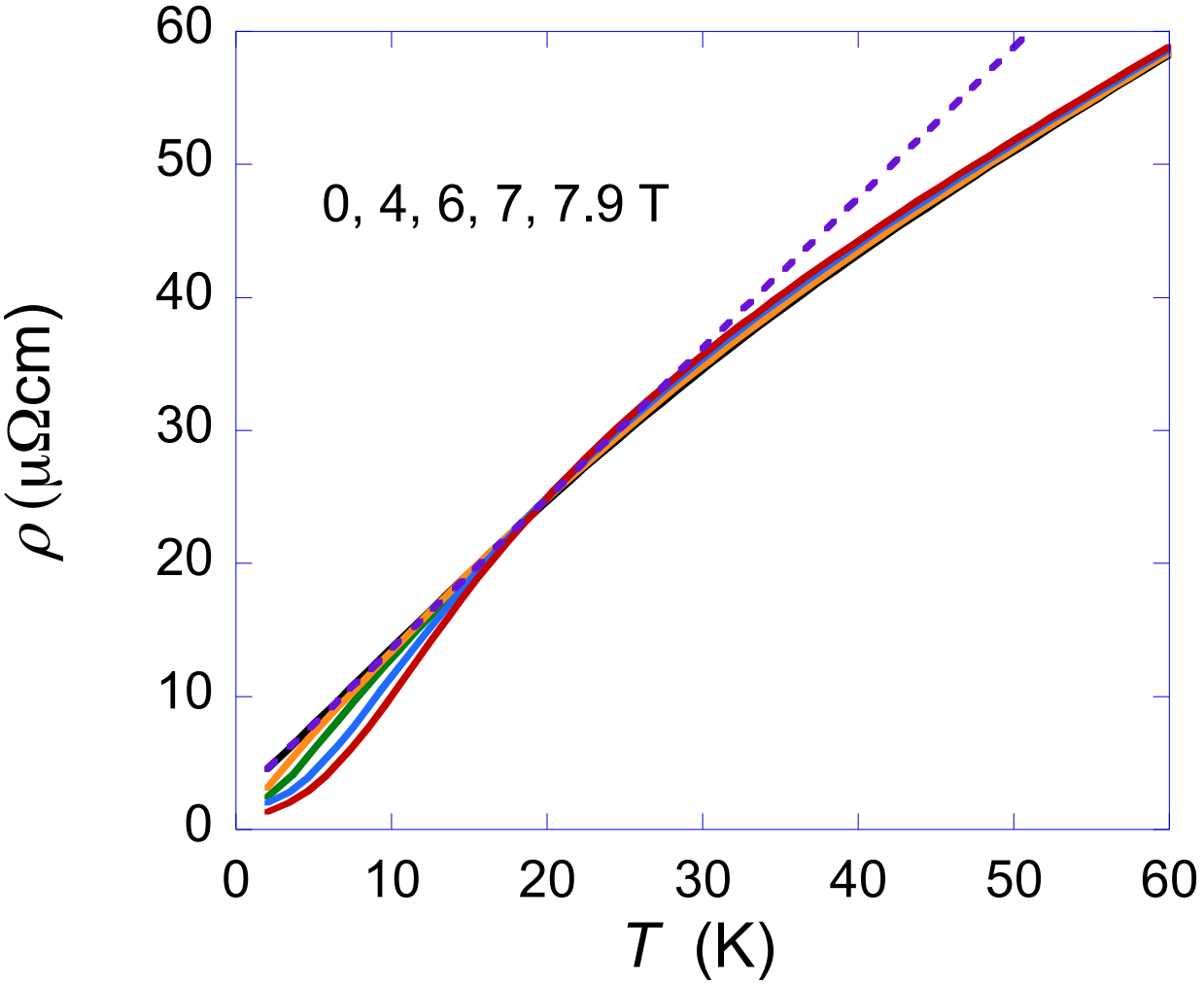
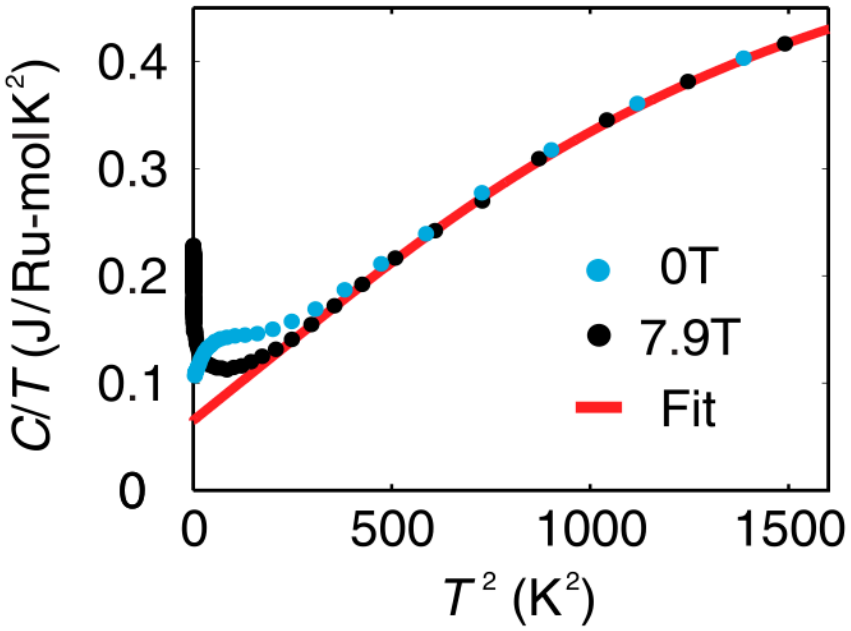
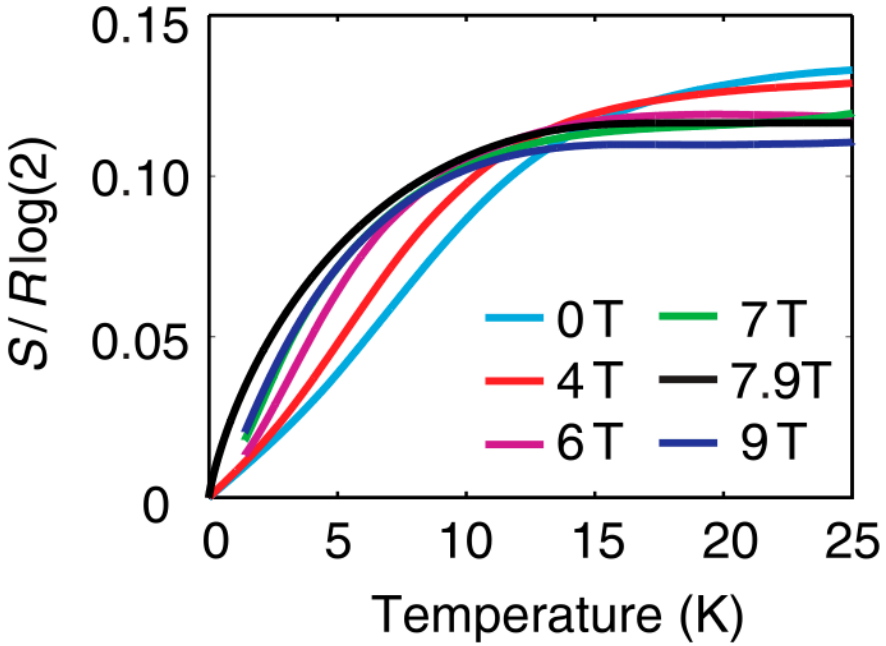


In spite of small negative MR at higher *T*, extrapolation of low temperature linear *T* term remains approximately an upper bound on resistivity.

Hydrodynamic in the Davison-Schalm-Zaanen sense?

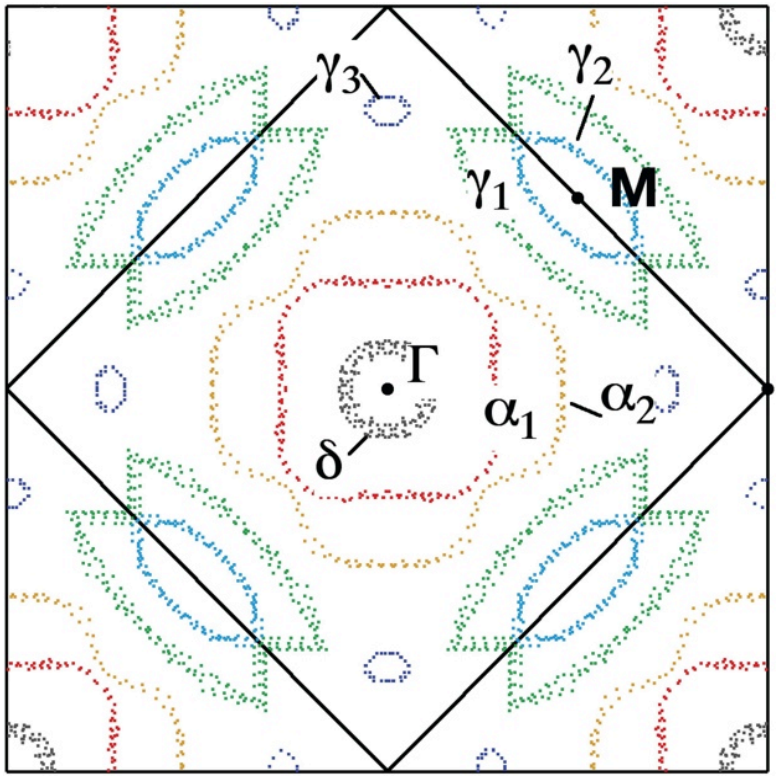




Recall their key prediction – resistivity should get its *T*-dependence from the entropy, for a strongly disordered system.

$Sr_3Ru_2O_7$ is *not* strongly disordered, so doubtful if the theory should apply, and empirically it does not (entropy is non-linear over the relevant range).

Why then is a *T*-linear resistivity seen in $Sr_3Ru_2O_7$?



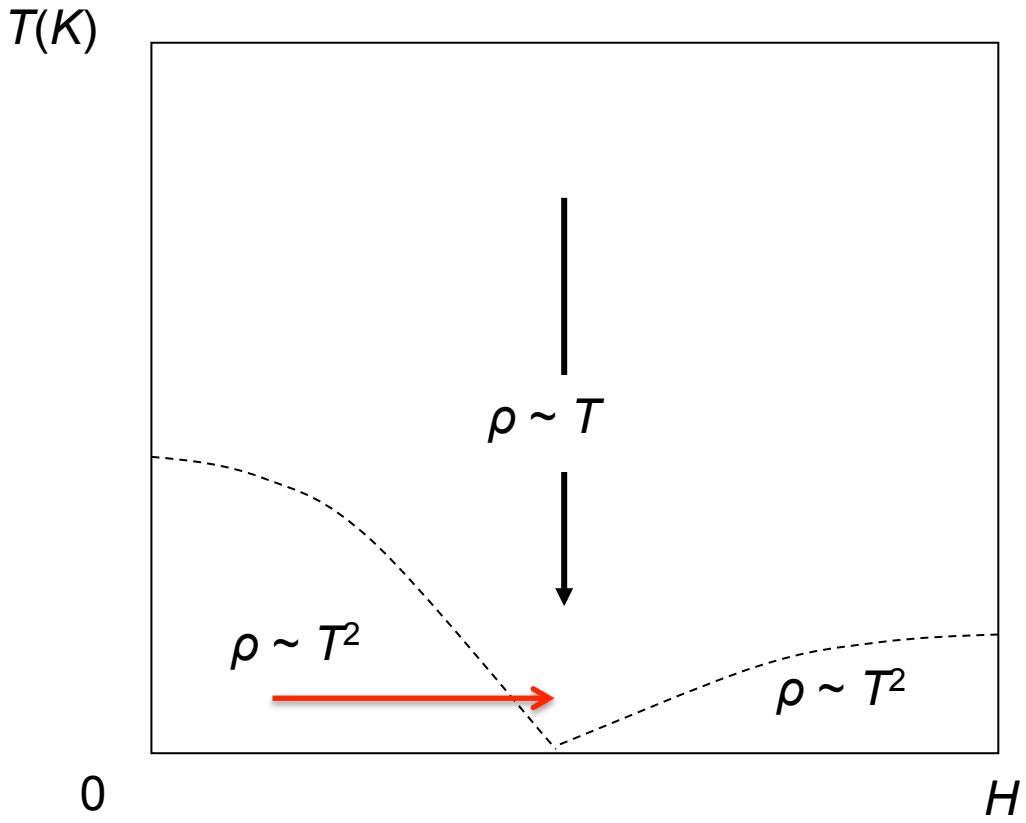
$Sr_3Ru_2O_7$ is a multi-band material in which one small Fermi surface sheet, γ_3, is thought to dominate the criticality.

At least 90% of the carriers are non-critical, or 'cold' (no mass enhancement as the critical field is approached, for example) but *all* seem to be subject to critical scattering!

A. Tamai et al., Phys. Rev. Lett. ***101****, 026407 (2008)*

J.F. Mercure et al., Phys. Rev. B ***81****, 235103 [2010]*

Rationale for quasiparticle analysis based on de Haas-van Alphen effect data



dHvA tells you the quasiparticle density and velocity as you tune towards the critical region, at low but finite temperature.

Well-formed quasiparticles do not just disappear suddenly at the crossover field.

What is the scale of the scattering rate at the quantum critical maximum in $Sr_3Ru_2O_7$?

Postulate: quantum criticality in $Sr_3Ru_2O_7$ provides a degree of freedom that defines the inelastic scattering rate of the 'cold' quasiparticles.

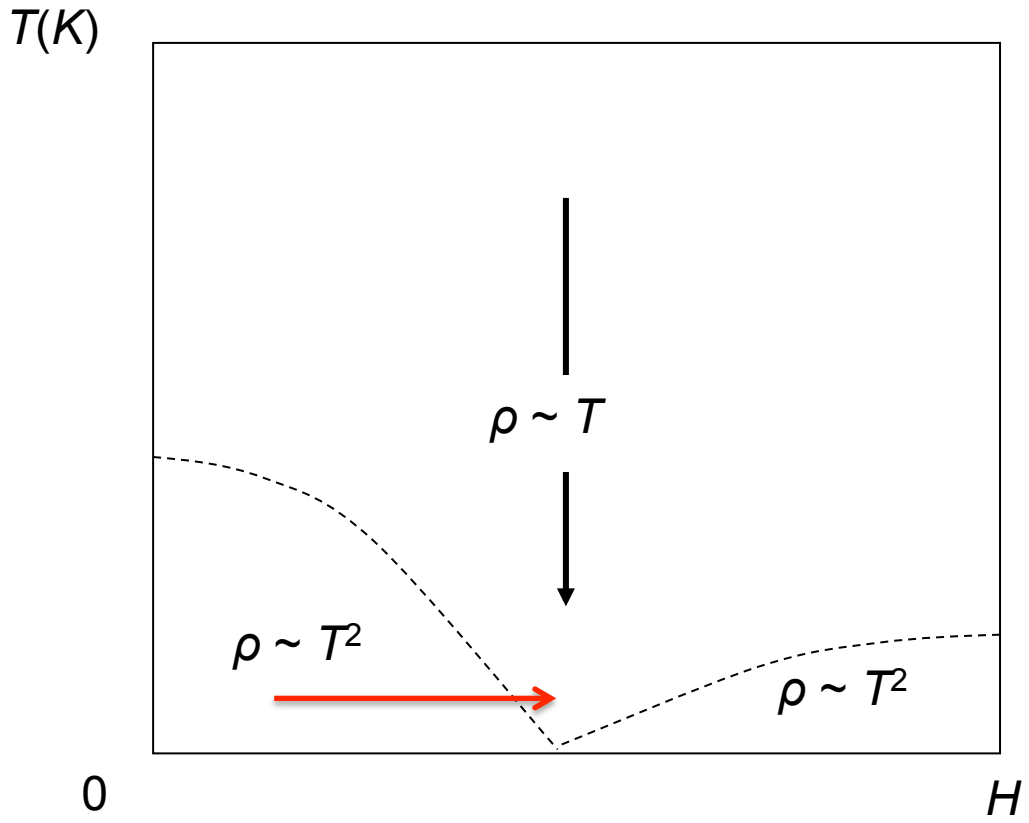
All Fermi surface pockets in $Sr_3Ru_2O_7$ are known from dHvA so

$$\frac{1}{\tau} = \frac{e^2\rho}{hd}\sum_i k_{Fi}v_{Fi}$$

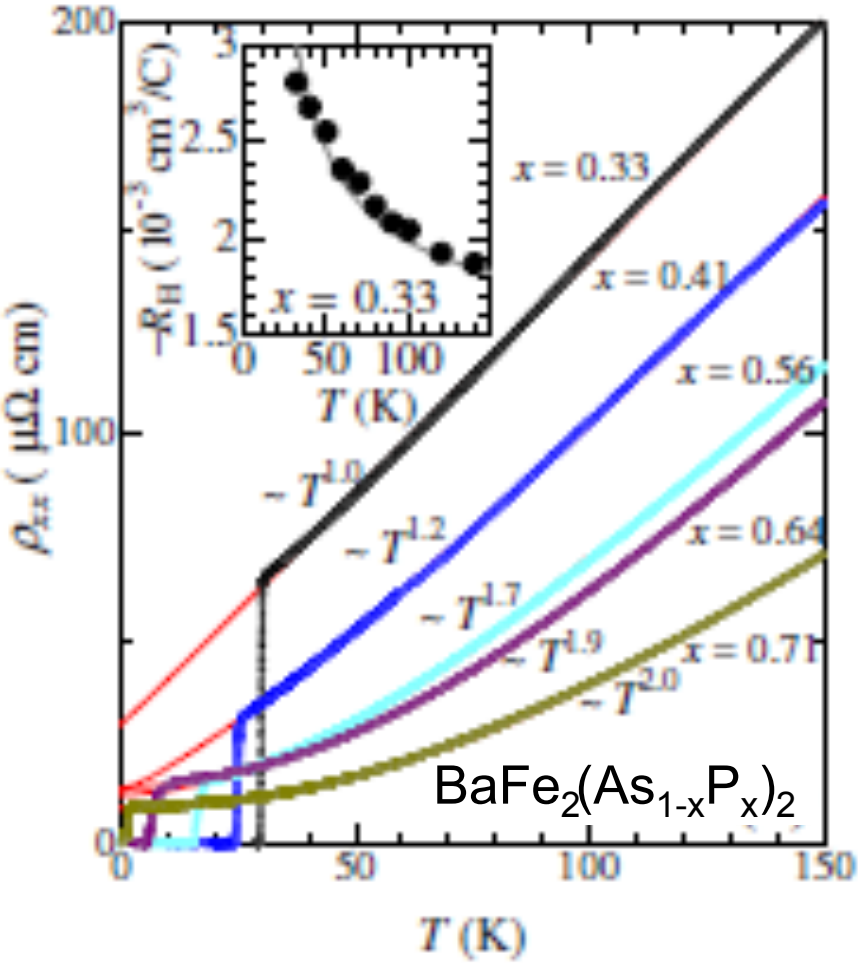
where *d* is the bilayer-bilayer spacing and *i* is the pocket index (taking into account multiplicities in the Brillouin zone – see *J.F. Mercure et al., Phys. Rev. B 81, 235103 [2010]*)

Result: $\alpha = 1.5$, i.e. $(T\tau)^{-1} = 1.5\ k_B/\hbar$

Repeat dHvA-based analysis on other systems for which data exists



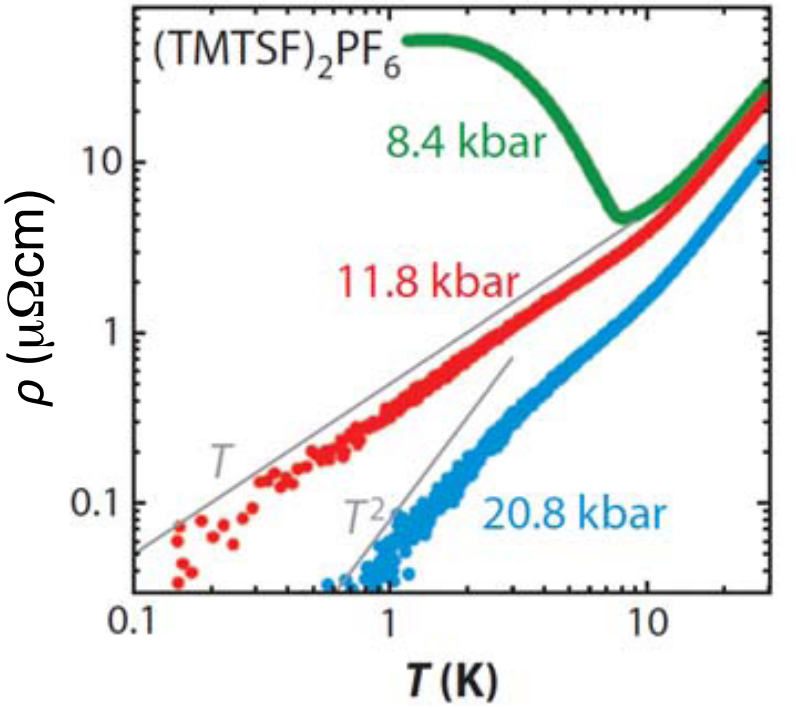
Pnictides, heavy fermions



S. Kasahara et al., Phys. Rev. B ***81****, 184519 (2010)*

H. Shishido et al., Phys. Rev. Lett. ***104****, 057008 (2010)*

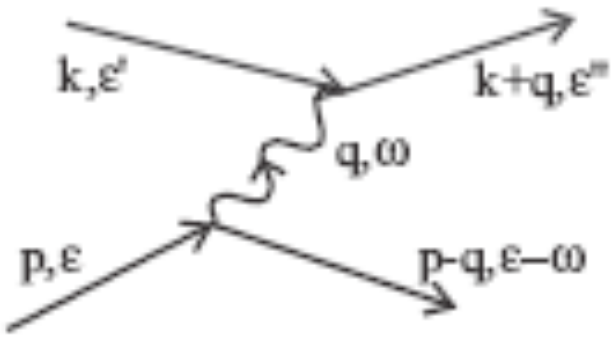
Organics

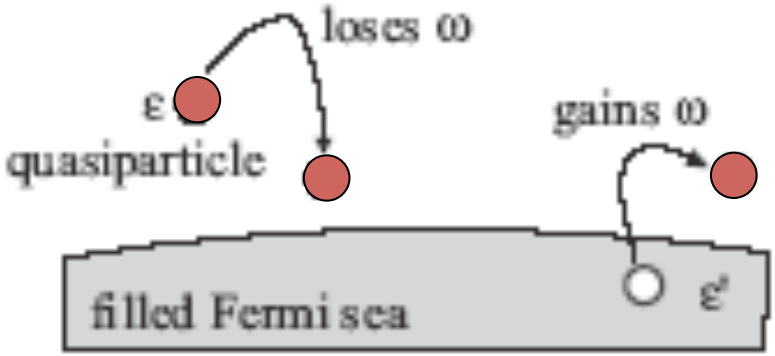


L. Taillefer, Ann. Rev. Cond. Matt. Phys ***1****, 51 (2010)*

Result: scattering rate when *T*-linear is always close to $k_B/\hbar$ per kelvin.

Not as crazy as it might at first seem: consider scattering in a Fermi liquid



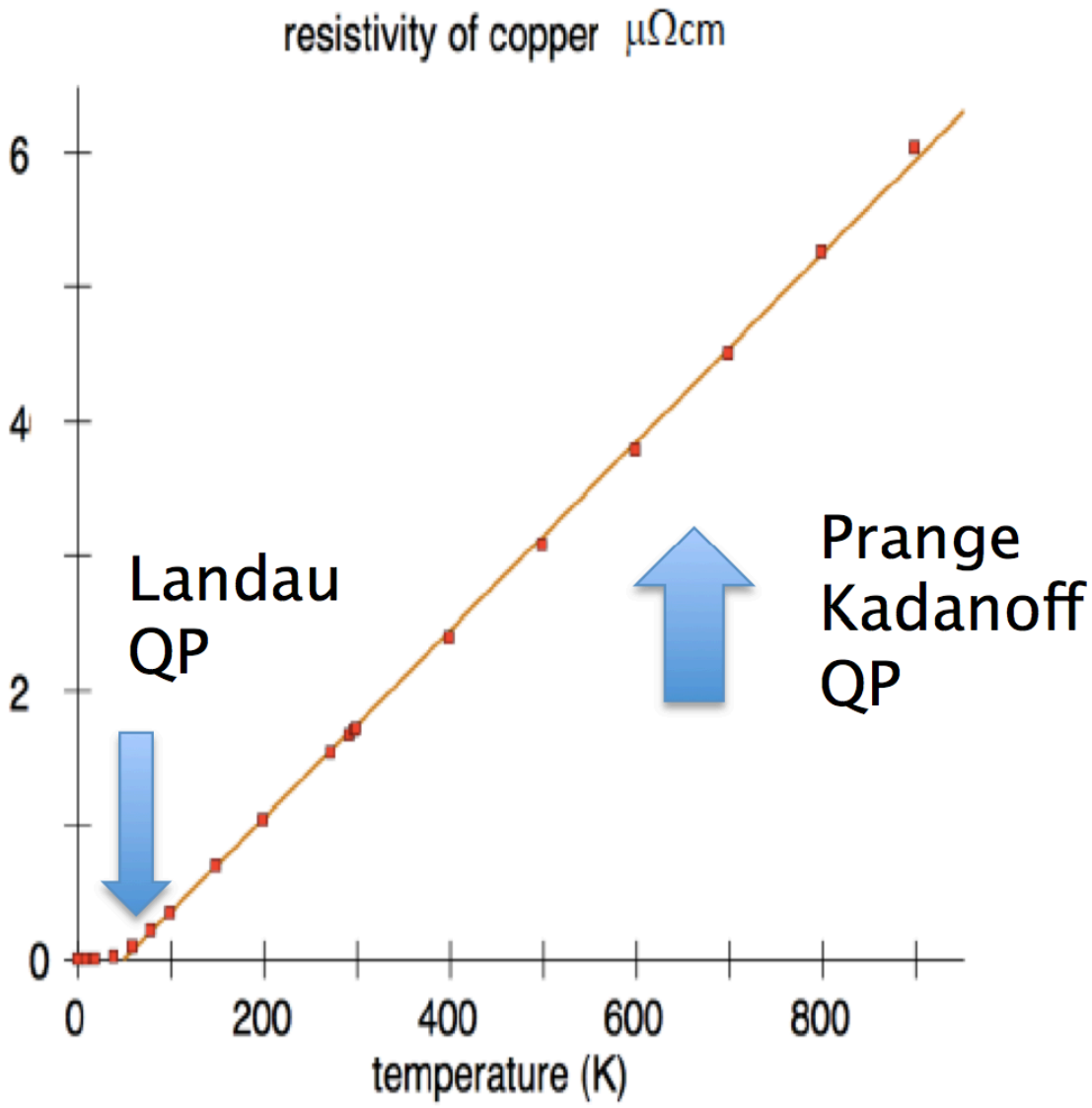


$$\tau_{ee}^{-1} \sim \frac{k_B T^2}{\hbar T_F}$$

Two final state restrictions on the scattering due to the quantum degeneracy, leading to two powers of *T*. One of these powers disappears if one party to the scattering is 'classical' at temperature *T*.

Not a scale-free situation but a two-scale situation; you could be well above the degeneracy temperature of the scatterer but still well below the degeneracy temperature of most of the electrons.

Similar to the situation in elemental metals at high *T*

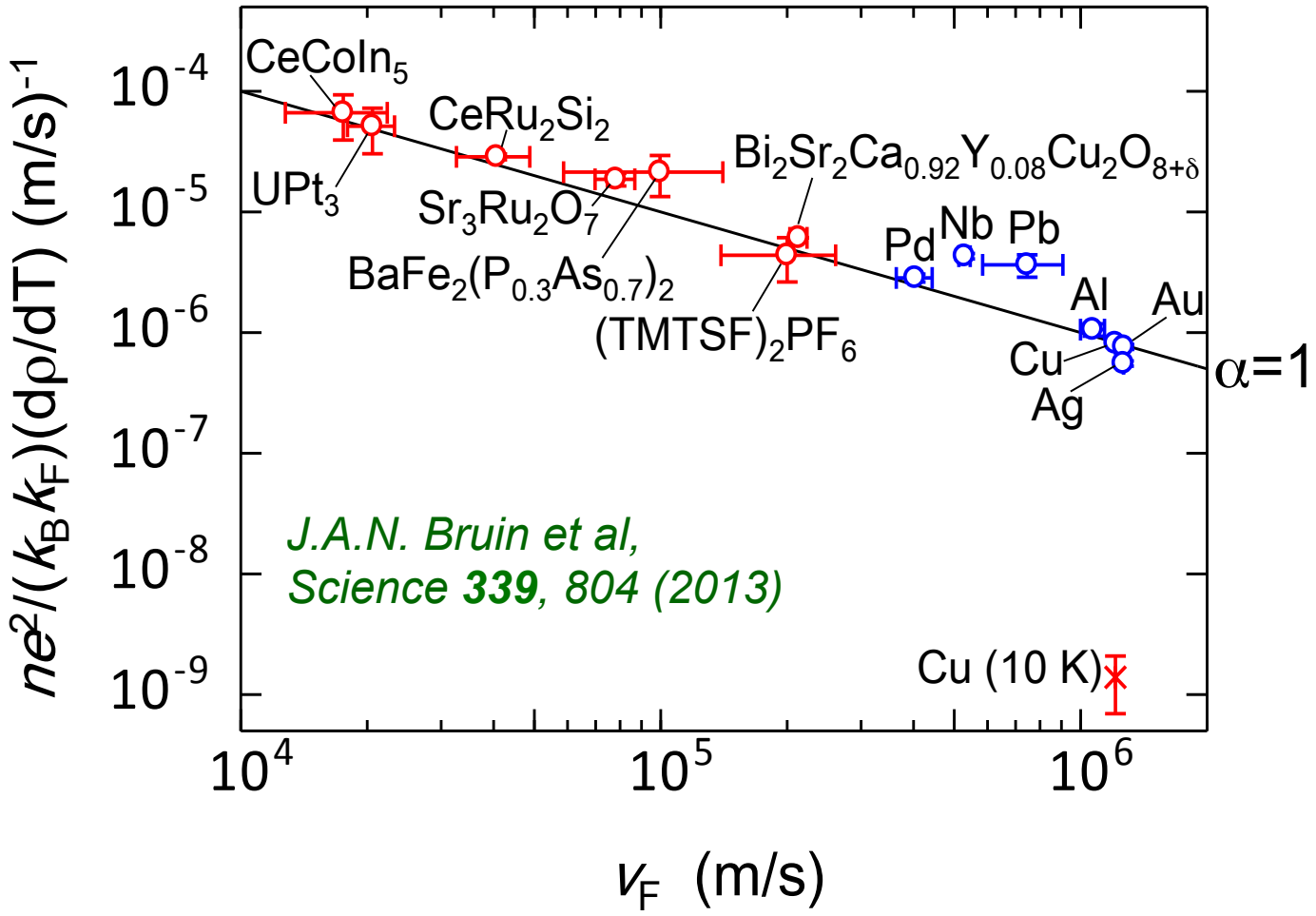


Prange-Kadanoff region: a two-scale situation.

Landau QPs form below T_F ~ 20000 K but phonons have T_{Debye} ~ 400 K. In the PK region you therefore have quasiparticles scattering from classical excitations.

Measured scattering rate in the PK region? Again, $k_B/\hbar$ per kelvin.

‘Universal’ behaviour: heavy fermions, oxides, pnictides and organics (scattering electronic in origin) and electron-phonon materials at high *T*



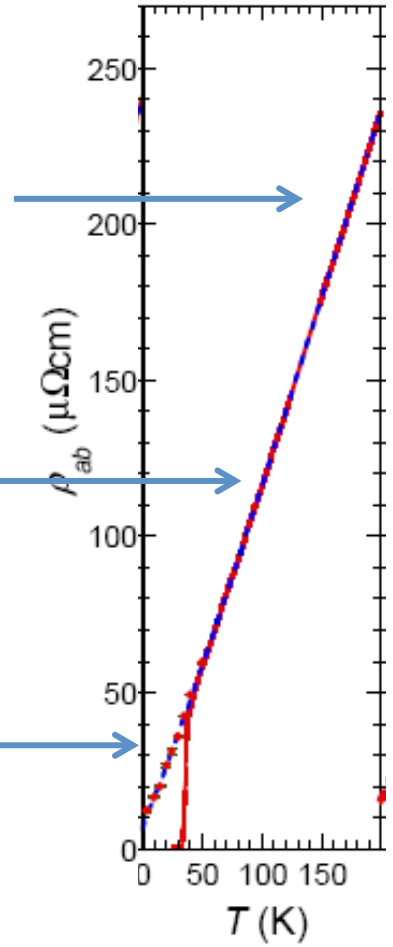
Different systems, different microscopics but always $(\tau T)^{-1} \sim k_B / \hbar$.

Circumstantial evidence for a bound: why do we not see signs of multiple scattering channels?

At higher *T* still, one presumably makes a crossover from a ‘coherent’ to an ‘incoherent’ metal, but no change of slope is seen.

At higher *T* there is known coupling to specific phonon modes but no sign of this is seen in the resistivity.

T-linear resistivity at low temperatures must result from scattering that is electronic in origin.



Message from experiments on strong-scattering systems:

There are probably a number of different microscopic mechanisms for creating *T*-linear resistivity.

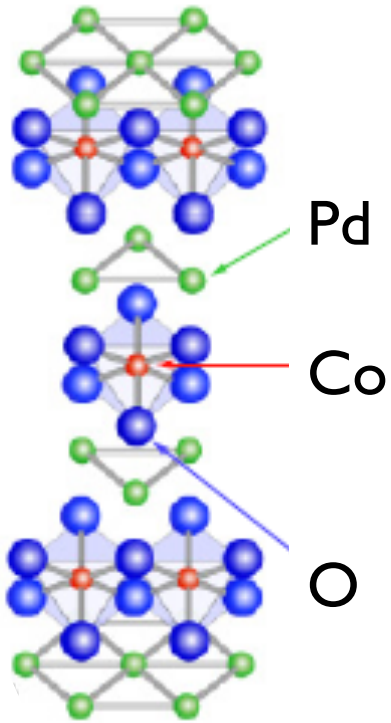
Some offer the promise of accessing the hydrodynamic limit; others are more conventional.

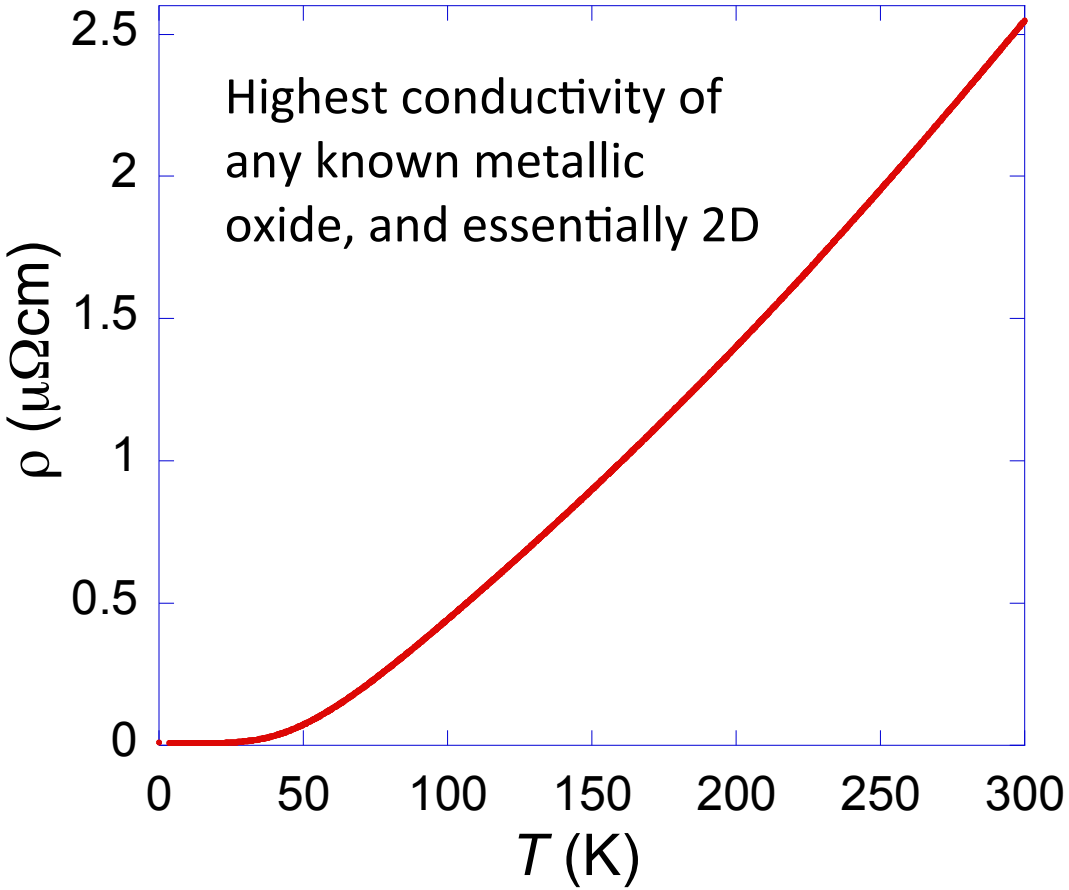
However, the same ‘universal’ scattering rate per kelvin is seen across the many material classes and microscopic situations – why?

What about the other ‘end of the spectrum’?

Special situations in which the momentum-relaxing scattering is suppressed.

$PdCoO_2$: a layered oxide metal with extraordinary conductivity

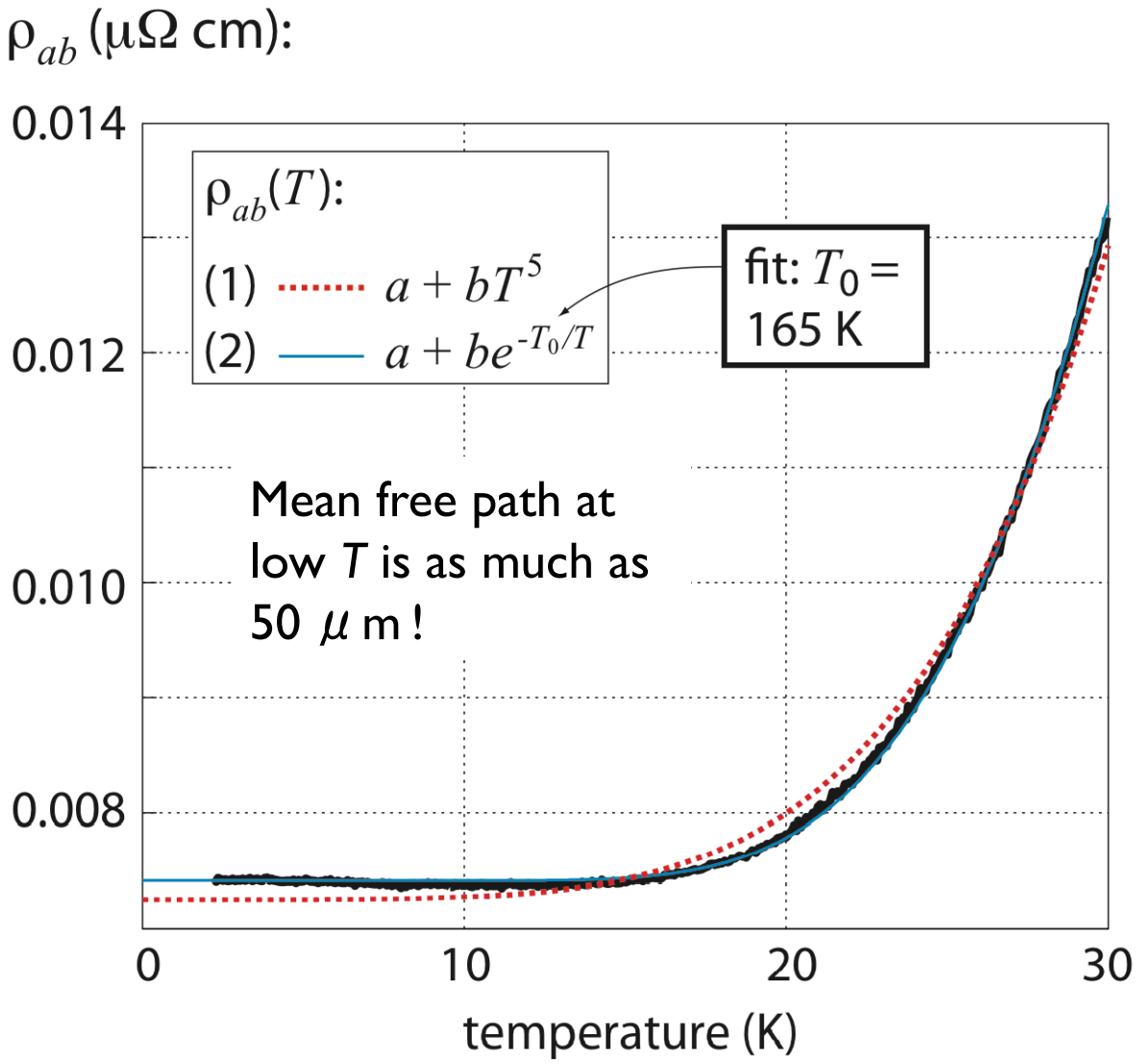




R.D. Shannon, D.B. Rogers and C.T. Prewitt, Inorg. Chem. ***10****, 713 (1971)*

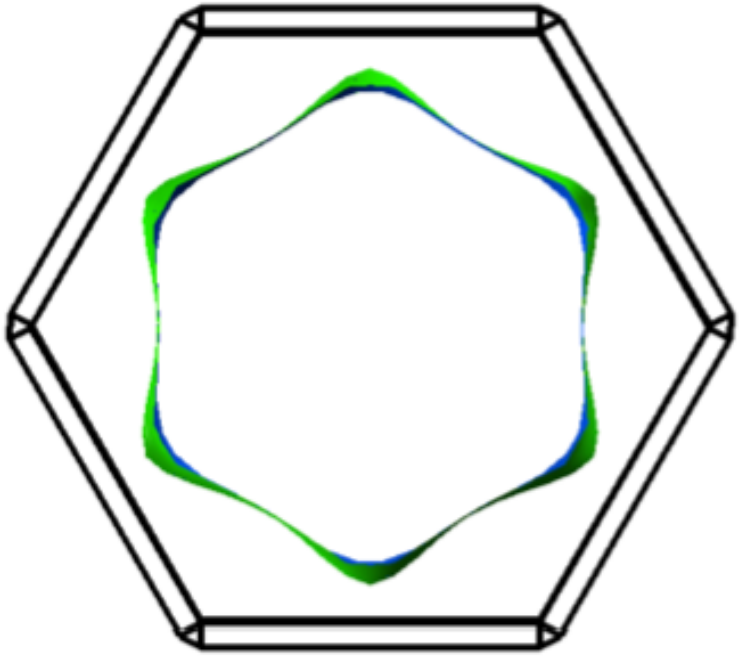
H. Takatsu, S. Yonezawa, S. Mouri, S. Nakatsuji, K. Tanaka and Y. Maeno, J. Phys. Soc. Jpn. ***76****, 104701 (2007)*

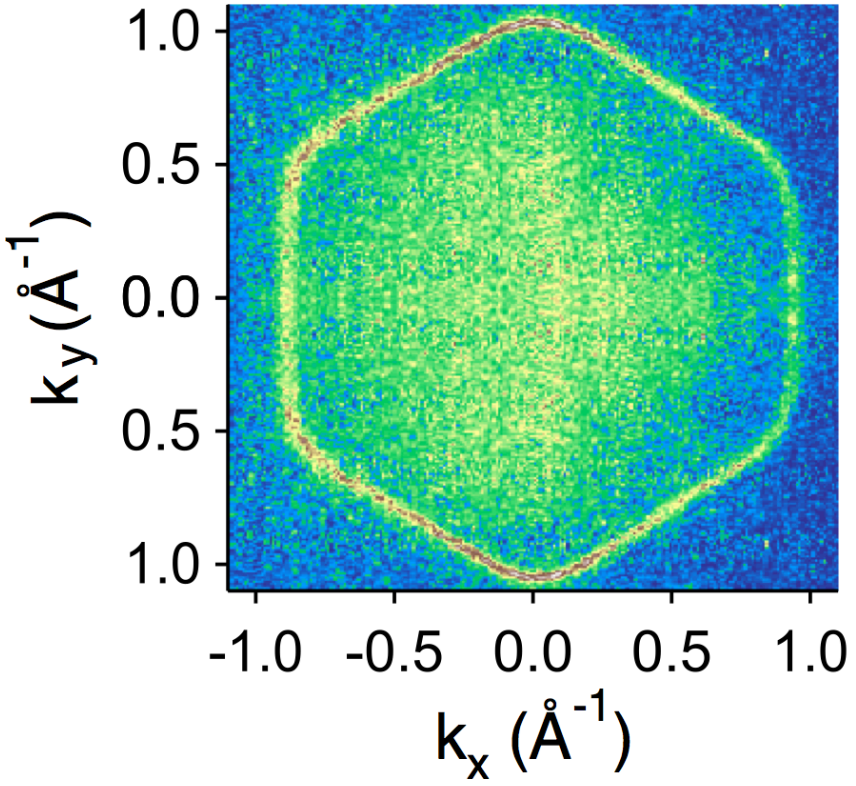
Exponential resistivity at low temperatures



C.W. Hicks, A.S. Gibbs, A.P. Mackenzie, H. Takatsu, Y. Maeno & E.A. Yelland Phys. Rev. Lett. ***109****, 116401 (2012)*

Fermi surface from calculation and ARPES





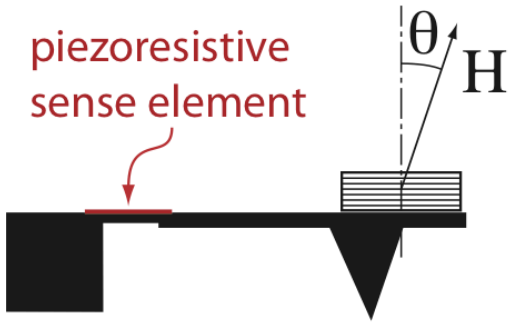
K.P. Ong, J. Zhang, J.S. Tse and P. Wu
Phys. Rev. B ***81****, 115120 (2010)*

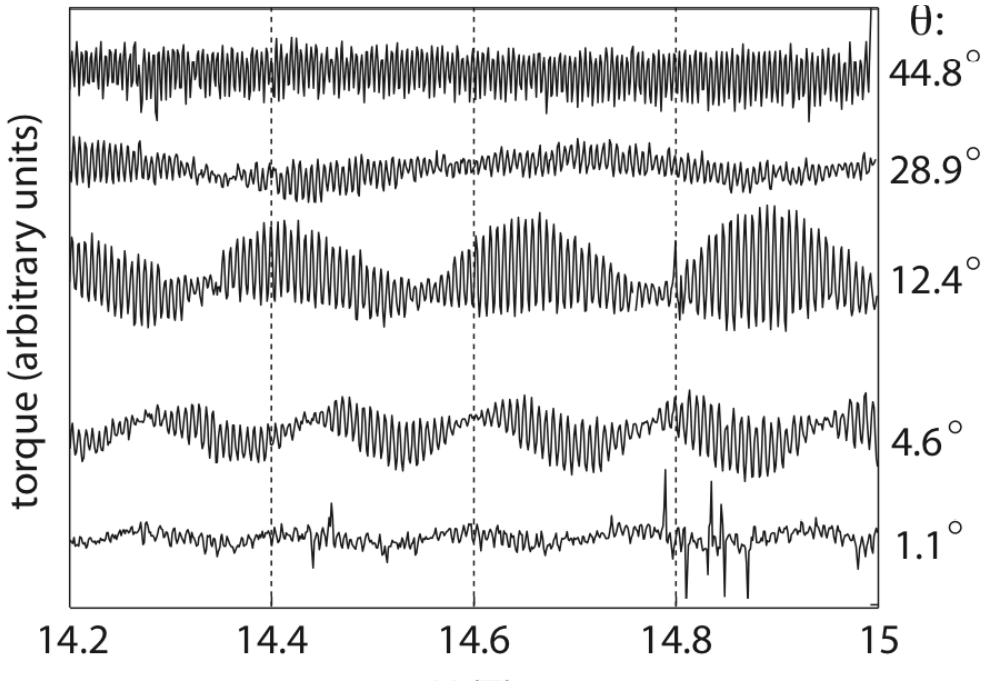
H.J. Noh, J. Jeong, J. Jeong, E.J. Cho,
S.B. Kim, K. Kim, B.I. Min and H.D. Kim,
Phys. Rev. Lett. ***102****, 256404 (2009)*

This looks highly nested – does a density wave form?

Consistent with the de Haas – van Alphen effect

Quantum oscillations by torque magnetometry





Single closed Fermi surface; no observed density wave gapping.

Conduction band clearly has some 5*s* character but enough 4*d* character to give the Fermi surface a hexagonal shape.

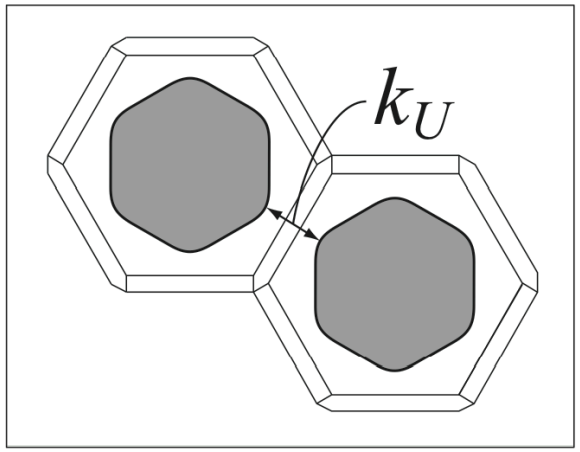
$\overline{k}_F = 0.97$ Å^{-1}

$\overline{m}^* = 1.5\ m_e$

$\overline{v}_F = 7.5 \times 10^5$ ms^{-1}

$T_F = 27000$ K

Could exponential resistivity be due to ‘phonon drag’?



Idea: phonons are dragged along by the electron distribution in an applied electric field at low temperatures, so normal el-ph scattering does not relax momentum.

Electron-phonon Umklapp processes then have an activation temperature $k_B T_U = \hbar c k_U$ where c is the sound velocity.

Estimating c from phonon specific heat and knowing k_U from the Fermi surface gives reasonable agreement between T_U and the measured T_o.

In $PdCoO_2$ below 15K

Electron-impurity processes relax momentum, but slowly: τ_{ei} is approx. 70 ps

Normal el-ph processes should exist, but apparently do not relax momentum

Umklapp electron-phonon processes seem to be entirely frozen out

Umklapp electron-electron processes should exist, but are not visible in ρ_{ab}

Is it possible that if Umklapp el-el processes and el-imp process are slow enough, there could be a crossover from Boltzmann-like T^2 to viscous $1/T^2$ behaviour? This would be a 'hydrodynamic Fermi liquid metal'.

Can other good examples be found of bulk materials in which phonon scattering can be made momentum-conserving like in $PdCoO_2$?

Extremely low carrier density metals might also offer a route to electron hydrodynamics if they can be made with sufficiently high purity.

A.V. Andreev, S.A. Kivelson and B. Spivak, Phys. Rev. Lett. ***106****, 256804 (2011)*

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Summary

1. *T*-linear resistivity can be found in a variety of different circumstances with different microscopic origins but always with a similar scattering rate. This 'universality' is not yet fully understood.

2. Some of these strong-scattering materials might host hydrodynamic fluids; investigation of low-disorder hydrodynamic predictions would be interesting.

3. Under some special circumstances it might be possible to produce 'hydrodynamic Fermi liquid metals' in systems whose momentum-relaxing scattering processes are strongly suppressed.