Low Energy Effective Theories for non-Fermi liquids

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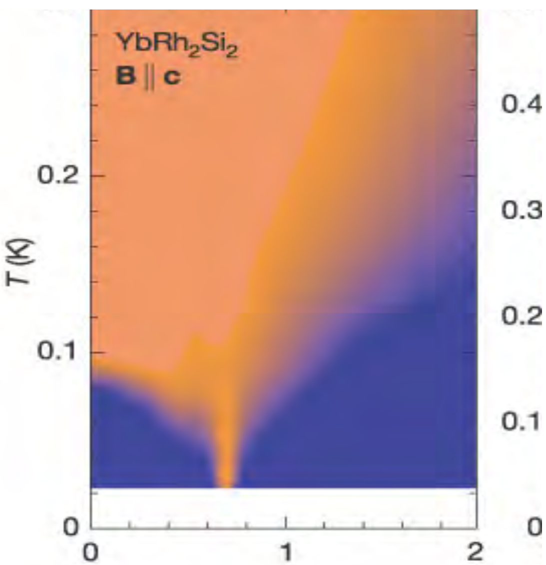
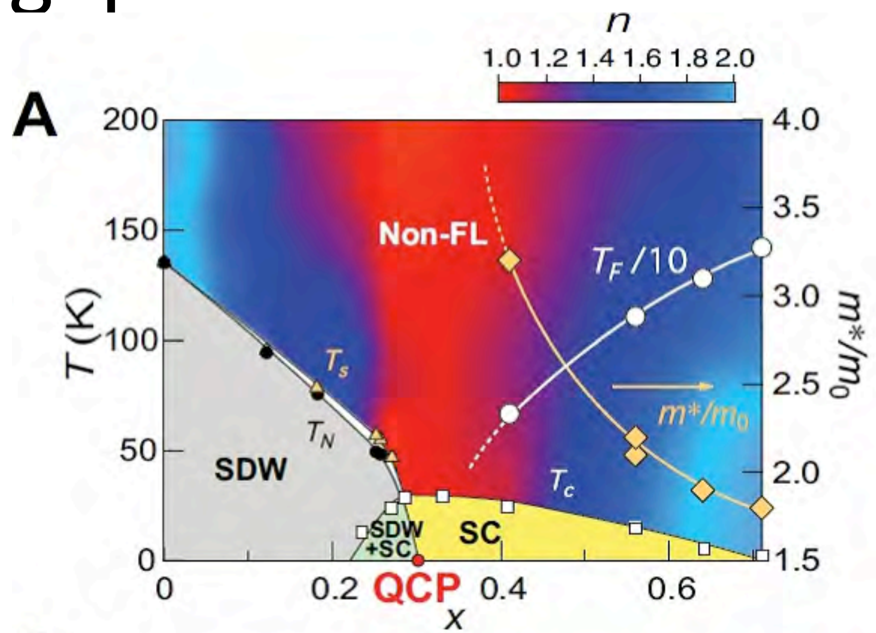
A route to **non-Fermi liquid** :

FS coupled with gapless collective mode



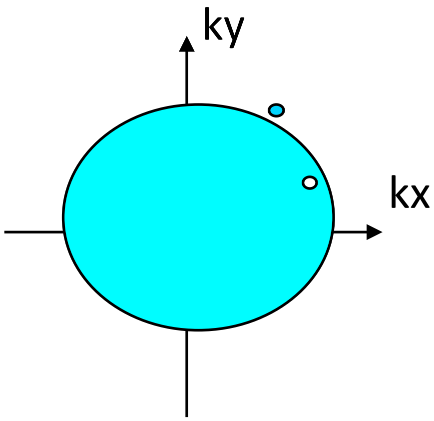
[Custers et al.(2003)]



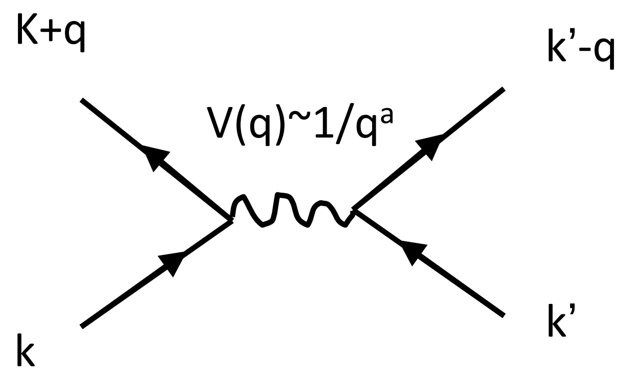
[Hashimoto et al. Science 336, 1554 (2012)]

- QCP in metal (AF, Nematic, CDW, …)
- Bose metal (Quantum spin liquid with spinon FS, …)

Long-range force destroys coherent quasiparticle



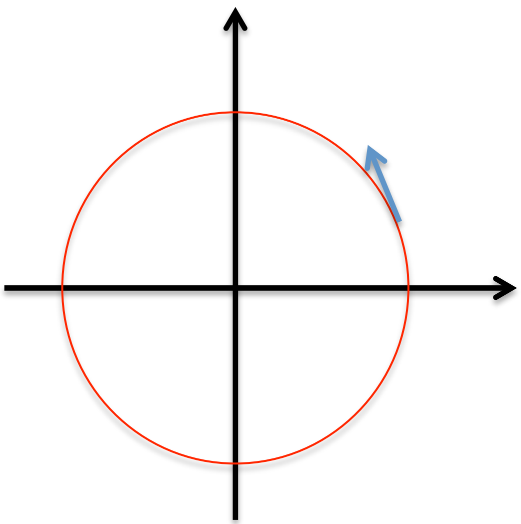
Fermi surface
+ gapless boson

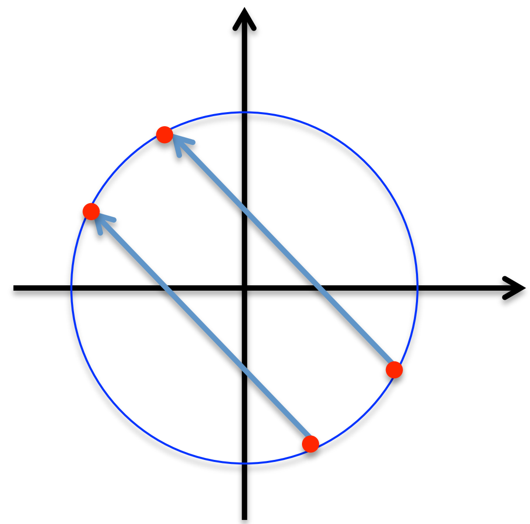


Non-forward scatterings are enhanced by long-range interactions (singular in momentum space) mediated by gapless boson

: bare fermion decays into a complicated superposition of states
single particle is no longer a good basis at low energies

Holy Grail : NFL in d=2





Q=0
Nematic, ferromagnetic QCP
Spin liquids with emergent gauge boson

$Q \neq 0$
Spin & CDW QCP

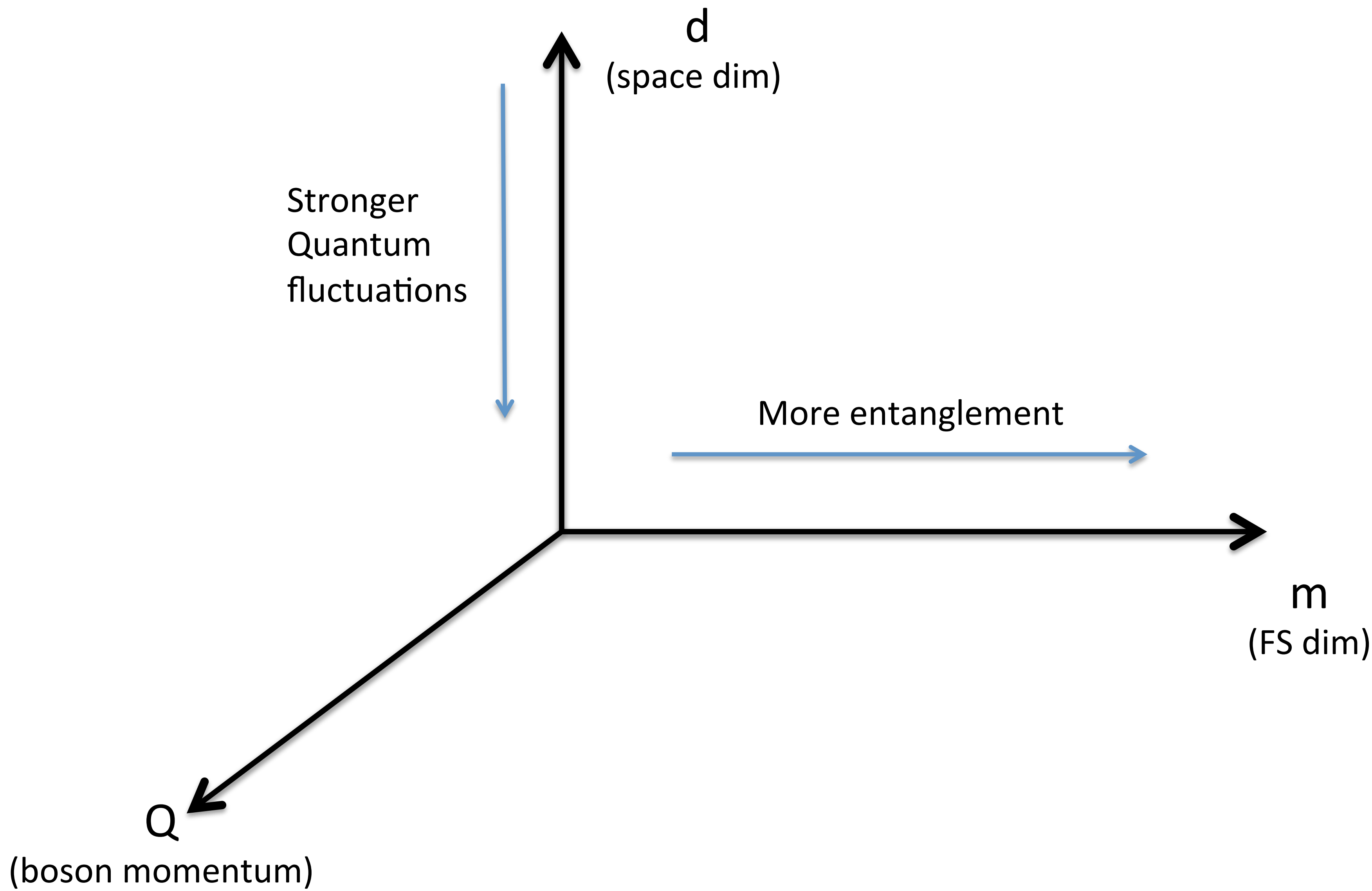
Theoretical status (Q=0)

- Coupling grows strong at low energies [Reizer (89); Nagaosa, Lee (92); Halperin, Lee, Read (93), Polchinski(93); Althsuler,Ioffe,Millis(94); Kim, Furusaki, Lee, Wen (94)]
- Even in the large N limit, the saddle point approximation breaks down [SL(09)]
- Exact scaling known for chiral non-Fermi liquids : genus expansion in large N [Sur, SL (14)]
- In non-chiral theories, even non-planar diagrams become important [Metlitski and Sachdev (10)]
- Perturbatively NFL with dynamical tuning (modified boson dispersion ~ $|k|^{1+\varepsilon}$) [Nayak, Wilczek(94); Mross, McGreevy, Liu, Senthil(10)]
- Perturbative NFL based on dim. Reg. [Dalidovich, SL (13); Mandal, SL(14)]

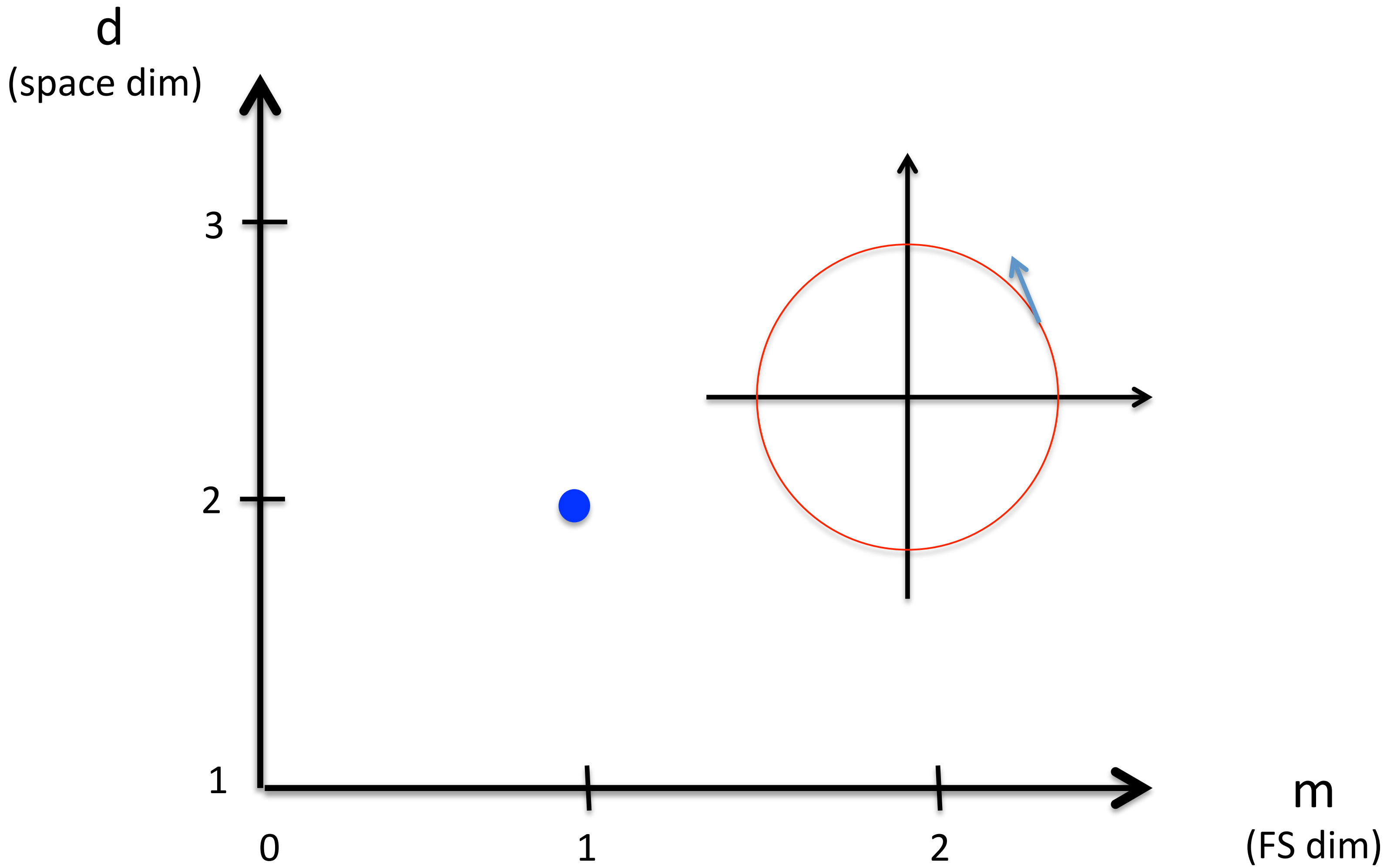
Theoretical status (Q≠0)

- Fermi surfaces tend to get nested, and quasiparticle is destroyed near the hot spots [Abanov, Chubukov]
- The theory flows to strong coupling regime even in the large N limit [Metlitski, Sachdev]
- Without considering flow of velocities, interacting fixed point found at one-loop [J. Lee, Strack, Sachdev]
- Strong SC and CDW fluctuations; some of them are degenerate due to pseudospin symmetry [Metlitski, Sachdev]
- The field theory can be regularized by a sign-problem-free lattice model : QMC shows strong enhancement of d-wave SC at QCP [Berg,Metlitski, Sachdev]
- Perturbative NFL based on dim. Reg. [Sur, SL(14)]

A classification of metal



A classification of metal (Q=0)



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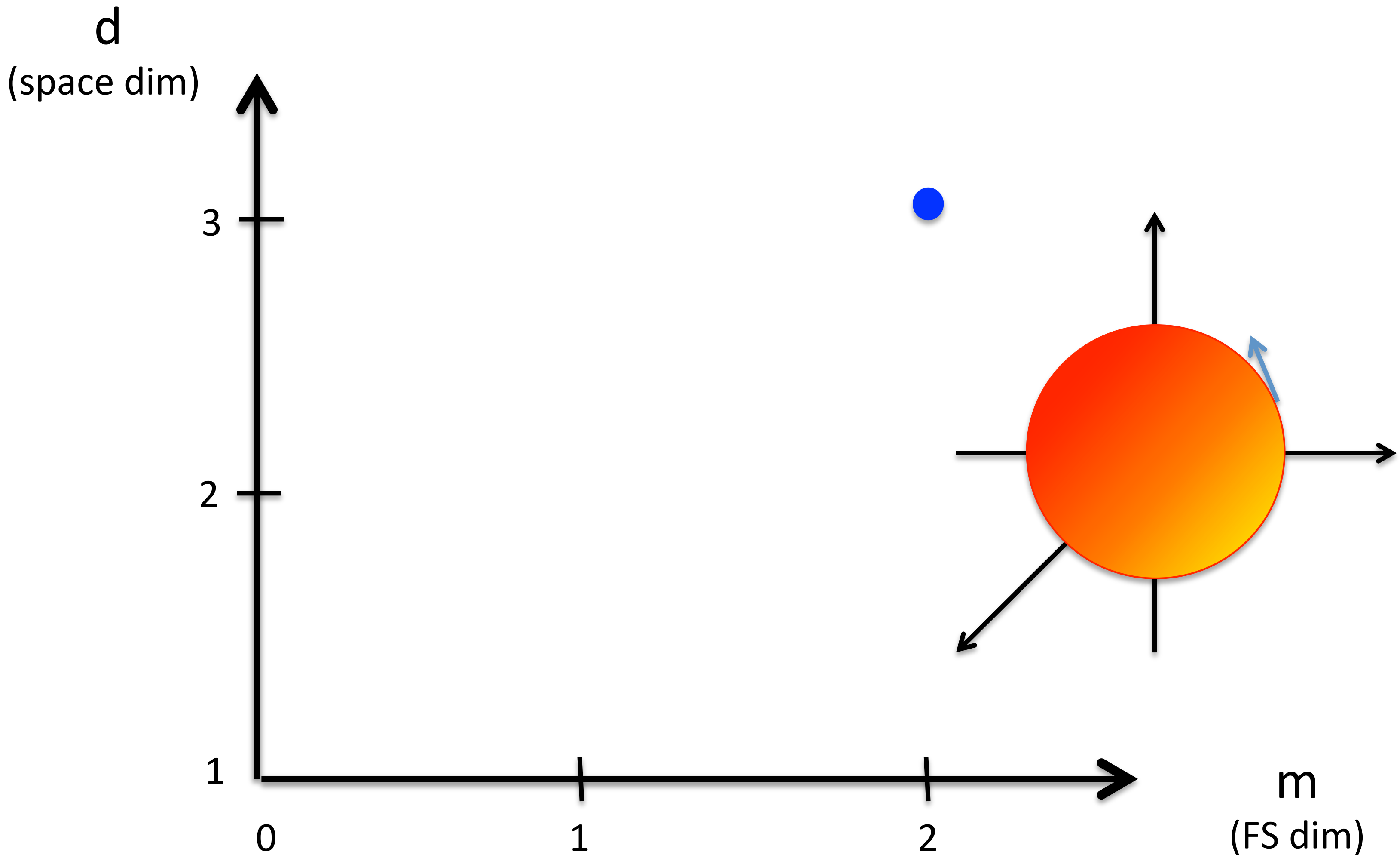

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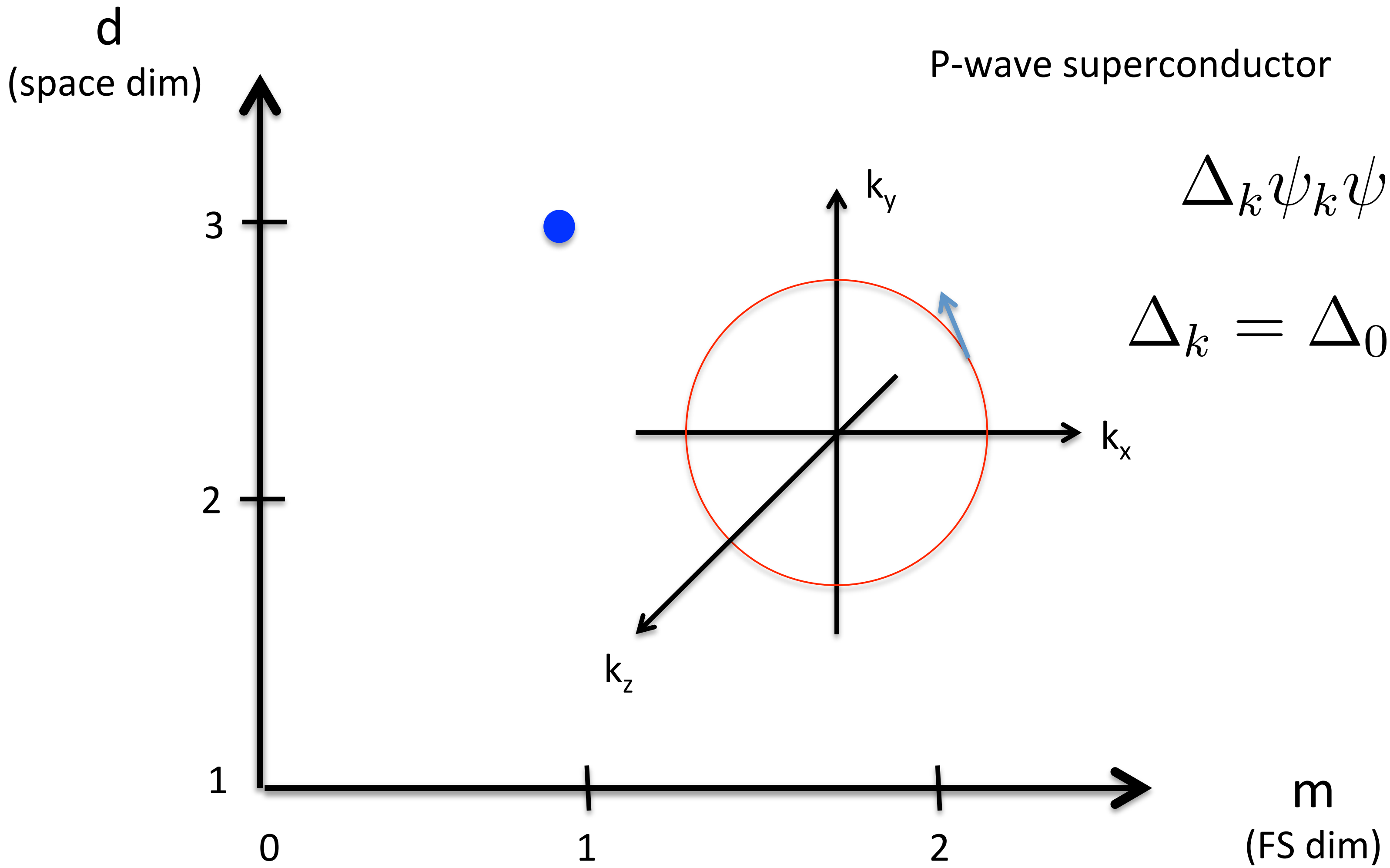
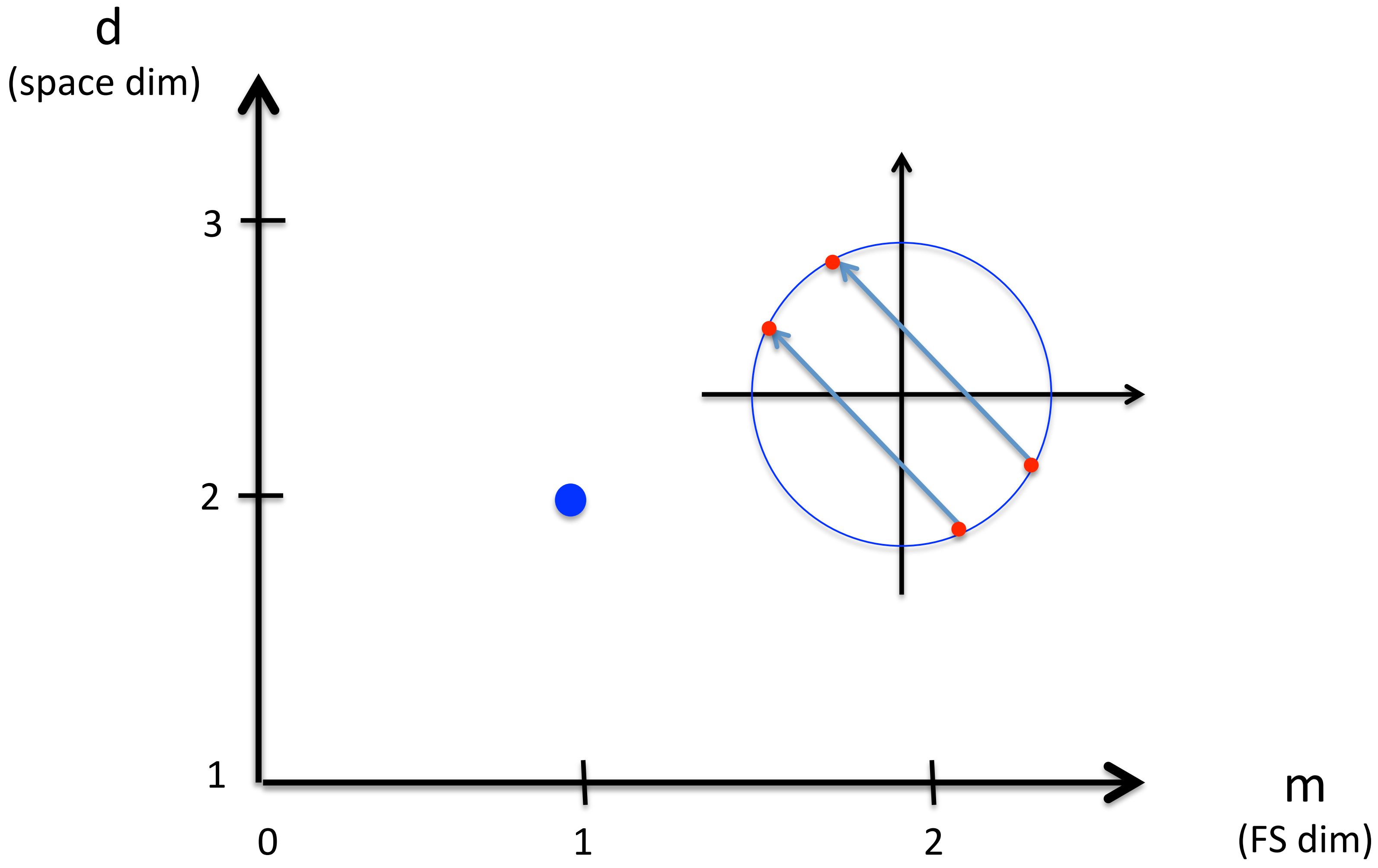


P-wave superconductor

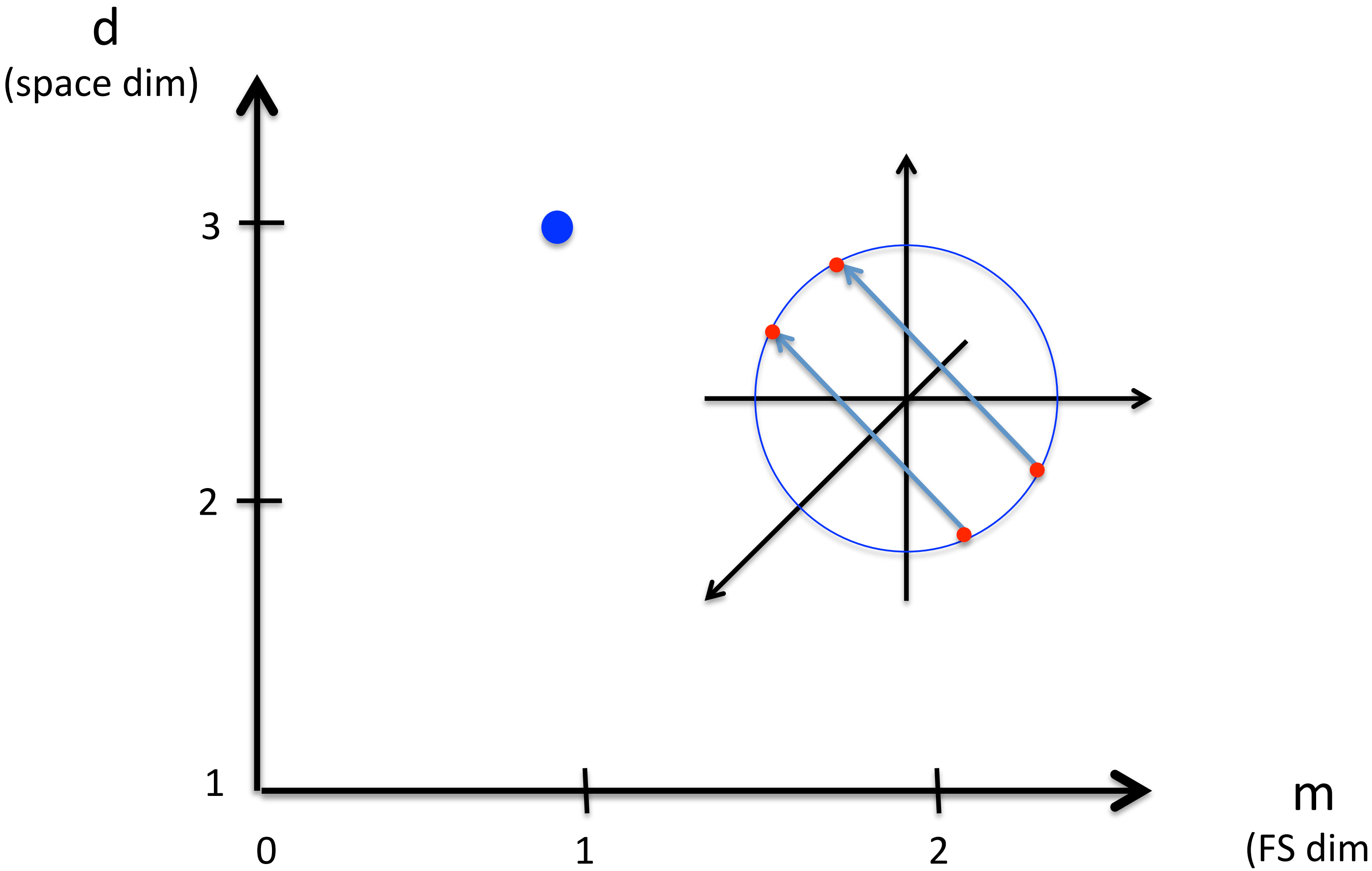
$$\Delta_k \psi_k \psi_{-k}$$

$$\Delta_k = \Delta_0 k_z$$

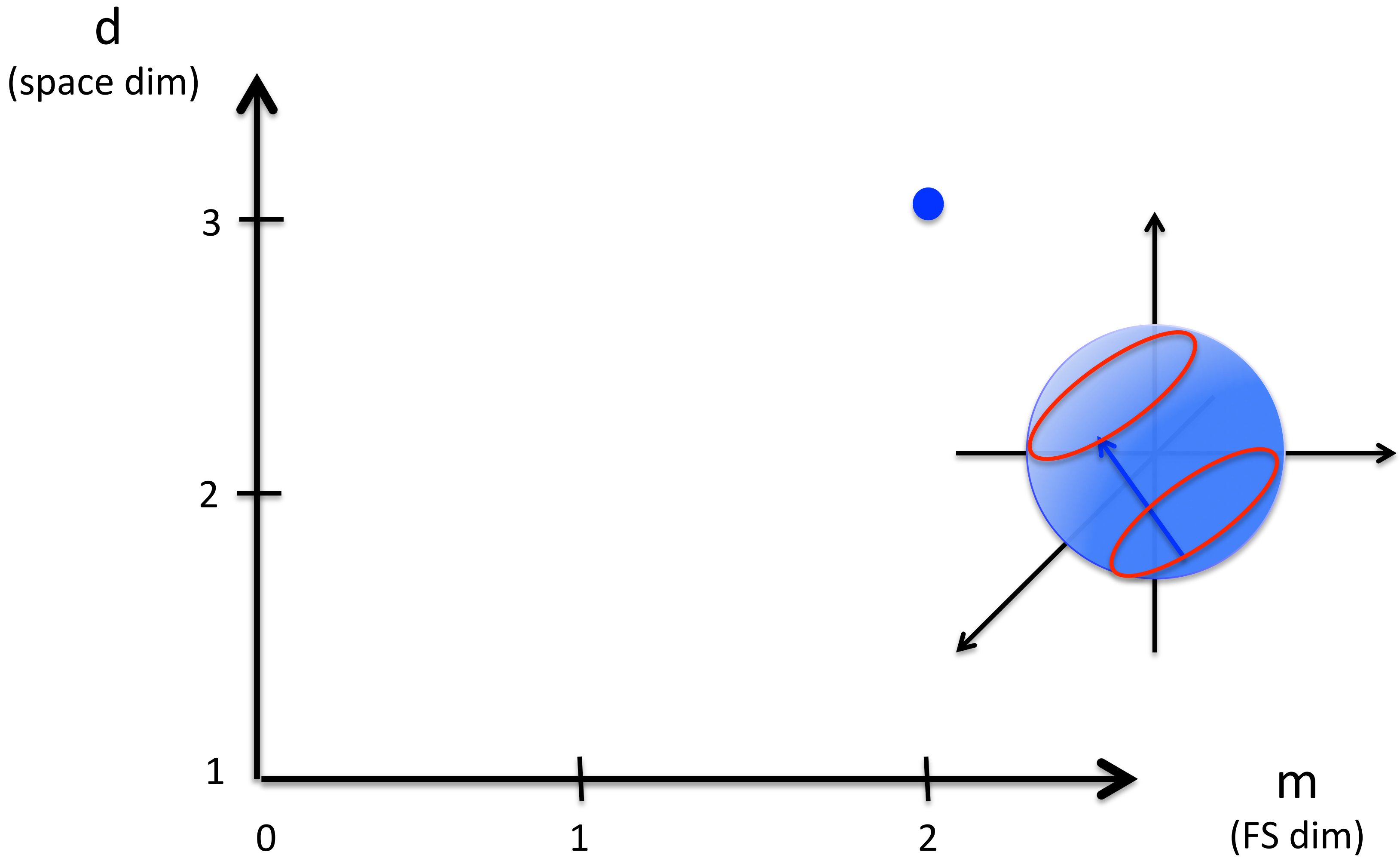
A classification of metal (Q≠0)



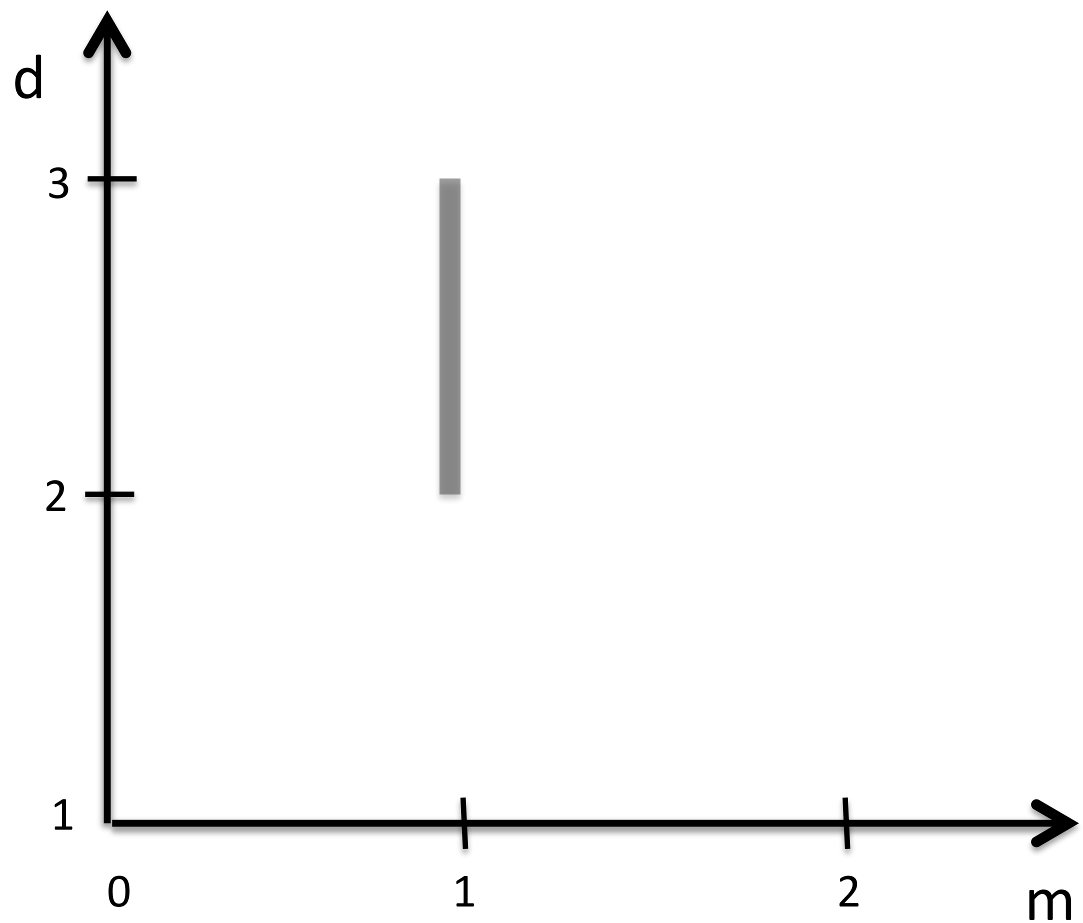
A classification of metal (Q≠0)



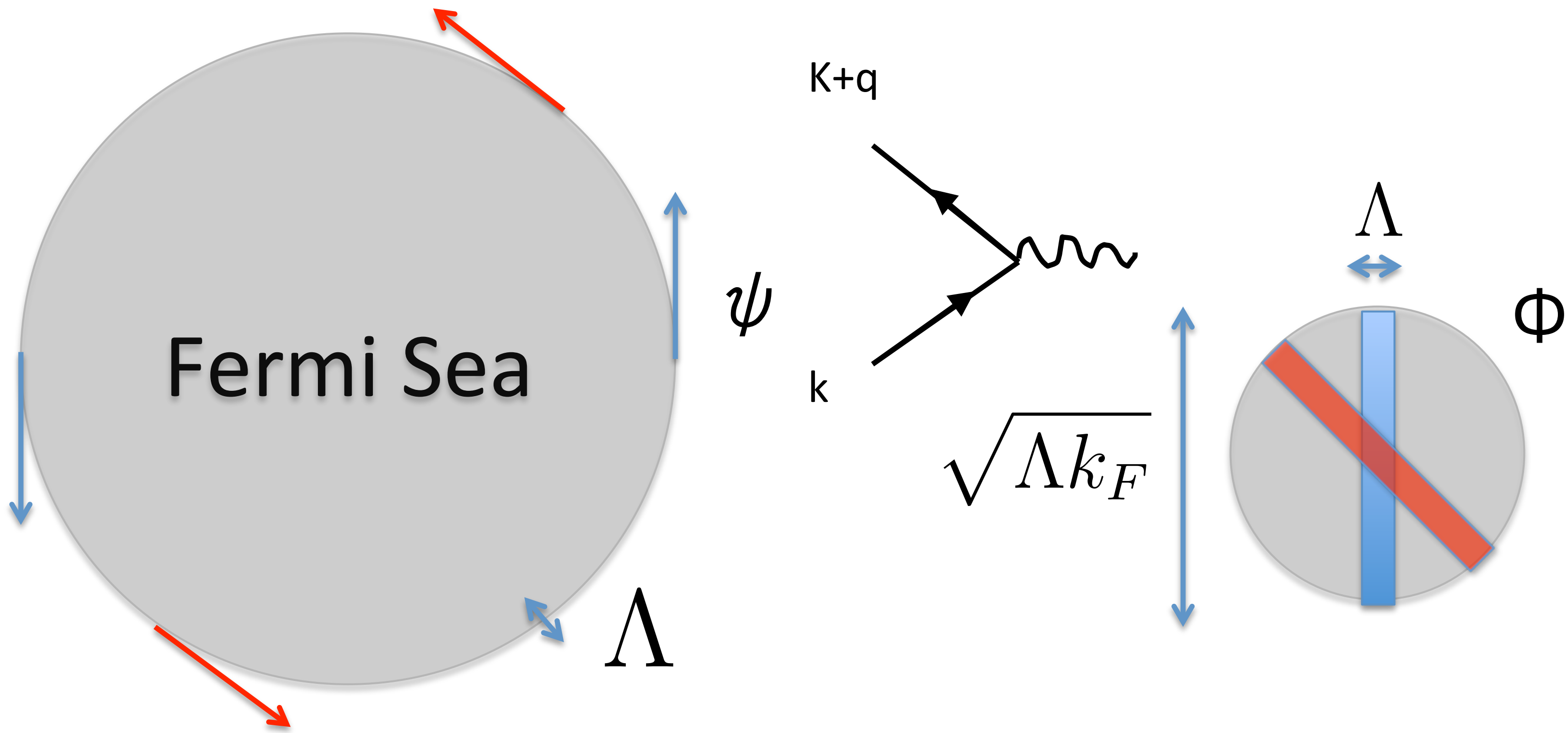
A classification of metal (Q≠0)



m=1, Q=0

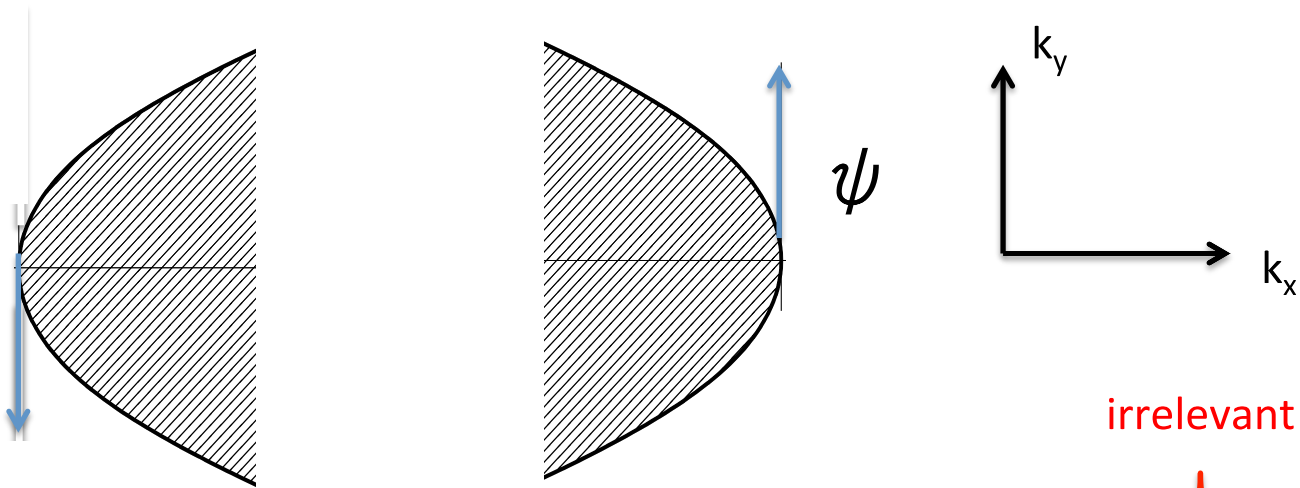


Emergent locality in momentum space for m=1 in any d



- Fermions are primarily scattered along the directions tangential to FS
- At low energies, fermions with different tangential vectors are decoupled from each other in the $\Lambda \rightarrow 0$ limit

Two-patch theory (m=1)

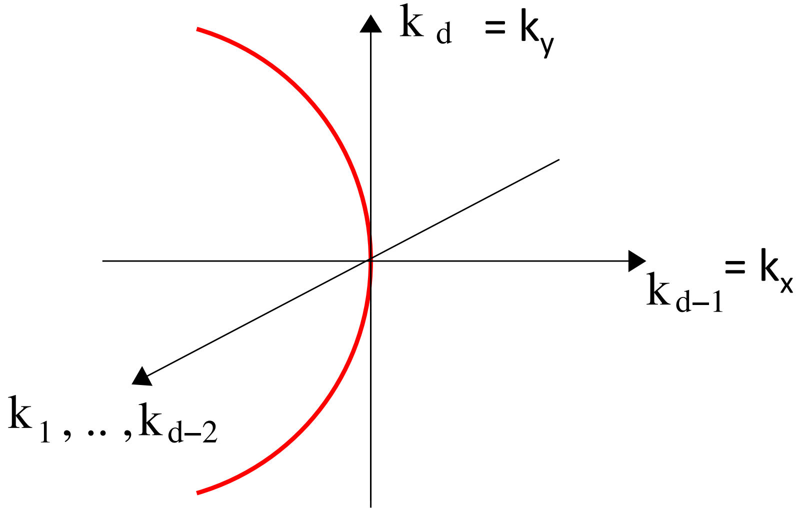


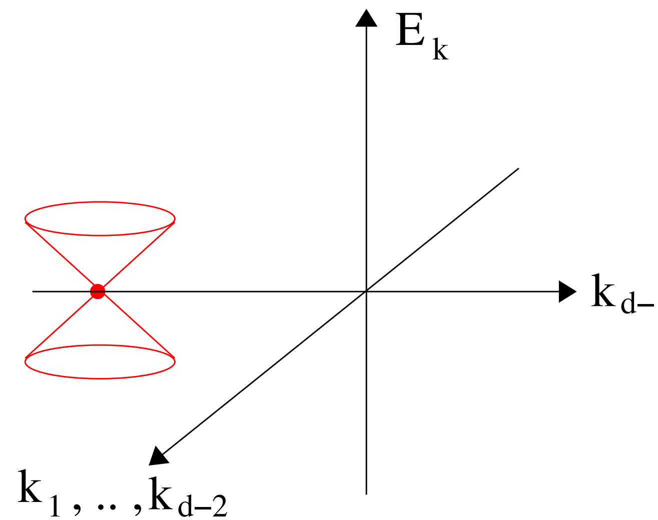
irrelevant

$$S_0 = \sum_{s=\pm,j} \int dk\, \psi^\dagger_{s,j}(k) \left[ik_0 + sk_x + k_y^2 + c_3 k_y^3/\sqrt{k_F} + .. \right]$$

- For those quantities that are local in momentum space (such as Green's function), the size of Fermi surface (k_F) does not enter
- Patch description with non-compact FS

1-dimensional line of Dirac points embedded in d-dimensional k-space

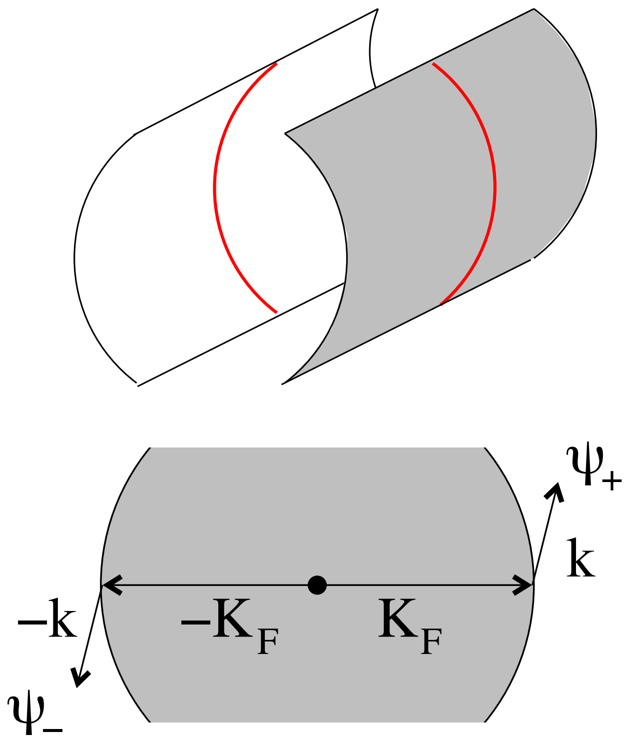


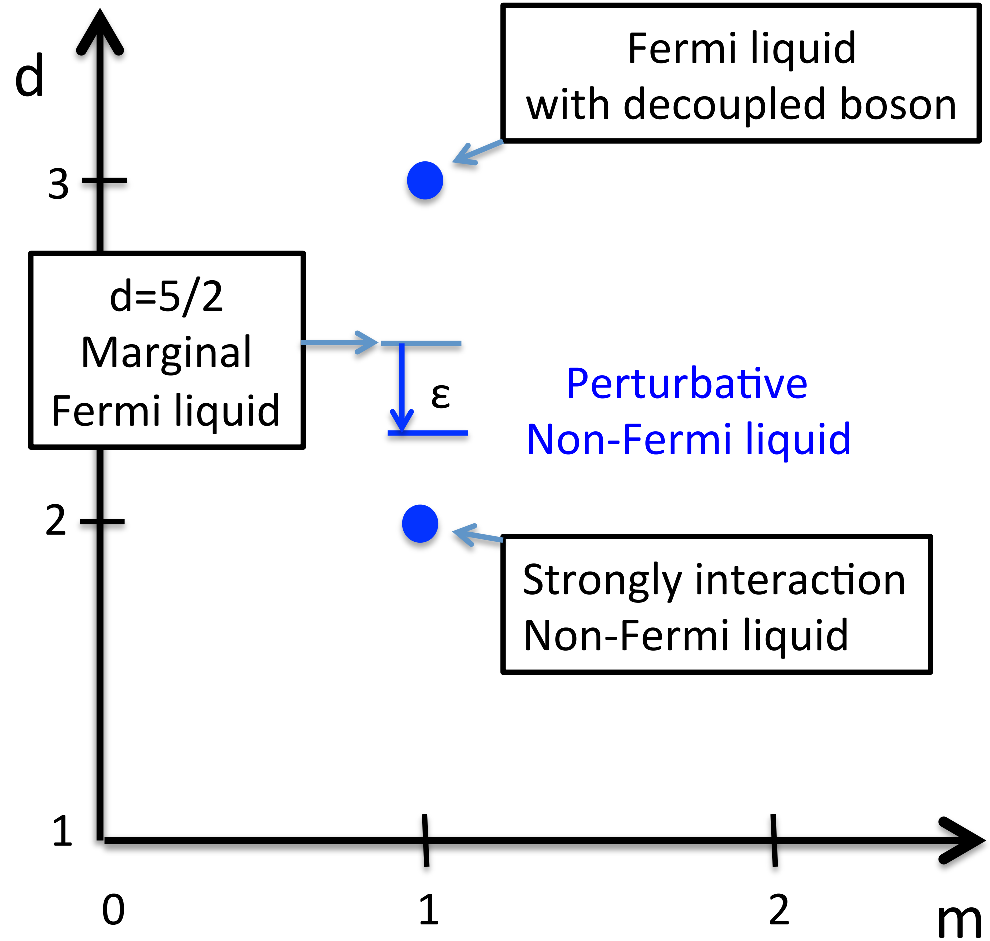


$$E_k = \pm\sqrt{k_1^2 + k_2^2 + .. + k_{d-2}^2 + \delta_k^2}$$

Non-local (in real space) implementation [Senthil & Shankar (09)]
Local (in real space) implementation : [Dalidovich & SL (13)]

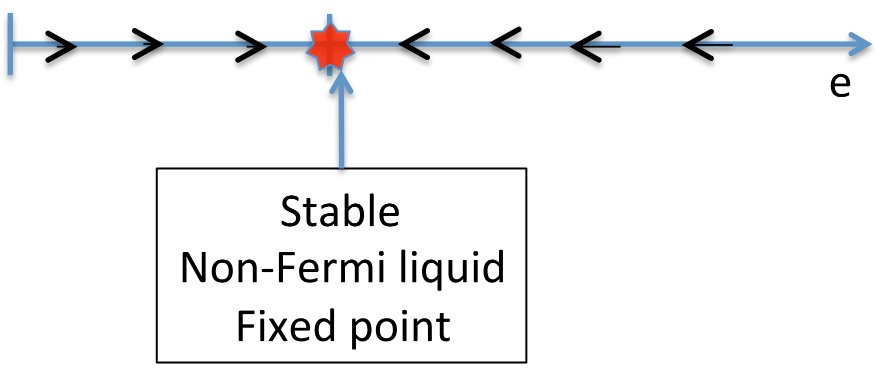
A continuous interpolation between 2d Fermi surface to 3d p-wave SC





Two-loop results

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} + 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$



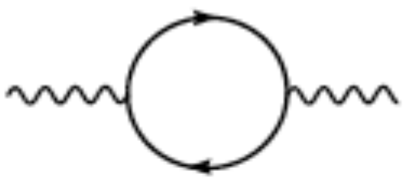
$$\frac{e^{*4/3}}{N} = 11.417\epsilon + 55.498\epsilon^2$$

$$z = \frac{3}{3-2\epsilon}$$

[D. Dalidovich, SL (13)]

Expansion in $e^{4/3}$ instead of e^2

$$\frac{de}{dl} = \frac{\epsilon}{2}e - 0.02920\left(\frac{3}{2}-\epsilon\right)\frac{e^{7/3}}{N} + 0.01073\left(\frac{3}{2}-\epsilon\right)\frac{e^{11/3}}{N^2}$$



$$D_1(k) = \frac{1}{\boxed{|\vec{K}|^2 + k_x^2} + k_y^2 + \beta_d e^2 \frac{|\vec{K}|^{d-1}}{|k_y|}}$$

irrelevant

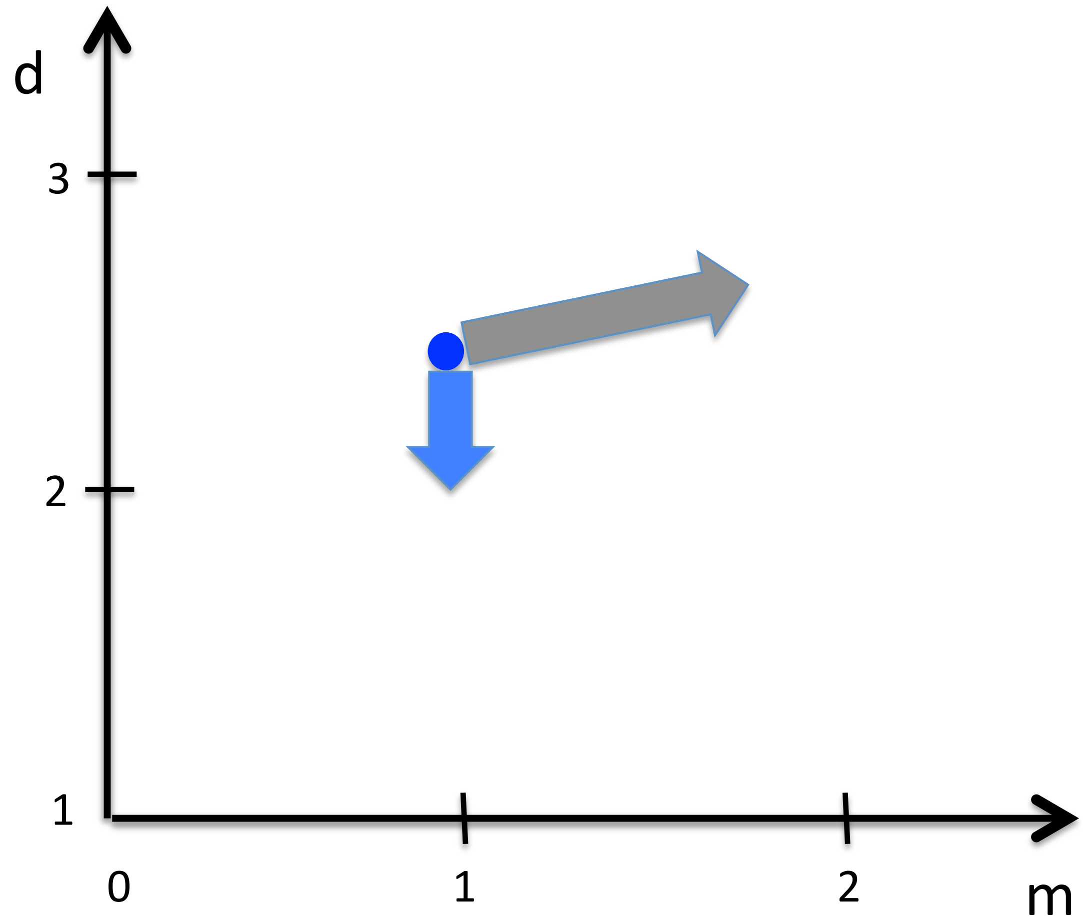
$$\vec{K} = k_0, k_1, .., k_{d-2}$$

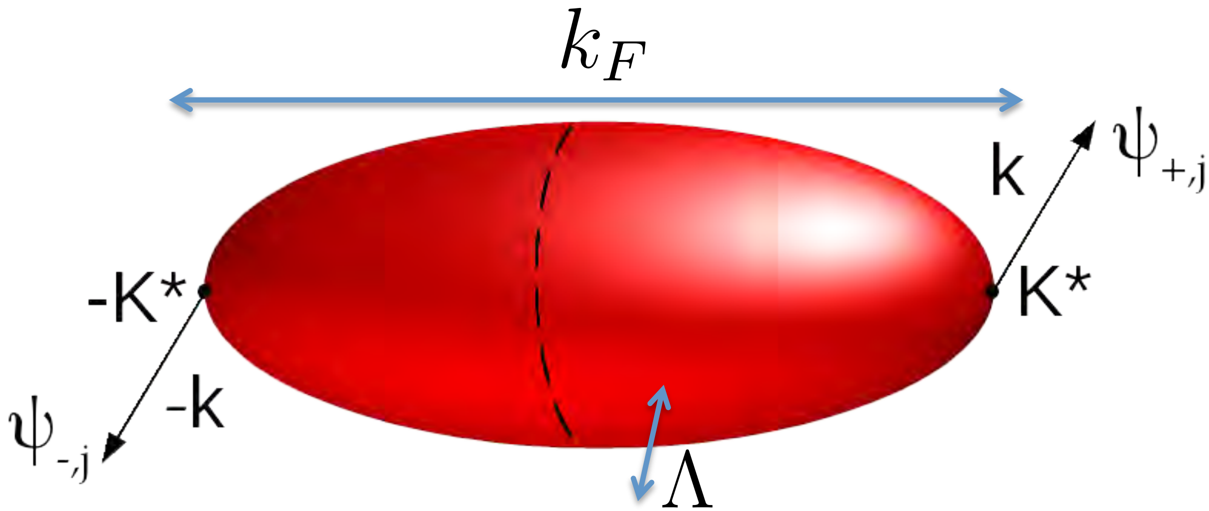
Landau damping, which is generated by interaction, dominates over the bare kinetic term

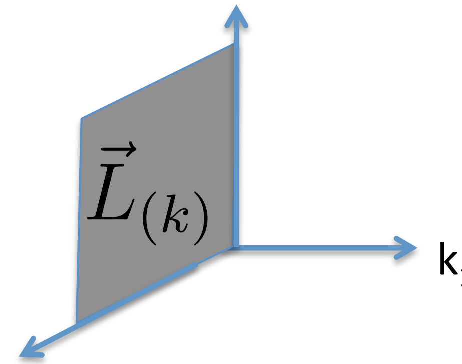
Physical properties

- Fermion Green fnt : $$G(k) = \frac{1}{|\delta_k|^{1-0.1508\epsilon^2}} \; g\left(\frac{|\vec{K}|^{1/z}}{\delta_k}\right)$$
- Boson Green fnt : $$D(k) = \frac{1}{k_d^2} \; f\left(\frac{|\vec{K}|^{1/z}}{k_d^2}\right)$$
- Specific heat : $$c \sim T^{(d-2)+\frac{1}{z}}$$
- Magnetic susceptibility : $$\chi_{ss} \sim T^{(d-1)-\frac{1}{z}}$$ $$\chi_{aa} \sim T^{(d-3)+\frac{1}{z}}$$

m>1, Q=0





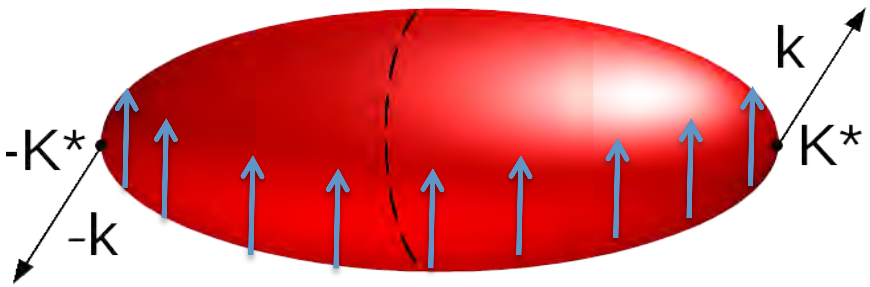


$$
\begin{aligned}
S \;=\; & \sum_{s=\pm,j} \int dk\, \psi^{\dagger}_{s,j}(k) \left[ik_0 + sk_1 + \vec{L}^2_{(k)} + H(\vec{L}^2_{(k)}) \right] \psi_{s,j}(k) \\
& + \frac{1}{2} \int dk \left[k_0^2 + k_1^2 + \vec{L}^2_{(k)} \right] \phi(-k)\, \phi(k) \\
& + \frac{1}{\sqrt{N}} \sum_{s=\pm,j} \int dk\, dq \; e_s \phi(q) \; \psi^{\dagger}_{s,j}(k+q)\, \psi_{s,j}(k)
\end{aligned}
$$

Irrelevant
By power counting

$$
H(\vec{L}^2_{(k)}) = \sum_{n=3}^{\infty} \sum_{i_1,..,i_n=2}^{d} \frac{c_{i_1,..,i_n}}{k_F^{\frac{n-2}{2}}} k_{i_1} .. k_{i_n}
$$

UV Sensitivity



$$D_1(k) = \frac{1}{\vec{L}_{(k)}^2 + \beta_d \, e^2 \, \mu^x \, \frac{(\mu \, \tilde{k}_F)^{\frac{m-1}{2}} \, |\vec{K}|^{d-m}}{|\vec{L}_{(k)}|}}$$

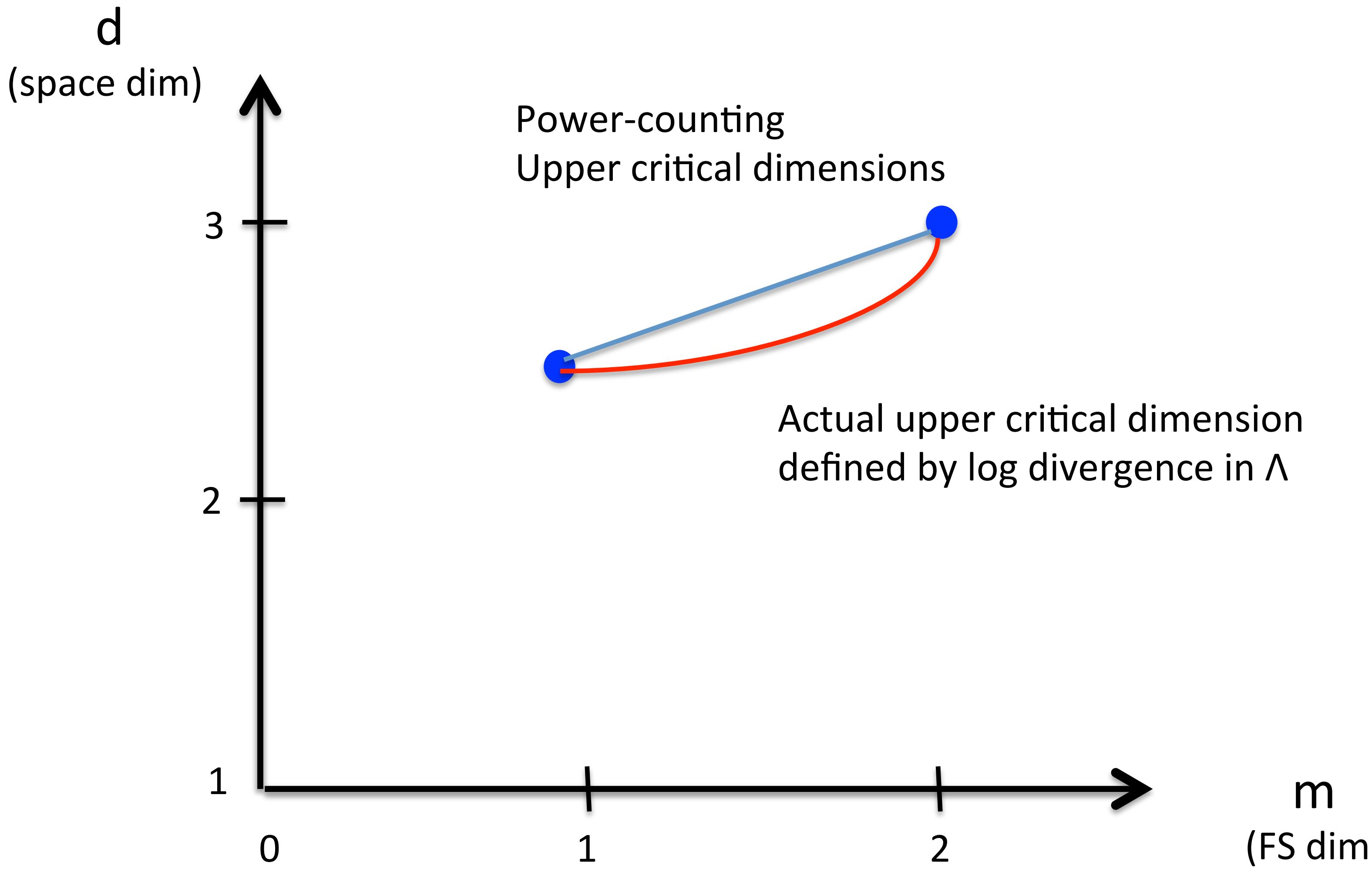
- Landau damping is divergent in the large k_F limit for m>1
- The `UV' divergence can not be subtracted by a local counter term

The Landau damping can be ignored over a finite energy window which can be made parametrically large in the presence of a large number of flavor of boson [Fitzpatrick, Kachru, Kaplan, Raghu (13)]
However, the system eventually flows to a regime dictated by the Landau damping for any fixed number of flavors

UV/IR mixing

- Modes with large momenta singularly affect low energy physics in the large k_F limit
- Low energy effective theory can not be specified without specifying the size/shape of FS
- Low energy limit and $k_F \to \infty$ limit do not commute
- k_F should be treated as a dimensionful `coupling' instead of UV cut-off
- Naïve patch scaling is `dressed' by k_F

Breakdown of patch scaling



Breakdown of patch scaling

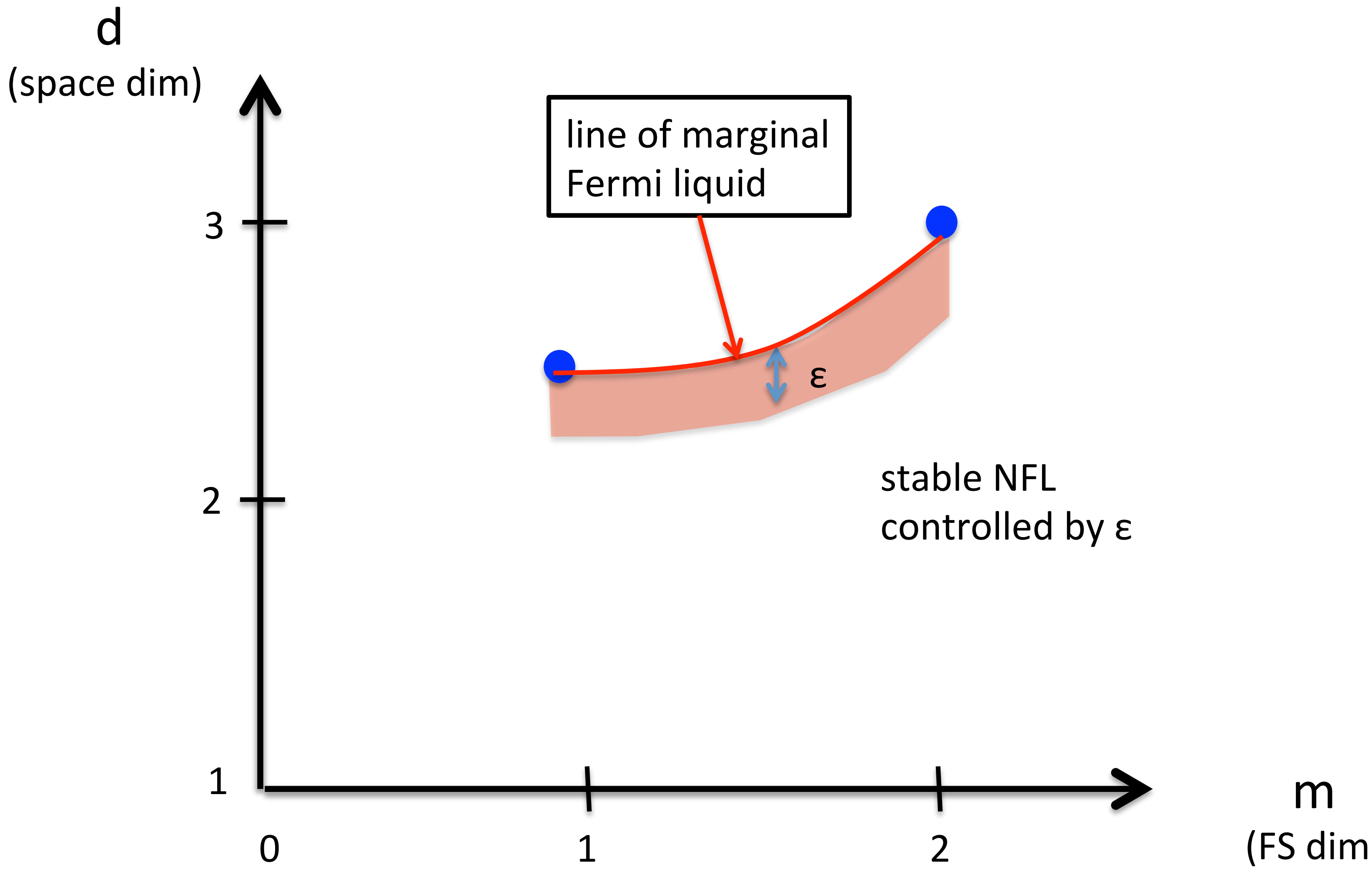
- Below the power counting upper critical dimension, the Yukawa coupling runs to infinity
- However, k_F also runs to infinity with increasing screening
- The actual perturbation series is controlled by a new parameter

$$\tilde{e} \equiv \frac{e^{2(m+1)/3}}{\tilde{k}_F^{\frac{(m-1)(2-m)}{6}}}$$

which flows to a finite value controlled by the distance from the actual upper critical dimension

[I. Mandal, SL (14)]

Breakdown of patch scaling



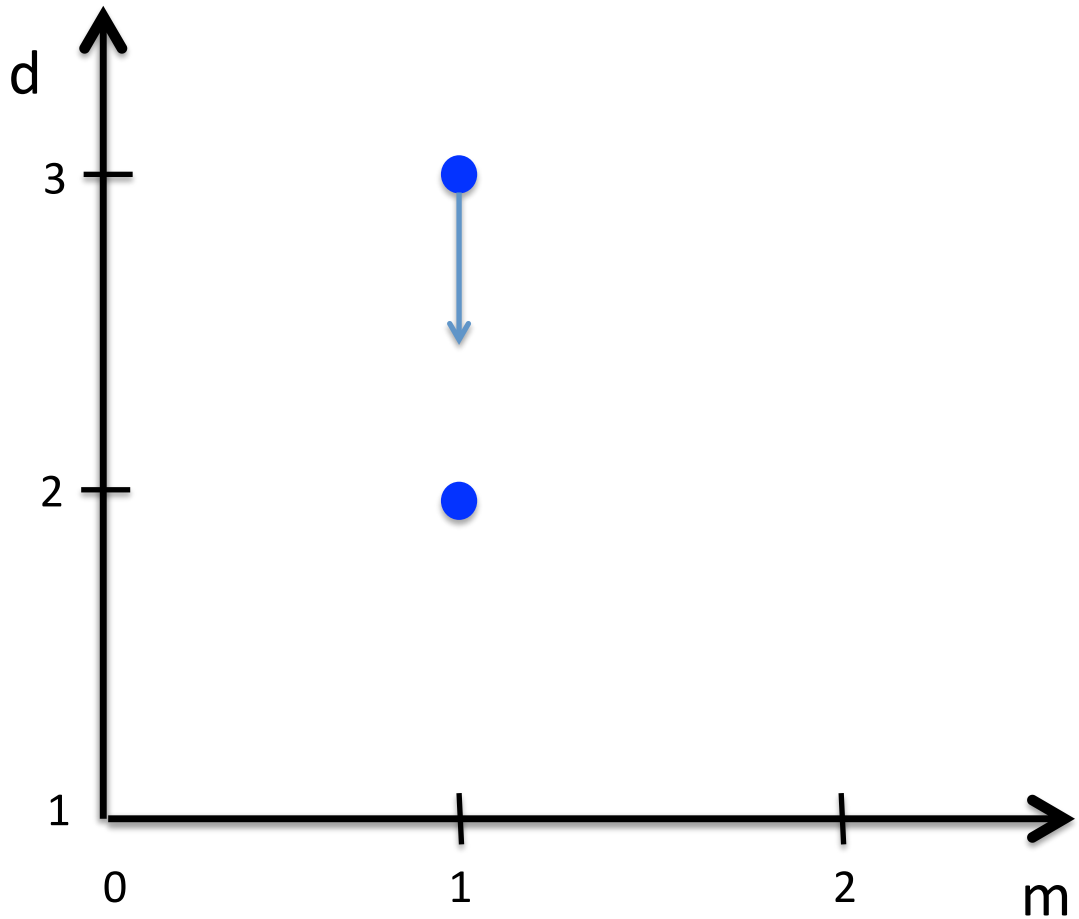
Correlation functions

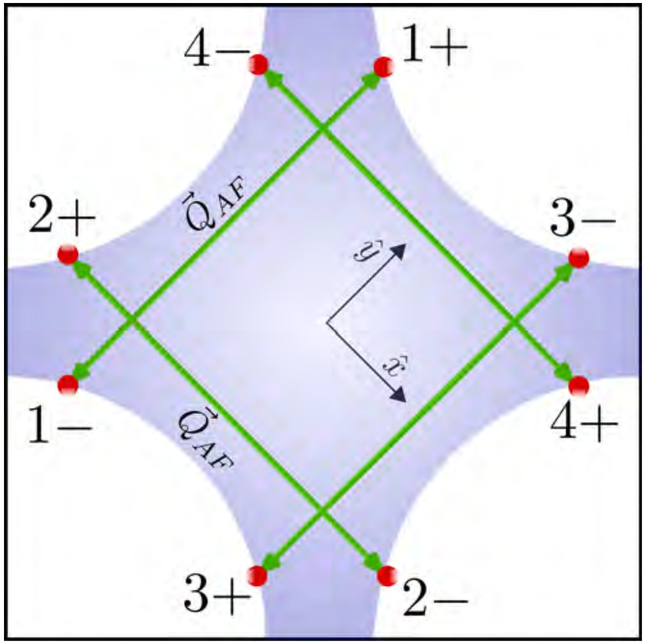
$$D = \frac{1}{\left(\vec{L}_{(k)}^2\right)^{2\Delta_\phi}} f_D\left(\frac{|\vec{K}|^{1/z^*}}{\vec{L}_{(k)}^2}, \frac{k_{d-m}}{k_F}, \frac{\vec{L}_{(k)}^2}{k_F}\right),$$

$$G = \frac{1}{|\delta_k|^{2\Delta_\psi}} f_G\left(\frac{|\vec{K}|^{1/z^*}}{\delta_k}, \frac{\delta_k}{k_F}, \frac{\vec{L}_{(k)}^2}{k_F}\right),$$

- Because of $k_{F,}$ boson and fermion appear to have different dynamical critical exponents
- However, there is only one dynamical critical exponent once running of k_F is taken into account

$m=1,\ Q\neq 0$





Minimal Theory for SDW in 2d

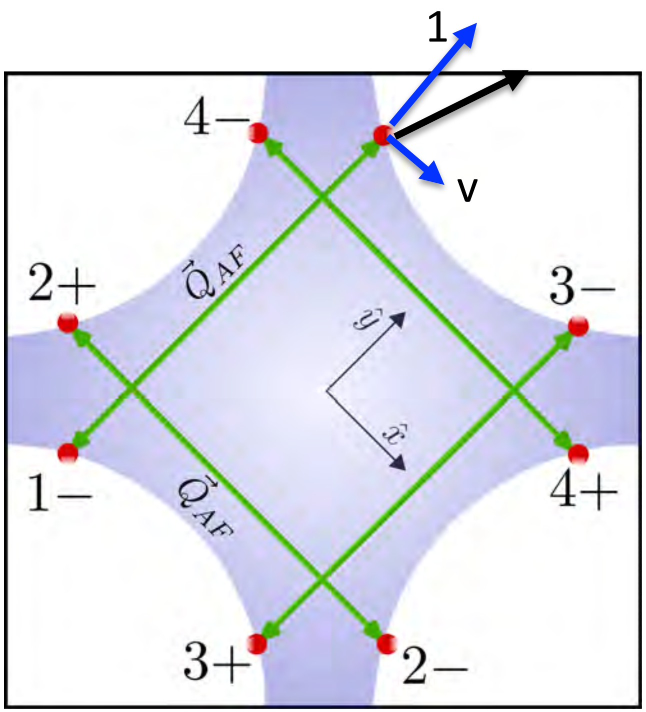
[Abanov, Chubukov]

$$e_1^{\pm}(\vec{k}) = -e_3^{\pm}(\vec{k}) = vk_x \pm k_y$$

$$e_2^{\pm}(\vec{k}) = -e_4^{\pm}(\vec{k}) = \mp k_x + vk_y$$

$$\begin{aligned}
\mathcal{S} = & \sum_{l=1}^{4} \sum_{m=\pm} \sum_{\sigma=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \, \psi_{l,\sigma}^{(m)*}(k) \left[ik_0 \; + e_l^m(\vec{k}) \right] \psi_{l,\sigma}^{(m)}(k) \\
& + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \left[q_0^2 + c^2 |\vec{q}|^2 \right] \vec{\Phi}(-q) \cdot \vec{\Phi}(q) \\
& + g_0 \sum_{l=1}^{4} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(q) \cdot \psi_{l,\sigma}^{(+)*}(k+q) \vec{\tau}_{\sigma,\sigma'} \psi_{l,\sigma'}^{(-)}(k) + c.c. \right] \\
& + \frac{u_0}{4!} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left[\vec{\Phi}(k_1+q) \cdot \vec{\Phi}(k_2-q) \right] \left[\vec{\Phi}(k_1) \cdot \vec{\Phi}(k_2) \right]
\end{aligned}$$

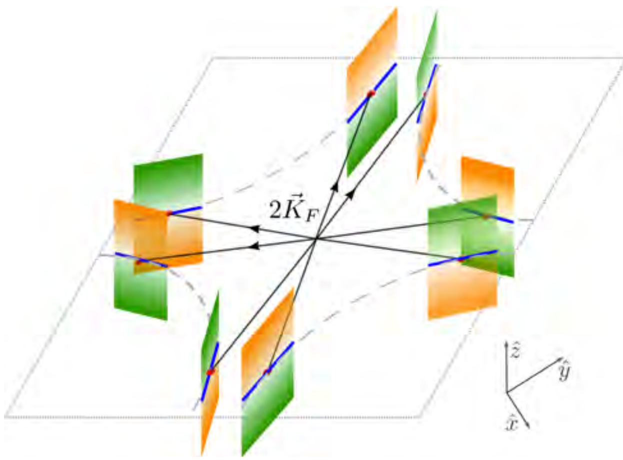
Parameters of the theory

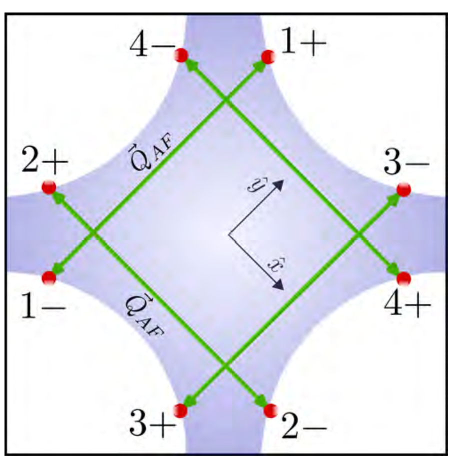


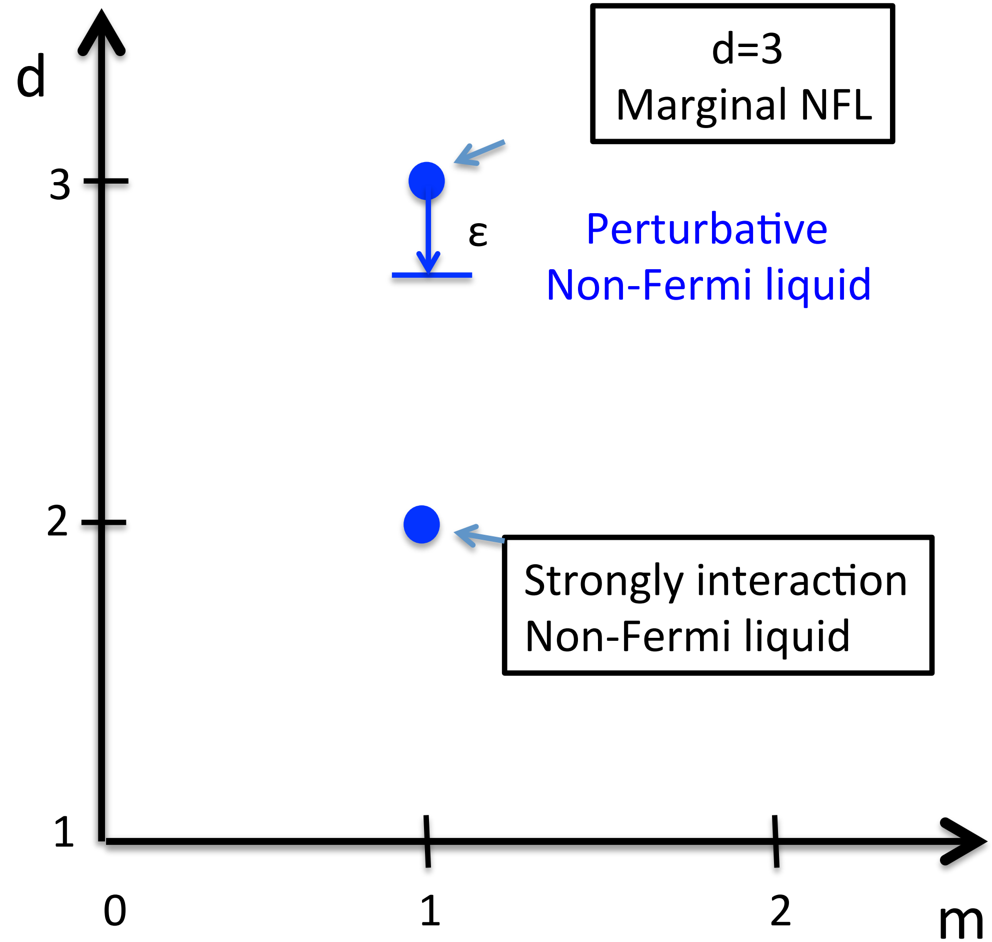
- v : Fermi velocity perpendicular to Q_{AF}
- c : boson velocity
- g : Yukawa coupling
- u : quartic boson coupling

- If v=0, hot spots connected by Q_{AF} are nested
- The four parameters can not be scaled away

A continuous interpolation between 2d Fermi surface and 3d metal with line nodes

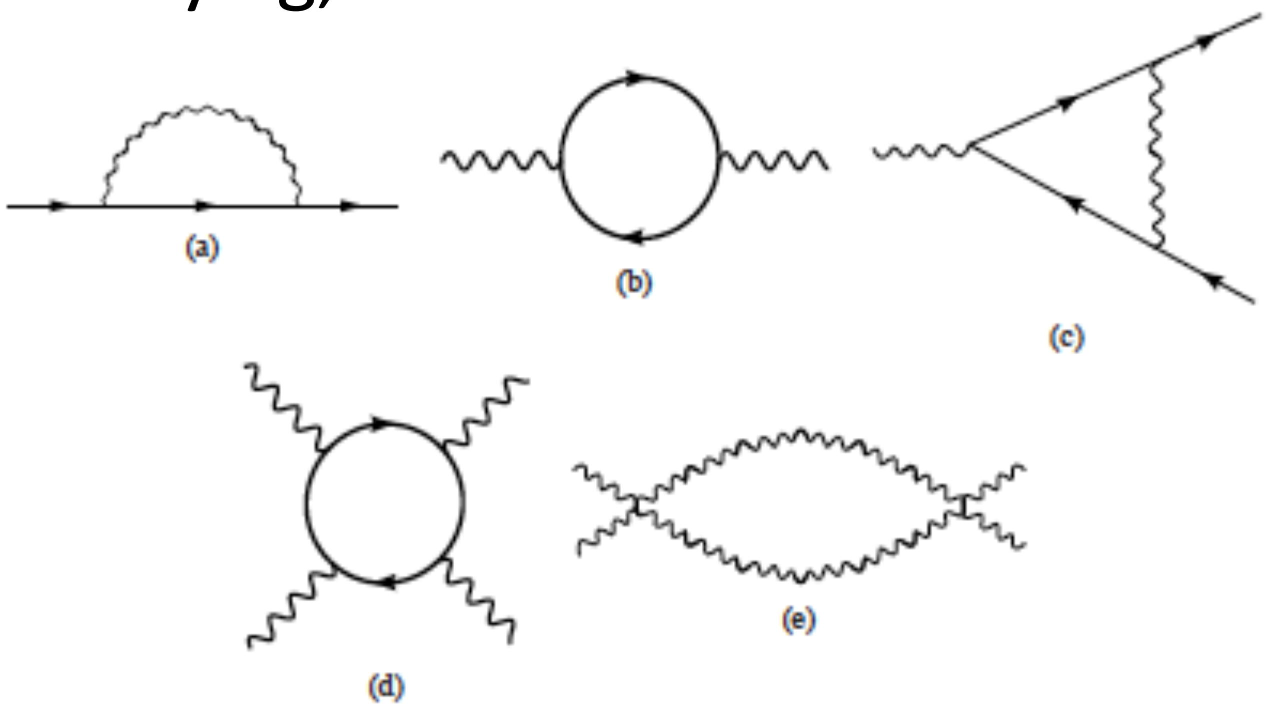




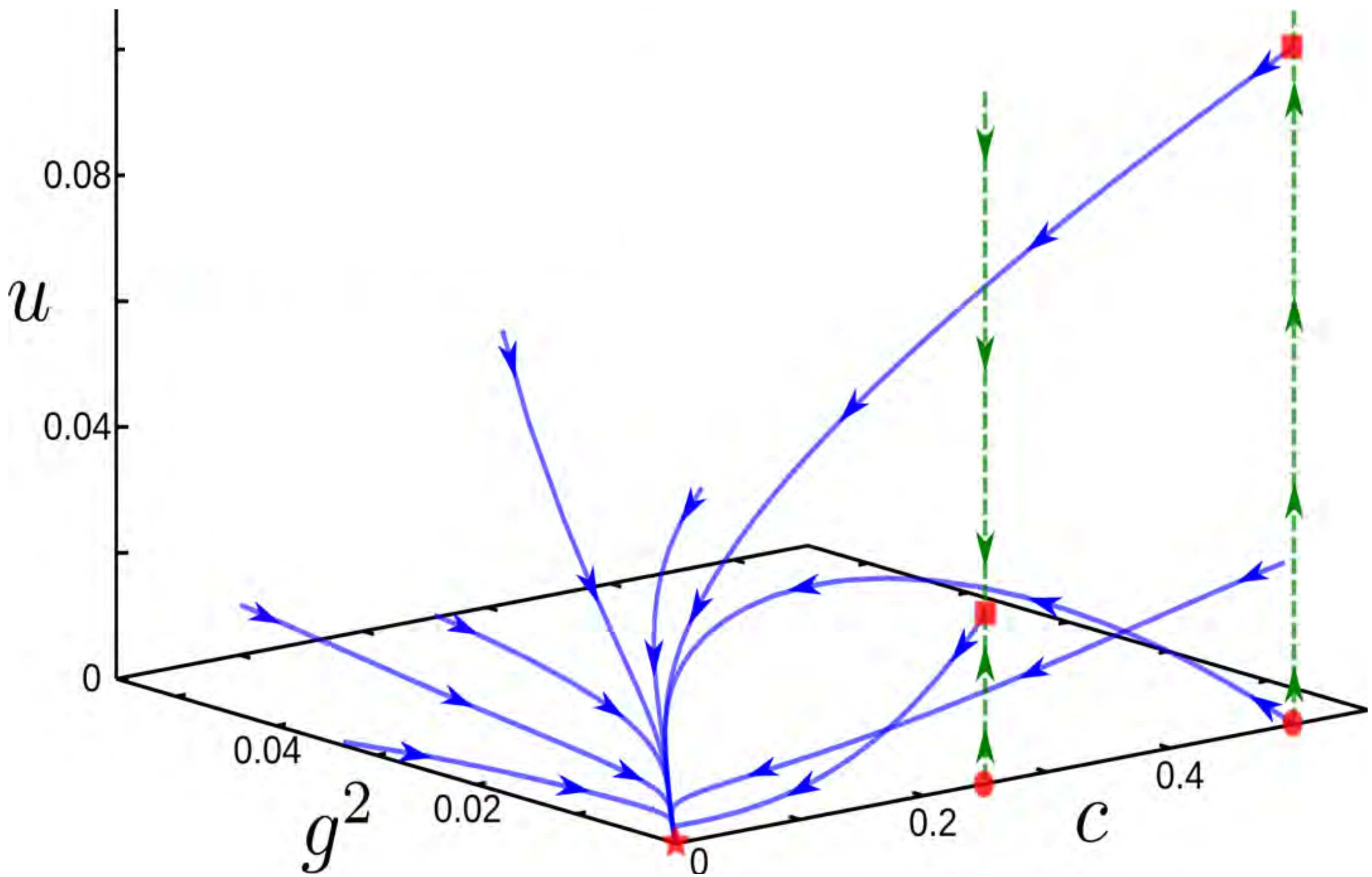


One-loop RG flow

- Yukawa coupling induces nesting : v ➘
- Nesting makes boson slower : c ➘
- Nested FS and slow boson screen more efficiently : g, u ➘

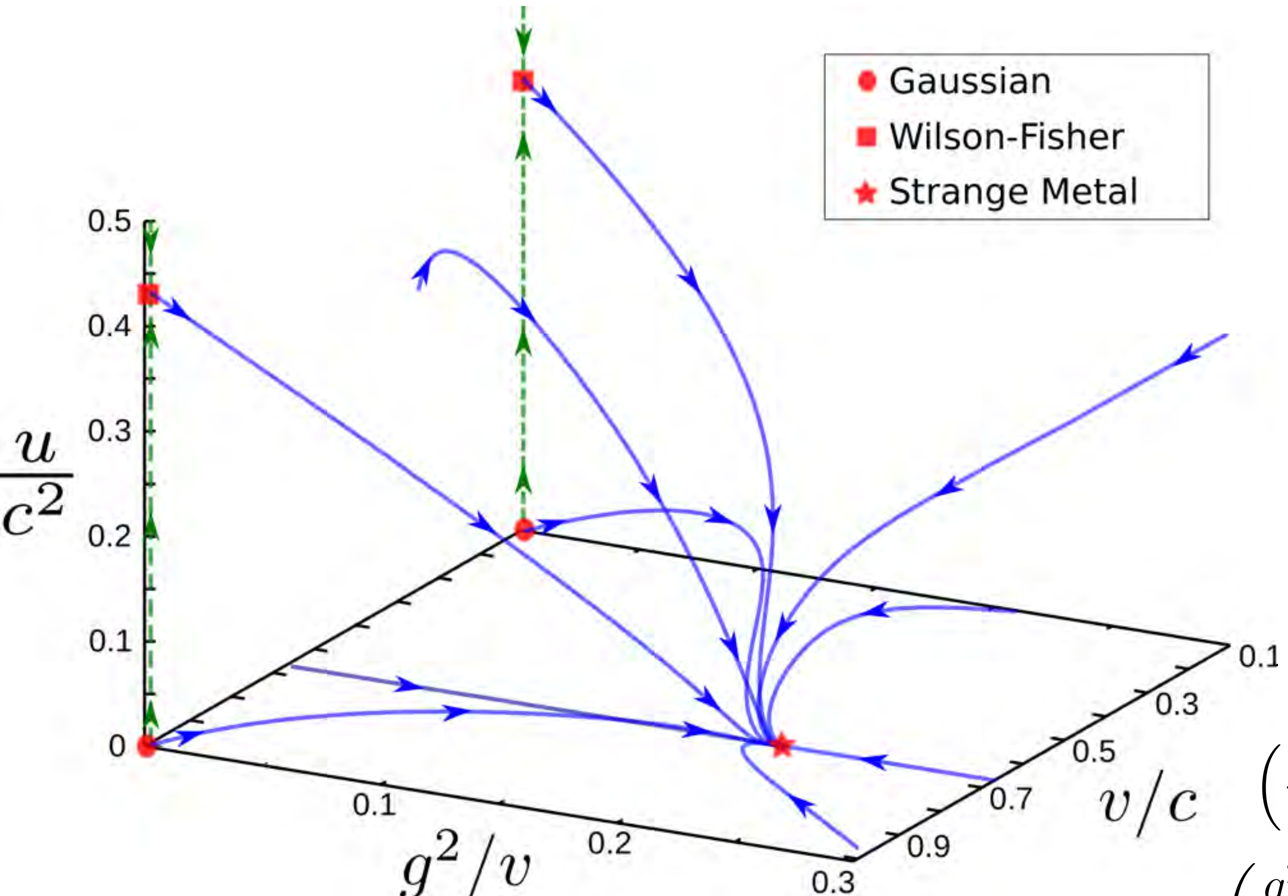


Cycle of negative feedback between (v,c) and (g,u) make them all flow to zero!



One-loop is asymptotically exact in the low energy limit in three dimensions

The kinetic energy & interactions maintain balance as they die



$$\left(\frac{v}{c}\right)^* = \frac{N_c N_f}{N_c^2 - 1}$$

$$\left(\frac{g^2}{v}\right)^* = \frac{4\pi(N_c^2 + N_c N_f - 1)}{N_c^2 + N_c N_f - 3}\epsilon$$

$$\left(\frac{u}{c^2}\right)^* = 0$$

Properties of the IR fixed point

- Interactionless
- Nested FS + dispersionless boson (quasi-local)
  - v,c flow to zero 1/log(L) for d<3
  - v,c flow to zero 1/log(log(L)) at d=3
- Breakdown of Fermi liquid (strange metal)
  - Non-Fermi liquid for d<3
  - Marginal Fermi liquid at d=3
- New form of stable metallic state :

Quasi-Local Strange Metal [S. Sur, SL (14)]

Spectral functions in QLSM

$$\mathcal{G}(k) \;=\; \frac{1}{|k_y|^{1-2\tilde{\eta}_\psi}} \tilde{G}\left(\frac{\mathbf{K}}{|k_y|^z}\right)$$

For hot spots 1,3

$$\mathcal{D}(k) \;=\; \frac{C}{|\mathbf{K}|^{\frac{2-2\tilde{\eta}_\phi}{z}}}$$

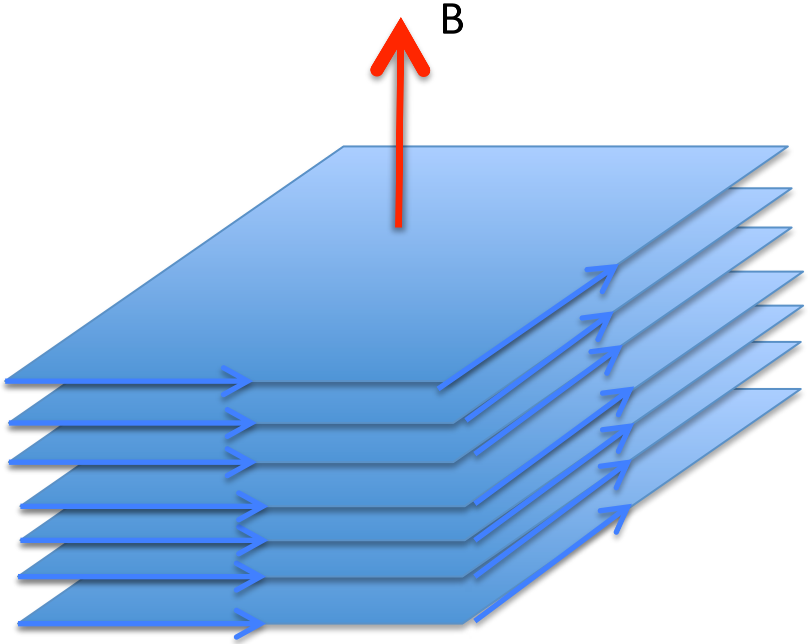
$$z = 1 + \frac{(N_c^2 + N_c N_f - 1)}{2(N_c^2 + N_c N_f - 3)}\epsilon$$

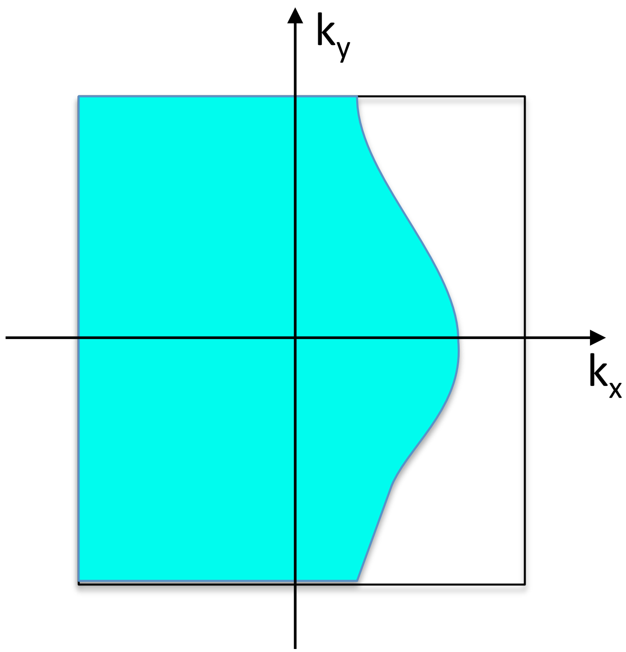
$$\tilde{\eta}_\psi, \tilde{\eta}_\phi \sim O(\epsilon^2)$$

Q. Can one find a strongly interacting 2d non-Fermi liquid state that can be accessed non-perturbatively ?

CHIRAL NON-FERMI LIQUID

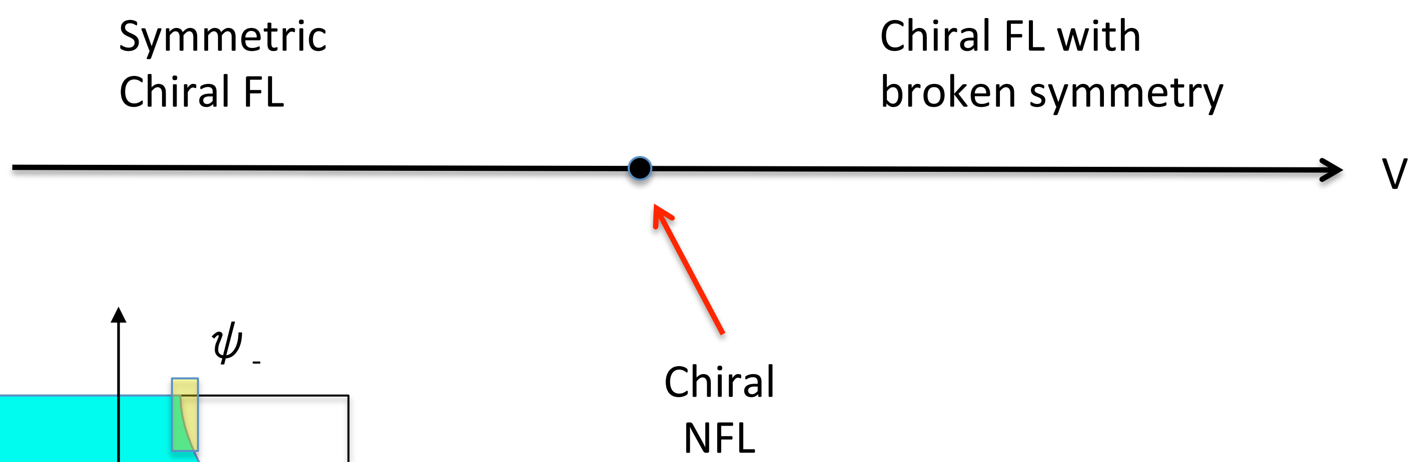
Chiral Metal

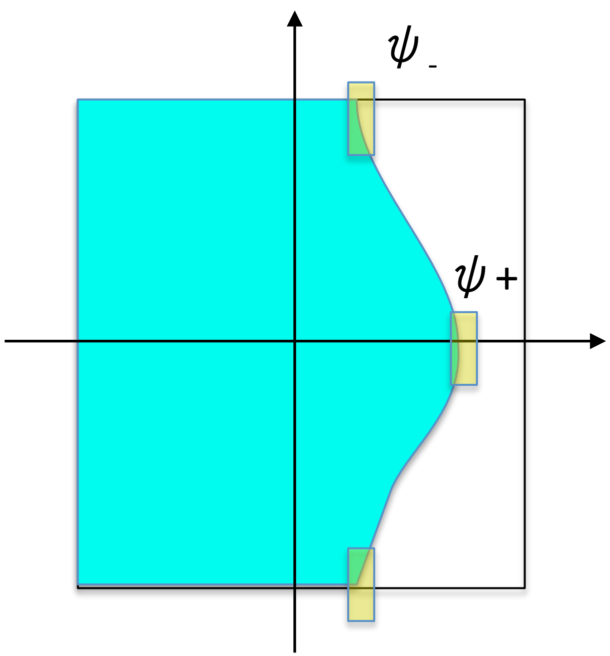




A stack of quantum Hall layers creates a two-dimensional chiral Fermi surface [Balents and Fisher (96)]

Minimal theory for QPT in chiral metal : chiral patch theory





+ gapless boson

Interaction driven scaling (as opposed to the Gaussian scaling)

irrelevant

$$S = \int dk \, \left(i \frac{k_0}{\Lambda^{1/2}} + k_x + \gamma k_y^2\right) \psi_j^*(k) \psi_j(k) + \int dk \, k_y^2 \, \phi_\alpha(-k) \phi_\alpha(k) + g \int dk \, dq \, \phi_\alpha(k) \, \psi_i^*(k+q) \, T_{ij}^\alpha \, \psi_j(q)$$

marginal

- The interaction is kept as a marginal term while one of the quadratic term is deemed irrelevant
- Irrelevant term enters as a scale

Wilsonian effective action with running length scale X_0

$$S_{X_0}(\Lambda, g)$$

dimensionful (Λ) — dimensionless (g)

- The Wilsonian effective action depends on all parameters of the theory
- In non-chiral case, divergence in $\Lambda \to \infty$ alter the naïve scaling
- In this case, thanks to chirality, the theory is UV finite : Λ can be dropped !

Stable fixed point

- UV/IR finiteness + absence of scale : the interaction driven scaling gives the exact scaling
- Exact Scaling form of the Green's function :

$$G^{-1}(k) = (k_x + k_y^2) g(|\omega|^{2/3}/(k_x + k_y^2))$$

- However, the full Green's function can not be computed perturbatively

Summary

- Perturbative non-Fermi liquids based on dimensional regularization
- Patch description for m=1
- UV/IR mixing for m>1
- Quasi-local non-Fermi liquids for SDW QCP
- Exact solution for chiral NFL at d=2