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# Gravity dual of spin and charge density waves

Matti Järvinen

University of Crete

Quantum Field theory, String Theory and Condensed Matter Physics

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# Literature

The D3-D7' model:

- First study $\rightarrow$ Quantum Hall state
  [Bergman, Jokela, Lifschytz, Lippert arXiv:1003.4965]
- Fluctuation analysis $\rightarrow$ Magneto-roton
  [Jokela, Lifschytz, Lippert arXiv:1012.1230]
- Quasi normal modes $\rightarrow$ Striped instability
  [Bergman, Jokela, Lifschytz, Lippert arXiv:1106.3883]
  [Jokela, Lifschytz, Lippert arXiv:1204.3914]
- Inhomogeneous ground state $\rightarrow$ Spin and charge density wave
  (This talk) [MJ, Jokela, Lippert arXiv:1408.1397]
- Alternative quantization $\rightarrow$ Anyonic superfluid
  (Talk by Matt Lippert on Saturday)
  [Jokela, Lifschytz, Lippert arXiv:1307.6336, 1407.3794]

The D2-D8' model $\rightarrow$ similar results, integer QH state

[MJ, Jokela, Lippert arXiv:1101.3329, 1107.3836, 1211.1381]

# Outline

1. Motivation
2. The D3-D7' model
3. Striped instability
4. Striped ground state
5. Outlook

# Motivation

Spatially modulated phases often appear in condensed matter systems

- E.g., pseudogap phase of high $T_c$ superconductors

Striped phases

- Charge density wave
- Spin density wave

Here: top-down holographic approach for condensed matter

- Concrete string models
- Control over dual field theory

# Holographic models of the QHE

Brane intersections with $\#\mathrm{ND} = 6$

- Fundamental fermions
- Probe $Dq$ in $Dp$ background
- No SUSY
- Chern-Simons terms

Familiar example: Sakai-Sugimoto D4-D8-$\overline{\text{D8}}$

QH models:

1. The D3-D7' model
   - 2+1 d defect, filling fraction $\nu$ irrational
2. The D2-D8' model
   - Fully 2+1 d, $\nu = 1$

# D3-D7' model

| | 0 | 1 | 2 | $z$ | $r$ | $\psi$ | $S^2_{(1)}$ | $S^2_{(2)}$ |
|---|---|---|---|---|---|---|---|---|
| $D3$ | X | X | X | X | | | | |
| $D7$ | X | X | X | | X | | X | X |

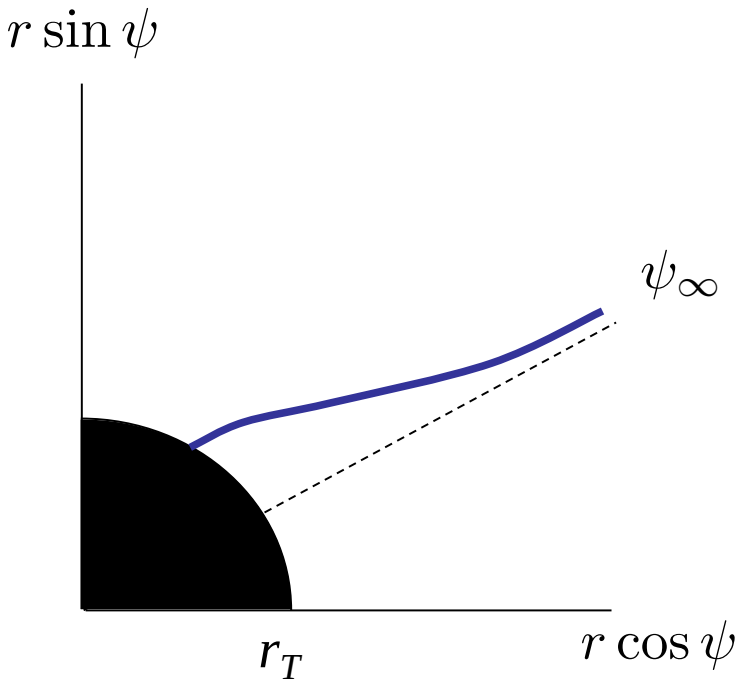
$$d\Omega_5^2 = d\psi^2 + \cos^2\psi \; d\Omega^2_{2(1)} + \sin^2\psi \; d\Omega^2_{2(2)}$$

Probe D7 brane

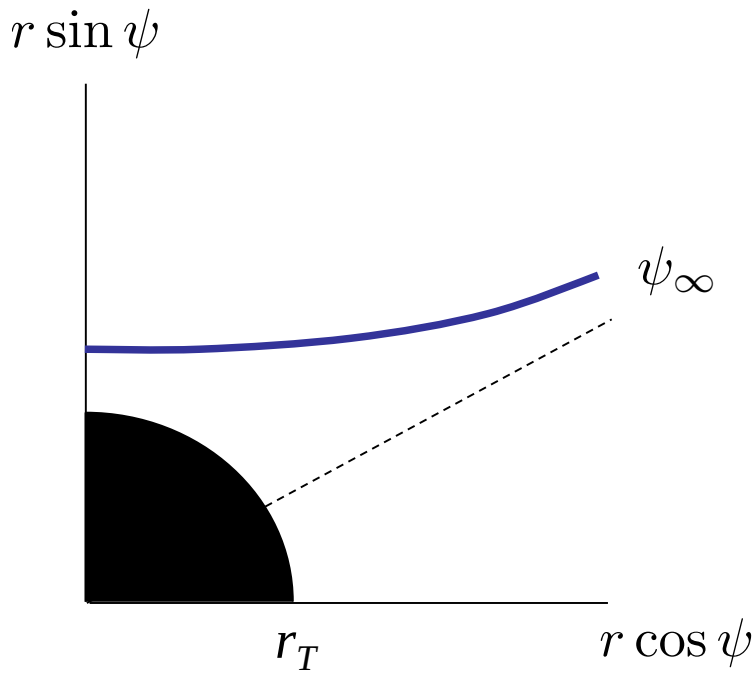
- Wraps $S^2 \times S^2 \subset S^5$
- Fermions on 2+1 d defect
- Embedding $\psi(r)$
  - is tachyonic
  - can be stabilized by wrapped flux on $S^2$'s

Add magnetic field $F_{12} = B$ and charge density $F_{0r} = A_0'(r)$

# Embeddings




- D7 enters horizon
- Metallic phase

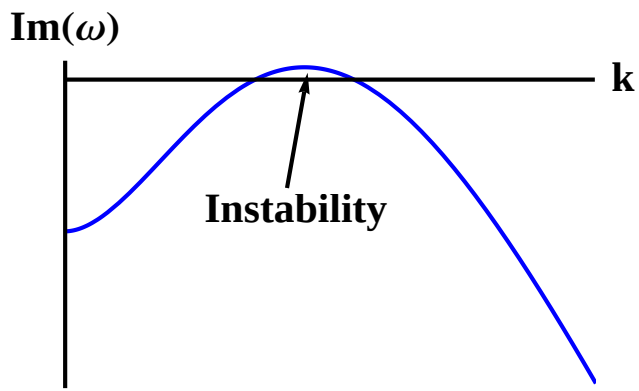


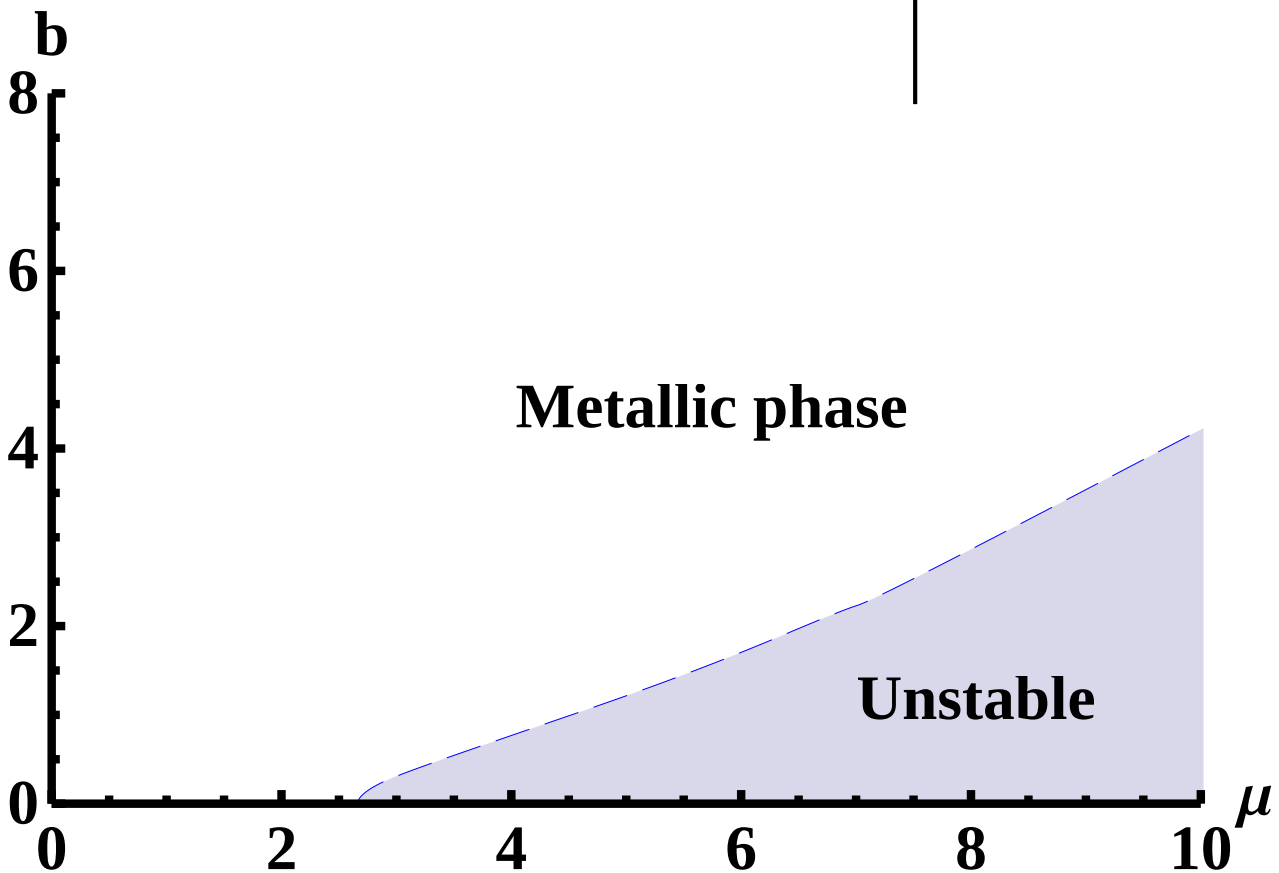

- D7 ends where $S^2$ shrinks
- QH phase

# Striped instability

Analysis of quasi normal modes $\Rightarrow$ instability

- In the metallic phase at large charge density
- Finite wave number $k \Rightarrow$ stripes



# Striped ground state

D3 background, $AdS_5 \times S^5$ (finite temperature)

$$ds_{10}^2 = r^2\left(-h(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{r^2}\left(\frac{dr^2}{h(r)} + r^2 d\Omega_5^2\right)$$

$$h(r) = 1 - \left(\frac{r_T}{r}\right)^4$$

D7 probe action

$$S = S_{\text{DBI}} + S_{\text{CS}} \quad = -T_7 \int d^8x\, e^{-\Phi}\sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$$
$$- \frac{(2\pi\alpha')^2 T_7}{2}\int P[C_4] \wedge F \wedge F$$

Look for inhomogeneous ground state

- Periodic with period $L$ in $x$-direction (wave number $k = 2\pi/L$)
- Inhomogeneous embedding of the D7 brane $\psi(x, r)$, $z(x, r)$ and gauge fields $a_0(x, r)$, $a_y(x, r)$

# Technical details

Equations of motion $\Rightarrow$ system of highly nonlinear PDEs

- Input: $\mu$, $b$ (Temperature scaled out, $m = 0$)

Solution to the PDEs:

- Discretization in both $r$ and $x$
- Pseudospectral derivatives
- Newton method (iteratively in grid size)

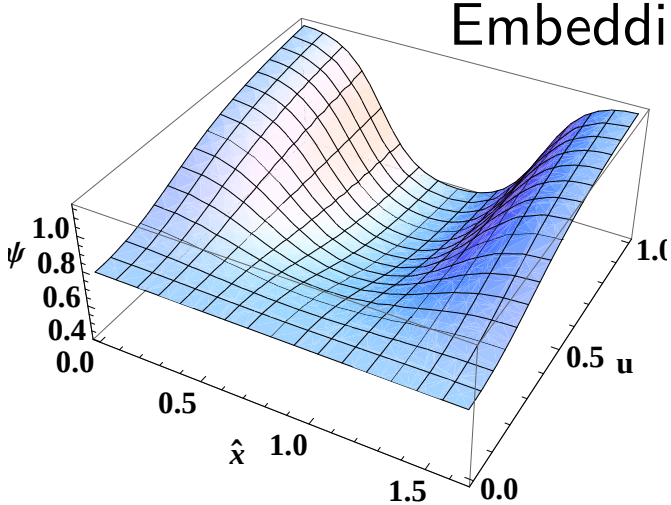
Difficulties due to DBI action:

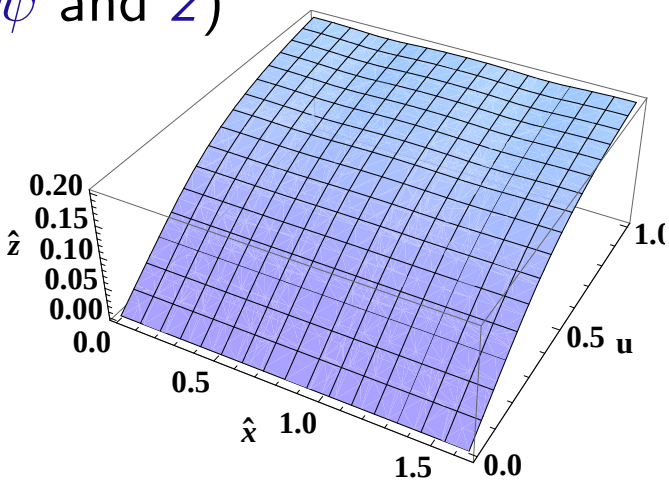
- Square root factors may become complex
  $\Rightarrow$ convergence issues $\Rightarrow$ careful choice of initial data

# Solutions

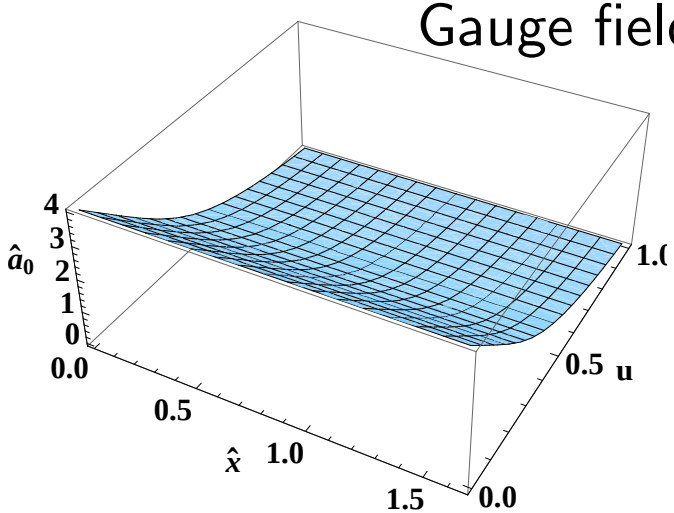
An example (with $u \sim 1/r$):

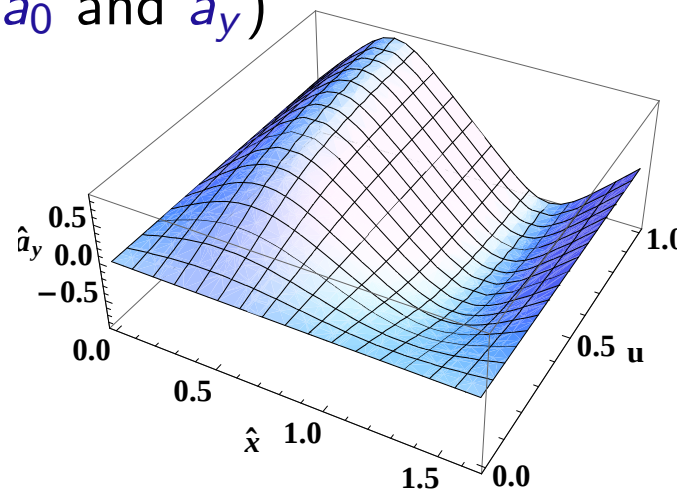
Embedding ($\psi$ and $z$)




Gauge fields ($a_0$ and $a_y$)

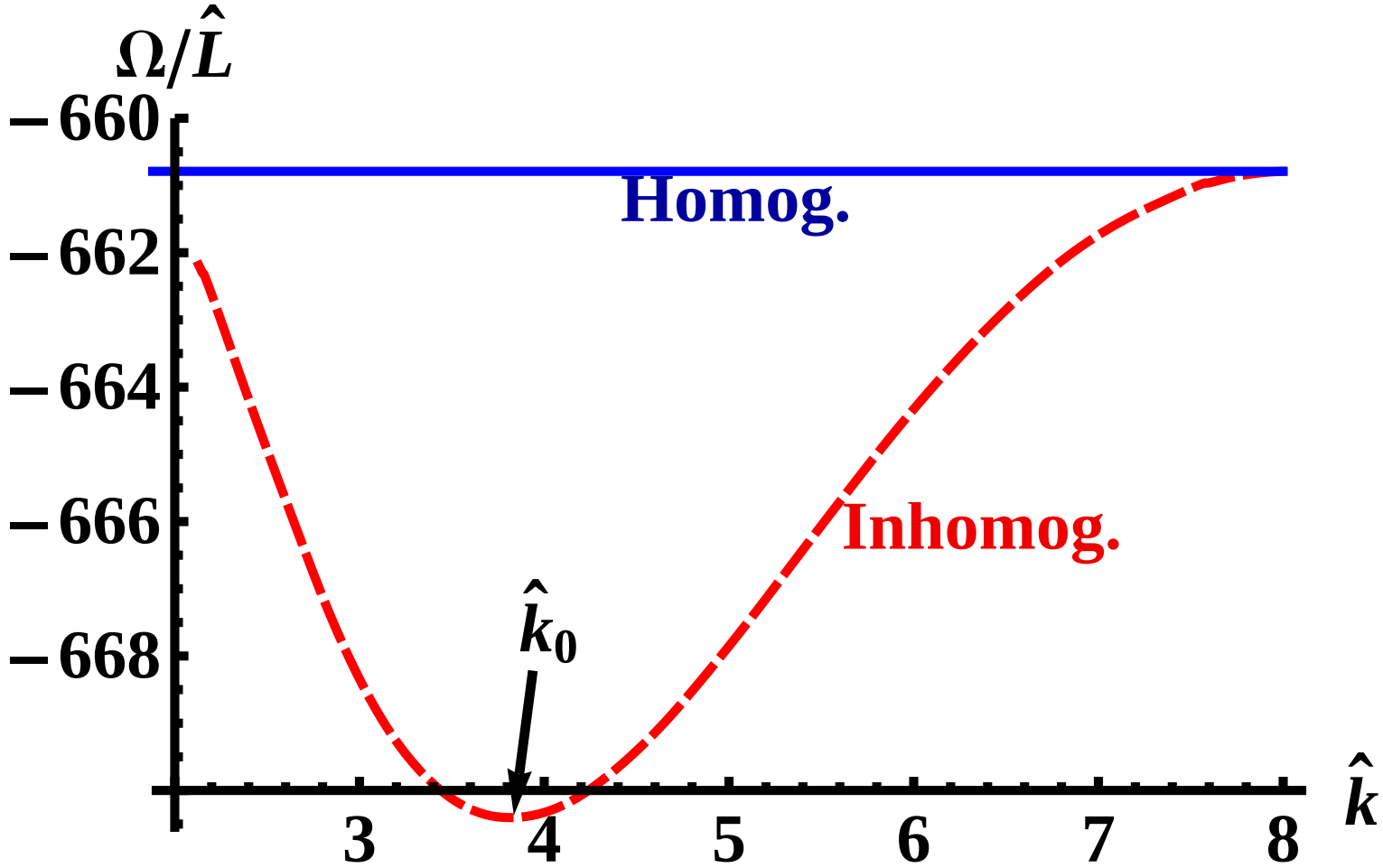



- "Spin density wave" $\gg$ charge density wave
- Modulated current

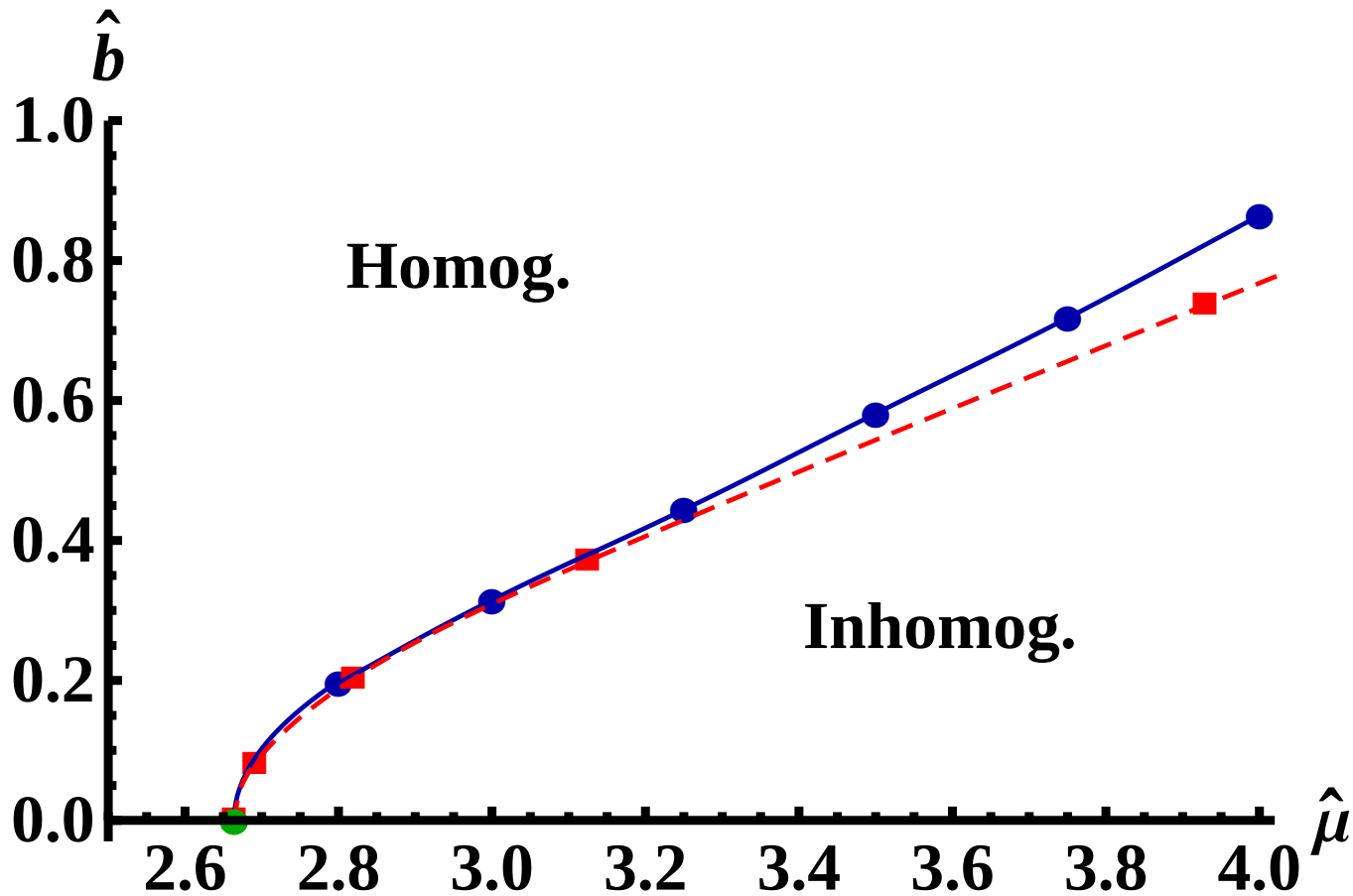
# Minimizing the energy

Grand canonical ensemble, $\Omega(\mu, b, L)/L$




- Inhomogeneous state usually preferred
- Energy minimized at $\hat{k} = \hat{k}_0(\mu, b)$

# Phase diagram




- 1st order transition
- 2nd order endpoint

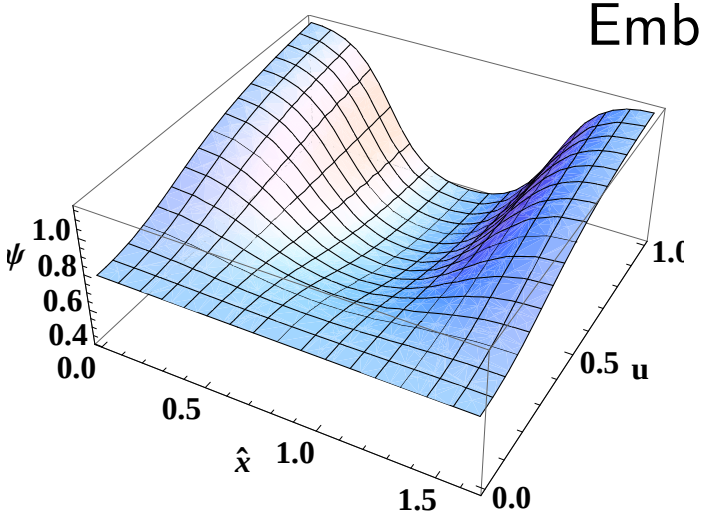
# Outlook

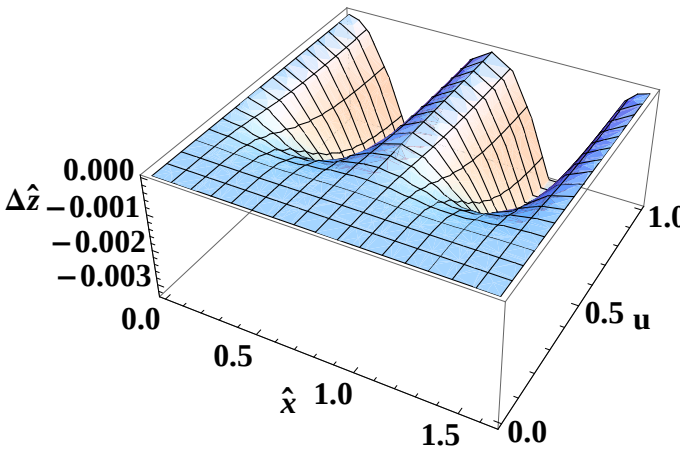
- Conductivities
- Impurities/disorder $\rightarrow$ effect on QH state?
- Break translation invariance completely: Stripes $\rightarrow$ lattices (?)
- Solve ground states in the D2-D8' model

# Extra slides

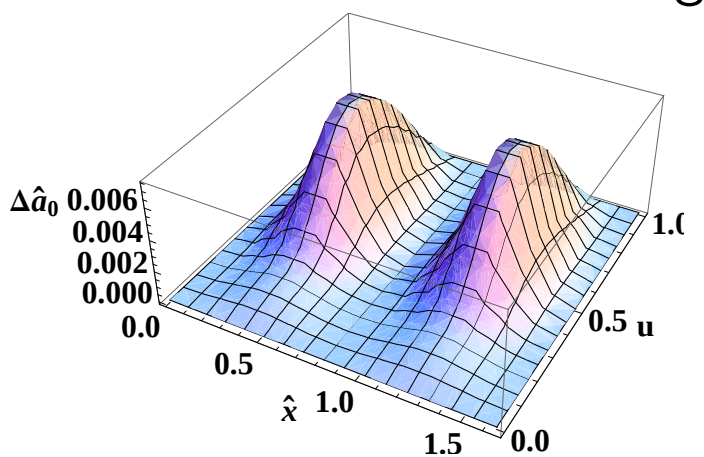
# Modulation

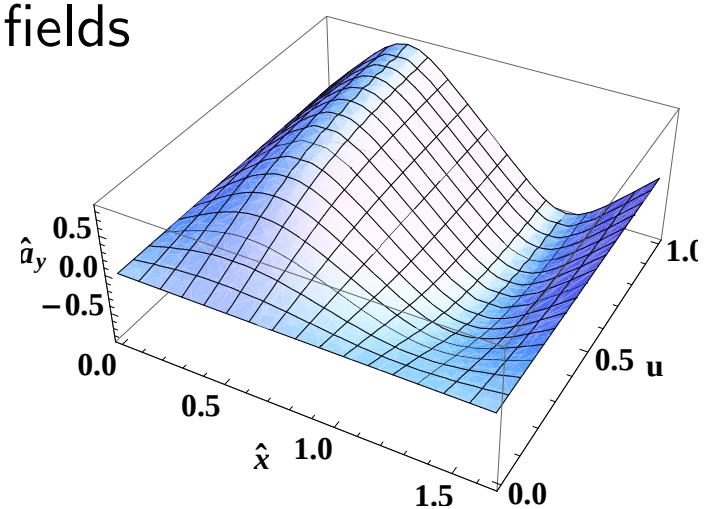
## Embedding



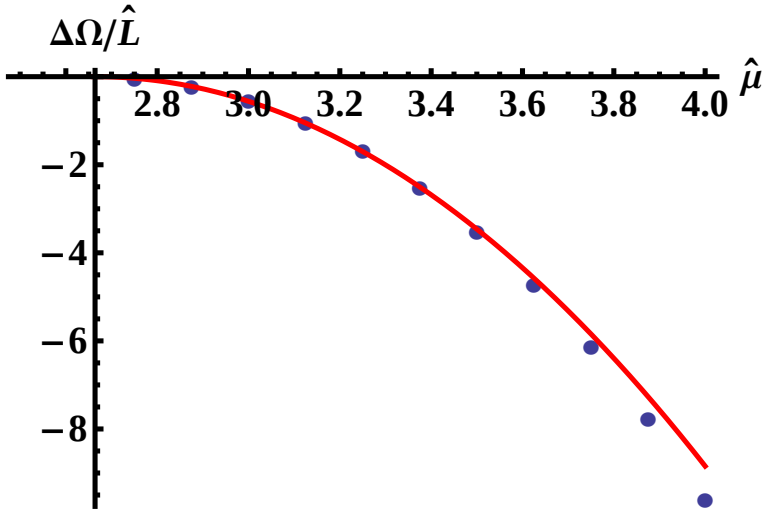



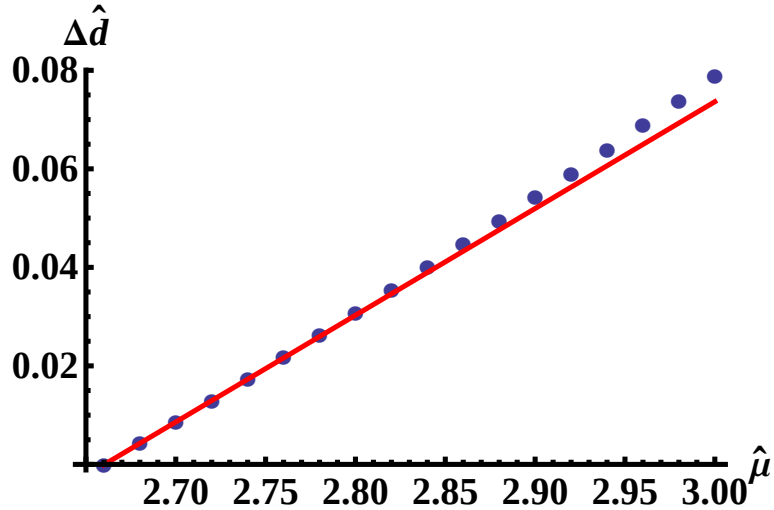

## Gauge fields

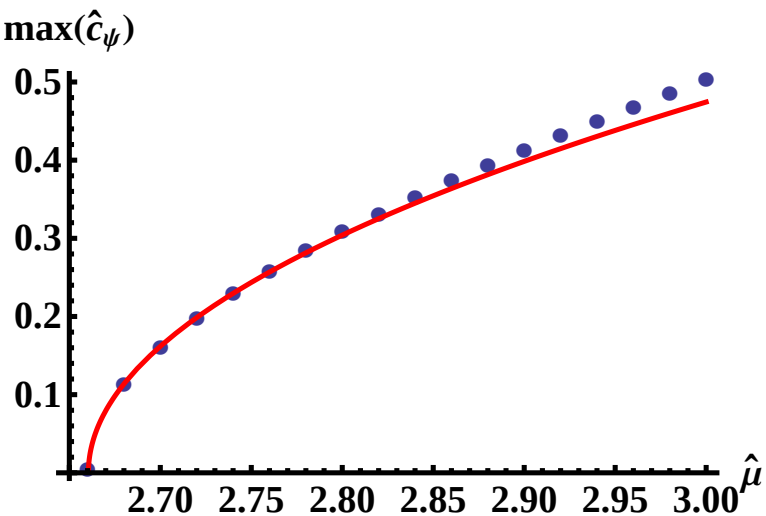


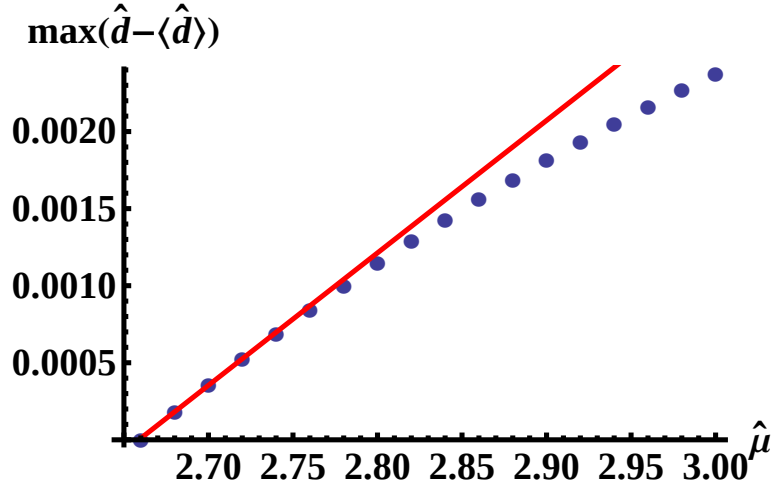



# Second order critical point



# First order transition