Charge transport with momentum relaxation in holography

Blaise Goutéraux

Nordita, Stockholm U. and KTH, Stanford U. and APC, CNRS Paris

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Based on

- *Charge transport in holography with momentum dissipation*, JHEP **1404** (2014) 181, arxiv:1401.5436

- *Holographic metals and insulators with helical symmetry*, arxiv:1406.6351, with A. Donos and E. Kiritsis

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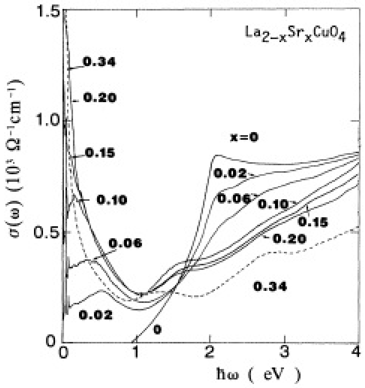
1. Features of conduction in real systems

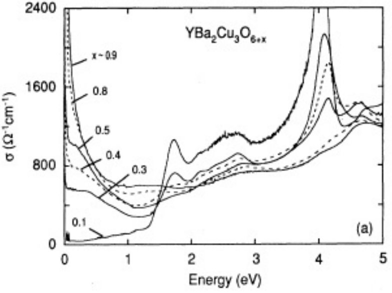
2. Computing finite DC conductivities

3. A holographic landscape of metals and insulators

In-plane metal/insulator transitions



[Uchida et al, PRB'91]



[Cooper et al, PRB'91]

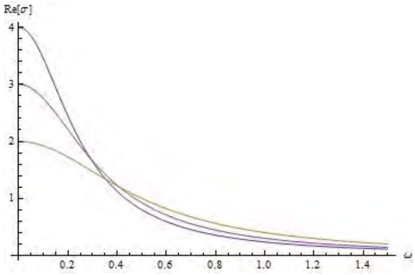
- **Drude-like feature** at low frequencies for high doping
- **Incoherent transport** at intermediate doping
- **Metal/insulator transition** at low doping
- Reorganisation of degrees of freedom assumed to be driven by strong correlations

Metallic transport with Drude peaks

- Drude model: postulates the **existence of (quasi)-particles** whose momentum relaxes slowly on a typical scale τ
- No quasi-particle assumption: use the memory matrix formalism [Forster'75], when there is an almost-conserved quantity (momentum) [Hartnoll & Hofman'12]: **'coherent' metals**

$$\sigma(\omega, T) = \frac{{\chi_{PJ}}^2 {\chi_{PP}}^{-1}}{1 - \imath\omega\tau}$$

$\sigma_{DC} \to +\infty$ when $\tau \to +\infty$ (no momentum relaxation)



Quantum critical thermal transport at zero density

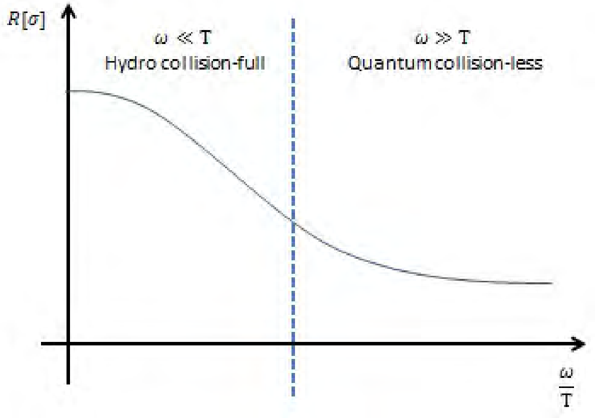
Above the Quantum Critical Point at the superfluid/insulator transition in d spatial dimensions, temperature is the only scale [DAMLE & SACHDEV'97]:

scale-covariant conductivity

$$\sigma \sim T^{\frac{d-2}{z}} \Sigma\left(\frac{\omega}{T}\right)$$

$$\Sigma(0) = ct$$

$$\Sigma(\infty) \sim ct' \left(\frac{\omega}{T}\right)^{-\frac{d-2}{z}}$$



No Drude peak: **'incoherent' metal**

Questions

- Can holography reproduce these possibilities (metals with and without Drude peaks, insulators)?
- Does weak momentum non-conservation (**'clean' limit**) always imply **coherent** transport (Drude peak)?
- Does strong momentum non-conservation (**'dirty' limit**) always imply **incoherent** transport (no Drude peak)?

Outline

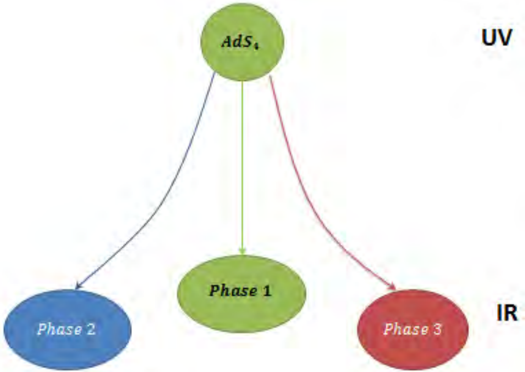
1. Features of conduction in real systems

2. Computing finite DC conductivities

3. A holographic landscape of metals and insulators

Philosophy

Assume the UV is Anti-de Sitter.
However, we wish to study the IR : **different IR phases can compete**.



How do we characterise these phases?

Effective holographic theories at low temperatures

Write down **effective holographic theories** describing the desired dynamics and characterize the possible IR phases by their symmetries and their behaviour under scaling transformations:

- Phases with unbroken U(1) symmetry (fractionalized phases), [Charmousis, **B.G.**, Kim, Kiritsis & Meyer '10, **B.G.** & Kiritsis'11]
- Phases with broken U(1) symmetry (e.g. cohesive phases like superfluids or electron stars), [**B.G.** & Kiritsis'12, **B.G.**'13];
- This talk: holographic metals and insulators with broken translation symmetry, [**B.G.**'14, Donos, **B.G.** & Kiritsis'14]

Effective holographic actions for momentum relaxation

Relax momentum by breaking translations to a helical Bianchi VII subgroup:

$$S = \int \mathrm{d}^5x\sqrt{-g}\left[R - \partial\phi^2 - Z_e(\phi){F_e}^2 - Z_m(\phi){F_m}^2 + V(\phi)\right]$$

- Contains gravity, an electric (finite density) and a magnetic field, a neutral scalar [Charmousis, **B.G.**, Kim, Kiritsis & Meyer '10].

- In the metric Ansatz, replace the $\mathbf{R}^d$ spatial factor by a helical, Bianchi VII symmetry in a 5D bulk [Iizuka et al'12, Donos & Gauntlett'12, Donos & Hartnoll'12], [Donos, **B.G.** & Kiritsis'14].

$$\mathrm{d}s^2 = -B\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B} + \sum_{i=1}^{3} C_i{\omega_i}^2\,, \qquad A_e = A_e(r)\,\mathrm{d}t\,, \quad A_m = A_m(r)\,\omega_2$$

$$\omega_1 = \mathrm{d}x\,, \quad \omega_2 = \cos(kx)\mathrm{d}y + \sin(kx)\mathrm{d}z\,, \quad \omega_3 = \sin(kx)\mathrm{d}y - \cos(kx)\mathrm{d}z$$

Other options to have finite DC conductivities

- Turn on massless scalars ('axions') $\psi_i = kx^i$ (preserves homogeneity of the eoms: ODEs) [Andrade & Withers'13, **B.G.**'14, Donos & Blake'14, Taylor & Woodhead'14].
  (similar mechanism to Q-lattices [Donos & Gauntlett'13,'14])

- Break diffeomorphism invariance in the bulk: massive gravity [Vegh'13, Davison'13, Blake & Tong'13, Davison, Schalm & Zaanen'13, Amoretti et al.'14]. Very similar to axions.

- Inhomogeneous lattices, [Horowitz, Santos & Tong'12, Donos & Hartnoll'12, Horowitz & Santos'13, Chesler, Lucas & Sachdev'13, Blake, Tong & Vegh'13, Withers'14, Jokela, Järvinen & Lippert'14].

- Random-field disorder, [Hartnoll & Herzog'08, Davison, Schalm & Zaanen'13, Lucas, Sachdev & Schalm'14].

The electric perturbation problem

The conductivity gives the response of the current to a small oscillating electric field

$$\sigma(\omega, T) = \frac{\delta J_x}{\delta E_x} = \frac{G^R_{J_x J_x}(\omega, \mathbf{q}=0)}{\imath\omega}$$

Turn on a small electric field along x on the boundary:

$$\delta A_x(r, \omega, \mathbf{q}=0) \sim \delta A_{x(0)} + r\, \delta A_{x(1)} + \ldots, \quad r \to 0$$

This couples to other (vector) perturbations: g_{tx}, δA_m, $g_{\omega_2\omega_3}\ldots$

The holographic dictionary tells us that the 2-point function (the conductivity) is the variation of the vev wrt the source:

$$\sigma(\omega, T) = \frac{\delta A_{x(1)}}{\imath\omega\, \delta A_{x(0)}}$$

The DC conductivity

- At $\omega = 0$, there is a **radially conserved quantity**:

$$\partial_r \left[\sigma(r)\right] = 0$$

- It can be evaluated at the horizon (IR)!

$$\sigma_{DC} = \lim_{r\to 0} \sigma(r) = \lim_{r\to r_h} \sigma(r) = \sigma_{pc} + \sigma_{mr}$$

[Iqbal & Liu'08, Blake & Tong'13, Andrade & Withers'13, **B.G.**'14, Donos & Gauntlett'14]

The DC conductivity: momentum relaxation term

$$\sigma_{DC} = \sigma_{pc} + \sigma_{mr}$$

- Model-dependent, dominates in the clean limit (momentum conservation weakly broken).
- Axions and massive gravity: [Blake & Tong'13], [Andrade & Withers'13, **B.G.**'14, Donos & Gauntlett'14]

$$\sigma_{mr} = \frac{{Q_e}^2}{s\, m_h^2}$$

- Bianchi VII_0: [Donos, **B.G.** & Kiritsis'14]

$$\sigma_{mr} = \frac{{Q_e}^2 \sqrt{C_2 C_3 / C_1}}{k^2 \left((C_2 - C_3)^2 + C_2 Z_m A_m^2 \right)}, \qquad \mathrm{d}s^2 = -B\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B} + \sum_{i=1}^{3} C_i {\omega_i}^2$$

- Similar result for random, perturbative disorder

  [Lucas, Schalm & Sachdev'14]

- Blows up if momentum is conserved (unless zero density)

The DC conductivity: pair creation term

$$\sigma_{DC} = \sigma_{pc} + \sigma_{mr}$$

$$\sigma_{pc} = Z_e(\phi)\left(\frac{C_2 C_3}{C_1}\right)^{1/2}\Bigg|_{r=r_h} \qquad \mathrm{d}s^2 = -B\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B} + \sum_{i=1}^{3} C_i {\omega_i}^2$$

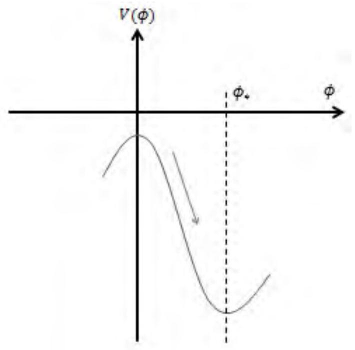
- Independent of the relaxation mechanism, dominates in the dirty limit (momentum conservation strongly broken);
- production of charged particles in an electric field without heat flow [Donos & Gauntlett '14].
- Unless there is no overlap between the current and momentum operator (CFT at zero density), does not give rise to finite conductivities by itself: $\sigma(\omega) \sim \sigma_{pc}\left(\delta(\omega) + \frac{i}{\omega}\right)$

Effective holographic actions for momentum relaxation

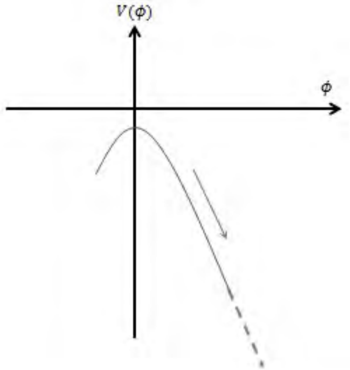
Momentum relaxation by **helical Bianchi VII lattices**

$$S = \int \mathrm{d}^5x\sqrt{-g}\left[R - \partial\phi^2 - Z_e(\phi){F_e}^2 - Z_m(\phi){F_m}^2 + V(\phi)\right]$$

In the IR, we can assume that the scalar ϕ



settles in an **extremum** of its effective potential, $\phi = \phi_\star$



has a **runaway behaviour** $\phi \to \pm\infty$.

$$Z_{e,m}(\phi) \sim e^{\gamma_{e,m}\phi}, \quad V(\phi) \sim V_0 e^{-\delta\phi}$$

Clean quantum critical phases

$$\mathrm{d}s_0^2 = r^{\frac{2\theta}{d}}\left(-\frac{\mathrm{d}t^2}{r^{2z}} + L^2\frac{\mathrm{d}r^2}{r^2} + \frac{\mathrm{d}\vec{x}_{(3)}{}^2}{r^2}\right), \quad \phi = \kappa \ln r\,, \quad s \sim T^{\frac{d-\theta}{z}}$$

- Translation symmetry is broken in the IR by **irrelevant** deformations
- $\theta = 0 \Rightarrow$ constant scalar, hyperscaling solutions

AdS$_5$ Lifshitz AdS$_2 \times \mathbf{R}^3$

- $\theta \neq 0$ captures **deviation** from symmetry under

$$t \to \lambda^z t\,, \qquad (r, x^i) \to \lambda(r, x^i)$$

hyperscaling violation $d_{eff} = d - \theta$

[**B.G.** & Kiritsis'11, Huijse, Sachdev & Swingle'11, Dong et al.'12]

The conduction (or vector hyperscaling violation) exponent

$$A_t \sim Q_e r^{\zeta - z}\,\mathrm{d}t$$

A **new exponent** parameterizes the scaling of the electric potential, [**B.G.** & KIRITSIS'12, **B.G.**'13]. It can behave in two ways:

- $T_{\mu\nu}^{maxwell}$ is subleading in r compared to $T_{\mu\nu}^{\phi}$ in Einstein's equations.

  Then $z = 1$ and the **charge density** Q_e sources an **irrelevant** mode:

  $$\mathrm{d}s^2 = \mathrm{d}s_0{}^2\left(1 + Q_e{}^2 r^{2\beta} + \cdots\right), \qquad \beta = \frac{\zeta + d - \theta}{2}$$

  β is the (anomalous) IR scaling dimension of the charge density [**B.G.**'13,'14, KARCH'14].
  $\zeta = d - \theta$ gives back the usual scaling in $d - \theta$ dimensions.

- $T_{\mu\nu}^{maxwell}$ scales with r like $T_{\mu\nu}^{\phi}$.

  Then $z \neq 1$ and the **charge density** Q_e sources a **marginal** mode of the solution and $\zeta = \theta - d$.

DC conductivity for clean helical quantum critical phases

Two translation-breaking modes from the metric and the magnetic field. The **momentum-relaxing** term dominates the DC conductivity

$$\sigma_{mr} = \frac{{Q_e}^2 \sqrt{C_2 C_3 / C_1}}{k^2 \left((C_2 - C_3)^2 + C_2 Z_m A_m^2 \right)}$$

- $0 < z < +\infty$: the modes are **exponentially suppressed**, as expected from the dispersion relation $\omega \sim k^z$ – No dofs at finite momentum if $z < +\infty$ [Hartnoll&Shaghoulian'12, Anantua et al.'12]

$$\sigma_{DC} \sim \frac{{Q_e}^2}{k^2} T^{\frac{z-1}{z}} \exp\left(2kT^{-\frac{1}{z}} \right)$$

- **Semi-locally critical limit** $z \to +\infty$, $\theta = \eta z$: the exponent $\beta(k)$ of the modes **depends on wavevector** k [Hartnoll&Hofman'12]

$$\sigma_{DC} \sim \frac{{Q_e}^2}{k^2} T^{\eta + \beta_m}$$

  where β_m is the irrelevant mode turning on the magnetic field.

- Always metals. Matches memory matrix prediction.

Dirty helical quantum critical phases

Anisotropic saddle points, with leading behaviour

$$\mathrm{d}s^2 = r^{2\theta/3}\left(-\frac{\mathrm{d}t^2}{r^{2z_1}} + \frac{L^2\mathrm{d}r^2 + {\omega_1}^2}{r^2} + \frac{{\omega_2}^2 + \lambda r^{-2}{\omega_3}^2}{r^{2z_2}}\right)\left(1 + O(k^2 r^2)\right)$$

$$\phi = \kappa \ln r + \cdots\,, \quad A_e = Q_e r^{\zeta - z_1} + \cdots\,, \quad A_m = Q_m \omega_2 + \cdots$$

$$z_1 < 0$$

The conductivity is **pair creation dominated**, and gives rise to **insulators** ($\zeta < 2$) or **metals** ($\zeta > 2$ + irrelevant density mode)

$$\sigma_{DC} \sim T^{\frac{\zeta-2}{z_1}} \qquad \neq \qquad \sigma_{DC} \sim T^{\frac{d-2}{z_1}} \quad (d = 3)$$

unless $\zeta = d - \theta$ (scale-invariant limit where the charge density does not have anomalous IR scaling dimension)

Universal scaling? Universal incoherent transport? [Hartnoll'14]

AC conductivity

At small frequency and zero temperature, there is a matched asymptotic argument relating IR and UV retarded Green's functions [Donos & Hartnoll'12]:

$$\Im\left[G^{R,UV}_{J_x J_x}(\omega, T)\right] = \sum_I d^I \Im\left[\mathcal{G}^{R,IR}_{\mathcal{O}_I\mathcal{O}_I}(\omega, T)\right]$$

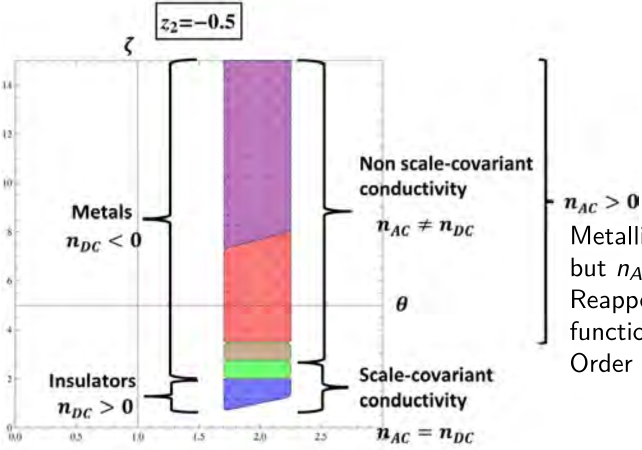
The scaling of the AC conductivity is then given by the scaling of the least irrelevant operator.

For the Bianchi VII solution: 3 propagating modes.

$T = 0$ IR asymptotics of the optical conductivity

Dirty, helical phase, pair-creation dominated

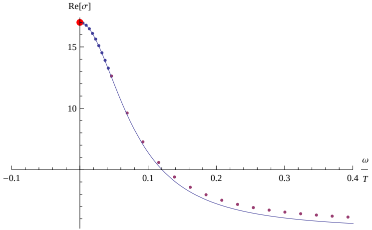
$$\sigma_{DC} \sim T^{n_{DC}}, \qquad \sigma_{AC}(T=0) \sim \omega^{n_{AC}}, \qquad n_{DC} = \frac{\zeta - 2}{z_1}$$



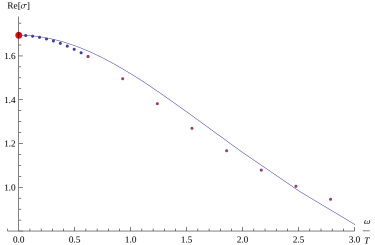
Metallic phases with $n_{DC} < 0$ but $n_{AC} > 0$.
Reappearance of a delta function?
Order of limits?

Clean/dirty limits vs coherent/incoherent transport

- What happens to the optical conductivity when either term dominates the DC conductivity? Is there always a Drude peak?
- Charged **axionic** black hole analogous to the Reissner-Nordström black hole with IR $AdS_2 \times \mathbf{R}^2$ [Bardoux, Caldarelli & Charmousis'12, Andrade & Withers '13] with $\sigma_{DC} = 1 + \mu^2/k^2$.



Weak momentum non-conservation
(clean limit, $k \ll \mu$)
Drude peak (coherent metal)
'Particle'-like physics



Strong momentum non-conservation
(dirty limit, $k \gg \mu$)
No Drude peak (incoherent metal)
'Unparticle'-like physics

Conclusions

- Recipe to compute the DC conductivity analytically.

$$\sigma_{DC} = \sigma_{pc} + \sigma_{mr}$$

- Either term can dominate: clean vs dirty limit.
- Does the clean/dirty limit always imply coherent/incoherent transport?
- Breaking translation symmetry allows $\sigma_{AC}(T = 0) \sim \omega^{n_{AC}}$ with $-1 < n_{AC} < 0$. But not a mid-IR scaling. See S. Cremonini's talk for intermediate scalings without breaking translation symmetry.
- **Insulators** seem to require $\mathbf{z} < 0$ – field theory interpretation? Clearly the non-relativistic scaling $\omega \sim k^z$ does not make sense. No apparent mass gap.