Negative magnetoresisitivity and dissipation effects in Weyl metal

Ya-Wen Sun, IFT-UAM/CSIC

Crete, Sep 7th, 2014,

based on: K. Landsteiner, Y. Liu, Y.W. Sun, in progress

Motivation: magneto-conductivity in Weyl metal

- 3+1 dimensional Weyl metal: chiral anomaly
- One signature of Weyl metal: *negative longitudinal magnetoresisitivity,* due to chiral anomaly;

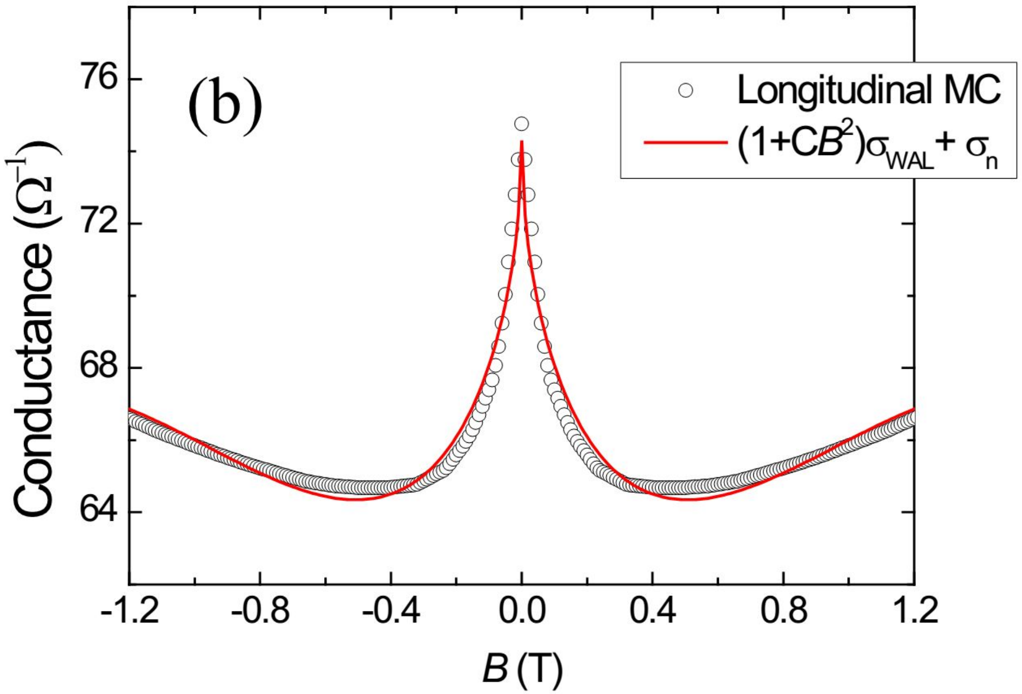
➢ the longitudinal electric conductivity with a background magnetic field;

➢ the resistivity decreases with increasing magnetic field *(H.B. Nielsen, et.al, 1983)*

- Dirac metal+ perturbations that break time reversal symmetry$\rightarrow$ Weyl metal

Motivation: magneto-conductivity in Weyl metal

- Experimental observations: features of Weyl metal (Kim et. al, 2013)



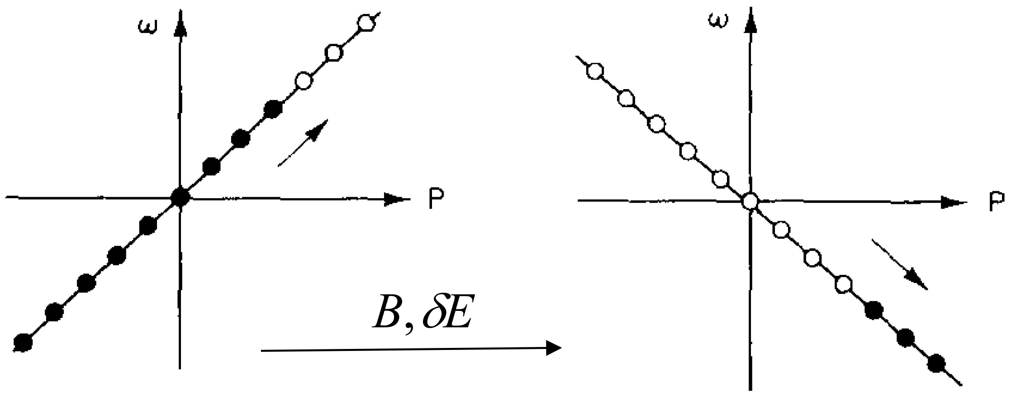
- Aim I: Can we produce the experimental result in holography? Negative magnetoresistivity?

Motivation: magneto-conductivity in Weyl metal

- Infinite DC magneto-conductivity due to chiral anomaly: the longitudinal electric conductivity with a background magnetic field

- Mechanism：3+1 dimensions, positive charge, chiral anomaly
- *(H.B. Nielsen, et.al, 1983)*

- RH



LH

Motivation: momentum dissipation

- Electric conductivity: $$J_\mu = \sigma E_\mu$$
- At zero frequency: delta function in the real part

$$\sigma = \sigma_E + \frac{i}{\omega}\frac{\rho^2}{\epsilon + P}$$

- Momentum dissipation: momentum relaxational time

$$\sigma = \sigma_E + \frac{i}{\omega + \frac{i}{\tau}}\frac{\rho^2}{\epsilon + P}$$

- **Aim II: Will momentum dissipation help to make this infinite conductivity due to chiral anomaly finite?**

Two aims:

- **A physical and finite longitudinal magnetoconductivity in Weyl metal**

  What kinds of dissipations do we need?

  Simplest way: hydrodynamic limit

- **The dependence on the magnetic field**

  "Negative" magneto-resistance?

  Holography?

Outline:

- I Magneto-conductivity in chiral anomalous fluid: universal hydrodynamic linear response result; adding all possible dissipations
- II Applying the conductivity formula to holographic system
- III Holographic checks: Schwarzschild black hole
- IV Generalizing to more natural system $U(1)_V \times U(1)_A$
- V Summary and ongoing work

I Magneto-conductivity in chiral anomalous fluid

- Consider one U(1) current with a chiral anomaly

- Hydrodynamics: conservation equations with external electromagnetic fields, small frequency limit

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha, \qquad \partial_\mu J^\mu = c E^\mu B_\mu,$$

- Constitutive equations:

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p g^{\mu\nu} + \tau^{\mu\nu},$$
$$J^\mu = \rho u^\mu + \nu^\mu,$$

- with

$$\nu^\mu = \sigma_E E^\mu - \sigma_E T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) + \sigma_B B^\mu + \ldots$$

I Magneto-conductivity in chiral anomalous fluid

- Linear response in hydrodynamics *(L. P. Kadanoff and P. C. Martin, 1963; S. Hartnoll, et.al, 2007)*: determine transport coefficients by solving initial value problems in hydrodynamic equations.
- Perturbations on a thermodynamic equilibrium:

$$\mu(\vec{x},t) = \mu + \delta\mu(\vec{x},t)$$
$$T(\vec{x},t) = T + \delta T(\vec{x},t)$$
$$u^\mu(\vec{x},t) = (1, \delta u_i(\vec{x},t))$$

- External magnetic field:

$$F_{12} = -F_{21} = B, \quad E^\mu = 0$$

- Electric conductivity:

$$\delta E_i = \delta F^{0i} = -\delta F^{i0}$$

I Magneto-conductivity in chiral anomalous fluid

- Linear response:

$$\begin{pmatrix} \delta T^{00} \\ \delta T^{0i} \\ \delta T^{ij} \\ \delta J^{0} \\ \delta J^{i} \end{pmatrix} \rightarrow \begin{pmatrix} \delta \mu^{0} \\ \delta T^{0} \\ \delta u_{j}^{0} \\ \delta E_{j}^{0} \end{pmatrix}$$

$$\delta\epsilon \equiv e_1\delta\mu + e_2\delta T = \left(\frac{\partial\epsilon}{\partial\mu}\right)\Big|_T \delta\mu + \left(\frac{\partial\epsilon}{\partial T}\right)\Big|_\mu \delta T,$$

$$\delta\rho \equiv f_1\delta\mu + f_2\delta T = \left(\frac{\partial\rho}{\partial\mu}\right)\Big|_T \delta\mu + \left(\frac{\partial\rho}{\partial T}\right)\Big|_\mu \delta T,$$

$$\delta p = \rho\delta\mu + s\delta T,$$

- Transport coefficients:

$$\delta J^\mu(\vec{k},\omega) = \frac{G_{J^\mu;J^\nu}(\vec{k},\omega) - G_{J^\mu;J^\nu}(\vec{k},0)}{i\omega}\delta E^0_\nu(\vec{k}) + \ldots$$

electric conductivity

I Magneto-conductivity in chiral anomalous fluid

- Dissipation terms:

$$\partial_\mu \delta T^{\mu 0} = \delta F^{0\mu} J_\mu + \frac{1}{\tau_e} \delta T^{\mu 0} u_\mu,$$

$$\partial_\mu \delta T^{\mu i} = \rho \delta E^i + F^{i\lambda} \delta J_\lambda + \frac{1}{\tau_m} \delta T^{\mu i} u_\mu,$$

$$\partial_\mu \delta J^\mu = c \delta E^\mu B_\mu + \frac{1}{\tau_c} \delta J^\mu u_\mu,$$

Energy, momentum, charge dissipations:

$$\omega_e \equiv \omega + \frac{i}{\tau_e}, \;\; \omega_m \equiv \omega + \frac{i}{\tau_m}, \;\; \omega_c \equiv \omega + \frac{i}{\tau_c}$$

I Magneto-conductivity in chiral anomalous fluid

- Longtitudinal Magneto-conductivity:

$$\Sigma = \sigma_E - \frac{i}{\omega + \frac{i}{\tau_e}} \frac{B^2 c\sigma_B}{2(e_2 f_1 - e_1 f_2)} Y_0 + \frac{i}{\omega + \frac{i}{\tau_m}} \frac{\rho}{\epsilon + p} \left[\rho - \frac{B^2 c\sigma_B}{2(e_2 f_1 - e_1 f_2)} Y_1 \right] + \frac{i}{\omega + \frac{i}{\tau_c}} \frac{B^2 c^2}{2(e_2 f_1 - e_1 f_2)} Y_1$$

$$Y_0 = \frac{1}{\epsilon + p} \left[2 f_2 T s - \frac{(e_2 f_1 - e_1 f_2) + (f_1 s - f_2 \rho)}{\epsilon + p} \mu^2 \rho \right]$$

$$Y_1 = \frac{1}{\epsilon + p} \left[2 e_2 T s - (e_2 f_1 - e_1 f_2) \mu^2 - \frac{e_1 s - e_2 \rho}{\epsilon + p} \mu^2 \rho \right]$$

- Some remarks:
- At zero c, reduces back to the ordinary electric conductivity

$$\sigma_E + \frac{i}{\omega_m} \frac{\rho^2}{\epsilon + p}$$

I Magneto-conductivity in chiral anomalous fluid

- Infinite DC conductivity even at zero density without dissipation terms:

$$\Sigma = \sigma_E + \frac{i}{\omega}\frac{B^2c^2}{(\partial\rho/\partial\mu)|_T}$$

- All dissipations needed

- momentum relaxation always associated with nonzero density

- Inifnite response in δJ^t , consistent with physical picture

II Applications of the formula to holographic systems

- The result involves thermodynamic quantities which depend on different systems
- Holographic system: Schwardzchild black hole in the probe limit, zero density

$$\epsilon = 3r_0^4, \quad s = 4\pi r_0^3, \quad T = \frac{r_0}{\pi}$$

- Chiral anomaly in AdS/CFT, represented by a Chern-Simons term

$$\mathcal{S} = \int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} - \frac{1}{4}F^2 + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu F_{\nu\rho}F_{\sigma\tau}\right]$$

- To solve for other thermodynamic quantities:

$$A_t = A_t(r),\ A_y = Bx,\ A_z = A_z(r)$$

II Applications of the formula to holographic systems

- Longitudinal conductivity:

$$\sigma_{zz} = \sigma_E + \frac{i}{\omega}\frac{B^2 c^2}{(\partial\rho/\partial\mu)|_T}$$

- With charge density

$$\rho = 4\mu r_0^2 \frac{\Gamma\left[\frac{5-\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]\Gamma\left[\frac{5+\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]}{\Gamma\left[\frac{3-\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]\Gamma\left[\frac{3+\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]}$$

- At small B, $\rho = 2\mu r_0^2 + \mathcal{O}(\alpha^2 B^2)$

- At large B, $\rho = \mu r_0^2 B + \mathcal{O}(\frac{1}{\alpha B})$

- Consistent with previous weakly coupled field theoretical results for chiral fermions at large B *(H. B. Nielsen, et.al,1983, D.T. Son, et.al, 2012)*

III holographic calculations: Schwarzschild

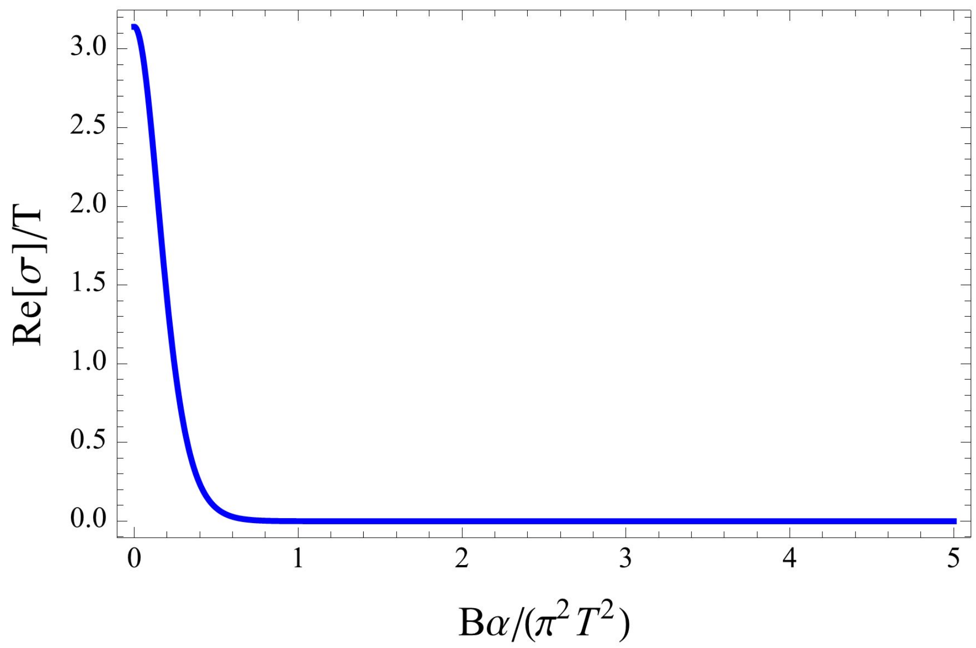
- Holographic calculations for the longitudinal DC conductivity in the probe limit:
- Fluctuactions: $\delta A_t(r)e^{-i\omega t}, \delta A_z(r)e^{-i\omega t}$
- By matching solutions at small frequency:

$$\sigma = \left(\frac{8\pi\alpha^2 B^2}{r_0^3}\sec\left(\frac{\pi}{2}\sqrt{1-(8B\tilde{\alpha})^2}\right) + \frac{i}{\omega}\frac{16B^2\alpha^2}{\pi^2 T^2}\right)\frac{\Gamma\left[\frac{3-\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]\Gamma\left[\frac{3+\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]}{\Gamma\left[\frac{5-\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]\Gamma\left[\frac{5+\sqrt{1-(8B\tilde{\alpha})^2}}{4}\right]}$$

- This is the same as the hydrodynamic result from the formula.
- The zero density conductivity gets affected by the background magnetic field
- small B $\sigma_E = \pi T - \frac{c^2 B^2 \log 2}{2\pi^3 T^3} + \mathcal{O}(\alpha^4 B^4)$ important small B behavior
- large B $\sigma_E = e^{-\frac{cB}{2\pi T^2}}\left(\frac{cB}{T} + \mathcal{O}\left(\frac{1}{cB}\right)\right)$

III holographic calculations: Schwarzschild

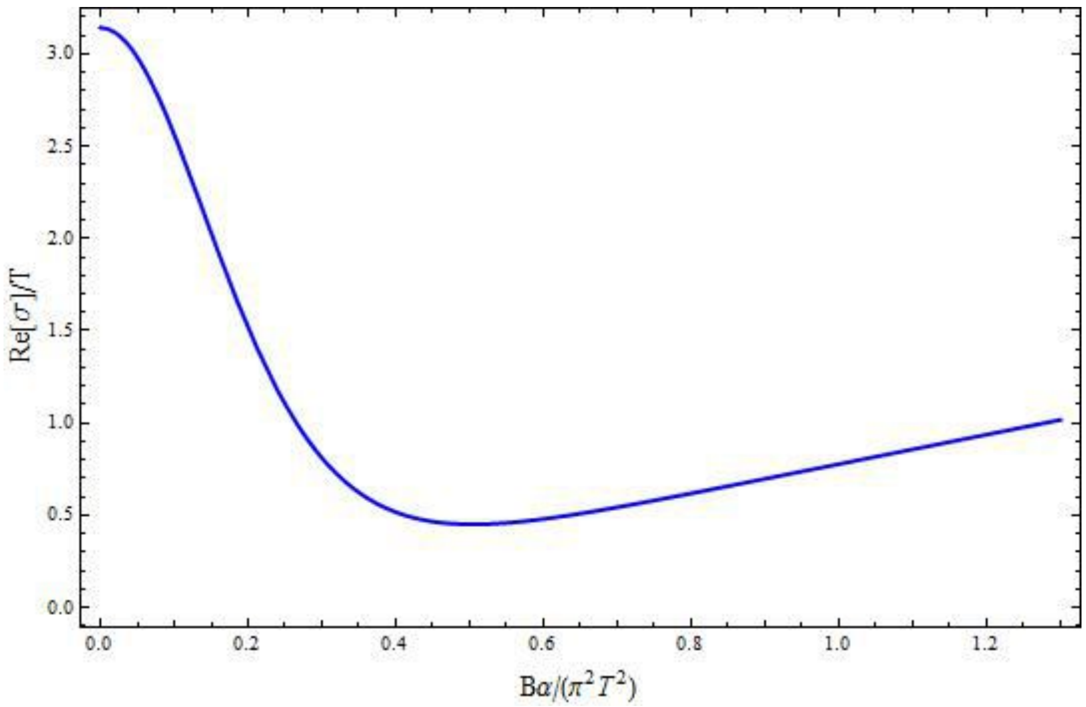
- Behavior of the first part as a function of B



III holographic calculations: Schwarzschild

$$\sigma_{zz} = \sigma_E + \tau_c \frac{B^2 c^2}{(\partial\rho/\partial/T)|_T}$$

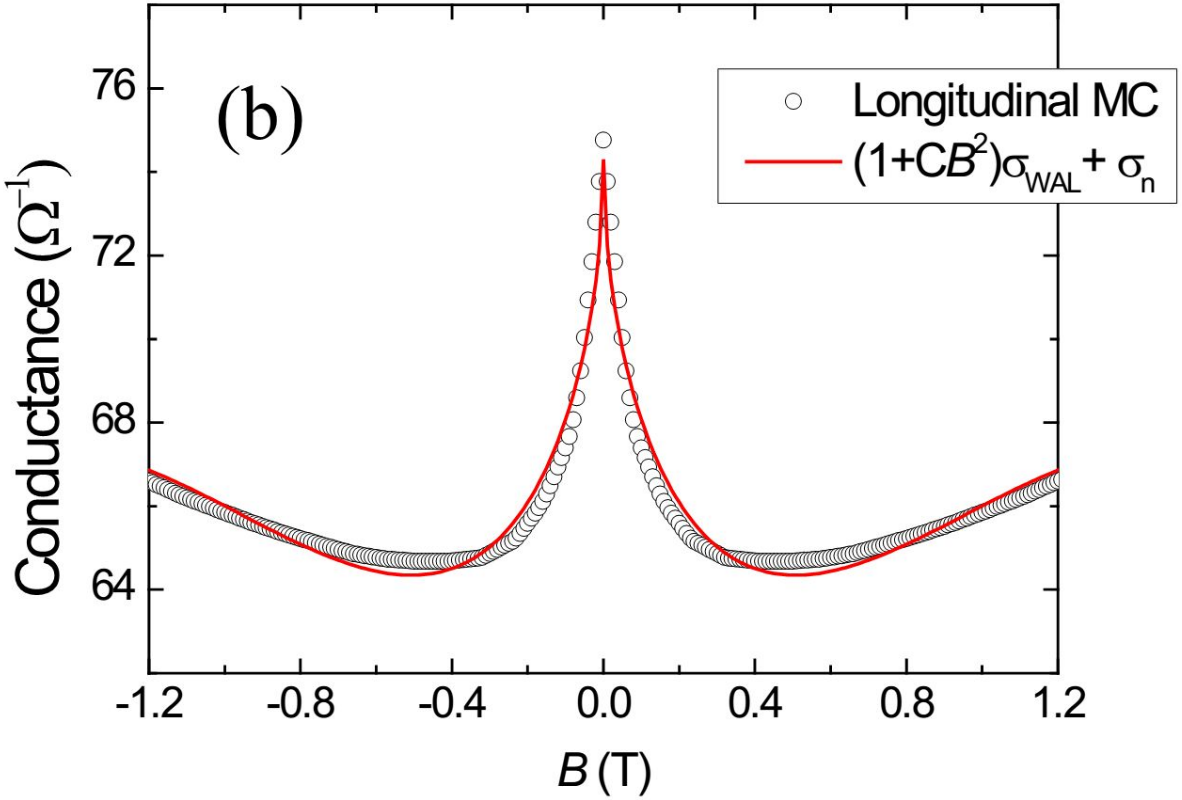
- Small B: first term dominates
- Large B: second term dominates,
- also depending on values of relaxational times



Comparison with experiments

$Bi_{1-x}Sb_x$

- Experimental data for MC(Kim et. al, 2013)
- Qualitatively similar: interesting small B upturn, a new explanation from holography: zero density conductivity due to chiral anomaly?



IV: A more natural system: $U(1)_V \times U(1)_A$

- Electric current, Axial current

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\alpha} J_\alpha, \\ \partial_\mu J^\mu &= 0, \\ \partial_\mu J_5^\mu &= cE^\mu B_\mu, \end{aligned}$$

- Constitute equations

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \Big(\frac{\mu}{T}\Big) - \sigma_5 T P^{\mu\nu} \partial_\nu \Big(\frac{\mu_5}{T}\Big) + \sigma^{(E)} E^\mu + \sigma^{(V)} \omega^\mu + \sigma^{(B)} B^\mu,$$
$$\nu_5^\mu = -\sigma_5 T P^{\mu\nu} \partial_\nu \Big(\frac{\mu}{T}\Big) - \sigma T P^{\mu\nu} \partial_\nu \Big(\frac{\mu_5}{T}\Big) + \sigma_5^{(E)} E^\mu + \sigma_5^{(V)} \omega^\mu + \sigma_5^{(B)} B^\mu,$$

- Dissipations, electric charge still conserved

$$\begin{aligned}
\partial_\mu \delta T^{\mu 0} &= \delta F^{0\mu} J_\mu + \frac{1}{\tau_e}\delta T^{\mu 0} u_\mu, \\
\partial_\mu \delta T^{\mu i} &= \rho \delta E^i + F^{i\lambda}\delta J_\lambda + \frac{1}{\tau_m}\delta T^{\mu i} u_\mu, \\
\partial_\mu \delta J^\mu &= 0, \\
\partial_\mu \delta J_5^\mu &= c\delta E^\mu B_\mu + \frac{1}{\tau_c}\delta J_5^\mu u_\mu.
\end{aligned}$$

- Electric conductivity

$$\Sigma = \sigma^{(E)} + \frac{i}{\omega + \frac{i}{\tau_e}} \frac{B^2 c \sigma^{(B)}}{D} K_0 + \frac{i}{\omega + \frac{i}{\tau_m}} \frac{\rho}{\epsilon + p} \left[\rho - \frac{B^2 c}{D} K_1 \right] + \frac{i}{\omega + \frac{i}{\tau_c}} \frac{B^2 c^2}{D} K_2$$

- Axial conductivity

$$\Sigma_5 = \sigma_5^{(E)} + \frac{i}{\omega + \frac{i}{\tau_e}} \frac{B^2 c \sigma^{(B)}}{D} W_0 + \frac{i}{\omega + \frac{i}{\tau_m}} \frac{\rho}{\epsilon + p} \left[\rho_5 - \frac{B^2 c}{D} W_1 \right] + \frac{i}{\omega + \frac{i}{\tau_c}} \frac{B^2 c^2}{D} W_2$$

VI Summary and ongoing work

- In chiral anomalous fluid, momentum, charge and energy dissipations are all needed to have a finite longitudinal DC magneto conductivity;
- We derived a universal formula for the longitudinal DC magneto conductivity in the hydrodynamic regime;
- For the probe Schwardzchild black hole background, we holographically checked the formula;
- The dependence of the longitudinal DC magneto conductivity on B is qualitatively the same as found in experiments; also consistent with previous weakly coupled results are large B: negative magnetoresistivity

VI Summary and ongoing work

- Ongoing work:
- *Holographic charge dissipation: massive gauge field (A. Jimenez-Alba, K. Landsteiner, L. Melgar, 2014)*

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{4}F^2 - \frac{1}{2}m^2A^2 + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu F_{\nu\rho}F_{\sigma\tau}\right)$$

The charge conservation equation should acquire a relaxational term for perturbations; fluid/gravity derivation;
- *Holographic check for the case without any dissipations;*
- *Holographic check for the case with momentum dissipation: massive gravity, Q lattice, ...*
- *Holographic energy dissipation*
- *Other transport coefficients: thermo-electric conductivity, violation of Wiedemann–Franz law*

Thank you!

A small advertisement:

Holographic duality for condensed matter physics
July 6-31, 2015, KITPC, Beijing