Non-equilibrium dynamics and compact stars in AdS/CFT

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Outline

1 Introduction

2 Perturbations of Schwarzschild and SUSY QM

3 Robinson-Trautman spacetimes

4 Holography

5 Conclusions

Gauge/gravity duality and non-equilibrium dynamics

- Gauge/gravity duality offers a new tool to study non-equilibrium dynamics at strong coupling.
- AdS black holes correspond to thermal states of the CFT.
- Black hole formation corresponds to thermalization.

Gauge/gravity duality and non-equilibrium dynamics

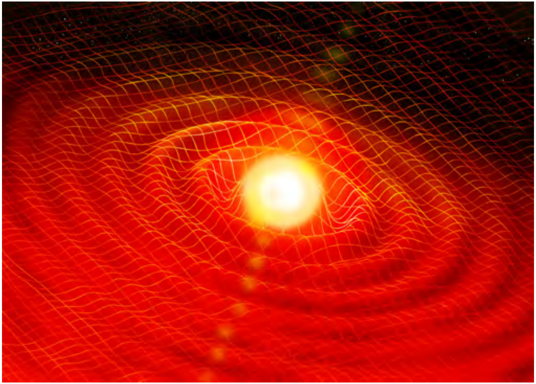
- Hydrodynamics captures the dynamics in the long wave-length, late time behavior of QFTs close to thermal equilibrium.
- On the gravitational side, one can construct bulk solutions in a gradient expansion that describe the hydrodynamic regime.
- Global solutions corresponding to non-equilibrium configurations should be well-approximated by the solutions describing the hydrodynamic regime at sufficiently long distances and late times.

Gauge/gravity duality and non-equilibrium dynamics

- Almost all work on global solutions is numerical.
- In this work we aim at obtaining analytic solutions describing out-of-equilibrium dynamics.
- We will discuss this in the context AdS_4/CFT_3.

Compact stars and AdS/CFT

- Another motivation for this work is to initiate the study of compact objects using AdS/CFT.



References

- This talk is based on work done with I. Bakas, 1404.4824.
- Related work appeared in [G. de Freitas, H. Reall, 1403.3537]

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Equilibrium configuration

- The thermal state corresponds to the AdS Schwarzschild black hole

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right),$$

with $f(r) = 1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2$.

- Linear perturbations around the Schwarzschild solution describe holographically thermal 2-point functions in the dual QFT.
- From those, using linear response theory, one can obtain the transport coefficients entering the hydrodynamic description close to thermal equilibrium.
- To describe out-of-equilibrium dynamics we need to go beyond linear perturbations.

Strategy

To describe analytically non-equilibrium phenomena and their approach to equilibrium we need

- Exact time-dependent solutions of Einstein equations.
- These solutions should limit at late times to the Schwarzschild solution.

➢ Can we find analytically exact solutions corresponding to linear perturbations of the Schwarzschild solution?

Linear perturbations of AdS Schwarzschild

Linear perturbations can be classified into parity even and parity odd. The parity even are parametrized by

$$\begin{pmatrix} f(r)H_0(r) & H_1(r) & 0 & 0 \\ H_1(r) & H_0(r)/f(r) & 0 & 0 \\ 0 & 0 & r^2K(r) & 0 \\ 0 & 0 & 0 & r^2K(r)\sin^2\theta \end{pmatrix} e^{-i\omega t} P_l(\cos\theta) ,$$

where $P_l(\cos\theta)$ are Legendre polynomials. (For simplicity we only display axially symmetric perturbations.)

- There is a similar formula for parity odd perturbations.

Effective Schrödinger problem

- The study of these perturbations can be reduced to an effective Schrödinger problem [Regge, Wheeler] [Zerilli] ...

$$\left(-\frac{d^2}{dr_\star^2} + W^2 \pm \frac{dW}{dr_\star}\right)\Psi(r_\star) = E\,\Psi(r_\star)\ .$$

- The two signs correspond to the parity even and odd cases.
- $E = \omega^2 - \omega_{\rm s}^2, \qquad \omega_{\rm s} = -\frac{i}{12m}(l-1)l(l+1)(l+2)\ .$
- $\Psi_{even}(r) = \frac{r^2}{(l-1)(l+2)r+6m}\left(K(r) - i\frac{f(r)}{\omega r}H_1(r)\right)$ and there is a similar formula for the odd case.
- $r_\star$ is the tortoise radial coordinate, $dr_\star = dr/f(r)$.
- $W(r) = \frac{6mf(r)}{r[(l-1)(l+2)r+6m]} + i\omega_{\rm s}$

Supersymmetric Quantum mechanics

- There is an underlying supersymmetric structure with W being the superpotential,

$$H_{even} = Q^{\dagger}Q + \omega_s^2, \qquad H_{odd} = QQ^{\dagger} + \omega_s^2$$

with

$$Q = \left(-\frac{d}{dr_\star} + W(r_\star)\right), \qquad Q^{\dagger} = \left(\frac{d}{dr_\star} + W(r_\star)\right)$$

- Forming

$$H = \begin{pmatrix} H_{even} & 0 \\ 0 & H_{odd} \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 0 & 0 \\ Q & 0 \end{pmatrix}$$

one finds that they form a SUSY algebra, $\{\mathbf{Q}, \mathbf{Q}^{\dagger}\} = H$ etc.

Remarks

- The Hamiltonian is only formally hermitian.
- Boundary conditions break supersymmetry.
- E is not bounded from below, it is not even real.

Zero energy solutions

- A special class of solutions are those with zero energy,

$$E = 0 \qquad \Leftrightarrow \qquad \omega = \omega_s$$

- These modes satisfy a first order equation

$$Q\Psi_0 = \left(-\frac{d}{dr_\star} + W(r_\star) \right) \Psi_0 = 0$$

They are the supersymmetric ground states of supersymmetric quantum mechanics.

- These are the so-called algebraically special modes [Chandrasekhar].
- It is these modes that we would like to study at the non-linear level.

Properties of the zero energy solutions

- Ψ_0 vanishes at the horizon.
- It satisfies mixed boundary conditions at the conformal boundary,

$$\frac{d}{dr_\star}\Psi_0(r_\star)\mid_{r_\star=0} = \left(i\omega_{\rm s} - \frac{2m\Lambda}{(l-1)(l+2)} \right) \Psi_0(r_\star = 0) \ .$$

- It is normalizable,

$$\int_{-\infty}^{0} dr_\star \mid \Psi_0(r_\star) \mid^2 < \infty \ .$$

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Robinson-Trautman spacetimes

- The metric is given by

$$ds^2 = 2r^2 e^{\Phi(z,\bar{z};u)} dz d\bar{z} - 2du dr - F(r,u,z,\bar{z}) du^2$$

- The function F is uniquely determined in terms of Φ,

$$F = r\partial_u \Phi - \Delta\Phi - \frac{2m}{r} - \frac{\Lambda}{3} r^2$$

  where Λ is the cosmological constant and $\Delta = e^{\Phi}\partial_z \partial_{\bar{z}}$ is the Laplace-Beltrami operator on S^2.
- The function $\Phi(z,\bar{z};u)$ should solve the *Robinson-Trautman equation*,

$$3m\partial_u \Phi + \Delta\Delta\Phi = 0.$$

Robinson-Trautman equation and the Calabi flow

- The Robinson-Trautman equation governs a particular geometric flow on S^2: the Calabi flow

$$\partial_u g_{z\bar{z}} = \frac{\partial^2 R}{\partial z \partial \bar{z}}, \qquad g_{z\bar{z}} = e^{\Phi(z,\bar{z};u)}$$

The Calabi flow is defined more generally for a metric $g_{a\bar{b}}$ on a Kähler manifold M .

- The area of S^2 and the average curvature $< R >$[1] are fixed along the flow, while $< R^2 >$ acts as an "entropy functional": it decreases monotonically along the flow.

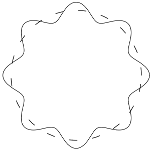
[1] $< A >= \int_{S^2} \sqrt{g} A$

Calabi flow on S^2

- The Calabi flow can be regarded as a non-linear diffusion process on S^2.
- Starting from a general initial metric $g_{a\bar{b}}(z, \bar{z}; 0)$, the flow deforms the metric to the constant curvature metric on S^2, described by

$$e^{\Phi_0} = (1 + z\bar{z}/2)^{-2} \ .$$

- Curvature perturbations of the round sphere dissipate under Calabi flow.



AdS Schwarzschild as Robinson-Trautman

- Using the fixed point solution of the Robinson-Trautman equation

$$e^{\Phi_0} = \frac{1}{(1 + z\bar{z}/2)^2} \, .$$

the metric becomes

$$ds^2 = \frac{2r^2}{(1 + z\bar{z}/2)^2} dz d\bar{z} - 2du dr - \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right) du^2$$

which is the Schwarzschild metric in the Eddington - Filkenstein coordinates.

Zero energy solutions as Robinson-Trautman

- Perturbatively solving the Robinson-Trautman equation around the round sphere

$$ds_2^2 = [1 + \epsilon_l(u) P_l(\cos\theta)] \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

one finds

$$\epsilon_l(u) = \epsilon_l(0) e^{-i\omega_{\rm s} u}$$

with

$$\omega_{\rm s} = -i \frac{(l-1)l(l+1)(l+2)}{12m}$$

- This is exactly the frequency of the zero energy solutions we found earlier!
- Inserting in the Robinson-Trautman metric we find the zero energy perturbations of AdS Schwarzschild we discussed earlier.

Summary

The Robinson-Trautman solution is a non-linear version of the algebraically special perturbations of Schwarzschild.

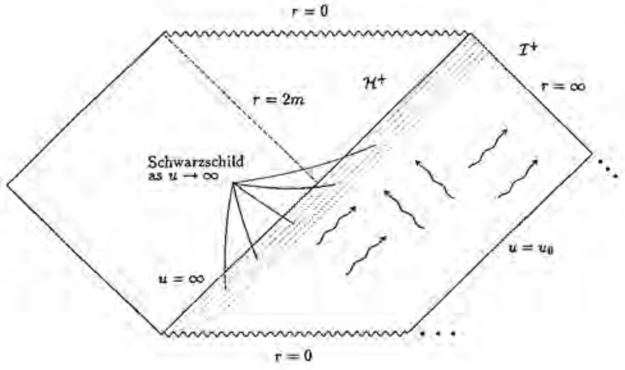
Late-time behavior of solutions [Chruściel, Singleton]

- We parametrize the conformal factor of the S^2 line element as
$$e^{\Phi(z,\bar{z};u)} = \frac{1}{\sigma^2(z,\bar{z};u)\,(1+z\bar{z}/2)^2}\,.$$
- $\sigma(z,\bar{z};u)$ has the following asymptotic expansion

$$1+\sigma_{1,0}(z,\bar{z})e^{-2u/m}+\sigma_{2,0}(z,\bar{z})e^{-4u/m}+\cdots+\sigma_{14,0}(z,\bar{z})e^{-28u/m}$$
$$+[\sigma_{15,0}(z,\bar{z})+\sigma_{15,1}(z,\bar{z})u]e^{-30u/m}+\mathcal{O}\left(e^{-32u/m}\right).$$

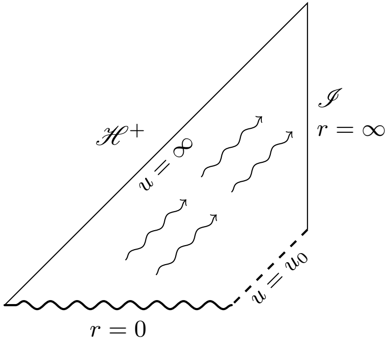
- The terms with $\sigma_{1,0},\sigma_{5,0},\sigma_{15,0},\ldots$ are due to the linear algebraically special modes with $l=2,3,4,\ldots$.
- The other terms are due to non-linear effects.

Global aspects: $\Lambda = 0$



- The extension across the horizon $\mathcal{H}^+$ is not smooth: the metric is C^{117}-differentiable not C^∞ [Chruściel, Singleton].

Global aspects: $\Lambda < 0$



- With $\Lambda < 0$ things are worse: for large black holes, the solution does not appear to have a smooth extension beyond $u \to \infty$ [Bicak, Podolsky].

Bondi mass

- In asymptotically flat spacetimes, a measure of mass at null infinity is the Bondi mass.
- For the Robinson-Trautman spacetimes it is equal to [Singleton]

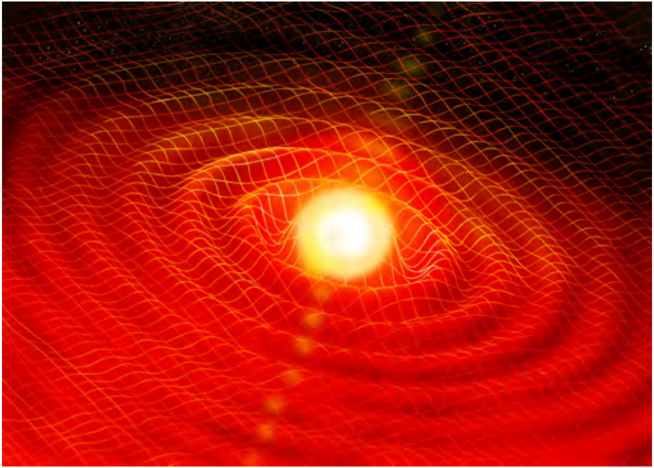
$$\mathcal{M}_{\rm Bondi}(u) = \frac{m}{4\pi} \int_{S^2} d\mu_0 \frac{1}{\sigma^3} \ ,$$

  where the integral with respect to the unit round metric on S^2.
- $\mathcal{M}_{\rm Bondi}$ has the properties

$$\mathcal{M}_{\rm Bondi} \geq m, \quad \frac{d}{du}\mathcal{M}_{\rm Bondi} \leq 0.$$

Interpretation of the solution when $\Lambda = 0$

- The solution describes the gravitational field due to a compact star which relaxes to equilibrium by radiating away its excess energy and asymmetry.
- As $u \to \infty$, we approach a spherically symmetric configuration and the gravitational field becomes that of the Schwarzschild solution.
- As the system radiates away the excess energy, $\mathcal{M}_{\rm Bondi}$ decreases and becomes equal to m as $u \to \infty$.
- There is only out-going radiation.



Bondi mass for Asymptotically AdS spacetimes?

- Asymptotically AdS spacetimes do not have null infinity, so a priori it is not clear if there is an analogue of the Bondi mass.
- Here we take the perspective that the formula for $\mathcal{M}_{\rm Bondi}$ in Robinson-Trautman spacetimes makes sense also for $\Lambda < 0$.
- The properties

$$\mathcal{M}_{\rm Bondi} \geq m, \quad \frac{d}{du}\mathcal{M}_{\rm Bondi} \leq 0.$$

continue to hold.

Penrose inequality and Hoop conjecture

- The Bondi mass satisfies a Penrose inequality

$$16\pi\mathcal{M}_{\text{Bondi}}^2 \geq \text{Area}(\Sigma)\left(1 - \frac{\Lambda}{3}\frac{\text{Area}(\Sigma)}{4\pi}\right)^2 .$$

where $\text{Area}(\Sigma)$ is the area of the past apparent horizon.

- Hoop conjecture (Thorne): *Horizons form when and only when a mass E gets compacted into a region whose circumference C in every direction is $4\pi E \geq C$.*
- We formulated (and proved for some cases) a version of this conjecture.

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R-T as an Asymptotically locally AdS solution

- When $\Lambda < 0$ the solution is asymptotically locally AdS. This means that near the conformal boundary the metric takes the Fefferman-Graham form:

$$ds^2 = \frac{d\varrho^2}{\varrho^2} + \frac{1}{\varrho^2}\left(g_{(0)ab}(x) + \varrho^2 g_{(2)ab}(x) + \varrho^3 g_{(3)ab}(x) + \cdots\right) dx^a dx^b$$

- One can reach this gauge by a coordinate transformation $(r^* = u - t, t, z, \bar{z}) \to (\varrho, t, z, \bar{z})$:

$$\begin{aligned} r_\star &\to \varrho + \qquad\qquad (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \\ t &\to t + (\quad)\varrho^2 + (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \\ z &\to z + \qquad\qquad (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \\ \bar{z} &\to \bar{z} + \qquad\qquad (\quad)\varrho^3 + (\quad)\varrho^4 + \mathcal{O}(\varrho^5)\,, \end{aligned}$$

Asymptotic structure

- The boundary metric is time-dependent and it is not conformally flat

$$ds_0^2 = -dt^2 - \frac{6}{\Lambda} e^{\hat{\Phi}(z,\bar{z};t)} dz d\bar{z} \ .$$

where $\hat{\Phi}(z,\bar{z};t) = \Phi(z,\bar{z};u = t - r^\star)|_{r^\star=0}$.

- $g_{(2)ab} = -\mathcal{R}_{ab} + \frac{1}{4}\mathcal{R} g_{(0)ab}$, where $\mathcal{R}_{ab}$ is the Ricci tensor of $g_{(0)}$, as expected [de Haro, Solodukhin, KS].

Energy-momentum tensor

- Asymptotically locally AdS spacetime have a conserved T_{ab} which in even dimension is traceless,

$$\nabla^b T_{ab} = 0, \qquad T^a_a = 0$$

- The tensor can be extracted from the asymptotics of the solution [Henningson, KS][de Haro, Solodukhin, KS]

$$T_{ab} = -\frac{3}{2\kappa^2}\left(-\frac{3}{\Lambda}\right) g_{(3)ab}$$

- For the R-T solution we obtain,

$$\kappa^2 T_{tt} = -\frac{2m\Lambda}{3}\,, \qquad \kappa^2 T_{tz} = -\frac{1}{2}\partial_z(\hat{\Delta}\hat{\Phi})$$
$$\kappa^2 T_{z\bar{z}} = m e^{\hat{\Phi}}\,, \qquad \kappa^2 T_{zz} = -\frac{3}{4\Lambda}\partial_t\left((\partial_z\hat{\Phi})^2 - 2\partial_z^2\hat{\Phi}\right),$$

Hydrodynamics

- As $t \to \infty$ the stress energy tensor approaches that of Schwarzschild AdS.
- Including the late time corrections, it takes the hydrodynamic form:

$$T^{ab} = (\rho + p)u^a u^b + p g^{ab} + \Pi^{ab}$$

  ρ is the energy density, p is the pressure. Conformal invariance implies

$$p = \rho/2$$

- Π^{ab} encodes the dissipation and it satisfies the Landau condition

$$u_a \Pi^{ab} = 0.$$

Late-time expansion

- One can obtain the late-time corrections to ρ, u^a, Π^{ab} to all orders:

$$\rho = \rho_0 + \rho_1 + \cdots\,, \quad u^a = u_0^a + u_1^a + \cdots\,, \quad \Pi^{ab} = \Pi_1^{ab} + \cdots$$

  where ρ_0, u_0^α are the equilibrium values.
- One can also define an entropy current

$$s^a = s u^a, \qquad s = s_0 + s_1 + \cdots$$

  where s_0 is the entropy at equilibrium which has a non-negative divergence

$$\nabla_a s^a \geq 0$$

Violation of KSS bound

➢ Specialising to linear algebraically special modes we obtain

$$\Pi_1^{ab} = -\eta\sigma^{ab}, \quad \eta = \frac{1}{4\kappa^2} l(l+1)$$

which then leads to

$$\frac{\eta}{s} = \frac{l(l+1)}{8\pi} \frac{r_{\rm h}}{2m - r_{\rm h}}$$

➠ The bound $\eta/s \geq 1/4\pi$ is violated for large black holes and small enough l.

Violation of KSS bound

- These modes however do not satisfy Dirichlet boundary conditions.
- The modes are out-going rather than in-going at the horizon.
- All modes that violate the bound do not extend smoothly beyond $u = \infty$ (however there are modes that do not have smooth extension but nevertheless satisfy the bound).

KSS bound and normalised Ricci-flow

- The result for η/s can be rewritten as

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\omega_s}{\Omega_S}$$

where

$$\Omega_S = -i\frac{(l-1)(l+2)}{3r_h}$$

- Curiously, these modes are associated with the late time expansion of a different geometric flow: the normalised Ricci flow.
- However, there are no (known) non-linear gravitational solutions associated with this flow.

Remarks

- The physics of the R-T solutions is different than the cases studied in the literature.
- In the usual cases: one injects energy to the system from the boundary and this energy is dissipated by getting absorbed by the black hole.
- In this case: nothing is absorbed by the black hole. There is only out-going radiation (coming from an isolated star which sheds its asymmetry by radiation) and the radiation in absorbed by the boundary. Dissipation is due to coupling to external sources.

Entropy production

- Computing the leading correction to the entropy we find

$$s_1 = \frac{\rho_1}{T_0}.$$

- ρ_1 is negative and goes to zero as $t \to \infty$.
- ⇒ As gravitational radiation is absorbed at the boundary the energy increases and there is entropy production.

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Conclusions/Outlook

- The Robinson-Trautman solution is a non-linear version of the algebraically special perturbation of Schwarzschild.
- One can study quantitatively and analytically the approach to equilibrium and the effects of non-linear terms.
- It would be interesting to understand better holography for these solutions, in particular the implications of the unusual boundary conditions, the holographic meaning of the Bondi mass, the Penrose inequality, the hoop conjecture, etc. ...