Duality Between Zeroes and Poles in Holographic Systems

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Outline

1. Introduction
2. Holographic Description of Zeros and Poles
3. Duality Between Zeroes and Poles
4. Conclusions

Introduction

The application of the AdS/CFT correspondence to condense matter physics has developed into one of the most productive topics of string theory.

Holographic principle: understanding strongly coupled phenomena of condensed matter physics by studying their weakly coupled gravity duals.

Applications to:

- Conventional and unconventional superfluids and superconductors
  [S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008)]
- Fermi liquids
  [S. Bhattacharyya, V. E. Hubeny, S. Minwalla and M. Rangamani, JHEP **0802**, 045 (2008)]
- Quantum phase transitions
  [M. Cubrovic, J. Zaanen and K. Schalm, Science **325**, 439 (2009)]

Application of the Holographic Principle to Zeros and Poles

The charge density remains fixed under renormalization from high (UV) to low (IR) energy, however the carriers can change drastically.

In QCD: free quarks at UV scales, bound states in the IR scales.

- Free quarks at UV scales leads to the appearance of a pole in the propagator.
- In the IR the pole in the propagator is converted to a zero which is the signature of bound quark states.

  [G. 't Hooft, N. Phys. B **75**, 461 (1974)]

The conversion of poles to zeros of the single-particle Green function also appears in superconductivity.

Free quarks of QCD correspond to free electrons in superconductivity.

The question of how to compute the number of low-energy charged particle states is then problematic because what was a particle (pole) at high energy is no longer so at low energy.

In condensed matter physics the particle density in a fermionic system is given by Luttinger's theorem:
The volume enclosed by a material's Fermi surface is directly proportional to the particle density

$$n = 2 \int_{G(k,\omega=0)>0} \frac{d^D k}{(2\pi)^D}$$

where $G(k, \omega = 0)$ is the Green Function. Then the Green Function can be zero or infinite.

[J. M. Luttinger, Phys. Rev. **119**, 1153 (1960)]

However:

- Poles of the single-particle Green function represent quasiparticles.
- Zeros indicate the presence of a gap or equivalently the self energy diverges indicating a breakdown of perturbation theory.

Result:
The Luttinger's theorem is valid only if the volume of the Fermi surface is independent of the interactions.

[K. B. Dave, P. W. Phillips and C. L. Kane, Phys. Rev. Lett. **110**, 090403 (2013)]

Holographic Description of Zeros and Poles

Consider the action

$$S = \int d^4x\sqrt{-g}\left[\frac{R+6/L^2}{16\pi G} - \frac{1}{4}F_{MN}F^{MN}\right]$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ and for convenience $L = 4\pi G = 1$.
The Einstein-Maxwell equations admit a charged four-dimensional AdS black hole solution,

$$ds^2 = \frac{1}{z^2}\left[-h(z)dt^2 + \frac{dz^2}{h(z)} + dx^2 + dy^2\right].$$

The metric function is given by

$$h(z) = 1 - \left(1+\mu^2\right)z^3 + \mu^2 z^4$$

with the horizon radius set at $z = 1$, and the $U(1)$ potential is

$$A_t = \mu\left(1 - z\right)\ ,\quad A_z = A_x = A_y = 0$$

corresponding to a non-vanishing electric field in the radial z direction,

$$F_{tz} = -F_{zt} = \mu.$$

The Hawking temperature is given by

$$T = -\frac{h'(1)}{4\pi} = \frac{3-\mu^2}{4\pi}\ ,$$

with $\mu^2 = 3$ providing the zero temperature limit.

Next add a massless fermion with charge q. The action is

$$S_{\text{fermion}} = i \int d^4x \sqrt{-g} \bar{\Psi} \left[\not{D} - p \Sigma^{MN} F_{MN} \right] \Psi$$

and included a dipole coupling to the $U(1)$ field.

[M. Edalati, R. Leigh, and P. W. Phillips, Phys. Rev. Lett. **106**, 091602 (2011); Phys. Rev. D **83**, 046012 (2011)]

The various terms in the action are

$$\begin{aligned}
\not{D} &= e_a^M \Gamma^a \left(\partial_M + \Omega_M - iqA_M \right) \ , \\
\Omega_M &= \frac{1}{8} \omega_{abM} \left[\Gamma^a, \Gamma^b \right] \ , \\
\omega_{abM} &= \eta_{ac} \omega^c_{bM} \ , \\
\omega^a_{bI} &= e^a_M \partial_I e^M_b + e^a_M e^N_b \Gamma^M_{NI} \ , \\
\Sigma^{MN} &= \frac{i}{4} \left[\Gamma^a, \Gamma^b \right] e^M_a e^N_b
\end{aligned}$$

with spin connection ω_{abM} and vierbein e_a^M, and lower-case indices a, b belong to the tangent space.

Notice:

- In the case, $p = 0$, this is a system of two non-interacting Weyl fermions of opposite chiralities.
- With $p \neq 0$, the dipole interaction term introduces an interaction between the two Weyl fermions. The system corresponds to an order parameter in the dual gauge theory of conformal dimension $\Delta = \frac{3}{2}$.

  To solve the Dirac equation, we choose the *ansatz*

$$\Psi = e^{-i\omega t - ikx^1} \sqrt{\frac{z^3}{h}} \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix} \quad , \qquad \psi_\pm = \begin{pmatrix} \psi_{\pm 1} \\ \psi_{\pm 2} \end{pmatrix}$$

and an appropriate basis of the Dirac matrices. At the horizon, we choose in-going boundary conditions,

$$\psi_{\pm,12} = (1-z)^{-i\omega/(4\pi T)} \mathcal{F}_{\pm,12}$$

Defining the ratios

$$\xi_\pm = \frac{\mathcal{F}_{\pm 1}}{\mathcal{F}_{\pm 2}}$$

we write the non-linear flow equations

$$h\xi'_\pm + \left[\omega + \mu q(1-z) + \sqrt{h}\,(\pm k + \mu p z)\right]\xi_\pm^2$$
$$+\omega + \mu q(1-z) + \sqrt{h}\,(\mp k - \mu p z) = 0$$

together with the in-going boundary conditions,

$$\xi_\pm = \begin{cases} i & , \quad \omega \neq 0\,, \\ i\sqrt{\frac{q/\sqrt{2}+\sqrt{3}p\pm k}{q/\sqrt{2}-\sqrt{3}p\mp k}} & , \quad \omega = 0 \end{cases}$$

[S. S. Lee, Phys. Rev. D **79**, 086006 (2009)]
[H. Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 065029 (2011)]

The solution to the flow equations determines the retarded Green function as

$$G_R(\omega, k) = \begin{pmatrix} G_+ & 0 \\ 0 & G_- \end{pmatrix}, \quad G_\pm(\omega, k) = \xi_\pm|_{z\to 0}$$

From the symmetries of the Dirac equation we deduce the relation between the Green functions

$$G_\pm(\omega, k) = G_\mp(\omega, -k).$$

The Fermi momentum is found as the pole

$$G_\pm^{-1}(\omega = 0, k = k_F) = 0.$$

Near the holographic Fermi surface, the structure of the poles can be expanded as

$$G_R^{-1}(\omega, k) \sim k - k_F - \omega/v_F - \Sigma(\omega, k)$$

with self-energy Σ

The excitations near the Fermi surface were shown to consist of a Fermi liquid, marginal Fermi liquid, and non-Fermi liquid, dependent on the mass and charge of the fermion.

[T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, Phys. Rev. D **83**, 125002 (2011)]

To identify the type of (non-) Fermi fluid, we look at the near-horizon region.

Define the scaling dimension $\nu_k^{\pm}$. It can be shown to be

$$6\left(\nu_k^{\pm}\right)^2 = \left(\sqrt{3}p \pm k\right)^2 - \frac{q^2}{2}$$

The self energy Σ at the Fermi surface in the Green function behaves as

$$\Sigma(\omega = 0, k) \sim \omega^{2\nu_k}$$

It is this scaling dimension that dictates the dispersion relation at the Fermi surface, and hence the type of fluid.

We get:

- For scaling dimension $\nu_k < 1/2$, the pole of G_R corresponds to an unstable quasi-particle. This is identified as a non-Fermi fluid.
- With the value $\nu_k = 1/2$, the excitations are of marginal Fermi fluid type.
- For $\nu_k > 1/2$ the dispersion relation is linear. This is the Fermi (non-Landau) fluid.
- Imaginary ν_k corresponds to "log oscillatory" solutions.

Result:

For fixed fermion charge and ν_k the quasi-particle character is determined by the strength of the dipole interaction p.

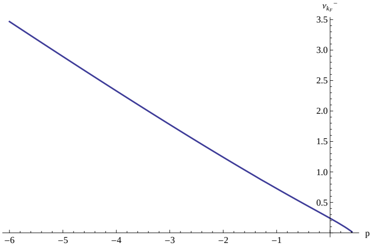

Figure: Scaling dimension ν_k^- *vs.* p for $q = 1$. ν_k^+ is deduced by reflecting $k \to -k$.

- Fermi liquid ($p \lesssim -.53$)
- Marginal Fermi liquid ($p \sim -.53$)
- Non-Fermi liquid ($-.53 \lesssim p \lesssim .41$)
- Log-oscillatory ($p \gtrsim .41$)

Duality Between Zeroes and Poles

It is known that in the conventional case, $p = 0$,

$$\det G_R(\omega = 0, k; p = 0) = 1$$

therefore it possesses neither poles nor zeroes.

This is because poles (zeroes) of G_+ are cancelled by zeroes (poles) of G_- at the same momentum.

If $p \neq 0$ then the coincidence of poles and zeroes is lifted.

Observe that if we define the reciprocal of $\xi_\pm$,

$$\zeta_\pm = \frac{1}{\xi_\pm}$$

then the flow equation becomes

$$h\zeta'_\pm + \left[-\omega - \mu q(1 - z) + \sqrt{h}\,(\pm k + \mu p z)\right]\zeta_\pm^2$$
$$-\mu q(1 - z) + \sqrt{h}\,(\mp k - \mu p z) - \omega = 0$$

However, looking at the two flow equations we see that $\zeta_\pm$ solves the same equation as $-\xi_\pm$ under the change of parameters $k \to -k$ and $p \to -p$.

It follows that the inverse Green function $G_\pm^{-1}(0, k)$ at p is identified with $-G_\pm(0, -k)$ at opposite dipole coupling $-p$.

Using the relation between the two Green functions we get

$$\det G_R(\omega = 0, k; p) = \frac{1}{\det G_R(\omega = 0, k; -p)}$$

Important result:

There is a relation between poles and zeroes of the determinant of the Green function at zero frequency between systems of opposite dipole coupling.

[J. Alsup, E. Papantonopoulos, G. Siopsis and K. Yeter, arXiv:1404.4010 [hep-th]]

[G. Vanacore and P. W. Phillips, arXiv:1405.1041 [cond-mat.str-el]]

For large negative dipole coupling strength, i.e., $p \lesssim -.53$, we are in the Fermi liquid phase. Then for $p = -5$ we find two poles at $k = k_F \approx \pm 1.5$, and no zeroes.

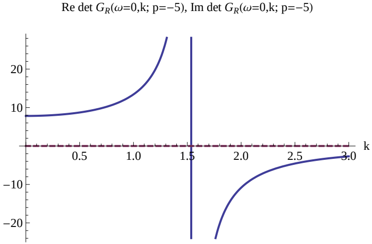

Figure: Plot of $\Re \det G_R$ ($\Im \det G_R = 0$) with $q = 1$ for $p = -5$ showing a pole at $k = k_F \approx 1.5$.

According to the duality we expect to see two zeroes at $k = k_L \approx \pm 1.5$ and no poles for a system with $p = 5$.

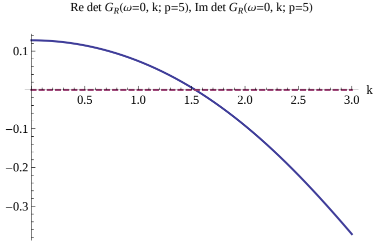

Figure: Plot of $\Re \det G_R$ ($\Im \det G_R = 0$) with $q = 1$ for $p = 5$ showing a zero at $k = k_L \approx 1.5$.

This is in the Mott insulator regime and no Fermi surface is expected.

For small values of the dipole coupling strength ($|p| \lesssim .41$), we expect both zeroes and poles.

Let us consider the case for $p = \pm .1$. Unlike in the case of large p, $\Im \det G_R$ does not vanish. Instead, the zeroes for $p = .1$ found at $k = k_L \approx \pm .8$ are isolated zeroes of both $\Re \det G_R$ and $\Im \det G_R$.

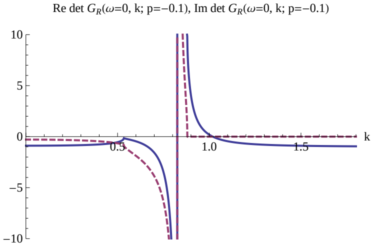

Figure: Plots of $\Re \det G_R$ (solid lines) and $\Im \det G_R$ (dashed lines) with $q = 1$ for $p = -0.1$ showing a pole at $k = k_F \approx 0.8$ and a zero at $k = k_L \approx 1.0$.

Correspondingly, the same is true for the poles at $k = k_F \approx \pm.8$ for $p = -.1$. At the other set of zeroes at $k = k_L \approx \pm 1.0$ for $p = -.1$ (and correspondingly poles at $k = k_F \approx \pm 1.0$ for $p = .1$), $\Im \det G_R$ vanishes over a range containing the zeroes (poles).

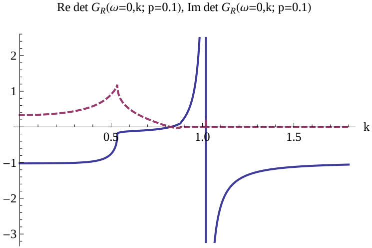

Figure: Plots of $\Re \det G_R$ (solid lines) and $\Im \det G_R$ (dashed lines) with $q = 1$ for $p = 0.1$ showing a pole at $k = k_F \approx 1.0$ and a zero at $k = k_L \approx 0.8$.

Let us show the location of poles ($k = k_F$) and zeroes ($k = k_L$) as the dipole coupling strength p varies. Notice the symmetry under $k \to -k$, as well as the interchange $k_L \leftrightarrow k_F$ under the mapping $p \to -p$.

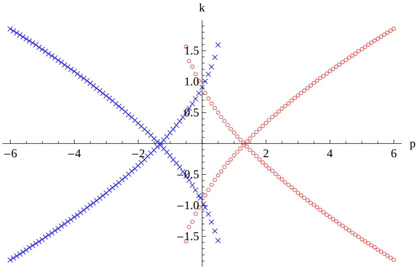

Figure: Poles at $k = k_F$ (blue lines) and zeroes at $k = k_L$ (red lines) *vs.* p with $q = 1$. Notice the symmetry under $k \to -k$, and the duality of poles and zeroes under $p \to -p$.

Conclusions

We studied a holographic theory with a bulk dipole interaction between a massless fermion and gauge field.

We found

- The system possesses a robust phase diagram including Fermi and non-Fermi liquids, insulating Mott state and pseudo-gap state.
- The identifying feature of these is the behavior of the Green function.
- A pole is indicative of a (non-) Fermi fluid while a zero is responsible for an insulating phase. It is the coexistence of both that underlies the holographic pseudo-gap state.
- A duality exists relating systems of opposite dipole coupling strength p. This duality maps zeroes to poles and vice versa.

Further work

- Explore further the pseudo-gap state and other aspects that may be recovered by the evolution of the Fermi and Luttinger surfaces with $\vec{k}$ dependence.
- Consider how a bulk scalar responsible for superconductivity will influence the system.
- Try to understand how the entanglement entropy behaves in these systems. The entanglement entropy counts the degrees of freedom of the system. We expect that entanglement entropy will vary as we vary the dipole moment p.