Holography as a special RG flow

Ayan Mukhopadhyay
(CPHT Ecole Polytechnique and IPhT CEA-Saclay, France)

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A fundamental question : Why the holographic principle may be applicable beyond those explicit examples obtained from string theory?

Intuitive answer : The radial dimension in the dual theory of gravity is the scale of a RG flow for (a subset of) degrees of freedom in the QFT.

Can we make this precise and construct gravity from quantum field theory?

Need to work on two fronts:

(i) Learn how to extract scale-dependent observables in field theory from gravity

Questions:

Can we write equations of gravity directly as a first order RG flow of field theory observables?

What conditions on this (geometric) RG flow leads to regular space-time (that determines right values of field theory observables)?

(ii) Decode the geometric RG flow as a coarse-graining in field theory

Questions:

How to understand emergence of bulk diffeomorphism symmetry?

What are the field-theoretic principles that determine this RG flow which reproduces gravity?

In this talk, we will show these questions can be answered in a special dynamical sector.

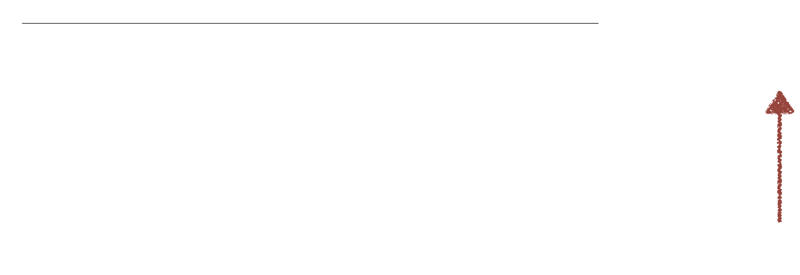
References:

(i) S. Kuperstein and AM, arXiv:1105.4530 and arXiv:1307.1367 (published in JHEP)
(ii) N. Behr and AM, arXiv:1409.xxxx

Related approaches : Polchinski et. al.; Liu, Rangamani et. al. (gravity side); S. S. Lee; R. Leigh et. al. (path-integral); B. Swingle and S. Ryu et. al. (entanglement renormalisation)

STRONGLY INTERACTING AND LARGE N LIMIT

Strongly interacting - a large gap in the scaling dimensions of operators



$$\Delta_{\text{gap}} \sim \lambda^k, k > 0$$

This gap is tuneable via some parameters in the theory

All these operators form an algebra generated via FINITE number of elements:

$T_{\mu\nu}$	energy-momentum tensor
j_μ	conserved currents
$\mathcal{O}$	order parameters for various phases, etc.

Natural in supersymmetric theory — otherwise can be realised via consistent truncation of SUGRA in holography

These generators are the abstract versions of “single-trace operators”.

There is another tuneable parameter “N” which leads to factorisation of expectation values in ALL states

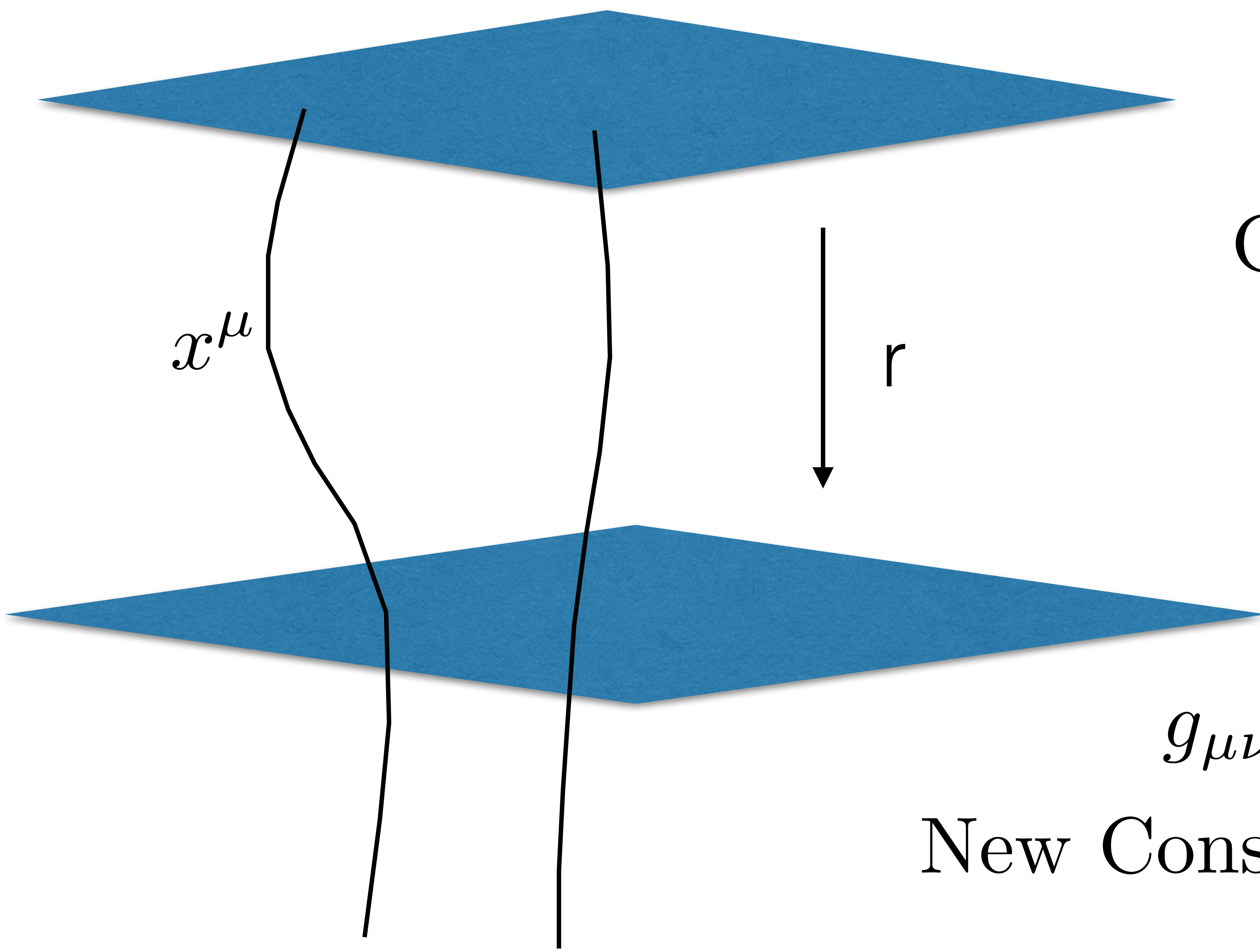
$$\langle T_{\mu\nu} T_{\rho\sigma} \rangle = \langle T_{\mu\nu} \rangle \langle T_{\rho\sigma} \rangle + O(\frac{1}{N^2}), \text{ etc.}$$

Strongly interacting and large N limit implies that the dual gravity theory (if it exists) is similar to classical Einstein’s equations and tractable.

Note single trace operators will mix with all multi-trace operators under RG flow.

This is why the dual gravity theory is non-linear.

Basic Principle



UV (Boundary)

$$\eta_{\mu\nu}, < T_{\mu\nu} >$$

$$\text{Constraint} : \partial^\mu \langle T_{\mu\nu} \rangle = 0$$

Finite scale

$$g_{\mu\nu}, < T_{\mu\nu}(\Lambda) > \;\; (r = \Lambda^{-1})$$

$$\text{New Constraint} : \nabla^\mu \langle T_{\mu\nu}(\Lambda) \rangle = 0$$

Hypersurface foliation: Σ_r
Radial coordinate: r

Congruence of curves: $\mathcal{C}$
Field-theory coordinates: x^μ

The background metric becomes dynamical in RG flow in order to absorb effect of mixing of single-trace operators with multi-trace operators. (Polchinski et. al; Liu et. al.; Lee)

Here we will understand this in the Heisenberg picture. Advantage : Works for all states.

Holography = A “highly efficient” RG flow which keeps the operator equations for “single trace” operators invariant in form by suitably redefining the background metric and sources.

Necessary but not sufficient condition to see emergence of gravity

Consider projecting out certain fast degrees of freedom above a certain scale

$$T_{\mu\nu}(\Lambda) = \mathcal{P}^\dagger(\Lambda) T^{\mathrm{UV}}_{\mu\nu} \mathcal{P}(\Lambda)$$

$$\partial^\mu T^{\mathrm{UV}}_{\mu\nu} = 0$$

Can we define a sequence of projection operators such that

$$\partial^\mu T_{\mu\nu}(\Lambda) = \frac{1}{4\Lambda^4} \partial_\nu \left(T_{\alpha\beta}(\Lambda) T^{\alpha\beta}(\Lambda) \right) - \frac{1}{2\Lambda^4} \left(\partial^\alpha \operatorname{Tr} T(\Lambda) \right) T_{\alpha\nu}(\Lambda) + O(|T_{\alpha\beta}(\Lambda)|^3)$$

Then redefining the background metric as

$$g_{\mu\nu}(\Lambda) = \eta_{\mu\nu} + \Lambda^{-4} T_{\mu\nu} + \cdots$$

the operator equation becomes:

$$\nabla^{\mu}_{(\Lambda)} T_{\mu\nu}(\Lambda) = 0$$

Take expectation values, assume large N factorisation and $r = \Lambda^{-1}$

This reduces to the constraint of classical gravity equation at the hyper-surface.

A major issue:

$g_{\mu\nu}(\Lambda)$ can be constructed from $T_{\mu\nu}(\Lambda)$

Physical parameters in $T_{\mu\nu}(\Lambda)$ should have first order RG evolution

Therefore we should re-package classical gravity equations as:

(i) **First order** evolution of physical parameters in $T_{\mu\nu}(\Lambda)$

(ii) Prove that an **appropriate infrared fixed point** determine the physical parameters (giving regularity of horizon) !!

(iii) Then r**econstruct** $g_{\mu\nu}(\Lambda)$ from these physical parameters and the **boundary metric**

However we require two additional conditions:

(i) The RG flow has an automorphism of a special type that lifts Weyl symmetry at UV to an arbitrary scale — (this automorphism has full information about choice of $\Sigma_r, \mathcal{C}$)

(ii) The flow ends at a good infrared fixed point (could end at a finite scale as in MERA approach).

Conjecture : This “highly efficient” RG flow reproduces classical gravity equations.

We have constructed the explicit projection operators in a dynamical sector, but we have yet not proven this is the uniquely allowed construction.

Exact asymptotic hydrodynamic expansion

At a sufficiently high temperature (greater than any microscopic or dynamically generated scale) the low energy modes of any system are hydrodynamic modes.

The expectation values of **all** operators can be parametrised by hydrodynamic variables (velocity and temperature in locally charge neutral states) near equilibrium.

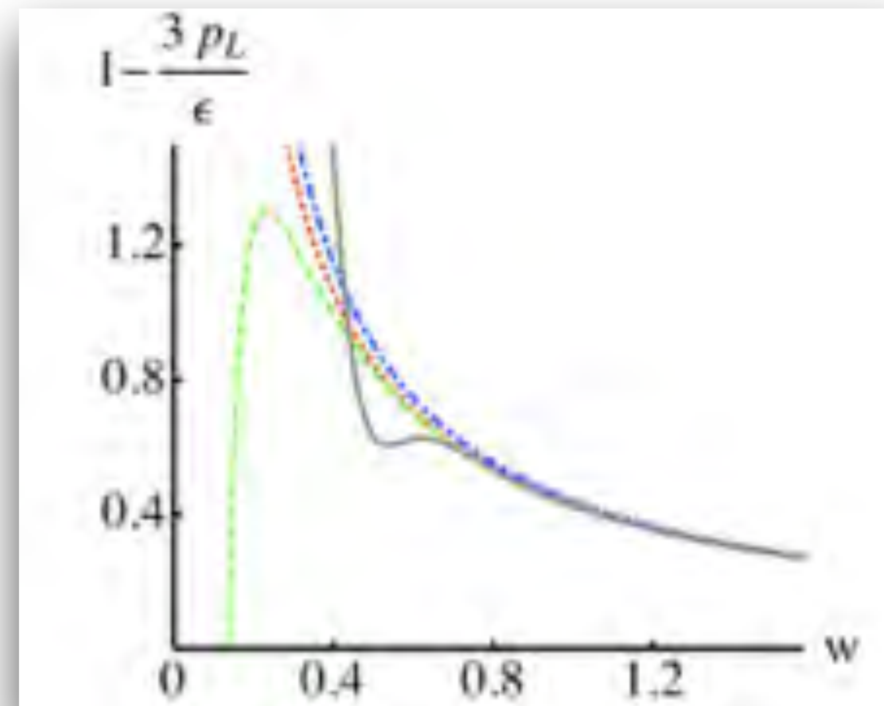
Example : Normal solutions of Boltzmann equation (Chapman and Enskog 1917)

This is an **asymptotic series** in the derivative expansion involving infinitely many terms — it is **exact and involves no direct coarse graining**.

The exact asymptotic hydrodynamic expansion is obtained by holography via fluid/gravity solutions (Son, Starinets, Policastro; Janik, Heller; Minwalla, Rangamani, Hubeny, Bhattacharyya).

Transport coefficients are determined by regularity of horizon.

The asymptotic series reproduces “hydrodynamic tails” of numerics (Janik and Heller).



We can "freeze" all other operators consistently so that only em-tensor has non-trivial state-dependence (pure gravity sector).

In this case, the only relevant equation is $\partial^\mu T^{\mathrm{UV}}_{\mu\nu} = 0$

This gives hydrodynamic equations with infinite number of derivative corrections to Navier-Stokes.

Coarse-graining should improve the convergence of the asymptotic series. This justifies a RG flow with scale-dependent equation of state, transport coefficients, etc.

This RG flow of exact asymptotic hydrodynamic expansion should end at incompressible non-relativistic Navier Stokes. In holography this is borne out by "membrane paradigm" (Damour, Thorne et. al.)

Notations and basic expressions

in flat space

generally

$$u^\mu \to \frac{1}{\sqrt{1-\mathbf{v}^2}}(1, v^i) \approx (0, v^i)$$

$$u^\mu g_{\mu\nu} u^\nu = -1$$

acceleration

$$a^\mu = (u\cdot\nabla)u^\mu \approx (0, a^i)$$

$$a^\mu u_\mu = 0$$

$$\Delta_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu} \approx \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} \end{pmatrix}$$

$$\Delta^\mu_\rho \Delta^\rho_\nu = \Delta^\mu_\nu$$

$$\Delta^\mu_\rho u^\rho = 0$$

$$<< A_{\mu\nu} >>= \frac{1}{2}\Delta^\rho_\mu \Delta^\sigma_\nu (A_{\mu\nu} + A_{\nu\mu}) - \frac{1}{d-1}\Delta_{\mu\nu}\Delta^{\alpha\beta}A_{\alpha\beta}$$

$$\sigma_{\mu\nu} =<< \nabla_\nu u_\nu >>\approx \frac{1}{2}(\partial_i v_j + \partial_j v_i)$$

shear

$s^{(n,m)}$	Hydro scalars with n derivatives	$\nabla \cdot u$, R, $(\nabla \cdot u)^2$, $\sigma_{\mu\nu}\sigma^{\mu\nu}$, etc. $m = 1, \cdots, n_s$
$v^{(n,m)\mu}$	Hydro transverse vectors with n derivatives	a^μ, $\Delta^{\mu\rho}\nabla_\alpha \sigma_{\alpha\rho}$, etc. $m = 1, \cdots, n_v$
$t^{(n,m)}_{\mu\nu}$	Hydro traceless transverse tensors with n derivatives	$\sigma_{\mu\nu}$, $<< R_{\mu\nu} >>$, $<< \sigma_\mu^{\ \rho}\sigma_{\rho\nu} >>$, etc. $m = 1, \cdots, n_t$

Magic Ansatz for Gravity : Beta function formulation

$$\mathrm{u}^{\mu\prime} = \alpha^{(0)}\mathrm{u}^{\mu} + \alpha_{\mathrm{s}}^{(1)}(\nabla\cdot\mathrm{u})\mathrm{u}^{\mu} + \alpha_{\mathrm{v}}^{(1)}(\mathrm{u}\cdot\nabla)\mathrm{u}^{\mu} + \sum_{n=2}^{\infty}\sum_{m=1}^{n_s}\alpha_s^{(\mathrm{n,m})}\, s^{(\mathrm{n,m})}u^{\mu} + \sum_{n=2}^{\infty}\sum_{m=1}^{n_v}\alpha_v^{(\mathrm{n,m})}\, v^{(\mathrm{n,m})\mu}$$

$$(\ln T)' = \lambda^{(0)} + \lambda^{(1)}(\nabla\cdot\mathrm{u}) + \sum_{\mathrm{n}=2}^{\infty}\sum_{\mathrm{m}=1}^{\mathrm{n_s}}\lambda^{(\mathrm{n,m})}\,\mathrm{s}^{(\mathrm{n,m})}$$

$$\langle\mathrm{T}_{\mu\nu}^{\mathrm{phys}}(\Lambda)\rangle = \epsilon\mathrm{u}_{\mu}\mathrm{u}_{\nu} + \mathrm{P}\Delta_{\mu\nu} - \eta\sigma_{\mu\nu} - \zeta(\nabla\cdot\mathrm{u})\Delta_{\mu\nu} + \sum_{\mathrm{n}=2}^{\infty}\sum_{\mathrm{m}=1}^{\mathrm{n_t}}\gamma_{\mathrm{t}}^{(\mathrm{n,m})}\ \mathrm{t}_{\mu\nu}^{(\mathrm{n,m})} + \sum_{\mathrm{n}=2}^{\infty}\sum_{\mathrm{m}=1}^{\mathrm{n_s}}\gamma_{\mathrm{s}}^{(\mathrm{n,m})}\ \mathrm{s}^{(\mathrm{n,m})}\Delta_{\mu\nu}$$

$$r = \Lambda^{-1}, \quad {}' = \partial_r = -\Lambda^2\partial_{\Lambda}$$

In Fefferman-Graham gauge this anstaz leads directly to the following without requiring us to solve Einstein's equations explicitly:

(i) elimination of redundant variables in the evolution of hydro variables
(ii) first order evolution of transport coefficients.

Requiring that the flow ends at **incompressible non-relativistic Navier Stokes at a finite scale (horizon)** gives **unique solutions to all transport coefficients**.

Remarkably at the boundary we reproduce exactly those values of transport coefficients obtained via fluid/gravity solutions from **regularity of horizon**.

The space-time metric can be reconstructed from RG flow once boundary metric is specified.

$$\epsilon' = \left(\frac{\mathrm{r}^{\mathrm{d}-1}}{2(\mathrm{d}-1)}\epsilon - \frac{1}{\mathrm{r}}\right)\mathrm{Tr}\,\mathrm{T}$$

$$(\mathrm{Tr}\,\mathrm{T})' = \left(\frac{\mathrm{r}^{\mathrm{d}-1}}{2(\mathrm{d}-1)}\mathrm{Tr}\,\mathrm{T} + \frac{\mathrm{d}}{\mathrm{r}}\right)\mathrm{Tr}\,\mathrm{T}$$

$$\zeta' - \left(\frac{\mathrm{d}-1}{\mathrm{r}}(\mathrm{c}_\mathrm{s}^2+1) + \mathrm{r}^{\mathrm{d}-1}\left(\mathrm{P} - \frac{1}{2}\mathrm{c}_\mathrm{s}^2\epsilon\right)\right)\zeta = 0$$

$$\left(\frac{\eta}{s}\right)' = 0$$

Liu, Iqbal

Similarly,

$$\gamma_3' + \frac{d-2}{2(d-1)}r^{d-1}\epsilon\gamma_3 = 2\frac{d+1}{d-1}r^{d-1}\eta^2 + \frac{r^{d-1}}{2}(P+\epsilon)$$
$$\left(\left(c_s^2 + \frac{3}{d-1}\right)\left(\gamma_1 - \frac{2r^{2-d}}{d-2}\right)\right.$$
$$\left. + \left(c_s^2 + \frac{d}{d-1}\right)\gamma_2\right)$$

Incompressible Navier-Stokes at the horizon

Horizon is the scale **r = r_H** where **P** blows up.

$\epsilon,\ \eta,\ \zeta$ have to be finite at horizon

For second order scalar transport coefficients:

Transport coefficient	Corresponding scalar	Allowed leading order near-horizon behaviour
δ_3	$(\nabla \cdot u)^2$	$(r_{\rm H} - r)^{-1}$
δ_4	$\nabla_\perp{}^\mu \nabla_{\perp\mu} \ln s$	$(r_{\rm H} - r)^{-3}$
δ_5	$\nabla_\perp{}^\mu \ln s \nabla_{\perp\mu} \ln s$	$(r_{\rm H} - r)^{-7}$
δ_6	$\sigma^\mu{}_\nu \sigma^\nu{}_\mu$	$(r_{\rm H} - r)^{-1}$
δ_7	$\omega^\mu{}_\nu \omega^\nu{}_\mu$	$(r_{\rm H} - r)^{-1}$

For second order tensor transport coefficients:

Transport coefficient	Corresponding tensor	Near-horizon behaviour will be weaker than
γ_3	$(\nabla \cdot u)\, \sigma^{\mu}_{\nu}$	$(r_{\rm H} - r)^{-1}$
γ_4	$\langle \nabla_{\perp}{}^{\mu} \nabla_{\perp \nu} \ln s \rangle$	$(r_{\rm H} - r)^{-3}$
γ_5	$\langle \nabla_{\perp}{}^{\mu} \ln s \nabla_{\perp \nu} \ln s \rangle$	$(r_{\rm H} - r)^{-7}$
γ_6	$\langle \sigma^{\mu}{}_{\tau} \sigma^{\tau}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$
γ_7	$\langle \omega^{\mu}{}_{\tau} \omega^{\tau}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$
γ_8	$\langle \sigma^{\mu}{}_{\tau} \omega^{\tau}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$

Remarkably evidence shows all **geometric counter-terms** that define the renormalised em-tensor **also get fixed — for every gauge there is a UNIQUE geometric RG flow**.

Field- theory Interpretation

The most general coarse graining consistent with derivative expansion:

$$u^{\mu}_{(\Lambda)}(x) = \int \mathrm{d}^d k\, \Theta\left(1 - \frac{k^2}{\Lambda^2}\right) e^{ik\cdot x} u^{\nu}(k)$$

$$\left(v_0\left(\frac{T_{\mathrm{eq}}}{\Lambda}\right)\delta^{\mu}_{\nu} + i v_{11}\left(\frac{T_{\mathrm{eq}}}{\Lambda}\right) u^{\mu}_{(\Lambda)}(x)\frac{k_{\nu}}{T_{\mathrm{eq}}} + i v_{12}\left(\frac{T_{\mathrm{eq}}}{\Lambda}\right)\delta^{\mu}_{\nu} u^{\alpha}_{(\Lambda)}(x)\frac{k_{\alpha}}{T_{\mathrm{eq}}} + O(k^2)\right)$$

$$T_{(\Lambda)}(x) = \int \mathrm{d}^d k\, \Theta\left(1 - \frac{k^2}{\Lambda^2}\right) e^{ik\cdot x} T(k)$$

$$\left(\tilde{v}_0\left(\frac{T_{\mathrm{eq}}}{\Lambda}\right)\delta^{\mu}_{\nu} + i\tilde{v}_1\left(\frac{T_{\mathrm{eq}}}{\Lambda}\right) u^{\alpha}_{(\Lambda)}(x)\frac{k_{\alpha}}{T_{\mathrm{eq}}} + O(k^2)\right)$$

After lot of algebraic manipulations one can solve all the unknown functions to match evolution by classical gravity equations in FG gauge.

Only if equation of state and transport coefficients via FG geometric RG flow :

There is a unique $g_{\mu\nu}(\Lambda)$ such that $\nabla^{\mu}_{(\Lambda)}\langle T_{\mu\nu}(\Lambda)\rangle = 0.$

We reproduce classical gravity completely but yet we have to find sufficient set of principles.

Emergence of bulk diffeomorphism symmetry

The bulk diffeomorphisms are the most general transformations for:

$$r = \hat{r} + \rho(\hat{r}, \hat{x}) \qquad \Lambda = \hat{\Lambda}\left(1 - \hat{\Lambda}\rho(\hat{\Lambda}, \hat{x})\right)$$

$$x^\mu = \hat{x}^\mu + \chi^\mu(\hat{r}, \hat{x})$$

such that at all scales Weyl transformations in UV is lifted efficiently

$$\hat{\mathrm{g}}_{\mu\nu} = \mathrm{g}_{\mu\nu} + \rho\, \mathrm{g}'_{\mu\nu} - 2\frac{\rho}{\hat{\mathrm{r}}}\, \mathrm{g}_{\mu\nu} + \mathcal{L}_\chi \mathrm{g}_{\mu\nu}$$

$$\hat{\mathrm{T}}_{\mu\nu} = \mathrm{T}_{\mu\nu} + \rho\, \mathrm{T}'_{\mu\nu} + (\mathrm{d} - 2)\frac{\rho}{\hat{\mathrm{r}}}\mathrm{T}_{\mu\nu} + \mathcal{L}_\chi \mathrm{T}_{\mu\nu} + \mathrm{O}(\hat{\nabla}^2)$$

$$\hat{\nabla}^\mu \hat{\mathrm{T}}_{\mu\nu} = 0$$

Furthermore in Fefferman-Graham gauge there is an automorphism of the RG flow for Penrose-Brown-Henneaux transformations $\mathcal{P}$

$$\rho = \hat{r}\sigma(\hat{x}), \quad \chi^{\mu} = \int_0^{\hat{r}} \mathrm{d}\tilde{r}\, \tilde{r} g^{\mu\nu} \hat{\partial}_{\nu}\sigma$$

In other coordinates related by a bulk-diffeo transformation $\mathcal{G}$ from FG, the automorphism is $\mathcal{G}^{-1}\mathcal{P}\mathcal{G}$

The automorphism of the RG flow knows completely about the corresponding choice of gauge in the bulk!!!

Conjecture

It is sufficient to reproduce the Fefferman-Graham RG flow as we have justified bulk diffeomorphisms from "efficiency" principles.

Conjecture: Our construction of the coarse graining in field theory is the unique one such that

(i) the effective equations can be rewritten as a conservation of energy and momentum in a redefined background metric

(ii) the PBH transformations are automorphisms

(iii) the flow ends at incompressible non-relativistic Navier-Stokes at thermal scale

Proving this will amount to constructing gravity from field theory in a specific dynamical sector.

Summary

We have repackaged classical gravity directly into first order RG flow of physical observables. There is strong evidence for unique choice of geometric counter-terms that leads to well defined infrared.

We have constructed a coarse graining in QFT which reproduces classical gravity equations.

We have conjectured a set of sufficient principles for which this coarse graining is the unique answer.

Concluding Comments

This “highly efficient” RG flow may not be constructible in all strongly interacting large N theories — what are the constraints on transport coefficients under which our three principles can be realised?

This will amount to understanding which QFTs may have pure gravity duals.

We should try to go beyond the hydrodynamic sector and understand how the reconstruction of gravity from QFT works.

THANK YOU