Charge density wave instability in holographic D-wave superconductor

JHEP 1404 (2014) 135
arXiv: 1312.1588

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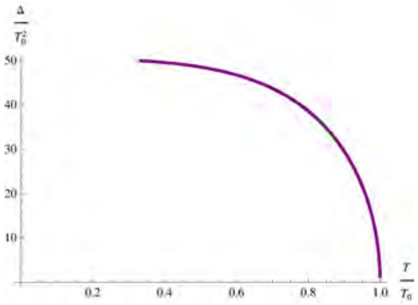
Kolymbary,
07 september 2014

Outline

- Introduction to holographic superconductivity
- D-wave holographic superconductor
- Charge density wave instabitlity
- Phase diagram

Superconductivity

Superconductivity
$\equiv$ charged order parameter
$\equiv$ vacuum expectation value of quantum operator



AdS/CFT duality

"CFT"	$\leftrightarrow$	"AdS"
$\lambda \ll 1$		$g_s \to 0$
$Mink_d$		AdS_{d+1}
Operators O_i	$\leftrightarrow$	Fields ϕ_i
Partition function		Action
$\int \mathcal{D}\psi \;\; \exp\left(S[\psi] + J_i O_i[\psi]\right)$	$=$	$\exp\left(S_{cl}[\phi_i]\right)\big\rvert_{\phi_i(\partial AdS)=J_i}$
		$\phi_i\rvert_{r\to\infty} = a r^{\Delta - d} + b r^{-\Delta}$
J_i	$=$	a
$\langle O_i \rangle$	$\sim$	b

Holographic superconductor

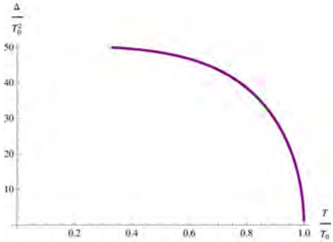
"Superconductor"	$\leftrightarrow$	"AdS"
Temperature	$\leftrightarrow$	Black hole
$T = \frac{3}{4\pi}\frac{1}{z_0}$		$f(z) = 1 - \frac{z^3}{{z_0}^3}$
Operators		Fields
$J_\mu \sim p_\mu c_p^\dagger c_p$	$\leftrightarrow$	A_μ
$\Delta \sim c_p^\dagger c_{-p}^\dagger$	$\leftrightarrow$	ϕ
$\mu, \ \rho$		$A_t(z) = \mu + \rho z$
$\langle\Delta\rangle$		$\phi(z) = z^3 \langle\Delta\rangle$

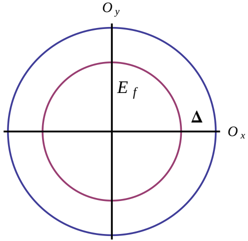
Holographic superconductor

At low temperature the scalar field develops nontrivial profile, associated with nonzero superconducting order parameter

$$\phi'' + \left(\frac{f'}{f} - \frac{2}{z}\right)\phi' + \frac{A_t^2}{f(z)^2}\phi - \frac{m^2 L^2}{z^2 f(z)}\phi = 0$$

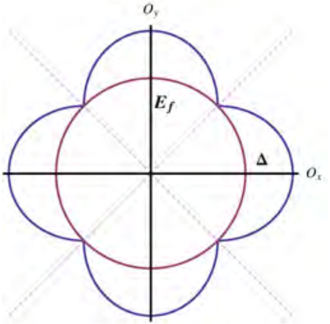
$$A_t'' - \frac{2q^2 L^2}{f(z)^2}\phi^2 A_t = 0$$



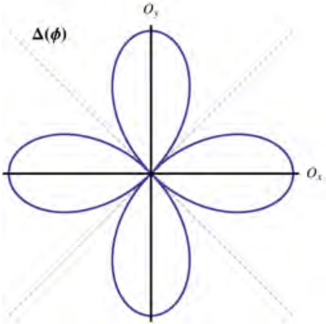


*S. A. Hartnoll, C. P. Herzog, G. T. Horowitz, Phys.Rev.Lett.***101***, 031601(2008)*

D-wave superconductor



$\Rightarrow$



D-wave: spin 2 symmetry $\Rightarrow$ order parameter is a symmetric tensor

$$\Delta_{xy} \leftrightarrow \phi_{xy}$$

The action

Lagrangian is a generalization of Fierz-Pauli Lagrangian for massive gravity

F. Benini, C. P. Herzog, R. Rahman and A. Yarom, JHEP 1011, 137 (2010)

$$\begin{aligned}\mathcal{L} = & - |D_\rho \phi_{\mu\nu}|^2 + 2|D_\mu \phi^{\mu\nu}|^2 + |D_\mu \phi^\nu_\nu|^2 - [D_\mu \phi^{*\mu\nu} D_\nu \phi^\rho_\rho + c.c.] \\ & - m^2(|\phi_{\mu\nu}|^2 - |\phi^\mu_\mu|^2) \\ & + 2R_{\mu\nu\rho\lambda}\phi^{*\mu\rho}\phi^{\nu\lambda} - \frac{1}{4}R|\phi^\mu_\mu|^2 \\ & - iqF_{\mu\nu}\phi^{*\mu\lambda}\phi^\nu_\lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

Consistent if gravity is non-dynamical

$$ds^2 = \frac{L^2}{z^2}\left(-f(z)dt^2 + f(z)^{-1}dz^2 + dx^2 + dy^2\right), \qquad f(z) = 1 - \frac{z^3}{z_0^3}.$$

Equations of motion

The ansatz for for homogeneous static solution

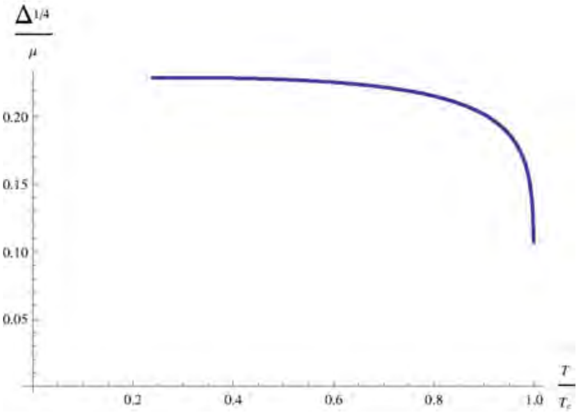
$$\phi_{xy} = \frac{1}{2z^2}\tilde{\psi}, \qquad A_t = \tilde{A}_t$$

Leads to familiar equations of holographic superconductor

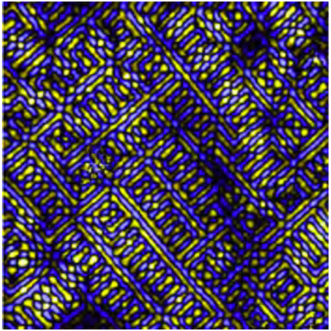
$$0 = \partial_z^2 \tilde{A}_t(z) - \frac{q^2}{z^2 f(z)}\tilde{\psi}^2 \tilde{A}_t,$$

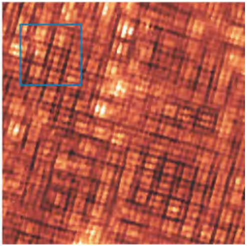
$$0 = \partial_z^2 \tilde{\psi}(z) + \left(\frac{f'(z)}{f(z)} - \frac{2}{z}\right)\partial_z \tilde{\psi}(z) + \left(\frac{q^2 \tilde{A}_t^2}{f(z)^2} - \frac{m^2}{z^2 f(z)}\right)\tilde{\psi}(z).$$

Order parameter



Stripes and checkerboards in cuprates





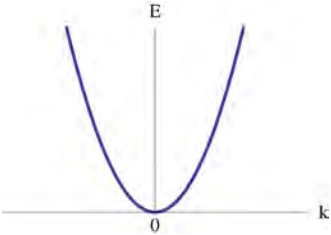
The checkerboard pattern is seen in the pseudogap phase of cuprates. It might be connected with the superconducing order parameter.

K. B. Efetov, H. Meier and C. Pépin, Nature Physics **9**, *442–446 (2013)*

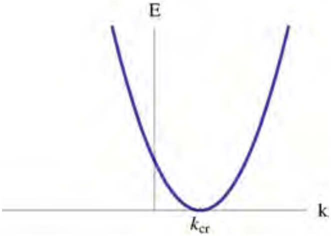
Spontaneous translation symmetry breaking

Mixing of modes can lead to the shift of dispersion relation of fluctuations. It is the sign of the onset of dynamical instability.

A.Donos and J. P. Gauntlett, JHEP 1108, 140 (2011)



$\Rightarrow$



Spontaneous translation symmetry breaking

Looking for terms in the action, which mix A_t with ϕ at nonzero momentum (k, A_t, ϕ)

$$\begin{aligned}\mathcal{L} = &- |D_\rho \phi_{\mu\nu}|^2 + 2|D_\mu \phi^{\mu\nu}|^2 + |D_\mu \phi^\nu_\nu|^2 - [D_\mu \phi^{*\mu\nu} D_\nu \phi^\rho_\rho + c.c.] \\ &- m^2(|\phi_{\mu\nu}|^2 - |\phi^\mu_\mu|^2) \\ &+ 2R_{\mu\nu\rho\lambda}\phi^{*\mu\rho}\phi^{\nu\lambda} - \frac{1}{4}R|\phi^\mu_\mu|^2 \\ &- iqF_{\mu\nu}\phi^{*\mu\lambda}\phi^\nu_\lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

$$D_\rho \phi_{\mu\nu} = \nabla_\rho \phi_{\mu\nu} - iqA_\rho \phi_{\mu\nu}$$

In symmetric (non-condensed) phase there is no mixing as $\phi = 0$

Spontaneous translation symmetry breaking

Hence one need to study the fluctuations in condensed phase
The charge density wave mode should include fluctuations of A_t.

$$\delta\phi_{xy} \sim \cos(k_y y)\,\psi^1_{xy}(z), \qquad \delta\phi_{tx} \sim \sin(k_y y)\,\psi^2_{tx}(z),$$
$$\delta A_t \sim \cos(k_y y)\,a_t(z), \qquad \delta\phi_{zx} \sim \sin(k_y y)\,\psi^1_{zx}(z).$$

Linearized equations of motion

In condensed phase the mixing of modes is found indeed

$$0 = \left[\partial_z^2 + \left(\frac{f'(z)}{f(z)} - \frac{2}{z}\right)\partial_z - \frac{m^2}{z^2 f(z)} + \frac{(\tilde{A}_t)^2}{f(z)^2}\right]\psi^1_{xy}$$

$$+ k_y 2z^2\left[\partial_z + \frac{f'(z)}{f(z)}\right]\psi^1_{zx} - k_y\frac{\tilde{A}_t}{f(z)^2}\psi^2_{tx} + 2\frac{\tilde{A}_t\tilde{\psi}}{f(z)^2}a_t$$

$$0 = \left[\partial_z^2 - \frac{2}{z}\partial_z - \frac{m^2 + z^2 {k_y}^2}{z^2 f(z)}\right]\psi^2_{tx} + k_y\frac{\tilde{A}_t}{f(z)}\psi^1_{xy}$$

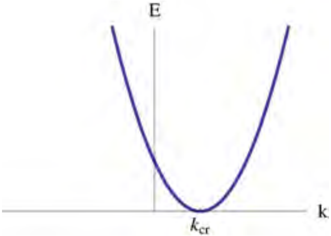
$$+ 2z^2\left[\tilde{A}_t\partial_z + \frac{1}{2}\partial_z\tilde{A}_t\right]\psi^1_{zx} + k_y\frac{\tilde{\psi}}{2f(z)}a_t$$

$$0 = \left[-m^2 + \frac{z^2}{f(z)}(\tilde{A}_t)^2 - z^2 {k_y}^2\right]\psi^1_{zx} + \frac{1}{2f(z)}\left[\tilde{A}_t\partial_z + \frac{1}{2}\partial_z\tilde{A}_t\right]\psi^2_{tx} - \frac{k_y}{2}\partial_z\psi^1_{xy}$$

$$0 = \left[\partial_z^2 - \frac{{k_y}^2}{f(z)} - \frac{(\tilde{\psi})^2}{z^2 f(z)}\right]a_t - 2\frac{\tilde{A}_t\tilde{\psi}}{z^2 f(z)}\psi^1_{xy} + k_y\frac{\tilde{\psi}}{2z^2 f(z)}\psi^2_{tx}$$

Search for nontrivial solutions

We need to study, whether the nontrivial mode with $\omega = 0$ and $k \neq 0$ exist.



That means we need to solve Sturm-Liuvile problem and obtain values of k_{cr} for which the mode exist.

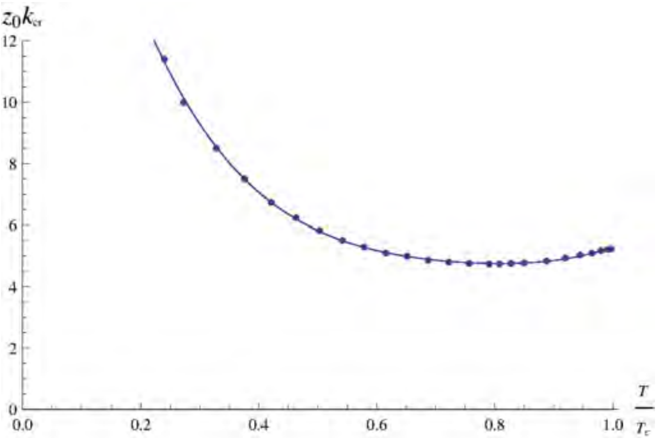
Search for nontrivial solutions

We check Wronskian of the modes obatained by the numarical shooting from both black hole horizon and AdS boundary

$$W(z) = \begin{vmatrix} \xi_1^1(z) & \dots & \xi_1^3(z) & \eta_1^1(z) & \dots & \eta_1^3(z) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \xi_3^1(z) & \dots & \xi_3^3(z) & \eta_3^1(z) & \dots & \eta_3^3(z) \\ \partial_z\xi_1^1(z) & \dots & \partial_z\xi_1^3(z) & \partial_z\eta_1^1(z) & \dots & \partial_z\eta_1^3(z) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \partial_z\xi_3^1(z) & \dots & \partial_z\xi_3^3(z) & \partial_z\eta_3^1(z) & \dots & \partial_z\eta_3^3(z) \end{vmatrix}$$

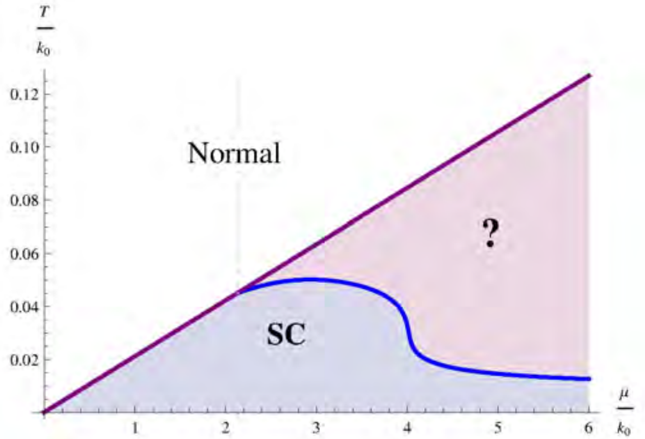
Critical momenta

At all temperatures we find the spacially modulated static mode

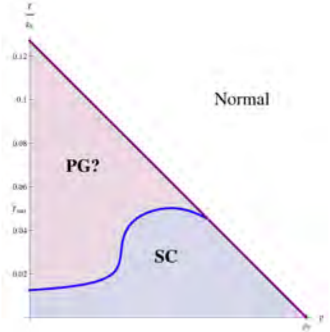


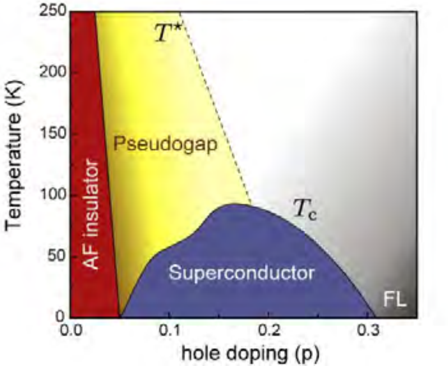
Phase diagram

Fixing the unique allowed value for k_{cr} one obtains the interesting phase diagram



Premature speculation!



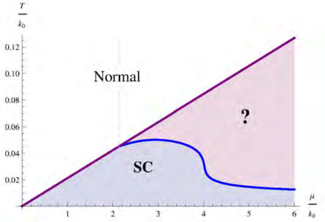


Surprisingly, the estimate for the maximum superconducting temperature, based on the data of cuprates, gives reasonable value.

$$T_{max} = 0.05 \frac{\hbar c}{K_b} k_0 \approx 140^\circ$$

Conclusion

- The condensed phase of D-wave holographic superconductor is unstable to the formation of spacialy modulated phase
- The phase includes charge density waves without current density
- May be a candidate for a description of the pseudogap state



Problems:

- PDE need to be solved in order to study nonlinear solution
- Backreaction of the metric may not be included in present model

Additional slides

Current density wave

$$0 = \left[\partial_z^2 + \left(\frac{f'(z)}{f(z)} - \frac{2}{z}\right)\partial_z - \frac{m^2}{z^2 f(z)} + \frac{(\tilde{A}_t)^2}{f(z)^2}\right]\psi^2_{xy} + 2k_y z^2 \left[\partial_z + \frac{f'(z)}{f(z)}\right]\psi^2_{zx}$$
$$- k_y \tilde{A}_t \frac{1}{f(z)^2}\psi^1_{tx} - i\left[\tilde{\psi}\partial_z + 2\partial_z\tilde{\psi} + \left(\frac{f'(z)}{f(z)} - \frac{2}{z}\right)\tilde{\psi}\right]A_z$$

$$0 = \left[-m^2 + \frac{z^2}{f(z)}(\tilde{A}_t)^2 - z^2 k_y^2\right]\phi^2_{zx} + \frac{1}{2f(z)}\left[\tilde{A}_t\partial_z + \frac{1}{2}\partial_z\tilde{A}_t\right]\psi^1_{tx}$$
$$- k_y\frac{1}{2}\partial_z\psi^2_{xy} - \frac{1}{4}\left[\tilde{\psi}\partial_z + 2\partial_z\tilde{\psi}\right]A_y + ik_y\frac{1}{2}\tilde{\psi}A_z$$

$$0 = \left[\partial_z^2 - \frac{2}{z}\partial_z - \frac{m^2 + z^2k_y^2}{z^2 f(z)}\right]\psi^1_{tx} + 2z^2\left[\tilde{A}_t\partial_z + \frac{1}{2}q\partial_z\tilde{A}_t\right]\psi^2_{zx} + k_y\frac{\tilde{A}_t}{f(z)}\psi^2_{xy} + \frac{\tilde{\psi}\tilde{A}_t}{f(z)}A_y$$

$$0 = \left[\partial_z^2 + \frac{f'(z)}{f(z)}\partial_z\right]A_y + ik_y\left[\partial_z + \frac{f'(z)}{f(z)}\right]A_z + \tilde{\psi}\left[\partial_z + \frac{f'(z)}{f(z)} - \frac{2}{z}\right]\psi^2_{zx} + \frac{\tilde{A}_t\tilde{\psi}}{z^2 f(z)}\psi^1_{tx}$$

$$0 = i\left[k_y^2 + (\tilde{\psi})^2\frac{1}{z^2}q\right]A_z + k_y\partial_z A_y - \frac{1}{z^2}\left[\tilde{\psi}\partial_z - \partial_z\tilde{\psi}\right]\psi^2_{xy} - k_y\tilde{\psi}\phi^2_{zx}$$