07.09.2014

Holographic Superconductors
...In Helical Backgrounds & Homes' Relation

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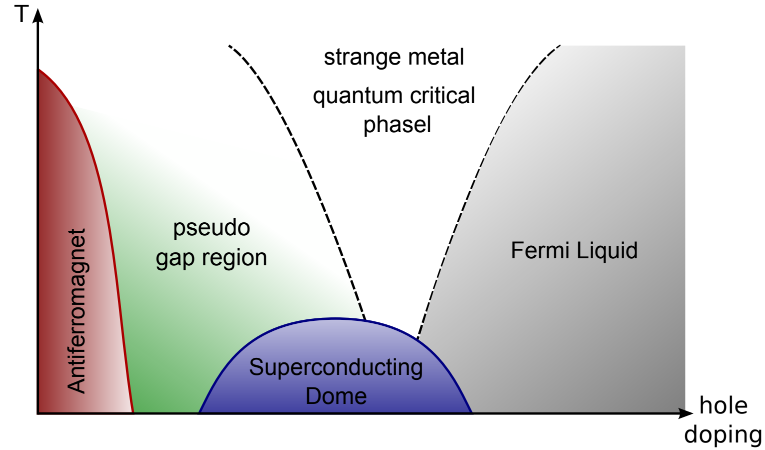
Quantum Field Theory, String Theory & Condensed Matter Physics
Κολυμβαρι

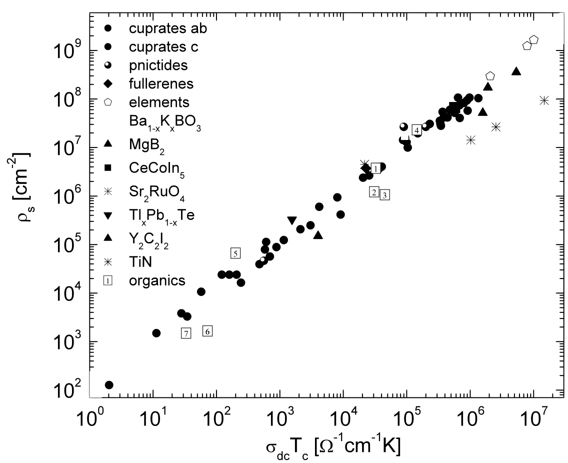
Motivation

- Holography is a powerful tool to describe strongly correlated systems

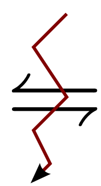
- Experimental puzzling results: Homes' Relation

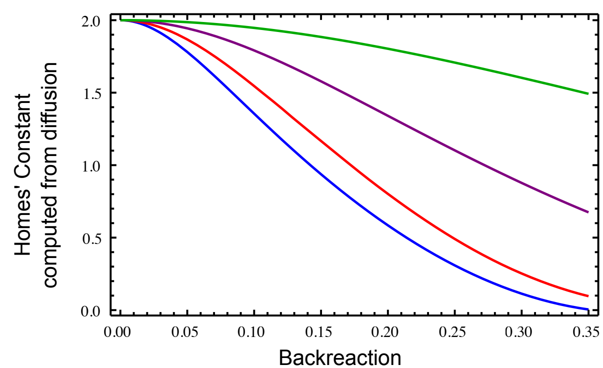


$$\rho_s = C \sigma_{DC} T_c$$



[Dordevic,Basov,Homes '12]





[Erdmenger,Kerner,SK '12]

Motivation

- Homes relation cannot hold in "pure" holographic s-wave superconductors, where transport is related to diffusion

- Momentum is intrinsically conserved in Holography

  ↳ Impossible to discern an ideal metal from a superconductor

- How to incorporate momentum relaxation?

Q-Lattice
[Donos & Gauntlett '13]

Massive Gravity
[Vegh '13]

Helical Lattice
[Donos & Hartnoll '12]

Inhomogenous Background
[Hartnoll & Hofman '12]
[Horowitz, Santos &Tong '12]

Outline

Outline

- Holographic Setup
- Thermodynamics of the Superfluid Insulator Transition
- Optical Conductivity at Finite Temperature
- Homes' Relations of high Tc Superconductors
- Zero Temperature Solutions

Part I:

Holographic Setup

Holographic Setup

Holographic Setup

- Action

$$S = \int \Big[R e^{\wedge 3} + \star 12 - \frac{1}{4} F \wedge \star F - \frac{1}{4} W \wedge \star W - \cancel{m^2 B \wedge \star B}$$
$$- \int \mathrm{d}^{4+1}\mathbf{x} \Big[|\partial \chi - \mathrm{i} q A \chi|^2 + \cancel{V(|\chi|)} \Big]$$
$$- \frac{\kappa}{2} \int B \wedge F \wedge W$$

Gauge fields

$$F = \mathrm{d}A$$
$$W = \mathrm{d}B$$

Gravity/Geometry

$$0 = \mathrm{d}e + \Omega \wedge e$$
$$R = \mathrm{d}\,\Omega + \Omega \wedge \Omega$$
$$\mathrm{d}s^2 = \text{"}e * e\text{"}$$

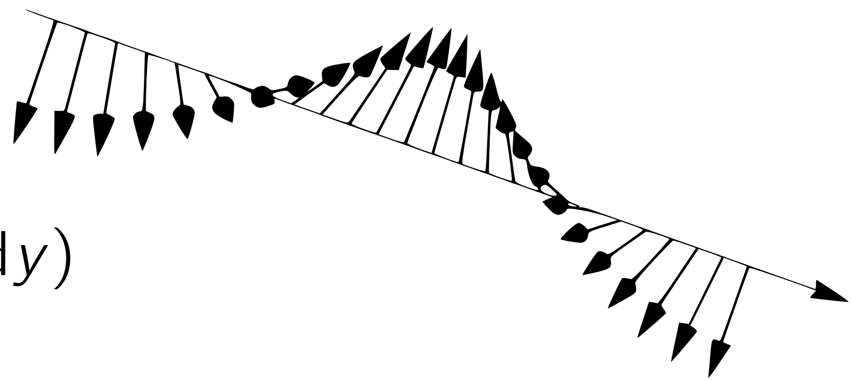
Holographic Setup

- Bianchi VII Ansatz for the metric

$$ds^2 = -U(r)\,dt^2 + \frac{dr^2}{U(r)} + e^{2v_1(r)}\,\omega_1^2 + e^{2v_2(r)}\,\omega_2^2 + e^{2v_3(r)}\,\omega_3^2$$

$$\omega_1 = dx$$

$$\omega_2 + \omega_3 = e^{ip\cdot x}\,(dx + i\,dy)$$



- Asymptotically AdS spacetime $r \to \infty$

$$U(r) = r^2 \qquad v_i(r) = \log(r) \quad \text{for } i = 1, 2, 3$$

Holographic Setup

- Ansatz for the gauge fields

encodes finite density

$$A = a(r)\,\mathrm{d}t \qquad a(\infty) = \mu$$

$$B = w(r)\omega_2 \qquad w(\infty) = \lambda$$

generates helix
with "lattice strength" λ

- Homogenous charged scalar field

$$\chi = \chi(r)$$

Equation of Motion

$$
\left.
\begin{aligned}
0 &= a'' + a'\left(v_1' + v_2' + v_3'\right) - \frac{2aq^2\rho^2}{U} + \kappa p e^{-v_1-v_2-v_3} w w', \\
0 &= w'' + w'\left(\frac{U'}{U} + v_1' - v_2' + v_3'\right) + \frac{w}{U}\left(\kappa p e^{-v_1+v_2-v_3} a' - m^2 - p^2 e^{-2(v_1-v_2+v_3)}\right), \\
0 &= 2\rho^2\left(m_\rho^2 - \frac{a^2q^2}{U}\right) + a'^2 + w^2\left(m^2 e^{-2v_2} + p^2 e^{-2(v_1+v_3)}\right) + 4p^2 e^{-2v_1}\sinh^2\left(v_2 - v_3\right) \\
&\quad + 2U'\left(v_1' + v_2' + v_3'\right) - U\left(2\rho'^2 + e^{-2v_2} w'^2 - 4v_1'v_2' - 4v_1'v_3' - 4v_2'v_3'\right) - 24, \\
0 &= 2\rho^2\left(\frac{m_\rho^2}{U} - \frac{a^2q^2}{U^2}\right) - \frac{a'^2}{U} + \frac{w^2}{U}\left(m^2 e^{-2v_2} - p^2 e^{-2(v_1+v_3)}\right) \\
&\quad + \frac{p^2}{U}\left(-2e^{-2v_1} + 3e^{-2(v_1+v_2-v_3)} - e^{-2(v_1-v_2+v_3)}\right) + 2\rho'^2 + \frac{2U''}{U} \\
&\quad + 4\left(\frac{U'}{U}\left(v_1' + v_2'\right) + v_1'^2 + v_2'v_1' + v_2'^2\right) - \frac{24}{U} + e^{-2v_2} w'^2 + 4\left(v_1'' + v_2''\right), \\
0 &= \frac{2w^2}{U}\left(p^2 e^{-2(v_1+v_3)} - m^2 e^{-2v_2}\right) + \frac{4p^2}{U}\left(e^{-2(v_1-v_2+v_3)} - e^{-2(v_1+v_2-v_3)}\right) \\
&\quad + \frac{4U'}{U}\left(v_3' - v_2'\right) - 2e^{-2v_2} w'^2 + 4\left(-v_2'^2 - v_1'v_2' + v_3'^2 + v_1'v_3'\right) + 4\left(v_3'' - v_2''\right), \\
0 &= \frac{p^2}{U}\left(e^{-2v_1} - e^{-2(v_1+v_2-v_3)}\right) + \frac{U'}{U}\left(v_3' - v_1'\right) - v_1'^2 + v_3'^2 - v_1'v_2' + v_2'v_3' - v_1'' + v_3'', \\
0 &= \rho'' + \rho'\left(\frac{U'}{U} + v_1' + v_2' + v_3'\right) + \rho\left(\frac{a^2q^2}{U^2} - \frac{m_\rho^2}{U}\right)
\end{aligned}
\right\}
$$

Only ODEs as promised by Bianchi VII

Numerical Solution

- Expansion at black hole horizon

$$
\left.
\begin{aligned}
a &= a_1^h\,(r-r_h) + a_2^h\,(r-r_h)^2 + \dots,\\
w &= w_0^h + w_1^h\,(r-r_h) + \dots,\\
U &= U_1^h\,(r-r_h) + U_2^h\,(r-r_h)^2 + \dots,\\
v_i &= v_{(i,0)}^h + v_{(i,1)}^h\,(r-r_h) + \dots,\\
\rho &= \rho_0^h + \rho_1^h\,(r-r_h) + \dots,
\end{aligned}
\right\}
$$

$$(a_1^h, w_0^h, \rho_0^h, U_1^h, v_{(i,0)}^h)$$

$$T = \frac{U_1^h}{4\pi}$$

- Expansion at AdS boundary

$$
\left.
\begin{aligned}
a &= \mu + \frac{\nu}{r^2} + \dots,\\
w &= \lambda + \frac{\beta - p^2\lambda\log(r)/2}{r^2} + \dots,\\
\rho &= \frac{\rho_b}{r^4} - \frac{q^2\mu^2\rho_b}{12r^6} + \dots,\\
U &= r^2 - \frac{\epsilon/3 + p^2\lambda^2\log(r)/6}{r^2} + \dots,\\
v_i &= \log(r) + \frac{g_i - (-1)^i p^2\lambda^2\log(r)/24}{r^4} + \dots,
\end{aligned}
\right\}
$$

$$(\epsilon, g_1, g_2, \mu, \nu, \lambda, \beta, \rho_b)$$

Shooting Method

κ

p

q

Part I:

Superfluid Phase Transition

Thermodynamics

- Free energy density evaluated at the boundary

$$\frac{\Omega}{V} = \epsilon + \mu \underbrace{\left(2\nu - \frac{1}{2}\kappa\lambda^2 p\right)}_{=n} - \underbrace{e^{v_1(r_H)+v_2(r_H)+v_3(r_H)}\, U'(r_H)}_{=Ts}$$

$$= 4g_1 + \frac{\alpha^4}{16} - \frac{\epsilon}{3} - \beta\lambda - \frac{\lambda^2 p^2}{8}$$

shift symmetry $\longrightarrow$ $r \to r + \alpha/2$

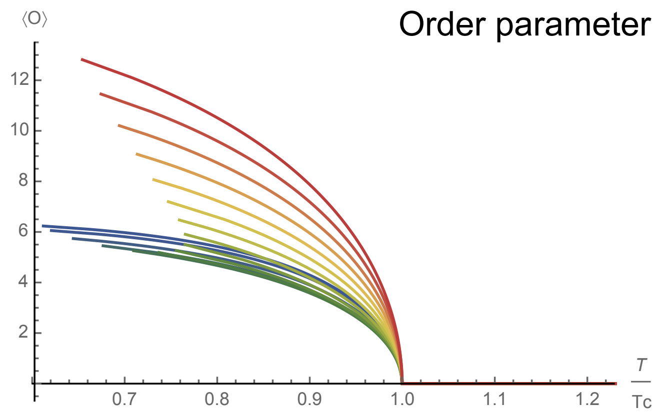
$\longrightarrow$ $4\langle T^a{}_a \rangle$ conformal anomaly

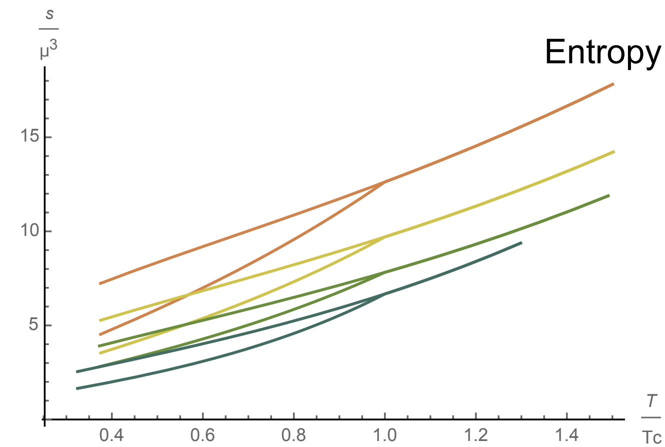
- Physical parameters we can freely tune

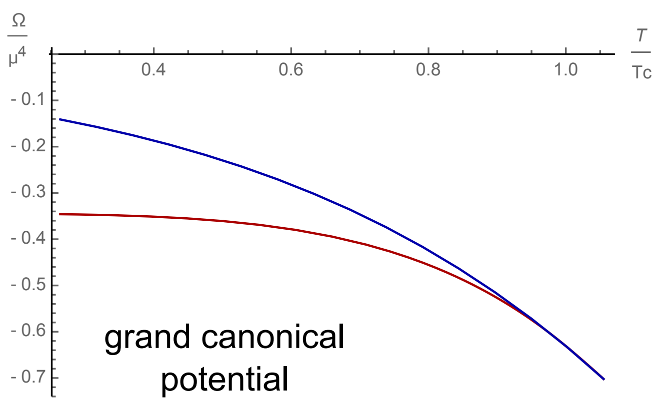
$$(\kappa, q, T/\mu, p/\mu, \lambda/\mu)$$

Phase Transition

- Mean field insulator-superfluid transition



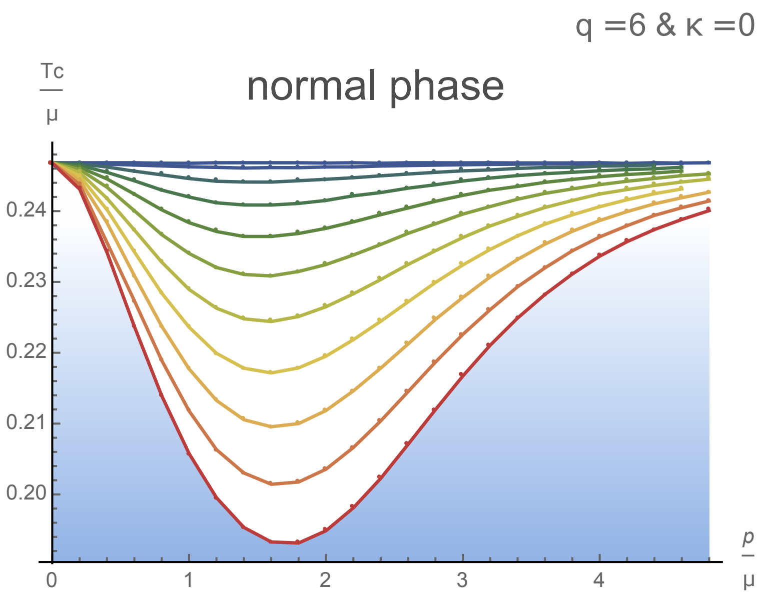


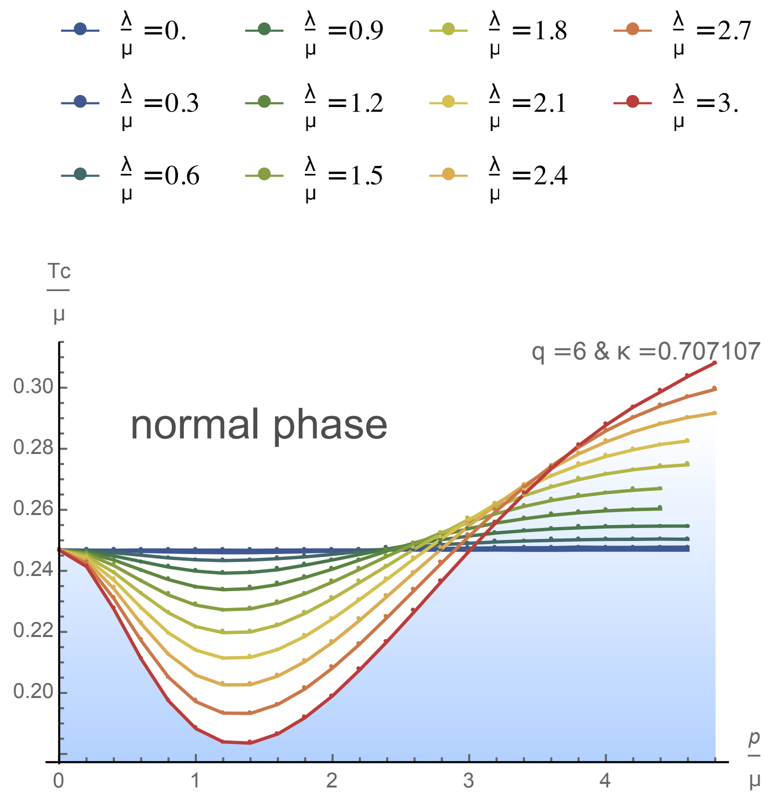


Large N mean field
second order
phase transition

Phase Diagram

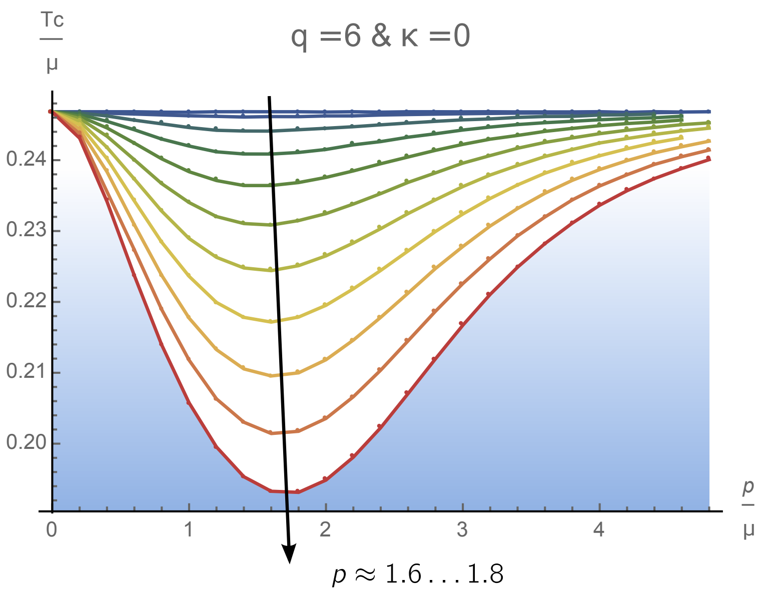
Phase Diagram

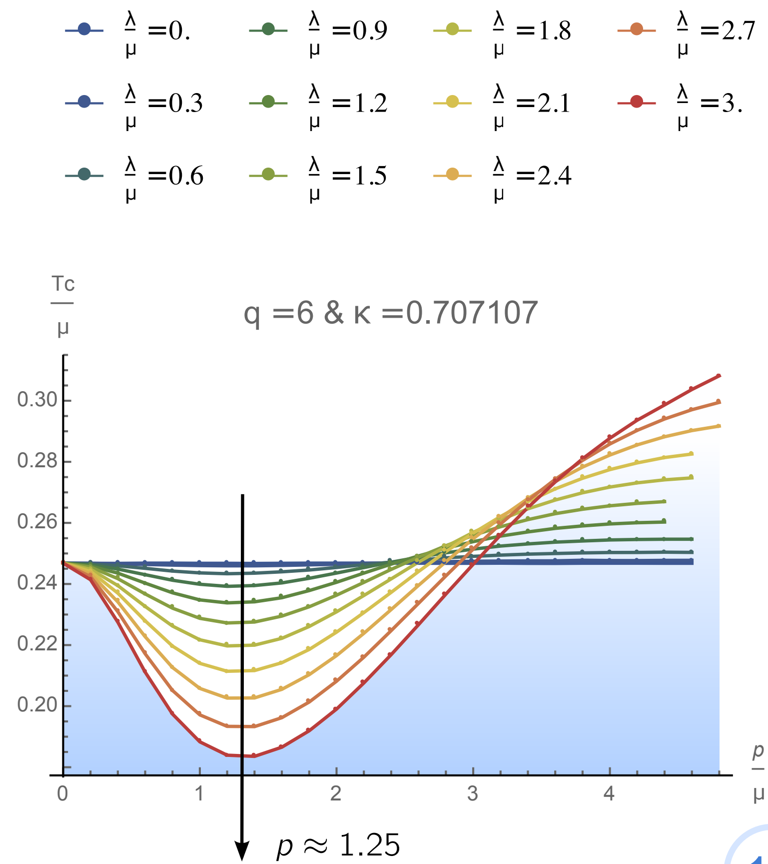




condensed phase

Minimal Tc @ Fixed p

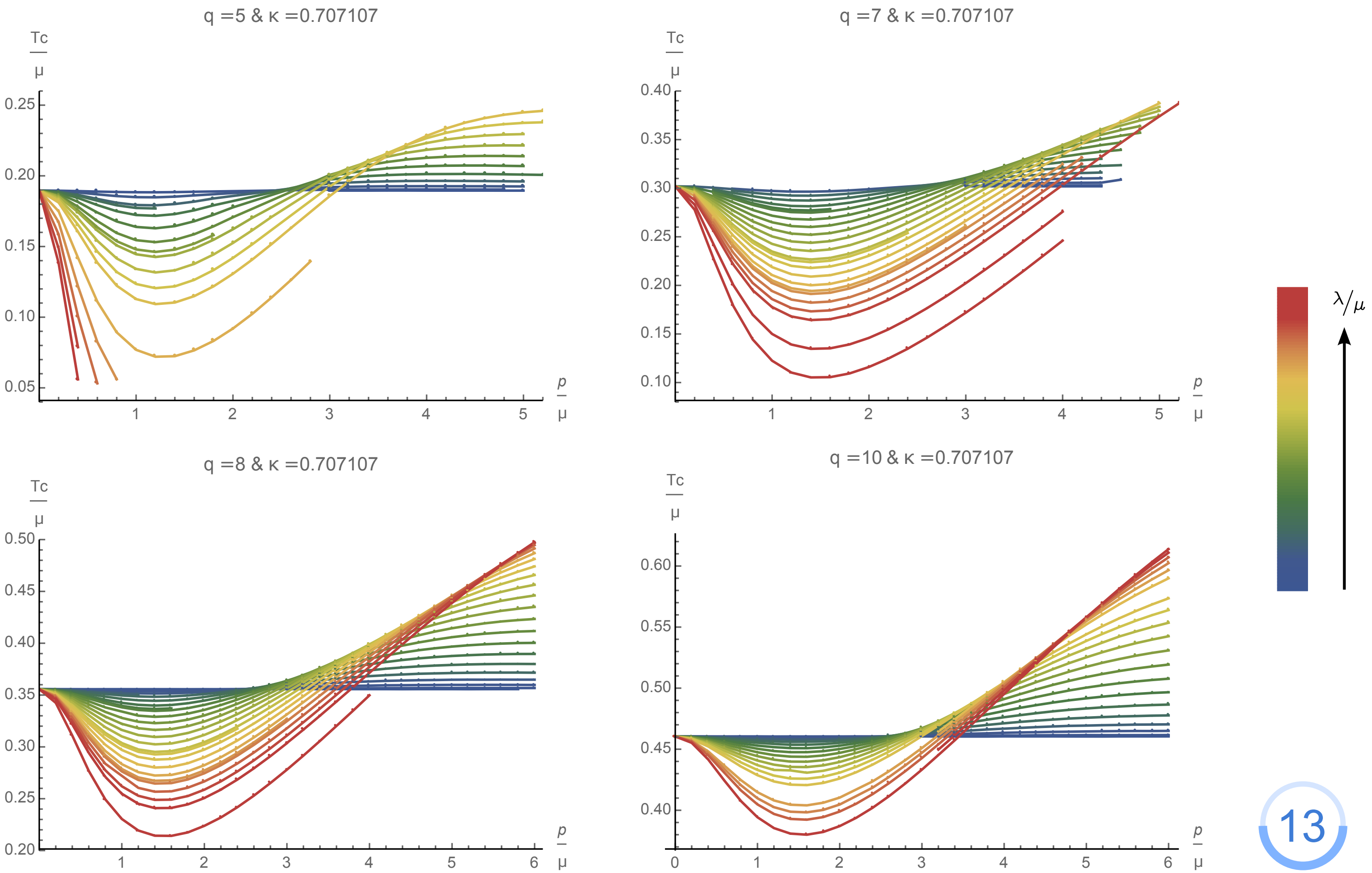




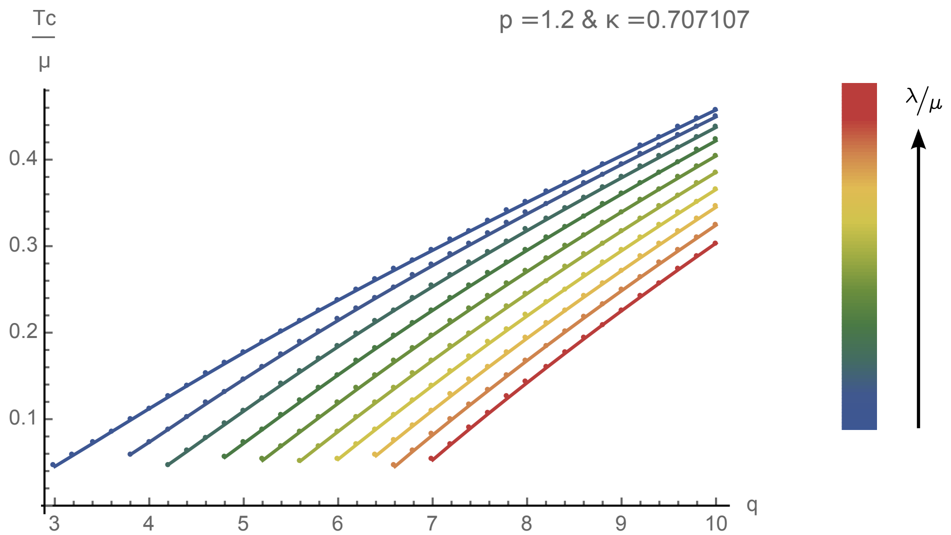
- Minimal Tc occurs at a fixed p for all λ
- p-value increases with decreasing q

Phase Diagram

Phase Diagram



Variation of Tc with q



- Tc is reduced when increasing backreaction controlled by q

  ⤷ matches known behavior of holographic s-wave superconductor in pure AdS-RN background

Part II:

Optical Conductivity

Fluctuation Equations of Motion

Fluctuation Equations of Motion

	h_{tt}	h_{rr}	h_{11}	h_{22}	h_{33}	h_{tr}	h_{t1}	h_{t2}	h_{t3}	h_{r1}	h_{r2}	h_{r3}	h_{12}	h_{13}	h_{23}	A^f_t	A^f_r	A^f_1	A^f_2	A^f_3	B^f_t	B^f_r	B^f_1	B^f_2	B^f_3	ρ^f	$(\rho^f)^*$
h_{tt}	•	•	•	•	•	•										•	•							•		•	•
h_{rr}		•	•	•	•	•										•	•							•		•	•
h_{11}			•	•	•	•										•	•							•		•	•
h_{22}				•	•	•										•	•							•		•	•
h_{33}					•	•										•	•							•		•	•
h_{tr}						•											•							•		•	•
h_{t1}							•			•					•			•							•		
h_{t2}								•			•			•					•		•	•					
h_{t3}									•			•	•							•			•				
h_{r1}										•					•			•							•		
h_{r2}											•			•					•			•					
h_{r3}												•	•							•			•				
h_{12}													•										•				
h_{13}														•													
h_{23}															•										•		
A^f_t																•	•							•		•	•
A^f_r																	•							•		•	•
A^f_1																		•							•		
A^f_2																			•		•	•					
A^f_3																				•			•				
B^f_t																					•	•					
B^f_r																						•					
B^f_1																							•				
B^f_2																								•			
B^f_3																									•		
ρ^f																											•
$(\rho^f)^*$																											

$$\delta A = \mathcal{A}(t,r)\omega_1$$

$$\delta B = \mathcal{B}(t,r)\omega_3$$

$$\delta\left(\mathrm{d}s^2\right) = h_{t1}(t,r)\,\mathrm{d}t\otimes\omega_1 + h_{23}(t,r)\omega_2\otimes\omega_3 + h_{r1}(t,r)\,\mathrm{d}r\otimes\omega_1$$

Transport Properties

- Horizon expansion of fluctuation equations

$$\mathcal{A} = (r - r_h)^{\pm i\omega/(4\pi T)} \left(A_0^h + A_1^h\,(r - r_h) + \ldots\right)$$

$$\mathcal{A} = A_0^b + \frac{A_2^b + A_0^b\,\omega^2\,\log(r)/2}{r^2} + \ldots$$

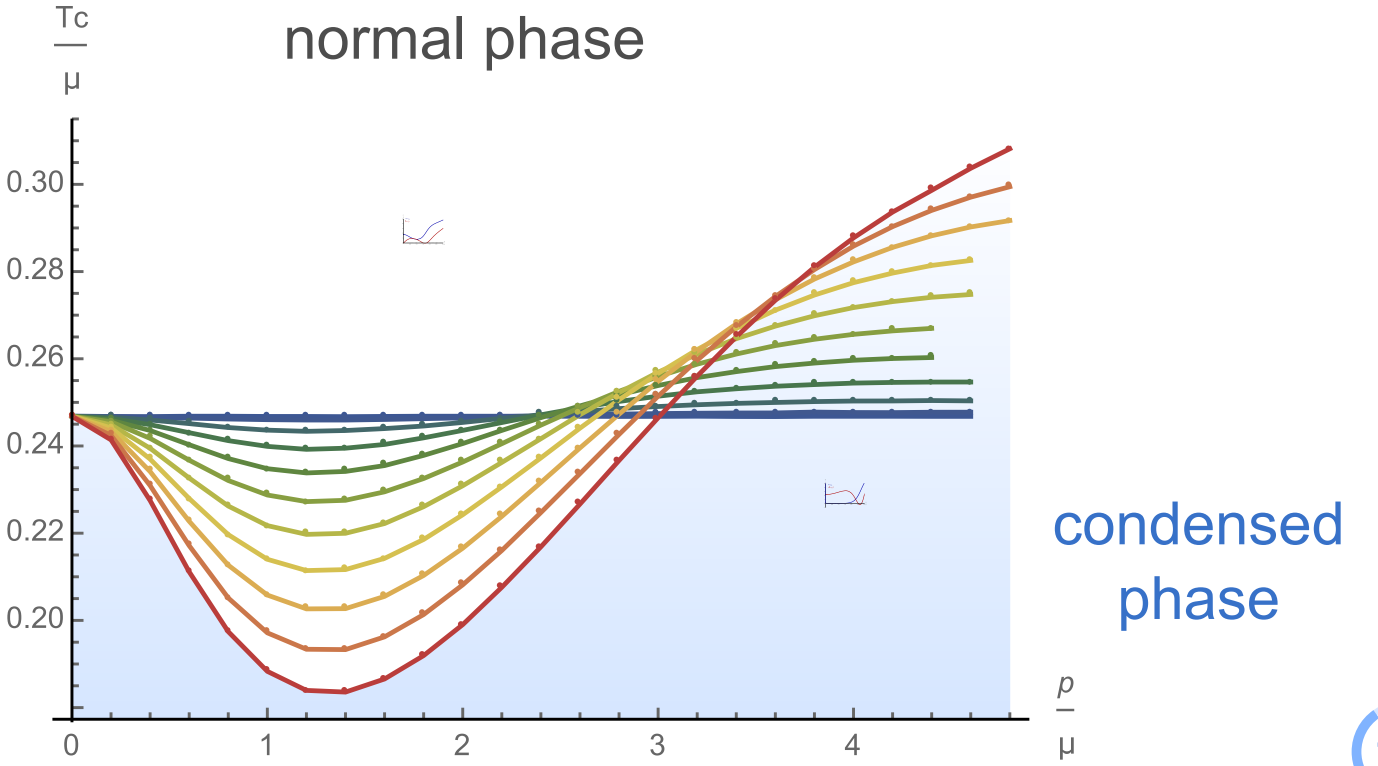
- Photon propagator at zero momentum

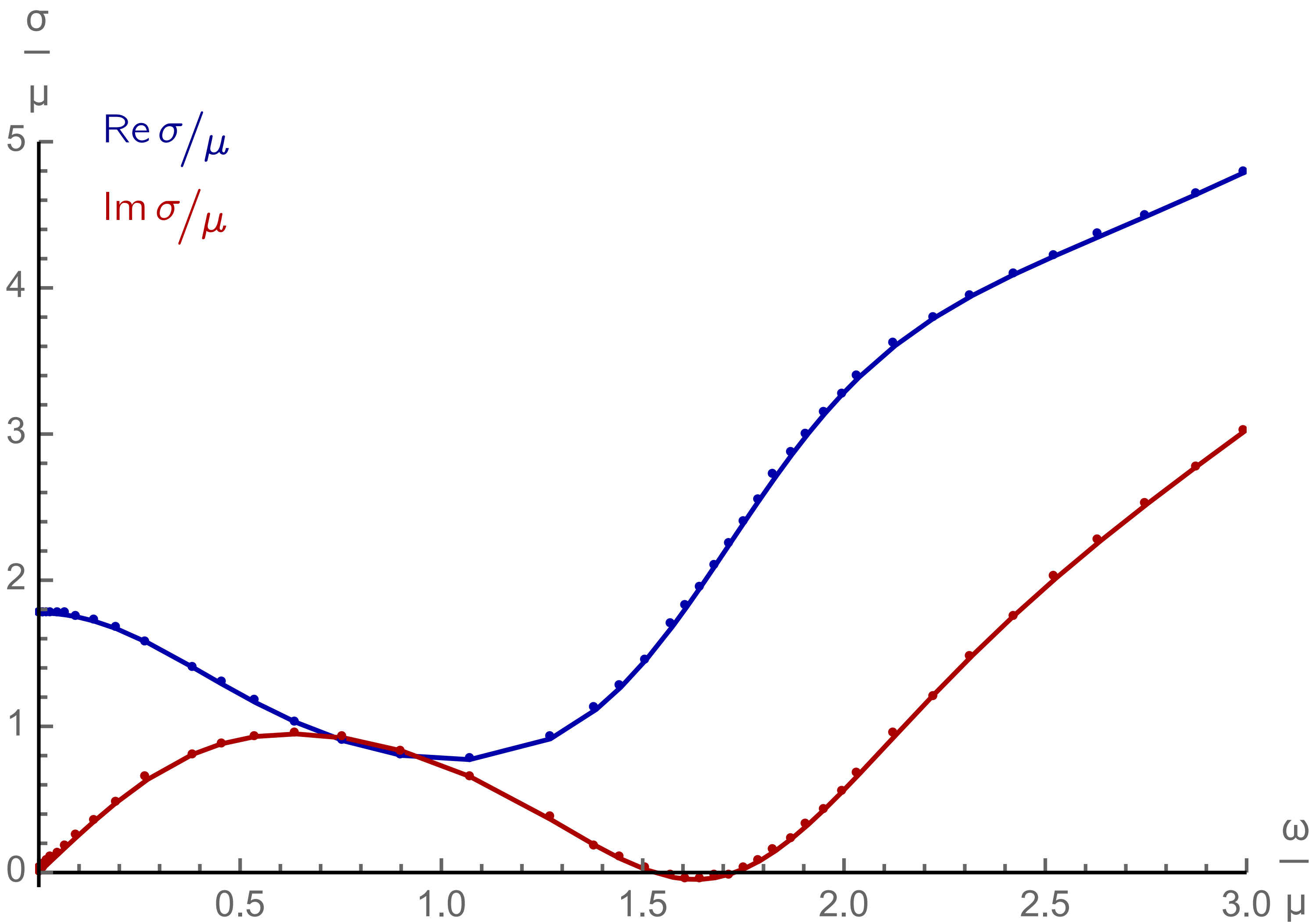
$$\sigma_x(\omega) = \lim_{\mathbf{k}\to 0} \frac{G^R_{xx}(\omega, \mathbf{k})}{i\omega} \qquad G^R_{xx}(\omega, 0) = 2\left(\frac{A_2^b(\omega)}{A_0^b(\omega)} - \frac{\omega^2}{4}\right)$$

Optical Conductivity

Optical Conductivity

- Optical conductivity in normal and condensed phase

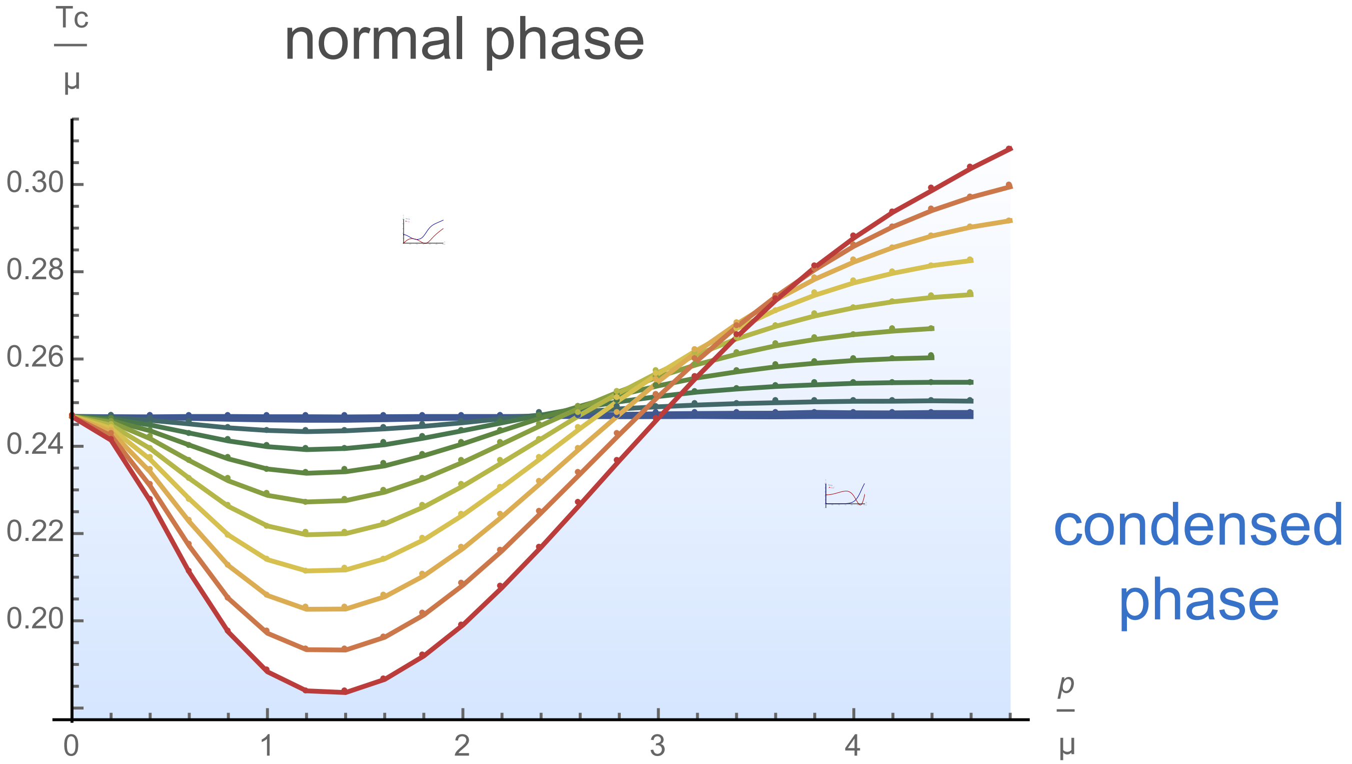


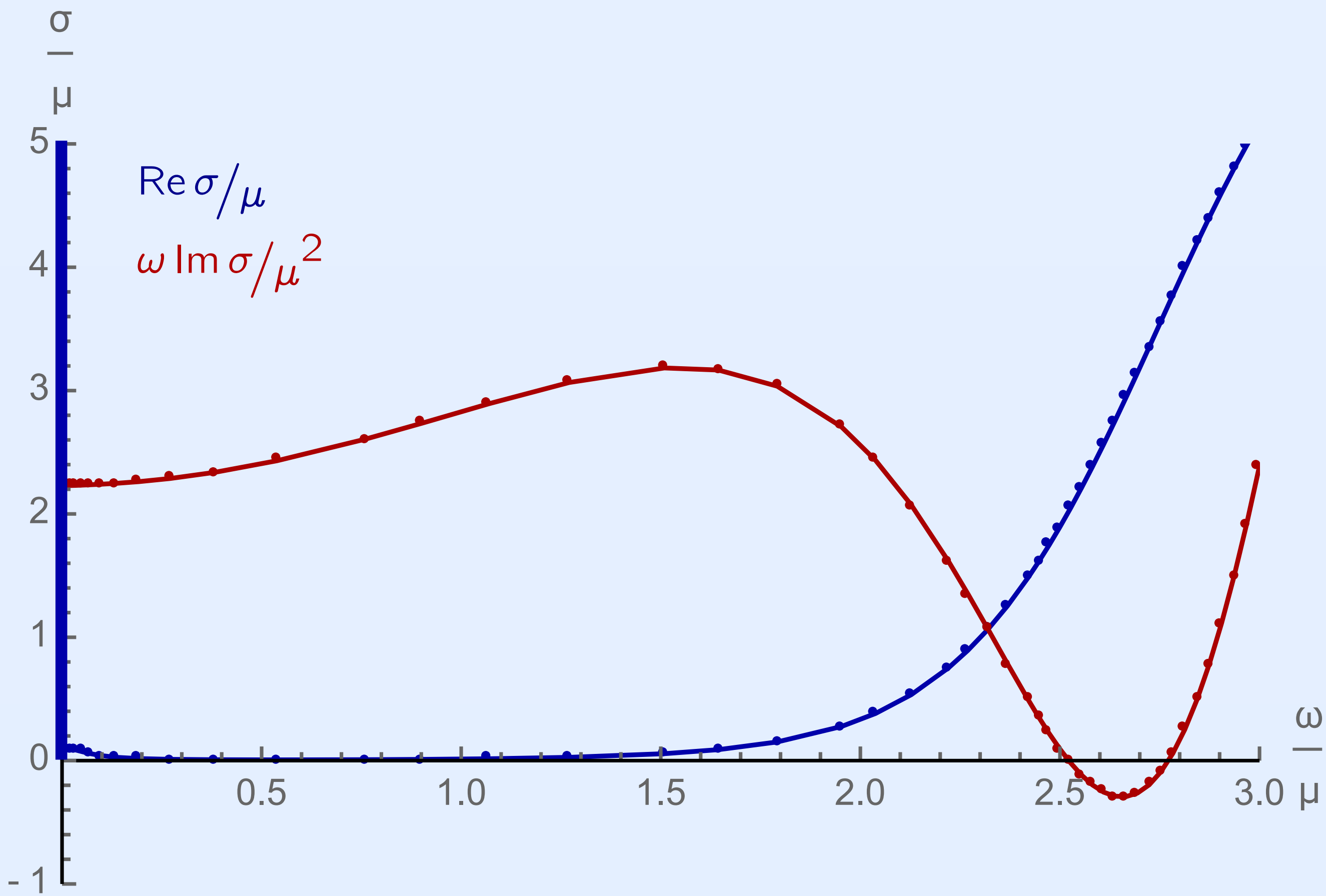


σ/μ
Re σ/μ
Im σ/μ
5
4
3
2
1
0
0.5
1.0
1.5
2.0
2.5
3.0
ω/μ

Optical Conductivity

- Optical conductivity in normal and condensed phase





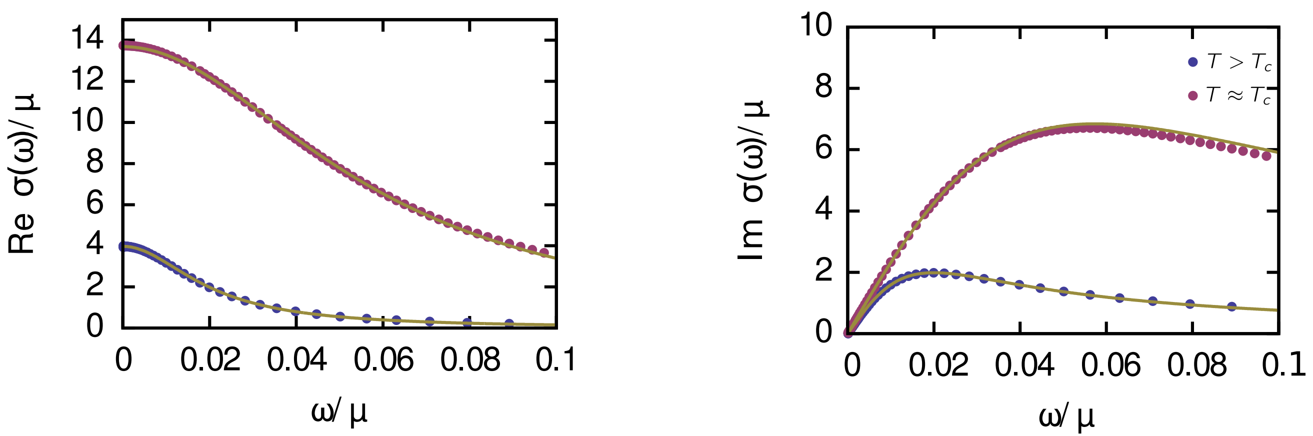
σ/μ
Re σ/μ
ω Im σ/μ²
5
4
3
2
1
0
- 1
0.5
1.0
1.5
2.0
2.5
3.0
ω/μ

Drude Model

Drude Model

- Drude model

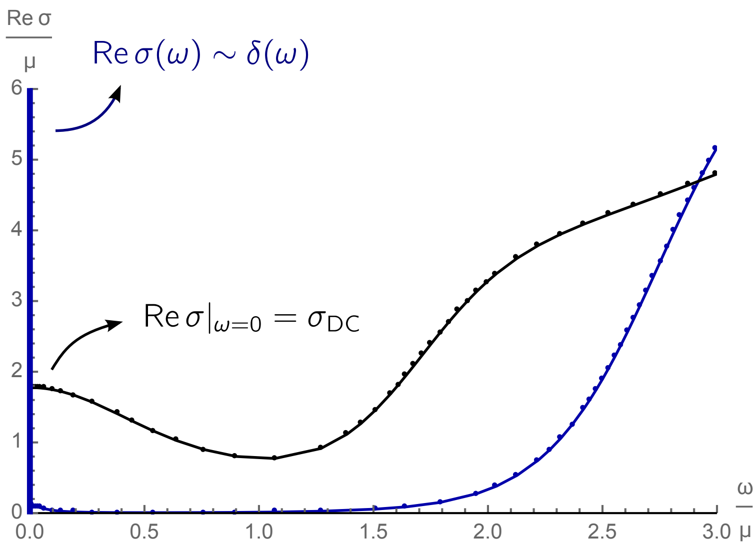
$$\sigma(\omega) = \frac{\sigma_{\mathrm{DC}}}{1 - \mathrm{i}\omega\tau} = \frac{n_{\mathrm{n}} e^2 \tau}{m^*} \frac{1}{1 - \mathrm{i}\omega\tau} = \frac{\omega_{\mathrm{P}}^2}{4\pi} \frac{1}{1/\tau - \mathrm{i}\omega}$$



$\hookrightarrow$ $$\mathrm{Re}\,\sigma(\omega) = \frac{\sigma_{\mathrm{DC}}}{1 + \omega^2\tau^2}$$

$\hookrightarrow$ $$\mathrm{Im}\,\sigma(\omega) = \frac{\omega\tau}{1 + \omega^2\tau^2}\sigma_{\mathrm{DC}}$$

Optical Conductivity Superfluid/Insulator

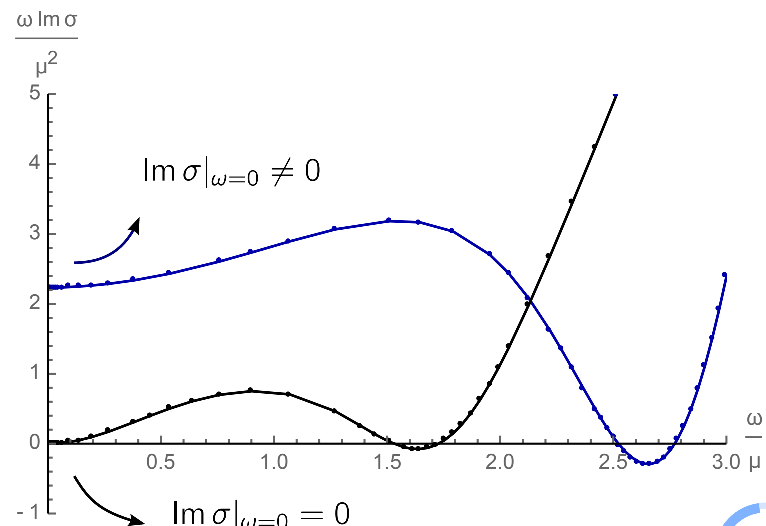


soft gap

Kramers-Kronig relations

$$\mathrm{Re}\, f(\omega) = \frac{2}{\pi}\mathcal{P}\int_0^\infty d\omega' \frac{\omega' \,\mathrm{Im}\, f(\omega')}{\omega'^2 - \omega^2}$$

$$\mathrm{Im}\, f(\omega) = -\frac{2\omega}{\pi}\mathcal{P}\int_0^\infty d\omega' \frac{\mathrm{Re}\, f(\omega')}{\omega'^2 - \omega^2}$$



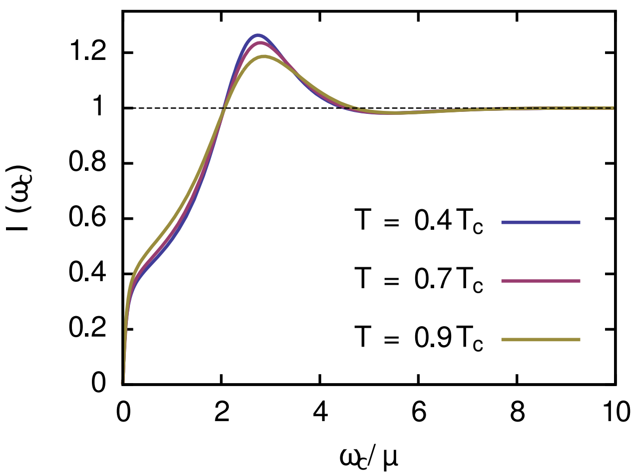
$$\mathrm{Re}\,\sigma(\omega) \sim \delta(\omega) \quad \Leftrightarrow \quad \mathrm{Im}\,\sigma(\omega) \sim \frac{i}{2}\frac{1}{\omega}$$

F-Sum Rule

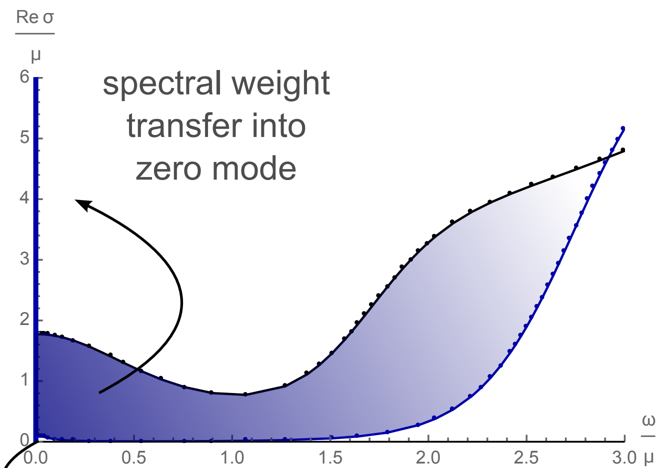
- Plasma frequency
  ~ Superfluid strength

$$\omega_{Ps}^2 = 8\int_0^\infty d\omega \ \mathrm{Re}\,\sigma_s(\omega)$$

$$= 4\pi\frac{n_s e^2}{m} = \lambda_L^{-2} \equiv \rho_s$$



- Ferrell-Glover-Tinkham sum rule



$\sigma_{reg} \neq 0$ regular contribution due to soft gap
c.f high Tc superconductors

$$\rho_s = \int_{0^+}^\infty d\omega \left[\mathrm{Re}\,\sigma_n(\omega) - \mathrm{Re}\,\sigma_s(\omega)\right]$$

assuming no residual regular
or high frequency contribution

Two Fluid Model

- Two fluid model

$$\mathrm{Re}\,\sigma(\omega) = \frac{e}{m^*}\left(\chi_\mathrm{n}(T)\frac{\tau}{1+\omega^2\tau^2} + \frac{\pi}{2}\chi_\mathrm{s}(T)\delta(\omega)\right)$$

regular metallic part — superfluid part

- χ is controlling the strength

normal phase:

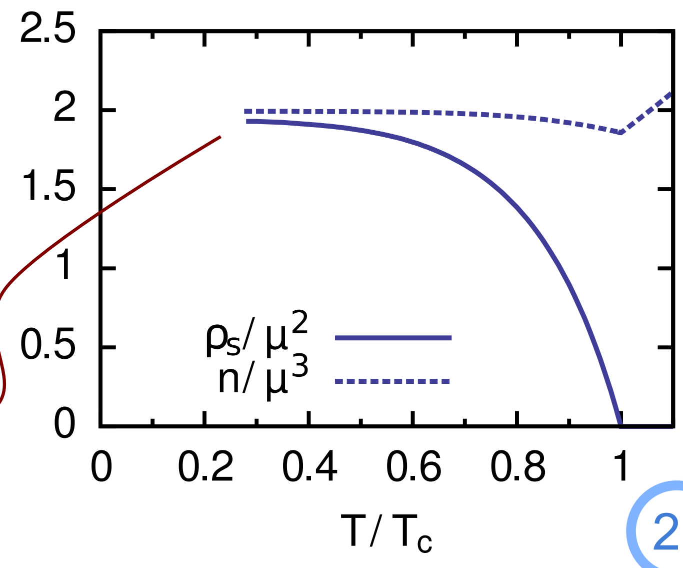
$$\chi_\mathrm{n}(T > T_c) = n_\mathrm{n}$$
$$\chi_\mathrm{s}(T > T_c) = 0$$

superconducting phase:

$$\chi_\mathrm{n}(T = 0) = 0$$
$$\chi_\mathrm{s}(T = 0) = n_\mathrm{s}$$

$$\rho_\mathrm{s}(T \to 0) \approx n_\mathrm{s}(T \to 0)$$

$$\chi_\mathrm{n}(T \to 0) \approx 0 \qquad \chi_\mathrm{s}(T \to 0) \approx n_\mathrm{s}$$



Part IV:
Homes' & Uemura's Relation

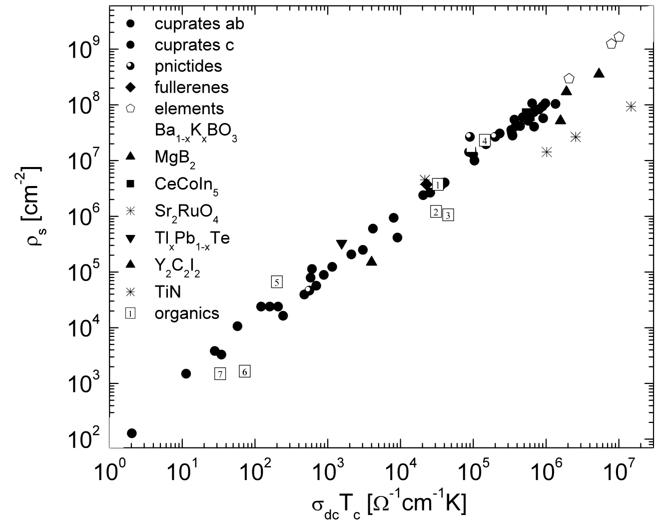
Homes Relation

- **Homes' relation**

$$\rho_s = C \sigma_{DC}(T_c) T_c$$

Insert Drude model:

$$\rho_s = C \frac{e}{m^*} n_n(T_c) \tau(T_c) T_c$$



[Dordevic,Basov,Homes '12]

- **Perfect fluids / "Planckian Dissipators**

$$\tau_\hbar(T) \sim \frac{\hbar}{k_B T} \qquad \frac{\eta}{Ts} = \frac{1}{4\pi}\frac{\hbar}{k_B T} = \frac{\tau_\hbar}{4\pi}$$

holographic superconductors
are
Planckian dissipators

Homes Relation

Homes Relation

- Holographic Version of Homes' Relation

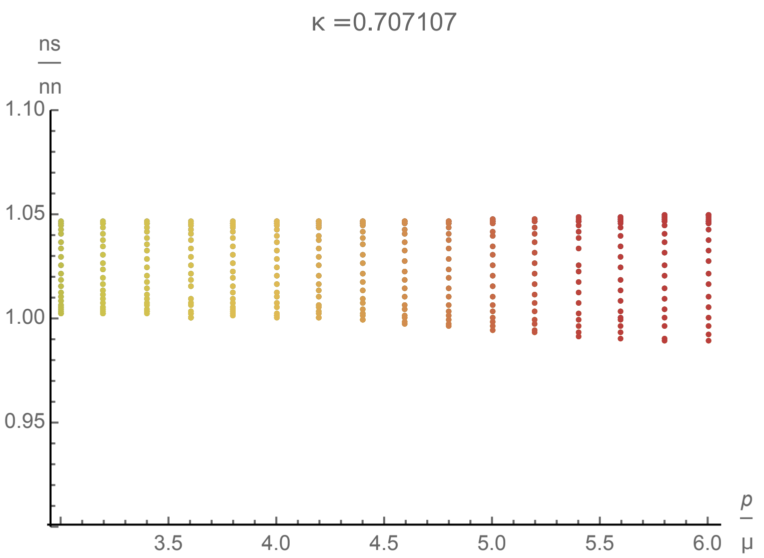
$$\rho_s = C\frac{e}{m^*}n_n(T_c)\tau(T_c)T_c$$

all charge carrier condense
in zero mode

$$\rho_s(T\to 0)\approx n_s(T\to 0)$$

quantum crictical system

$$\tau(T_c)T_c\sim 1$$



$$\Rightarrow \quad n_s(T\approx 0) = C'\frac{e}{m^*}n_n(T_c) \qquad \Rightarrow \quad \frac{n_s(T\approx 0)}{n_n(T_c)} = \text{const.}$$

Part V:
Zero Temperature Solutions

Zero Temperature Solution

Zero Temperature Solution

- Scaling solution in the deep IR (no horizon!)

$$a_0 = \frac{9\,\kappa\,|p|p\,e^{-2v_{10}}}{5\,(6\kappa^2 + q^2\rho_0^2 - 4)} \qquad w_0 = \sqrt{3}\left(\frac{|p|}{2}\right)^{-2} e^{2v_{10}+v_{20}}$$

$$\rho_1 = -\left(\frac{|p|}{2}\right)^4 \frac{\kappa^2 q^2 \rho_0 e^{-4v_{10}}}{(6\kappa^2 + q^2\rho_0^2 - 4)^2} \qquad w_1 = \left(\frac{|p|}{2}\right)^2 \frac{\sqrt{3}\left(q^2\rho_0^2 - 4\right) e^{v_{20}-2v_{10}}}{2\left(6\kappa^2 + q^2\rho 0^2 - 4\right)}$$

$$e^{v_{30}} = \frac{2}{|p|} e^{v_{10}+v_{20}}$$

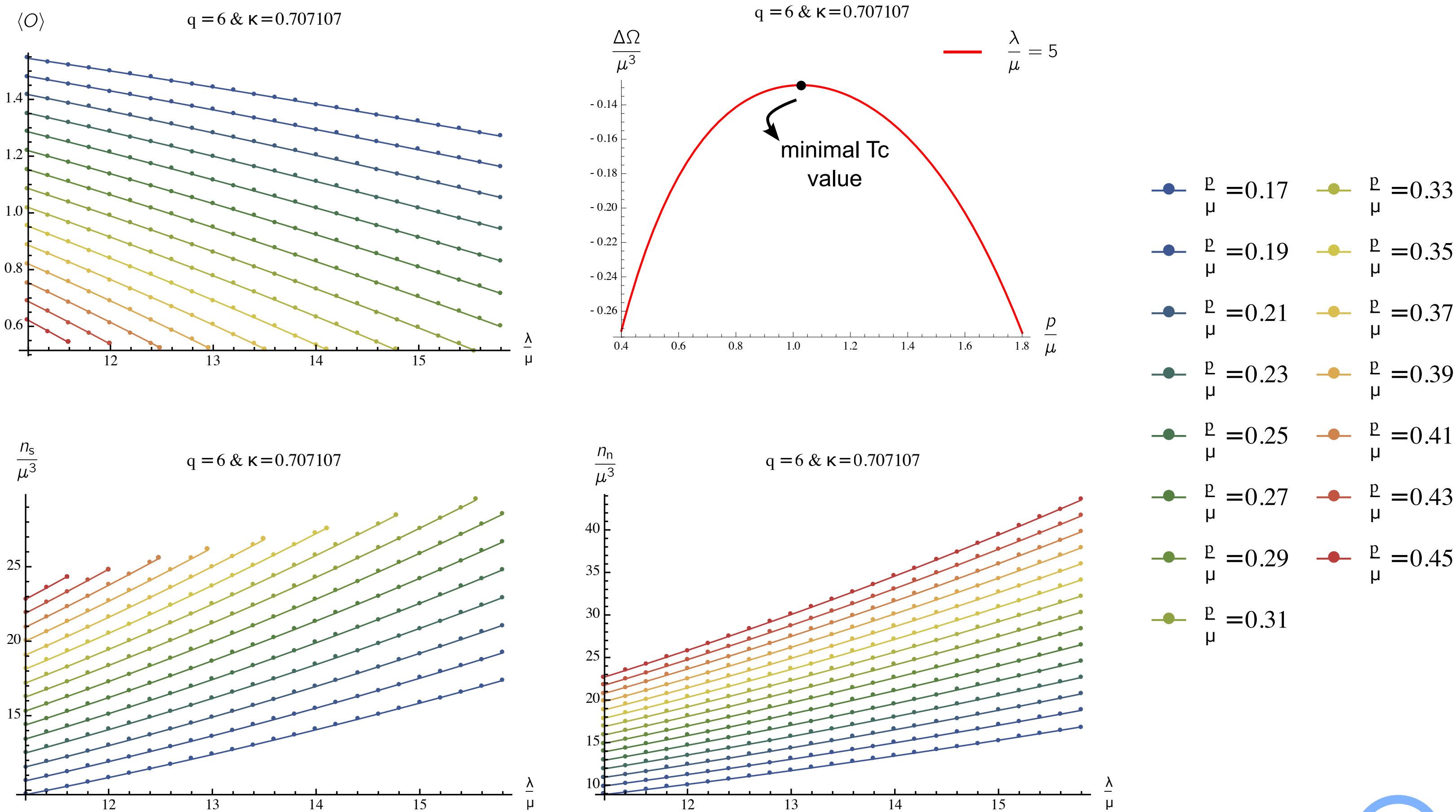
- Expansion about IR geometry

$$w = w_0 + w_1 r^{4/3}(1 + c_w r^\delta) \qquad \rho = \rho_0 + \rho_1 r^{4/3}(1 + c_\rho r^\delta) \qquad a = a_0 r^{5/4}(1 + c_a r^\delta)$$

$$v_1 = v_{10} + \log(r^{-1/3}) + c_1 r^\delta \qquad v_2 = v_{20} + \log(r^{2/3}) + c_2 r^\delta \qquad U = \frac{18}{5} r^2 (1 + c_U r^\delta)$$

$$v_3 = v_{30} + \log(r^{1/3}) + c_3 r^\delta$$

Zero Temperature Thermodynamics



Outlook

- Check other relations such as Uemura's or Tanner's relation
- Scaling behavior of optical conductivity with temperature and frequency
- Zero temperature solutions deserve closer investigation
  - ↪ Determine zero temperature quantities e.g. ρ, ns, σ,...
  - ↪ Compute zero temperature normal phase diagram

Thank You!

for listening...