AC conductivities in a holographic model of momentum relaxation

1409.XXXX

K-Y Kim, Kyung Kiu Kim, Yunseok Seo, and Sang-Jin Sin

Keun-Young Kim
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In the context of this meeting

AC conductivities in a holographic model of momentum relaxation

Universal incoherent metallic transport

Sean Hartnoll (Stanford)

Charge transport with momentum relaxation in holography

Blaise Goutéraux

Holographic Lattices, Metals and Insulators

Jerome Gauntlett

Strongly Coupled Anisotropic Fluids From Holography

Sandip Trivedi

The thermoelectric properties of inhomogeneous holographic lattices

Aristomenis Donos

In the context of arXive

1311.5157	**A simple holographic model of momentum relaxation** Tomás Andrade[1] and Benjamin Withers[2]	Electric DC
1406.4870	**Inhomogeneity simplified** **Marika Taylor and William Woodhead**	Extended model + Electric AC
1409.XXXX	AC conductivities in a holographic model of momentum relaxation K-Y Kim, Kyung Kiu Kim, Yunseok Seo, and Sang-Jin Sin	Electric AC in more detail + Thermal/Thermoelectric

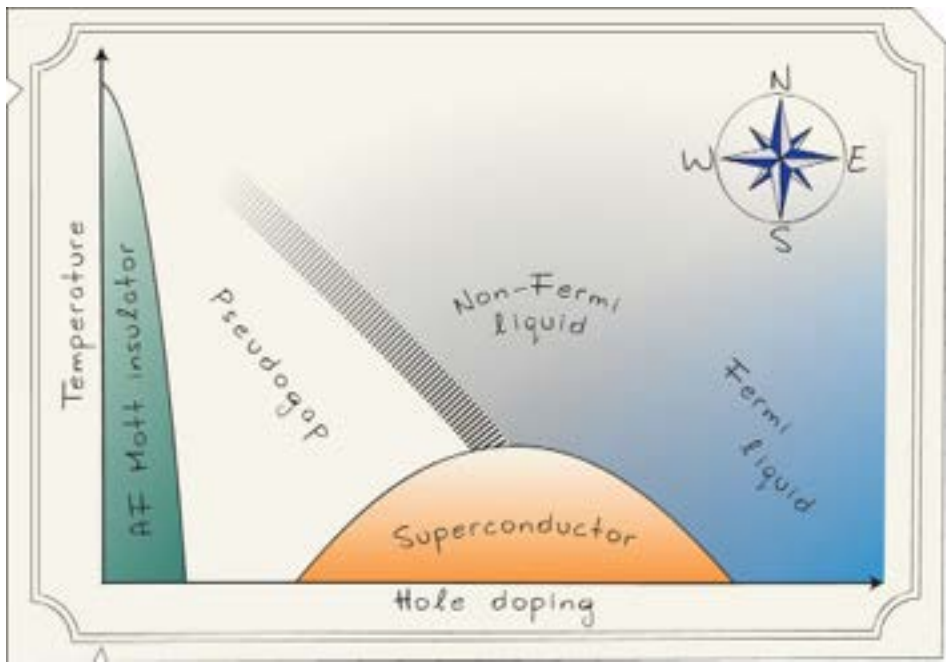
Outline

1. Motivations and objects
2. Methodology: RN AdS black holes
   - Review
   - Numerical recipe
3. RN AdS black holes + momentum relaxation
4. Summary and future plans

Motivations: Phenomenology

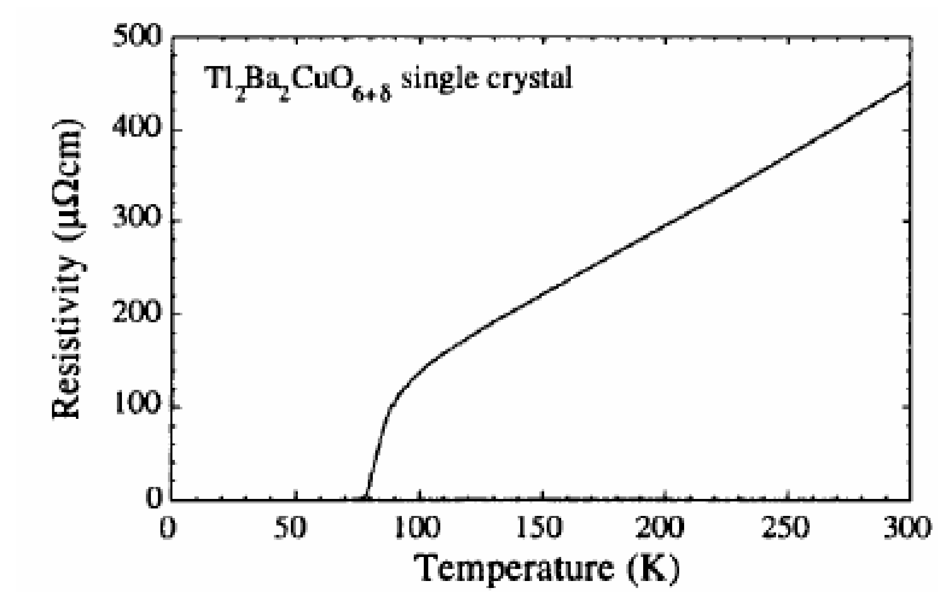
Anomalous conductivities of strongly interacting system

- Cuprate phase diagram

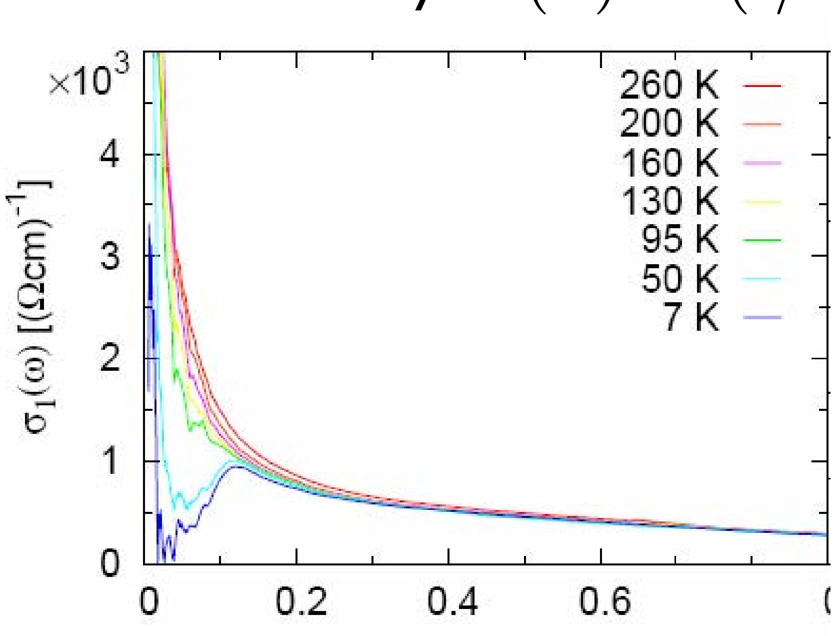


Peter Wahl, 2012, Nature Physics

- DC resistivity $\rho \sim T$



- AC conductivity $\sigma(\omega) \to (i/\omega)^{\nu}$

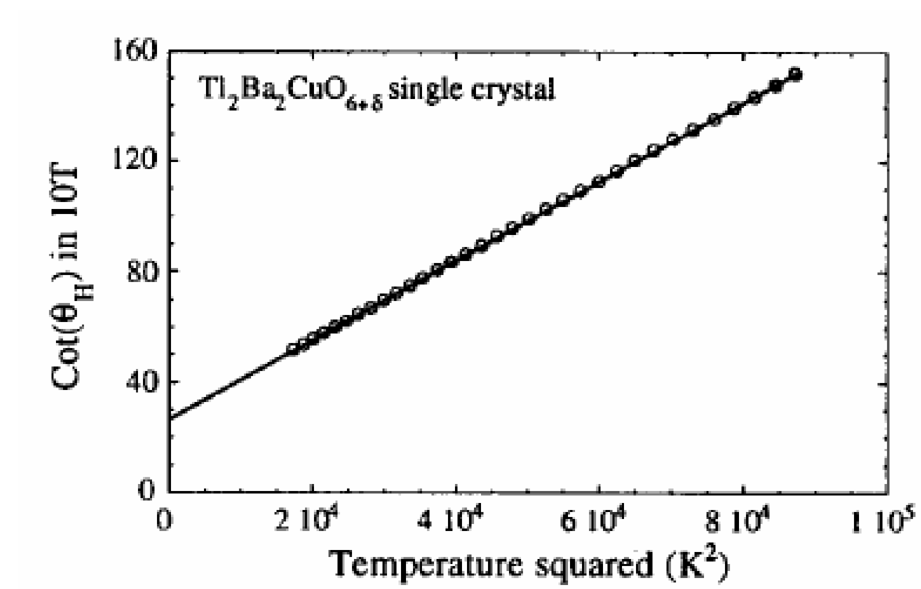


$\nu \approx 0.65$

Van der Marel et al., 2003

Jerome, Aristos, Blaise, Sera

- Hall angle $\sigma_{xx}/\sigma_{xy} \sim T^2$



Mike Blake

Mackenzie, 1997

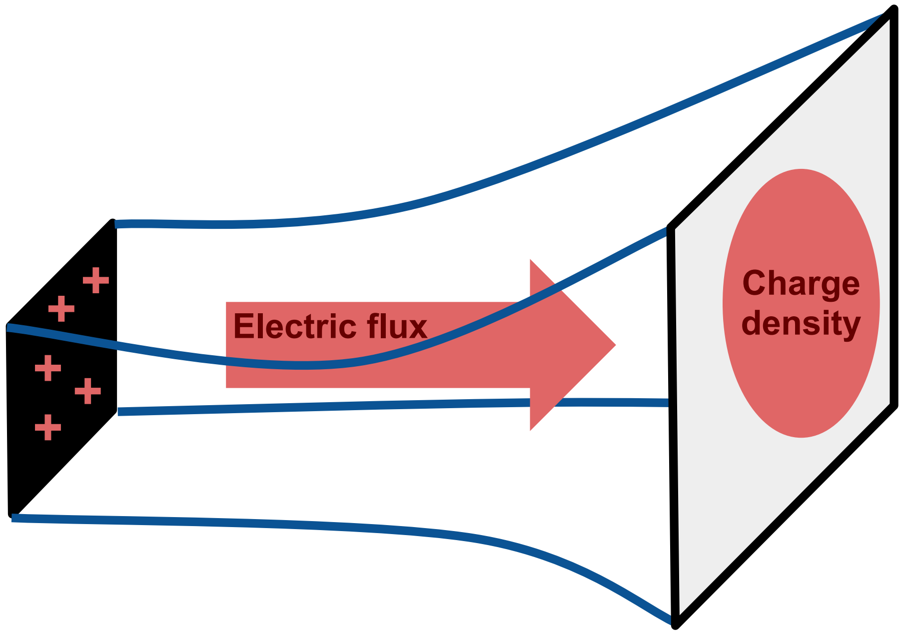
Motivations: Holographic model

Conductivity of strongly interacting system by holography

- Einstein-Maxwell system

$$S_{\rm EM} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M}\mathrm{d}^3x\sqrt{-\gamma}K$$

- Reissner-Nordstrom-AdS black hole
  ~ Boundary field theory at finite temperature and density



Hartnoll, 1106.4324

$$+ \quad (f\delta A_x')' + \frac{\omega^2}{f}\delta A_x - \frac{4\mu^2 r^2}{\gamma^2 r_+^2}\delta A_x = 0$$

- Electric conductivity

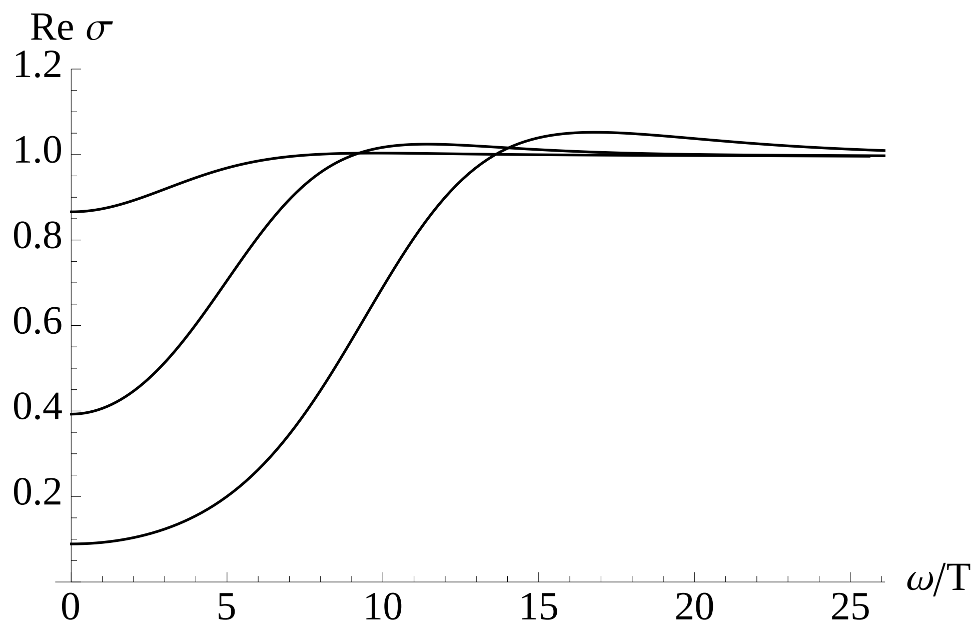
$$\delta A_x(r,\omega) = \frac{E}{i\omega} + \frac{J_x(\omega)}{r} + \cdots \qquad J_x = \sigma E_x$$

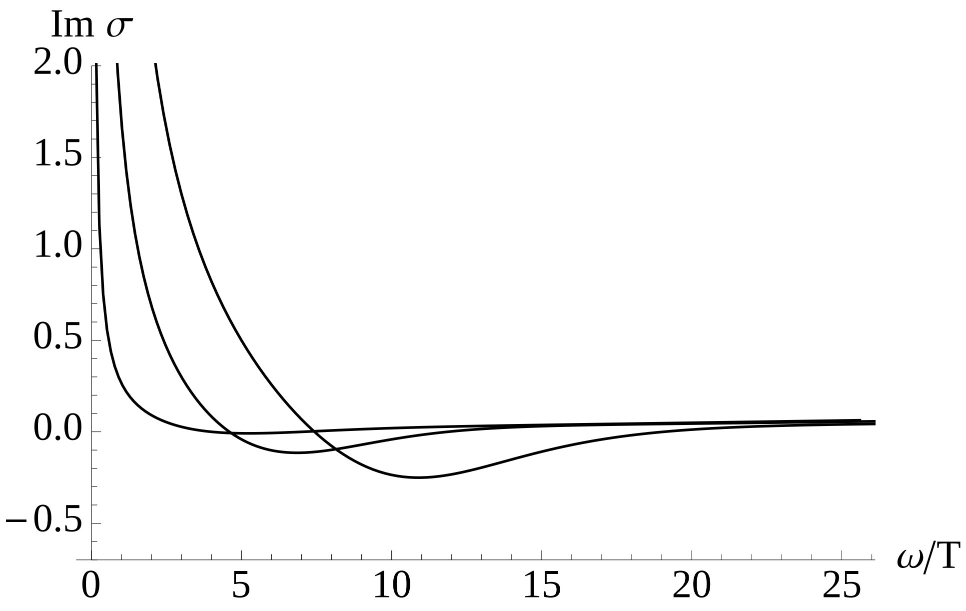
Motivations: Holographic model

- Conductivity

Herzog, Kovtun, Sachdev and Son; Hartnoll 2007

$$A_x = \frac{E_x}{i\omega} e^{i\omega t} + \frac{J_x}{r} + \cdots \qquad J_x = \sigma E_x$$





- Kramers-Kronig relation

$$\mathrm{Im}\,\sigma \sim 1/\omega \quad \Leftrightarrow \quad \mathrm{Re}\,\sigma(\omega) \sim \delta(\omega)$$

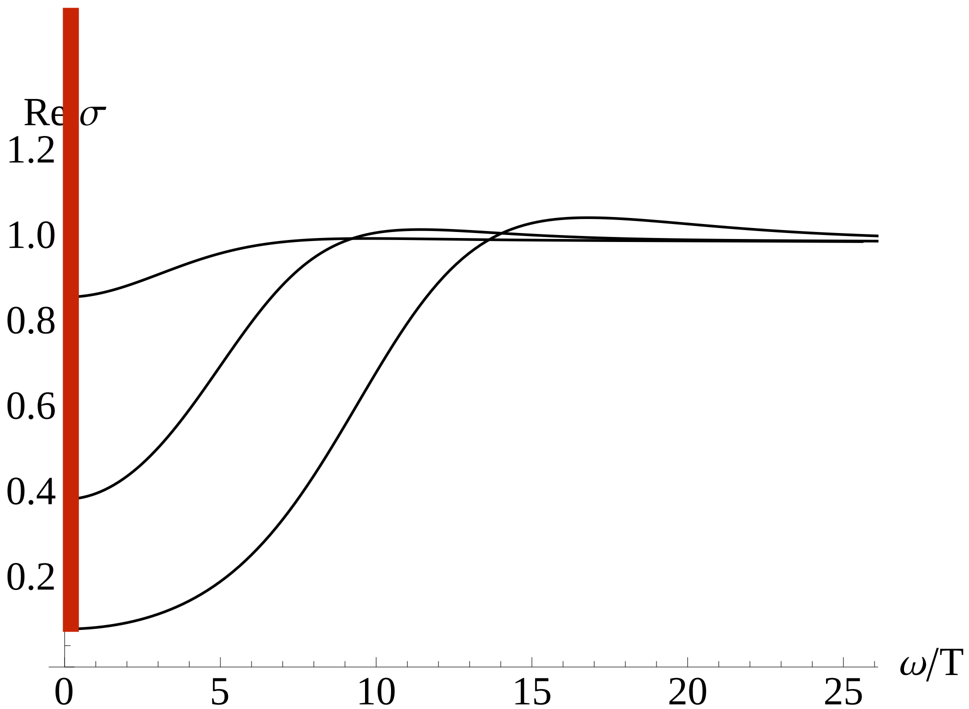
Translation invariance + finite density

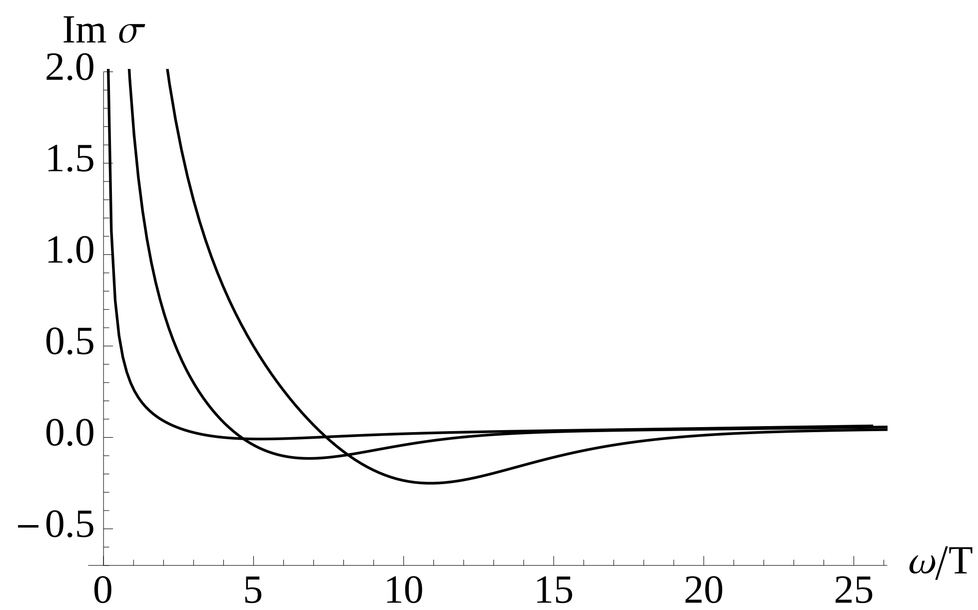
Motivations: Holographic model

- Conductivity

Herzog, Kovtun, Sachdev and Son; Hartnoll 2007

$$A_x = \frac{E_x}{i\omega} e^{i\omega t} + \frac{J_x}{r} + \cdots \qquad J_x = \sigma E_x$$





- Kramers-Kronig relation

$$\mathrm{Im}\,\sigma \sim 1/\omega \quad \Leftrightarrow \quad \mathrm{Re}\,\sigma(\omega) \sim \delta(\omega)$$

Translation invariance + finite density

Important to have a model without a delta function

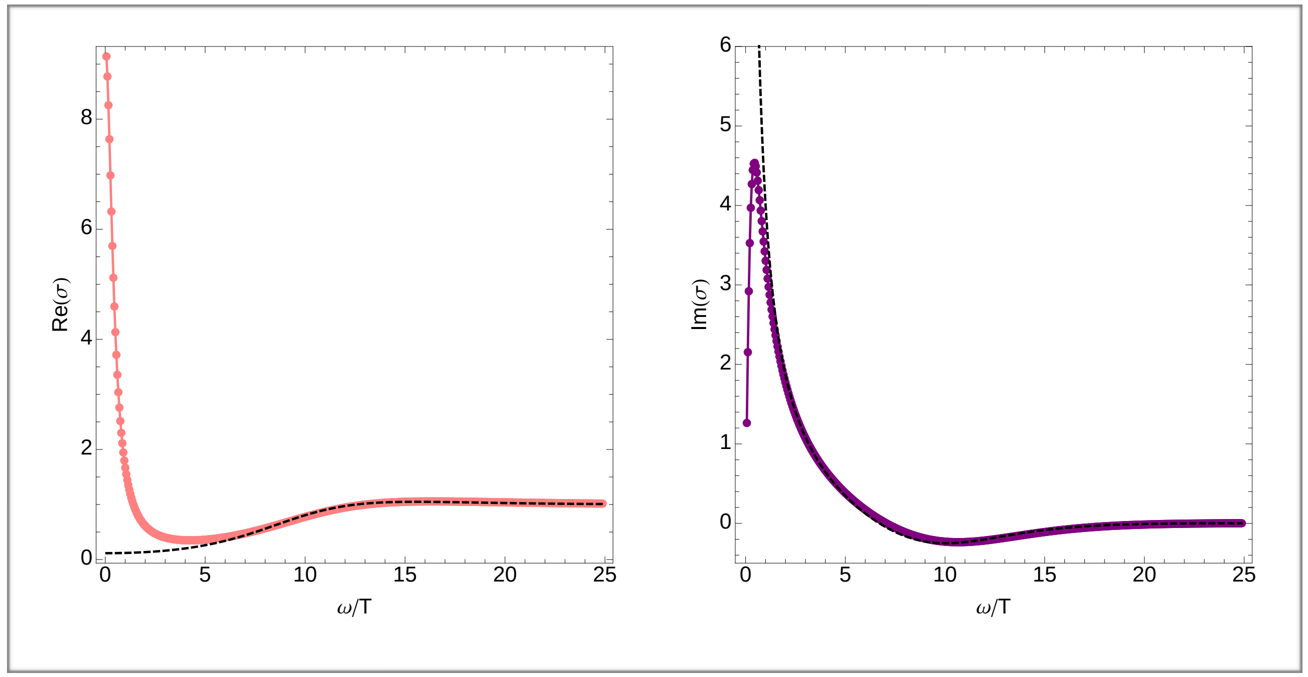
Motivations: Holographic model

- Momentum relaxation

'Ionic' Lattice

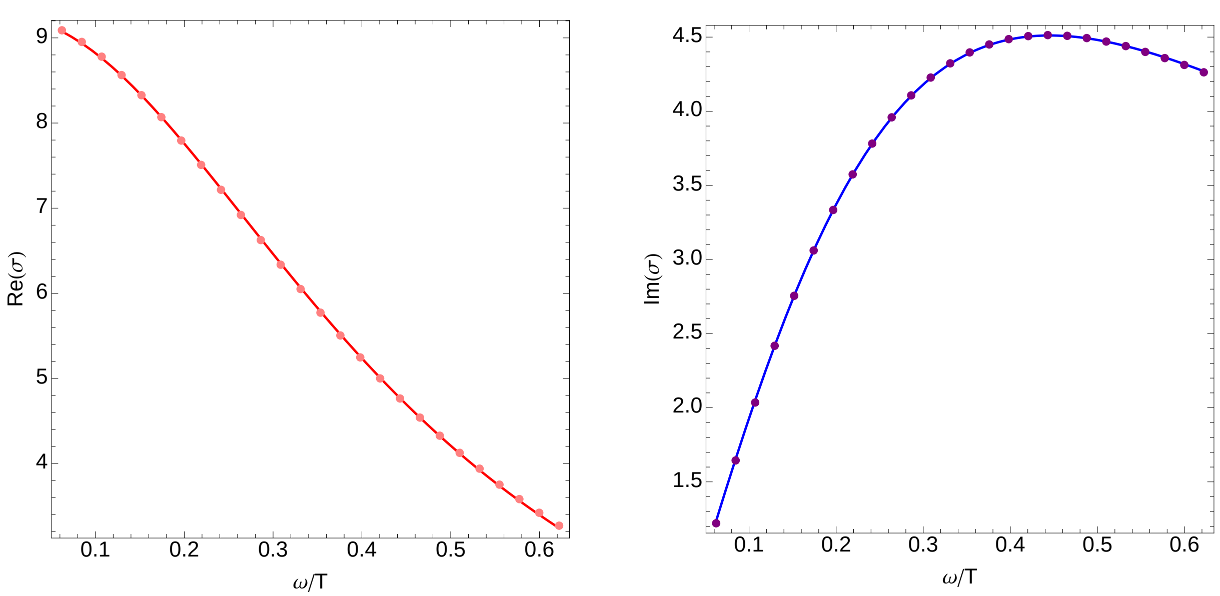
$$A_t \sim 1 + A_0 \cos(k_0 x)$$
$$\phi \sim A_0 \cos(k_0 x)$$

Horowitz, Santos, Tong: 1209.1098
Horowitz, Santos, Tong: 1204.0512



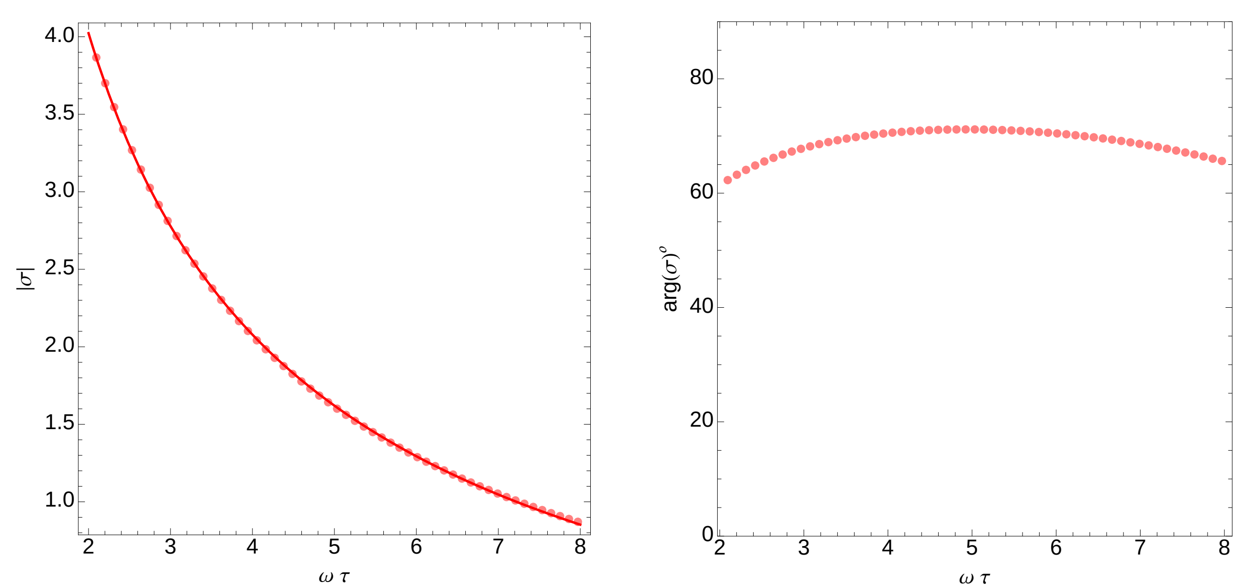
Low frequency

$\omega < T$ $\qquad$ $\sigma(\omega) = \dfrac{K\tau}{1 - i\omega\tau}$



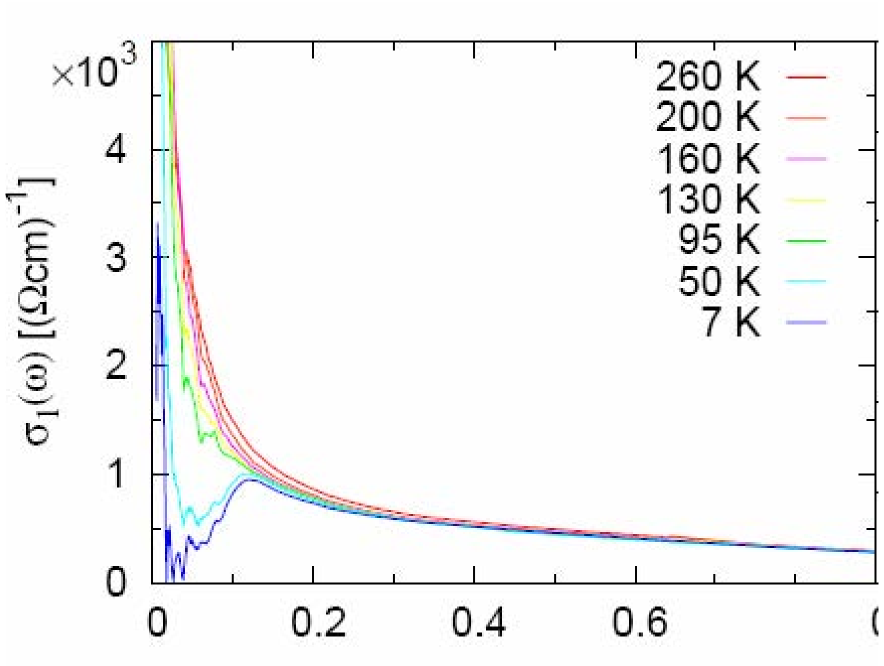
Intermediate frequency

$T < \omega < \mu$ $\qquad$ $|\sigma(\omega)| = \dfrac{B}{\omega^{2/3}} + C$

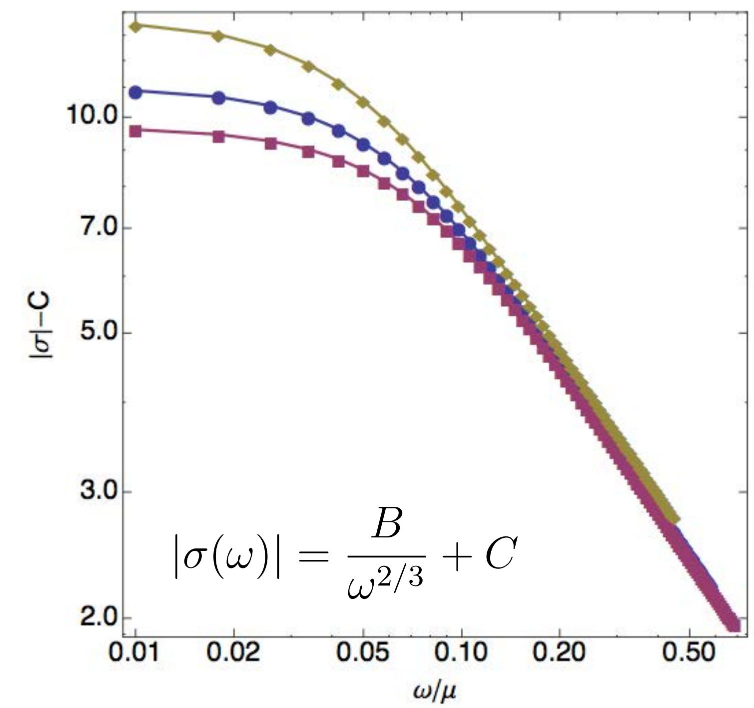


Motivations: Holographic model

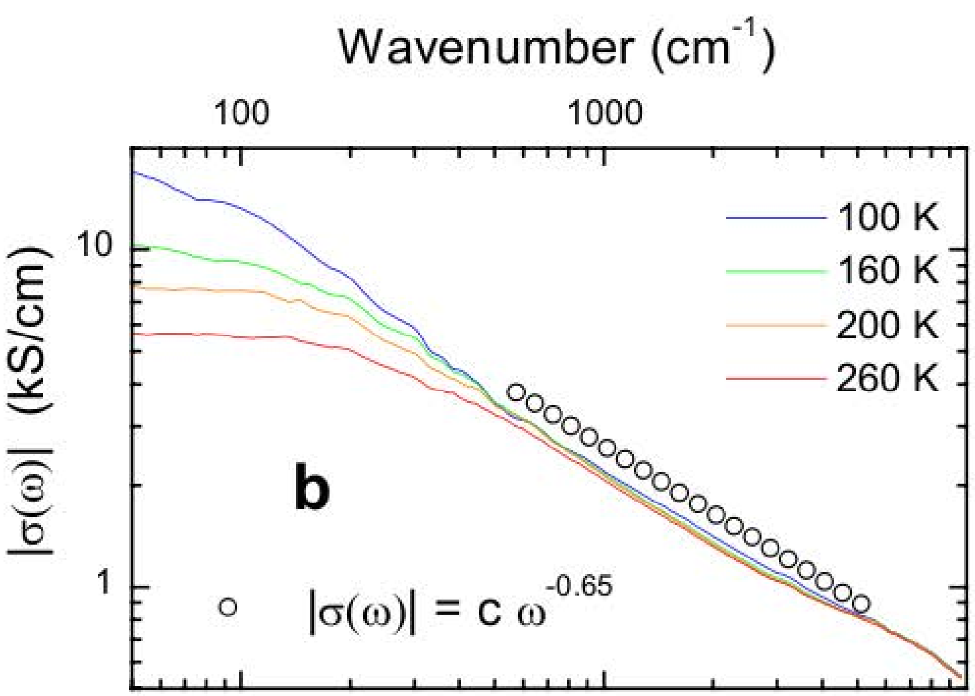
- AC conductivity $\sigma(\omega) \to (i/\omega)^\nu$ $\nu \approx 2/3$

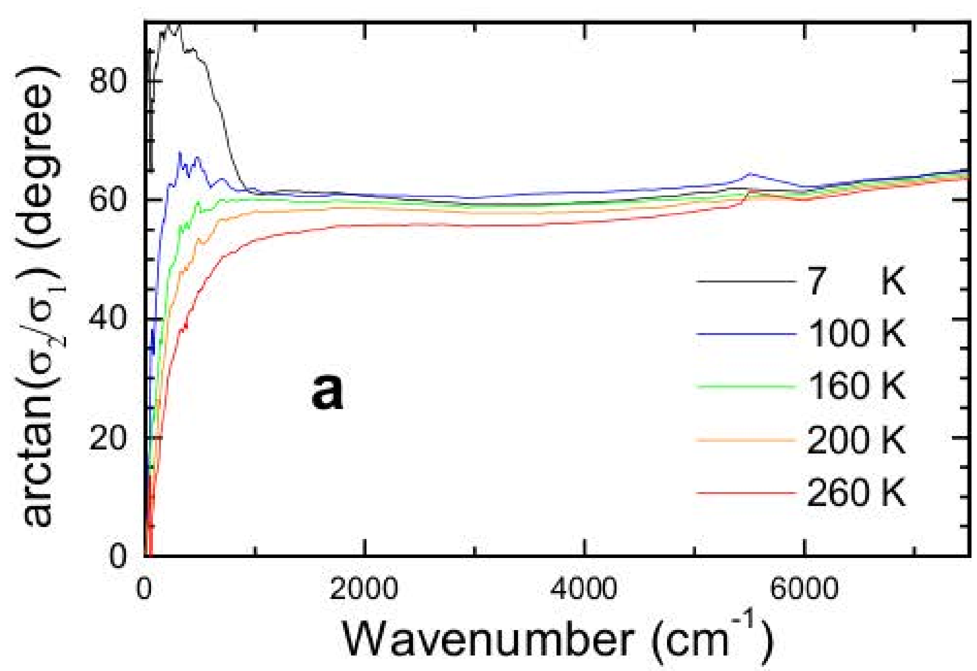


Van der Marel et al., 2003

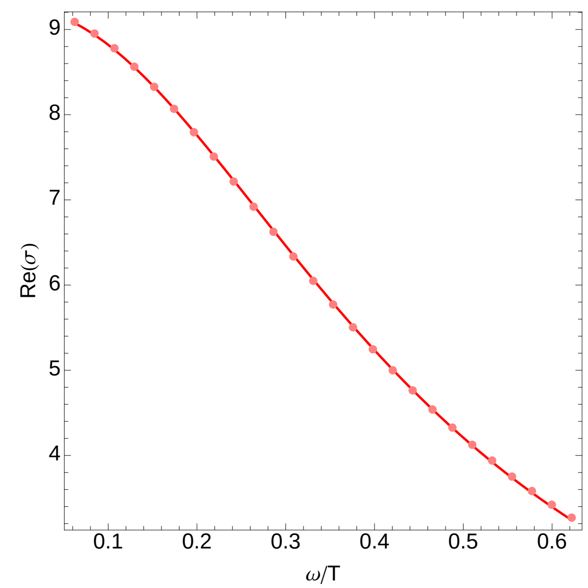


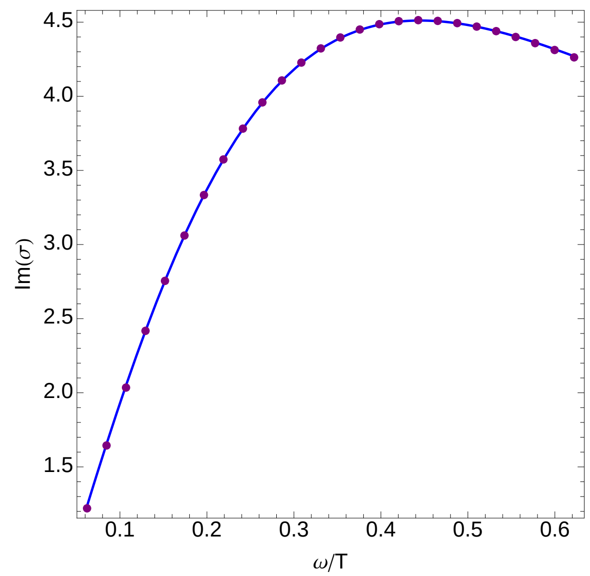
Horowitz, Santos, Tong: 1204.0512





Main questions



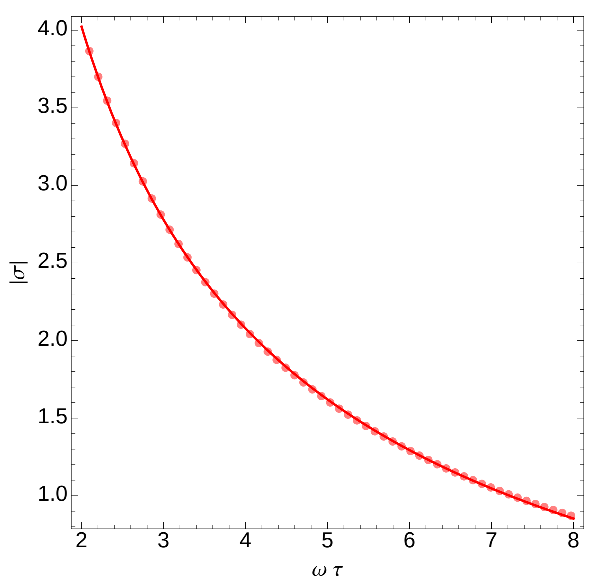


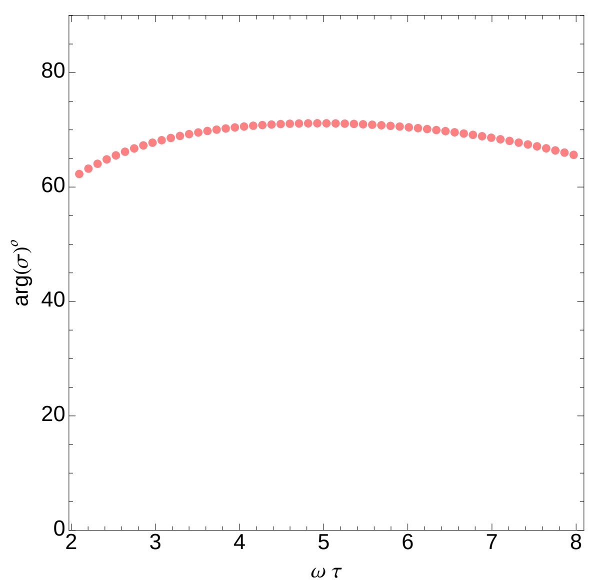
Q1. No contribution from pair creation?

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} \quad + \sigma_Q ?$$ ← pair creation

Q2. Drude peak without quasi particle?

1) Weak translation symmetry breaking(coherent metal)
Yes by Hartnoll and Hofman(1201.3917)
2) Strong translation symmetry breaking(incoherent metal)
?





Q3. Origin of scaling?

Q4. B and C ?

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$

Q5. Thermoelectric and thermal conductivity?

Q0. Easier model capturing essential physics?

Models

- Momentum relaxation

'Ionic' Lattice $A_t \sim 1 + A_0 \cos(k_0 x)$ Horowitz, Santos, Tong: 1209.1098

$\phi \sim A_0 \cos(k_0 x)$ Horowitz, Santos, Tong: 1204.0512

- Background: 7 PDEs in two variables

$$ds^2 = \frac{L^2}{z^2}\left[-(1-z)P(z)Q_{tt}dt^2 + \frac{Q_{zz}dz^2}{P(z)(1-z)} + Q_{xx}(dx + z^2 Q_{xz}dz)^2 + Q_{yy}dy^2\right]$$

$$A = (1-z)\,\psi(x,z)\,dt \qquad \Phi = z\,\phi(x,z)$$

- Fluctuations:11 PDEs in two variables

$$\{\tilde{h}_{tt}, \tilde{h}_{tz}, \tilde{h}_{tx}, \tilde{h}_{zz}, \tilde{h}_{zx}, \tilde{h}_{xx}, \tilde{h}_{yy}, \tilde{b}_t, \tilde{b}_z, \tilde{b}_x, \tilde{\eta}\}$$

- Momentum relaxation simplified (ODE)

$\psi_I = \beta_{Ii} x^i = \beta \delta_{Ii} x^i$ Andrade and Withers 1311.5157 Sandip Trivedi

$\sigma_{DC} = 1 + \frac{\mu^2}{\beta^2}$

Other methods

Massive gravity model: Vegh(1301), Davison(1306)

Q-lattice model: Donos and Gauntlett (1311)

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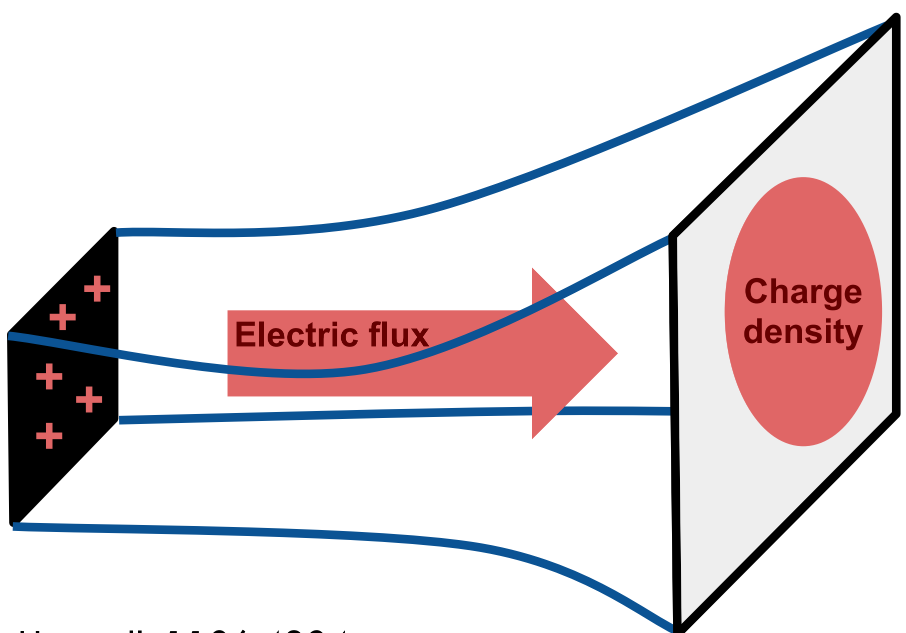
RN AdS black holes

- Einstein-Maxwell system

$$S_{\rm EM} = \int_M d^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M} d^3x\sqrt{-\gamma}K$$

- Reissner-Nordstrom-AdS black hole
  ~ Boundary field theory at finite temperature and density



Hartnoll, 1106.4324

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\delta_{ij}dx^i dx^j\,,$$

$$f(r) = r^2 - \frac{r_0^3}{r}\left(1 + \frac{\mu^2}{4r_0^2}\right) + \frac{\mu^2 r_0^2}{4r^2}\,,$$

$$A = \mu\left(1 - \frac{r_0}{r}\right)dt$$

$$T = \frac{f'(r_0)}{4\pi} = \frac{1}{4\pi}\left(3r_0 - \frac{\mu^2}{4r_0}\right)$$

Electric, Thermoelectric, Thermal conductivity

- Fluctuations

$$\delta g_{ti}(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} r^2 h_{ti}(\omega, r),$$
$$\delta A_i(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} a_i(\omega, r),$$

- EOMs

$$\begin{aligned} -\frac{\mu a_x'}{r^4} - \frac{4h_{tx}'}{r} - h_{tx}'' &= 0 \\ \frac{\mu a_x}{r^4} + h_{tx}' &= 0 \\ \frac{f' a_x'}{f} + \frac{\mu h_{tx}'}{f} + \frac{\omega^2 a_x}{f^2} + a_x'' &= 0 \end{aligned}$$

- Boundary action

$$S_{\mathrm{ren}}^{(2)} = \lim_{r\to\infty} V_2 \frac{1}{2} \int \mathrm{d}\omega \left[-m_0\, h_{tx} h_{tx} - \mu\, a_{tx} h_{tx} - f(r) a_{tx} a_{tx}' + r^4 h_{tx} h_{tx}'\right]$$

$$m_0 = \left(1 + \frac{\mu^2}{4}\right)$$

$$\begin{pmatrix} G^R_{J_x J_x} & G^R_{J_x T_{tx}} \\ G^R_{T_{tx} J_x} & G^R_{T_{tx} T_{tx}} \end{pmatrix} = \begin{pmatrix} \frac{a_x^{(1)}}{a_x^{(0)}} & -\mu \\ -\mu & -m_0 \end{pmatrix}$$

$$=: \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

Linear response

$$\begin{pmatrix} \langle J_x \rangle \\ \langle T_{tx} \rangle \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} \delta a_x^s \\ \delta h_{tx}^s \end{pmatrix}$$

$$\begin{pmatrix} \langle J_x \rangle \\ \langle Q_x \rangle \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar\alpha T & \bar\kappa T \end{pmatrix} \begin{pmatrix} E_x \\ -(\nabla_x T)/T \end{pmatrix}$$

- Two issues for generalisation
  1. more than one equation
  2. identify the sources and currents

Hartnoll 0903.3246

$$\begin{pmatrix} \sigma & \alpha T \\ \bar\alpha T & \bar\kappa T \end{pmatrix} = \begin{pmatrix} -\frac{iG_{11}}{\omega} & \frac{i(G_{11}\mu - G_{12})}{\omega} \\ \frac{i(G_{11}\mu - G_{21})}{\omega} & -\frac{i(G_{22} + \mu(-G_{12} - G_{21} + G_{11}\mu))}{\omega} \end{pmatrix}$$

Systematic numerical methods

- Fluctuations

$$\Phi^a(x,r) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \Phi^a_k(r) e^{-ikx}$$

- Boundary action

$$S_B = \lim_{r\to\infty} \frac{1}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left[\Phi^a_{-k}(r) A_{ab}(r,k) \Phi^b_k(r) + \Phi^a_{-k}(r) B_{ab}(r,k) \partial_r \Phi^b_k(r)\right]$$

- Solutions near boundary

$$\Phi^a_k(r) = \Phi^a_{k,i}(r) c^i \to \left(\Phi^{\mathrm{s},a}_{k,i} + \frac{\Phi^{\mathrm{o},a}_{k,i}}{r^{\delta_a}} + \cdots\right) c^i \qquad \text{(near boundary)}$$

$$J^a_k = \Phi^{\mathrm{s},a}_{k,i} c^i \qquad\qquad c^i = \Phi^{\mathrm{s},i}_{k,a} J^a_k$$

$$B_{ac}(r,k)\partial_r \Phi^c_k(r) = \left[-B_{ac}(r,k)(\delta_c r^{-\delta_c - 1} \Phi^{\mathrm{o},c}_{k,i}) \Phi^{\mathrm{s},i}_{k,b}\right] J^b_k + \cdots = \left[C_{ab}(r,k)\right] J^b_k + \cdots$$

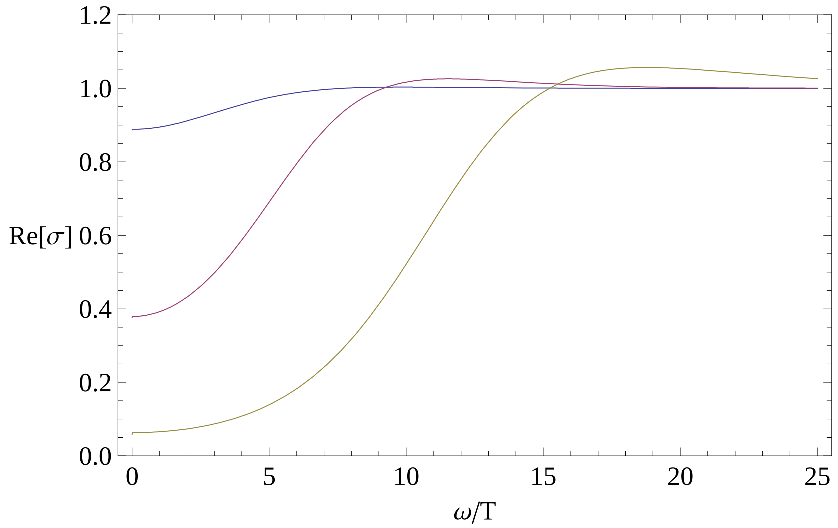
- Boundary action

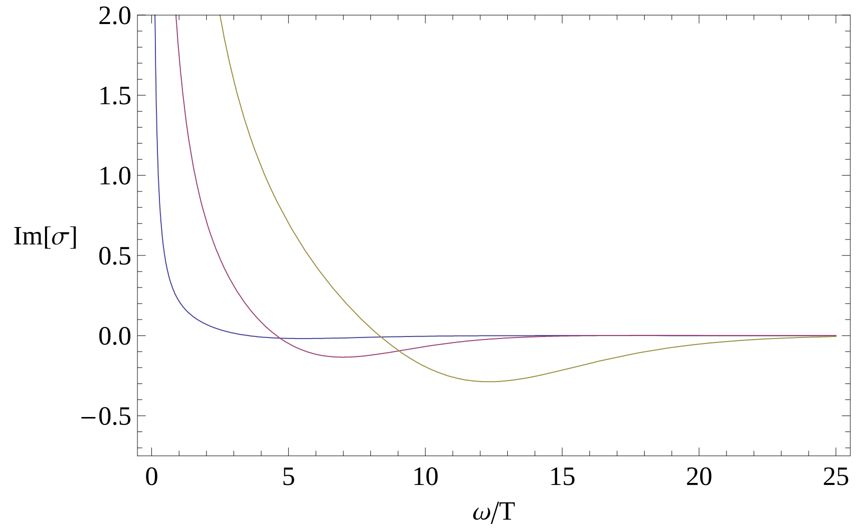
$$S_B = \frac{1}{2} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left[J^a_{-k} \left[A_{ab}(\infty,k) + C_{ab}(\infty,k)\right] J^b_k\right]$$

$$G_{ab} = A_{ab}(\infty,k) + C_{ab}(\infty,k)$$

Based on
Kaminski, Landsteiner, Mas, Shock, Tarrio(2009)

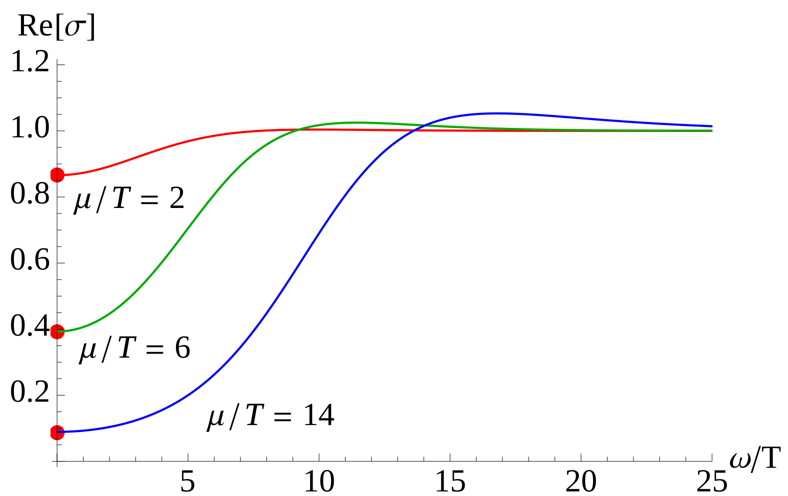
Checking numerical methods

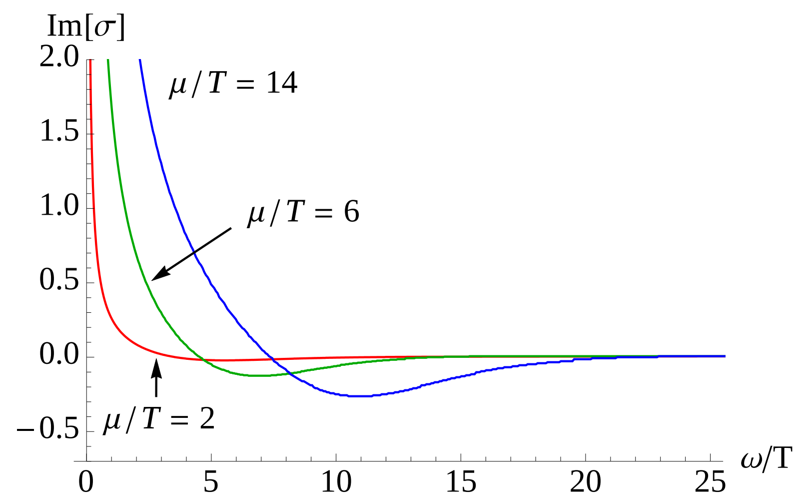


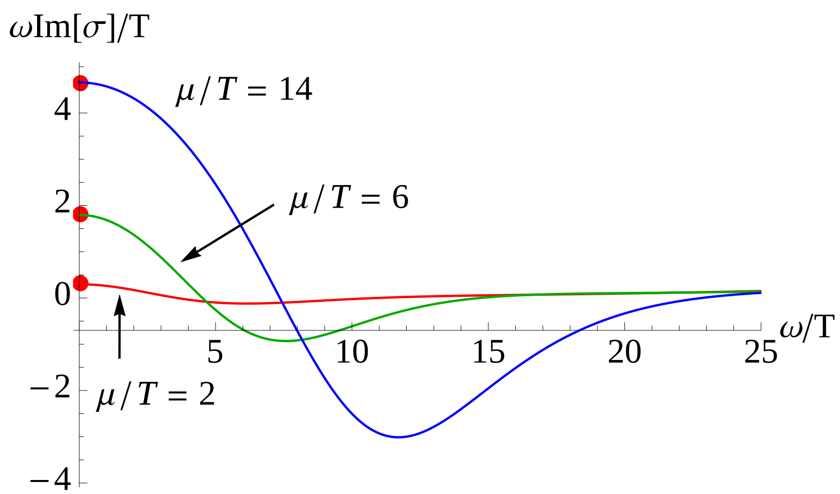


Hartnoll 0903.3234

Our results







$$\sigma = \sigma_Q + i\frac{K}{\omega}$$

$$\sigma_Q = \left(\frac{3 - \frac{\mu^2}{4r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}}\right)^2 \qquad K = r_0 \frac{\frac{\mu^2}{r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}}$$

$$r_0 = \frac{2\pi}{3}\left(T + \sqrt{T^2 + 3(\mu/4\pi)^2}\right)$$

Ge, Jo, and Sin, 1012.2515

$$\begin{pmatrix} G^R_{J_xJ_x} & G^R_{J_xT_{tx}} \\ G^R_{T_{tx}J_x} & G^R_{T_{tx}T_{tx}} \end{pmatrix} = \begin{pmatrix} \frac{a_x^{(1)}}{a_x^{(0)}} & -\mu \\ -\mu & -m_0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a_x^{(1)}}{a_x^{(0)}} & -\frac{\mu}{2} - \frac{h_{tx}^{(1)}}{a_x^{(0)}} \\ -\frac{\mu}{2} - \frac{h_{tx}^{(1)}}{a_x^{(0)}} & -m_0 \end{pmatrix}$$

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RN AdS black holes + scalar

Bardoux, Caldarelli, Charmousis (2012), Andrade, Withers(2013)

- Actions

$$S_{\rm EM} = \int_M \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{4}F^2\right] - 2\int_{\partial M}\mathrm{d}^3x\sqrt{-\gamma}K$$

$$S_\psi = \int_M \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right]$$

$$S_{\rm c} = \int_{\partial M}\mathrm{d}x^3\sqrt{-\gamma}\left(-4 - R[\gamma] + \frac{1}{2}\sum_{I=1}^{2}\gamma^{\mu\nu}\partial_\mu\psi_I\partial_\nu\psi_I\right)$$

- EOMs

$$R_{MN} = \frac{1}{2}g_{MN}\left(R - 2\Lambda - \frac{1}{4}F^2\right) + \frac{1}{2}\sum_I \partial_M\psi_I\partial_N\psi_I + \frac{1}{2}F_M{}^PF_{NP}$$

$$\nabla_M F^{MN} = 0\,,$$

$$\nabla^2\psi_I = 0\,.$$

- RN-AdS solution + two scalars

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\,,$$

$$f(r) = r^2 - \frac{\beta^2}{2} - \frac{m_0}{r} + \frac{\mu^2}{4}\frac{r_0^2}{r^2}\,,$$

$$A = \mu\left(1 - \frac{r_0}{r}\right)\mathrm{d}t\,,$$

$$\psi_I = \beta_{Ii}x^i = \beta\delta_{Ii}x^i\,,$$

$$m_0 = r_0^3\left(1 + \frac{\mu^2}{4r_0^2} - \frac{\beta^2}{2r_0^2}\right)$$

$$T = \frac{f'(r_0)}{4\pi} = \frac{1}{4\pi}\left(3r_0 - \frac{\mu^2 + 2\beta^2}{4r_0}\right)$$

$$r_0 = \frac{2\pi}{3}\left(T + \sqrt{T^2 + 3(\mu/4\pi)^2 + 6(\beta/4\pi)^2}\right)$$

RN AdS black holes + scalar

- Fluctuations

$$\delta g_{tx}(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} r^2 h_{tx}(\omega,r),$$
$$\delta A_x(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} a_x(\omega,r),$$
$$\delta \psi_x(t,r) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} \chi_x(\omega,r)$$

- EOMs

$$\frac{\beta^2 h_{tx}}{r^2 f} + \frac{i\beta\omega\chi_x}{r^2 f} - \frac{\mu a_x'}{r^4} - \frac{4h_{tx}'}{r} - h_{tx}'' = 0$$
$$\frac{i\beta f \chi_x'}{r^2\omega} + \frac{\mu a_x}{r^4} + h_{tx}' = 0$$
$$\frac{f' a_x'}{f} + \frac{\mu h_{tx}'}{f} + \frac{\omega^2 a_x}{f^2} + a_x'' = 0$$
$$\frac{f'\chi_x'}{f} - \frac{i\beta\omega h_{tx}}{f^2} + \frac{\omega^2\chi_x}{f^2} + \frac{2\chi_x'}{r} + \chi_x'' = 0$$

- Boundary action

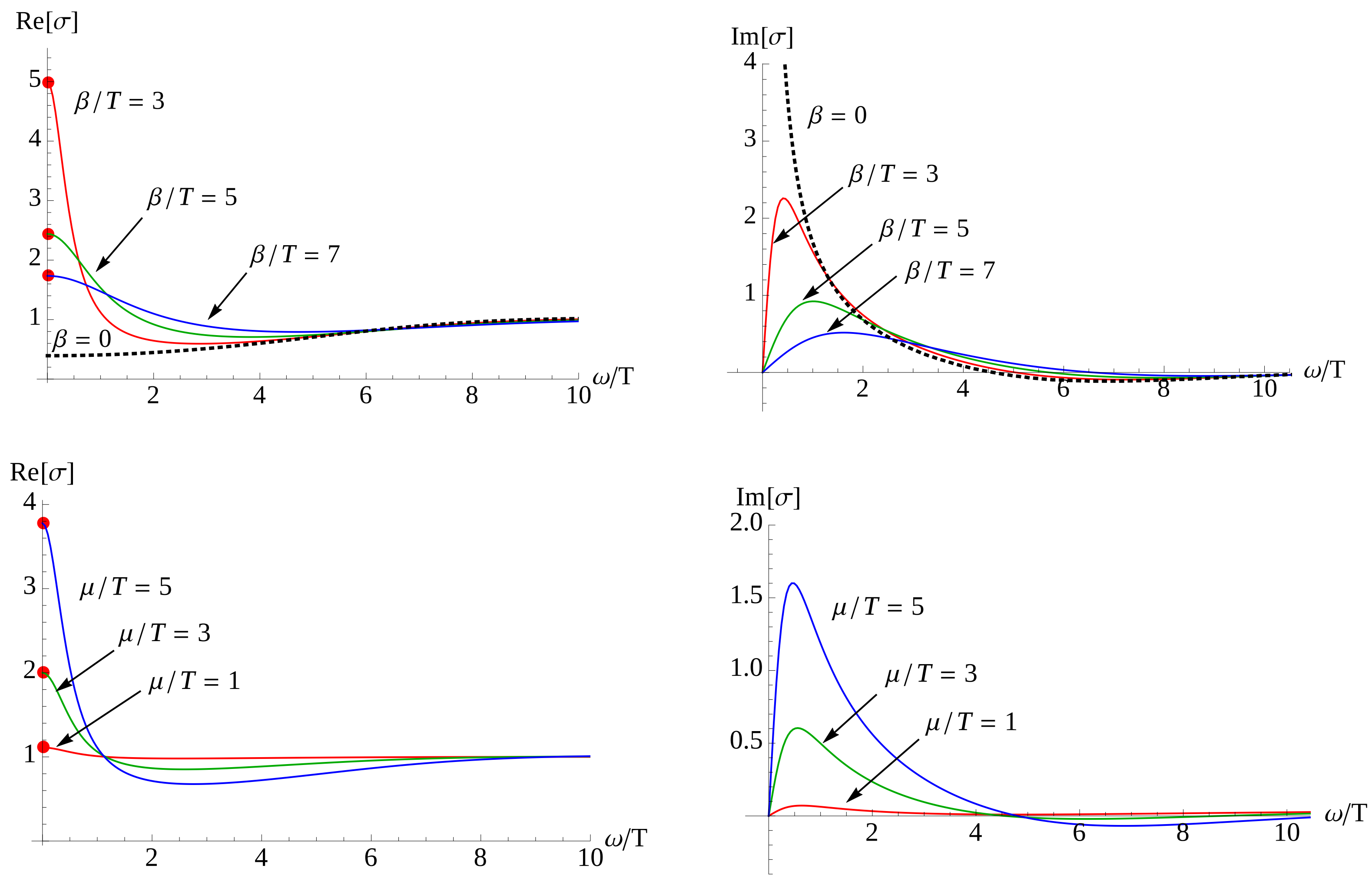
$$S^{(2)}_{\text{ren}} = \lim_{r\to\infty} V_2 \frac{1}{2}\int \mathrm{d}\omega \left[-m_0\, h_{tx}h_{tx} - \mu\, a_{tx}h_{tx} - f(r) a_{tx}a_{tx}' + r^4 h_{tx}h_{tx}' - r^2 f(r)\chi_x\chi_x'\right]$$

AC electric conductivity

DC limit

$$\sigma = 1 + \frac{\mu^2}{\beta^2}$$

Andrade and Withers
1311.5157

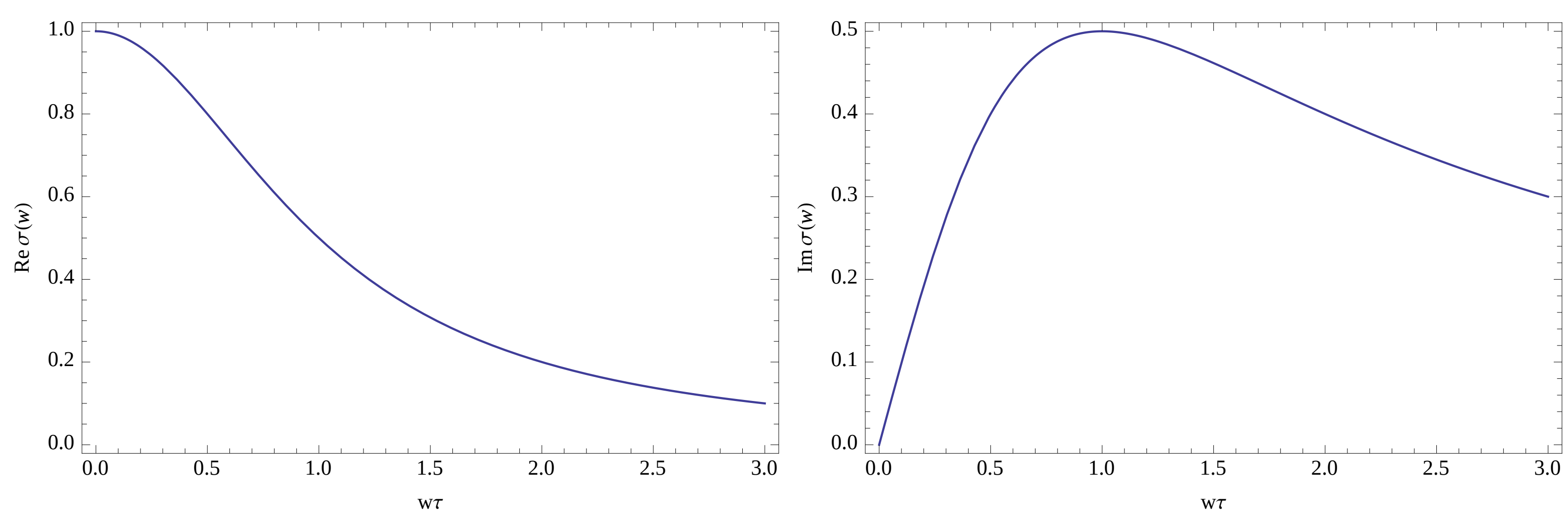


$\mu/T = 6$

$\beta/T = 3$

Drude like?

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$



Drude peak

- Drude model

$$\frac{dp}{dt} = -\frac{1}{\tau}p + qE$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

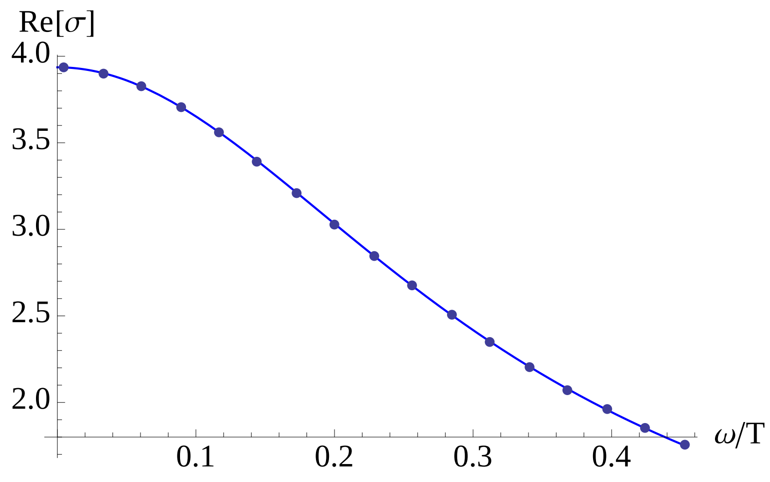
- Ward identity

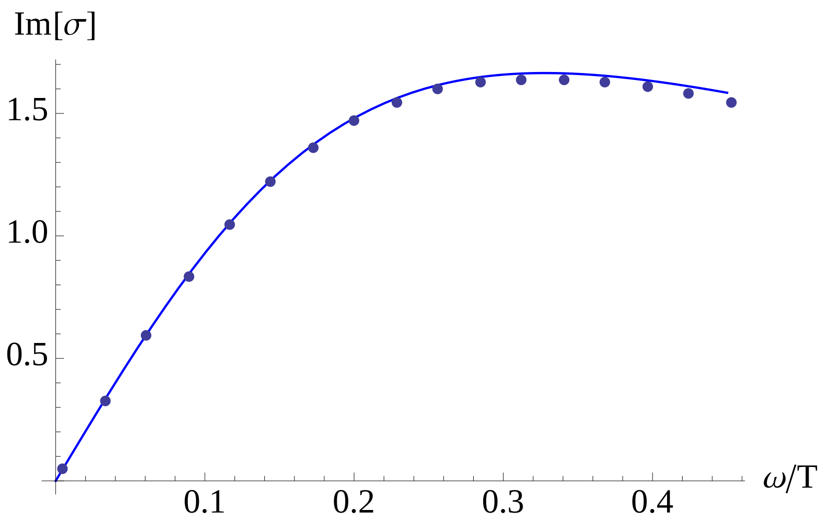
Andrade and Withers
1311.5157

$$\nabla^\nu \langle T_{\nu\mu} \rangle = \partial_\mu \phi \langle \mathcal{O} \rangle + F_{\mu\nu} \langle J^\nu \rangle$$

$$\partial_t \langle \delta p_x \rangle = \beta \langle \delta \mathcal{O} \rangle + \langle J^t \rangle \delta E_x$$

- Fitting $\sigma(\omega) = \frac{B\tau}{1 - i\omega\tau} + A$





Drude peak

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q$$

$\tau \to \infty \ (\beta \to 0)$

$$\sigma \to \sigma_Q + i\frac{K}{\omega}$$

$\omega \to 0$

$$\sigma \to K\tau + \sigma_Q = 1 + \frac{\mu^2}{\beta^2}$$

$$\sigma_Q = \left(\frac{3 - \frac{\mu^2}{4r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}}\right)^2$$

$$K = r_0 \frac{\frac{\mu^2}{r_0^2}}{3 + \frac{3\mu^2}{4r_0^2}}$$

$$r_0 = \frac{2\pi}{3}\left(T + \sqrt{T^2 + 3(\mu/4\pi)^2 + 6(\beta/4\pi)^2}\right)$$

$$\tau = \frac{1 + \frac{\mu^2}{\beta^2} - \sigma_Q}{K}$$

$$= \frac{1}{4\pi T} \cdot \frac{45\tilde{\beta}^4 + 36\tilde{\mu}^4 + 2(1+\Delta) + 6\tilde{\beta}^2(4 + 12\tilde{\mu}^2 + 3\Delta) + 3\tilde{\mu}^2(5 + 4\Delta)}{\tilde{\beta}^2(1+\Delta)(1 + 3\tilde{\beta}^2 + 6\tilde{\mu}^2 + \Delta)}$$

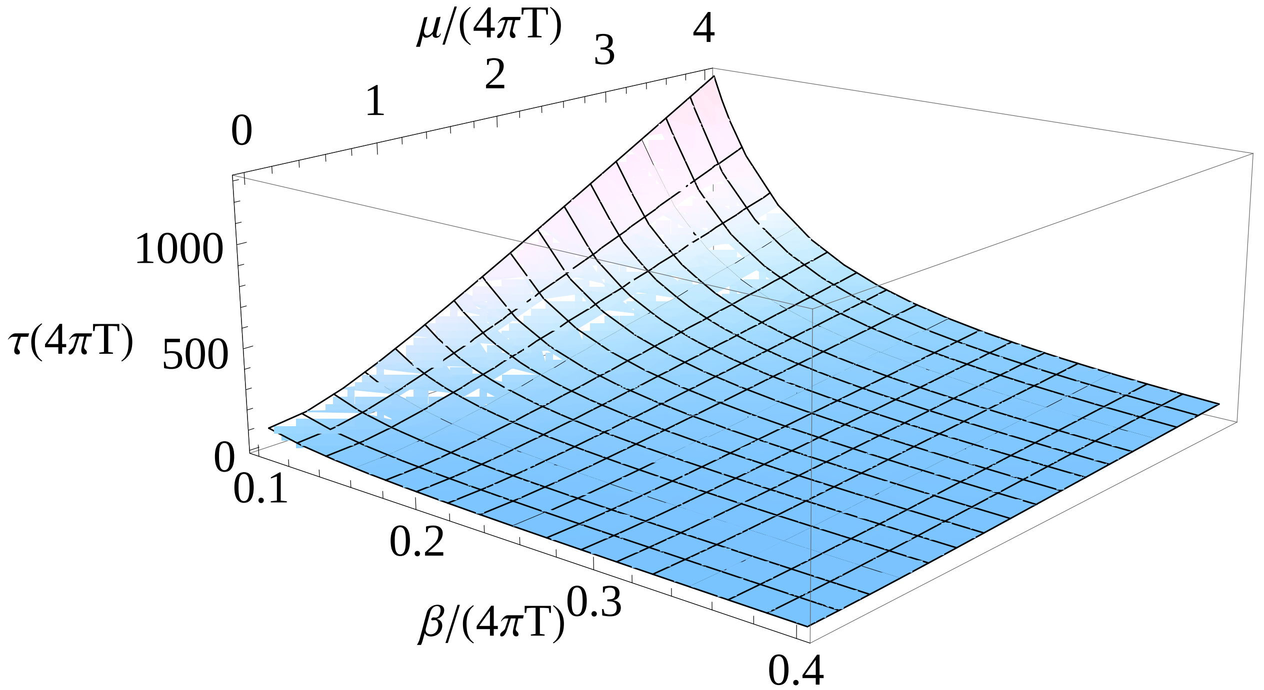
$$\Delta \equiv \sqrt{1 + 3\tilde{\mu}^2 + 6\tilde{\beta}^2}\,, \quad \tilde{\mu} \equiv \frac{\mu}{4\pi T}\,, \quad \tilde{\beta} \equiv \frac{\beta}{4\pi T}$$

Relaxation time

$$\tau = \frac{1 + \frac{\mu^2}{\beta^2} - \sigma_Q}{K} \quad = \frac{1}{4\pi T} \cdot \frac{45\tilde{\beta}^4 + 36\tilde{\mu}^4 + 2(1+\Delta) + 6\tilde{\beta}^2(4 + 12\tilde{\mu}^2 + 3\Delta) + 3\tilde{\mu}^2(5 + 4\Delta)}{\tilde{\beta}^2(1+\Delta)(1 + 3\tilde{\beta}^2 + 6\tilde{\mu}^2 + \Delta)}$$

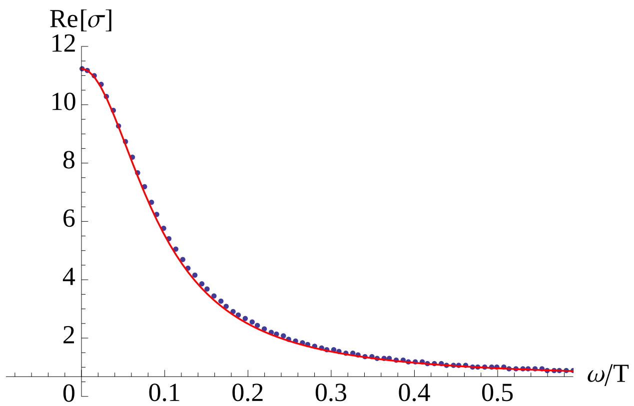
$$\Delta \equiv \sqrt{1 + 3\tilde{\mu}^2 + 6\tilde{\beta}^2}\,, \quad \tilde{\mu} \equiv \frac{\mu}{4\pi T}\,, \quad \tilde{\beta} \equiv \frac{\beta}{4\pi T}$$

$$\tau \approx \frac{\mu}{\beta^2}$$

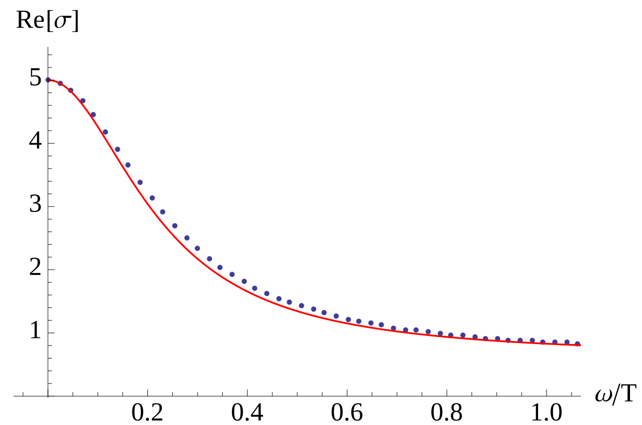


Drude peak

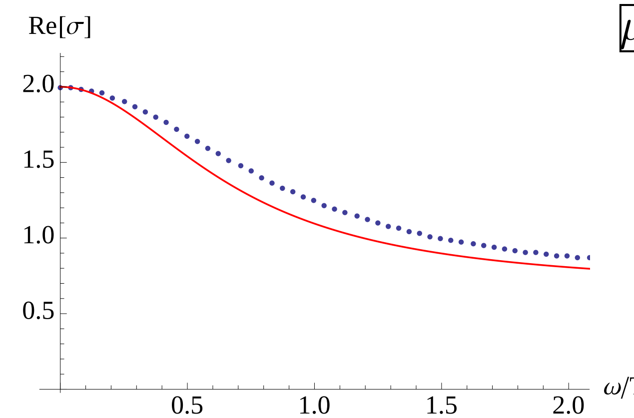
$\mu/T = 4$



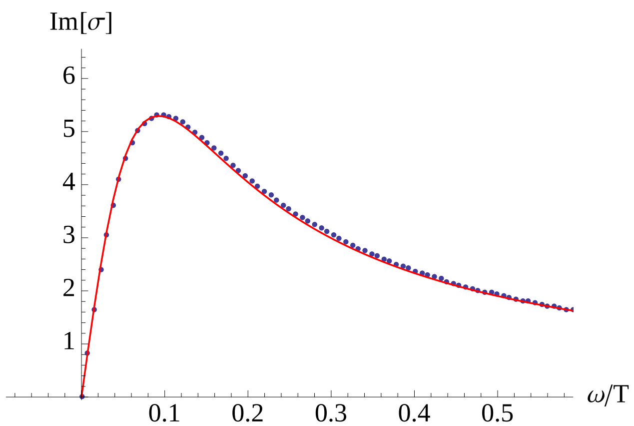
(a) Re σ, $\beta/T = 1$



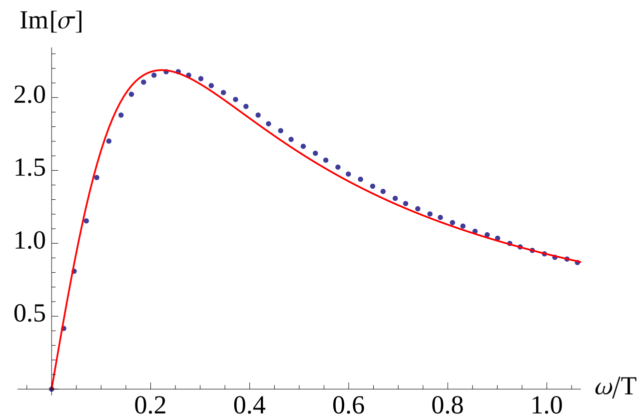
(b) Re σ, $\beta/T = 2$



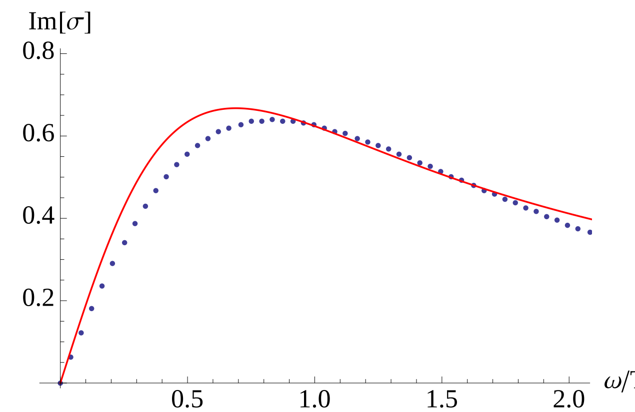
(c) Re σ, $\beta/T = 4$



(d) Im σ, $\beta/T = 1$



(e) Im σ, $\beta/T = 2$



(f) Im σ, $\beta/T = 4$

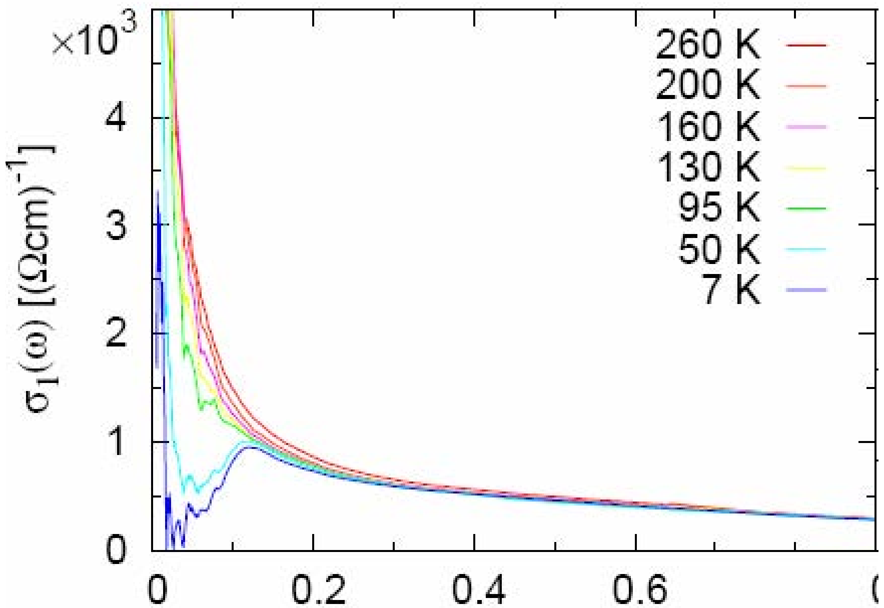
$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q$$

Low $T < \beta, \mu$

$\beta/\mu < 1$ 'Clean' region Drude Coherent metal $\tau \approx \frac{\mu}{\beta^2}$

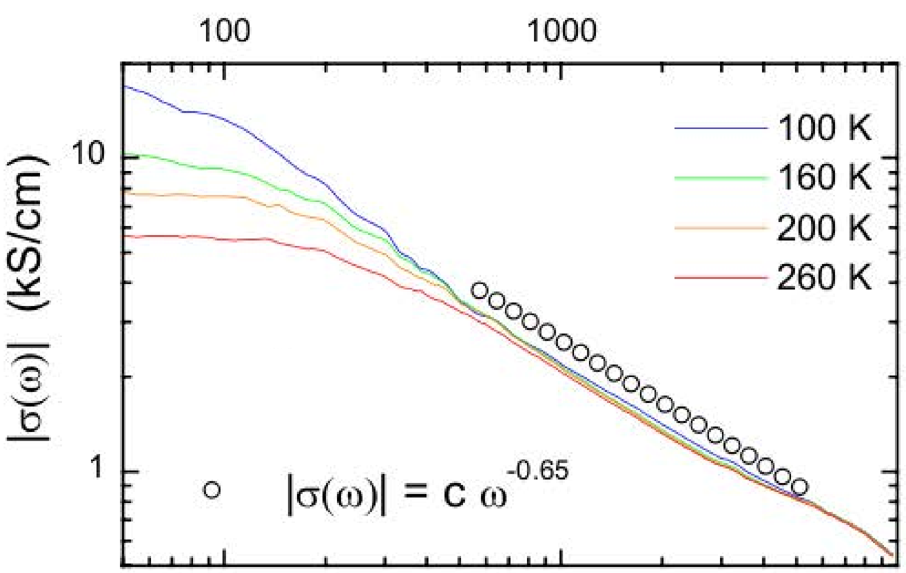
$\beta/\mu > 1$ 'Dirty' region ~~Drude~~ Incoherent metal

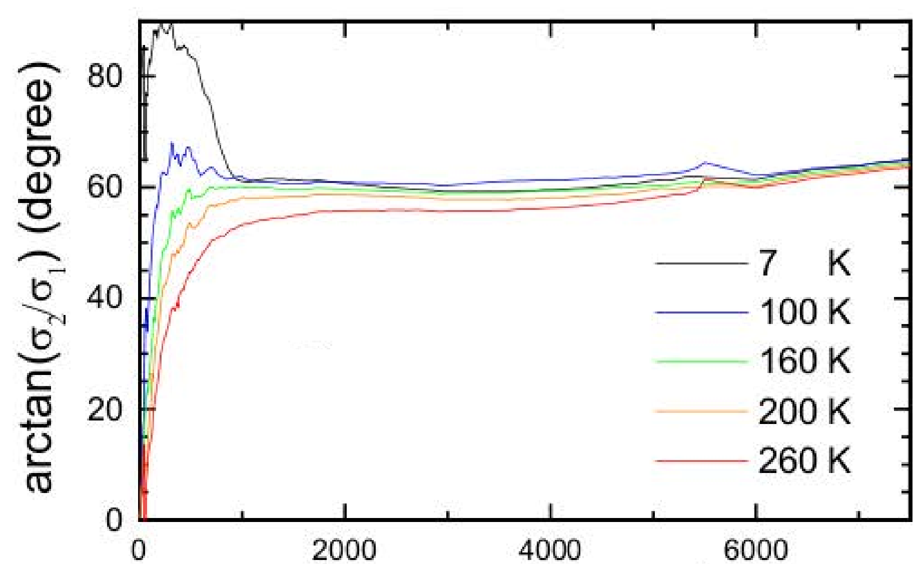
- AC conductivity $\sigma(\omega) \to (i/\omega)^\nu$



$\nu \approx 0.65$

Van der Marel et al., 2003

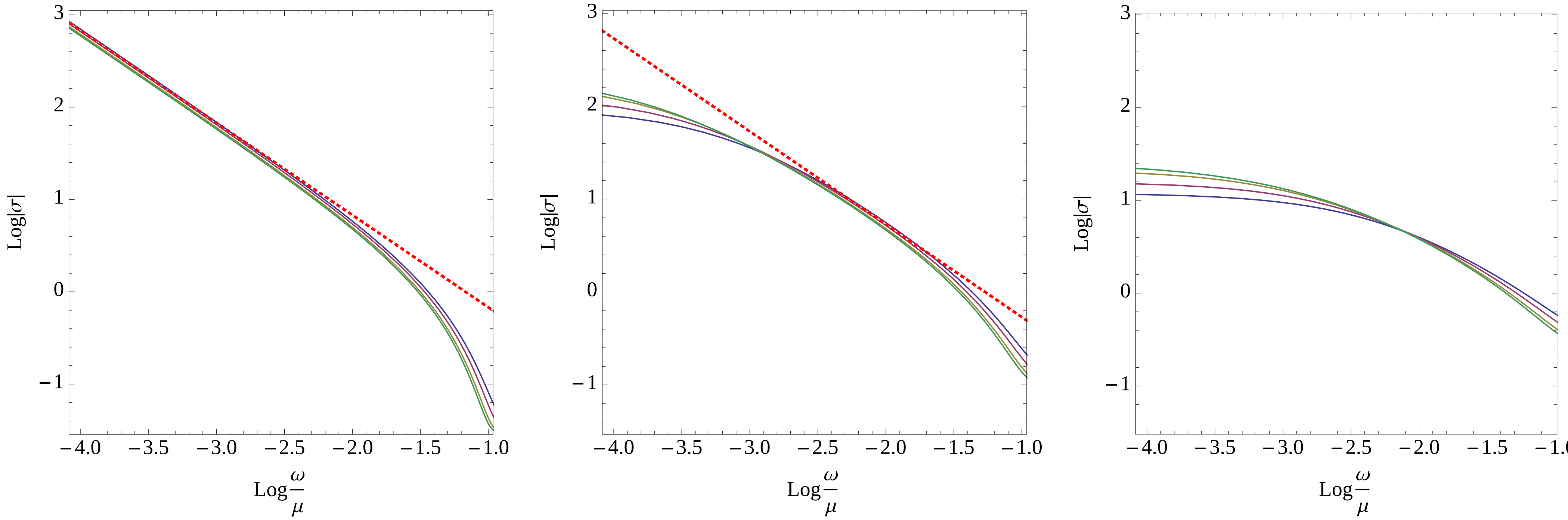




Intermediate frequency scaling

General feature

$$\sigma = \frac{B}{\omega^\gamma} e^{i\frac{\pi}{2}\gamma}$$

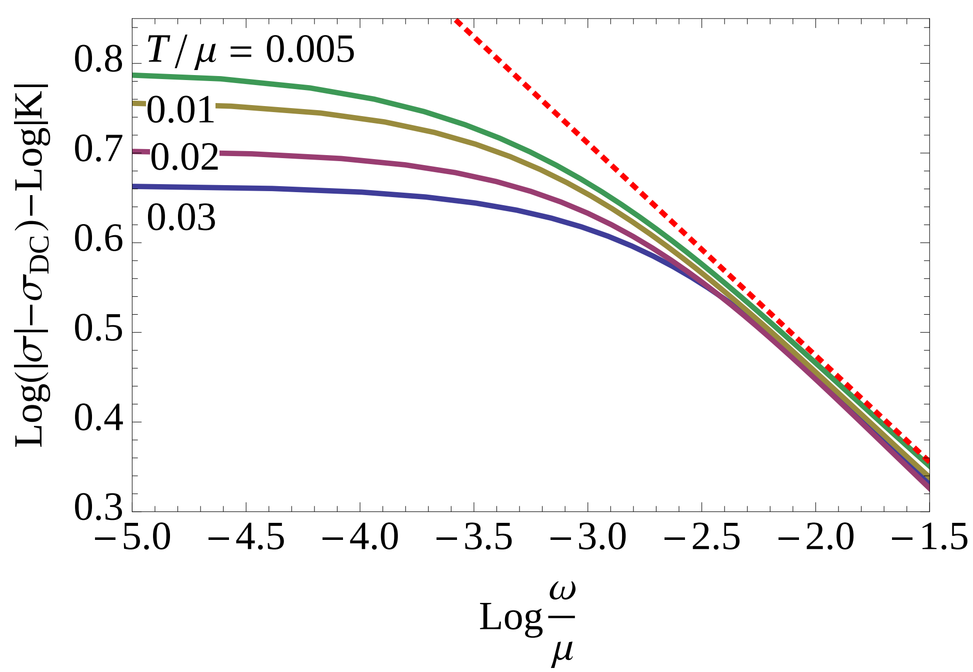


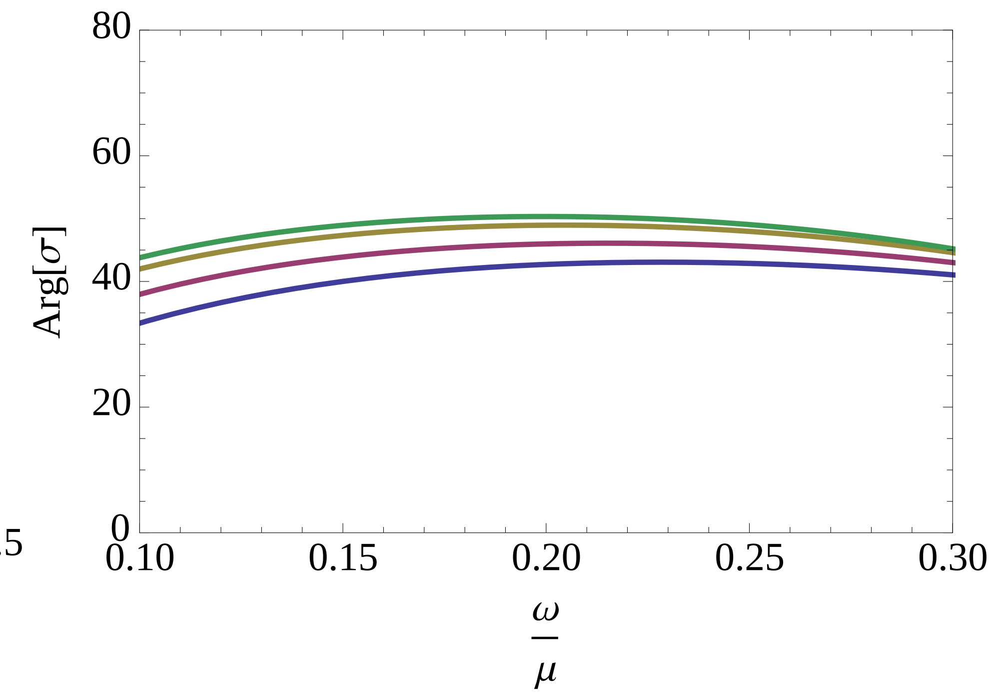
(a) $\beta/r_0 = 0.1$ (b) $\beta/r_0 = 1$ (c) $\beta/r_0 = 1.5$

The best we've found so far

$$\sigma = \left(\frac{B}{\omega^\gamma} + C\right) e^{i\frac{\pi}{2}\tilde{\gamma}}$$

$$\sigma = \left(\frac{K}{(\omega/\mu)^\gamma} + \sigma_{DC}\right)$$





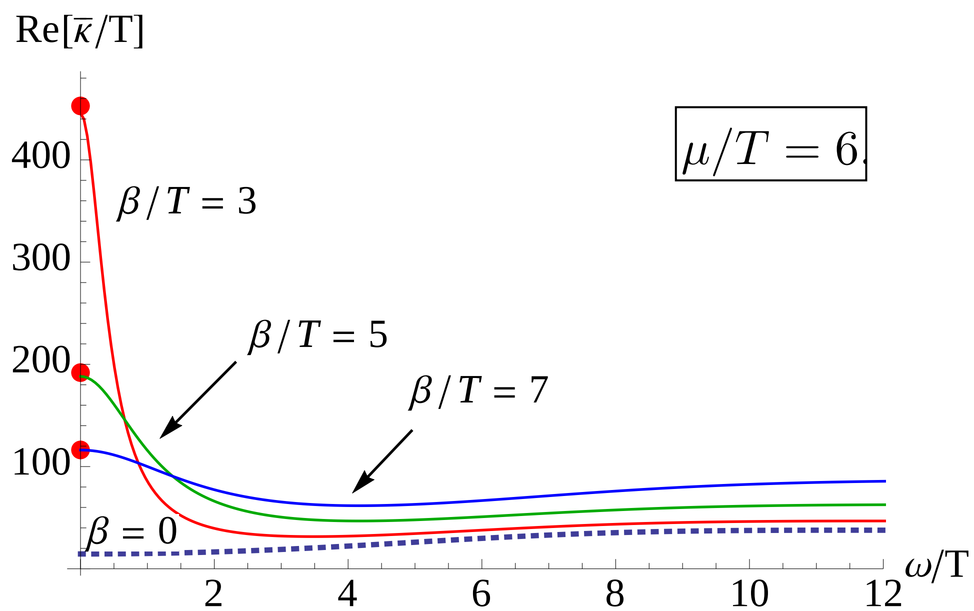
See also 1406.4870, Taylor and Woodhead

Thermal and thermoelectric conductivity

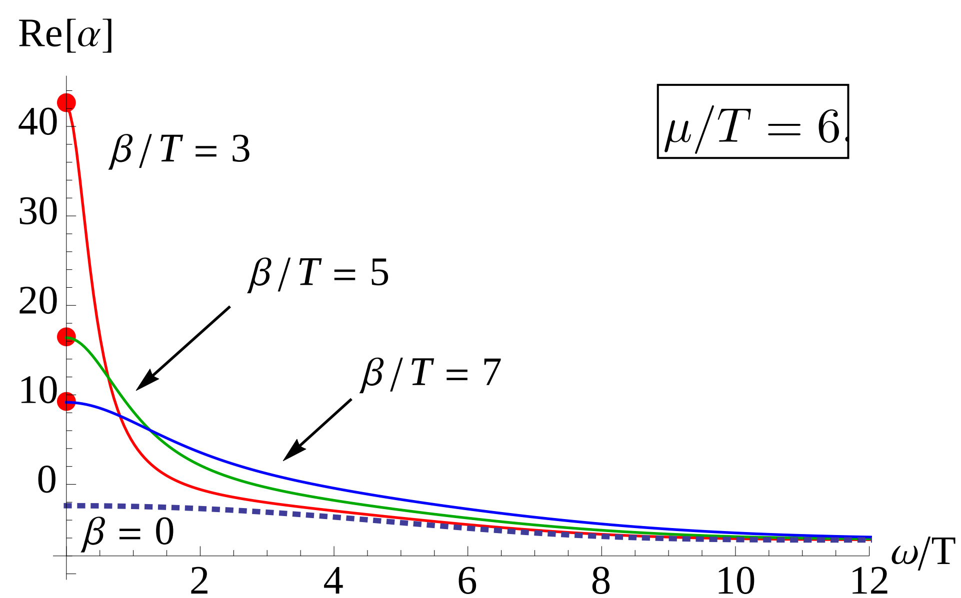
DC results:
Donos and Gauntlett
1406.4742

$$\frac{\bar{\kappa}}{T} = \frac{(4\pi)^2}{\beta^2} r_0^2$$

$$\alpha = \frac{4\pi\mu}{\beta^2} r_0$$



$$\frac{\bar{\kappa}}{T} \to \frac{\mu^2 + \beta^2}{T^2}$$



$$\alpha \to -\frac{\mu}{T}$$

Drude-like? Intermediate scaling?

Outline

1. Motivations and objects
2. Methodology: RN AdS black holes
   - Review
   - Numerical recipe
3. RN AdS black holes + momentum relaxation
4. Summary and future plans

Summary and plan

Summary

By using Andrade and Withers model $\psi_I = \beta_{Ii} x^i = \beta \delta_{Ii} x^i$

- AC electric conductivity

  - Coherent metal regime vs Incoherent metal regime

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau} + \sigma_Q \qquad \tau \approx \frac{\mu}{\beta^2}$$

  - No intermediate scaling yet

- Thermoelectric conductivity
- Systematic numerical recipe

Ongoing work

- Magnetic field: Dyonic black hole
- Holographic superconductor

Future plan

- Other models: Anisotropic case, Einstein-Maxwell-Dilaton, etc

Appendix

Hall conductivity