Simulating holographic correspondence in flexible graphene

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Kolumbari, Crete, 09/07/14

Outline

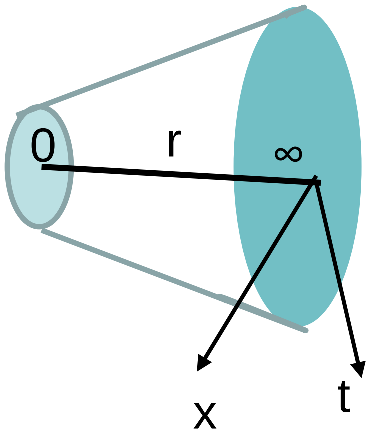
- Broadly defined holographic conjecture
- Holographic quantum systems and their gravity duals
- Analogue duality: testing the holographic conjecture in flexible graphene, metamaterials, and elsewhere

DVK, EPL, v.104, p.47002 (2013) and to appear.

Holographic correspondence

- Original context ('AdS/CFT'): d=4 N=4 SYM and d=5 type-IIB superstrings (G.'t Hooft, C.Thorn, L.Susskind; J.Maldacena; E.Witten, S.Gubser, I.Klebanov, A.Polyakov,…)

- Supporting evidence:
- thermodynamics of black holes and Hawking radiation,
- entanglement entropy,
- hydrodynamics of quark-gluon plasma and cold gases,…



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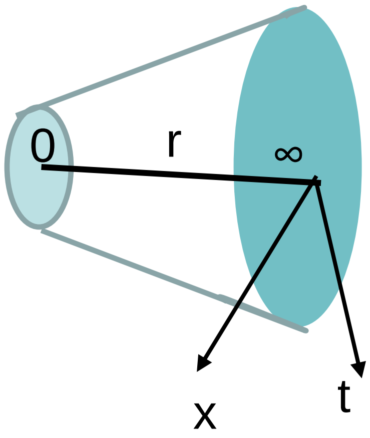

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- Further (‘non-AdS/non-CFT’) generalizations:
- non-SUSY,
- only a few-component (N~1),
- moderately interacting (T~U),
- Lorentz , scale, translationally and/or rotationally non-invariant

Open questions

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DVK, arXiv:1404.7000

- General:

-What is the status of the (broadly defined) holographic conjecture?

-If it is indeed valid, then WHY?

- Does EVERY system have a dual? Or only a precious few?

-How much symmetry is enough? Is large N necessary?

-AdS/CFT dictionary: is it written in stone?

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- Specific to CMP:

-Do the boundary theories of the previously studied gravity duals have physical realizations among the known materials?

-What are the gravity duals of the already documented 'strange metals'?

-**What can be tested in the lab (analogue holography)?**

Holography primer

- Fermion action $$S = \int dr dt d^d x \sqrt{|det \hat{g}|} \bar{\psi} \gamma_a e^a_\mu (i\partial_\mu + \frac{i}{8}\omega^{bc}_\mu [\gamma_b, \gamma_c] + A_\mu - m)\psi$$

- Background metric $$ds^2 = -f(z)dt^2 + g(z)dz^2 + h(z)d\vec{x}^2$$

- Radial Shroedinger equation $$\frac{\partial^2 \psi}{\partial r^2} = V(r)\psi \qquad V(r) = g(r)(m^2 + \frac{k^2}{h(r)} + \frac{\omega^2}{f(r)}) + \ldots,$$

z=1/r

- Fermion propagator $$G(\omega, k) = \frac{\psi_+(r, \omega, k)}{\psi_-(r, \omega, k)}|_{r \to R}$$

+/- solutions normalizable/non-normalizable at the boundary

Cf.: Sturm-Liouville Green function $$G(r, r', \omega, k) = \frac{\theta(r - r')\psi_+(r)\psi_-(r') + (r \leftrightarrow r')}{\psi_- \frac{d\psi_+}{dr} - \psi_+ \frac{d\psi_-}{dr}}$$

- **Computing Feynman diagrams → solving linear ODE**

Holographic propagator

- WKB solutions $$\psi_{\pm}(r,\omega,k) \sim \frac{1}{V^{1/4}(r)} e^{\mp \int_r^R dr' \sqrt{V}(r')}$$

- Asymptotic behavior of propagator $$G(\tau,x) \sim \exp(-S_0(\tau,x))$$

- Action $$S(\tau,x) = m \int dr \sqrt{g(r) + f(r)(d\tau/dr)^2 + h(r)(dx/dr)^2}$$

- Extremal value $$S_0(\tau,x) = 2m^2 \int_{r_t}^{R} dr \frac{\sqrt{g}(r)}{M(r)}$$

$$M(r) = \sqrt{m^2 - k^2/h(r) - \omega^2/f(r)}$$

- Equations of motion (geodesics) $$x = k\int_{r_t}^{R} \frac{dr\sqrt{g}(r)}{h(r)M(r)}, \quad \tau = \omega \int_{r_t}^{R} \frac{dr\sqrt{g}(r)}{f(r)M(r)}$$

- Different spins (0,1/2,1,…) – subdominant effects

$$mR \gg 1$$

AdS/CFT

- Einstein-Maxwell action $$S_g = \int \frac{1}{2\kappa^2}(\mathcal{R} + \frac{d(d+1)}{L^2}) - \frac{1}{2e^2}F_{\mu\nu}^2$$

- Static rotationally-invariant metrics $$ds^2 = -f(z)dt^2 + g(z)dz^2 + h(z)d\vec{x}^2$$

- Reissner-Nordstrom black hole $$f(z)(z/L)^2 = (L/z)^2/g(z) =$$ $$1-(1+\mu^2)(z/z_h)^{d+1} + \mu^2(z/z_h)^{2d}, h(z) = (L/z)^2$$

- Near-boundary and -horizon geometry $$AdS_{d+2} \longrightarrow AdS_2 \times R^d.$$

- Temperature $$T = (d+1-(d-1)\mu^2)/4\pi z_h$$

- ‘Searching where the lights are’: a number of historic BH metrics

- Not a true IR state - possibly, a crossover regime

Semi-local (non)Fermi liquid

• Boundary propagator

$$G(\omega, q) = \frac{1}{A(q) + B(q)\omega^{\nu_q}}$$

M.Cubrovic et al,'09;S.S.Lee,'09;H.Liu et al,'11;T.Faulkner et al,'11

$$G(\tau, x) \sim \exp(-S_0(\tau, x))$$

• Space-time dependence

$$S_{s-l}(\tau, x) = \sqrt{(1-\nu_0)^2(\ln \tau/a)^2 + m^2x^2}$$

• Superficially reminiscent of heavy fermions (Kondo lattices,…)

• Problematic: multiple Fermi surfaces, dispersionless peaks, finite entropy at T=0, nothing specific about v=1?, etc.

• And what about other types of holographic NFL?
Possibility of their systematic classification?

Graphene: scotch tape-induced relativity

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B

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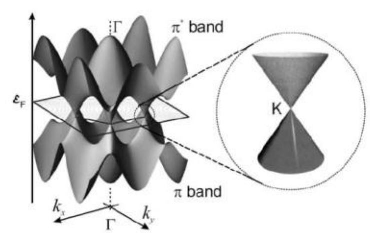

P. R. Wallace, ’47
G. W. Semenoff, ’84;
E. Fradkin, ’86;

pseudospin   2 valleys

$$-i\hbar v \begin{pmatrix} 0 & \partial_x - i\partial_y \\ \partial_x + i\partial_y & 0 \end{pmatrix} \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix} = E \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix}$$

$E = v_F\, p$ $v_F = 10^6$ m/s (= c/300)

- Spinor wavefunction (pseudospin ½) → Dirac equation

Graphene: scotch tape-induced relativity

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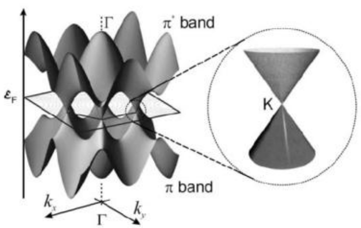

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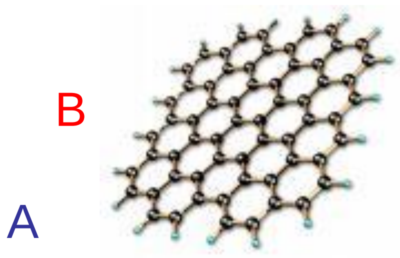

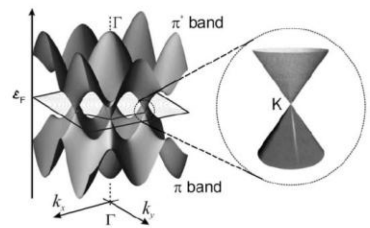

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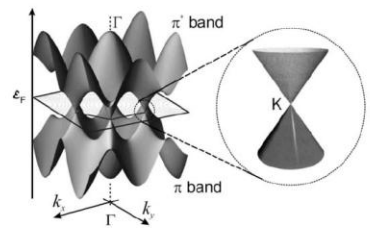

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•Analogs: molecular graphene, silicine, germanine, stanene, optical lattices…

Pseudo-relativistic effects in ‘Dirac materials’

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- Desktop realization of the previously predicted fundamental phenomena:
- Klein tunneling,
- ‘zitterbewegung’,
- Veselago lense,
- atomic collapse,
- chiral symmetry breaking (excitonic insulator) ,
- magnetic catalysis (Quantum Hall ferromagnetism),…

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- New challenges:
- (non-) abelian gauge fields and solitons,
- Mimicking gravity and cosmology,
- **Analogue holographic correspondence**,

….

Elastic strain in graphene

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- Hopping Hamiltonian $H = -\sum_{i,\mathbf{n}} t(\mathbf{r}_i, \mathbf{r}_i + \mathbf{n})\, a^\dagger_{\mathbf{r}_i} b_{\mathbf{r}_i+\mathbf{n}} + \mathrm{H.\,c.}$

- Strain tensor of flexible graphene $\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right) + \frac{1}{2}\left(\frac{\partial u_k}{\partial X_i}\frac{\partial u_k}{\partial X_j}\right).$

- Vector potential $A_x(\mathbf{R}) - iA_y(\mathbf{R}) = \frac{1}{qv_F}\sum_{n} \delta t(\mathbf{r}, \mathbf{r}+\mathbf{n}) e^{i\boldsymbol{K}\cdot\boldsymbol{n}} \simeq \frac{\hbar\beta}{2qa}(\epsilon_{xx} - \epsilon_{yy} + 2i\,\epsilon_{xy})$

$$\beta = -\partial \log t(r)/\partial \log r\Big|_{r=a}$$

- Higher order terms

$$A_x^{(c)} = -\frac{3a^2 V^0_{pp\pi}}{8qv_F}\left[\left(\frac{\partial^2 h}{\partial y^2}\right)^2 - \left(\frac{\partial^2 h}{\partial x^2}\right)^2\right],$$

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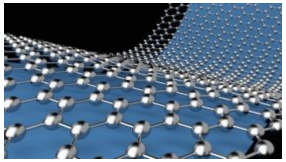
- Stress engineering



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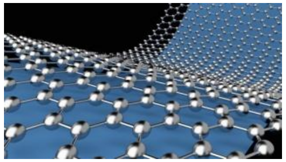

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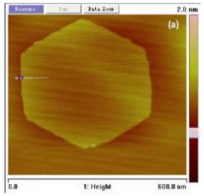
- Induced fermion mass



S.Tang et al, '13

Small deformations: pseudo-magnetic field

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- Effective vector potential

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\end{aligned}
$$

- Elastic energy

$$
\begin{aligned}
\mathcal{H}_{elastic} &= \frac{\kappa}{2}\int d^2\vec{r}\,\left[\nabla^2 h(\vec{r})\right]^2 + \\
&+ \int d^2\vec{r}\left\{\frac{\lambda}{2}\left[\sum_i u_{ii}(\vec{r})\right]^2 + \mu\sum_{ij}\left[u_{ij}(\vec{r})\right]^2\right\}
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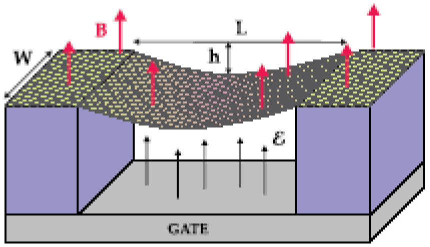
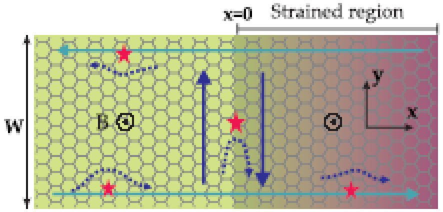
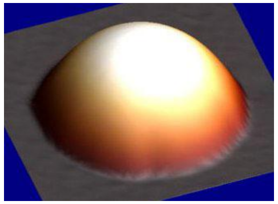




Nanobubbles: B~300 T



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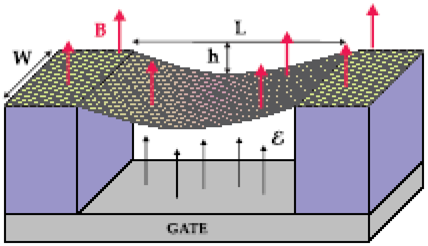
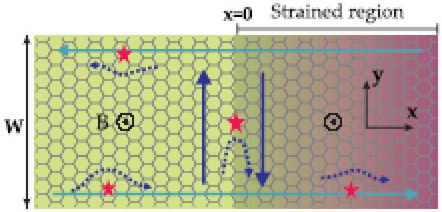
- Dirac fermion-phonon coupling

$$\begin{aligned}\mathcal{H}_{elec} &= v_F\int d^2\vec{r}\,\bar{\Psi}_1(\vec{r})\left\{\sigma_x\left[-i\partial_x - A_x(\vec{r})\right] + \right.\\ &+ \left.\sigma_y\left[-i\partial_y - A_y(\vec{r})\right]\right\}\Psi_1(\vec{r}) - \\ &- v_F\int d^2\vec{r}\,\bar{\Psi}_2(\vec{r})\left\{\sigma_x\left[-i\partial_x + A_x(\vec{r})\right] + \right.\\ &+ \left.\sigma_y\left[-i\partial_y + A_y(\vec{r})\right]\right\}\Psi_2(\vec{r})\end{aligned}$$





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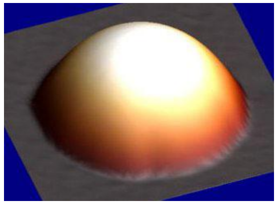

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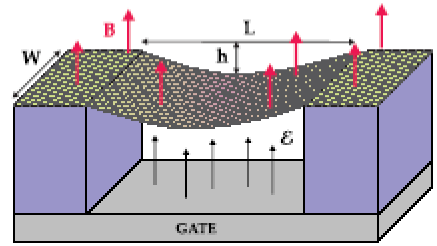
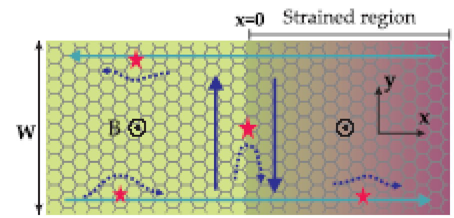
- Extra terms: position-dependent Fermi velocity,…





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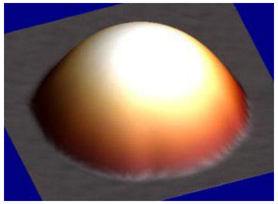

M.A.H.Vozmediano et al, ‘10;
A.L.Kitt et al,’12;
F. de Juan et al,’12

Large(r) deformations: emergent gravitational field

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- Emergent Riemann-Cartan gravity: Weitzenbock geometry

$$
\begin{aligned}
\mathbf{H}_- &= -\sigma^3\, \mathrm{f}_a^k \sigma^a [\partial_k + i\mathbf{A}_k], \quad a = 1,2; k = 1,2 \\
\mathbf{H}_+ &= -\sigma^2 \Big(\sigma^3\, \mathrm{f}_a^k \sigma^a [\partial_k - i\mathbf{A}_k]\Big)\sigma^2 .
\end{aligned}
$$

$$
\mathcal{H} = i\sigma^3 \mathbf{H}_- = -ie\, \mathbf{e}_a^k \sigma^a \circ [\partial_k + i\mathbf{A}_k]
$$

G.Volovik and M.Zubkov, ‘13

Large(r) deformations: emergent gravitational field

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$$\mathbf{H}_+ = -\sigma^2 \Big(\sigma^3\, \mathrm{f}_a^k \sigma^a [\partial_k - i\mathbf{A}_k]\Big)\sigma^2.$$

$$\mathcal{H} = i\sigma^3 \mathbf{H}_- = -ie\, \mathbf{e}_a^k \sigma^a \circ [\partial_k + i\mathbf{A}_k]$$

G.Volovik and M.Zubkov, '13

- Vielbein and gauge field

$$\mathbf{e}_a^i = \mathrm{f}_a^i / e, \quad e = [\det \mathrm{f}]^{1/2} = v_F(1 - \frac{1}{3}(\Delta_2 + \Delta_3 + \Delta_1))$$
$$\mathrm{f}_a^i = v_F\left(\delta_a^i - \begin{bmatrix} \Delta_1 & \frac{(\Delta_2 - \Delta_3)}{\sqrt{3}} \\ \frac{(\Delta_2 - \Delta_3)}{\sqrt{3}} & \frac{2}{3}(-\frac{1}{2}\Delta_1 + \Delta_2 + \Delta_3) \end{bmatrix}\right)$$
$$\mathbf{A}_1 = \frac{1}{2v_F a}(\mathbf{e}_2^1 + \mathbf{e}_1^2), \quad \mathbf{A}_2 = \frac{1}{2v_F a}(\mathbf{e}_1^1 - \mathbf{e}_2^2)$$

- Metric tensor $\quad g_{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab} \qquad g_{\mu\nu}^{\text{graphene}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & & g_{ij} \end{pmatrix}$

Large(r) deformations: emergent gravitational field

- Emergent Riemann-Cartan gravity: Weitzenbock geometry

$$\mathbf{H}_- = -\sigma^3\, \mathrm{f}_a^k \sigma^a [\partial_k + i\mathbf{A}_k], \quad a = 1,2; k = 1,2$$
$$\mathbf{H}_+ = -\sigma^2 \left(\sigma^3\, \mathrm{f}_a^k \sigma^a [\partial_k - i\mathbf{A}_k] \right) \sigma^2 .$$

$$\mathcal{H} = i\sigma^3 \mathbf{H}_- = -ie\, \mathbf{e}_a^k \sigma^a \circ [\partial_k + i\mathbf{A}_k]$$

G.Volovik and M.Zubkov, '13

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- GR-inspired LDOS $\quad \rho^{(B)}_{thermal}(E,u,r) = \frac{4}{\pi}\frac{1}{(\hbar v_F)^2}\frac{r^2}{\ell^2} e^{-2u/r} \frac{E}{\exp\left[E/(k_B T(u,r))\right] - 1}$

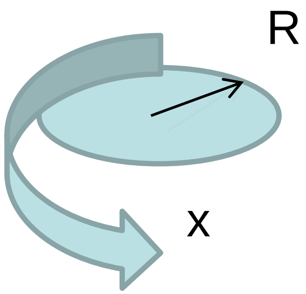
A.Iorio and G.Lambiase, '11;
G.Gibbons and M.Cvetic,'12

Bulk-edge correspondence

Bulk-edge correspondence

- Flat metric $dl^2_{flat} = dr^2 + r^2 d\phi^2$ $ds^2 = d\tau^2 + dl^2$

$$S_{flat}(\tau, x) = m\sqrt{\tau^2 + 4R^2 \sin^2(x/2R)}$$



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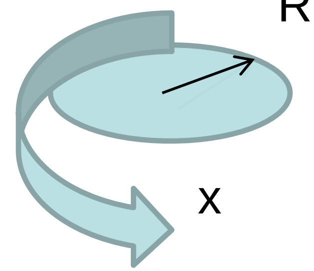
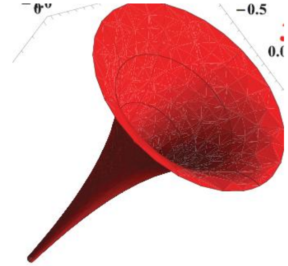


•Surface of rotation $dl^2_{sor} = dr^2[1 + (\frac{\partial h(r)}{\partial r})^2] + r^2 d\phi^2$

$$h(r) \sim (R/r)^\eta$$

$$S_{sor}(\tau, x) = m\sqrt{\tau^2 + (Rx^\eta)^{2/(\eta+1)}} \qquad z_{hol} = \eta/(\eta+1)$$



A.Iorio and G.Lambiase,'13

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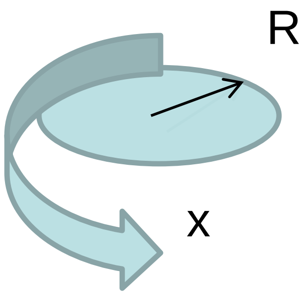

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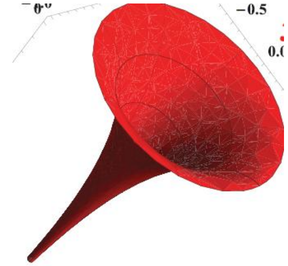

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• Boundary propagator: 1d bosonization

$$G^{\pm}_{bos}(\tau, x) \sim \exp[-\int \frac{dk}{2\pi} \frac{2+U_k}{\epsilon_k}(1 - e^{\pm ikx - \epsilon_k t})]$$

$$\epsilon_k = k\sqrt{1+U_k}$$

A.Iorio and G.Lambiase,'13

•Matching x-asymptotics $\eta = (1-\sigma)/(1+\sigma)$,

$$U(x) \sim 1/x^\sigma$$

while $z_{hol} = (1-\sigma)/2$, $z_{bos} = (1+\sigma)/2$

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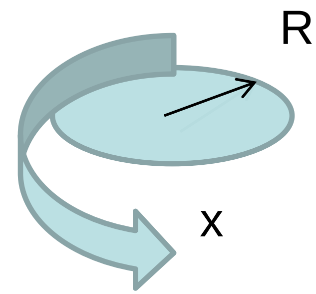

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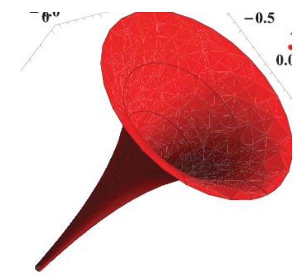

A.Iorio and G.Lambiase,'13

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• This behavior can be tested with time-of-flight, tunneling, capacitive, and noise power measurements

Bulk-edge correspondence: more examples

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• Generalized Beltrami trumpet: $dl^2_{log} = dr^2 + R^2 \exp(-2(r/R)^\lambda)d\phi^2$

$$dl^2 = d\rho^2/\rho^2 + \rho^2 d\phi^2$$

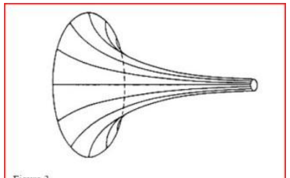
Figure 3

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$$S_{log}(\tau, x) = m\sqrt{\tau^2 + R^2(\ln x/a)^{2/\lambda}}$$

• Cf., semi-local regime: $S_{s-l}(\tau, x) = \sqrt{(1-\nu_0)^2(\ln \tau/a)^2 + m^2x^2}$

Bulk-edge correspondence: more examples

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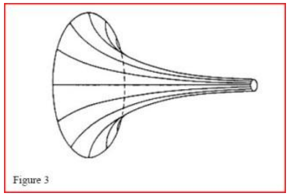

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• Cf., semi-local regime: $S_{s-l}(\tau, x) = \sqrt{(1-\nu_0)^2(\ln \tau/a)^2 + m^2 x^2}$

• $\lambda = 1$: Algebraic $G(0, x) \sim 1/x^{mR}$

• $\lambda = 2/3$: Coulomb interaction in 1d $G(0, x) \sim \exp(-const \ln^{3/2} x)$

1d charge density wave

• Generic λ : (Ir)relevant two-particle operator $U(x) \sim (\ln x)^{(2/\lambda)-3}/x$

•Bulk geometry → measured boundary correlator → dual boundary theory

Towards Lorentz-invariant boundary theory

Towards Lorentz-invariant boundary theory

$$f(r) \neq const$$

- AdS

$$ds^2_{AdS} = (d\tau^2 + dx^2)r^2 + \frac{dr^2}{r^2}$$

$$S_{AdS}(\tau, x) = 2mR\ln\left(\frac{\sqrt{\tau^2 + x^2}}{a} + \sqrt{\frac{\tau^2 + x^2}{a^2} + 1}\right)$$

$$G_{AdS}(\tau, x) \sim \frac{1}{(x - i\tau)^{2\Delta_+}(x + i\tau)^{2\Delta_-}}$$

Towards Lorentz-invariant boundary theory

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- Anomalous dimensions
 Luttinger regime

$$\Delta_\pm = mR/2 + 1/2 \pm 1/4$$

$$1/2 \leq \Delta_+ + \Delta_- = \frac{1}{4}(K + 1/K) \leq 5/8$$

K<1/2 unattainable for any local $U(x) \sim \delta(x)$
In carbon nanotubes K=0.2

- BTZ $$ds^2_{BTZ} = \frac{1}{\sinh^2 \rho/R}((\frac{a}{R})^2 d\tau^2 + d\rho^2 + a^2\cosh^2(\frac{\rho}{R})d\phi^2)$$

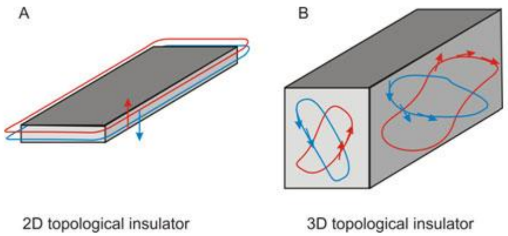
•**Underlying physics**: yet another manifestation of the equivalence principle?

Other desktop implementations of analogue holography

Other desktop implementations of analogue holography

- Topological insulators

  Problematic:
- Curved 3d space
- FL on a 2d boundary is more robust than in 1d



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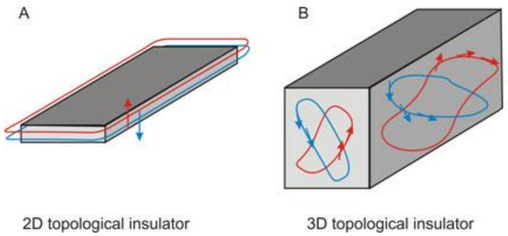

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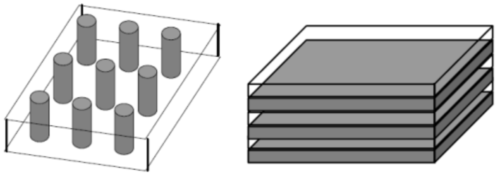
- Hyperbolic metamaterials

 - Artificial metrics,
- Rindler horizons and black/white/worm holes,
- Big Bangs and Crunches, wormholes,
- Double time, end of time, multiverses,…



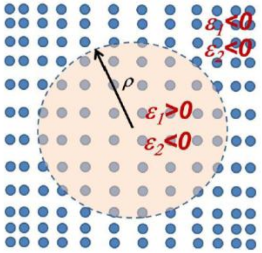
String holography meets its optical namesake

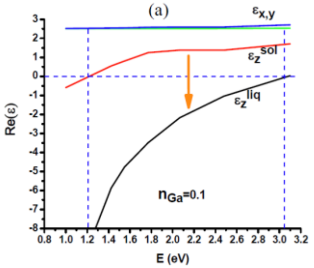
String holography meets its optical namesake

- Dispersion of extraordinary waves

$$\frac{\omega^2}{c^2}\vec{D}_\omega = \vec{\nabla}\times\vec{\nabla}\times\vec{E}_\omega \text{ and } \vec{D}_\omega = \overleftrightarrow{\varepsilon}_\omega \vec{E}_\omega$$

$$\frac{\omega^2}{c^2} = \frac{k_z^2}{\varepsilon_1} + \frac{k_x^2 + k_y^2}{\varepsilon_2} \qquad g_{00} = -\varepsilon_1 \text{ and } g_{11} = g_{22} = -\varepsilon_2$$





I.Smolyaninov, E.Narimanov,’09…

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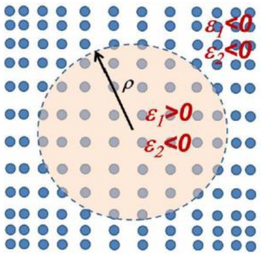
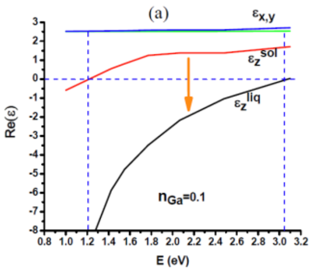
$g_{00}=-\varepsilon_1$ and $g_{11}=g_{22}=-\varepsilon_2$





- Inhomogeneous dielectric function

$$\varepsilon_2 = \varepsilon_z = n\varepsilon_m + (1-n)\varepsilon_d$$

$$\varepsilon_1 = \varepsilon_{x,y} = \varepsilon_d \frac{\varepsilon_m(1+n)+\varepsilon_d(1-n)}{\varepsilon_d(1+n)+\varepsilon_m(1-n)}$$

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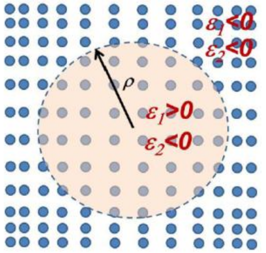

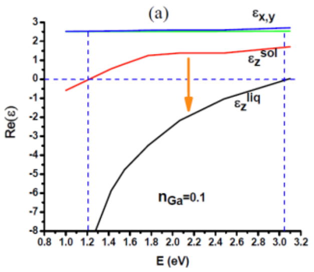

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I.Smolyaninov, E.Narimanov,'09…

- Noise power spectrum (e.g., T) and other moments of the boundary field distribution function can be related to the bulk 'metric' (DVK, to appear)

Conclusions

- The hypothesis of a broadly defined holographic correspondence still remains to be verified under such generic conditions as “non-CFT/non-AdS”.

- Deformed graphene and other 2d Dirac materials offer a testing ground for simulating certain holography-like effects whose physical nature can be elucidated more readily (and without invoking any new physical principles).

- The specific predictions of analogue holography can be probed with such established techniques as time-of-flight, tunneling, capacitive, and noise power measurements.

- Cf.: acoustic event horizons, Standard Model in a droplet of He 3,…