Holographic Signatures of Cosmological Singularities

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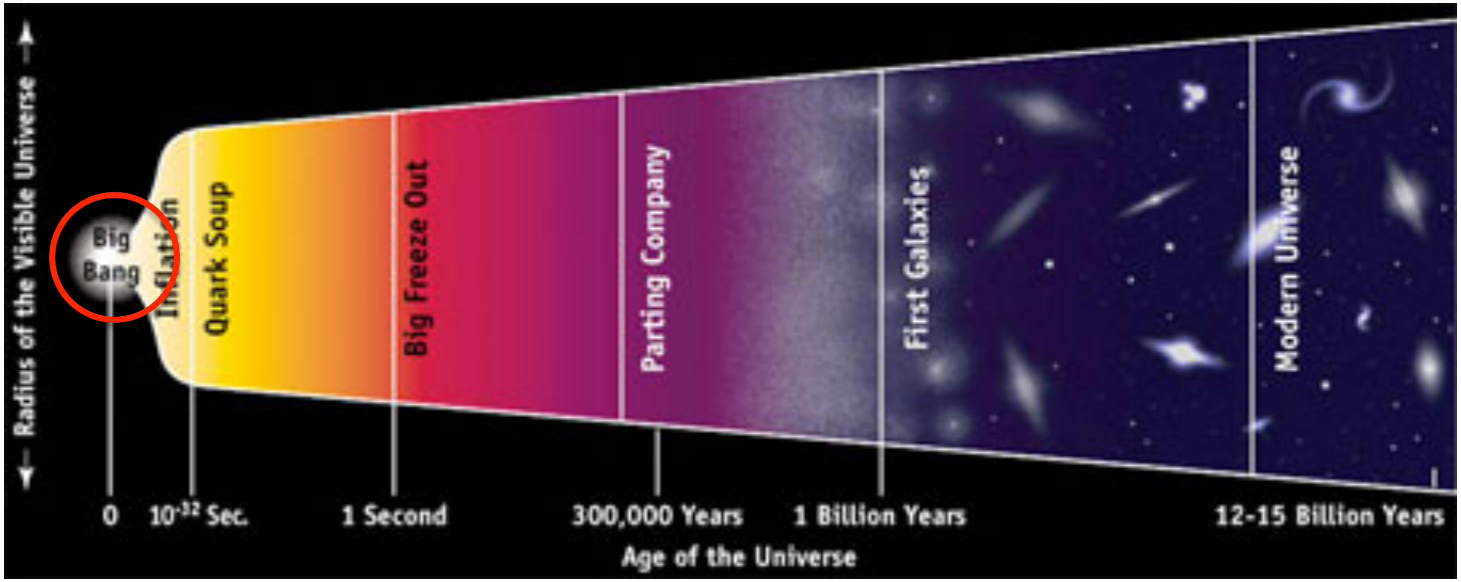
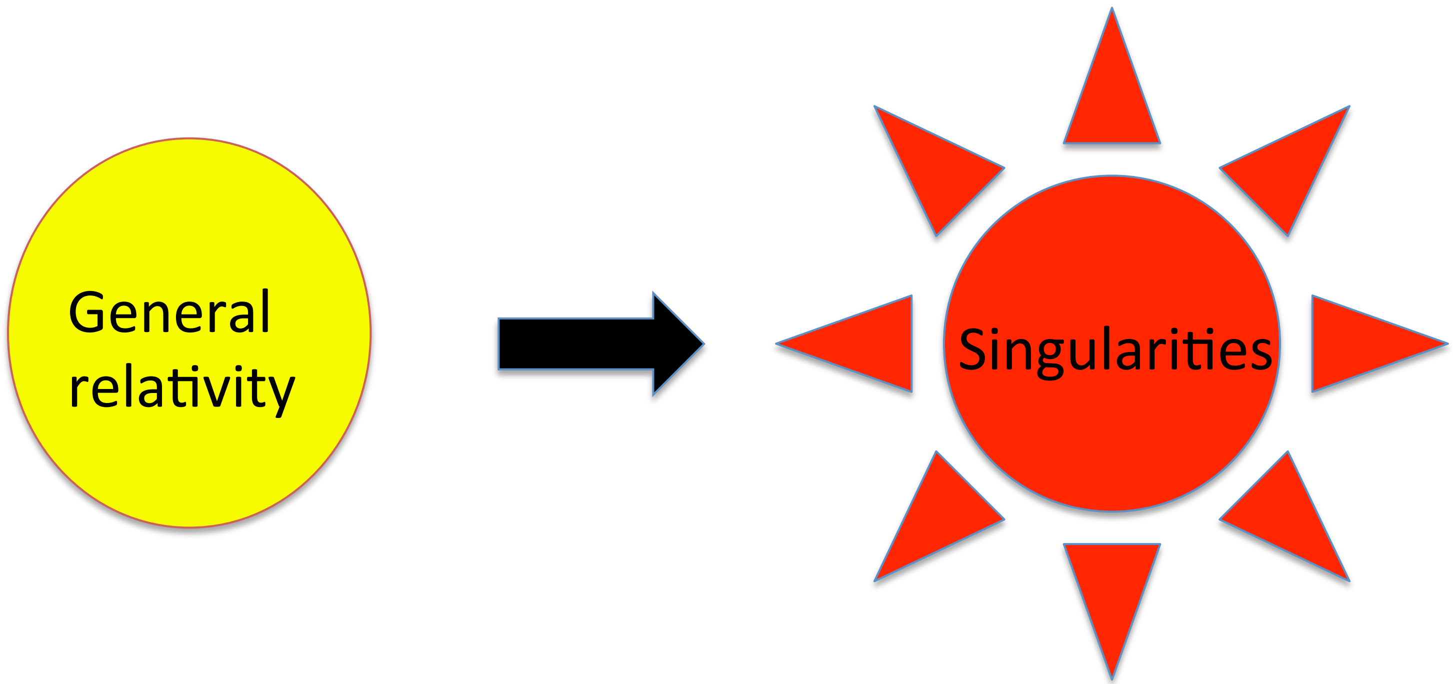
(N. Engelhardt, T. Hertog, G.H., 1404.2309
and to appear)

A brief history of our universe:



In general relativity, the big bang is a singularity.



Physics near these singularities is described by quantum gravity.

To study cosmological singularities using holography we need to construct asymptotically anti-de Sitter (AdS) solutions which evolve into (or from) a singularity that extends all the way out to infinity.

Payoff: It maps the problem into a problem in ordinary QFT.

(Earlier work by Hertog, G.H.; Craps, Rajaraman, Sethi; Das, Michelson, Narayan, Trivedi; Awad, Das, Nampuri, Narayan, Trivedi,...)

First holographic model of a cosmological singularity

(Hertog and GH, hep-th/0406134, hep-th/0503071)

Consider gravity in AdS_4 coupled to a scalar with potential $V(\phi)$ having $m^2 = -2$. One example coming from a truncation of N = 8 SUGRA is:

$$V(\phi) = -2 - \cosh\sqrt{2}\phi$$

Solutions must approach

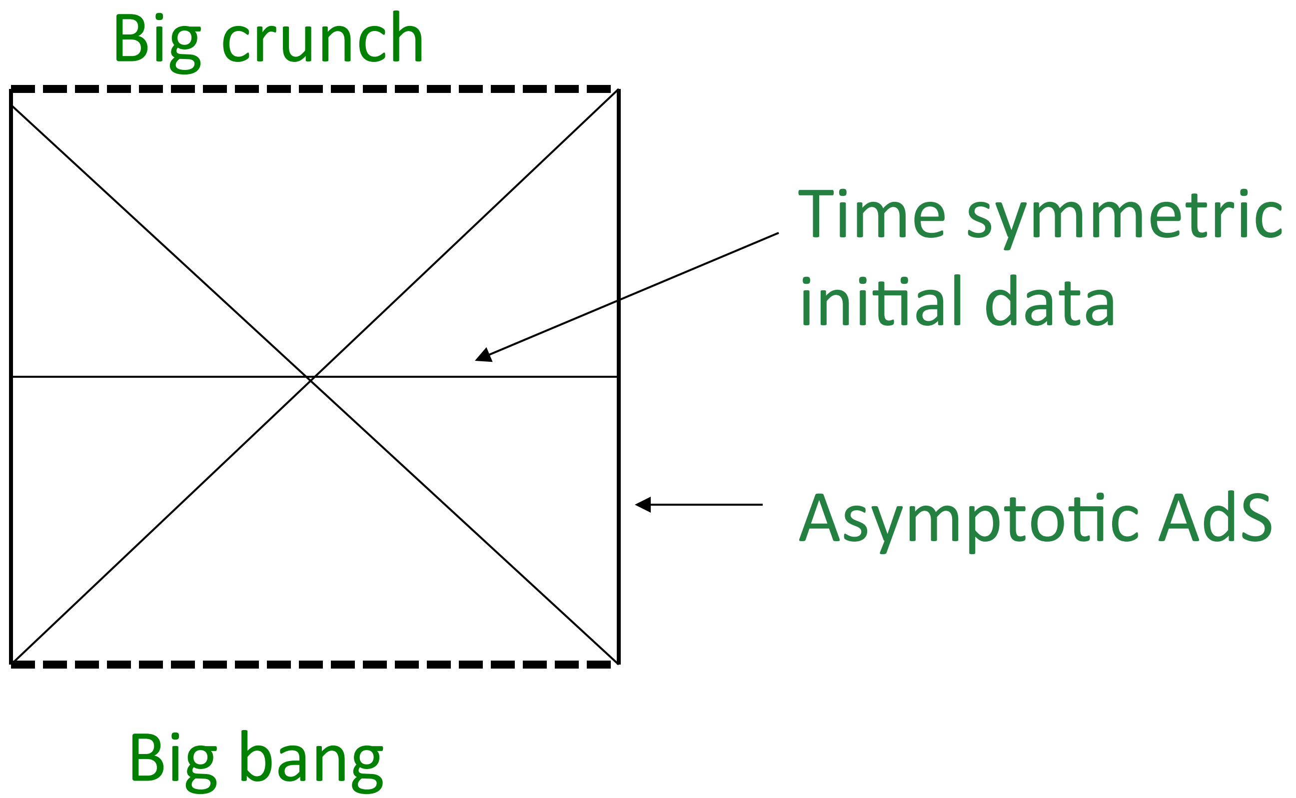
$$ds^2 = -(r^2+1)dt^2 + \frac{dr^2}{r^2+1} + r^2 d\Omega$$

In all asymptotically AdS solutions, the scalar field falls off like

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2}$$

If $\alpha = 0$ or $\beta = 0$, no solutions with cosmological singularities. Consider a new boundary condition $\beta = k\alpha^2$ (Hertog and Maeda). This is also invariant under all asymptotic AdS symmetries.

Claim: For all nonzero k, there are solutions that evolve from a big bang and into a big crunch.



This looks like Schwarzschild-AdS, but:

(1) Infinity is not complete.

(2) "Horizon" is just the lightcone of the origin.

CFT Description

This is the 2+1 theory on a stack of M2-branes (Aharony, Bergman, Jafferis, and Maldacena, 2008). The theory contains eight scalars. With β=0 boundary conditions, the bulk scalar ϕ is dual to the dimension one operator

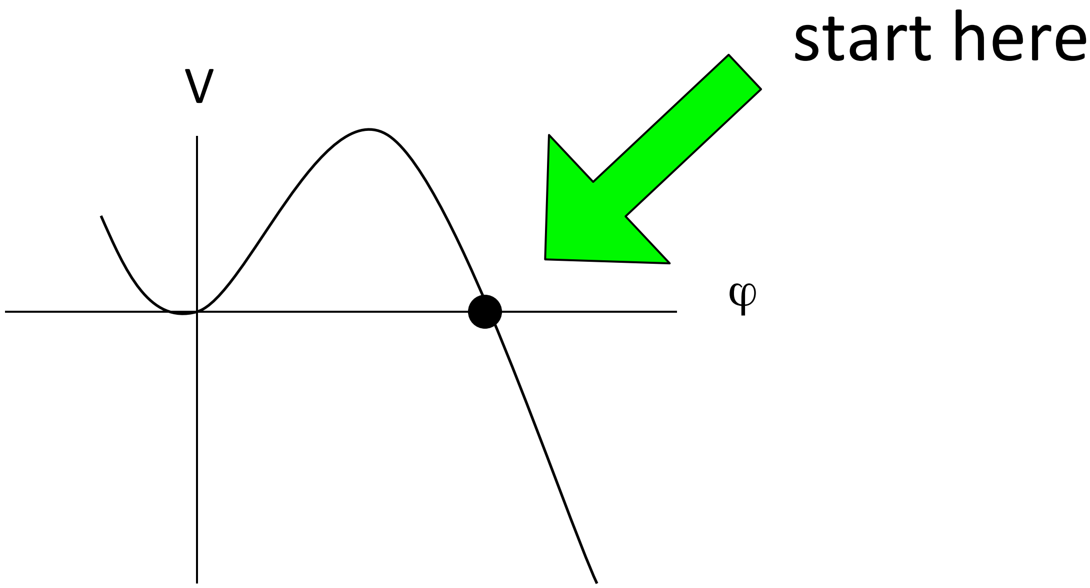
$$O = Tr(\varphi_1^2 - \varphi_2^2)$$

Our new boundary condition corresponds to adding to the field theory action the term (Witten; Sever and Shomer):

$$\frac{k}{3}\int O^3$$

CFT is like a 3D field theory with potential

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{k}{3}\varphi^6$$



This field theory is sick: φ rolls down the potential and reaches infinity in finite time. This is the analog of evolving to the singularity in the bulk.

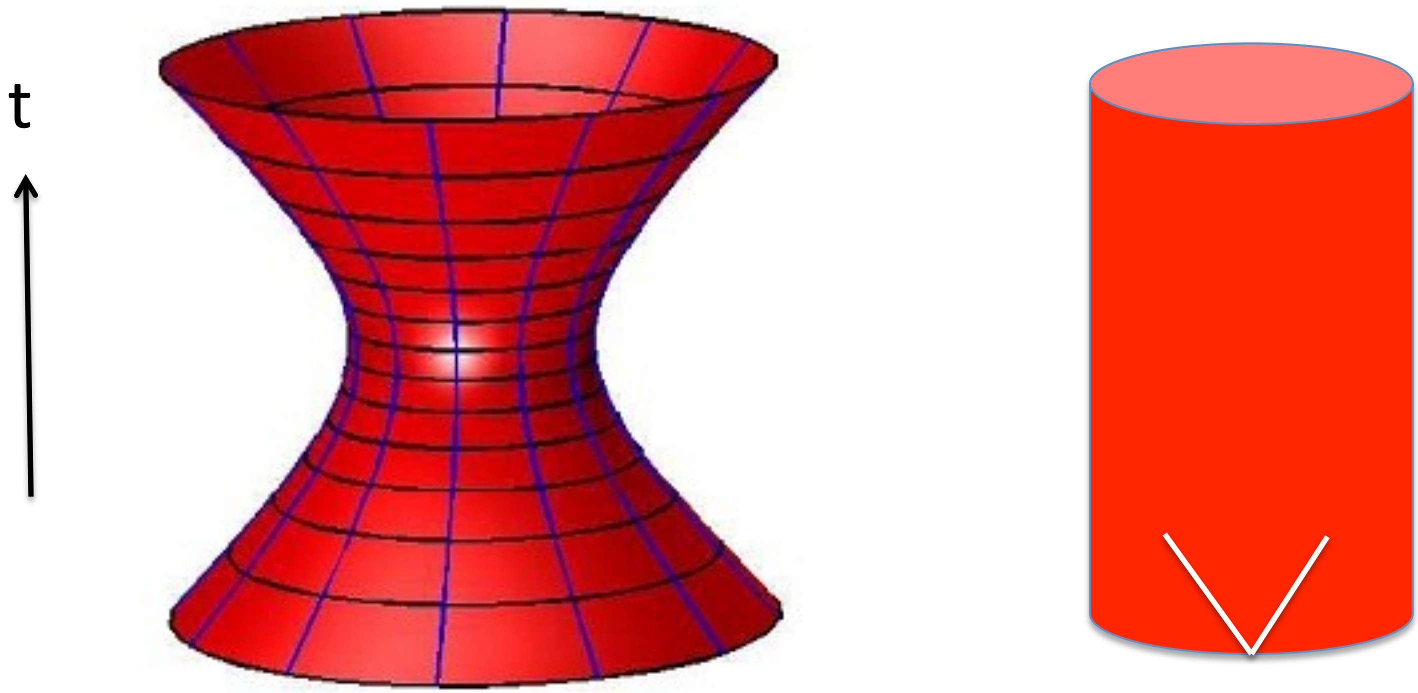
Update

Maldacena (1012.0274) pointed out that if we view the dual theory as living on de Sitter space, then our gravity solutions exist with boundary condition β = const.

This corresponds to adding a single trace operator *O* to the CFT on de Sitter. This is like a mass term.

Field theory remains well defined!

de Sitter space has constant positive curvature



Infinite hyperboloid can be conformally rescaled to a finite cylinder

Any renormalizable but not conformal deformation of the QFT on dS_3 will be dual to a crunch (Harlow and Susskind, 1012.5302).

Problem now is how to describe the region near the singularity in the dual theory.

If you rescale back to static cylinder, the mass deformation becomes time dependent and blows up when the singularity hits the boundary (Barbon and Rabinovici, 1102.3015).

Outline

1) Introduce new example of cosmological singularities in AdS

2) Describe the boundary observable and evaluate it in cases of interest

3) Find key signature of the singularity

4) Summary and future directions

The gravity solution

Solutions to Einstein's equation can be obtained by starting with AdS_5

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

and replacing $\eta_{\mu\nu}$ with any solution to 4D general relativity. We will use the Kasner solution:

$$ds^2 = -dt^2 + t^{2p_1}dx_1^2 + t^{2p_2}dx_2^2 + t^{2p_3}dx_3^2$$

$$\sum_i p_i = 1 = \sum_i p_i^2$$

$$ds^2 = \frac{t^2}{z^2}\left(\frac{-dt^2 + t^{2p_1}dx_1^2 + t^{2p_2}dx_2^2 + t^{2p_3}dx_3^2 + dz^2}{t^2}\right)$$

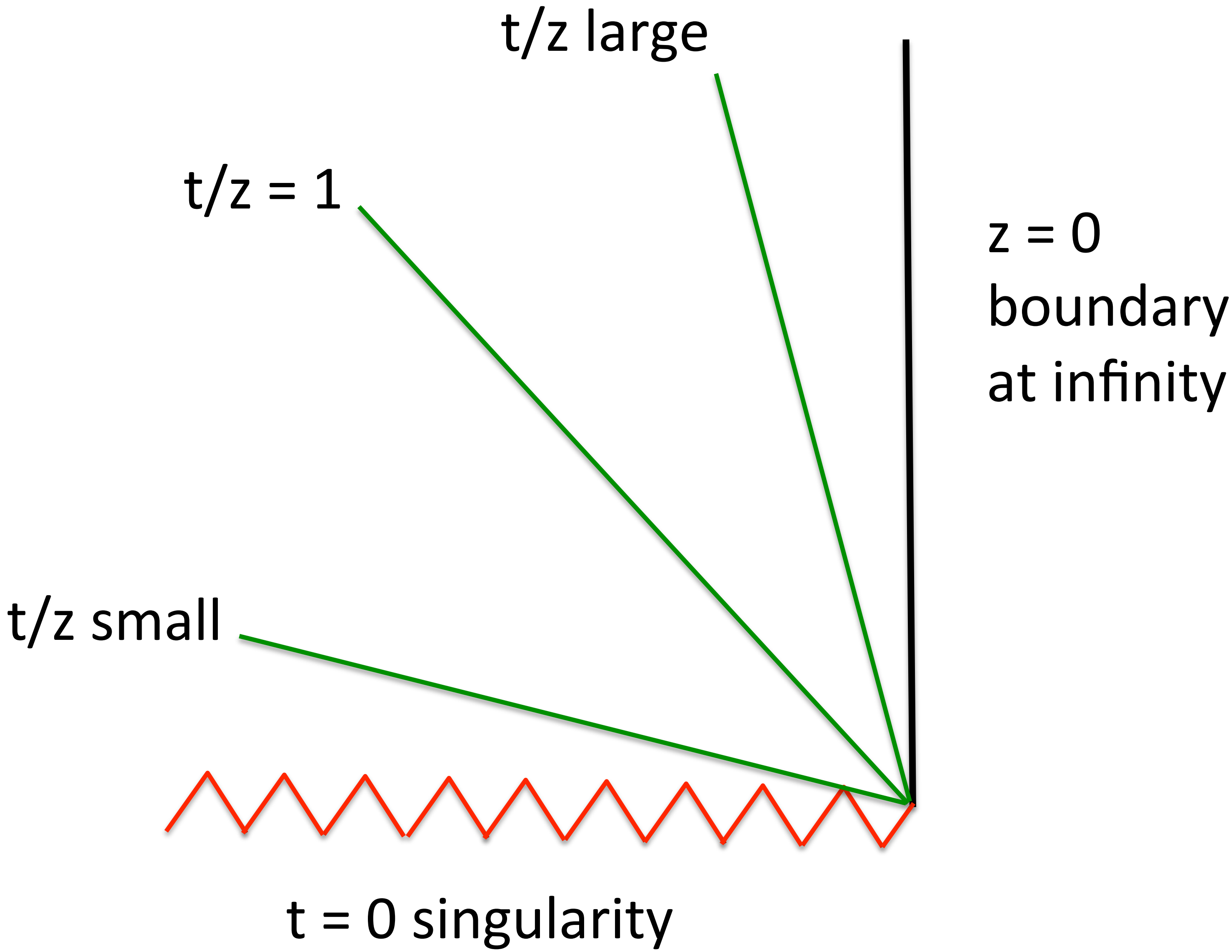
Letting t = e^{τ} , the boundary metric becomes an anisotropic version of de Sitter space

$$ds^2 = -d\tau^2 + \sum_i e^{-2(1-p_i)\tau}dx_i^2$$

Our bulk solution has a dilation symmetry

$$z \to \lambda z, \quad t \to \lambda t, \quad x_i \to \lambda^{(1-p_i)}x_i$$

which acts on each surface of constant t/z.


t/z large
t/z = 1
t/z small
z = 0
boundary
at infinity
t = 0 singularity

Observable: two-point function

In a quantum field theory, the two point function of a scalar field of mass m can be calculated from a path integral

$$\langle \Phi(x)\Phi(x')\rangle = \int \mathcal{D}x(\lambda) e^{-mL[x(\lambda)]}$$

where L is the length of the curve. When m is large, one can use a WKB approximation. Curves that extremize the length are geodesics.

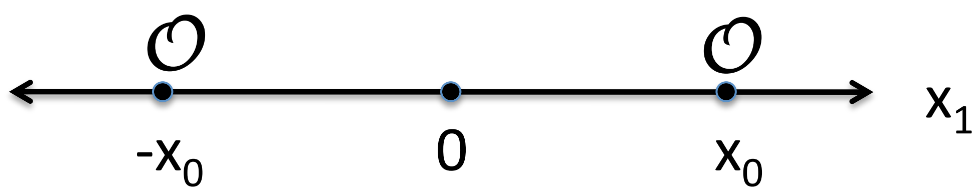
In holography:

The two point function of an operator in the CFT with large dimension Δ can be calculated using spacelike geodesics in the bulk with endpoints on the boundary:

$$\langle\psi|\,\mathcal{O}(x)\,\mathcal{O}(x')\,|\psi\rangle = e^{-L_{reg}(x,x')\Delta}$$

where L_{reg} is the regulated length of the geodesic.

$x_2 = x_3 = 0$
$t = 1$



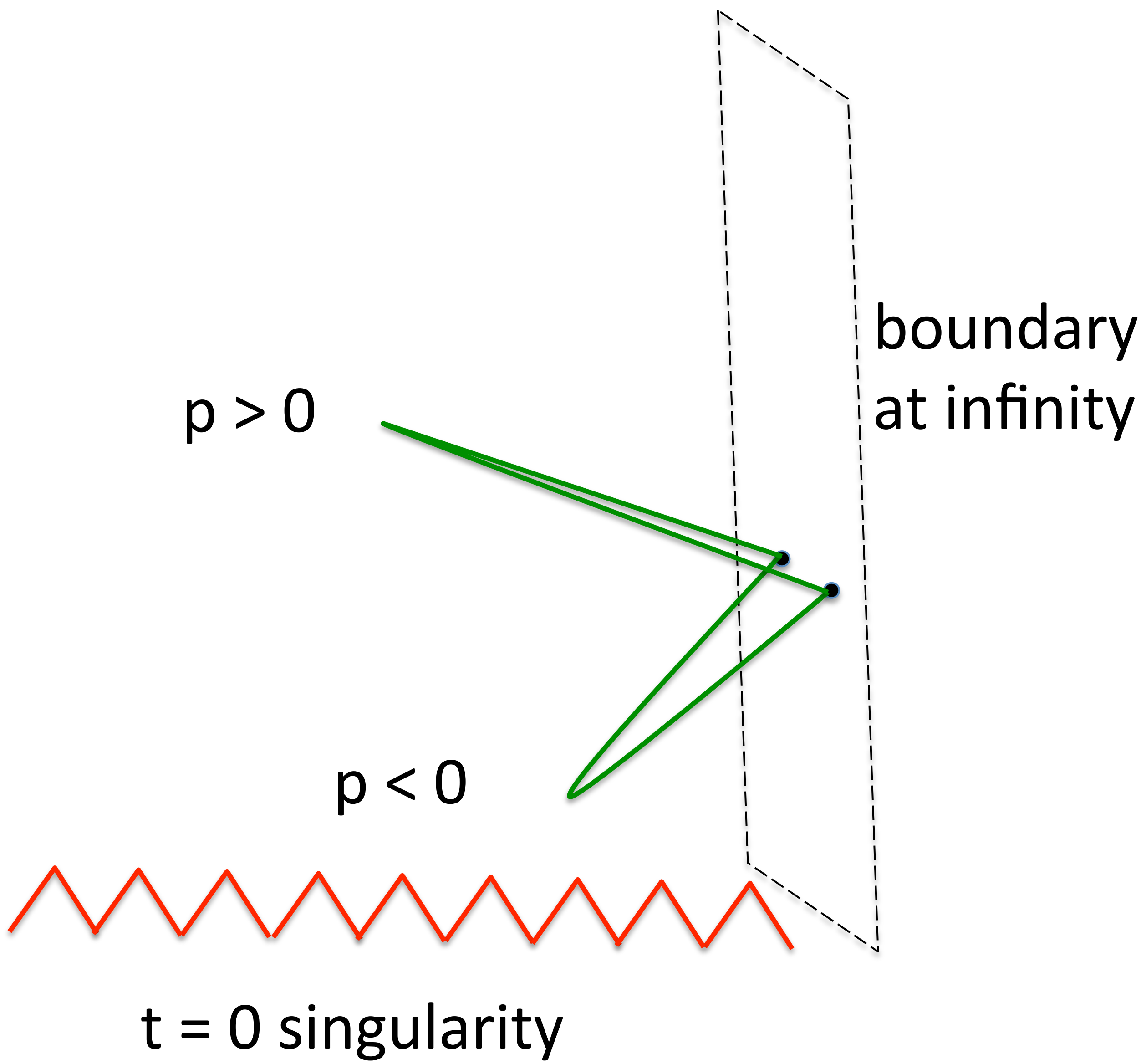
Bulk geodesics effectively travel in 3D spacetime

$$ds^2 = \frac{1}{z^2}(-dt^2 + t^{2p}dx^2 + dz^2)$$

Using x as a parameter along the geodesic, t(x) satisfies

$$t(x)t''(x) = p[2t'(x)^2 - t(x)^{2p}]$$

At turning point, dt/dx = 0, so for p < 0, geodesics bend toward the singularity and for p > 0 they bend away.



boundary
at infinity
p > 0
p < 0
t = 0 singularity

Simple example 1: Flat space

In pure AdS, the geodesics stay on a constant time surface. With UV cut-off z = ε, their length is

$$L = 2\ln\left(\frac{2x_0}{\epsilon}\right) = 2\ln(L_{bdy}) - 2\ln\epsilon,$$

We regulate this by dropping the last term. So

$$\langle\mathcal{O}(x_0)\mathcal{O}(-x_0)\rangle = L_{bdy}^{-2\Delta}$$

as expected for a CFT.

2: Isotropic de Sitter

Bulk is again pure AdS written in the form

$$ds^2 = \frac{H^2 t^2}{z^2}\left(\frac{-dt^2 + dx_i dx^i + dz^2}{H^2 t^2}\right)$$

With Hτ = ln(Ht), the boundary metric is

$$ds^2 = -d\tau^2 + e^{-2H\tau} dx_i dx^i$$

Geodesics are the same, but cut-off is now $\delta = \varepsilon/H$ and $L_{bdy} = 2x_0/H$, so effect of H cancels out.

$$L = 2\ln\left(\frac{2x_0}{H}\frac{H}{\epsilon}\right) = 2\ln L_{bdy} - 2\ln\delta$$

So in de Sitter space:

$$\langle \mathcal{O}(x_0)\mathcal{O}(-x_0)\rangle = L_{bdy}^{-2\Delta}$$

just like flat space. It is independent of the Hubble constant.

Note: If we change t, the UV cut-off $\delta = \varepsilon/Ht$ corresponds to a time independent proper length cut-off in the dual CFT.

Main example: p = -1/4

Recall: $$ds^2 = \frac{1}{z^2}(-dt^2 + t^{-1/2}dx^2 + dz^2)$$

Geodesic equation can be solved analytically using $w = t^{1/2}$ as the parameter:

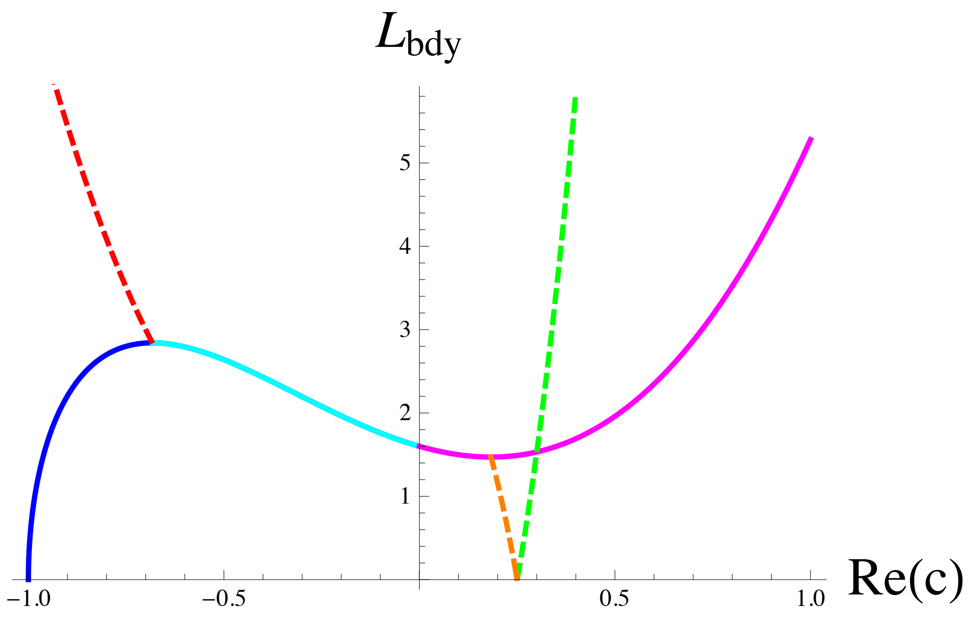
$$X(w) = \frac{4}{15}\sqrt{c+w}(8c^2 - 4cw + 3w^2)$$

$$Z(w) = \frac{4}{3}\left[c(1-w)[(3c-1)(w+1) - w^2)]\right]^{1/2}$$

c fixes the boundary separation:

$Z(1) = 0$, so $L_{bdy} = 2\,X(1)$

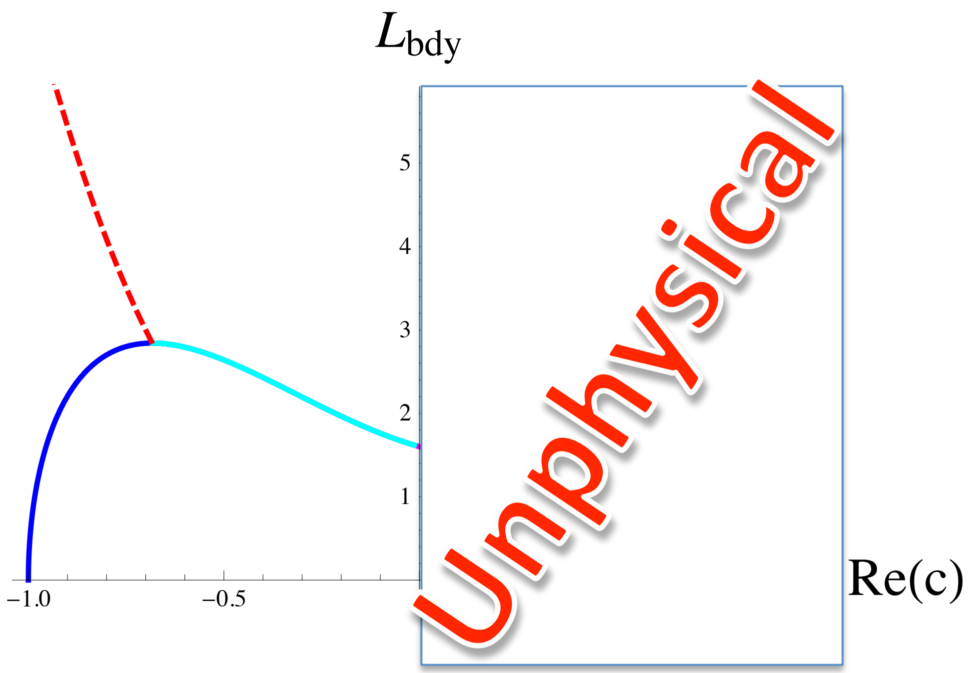
Fixing (real) L_{bdy}, there are 5 (complex) values for c.



Solid line: real c
Dashed lines: complex c

The geodesic turns around when X = 0, i.e., w = -c. For Re(c) > 0, geodesic hits the singularity at w = 0, but these don't contribute.

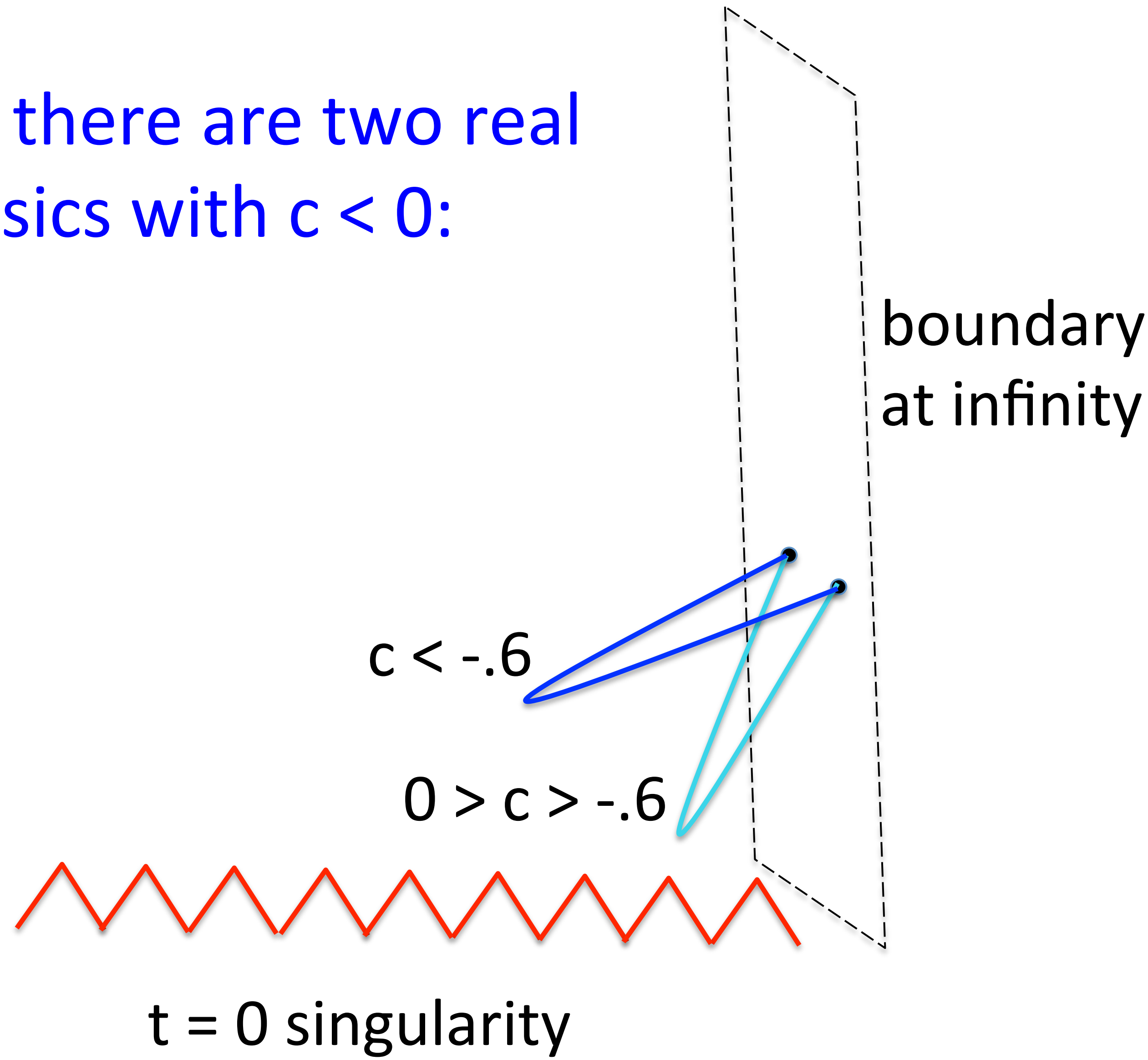
Fixing (real) L_{bdy}, there are 5 (complex) values for c.



Solid line: real c
Dashed lines: complex c

The geodesic turns around when X = 0, i.e., w = -c.
For Re(c) > 0, geodesic hits the singularity at w = 0, but these don't contribute.

When there are two real geodesics with c < 0:

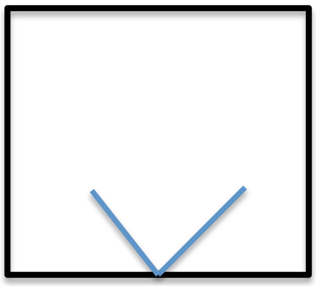


For Re c < 0, the regulated length is:

$$L_{reg} = \ln\left[-\frac{64}{9}c(1+c)(2c-1)^2\right]$$

For c close to -1, $L_{bdy} = 8\,(1+c)^{1/2}$ is small and $L_{reg} = 2\ln L_{bdy}$, just as in pure AdS.

As c -> 0, L_{reg} again diverges. This corresponds to $L_{bdy} \to L_{horizon}$. The bulk geodesics turn around close to the singularity and approach a null geodesic lying entirely on the boundary.



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This predicts a pole in the correlator at the horizon size.
This pole is a key signature of the bulk singularity in the dual field theory.

Maximum curvature along geodesic gets large as geodesic approaches the boundary.

This pole is weaker than the one at short distances:

$$< \mathcal{O}\mathcal{O} > \sim (L_{bdy} - L_{horizon})^{-\Delta} \quad \text{vs} \quad < \mathcal{O}\mathcal{O} > \sim L_{bdy}{}^{-2\Delta}$$

as required by QFT.

Common question: What about quantum or stringy corrections to the geometry near the singularity?

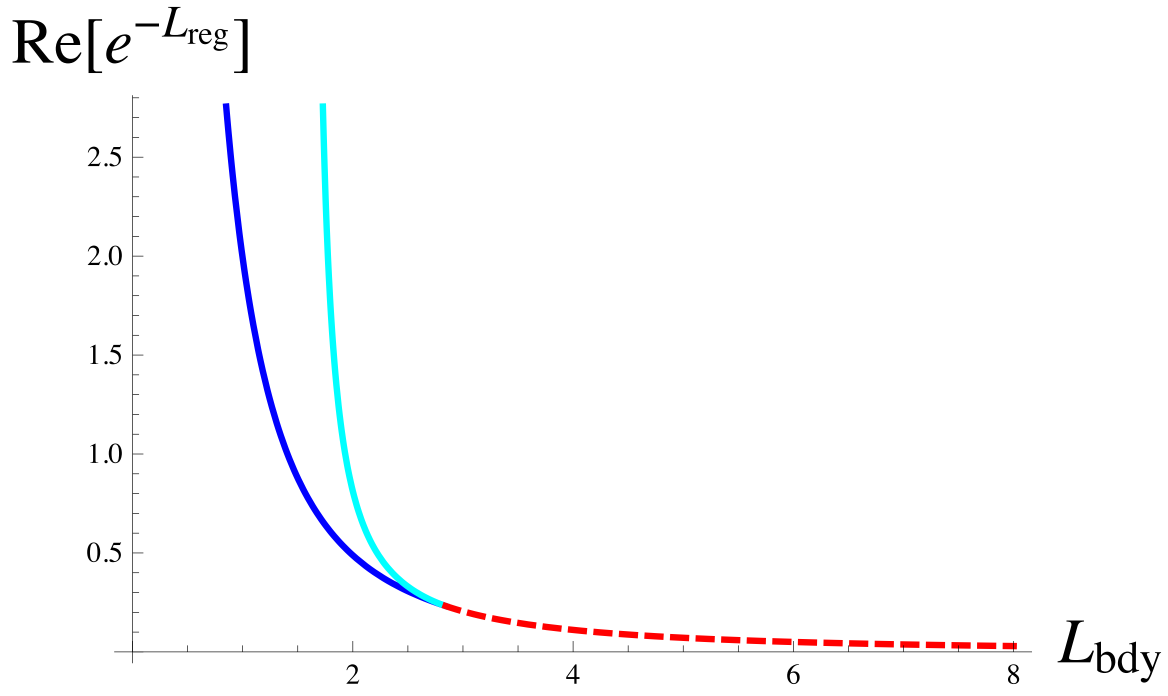
Answer: We are looking for a signature of the classical singularity in the large N strongly coupled CFT. After we find it, we can ask what happens at finite N or small coupling by studying the CFT.

$p = -1/4$ is typical of $p < 0$

One can solve the geodesic equations for general $p < 0$ and show that there always exist bulk geodesics which get close to the singularity and approach a null geodesic on the boundary.

All of these examples have a pole in the correlator at the horizon scale.

Final result for the p - -1/4 correlator:



For large L_{bdy}:

$$\langle \mathcal{O}\mathcal{O} \rangle \approx L_{bdy}^{-\frac{8\Delta}{5}}$$

The solutions we have examined lead us to conjecture that for general p, the large distance behavior is:

$$\langle \mathcal{O}(x_0)\mathcal{O}(-x_0)\rangle \approx L_{bdy}^{-\frac{2\Delta}{1-p}}$$

The fall-off of the 2pt function depends directly on the rate of expansion in that direction.

Suggestive reformulation of this result:

Our dilation symmetry implies that the general equal time correlator: $< \mathcal{O}(-x_0, t_0)\ \mathcal{O}(x_0, t_0) >$ is a function of only one variable: $\xi = t_0/x_0^{1/1-p}$

For small values of ξ, the correlator is:

$$< \mathcal{O}\ \mathcal{O} > \sim \xi^{2\Delta}$$

This is different from the short distance behavior.

This is probably due to particle creation: We have a CFT on a time dependent background.

Possible connection with inflation

Turning our model upside down, we have a CFT on an expanding (anisotropic) de Sitter space similar to standard models of inflation.

Modes are in their ground state at subhorizon scales and are highly excited at superhorizon scales.

Whenever you have particle creation in de Sitter, there seems to be a big crunch in the bulk.

If we compactify one spatial direction and put antiperiodic boundary conditions on the fermions there is a purely stringy resolution of the singularity.

The mass of a string wound around a circle of radius R (with antiperiodic fermions) has two contributions:

$$M^2 = \frac{R^2}{L_s^4} - \frac{1}{L_s^2}$$

usual tension

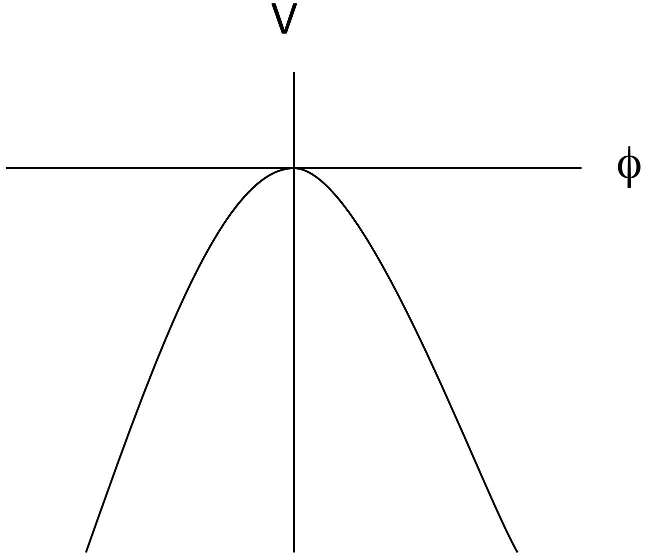
zero point energy

So if $R < L_s$, these wound strings become tachyonic. Tachyons should not be thought of as particles traveling faster than light.

Tachyons just indicate an instability.

In ordinary field theory, if

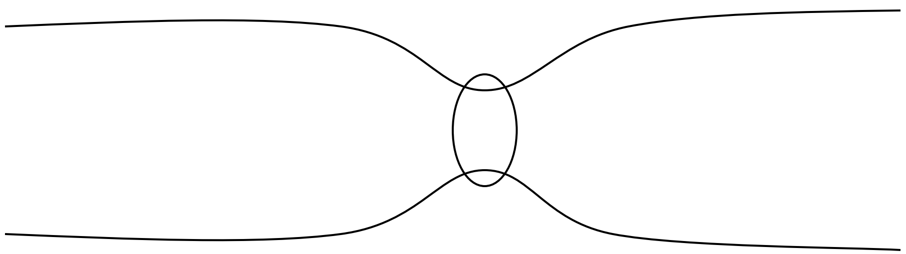
$$V(\phi) = -m^2\phi^2$$



$\phi = 0$ is unstable and ϕ rolls down the potential.
One says that "tachyons condense".

Winding string tachyon condensation

Consider a circle with radius R that shrinks below the string scale in a small region:



It has been shown that the outcome of this instability is that the circle smoothly pinches off, changing the topology of space (Silverstein et al., 2005).

If we compactify a direction with p > 0 in our bulk solution we get a Lorentzian cone. With antiperiodic boundary conditions for fermions, winding strings can become tachyonic before the curvature becomes large. The subsequent evolution is no longer given by supergravity, but rather by the physics of tachyon condensation. (McGreevy and Silverstein, 2005)

Summary

1. We have constructed a holographic dual of a cosmological singularity which is a CFT on an anisotropic de Sitter spacetime.
2. In some directions the 2pt function has a pole at the horizon scale which seems to be a unique signature of the singularity.
3. The asymptotic behavior of the 2pt function depends on the expansion rate in that direction.
4. With certain boundary conditions the singularity is resolved by tachyon condensation.

To Do:

1. Understand better the dual CFT state at weak coupling. Is the pole removed?
2. Calculate expectation values of Wilson loops (extremal 2-surfaces)
3. Calculate entanglement entropy (extremal 3-surfaces)
4. What is the sign of tachyon condensation in the dual CFT?