Black Funnels and Droplets

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V. Hubeny, D. Marolf, M. Rangamani, arXiv:0908.2270
J. Santos, B.W, arXiv:1207.4205
J. Santos, B.W, arXiv:1405.2078

AdS/CFT with Curved Boundary

To find gravity duals to holographic CFTs, we solve Einstein's Equations with negative cosmological constant

$$S = \int d^{d+1}x \sqrt{-g}\,(R - 2\Lambda)$$

Solutions have a boundary metric. Typically, this is

- Minkowski space (Poincaré AdS)
- Sphere (Global AdS)

We will take the boundary metric to contain a black hole

(e.g. Schwarzschild).

Holographic CFTs on Curved Spacetime

Hawking Radiation

- Hawking radiation is usually discussed in the context of perturbative fields.
- What are the differences when the fields are strongly coupled?

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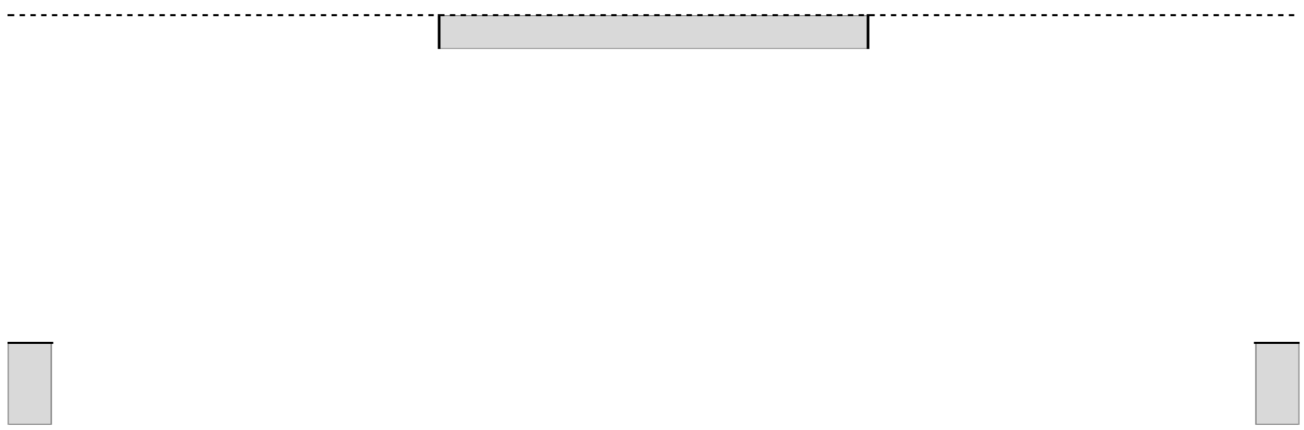
Heat Transport

- Black holes can play the role of a heat bath.
- Provides a natural setting to study heat transport.

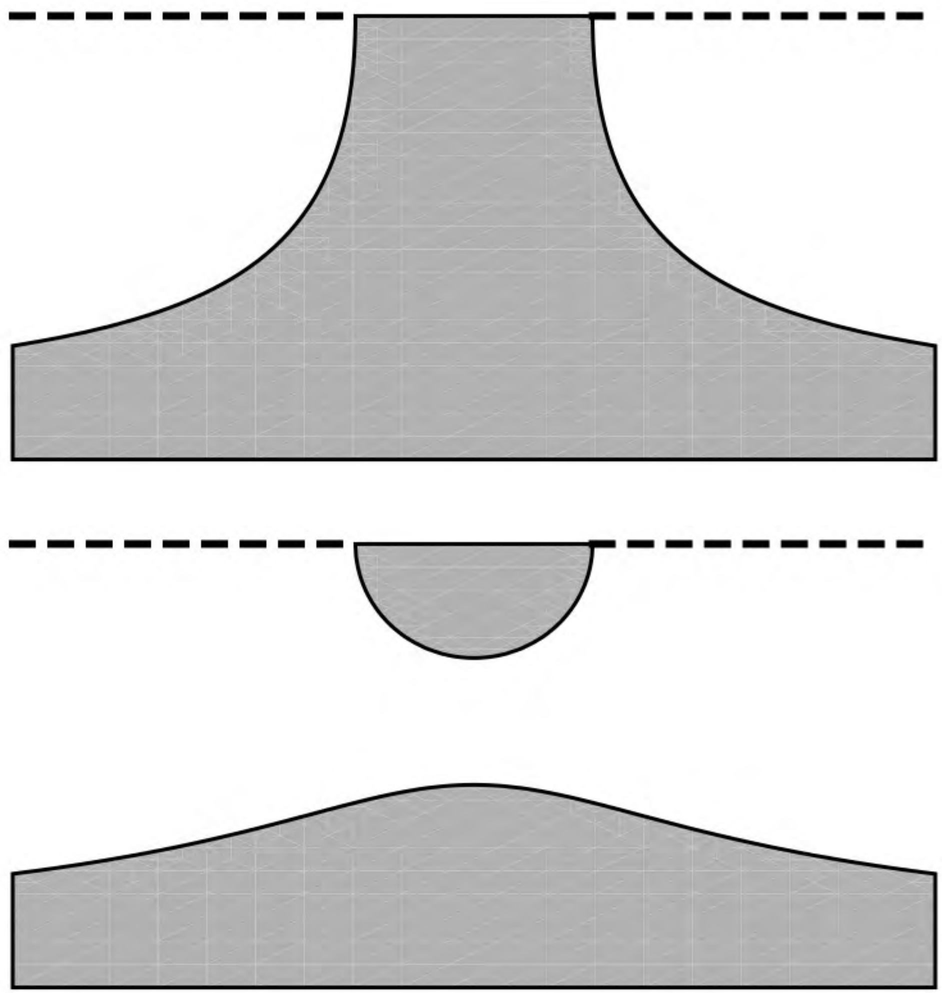
AdS/CFT with Curved Boundary

Black hole metric (e.g. Schwarzschild) on boundary with finite temperature at infinity.



Gravity Dual

Horizons are either connected or disconnected.

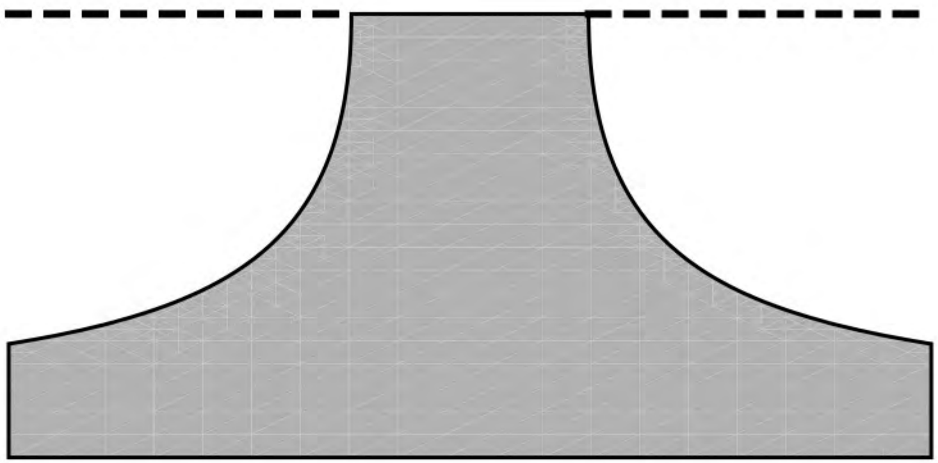


(Flowing) Black Funnel

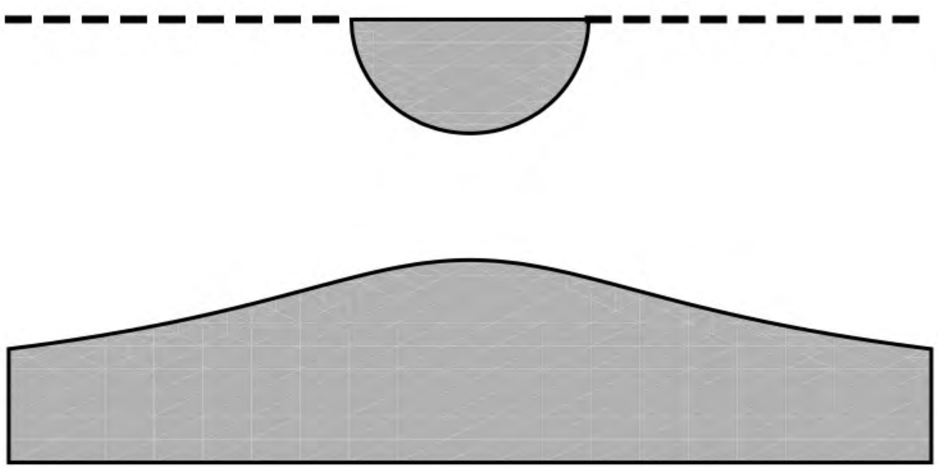
Black Droplet

(Over Planar Black hole)

Gravity Dual



Heat easily exchanged with infinity



Heat NOT easily exchanged with infinity

Droplet Field Theory Interpretation

From the field theory perspective, why are droplets static?

One possible answer:

- Suppose CFT far from black hole admits a useful quasiparticle description, and these quasiparticles have some preferred size $R_{\rm quasi} \sim 1/T_\infty$.
- Black holes of size $R_{\rm BH} \ll R_{\rm quasi}$ might have trouble absorbing/emitting such quasiparticles.

Droplet Field Theory Interpretation

From the field theory perspective, why are droplets static?

Another possible answer:

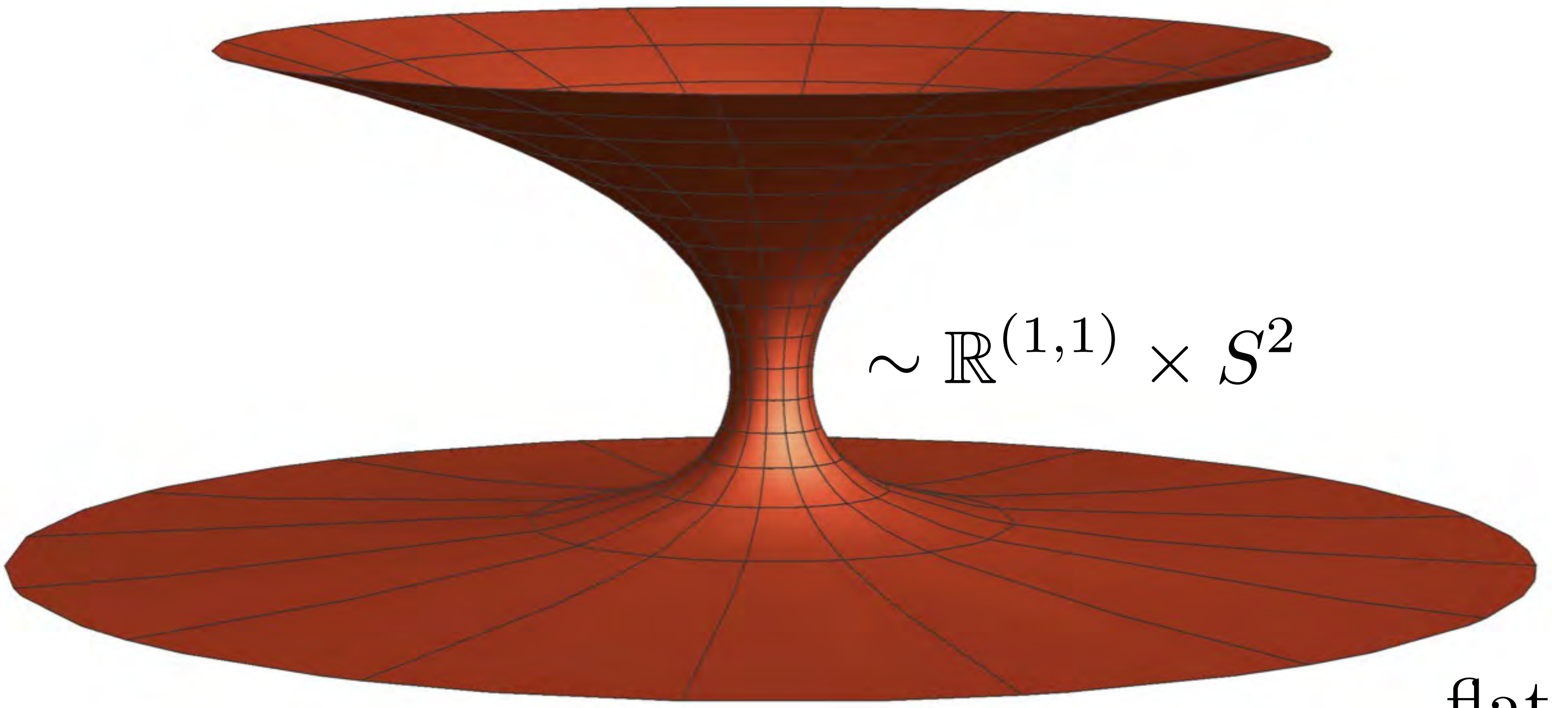
- Under a conformal transformation $ds^2 \to ds^2/f(r)$, the boundary black hole horizon becomes a hyperbolic region.

$$-f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega$$

$$\to -dt^2 + \frac{dr^2}{f^2} + \frac{r^2}{f} d\Omega$$

$$\sim -dt^2 + \frac{4}{|f'|^2}\left(\frac{dz^2}{z^2} + \frac{d\Omega}{z^2}\right) + \dots$$

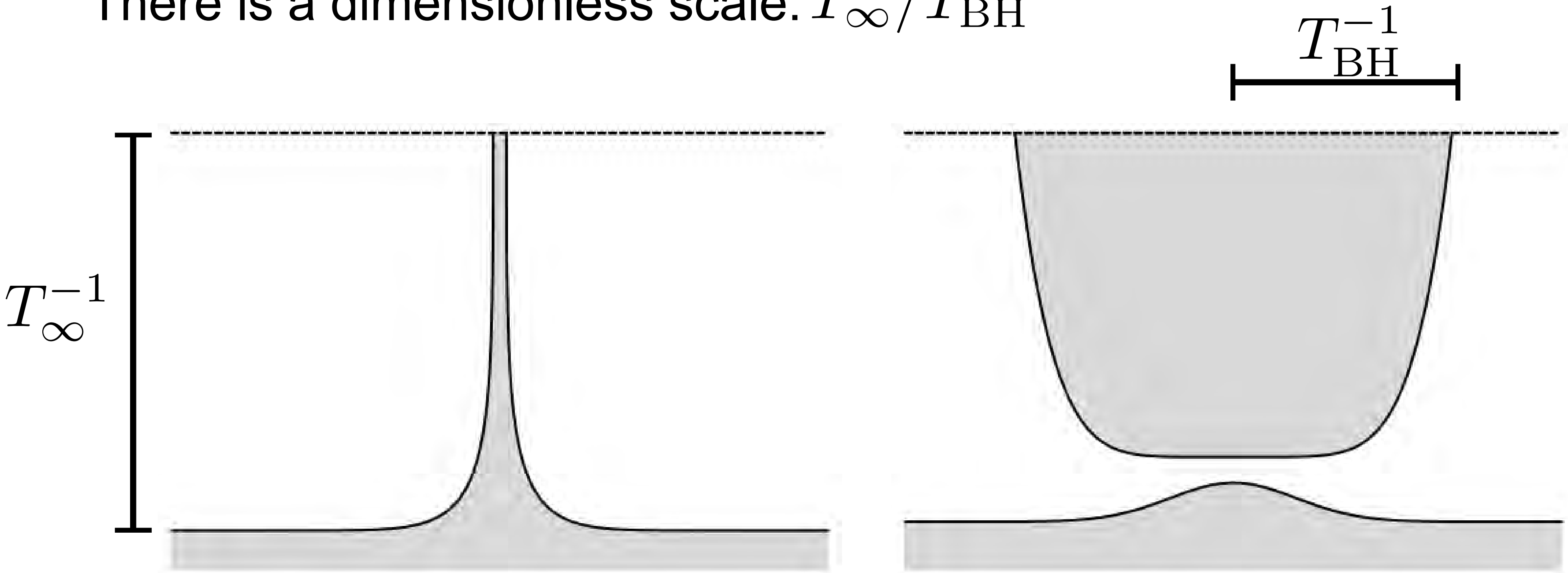
Ultrastatic Frame



Confining region prevents interaction between heat baths.

Phase Transition

There is a dimensionless scale: $T_\infty / T_{\rm BH}$



- These seem like 'unnatural' configurations.
- Conjecture: Funnel solutions preferred for $T_\infty / T_{\rm BH} \gg 1$, and droplet solutions preferred for $T_\infty / T_{\rm BH} \ll 1$.
- Cone Transition?

Static Solutions

For simplicity, we will focus on static solutions:

- Boundary metric is asymptotically flat Schwarzschild.
- Funnels have $T_\infty = T_{\rm BH}$.
- Droplets can have $T_\infty \neq T_{\rm BH}$.

- No analytic solutions, must rely on numerics.

The DeTurck Method

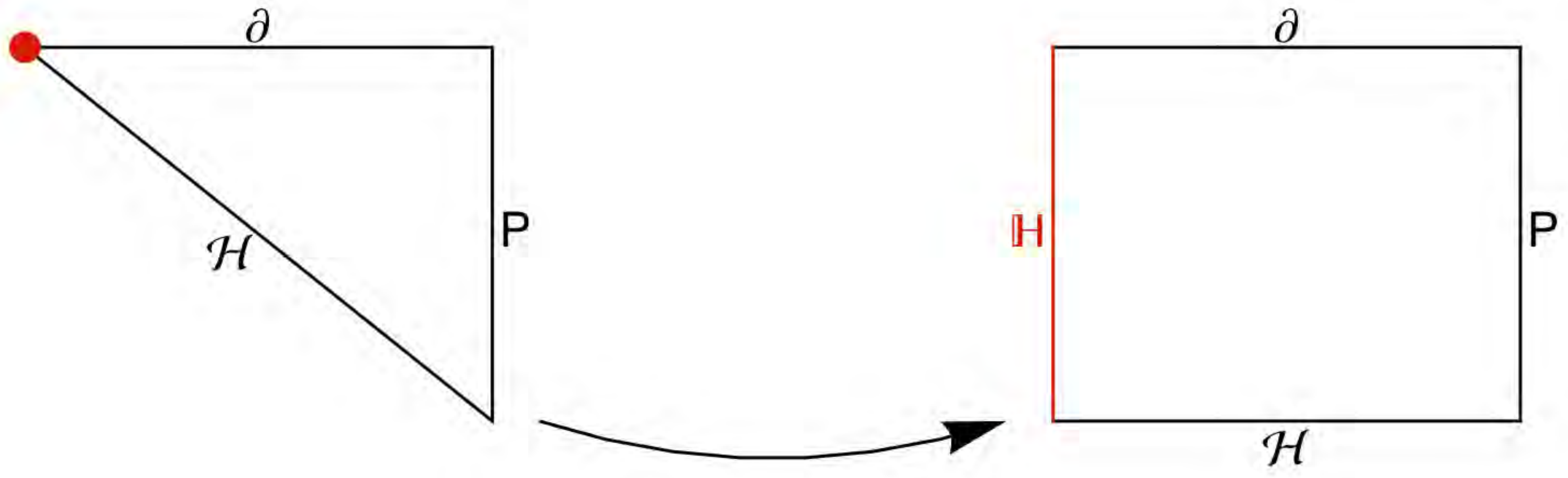
Instead of solving the usual Einstein's equations, solve the Einstein-Deturck equations:

$$R_{\mu\nu} = -\frac{d}{\ell^2} g_{\mu\nu} + \nabla_{(\mu}\xi_{\nu)} \qquad \xi^\mu = g^{\alpha\beta}\left(\Gamma^\mu_{\alpha\beta} - \bar{\Gamma}^\mu_{\alpha\beta}\right)$$

- Only a solution to Einstein's equations when $\xi^\mu = 0$.
- Can prove that solutions with $\xi^\mu \neq 0$ do not exist (in our case).
- Do not need to fix a gauge a priori. Solving the equations will give solution in the gauge $\xi^\mu = 0$.
- Equations are elliptic.
- Quantity $\chi \equiv \xi^\mu \xi_\mu$ can be used to monitor numerical error.

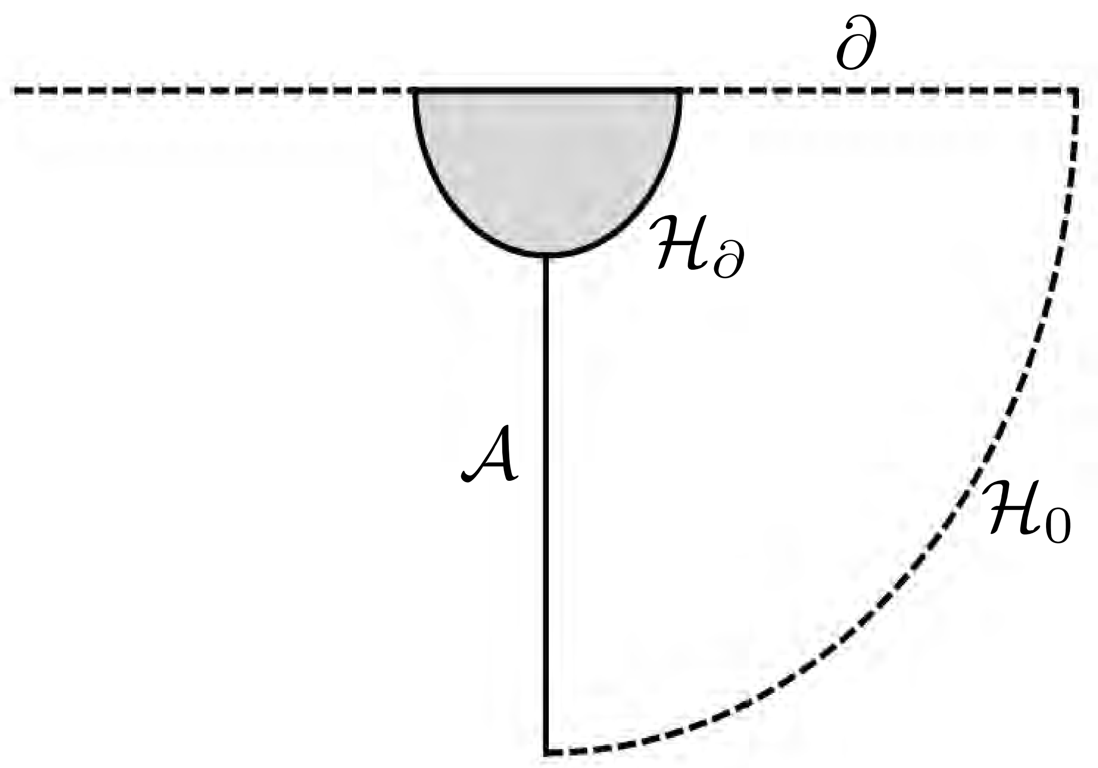
Funnel Integration Domain

- Natural integration domain is a triangle.
- Expand point where boundary meets the horizon.
- Point becomes a hyperbolic black hole.



Droplet Integration Domains

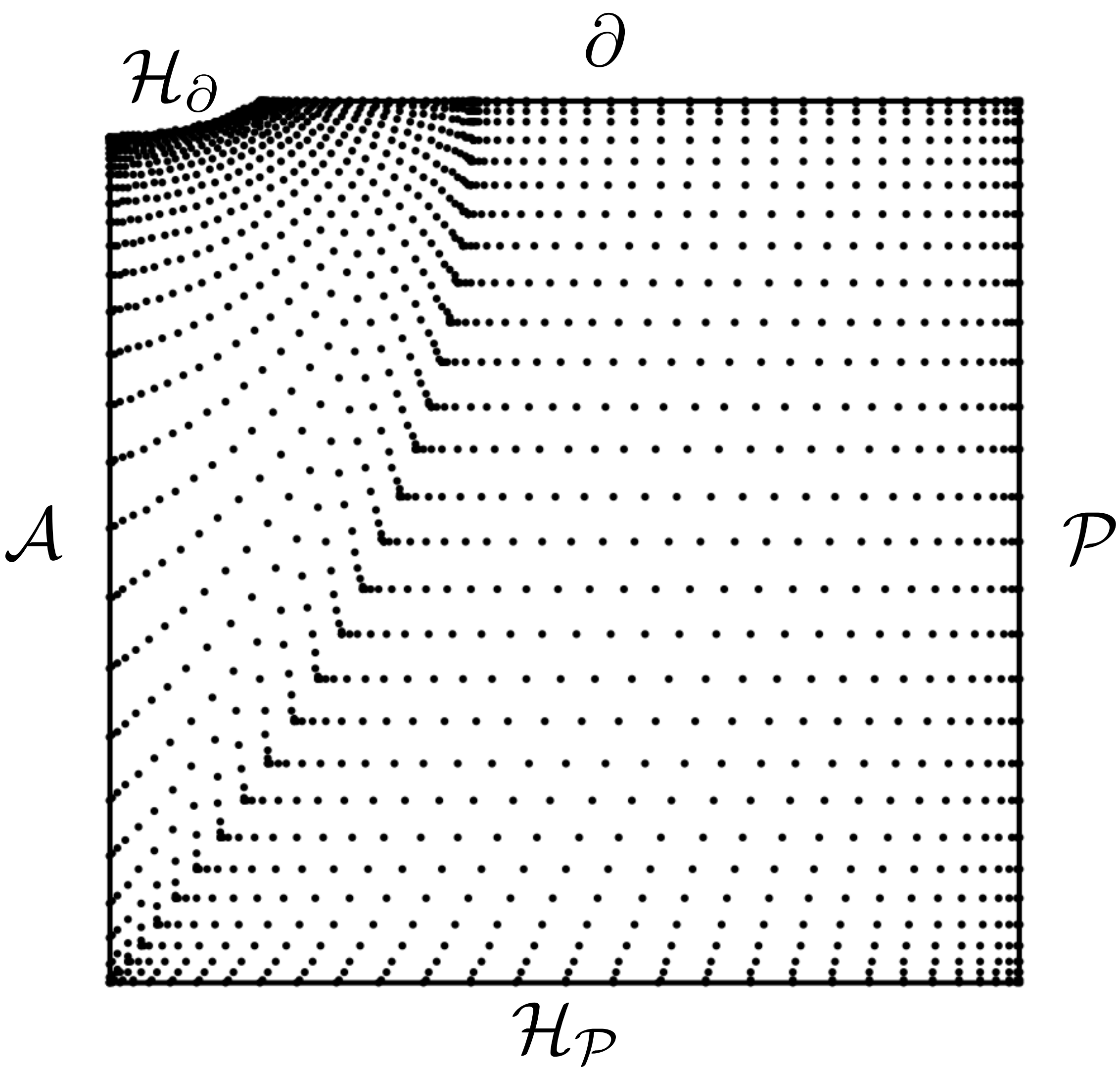
Droplets with $T_\infty = 0$ fit in 'polar' coordinates.



P. Figuras, J. Lucietti, T. Wiseman, arXiv:1104.4489

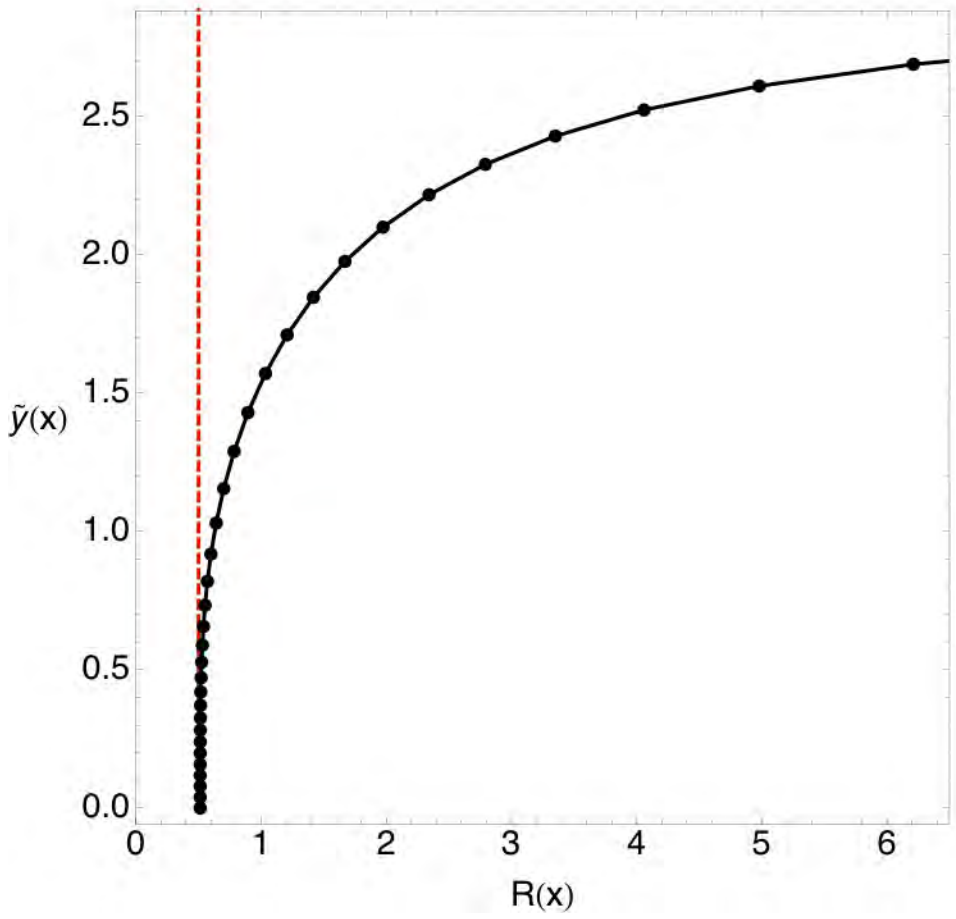
Droplet Integration Domains

Use patching with transfinite interpolation for droplets with planar black hole $(T_\infty \neq 0)$.



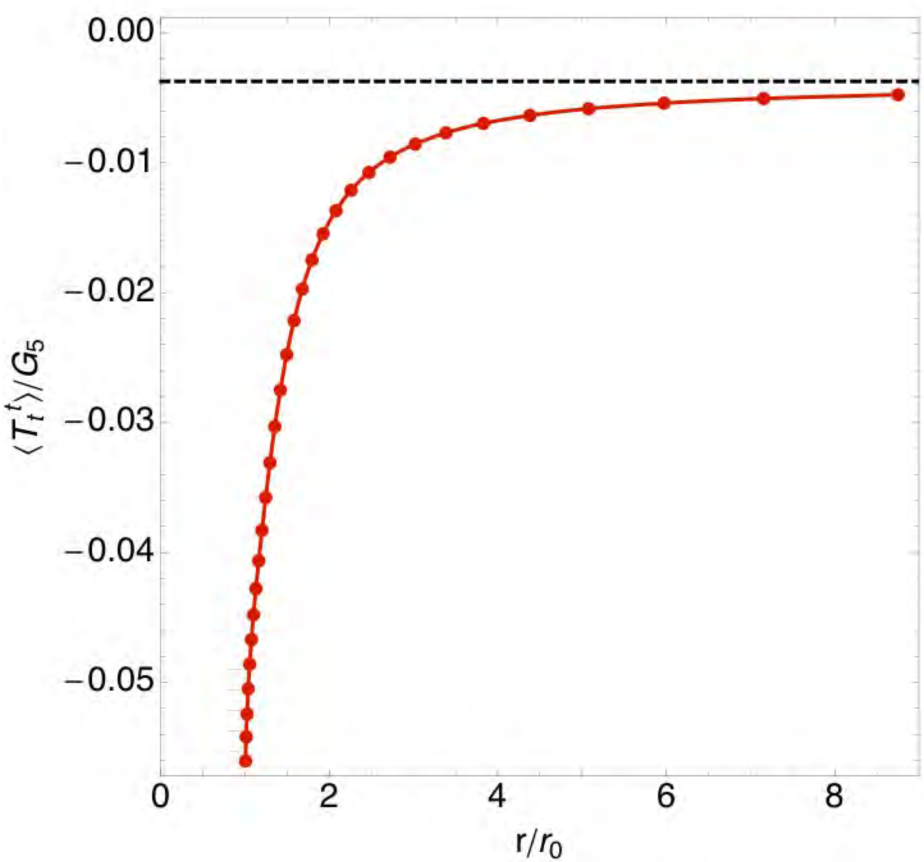
Funnel Embedding

Embed the funnel horizon in hyperbolic space.



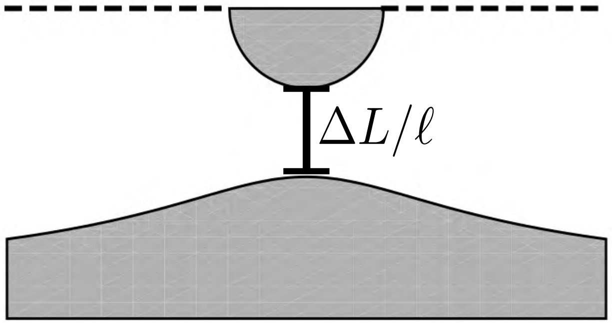
Funnel Stress Tensor

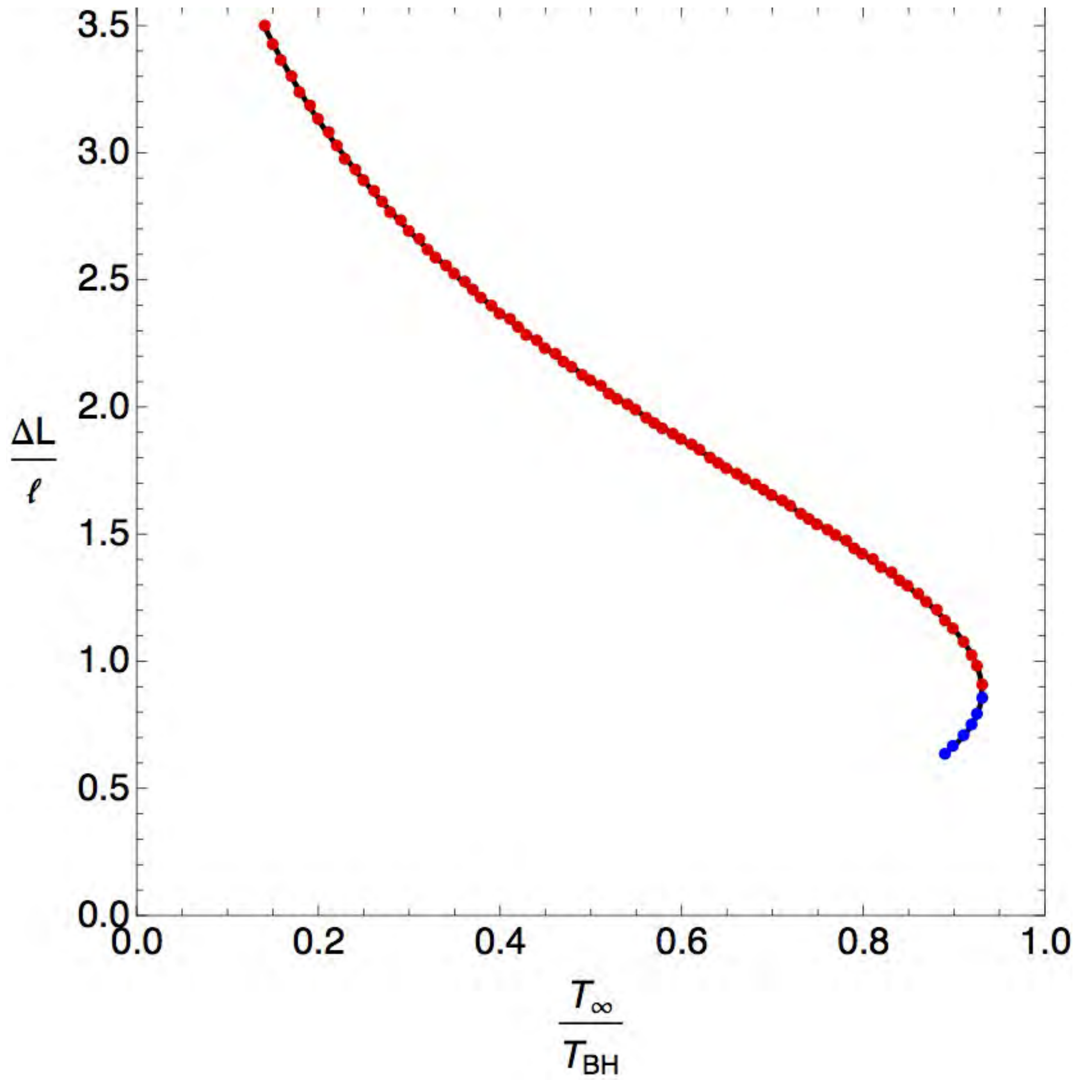
Compute the stress tensor by expanding off the boundary in Fefferman-Graham coordinates.



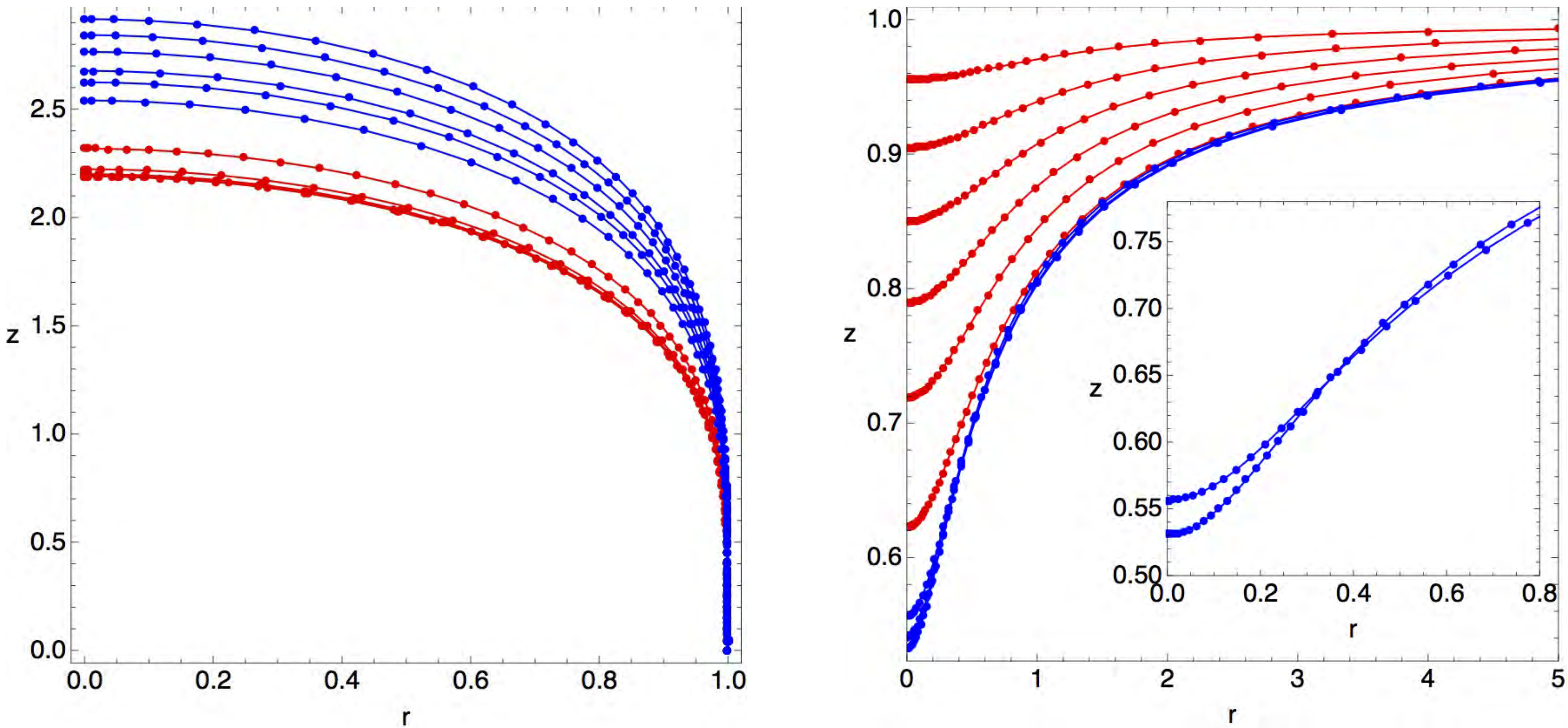
Energy-density positive, stress tensor falls off as 1/r.

Proper Distance



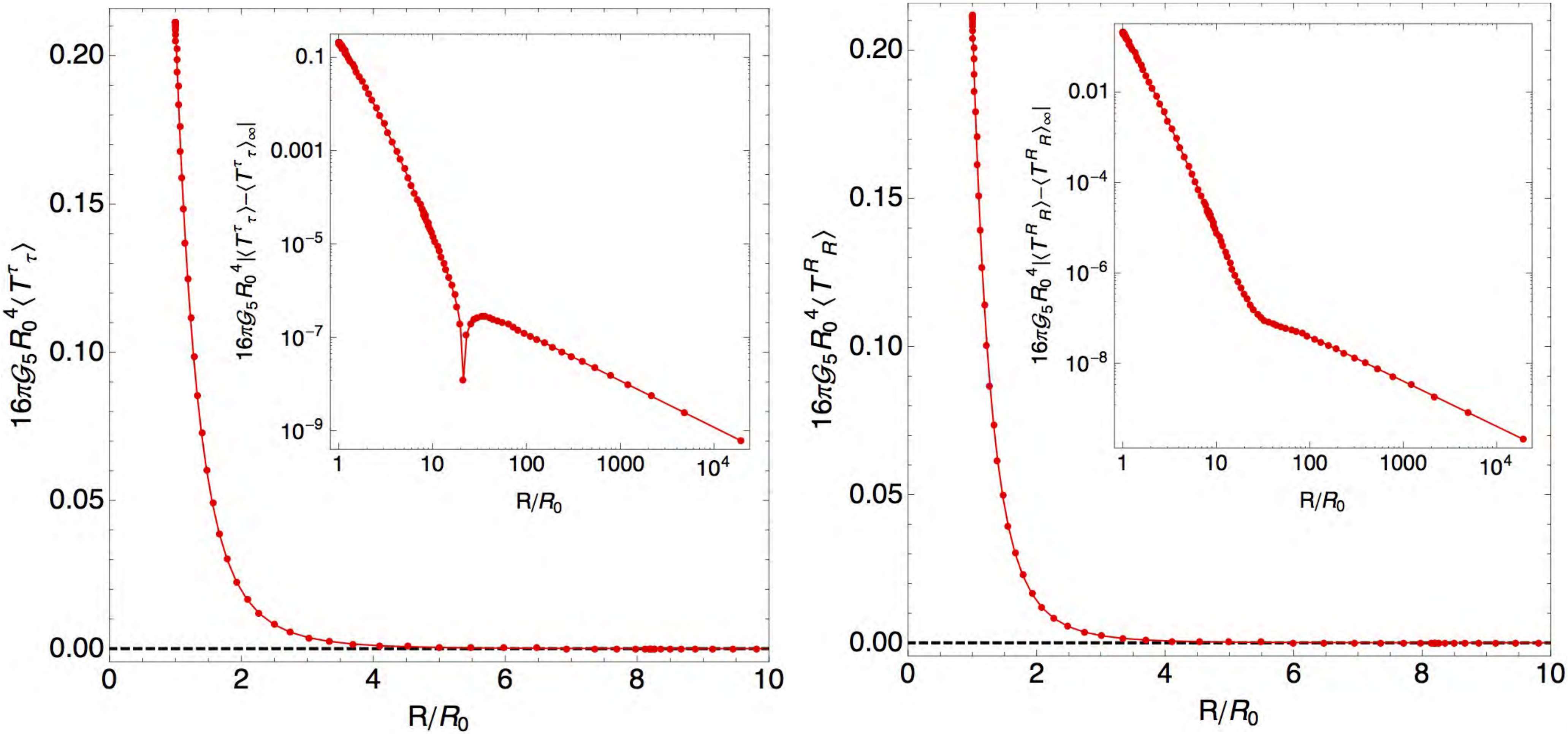


Embedding in Hyperbolic Space



Droplet Stress Tensor

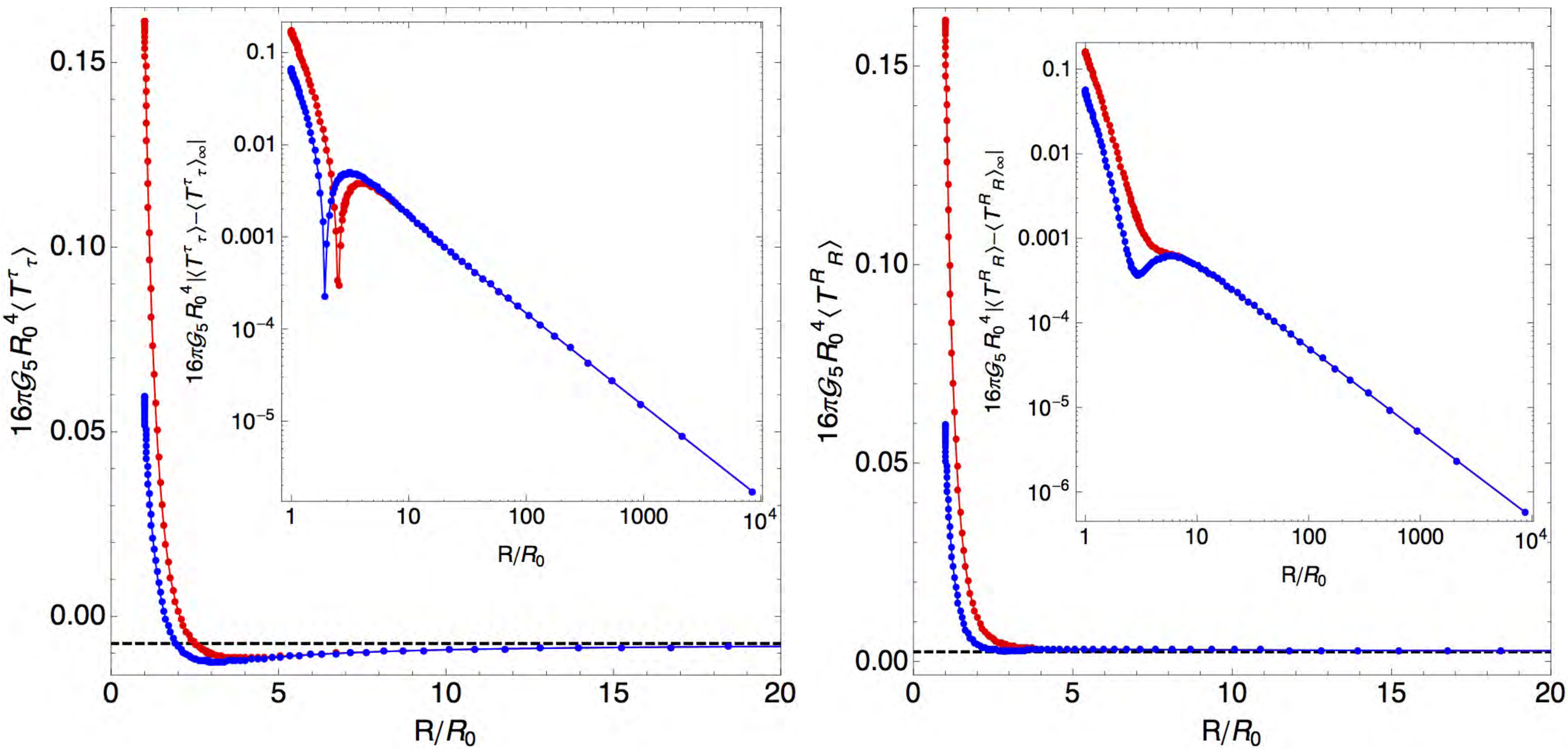
$$\frac{T_\infty}{T_{\rm BH}} = 0.15$$



Stress tensor exhibits two power-law regimes.

Droplet Stress Tensor

$$\frac{T_\infty}{T_{\rm BH}} = 0.89$$



Falloff at infinity is universal.

Summary

- Strongly coupled field theories on a black hole background can have two classes of solutions with different transport properties. These are black droplets and black funnels.

- In the 'droplet' phase, the field theory can be a poor conductor of energy-momentum and heat. These can be static, yet out of thermodynamic equilibrium.

- For a given $T_\infty / T_{\rm BH}$, there can be two droplet phases: a long droplet or a short droplet.

- The Hartle-Hawking state of Schwarzschild is likely dual to a funnel, and funnels dominate for large $T_\infty / T_{\rm BH}$.

Future/Related Work

- Is there a cone transition? To what?
- Stability? Short droplets and equilibrium funnel likely stable. Long droplets might be unstable.
- Boulware state with $T_{\rm bulk\ horizon} = T_\infty = 0$? Quantum energy inequalities?

Solutions with Non-Killing Horizons

- Flowing funnels needed to complete phase diagram.
- Existence of two kinds of funnels? (Wide neck/Narrow neck?)

- Extensions to Kerr/Rotating solutions.
- Equal-spinning Meyers-Perry droplets with $T_\infty = 0$ found.
- Rotating funnels (Black twisters) unknown.

- Test fluid approximation.
- Are there turbulent instabilities?

Thank You