Holographic Quantum Hall Ferromagnet

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Quantum Field theory, String Theory and Condensed Matter Physics
Kolumbari, Crete
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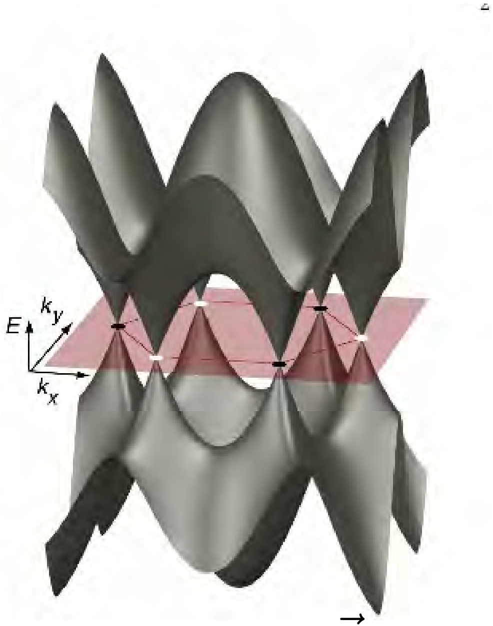
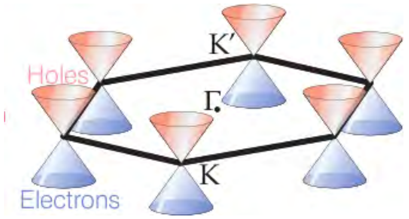
J.Hutchinson, C.Kristjansen, G.W.S. arXiv:1408.3320

C.Kristjansen, R. Pourhasan, G.W.S. arXiv:1311.6999

C.Kristjansen, G.W.S. arXiv:1212.5609

Graphene has relativstic 2+1-D fermions with emergent SU(4) symmetry





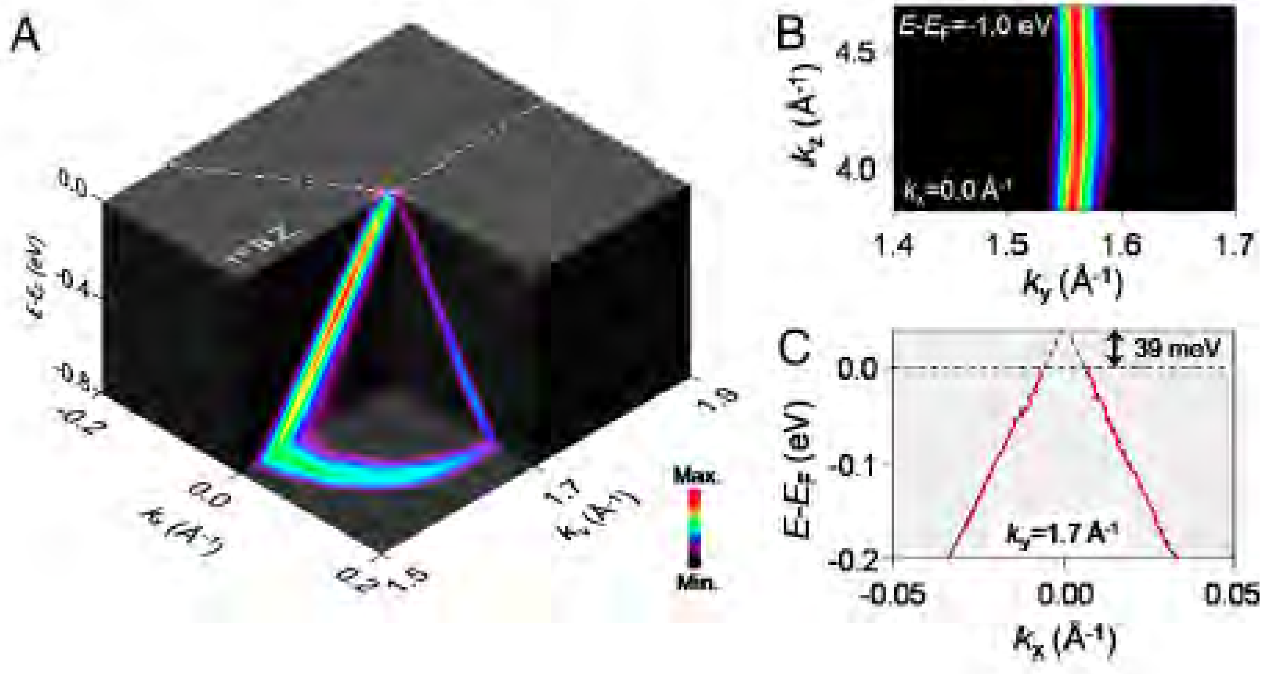
$E(k) = \hbar v_F |\vec{k}|$ $\quad v_F \sim c/300 \quad$ UV cutoff $\quad \sim 1ev$

2 valleys×2 spins

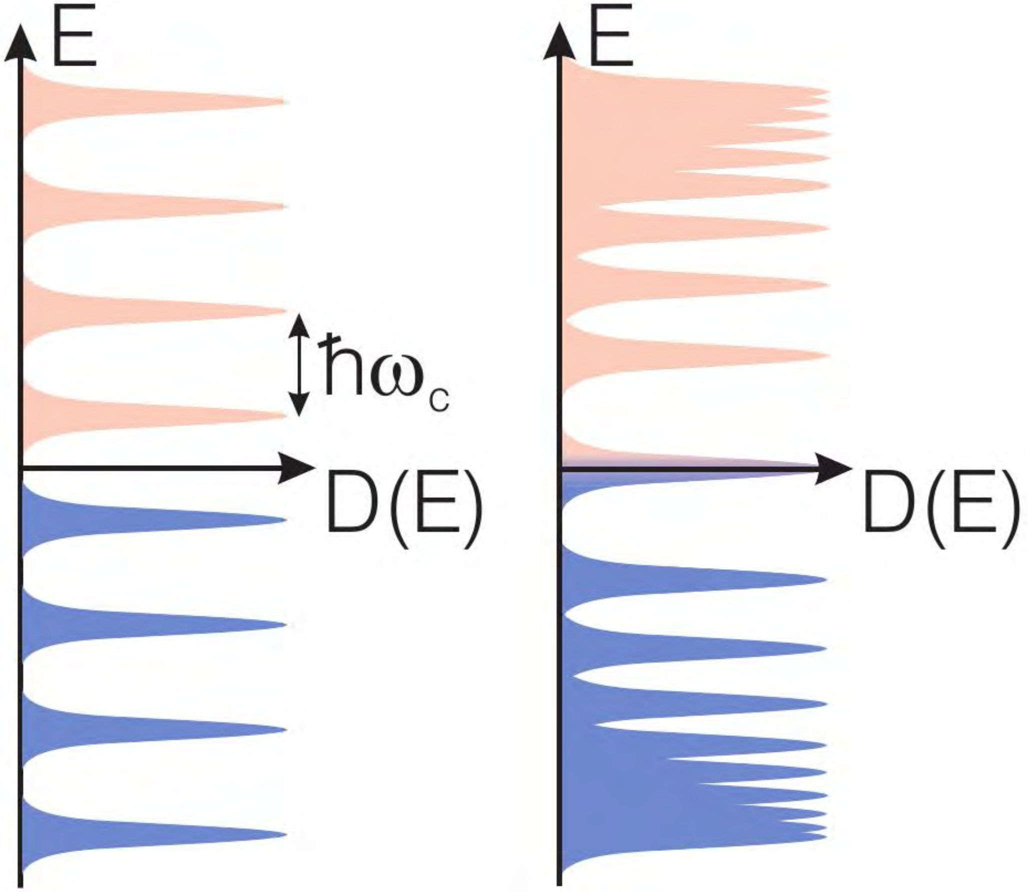
$$S = \int d^3x \sum_{\sigma=1}^{4} \bar{\psi}^{\sigma}\ i\gamma^{\mu}\partial_{\mu}\ \psi^{\sigma} + \text{interactions}$$

Electron dispersion relation with ARPES

D.A. Siegel et. al. PNAS,1100242108



Non-Relativistic and Relativistic Landau Levels

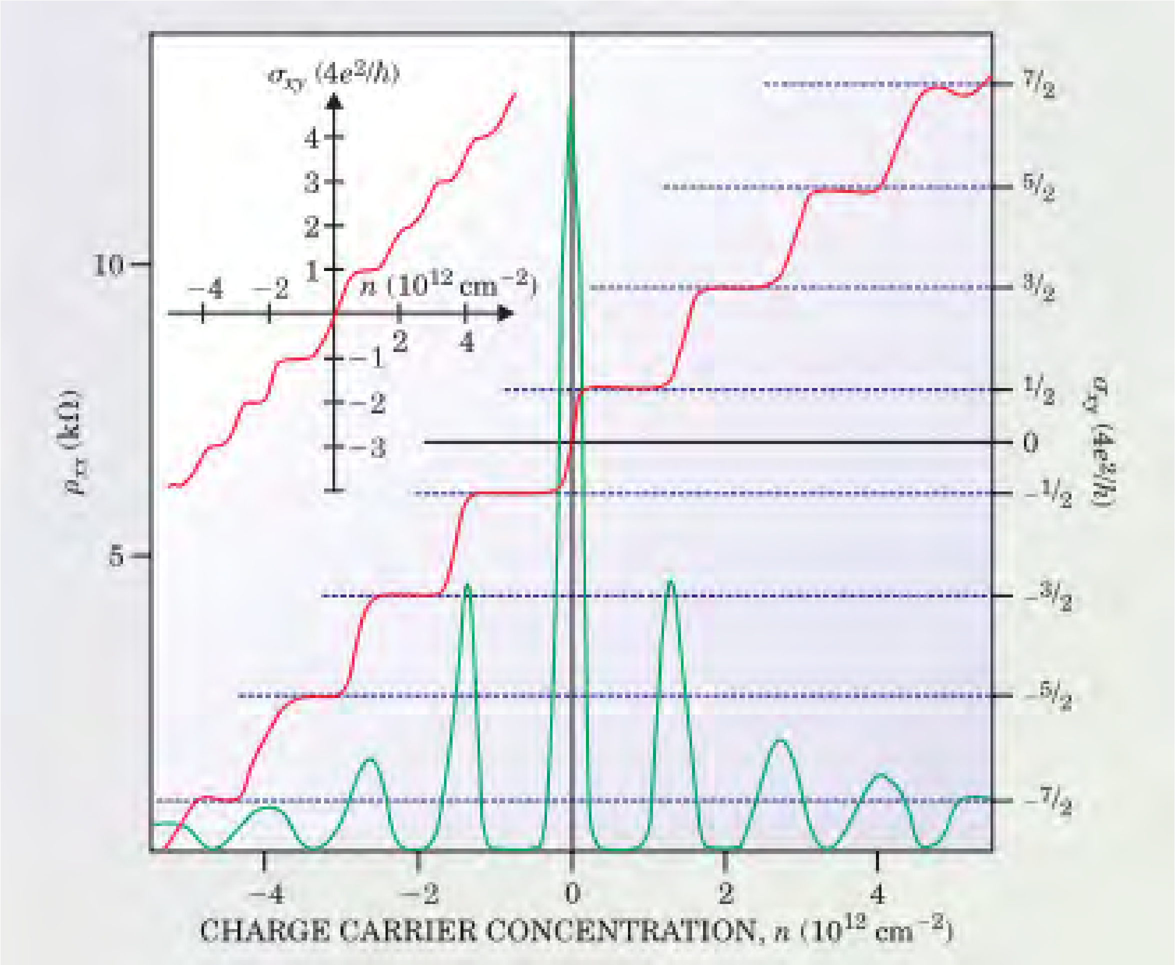


Non-relativistic: $E = \hbar\omega_C\left(n + \frac{1}{2}\right)$, $n = 0, 1, 2, \ldots$

Relativistic $E = \pm\hbar v_F\sqrt{2|B|n}$ degeneracy $= \frac{e|B|}{2\pi}$

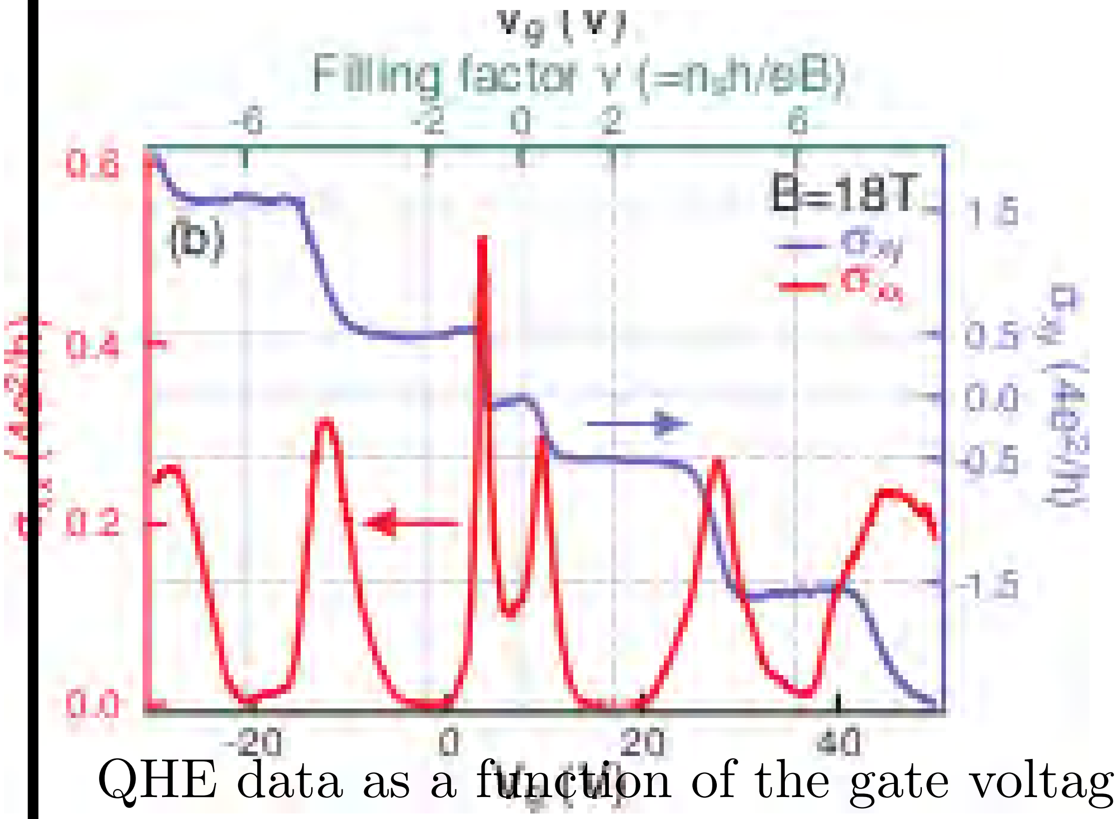
K. Novoselov et. al. *Nature 438, 197 (2005)*

Y. Zhang et. al. *Nature 438, 201 (2005)*



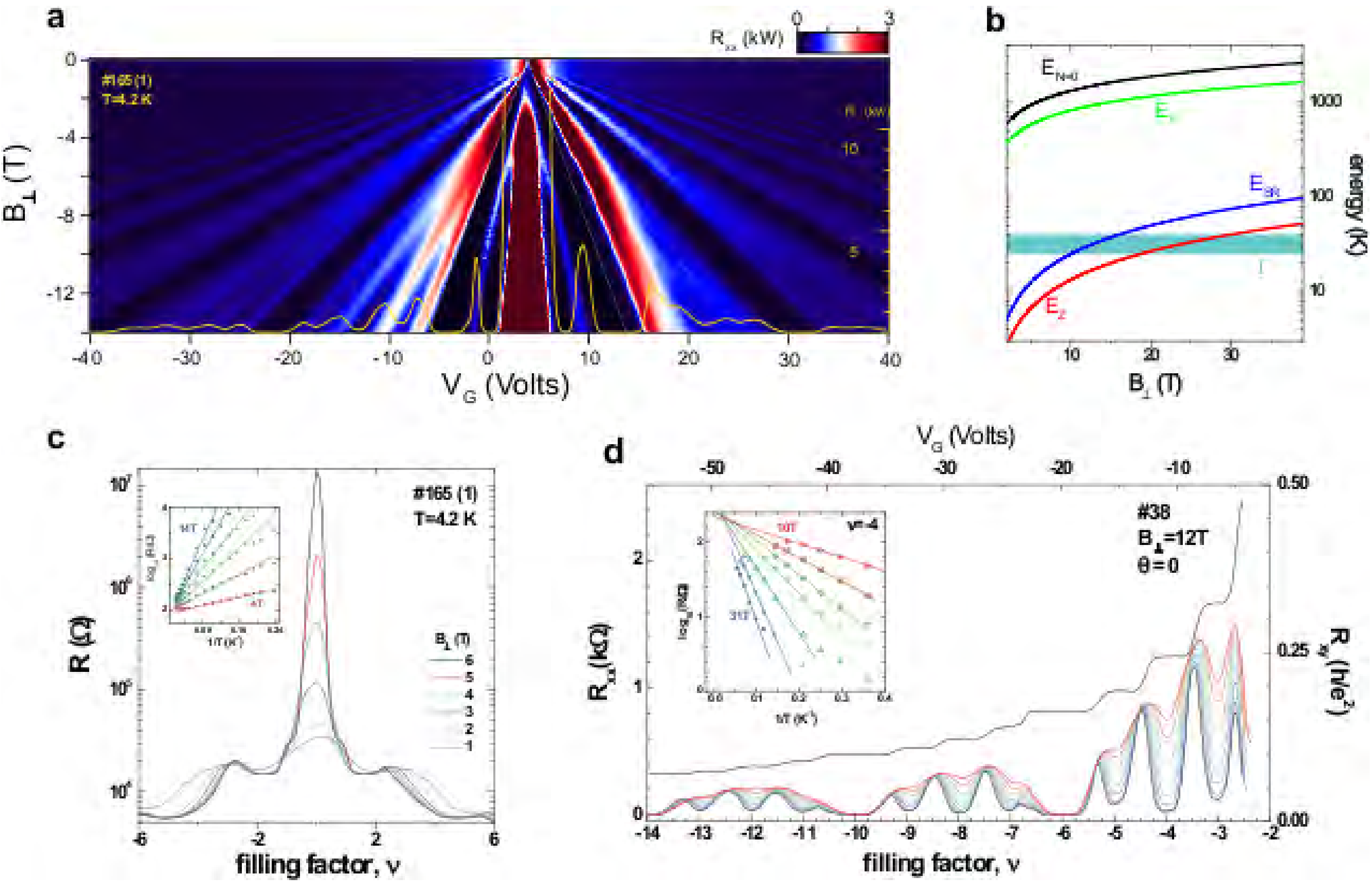
$$\sigma_{xy} = 4\frac{e^2}{h}\left(n+\frac{1}{2}\right)$$

Splitting of $\nu = 0$ Landau level Zhang et.al. arXiv:1003.2738



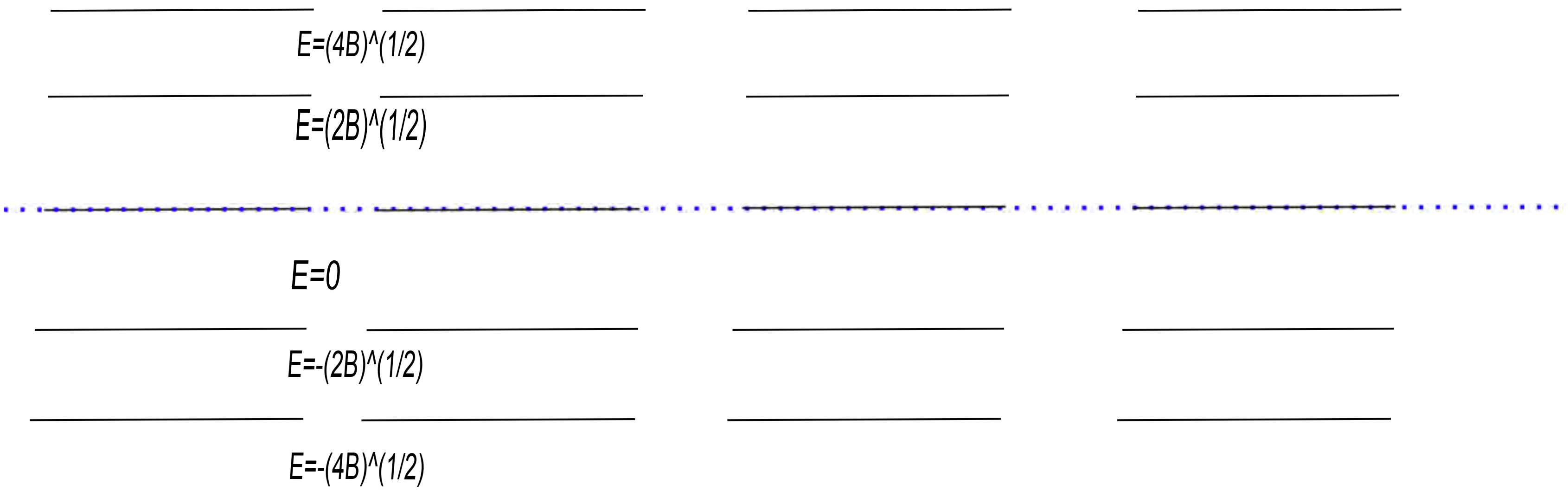
QHE data as a function of the gate voltage V_g, for B = 18 T at T = 0.25 K

Splitting of $\nu = 0$ Landau level A.F.Young et.al., Nat. Phys. 2012



Quantum Hall Ferromagnetism at Weak Coupling

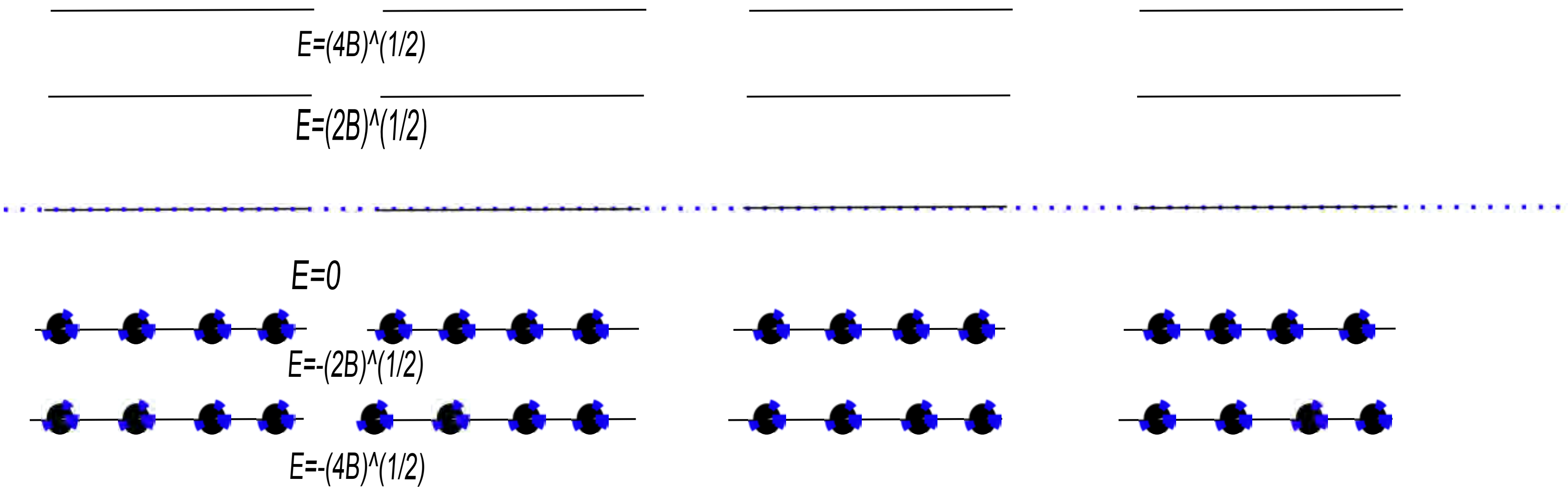
Consider a 4-fold degenerate spectrum of relativistic Landau levels



Quantum Hall Ferromagnetism at Weak Coupling

Consider a 4-fold degenerate spectrum of relativistic Landau levels

Ground state has negative energy levels filled

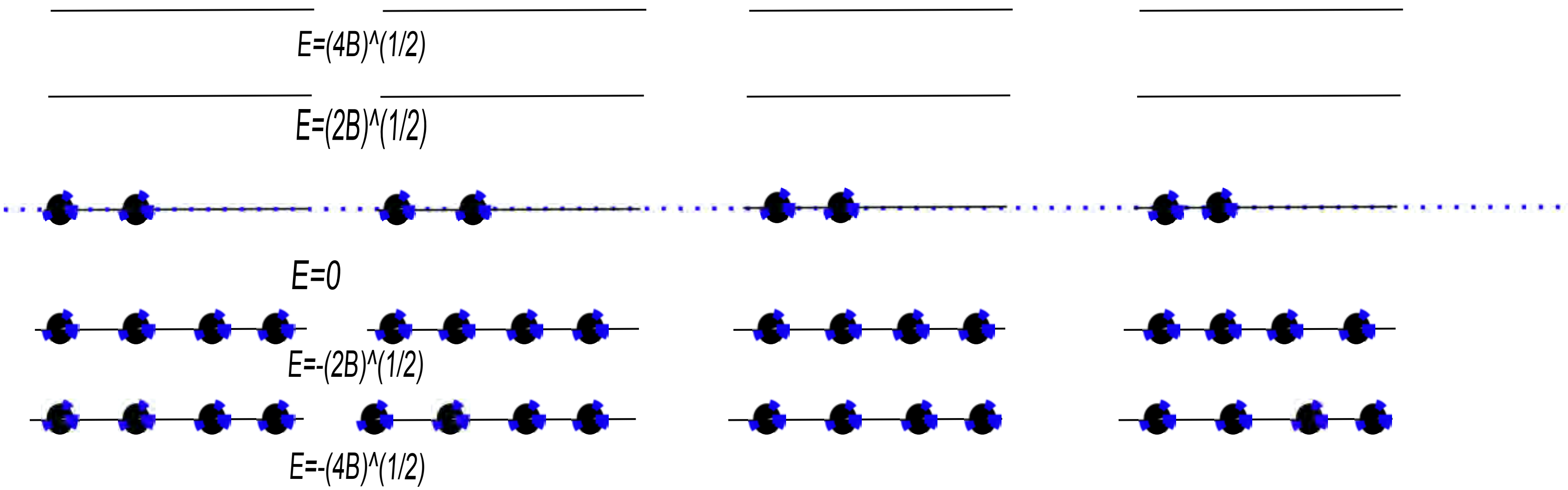


Quantum Hall Ferromagnetism at Weak Coupling

Consider a 4-fold degenerate spectrum of relativistic Landau levels

Ground state has negative energy levels filled

The zero energy states should be half-filled



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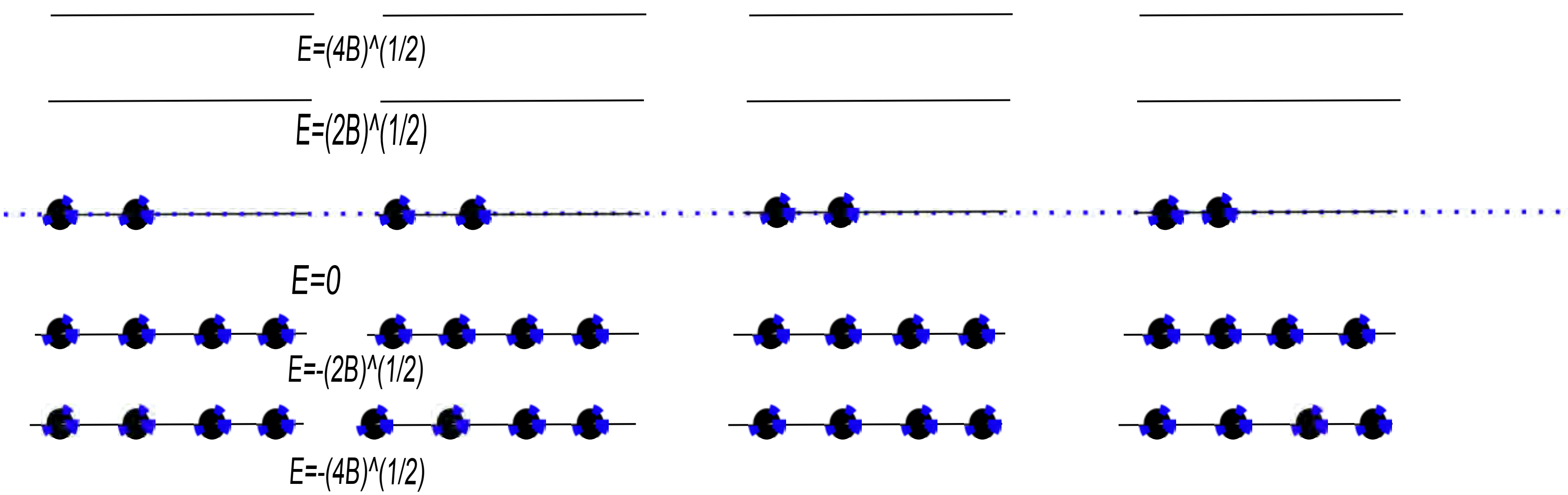
Highly degenerate ground state



Quantum Hall Ferromagnetism at Weak Coupling

Consider a 4-fold degenerate spectrum of relativistic Landau levels

Ground state has negative energy levels filled

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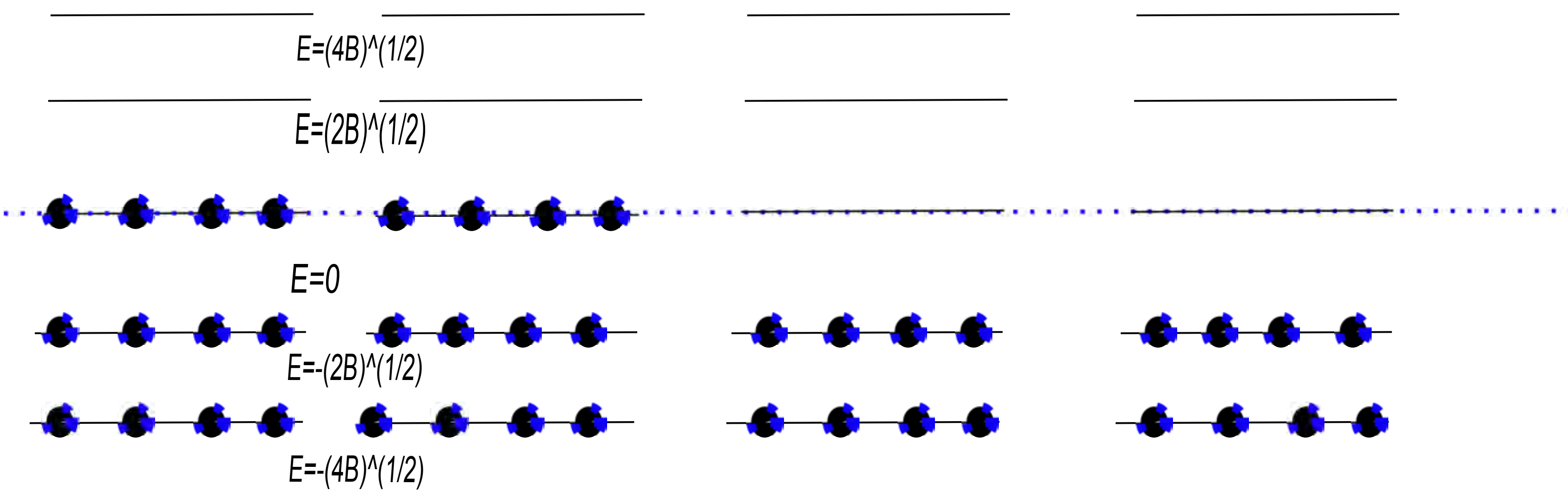
Highly degenerate ground state

Interaction resolves degenerary



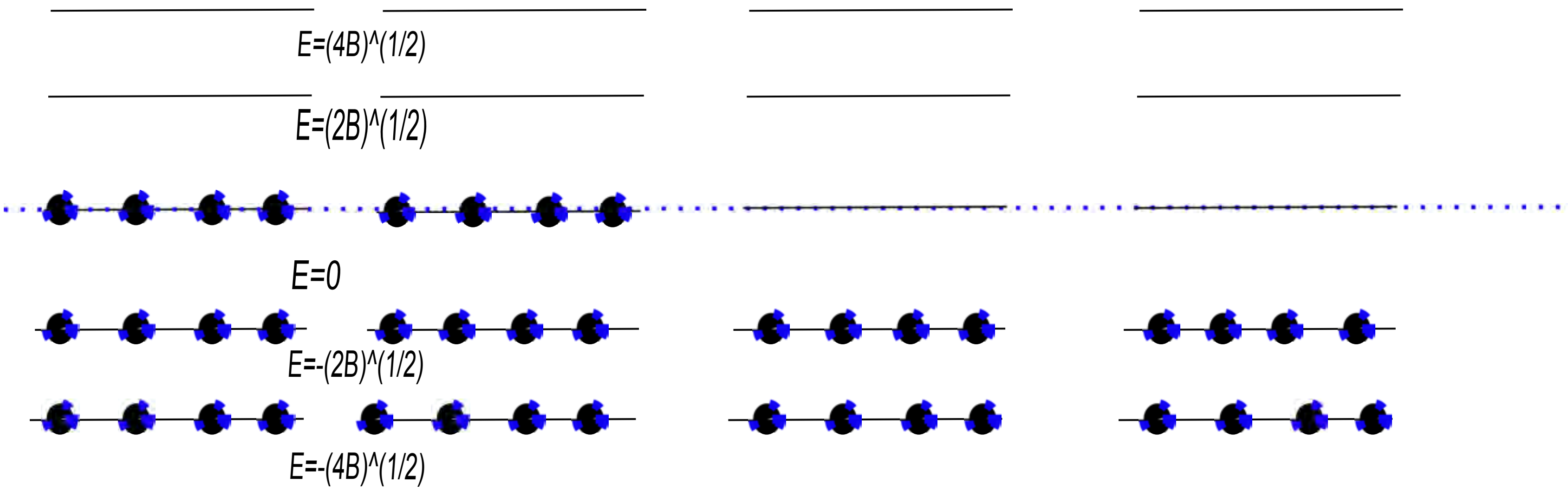
Quantum Hall Ferromagnet at Weak Coupling:

Spontaneous breaking $U(4) \to U(2) \times U(2)$

Coulomb interaction

$H_{\text{Coulomb}} = \frac{1}{2} \int \psi^\dagger(r)\psi(r) \frac{e^2}{4\pi|\vec{r}-\vec{r}'|} \psi^\dagger(r')\psi(r')$

$$\rho = \langle \psi^\dagger \psi \rangle = \frac{B}{4\pi}(1,1,-1,-1) \quad , \quad \langle \bar{\psi}\psi \rangle = \frac{B}{4\pi}(1,1,-1,-1)$$



Graphene with Coulomb interaction $V(r) = \frac{e^2}{4\pi r}$

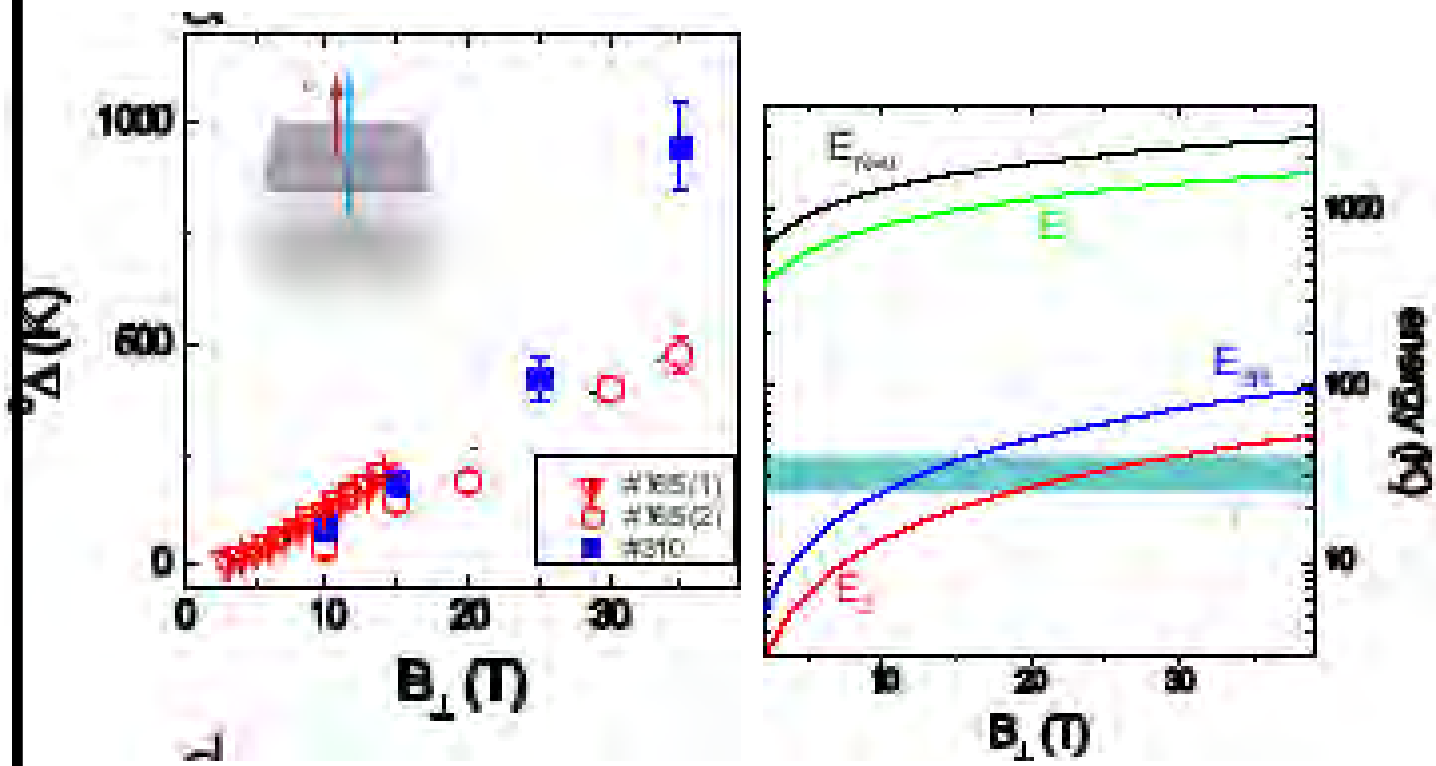
$$S = \int dtdxdy \sum_{k=1}^{4} \bar{\psi}_k \left[\gamma^t (i\partial_t - A_t) + v_F \vec{\gamma} \cdot (i\vec{\nabla} - \vec{A}) \right] \psi_k$$

$$+ \frac{1}{4e^2} \int dtdxdydz \left[\frac{1}{c} F_{0i}F_{0i} - cF_{ij}F_{ij} \right]$$

- The graphene fine structure constant

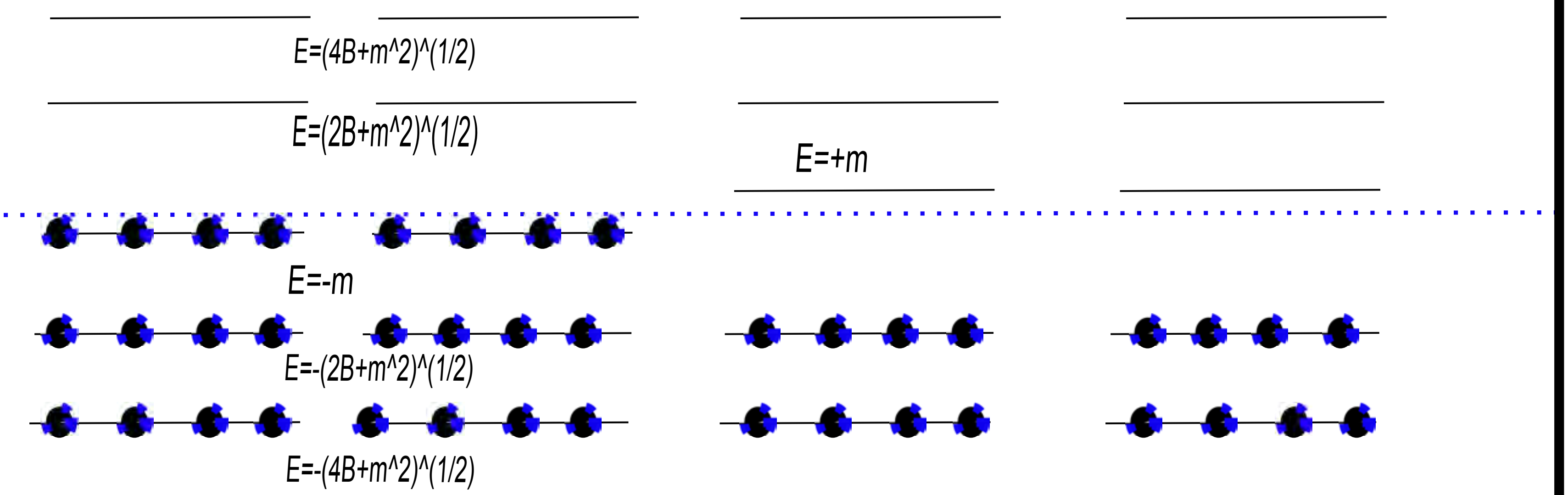
$$\alpha_{\text{graphene}} = \frac{\frac{e^2}{4\pi\lambda}}{\hbar v_F/\lambda} = \frac{e^2}{4\pi\hbar v_F} = \frac{e^2}{4\pi\hbar c}\frac{c}{v_F} \approx \frac{300}{137}$$

A.F.Young et.al., Nat. Phys. 2012



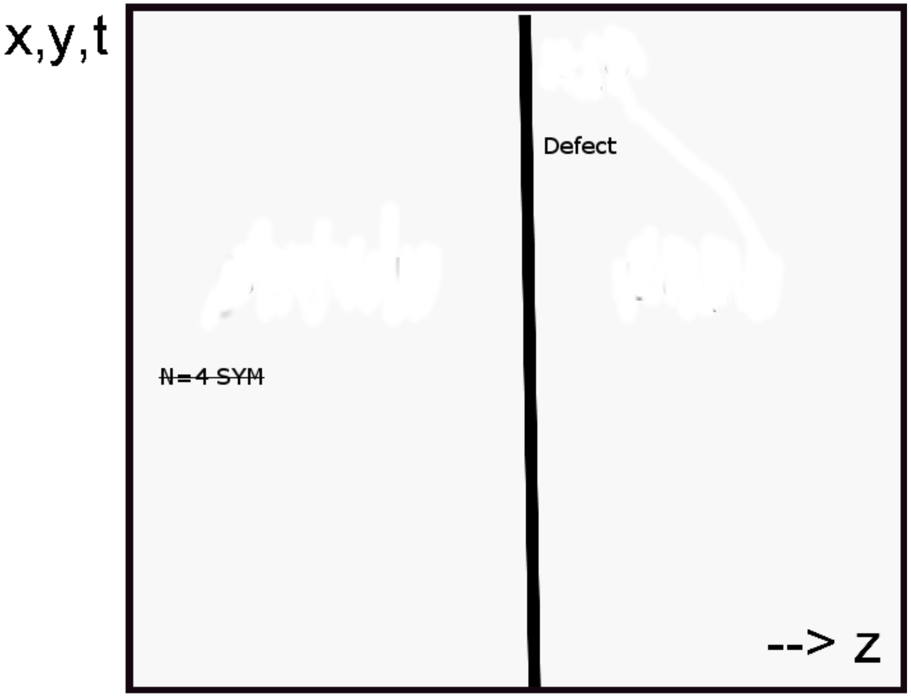
With a parity preserving mass term (breaking sublattice symmetry) F. Amet, J. R. Williams, K. Watanabe, T. Taniguchi, D. Goldhaber-Gordon, “Insulating Behavior at the Neutrality Point in Single-Layer Graphene” Physical Review Letters 110, 216601 (2013).

$U(4) \to U(2) \times U(2)$ chiral symmetry breaking



- **Can one use AdS/CFT holography to find a strong coupling realization of quantum Hall ferromagnetism?**
- **Top-down model: D3 - Probe D5 brane System**
- **Field Theory dual is known - deform with magnetic field, charge density, there will be quantum Hall ferromagnetism at weak couping**
- **At strong coupling we shall study the gravity dual $AdS_5 \times S^5$ with probe branes**
- **Mechanism for quantum Hall ferromagnetism with probe branes**
- **Properties of the ferromagnetic state: activation gap, conductivity, entropy**

Holographic Defect superconformal field theory



Bulk contains $\mathcal{N} = 4$ super Yang-Mills with SU(N) gauge group

Defect with “2DEG”

Introduce magnetic field, charge density, temperature.

Electronic properties

D3-D5 Defect superconformal field theory

- Field theory dual is bulk $\mathcal{N}=4$ Yang-Mills plus a hypermultiplet defect theory with SO(3)×SO(3) R-symmetry
  O.DeWolfe D.Z.Freedman H.Ooguri hep-th/0111135
  J.Erdmenger Z.Guralnik I.Kirsch hep-th/0203020

$$S = \int d^4x \left\{ -\frac{1}{2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \ldots \right\}$$

$$+ \int d^3x \sum_{\sigma=1}^{N_5} \sum_{\alpha=1}^{N} \left[\bar{\psi}^\sigma_\alpha i\gamma^\mu \partial_\mu \psi^\sigma_\alpha + \partial_\mu \bar{\varphi}^\sigma_\alpha \partial^\mu \varphi^\sigma_\alpha \right] + \text{interactions}$$

- Fermion ψ, scalar φ are SO(3) spinors (with different SO(3)'s), fundamental rep. of global $U(N_5)$ and fundamental rep. of SU(N) gauge group.

- Holographic description introduces temperature T, $U(1) \subset U(N_5)$ charge density ρ, magnetic field B

Weak Coupling

$$S = \int d^3x \sum_{\sigma=1}^{N_5} \sum_{\alpha=1}^{N} \left[\bar{\psi}^\sigma_\alpha i\gamma^\mu D_\mu \psi^\sigma_\alpha + D_\mu \bar{\varphi}^\sigma_\alpha D^\mu \varphi^\sigma_\alpha \right] + \text{interactions}$$

External Magnetic field

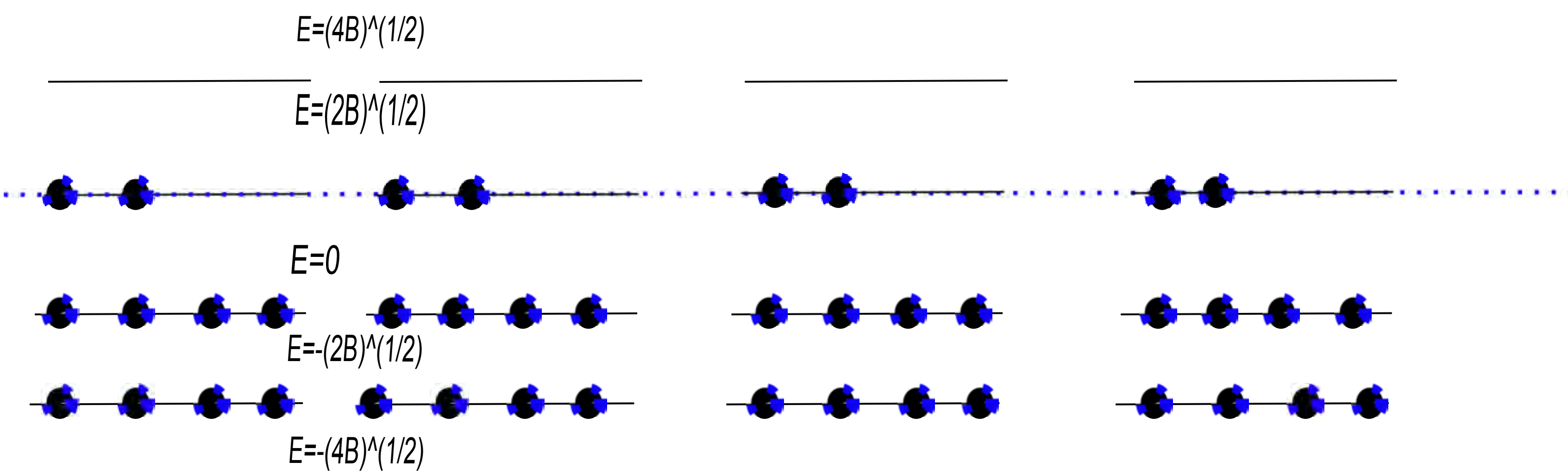
- $D_\mu = \partial_\mu + iA_\mu$ with a background magnetic field $\vec{\nabla} \times \vec{A} = B$
- Landau levels
  - Fermions $E_n = \sqrt{2Bn}$
  - Boson $\omega_n = \sqrt{(2n+1)B}$, $n = 0, 1, 2, ...$
- Magnetic scale $\sqrt{B}$
- supersymmetry is broken
- Lowest energy states are fermion zero modes
- The lowest energy non-zero modes are scalars.

Landau levels

fermions are $N_5 = 2$ $SO(3)$ doublets

Yang Mills at large N $\sim$Coulomb

Dynamical problem similar to graphene→gaps at $\nu = -2, -1, 0, 1, 2$



Hall States

The gapped states have charge densities and Hall conductivities

$$\nu \equiv \frac{2\pi}{N}\frac{\rho}{B} \; = \; 0, \pm 1, \pm 2, \ldots, \pm N_5$$

$$\sigma_{xy} = \frac{e^2 N}{h} \cdot (0, \pm 1, \pm 2, \pm 3, \ldots, \pm N_5)$$

All other quantum Hall states are beyond the threshold for creating scalars.

Do the quantum Hall states survive at strong coupling?

Hall States

The gapped states have charge densities and Hall conductivities

$$\nu \equiv \frac{2\pi}{N}\frac{\rho}{B} \;=\; 0, \pm 1, \pm 2, \ldots, \pm N_5$$

$$\sigma_{xy} = \frac{\lambda}{h} \cdot (0, \pm 1, \pm 2, \pm 3, \ldots, \pm N_5)$$

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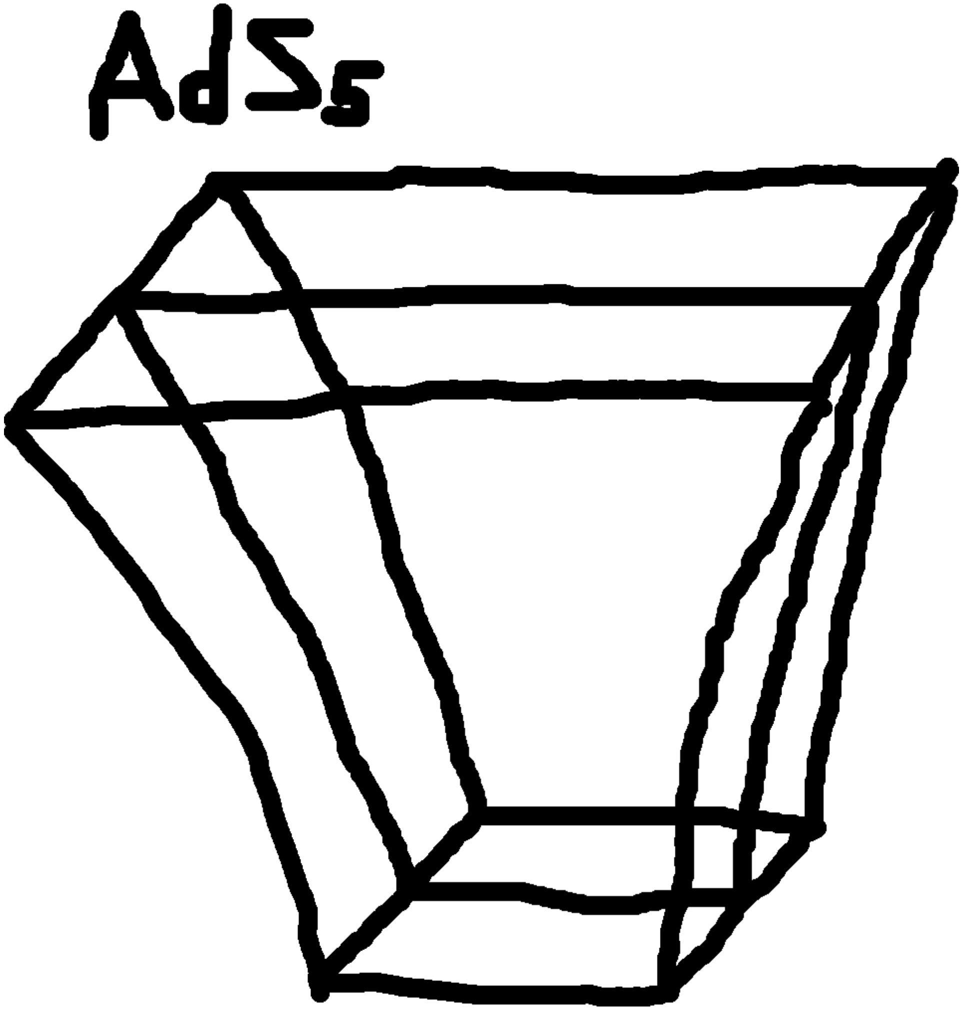
D3 - Probe D5 brane System



There are TWO solutions of the D5 brane:

1. D5 brane suspended in AdS_5 *with certain boundary conditions at boundary of* AdS_5
2. D7 brane which pinches down to a D5 brane at the boundary of AdS_5 *and identical boundary conditions to the D5 brane*
3. **There is a phase transition**
    - (a) D5 is stable when $0 \leq \nu <\sim 0.4$, never incompressible for any $\nu \neq 0$
    - (b) D7 is stable when $\sim 0.4 \leq \nu \leq 1$
    - (c) When $\nu = 1$ D7 has a charge gap, $\sigma_{xy} = \frac{e^2}{h}$
    - (d) When $\nu > 1$, gapped D7 + un-gapped D5 is stable
    - (e) in $\nu \in [1, 2]$ phase transition from (gapped D7+ un-gapped D5) to (gapped D7 + un-gapped D7)
    - (f) $\nu = 2$ two charge-gapped D7 branes with $\sigma_{xy} = 2\frac{e^2}{h}$ ...

D3 - Probe D5 brane System

- N coincident D3 branes and N_5 D5 branes oriented as

	0	1	2	3	4	5	6	7	8	9
$D3$	X	X	X	X	O	O	O	O	O	O
$D5$	X	X	X	O	X	X	X	O	O	O
$D7$	X	X	X	O	X	X	X	X	X	O

  brane extends in directions X, sits at point in directions O

- $\#ND = 4$ system – preserves $1/2$ of supersymmetries
- 't Hooft limit: $N \to \infty$, $\lambda = 4\pi g_s N$ fixed: D3's $\to AdS_5 \times S^5$
- probe limit $N_5 << N$ embed D5's in $AdS_5 \times S^5$
- flat space $\sim$ strong coupling $R^2 = \sqrt{\lambda}\alpha' >> 1$
- "2DEG" = D3-D5 strings - fund. reps. of $SU(N)$, $U(N_5)$

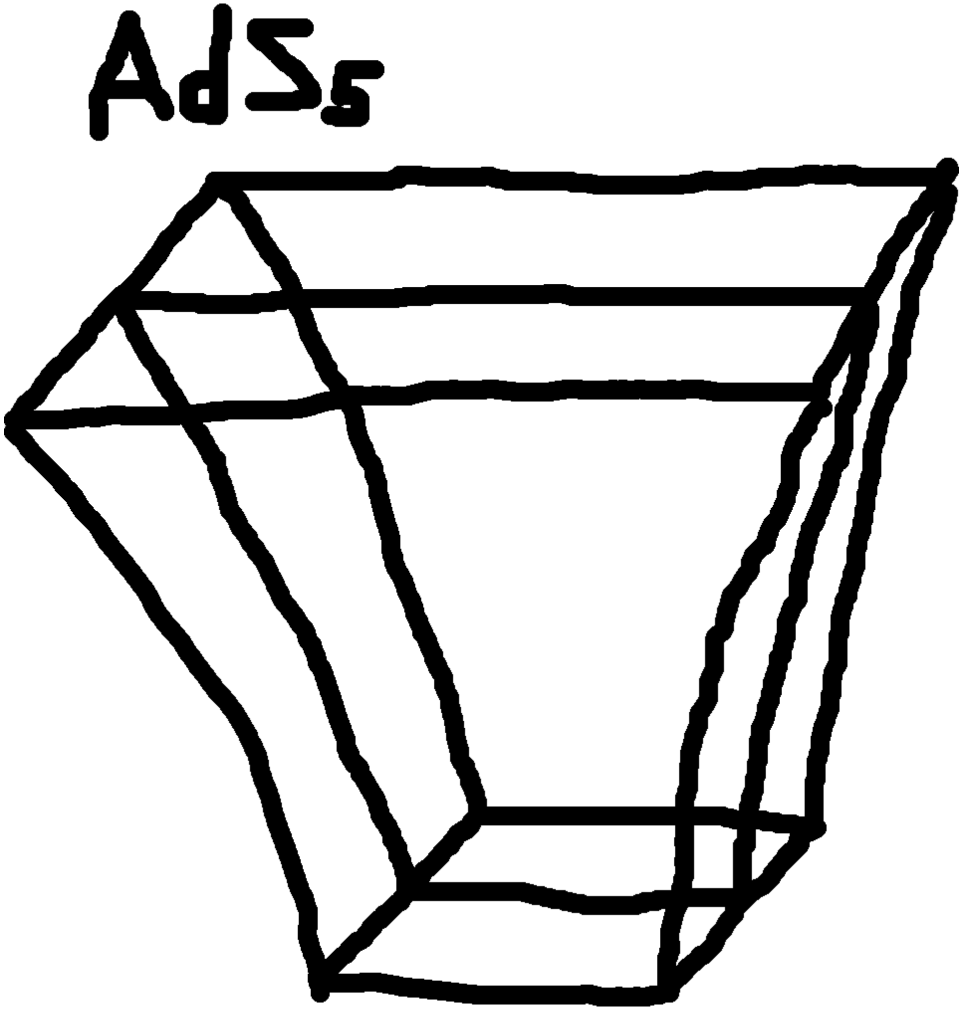
Probe D5 brane

Probe brane geometry from solving Dirac-Born-Infeld action plus Wess-Zumino terms

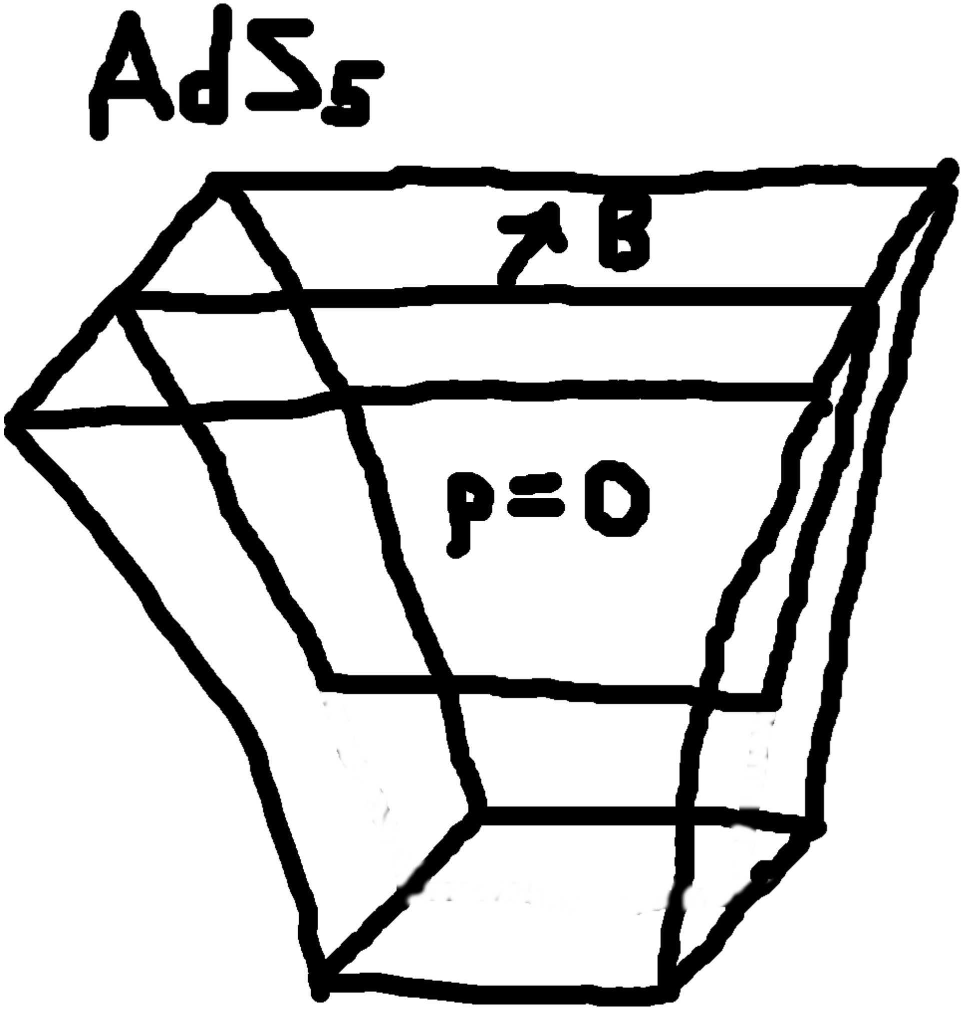
$$S_5 = N_5 T_5 \int d^6\sigma \left[-\sqrt{-\det(g + 2\pi\alpha' F)} + 2\pi\alpha' F \wedge \omega^{(4)} \right]$$

$$S_7 = T_7 \int d^8\sigma \left[-\sqrt{-\det(g + 2\pi\alpha' F)} + \frac{(2\pi\alpha')^2}{2} F \wedge F \wedge \omega^{(4)} \right]$$

$B = 0$, $\nu = 0$ **D5 brane**

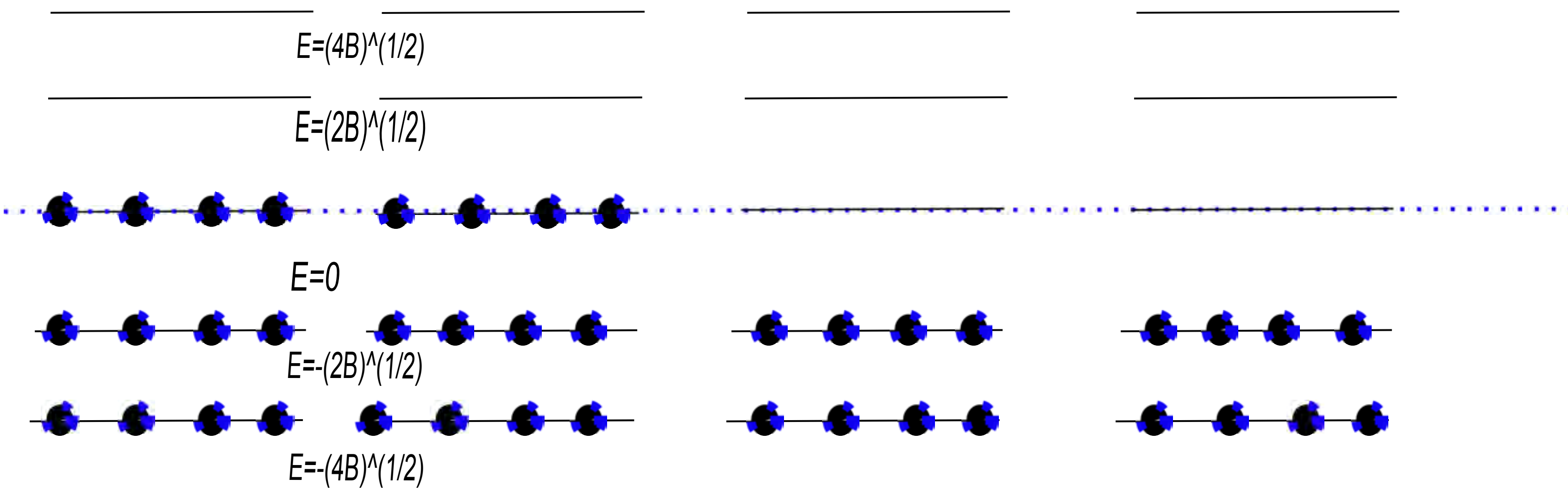


D5 brane with B turned on



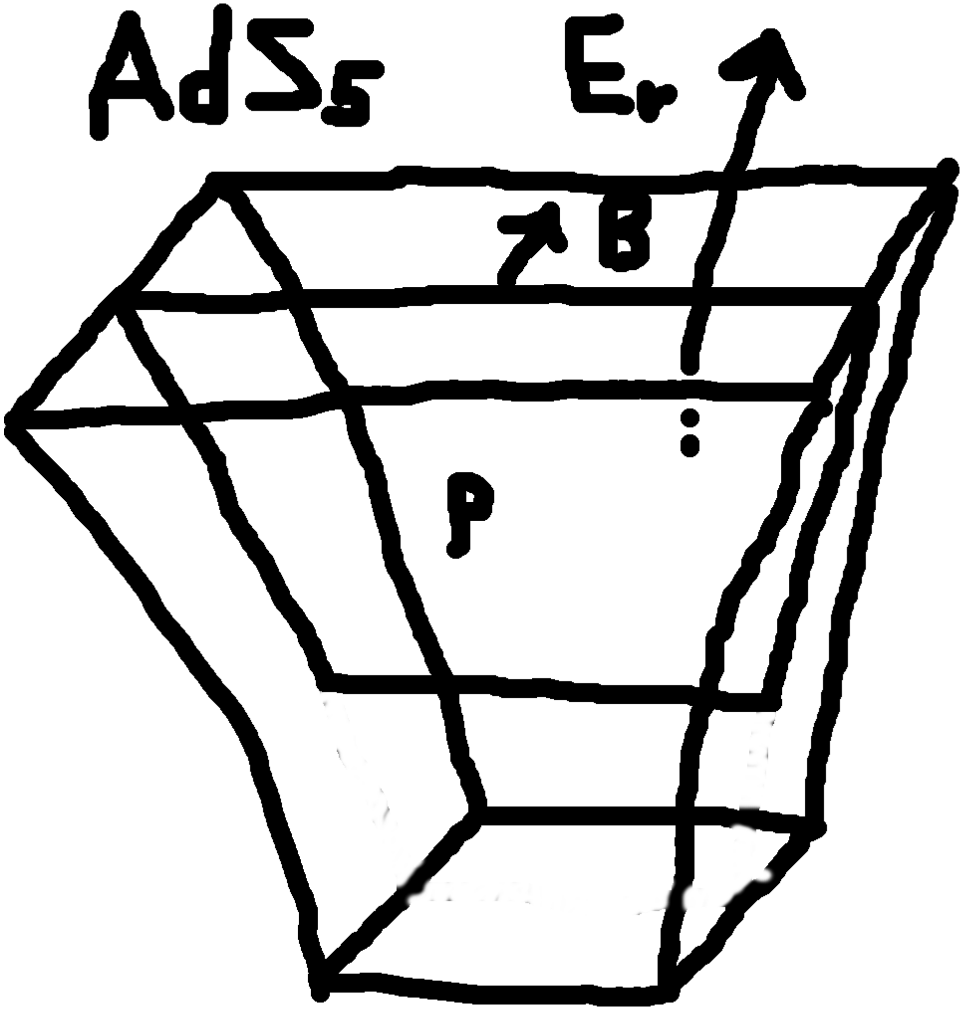
V.Filev C.Johnson J.Shock arXiv:0903.5345

Two times this charge neutral, gapped D5 brane is the strong coupling limit of this state:

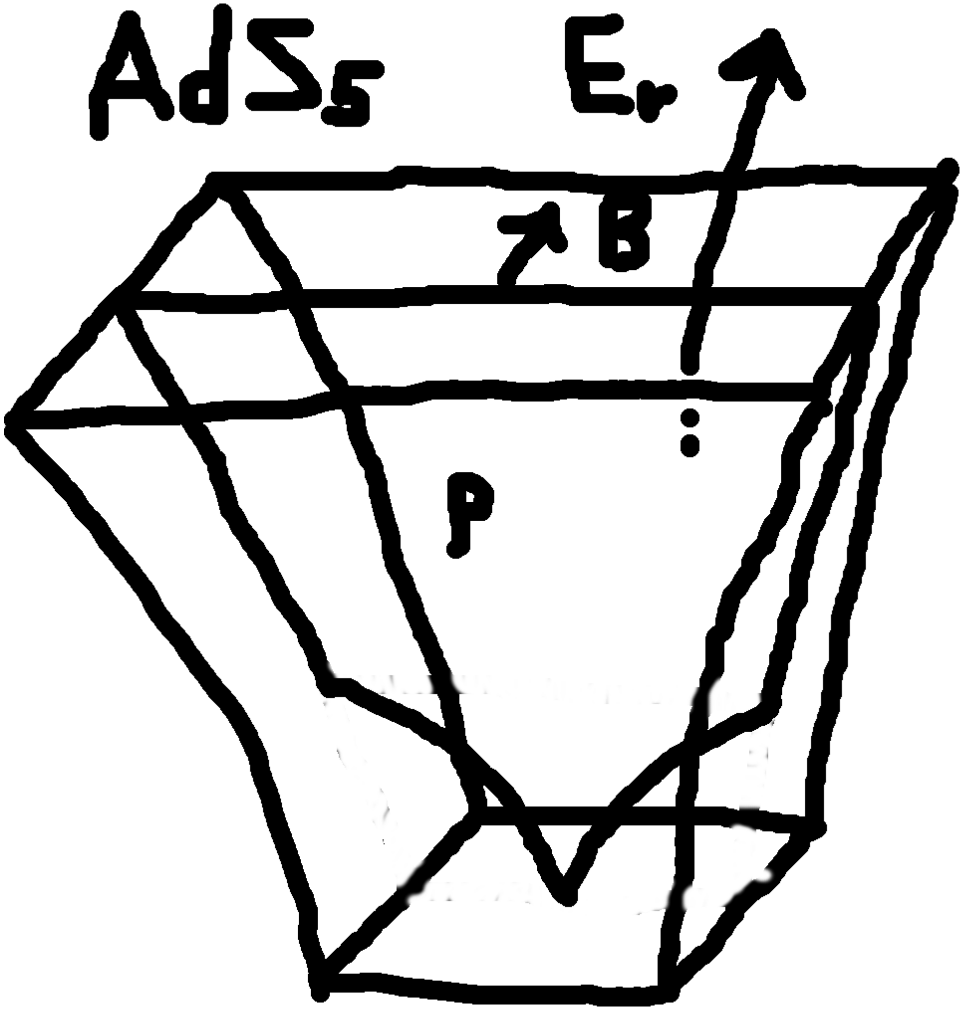


gap $\sim \; \frac{\sqrt{\lambda}}{2\pi}(0.4)\sqrt{B}$

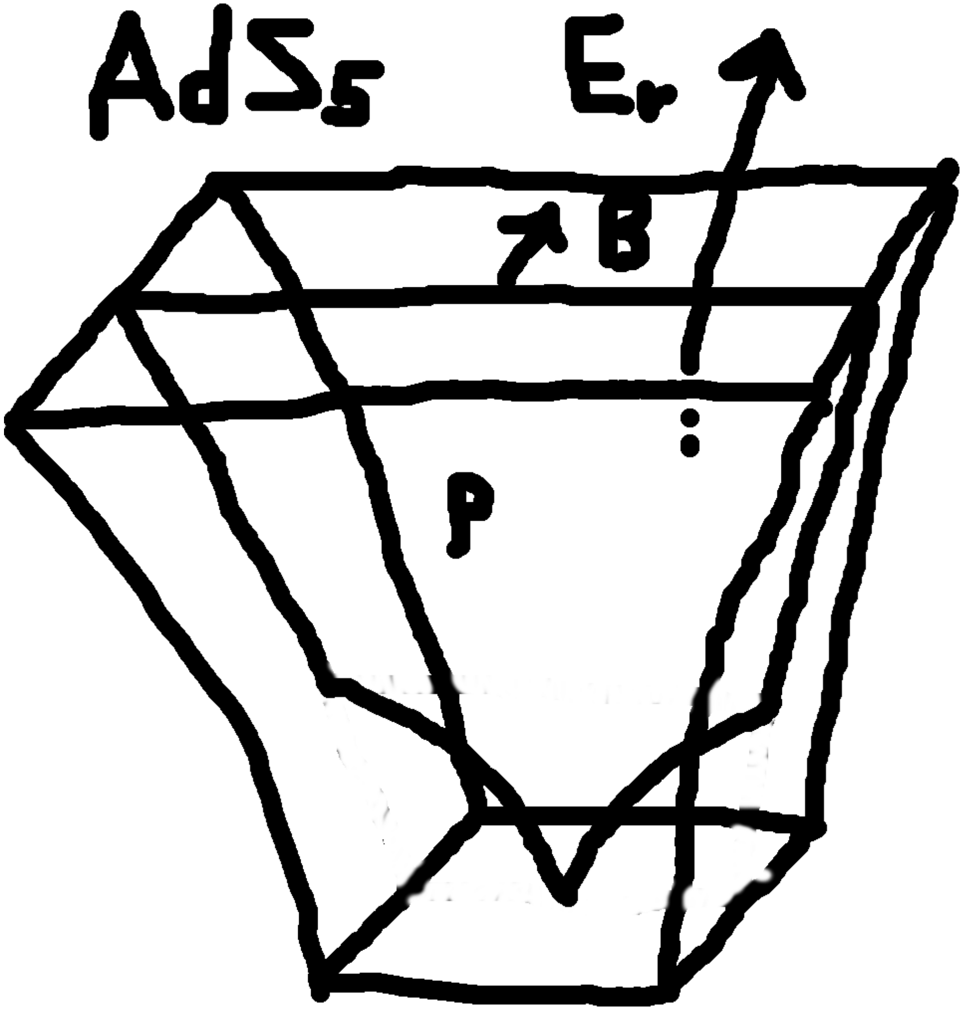
D5 brane with B turned on and $\rho < \rho_c$ turned on



D5 brane with B turned on and $\rho < \rho_c$ turned on can never have a charge gap



B and $\rho > \rho_c$ turned on: D5 replaced by D7 brane



World Volume Axion

$$S \sim \int \sqrt{\det(g + 2\pi\alpha' F)} + \int (2\pi\alpha')^2 F \wedge F \wedge C^{(4)}$$

Maxwell equations on the brane worldvolume ($\approx AdS_4$) have an axion term

$$\frac{d}{dr}\left[\frac{\partial}{\partial E_r}\sqrt{\det(g + 2\pi\alpha' F)} + \frac{1}{2\pi^2} B\Theta(r)\right] = 0$$

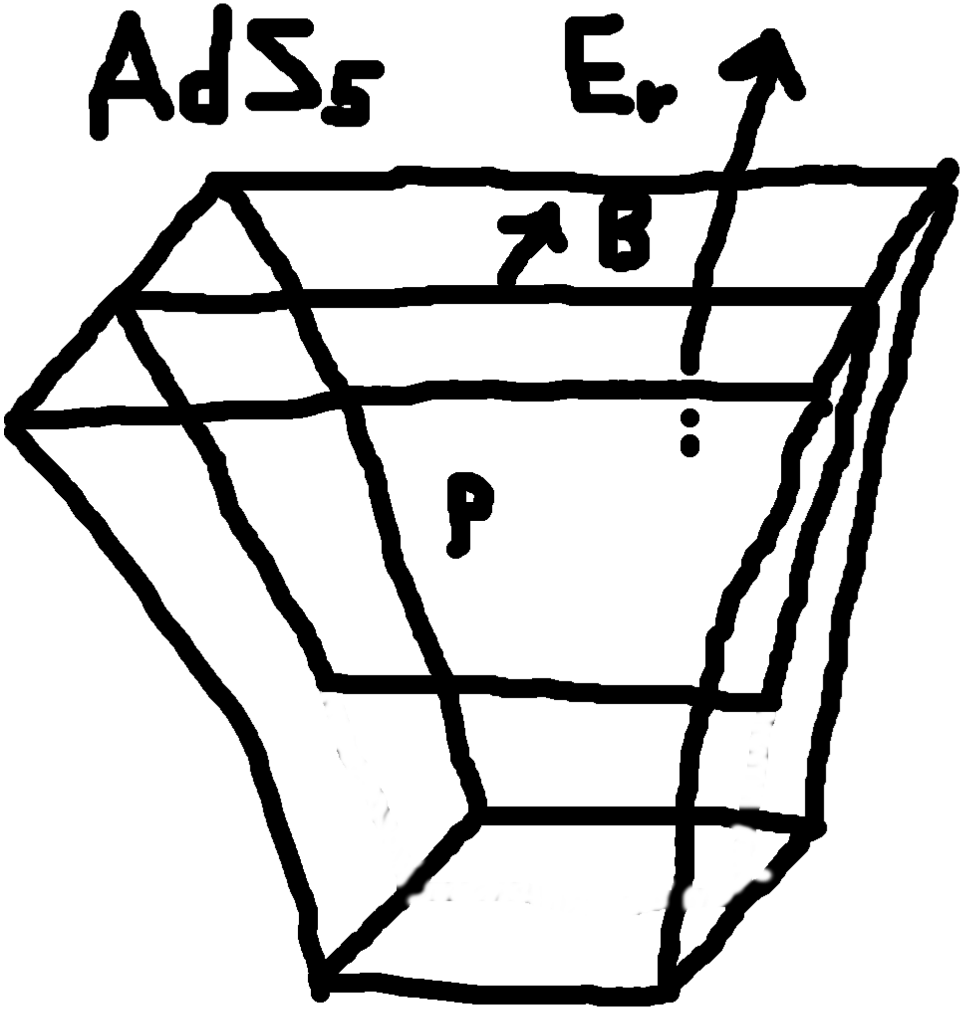
$$\frac{\partial}{\partial E_r}\sqrt{\det(g + 2\pi\alpha' F)} + \frac{1}{2\pi^2} B\Theta(r) = \rho$$

Gapped solution when

$$\rho \sim B \quad , \quad \nu = 1$$

$$\Theta(r) = \int C^{(4)}$$

D7 brane has incompressible state when $\nu = 1$



Conclusions

- $\exists$ quantum Hall ferromagnetism in the D5 brane
- $\nu = 0, \pm 1, \pm 2, ..., \pm N_5$ number of states which survive is not known
- When ν divides N_5, the Hall state has ν identical D7's → SU(ν) symmetry
- Activiation gap $\sim \frac{\sqrt{\lambda}}{2\pi}(0.4)\sqrt{B}$
- Other integer Hall states, fractional Hall states?
- D7 would be more like graphene
  O.Bergman, N.Jokela, G.Lifschytz, M.Lippert, arXiv:1003.4965
- Are there further instabilities? e.g. striped phase (D7$'$)
  O.Bergman N.Jokela G.Lifschytz M.Lippert arXiv:1106.3883