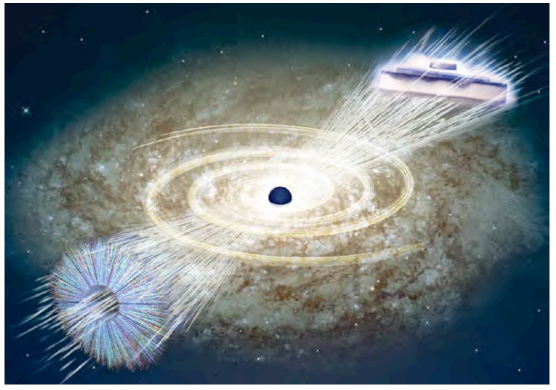
Holographic BCS

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University



Netherlands Organisation for Scientific Research

Kolymbari, Aug 2014

Andrey Bagrov

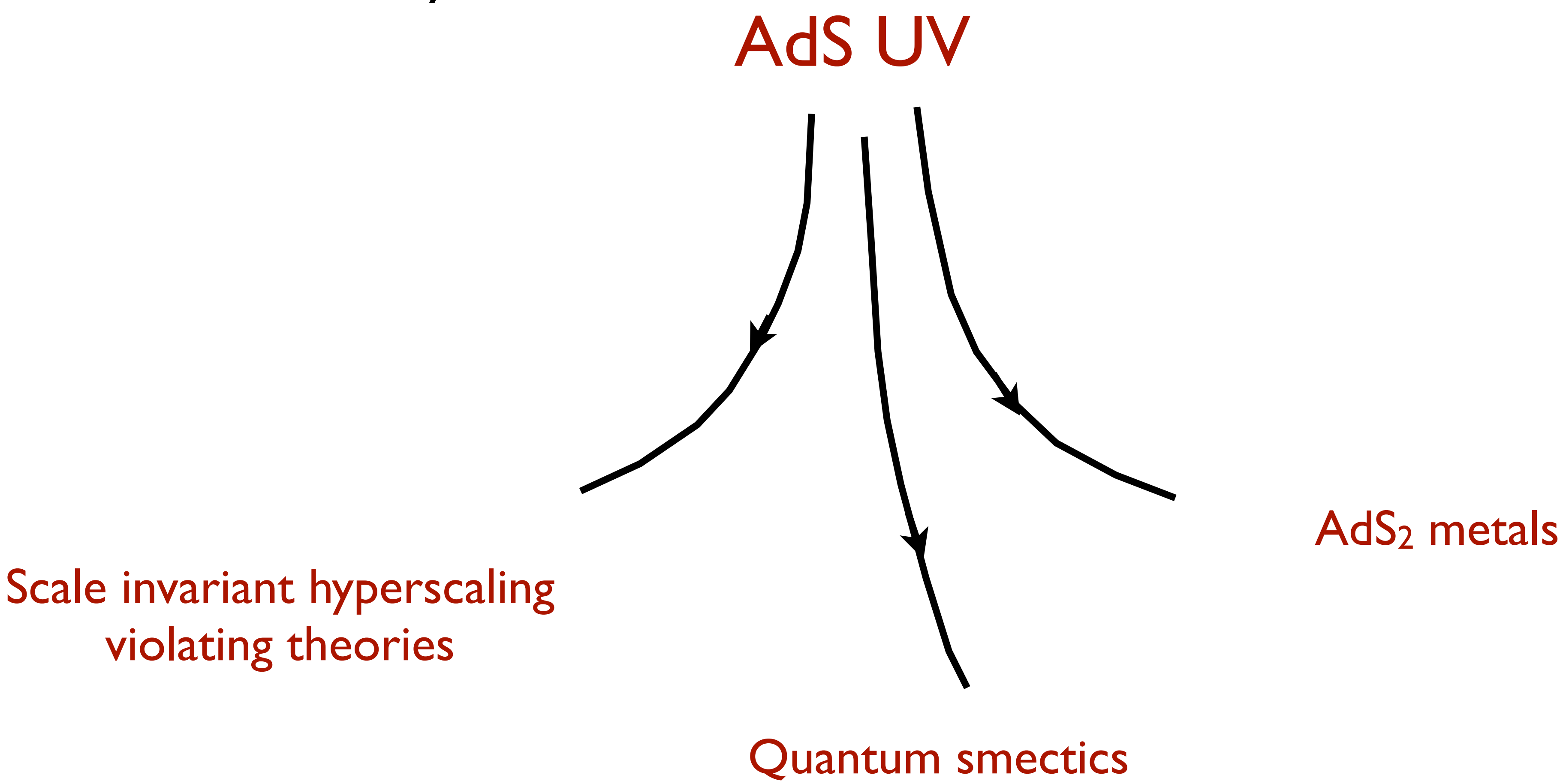
Balazs Meszena

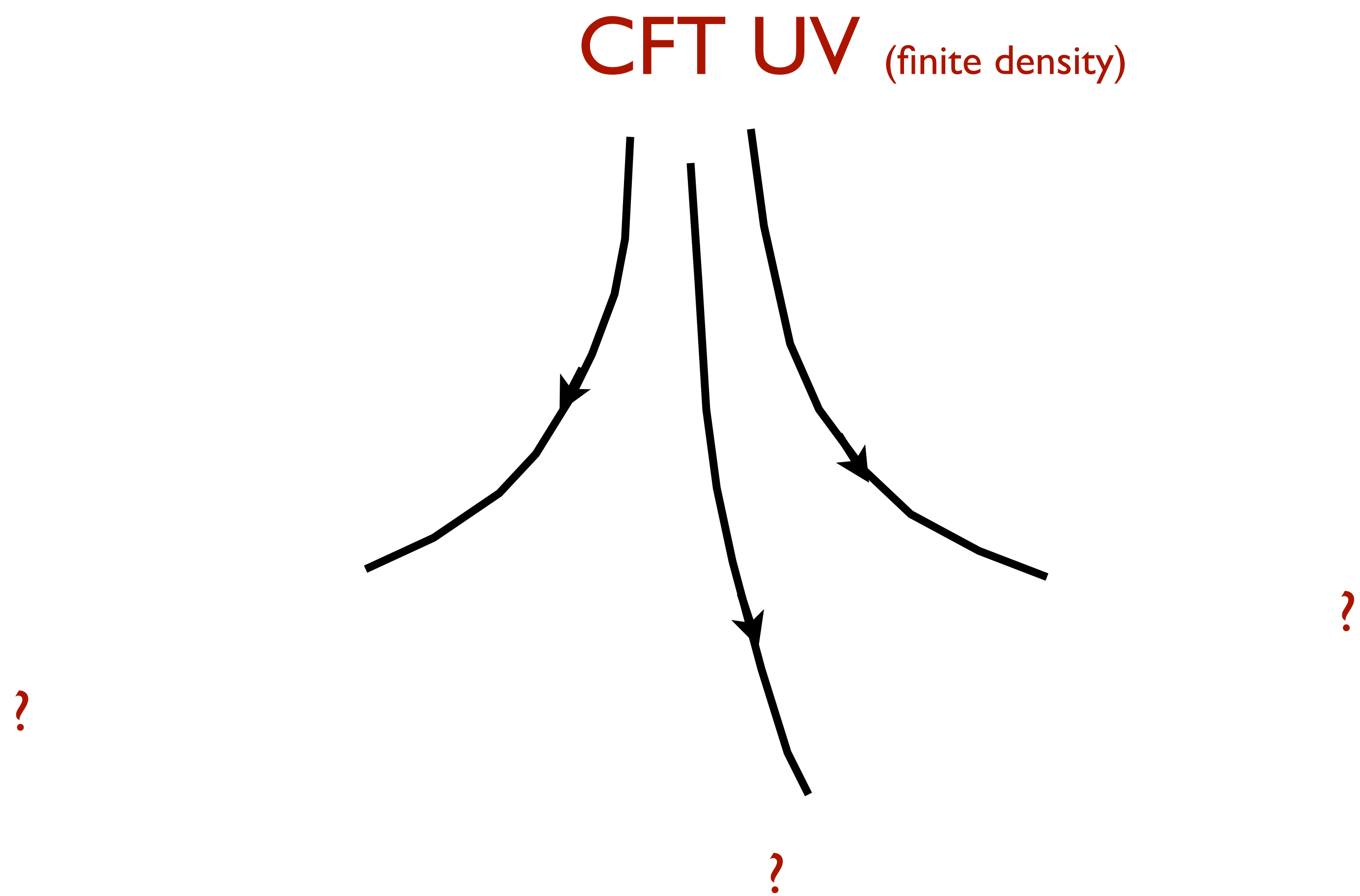
Yan Liu

Ya-Wen Sun

Jan Zaanen

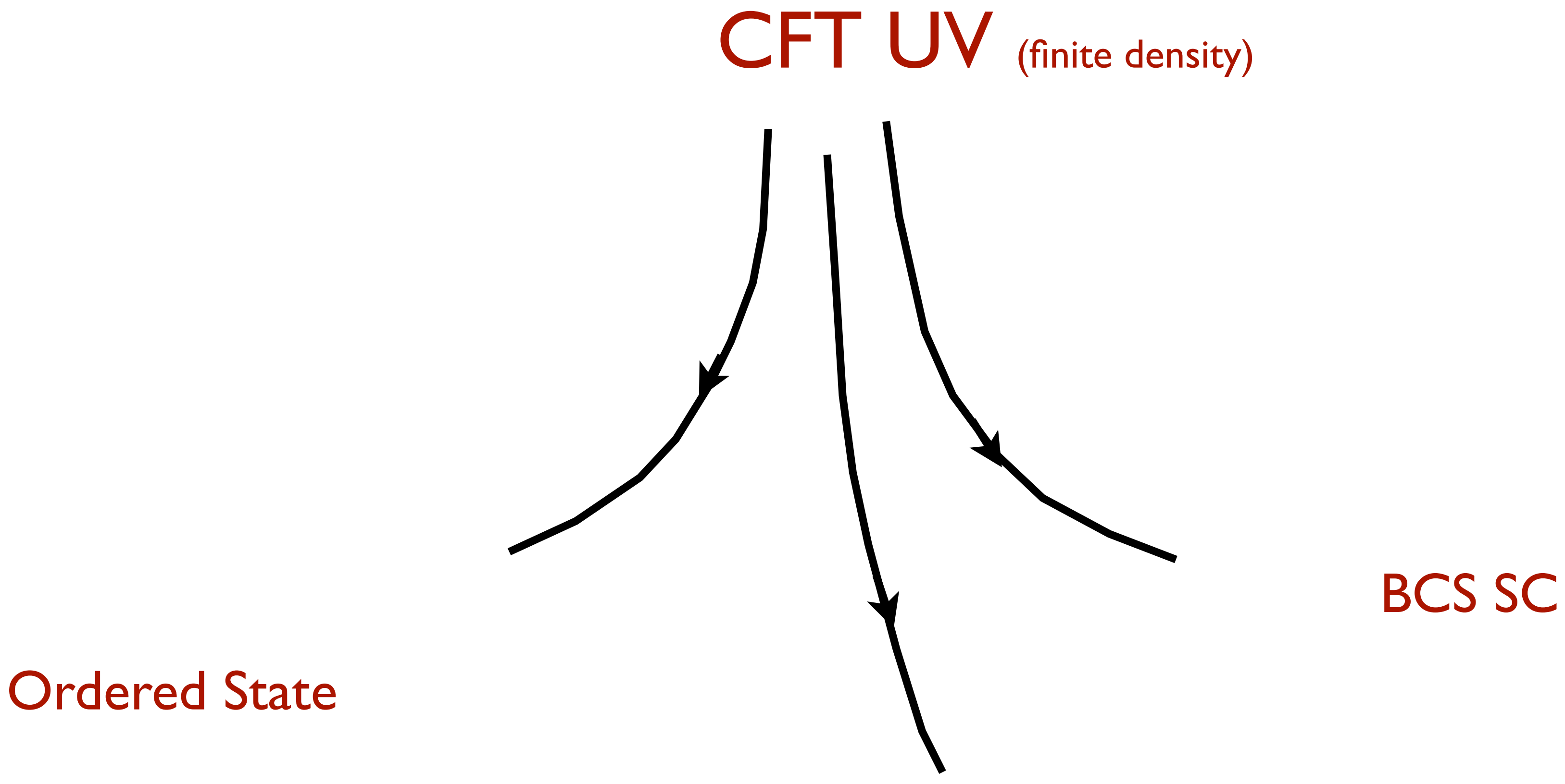
- AdS/CFT: new insight into strongly coupled systems at finite density:




CFT UV (finite density)
?
?
?

Quantum Condensed Matter Physics - Lecture Notes

Chetan Nayak


CFT UV (finite density)
Ordered State
Fermi liquid
BCS SC

- AdS/CFT is very non-generic

Quantum Condensed Matter Physics - Lecture Notes

Chetan Nayak

Holographic Superconductor

Hartnoll, Herzog, Horowitz

(Quantum) Electron Star

Hartnoll, Tavanfar

Goal here:

Pairing induced superconductivity in holography

Standard CMT vs Holography

- The Holographic Superconductor

AdS/CFT dictionary

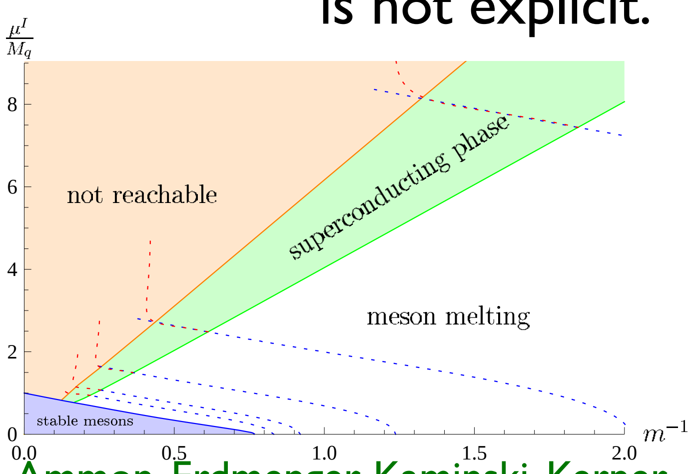
$$\mathcal{O} = \mathrm{Tr}\psi\psi$$

Order parameter is a composite of fundamental fields

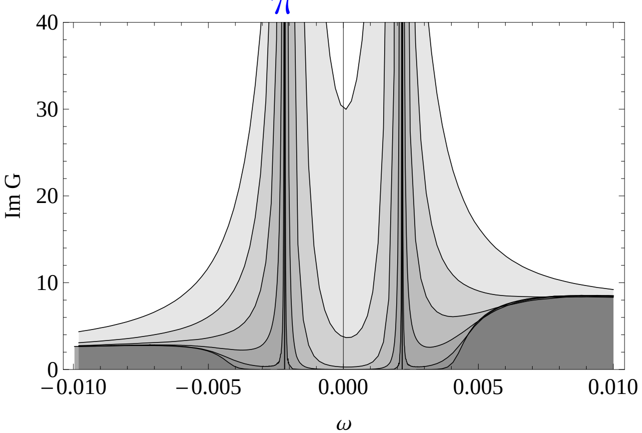
Familiar from BCS theory

In holography the pairing mechanism is not explicit.

$$\mathcal{A} = \frac{1}{\pi}\mathrm{Im}\langle\Psi\Psi\rangle_R$$



Ammon, Erdmenger, Kaminski, Kerner



Faulkner, Horowitz, McGreevy, Roberts, Vegh

Bosons and Fermions together

- AdS Einstein-Maxwell, scalars, fermions, Yukawa

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - 2iqA_\mu)\phi|^2 - m_\phi^2|\phi|^2$$
$$-i\bar{\Psi}(\Gamma^\mu(\partial_\mu - iqA_\mu) - m_\Psi)\Psi + \eta_5^*\bar{\phi}\bar{\Psi}^C\Gamma^5\Psi + \eta_5\phi\bar{\Psi}\Gamma^5\Psi^C$$

$$q_b = 2q_f$$

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$$q_b = 2q_f$$

  - Backgrounds:

AdS-RN/ Non-Fermi liquid AdS_2 metal	Cooper instability (absent for NFL)	Hartman, Hartnoll
Holographic Superconductor	BCS gap in fermion spectral function	Faulkner, Horowitz, McGreevy, Roberts, Vegh
Holographic Fermi liquid	*BCS instability and resulting background*	

- The Holographic Fermi Liquid

$$\text{AdS/CFT dictionary}$$

$$\Psi = \mathrm{Tr}\psi\phi$$

The fermion is a composite of fundamental fields.

For energies $E \ll E_{bind}$ composite operator acts a fundamental field.

Familiar from neutron stars.

Following textbook CMT this Fermi-liquid should have a BCS instability

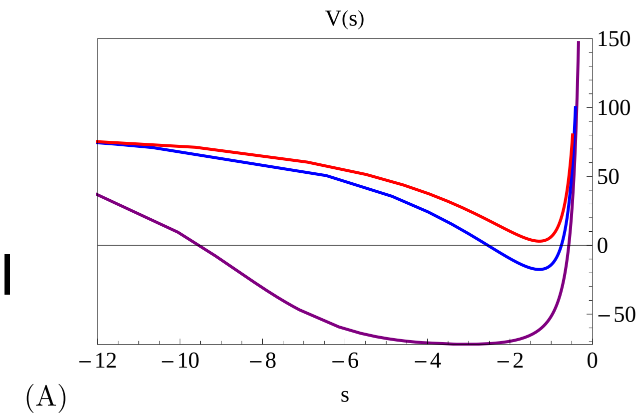
Along the way

- BCS theory in a box with self-consistent screening
- BCS theory in the fluid limit (Thomas-Fermi)

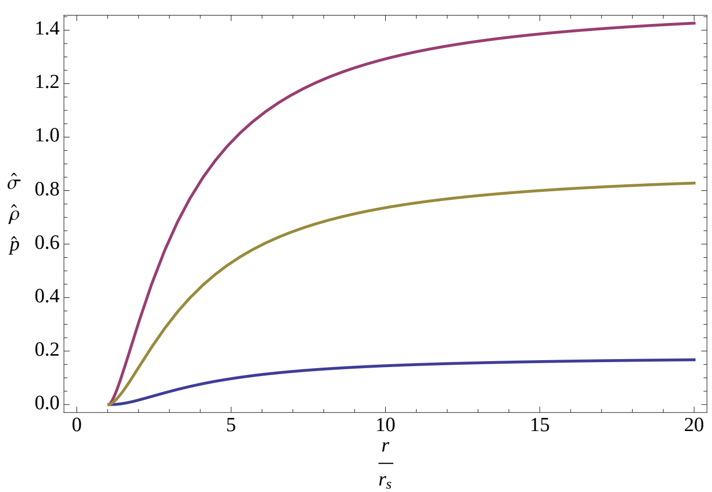
- Natural BEC-BCS crossover in holography

Hartnoll, Tavanfar;
Hartnoll, Hofman, Vegh;
Iqbal, Liu, Mezei;
Cubrovic, Liu, KS, Sun, Zaanen

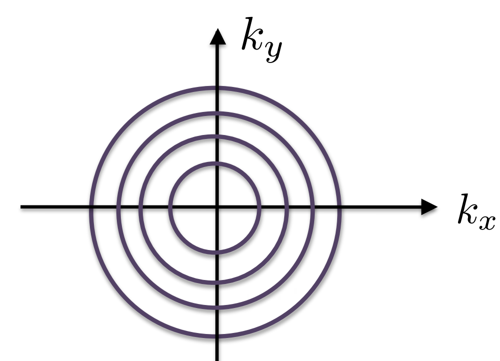
- The (fluid) Electron Star
  - Self-gravitating matter experiences a (self-consistent) box-potential
  - Matter are bound states in this potential
  - Thomas-Fermi approx:



sum over levels becomes an integral: level spacing goes to zero.



In the TF limit, star has an edge

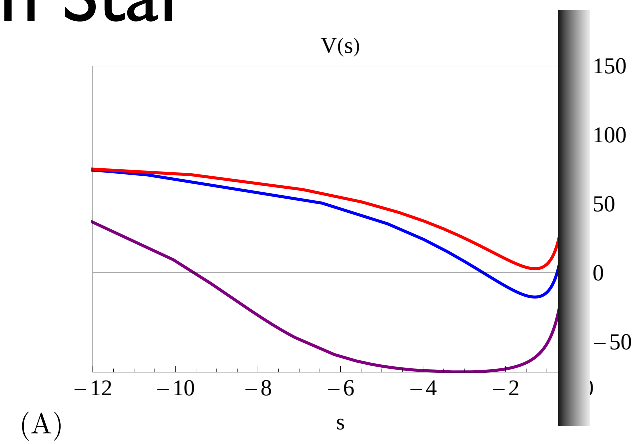


In Holography, each radial mode corresponds to a Fermi surface

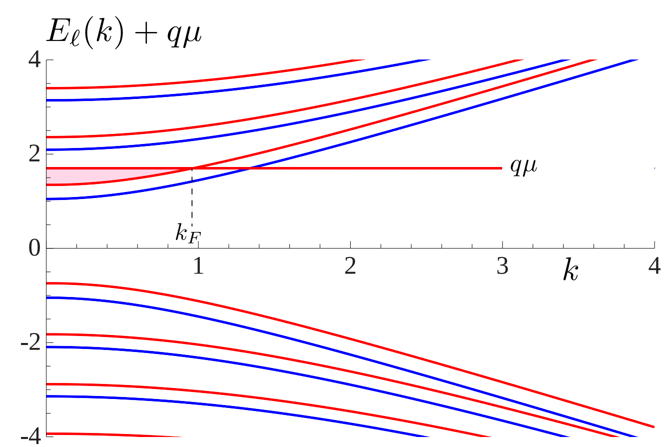
Sachdev

- The Hard wall quantum Electron Star
  - Artificial Hard wall box-potential

    Ignores the true IR



  - Select a single Fermi surface



Bosons and Fermions together

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Holographic Fermi liquid	*BCS instability and resulting background*	

- The (relativistic) BCS Lagrangian

$$\mathcal{L} = \qquad\qquad - m_\phi^2 |\phi|^2$$

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$$q_b = 2q_f$$

  - Nambu-Gorkov formulation

$$\Psi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \qquad \mathcal{L} = (\bar{\chi}_1, \chi_2^T C) \begin{pmatrix} \not{D} & \eta_5\phi \\ -\eta_5^*\bar{\phi} & \not{D} \end{pmatrix} \begin{pmatrix} \chi_1 \\ C\bar{\chi}_2^T \end{pmatrix} - m_\phi^2|\phi|^2$$

The EOM for ϕ = The BCS Gap equation

$$\phi = \frac{\eta_5^*}{m_\phi^2}\langle \bar{\Psi}^C \Gamma^5 \Psi \rangle_\phi$$

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$$\mathcal{L} = \qquad - m_\phi^2|\phi|^2$$

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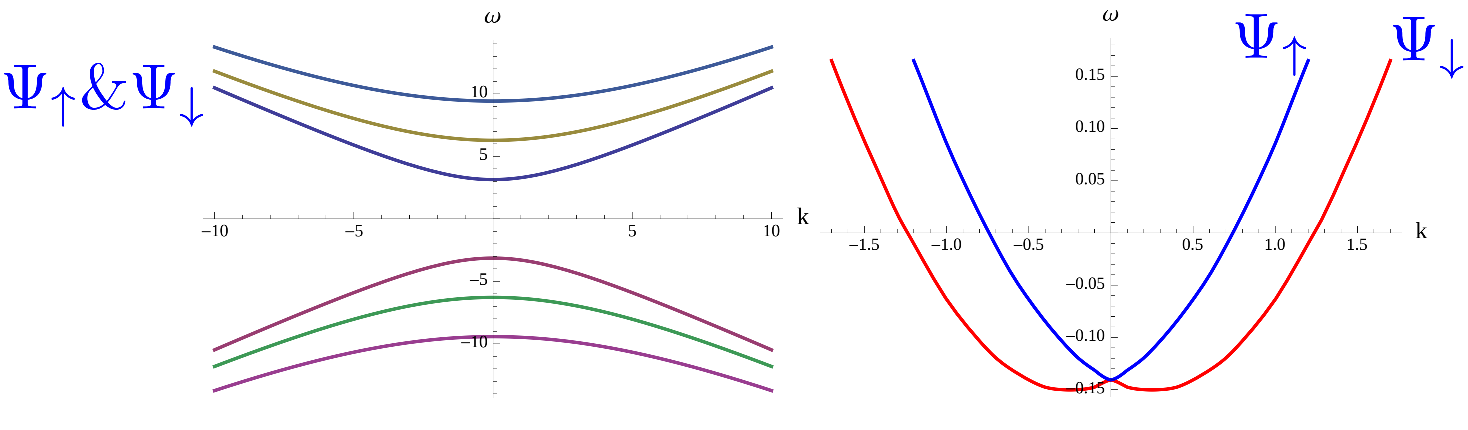
The EOM for ϕ = The BCS Gap equation

$$\phi = \frac{\eta_5^*}{m_\phi^2}\langle\bar{\Psi}^C\Gamma^5\Psi\rangle_\phi = \frac{1}{m_\phi^2}\int d^2k \int_{-\omega_D}^{\omega^D} d\omega \frac{-|\eta_5|^2\phi}{(\omega-\mu)^2 - k^2 - |\eta_5|^2|\phi|^2}$$

Pairing in the hard wall holography

- Spin splitting of holographic Fermions

Herzog, Ren;
Seo, Sin, Zhou



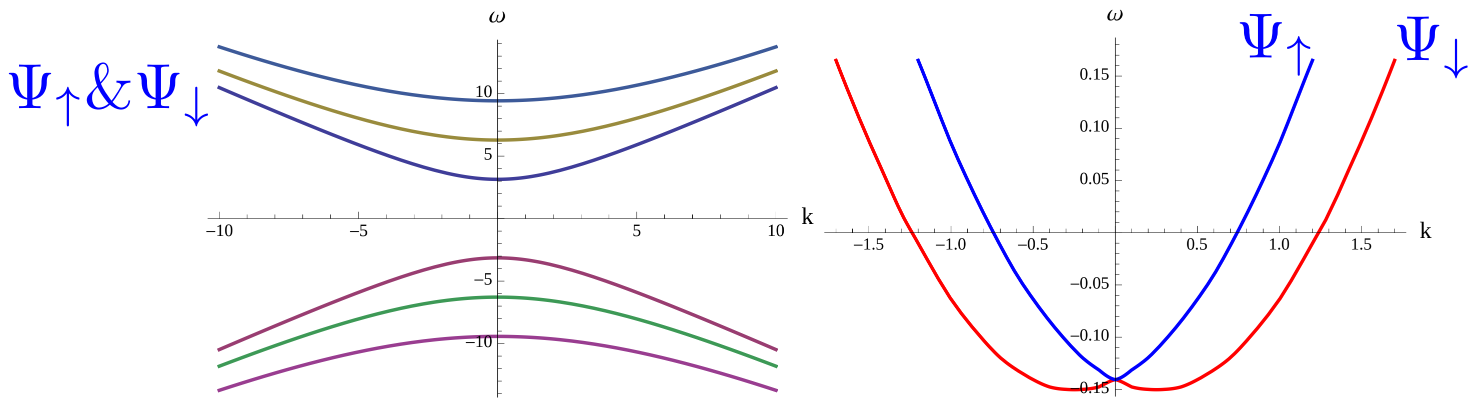
(a) (b)

Fermion spectra in hardwall at $\mu = 0$

Fermion spectra in hardwall at $\mu = a - bz$

- # Spin splitting of holographic Fermions

Herzog, Ren;
Seo, Sin, Zhou



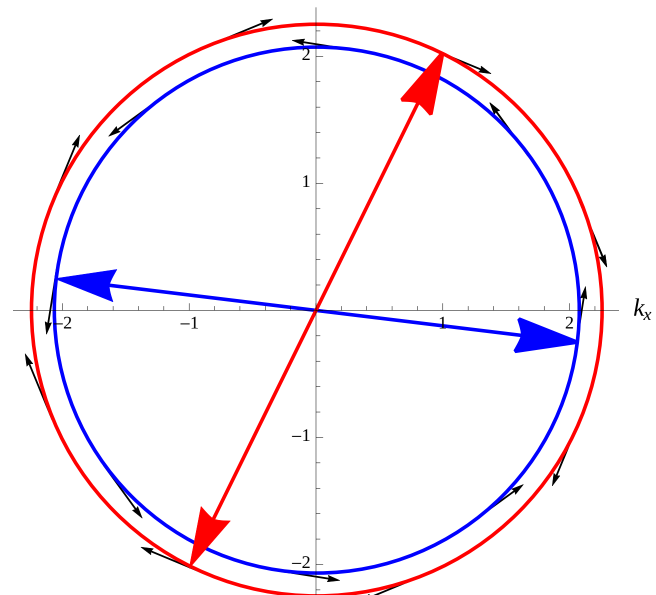
(a) (b)

Fermion spectra in hardwall at $\mu = 0$

Fermion spectra in hardwall at $\mu = a - bz$

True eigenstates are transverse helicity eigenstates:
There is zero-momentum pairing

At the same time there are two
non-degenerate BCS condensates



- Holographic Hard Wall BCS Lagrangian

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Hard-wall IR: Ignore gravitational backreaction

Include charge backreaction on the Maxwell-sector

- Holographic Hard Wall BCS Lagrangian

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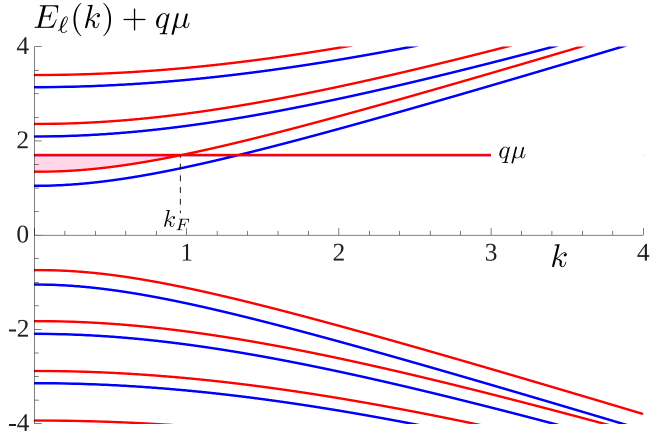
Sachdev



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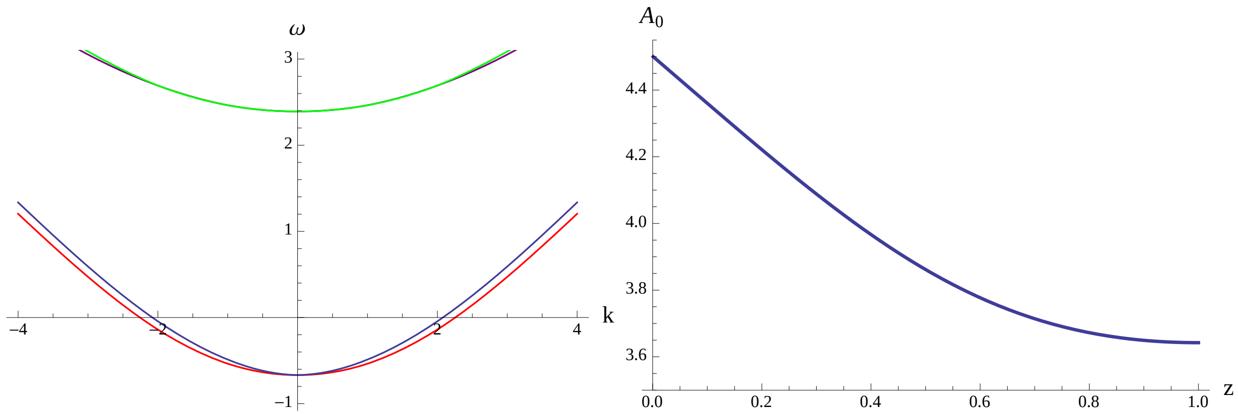

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Hard-wall IR: Ignore gravitational backreaction

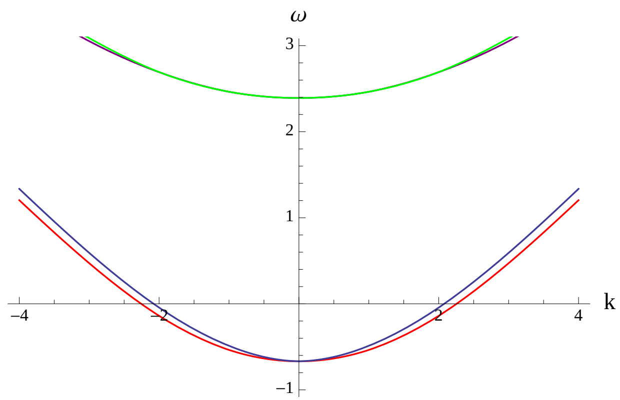
Include charge backreaction on the Maxwell-sector

Now also include pairing interaction .... solve with Hartree-Fock iteration

(BCS in a box with screening)

The BCS gap

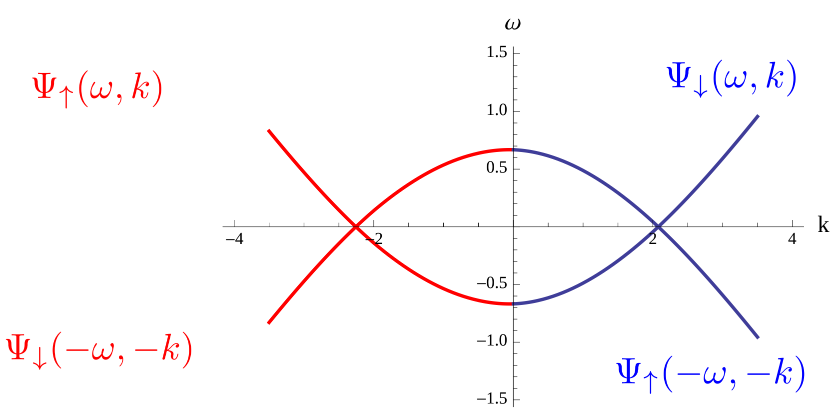
cf. Faulker, Horowitz, McGreevy, Roberts, Vegh



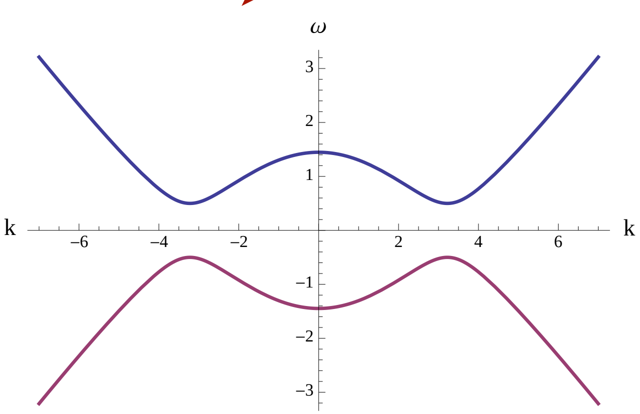
(a)

Spectrum redrawn

Increasing the BCS coupling

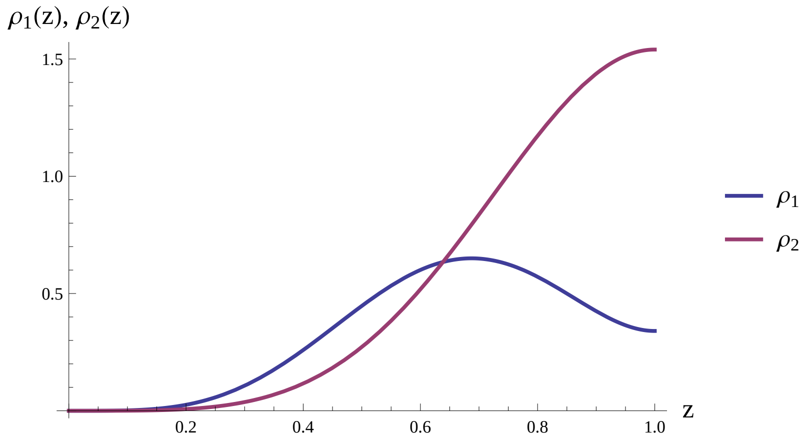


(a)



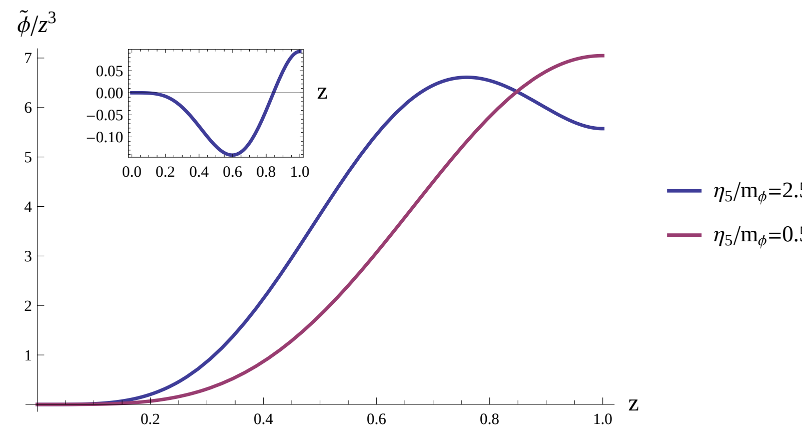
(b)

Two non-degenerate condensates



(a)

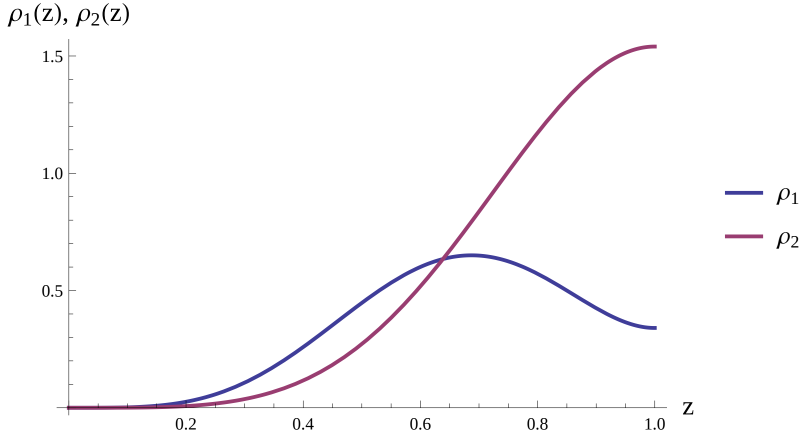
Profiles of the two different Fermion wavefunctions



(b)

Profiles of the condensate for different values of the BCS coupling

Two non-degenerate condensates



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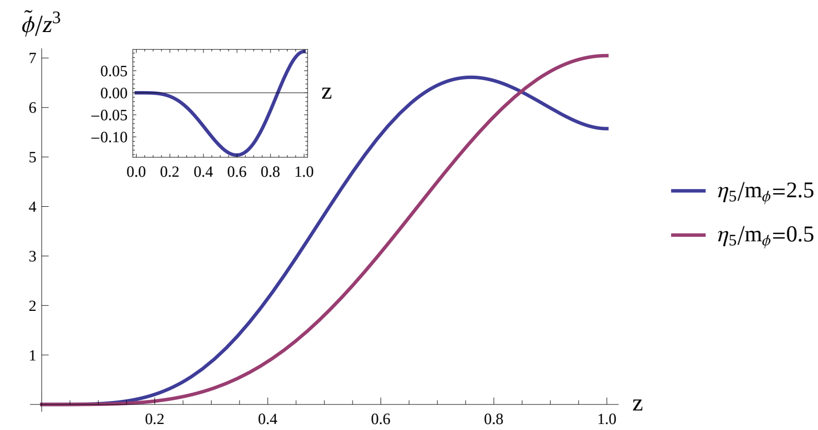

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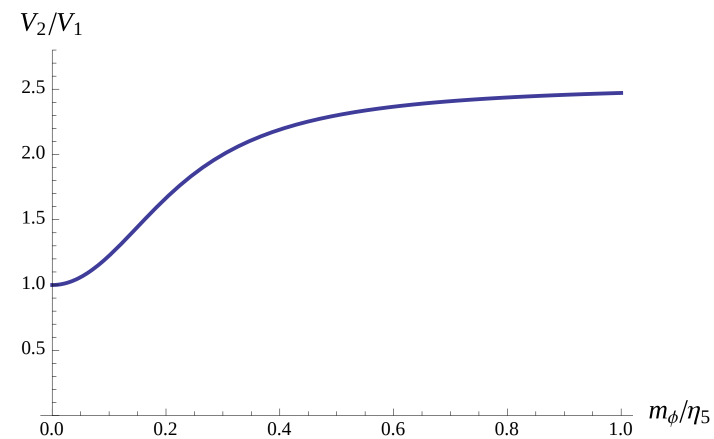
Profiles of the condensate for different values of the BCS coupling



(a)

Ratio of the gaps for different values of the BCS coupling

- Holographic Hard Wall BCS Lagrangian

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cf. Faulker, Horowitz, McGreevy, Roberts, Vegh

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  - including a dynamical scalar field

$$\frac{1}{m_\phi^2}\Box\phi - \underbrace{\phi}_{\mathcal{O}(N^2)} = -\frac{\eta_5^*}{m_\phi^2}\underbrace{\langle\bar{\Psi}^C\Gamma^5\Psi\rangle}_{\mathcal{O}(1)}$$

cf. Faulker, Horowitz, McGreevy, Roberts, Vegh

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$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |(\partial_\mu - 2iqA_\mu)\phi|^2 - m_\phi^2|\phi|^2$$
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$m_\phi^2 \to 0$

Holographic Superconductor
(Hard Wall)

$m_\phi^2 \to \infty$

Holographic BCS
(Hard Wall)

cf. Faulker, Horowitz, McGreevy, Roberts, Vegh

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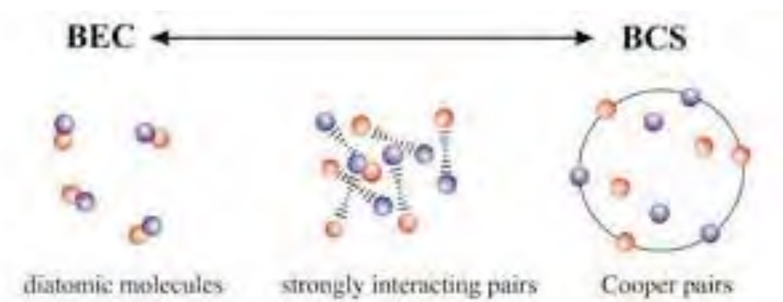
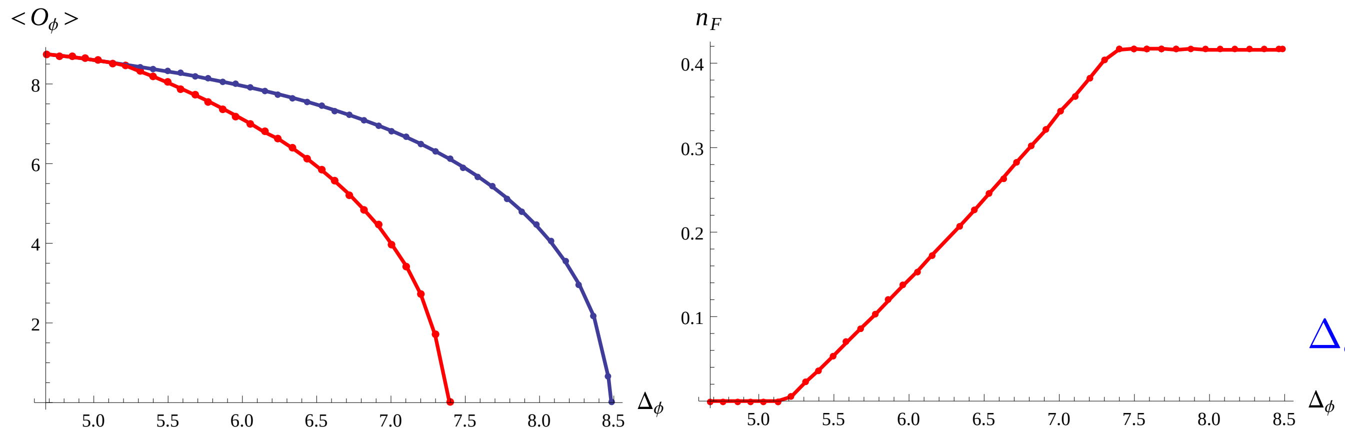
Holographic Superconductor
(Hard Wall)



Holographic BCS
(Hard Wall)

Hard-Wall Bose Fermi Competition

Nitti, Policastro, Vanel
Liu, Schalm, Sun, Zaanen



(a) (b)

$$\Delta_\phi = \frac{3}{2} + \sqrt{\frac{9}{4} + m_\phi^2}$$

Scalar condensate in pure HSC (blue)

Scalar condensate in Bose-Fermi system with no BCS coupling

(Hard Wall)

Fermion density in Bose-Fermi system with no BCS coupling

(Hard Wall)

Hard-Wall Bose Fermi Competition

Nitti, Policastro, Vanel
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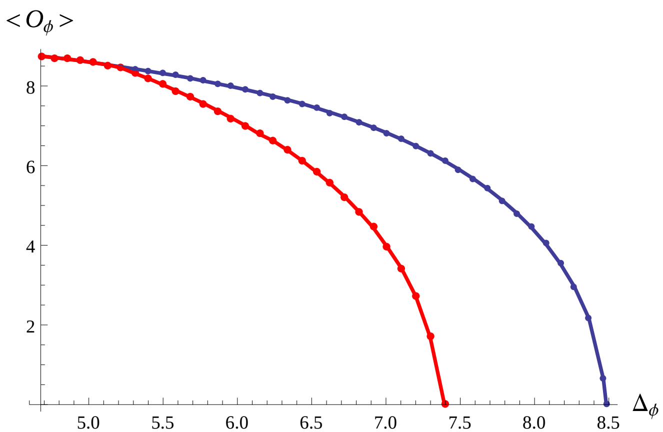
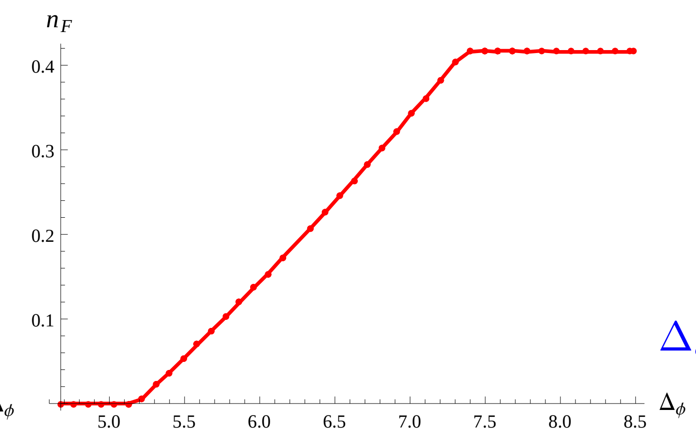


(a)



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Scalar condensate in pure HSC (blue)

Scalar condensate in Bose-Fermi system with no BCS coupling

(Hard Wall)

Fermion density in Bose-Fermi system with no BCS coupling

(Hard Wall)

Fluid approximation: A hairy Electron Star

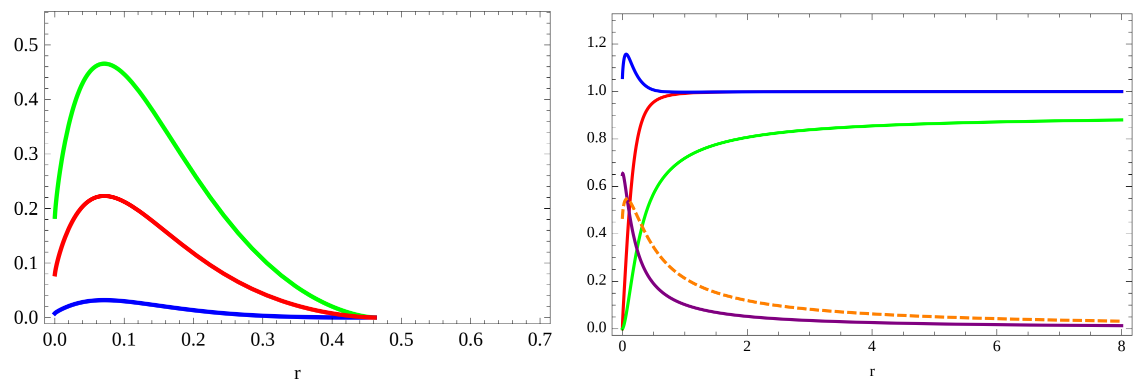

Figure 5. An example of the background of the single-edge hairy electron star solution. *Left:* fluid parameters as functions of the radial coordinate r: $\hat{\rho}$ (red), $\hat{n}$ (green), $\hat{p}$ (blue). It is easy to see that these functions are not monotonic along the radial coordinate as in the pure electron star case. *Right:* metric and scalar fields of the hairy electron star as functions of the radial coordinate r: f/c^2r^2(red), gr^2(blue), $h/c\mu$ (green), μ_{loc}(orange), $\hat{\phi}$ (purple).

Hard-Wall Bose Fermi Competition

Nitti, Policastro, Vanel
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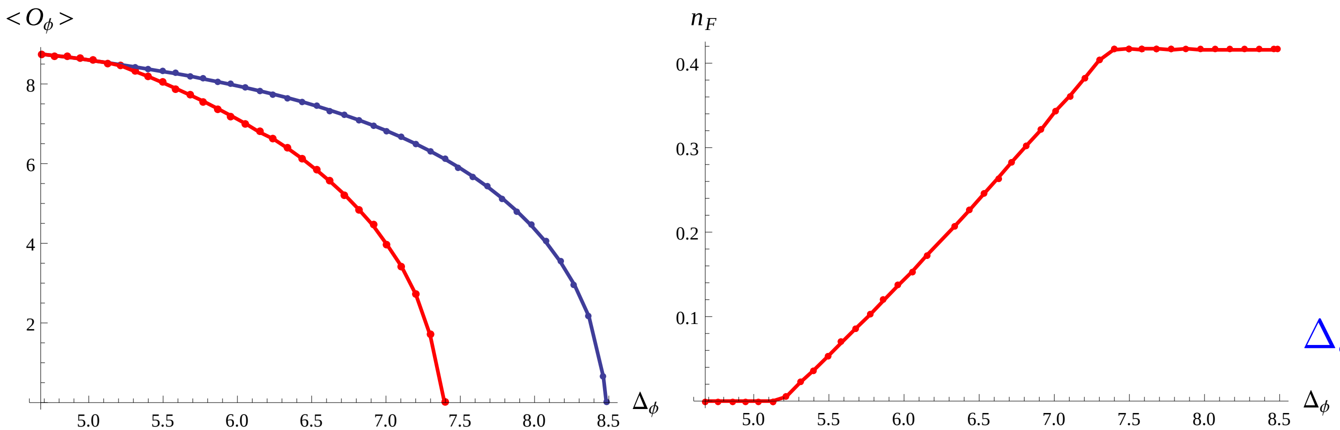

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Scalar condensate in pure HSC (blue)

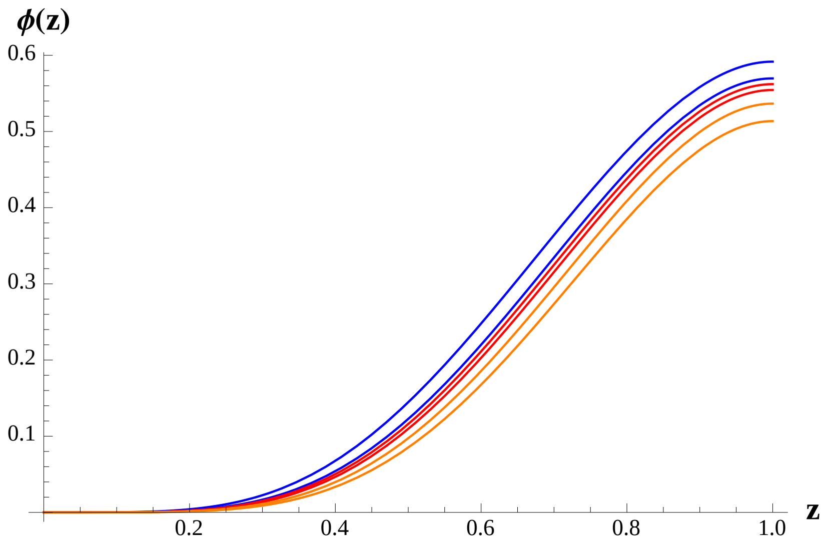
Scalar condensate in Bose-Fermi system with no BCS coupling

(Hard Wall)

Fermion density in Bose-Fermi system with no BCS coupling

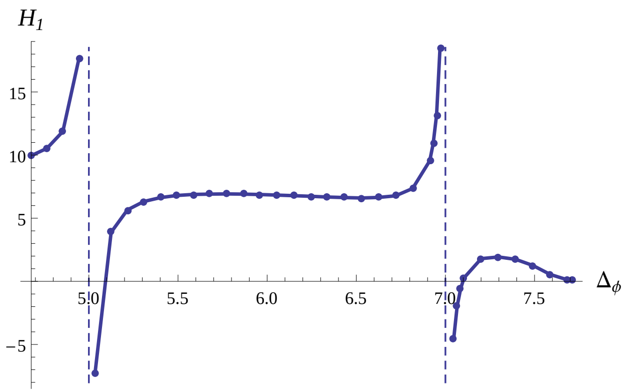
(Hard Wall)

- Holographic BEC-BCS system

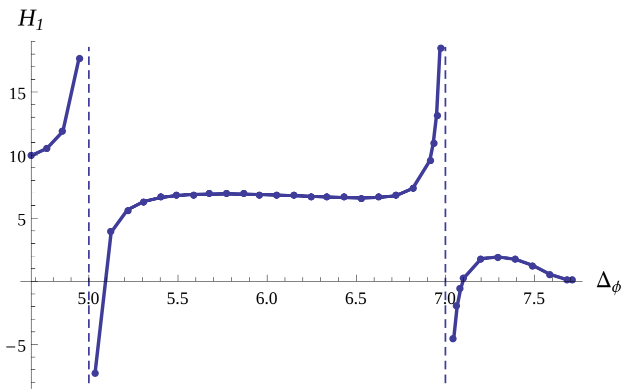


Normalizable Scalar wavefunctions for various values of m_ϕ^2

Reading off the condensate in the boundary...

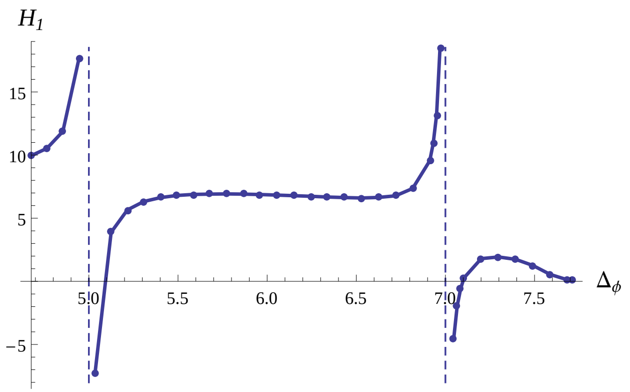


Reading off the condensate in the boundary...

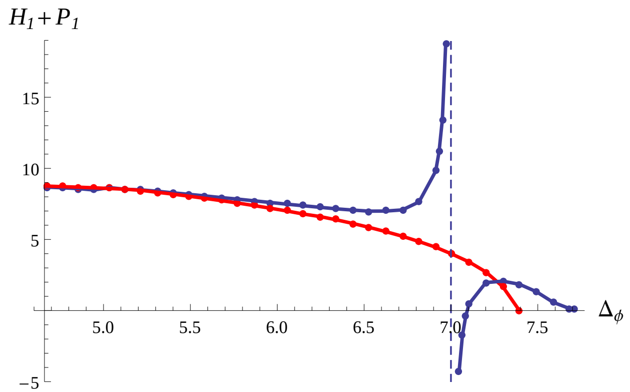


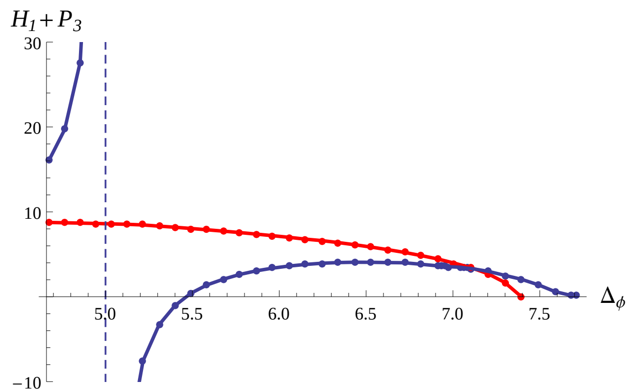
Recall that we should think of ϕ and $\langle \bar{\Psi}^C \Gamma^5 \Psi \rangle$ as independent condensates

Reading off the condensate in the boundary...



Recall that we should think of ϕ and $\langle \bar{\Psi}^C \Gamma^5 \Psi \rangle$ as independent condensates





A puzzle about AdS/CFT vevs

- Inhomogeneous differential equation

$$z^2\phi'' - 2z\phi' + 4q^2 z^2 A_0^2 \phi - m_\phi^2 = \eta_5 z^3 \langle \bar{\Psi}^C \Gamma^5 \Psi \rangle$$

A puzzle about AdS/CFT vevs

- Inhomogeneous differential equation

$$z^2\phi'' - 2z\phi' + 4q^2 z^2 A_0^2 \phi - m_\phi^2 = \eta_5 z^3 \langle \bar{\Psi}^C \Gamma^5 \Psi \rangle$$

$$\lim_{z\to 0} z^3 \langle \bar{\Psi}^C \Gamma^5 \Psi \rangle \sim z^{2\Delta_\Psi}$$

$$\phi(z) = \underbrace{\mathcal{H}_0 z^{d-\Delta_\phi} + \mathcal{H}_1 z^{\Delta_\phi} + \ldots}_{\text{Homogeneous solution}} + \underbrace{\mathcal{P}_1 z^{2\Delta_\Psi} + \mathcal{P}_2 z^{2\Delta_\Psi+1} + \mathcal{P}_3 z^{2\Delta_\Psi+2} + \ldots}_{\text{Particular solution}}$$

For the special case $\Delta_\phi = 2\Delta_\Psi + n$

$$\phi(z) = \mathcal{H}_0 z^{d-2\Delta_\Psi+n} + \mathcal{H}_1 z^{2\Delta_\Psi+n} + \ldots + \mathcal{P}_1 z^{2\Delta_\Psi} + \ldots + \mathcal{P}_n z^{2\Delta_\Psi+n} \ln(z) +$$

- Inhomogeneous differential equation

$$z^2\phi'' - 2z\phi' + 4q^2z^2A_0^2\phi - m_\phi^2 = \eta_5 z^3 \langle \bar{\Psi}^C \Gamma^5 \Psi \rangle$$

A puzzle about AdS/CFT vevs

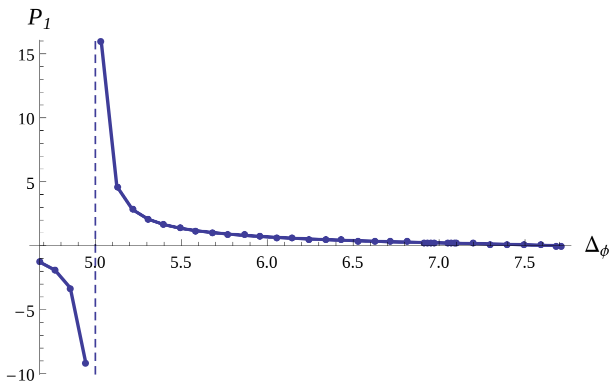
- Inhomogeneous differential equation

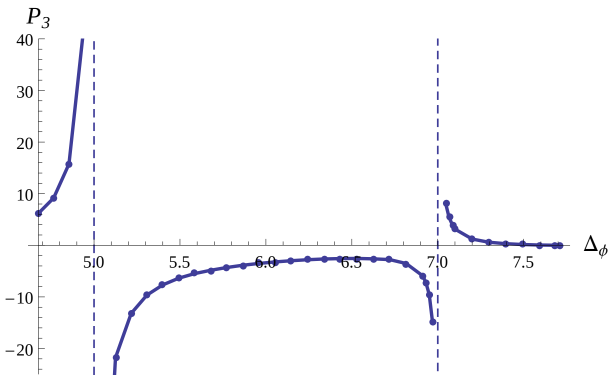
$$z^2\phi'' - 2z\phi' - m_\phi^2 = \mathcal{S}_1 z^{2\Delta_\Psi} + \mathcal{S}_3 z^{2\Delta_\Psi+2}$$

$$\mathcal{P}_1 = \frac{\mathcal{S}_1}{2\Delta_\Psi(2\Delta_\Psi-3) - \Delta_\phi(\Delta_\phi-3)}$$

$$\mathcal{P}_3 = \frac{\mathcal{S}_3}{(2\Delta_\Psi+2)(2\Delta_\Psi-1) - \Delta_\phi(\Delta_\phi-3)}$$

Visible in full numerical solution





- Composite double trace operators

$$\mathcal{O}_{\text{pair}} = \mathcal{O}_{\bar{\Psi}^C}\mathcal{O}_{\Psi} \quad = \mathrm{Tr}\phi\psi\mathrm{Tr}\phi\psi$$

  - The operator $\mathcal{O}^{(1)}_{\text{pair}} = \mathcal{O}_{\bar{\Psi}^C}(\overleftarrow{\partial}_\mu - \overrightarrow{\partial}_\mu)(\overleftarrow{\partial^\mu} - \overrightarrow{\partial^\mu})\mathcal{O}_{\Psi} - \text{traces}$ cannot be written as derivatives of $\mathcal{O}_{\text{pair}}$
  - Yet it has the same global quantum numbers
  - This operator mixes with $\mathcal{O}_{\text{pair}}$ under RG flow
  - There is a whole tower of such *conformal partial waves*

$$\mathcal{O}_{\text{pair}} = \mathcal{O}_{\bar{\Psi}^C}\mathcal{O}_{\Psi}$$

$$\mathcal{O}^{(1)}_{\text{pair}} = \mathcal{O}_{\bar{\Psi}^C}(\overleftarrow{\partial}_\mu - \overrightarrow{\partial}_\mu)(\overleftarrow{\partial^\mu} - \overrightarrow{\partial^\mu})\mathcal{O}_{\Psi} - \text{traces}$$

$$\mathcal{O}^{(2)}_{\text{pair}} = \mathcal{O}_{\bar{\Psi}^C}(\overleftarrow{\partial}_\mu - \overrightarrow{\partial}_\mu)(\overleftarrow{\partial^\mu} - \overrightarrow{\partial^\mu})(\overleftarrow{\partial}_\nu - \overrightarrow{\partial}_\nu)(\overleftarrow{\partial^\nu} - \overrightarrow{\partial^\nu})\mathcal{O}_{\Psi} - \text{traces}$$

$$\vdots$$

- Composite double trace operators

$$\mathcal{O}_{\text{pair}} = \mathcal{O}_{\bar{\Psi}^C}\mathcal{O}_{\Psi} \quad = \text{Tr}\phi\psi\text{Tr}\phi\psi$$

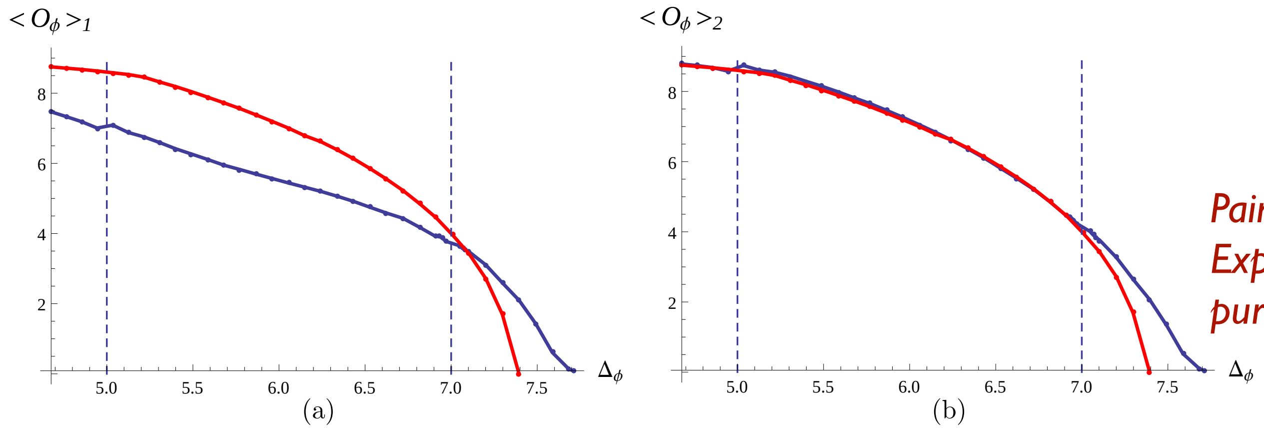
  - Higher order operators mixes with $\mathcal{O}_{\text{pair}}$ under RG flow
  - Postulate: Higher order moments in the particular solution should be seen as vevs of these higher order operators.

$$\phi(z) = \underbrace{\mathcal{H}_0 z^{d-\Delta_\phi} + \mathcal{H}_1 z^{\Delta_\phi} + \ldots}_{\text{Homogeneous solution}} + \underbrace{\mathcal{P}_1 z^{2\Delta_\Psi} + \mathcal{P}_2 z^{2\Delta_\Psi+1} + \mathcal{P}_3 z^{2\Delta_\Psi+2} + \ldots}_{\text{Particular solution}}$$

- Composite double trace operators: Consistency Check
  - A linear combination should exist without singularities (blue)

$$\langle\mathcal{O}_\phi\rangle_1 = \mathcal{H}_1 + \frac{1}{2}((2\Delta_\Psi + 2) - \Delta_\phi)\mathcal{P}_1 + \frac{1}{2}(\Delta_\phi - 2\Delta_\Psi)\mathcal{P}_3$$

$$\langle\mathcal{O}_\phi\rangle_2 = \mathcal{H}_1 + \frac{e^{-(2\Delta_\Phi - \Delta_\phi)}}{2}((2\Delta_\Psi + 2) - \Delta_\phi)\mathcal{P}_1 + \frac{e^{-(2\Delta_\Phi + 2 - \Delta_\phi)}}{2}(\Delta_\phi - 2\Delta_\Psi)\mathcal{P}_3$$



Pairing enhanced!
Exponentially asymptotes to pure BCS

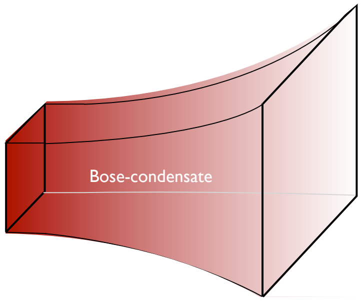
Comparison (red) is the holographic superconductor in the absence of BCS coupling

- BCS is faithfully reproduced in holography (hard wall)
    - with assumptions regarding operator mixing
    - Caution: the true IR is shielded

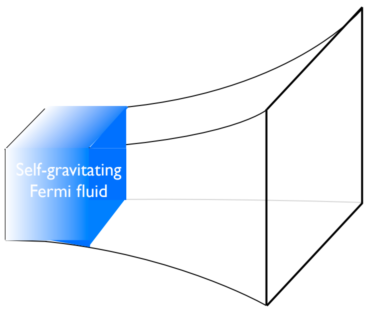
BCS instabilities of Electron Stars

Nitti, Policastro, Vanel
Liu, Schalm, Sun, Zaanen

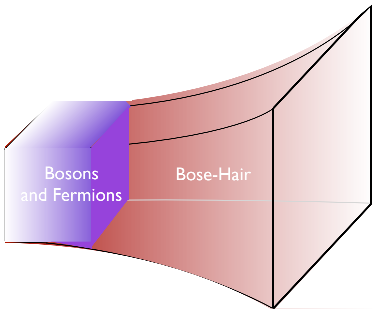
- AdS Stars with bosons and fermions
  - Pure Boson: Higgs star



  - Pure Fermion: Electron star

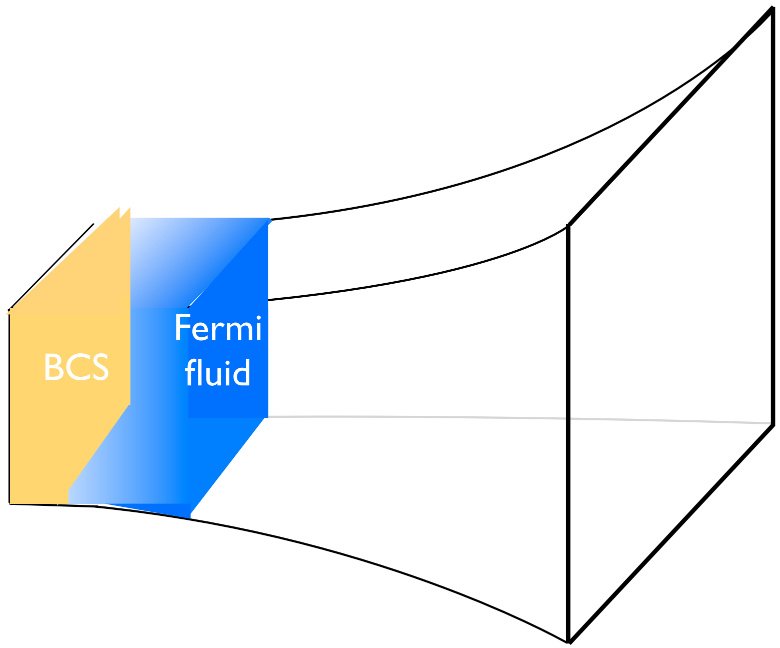


  - Non-interacting Bosons and Fermions: Hairy Electron Star



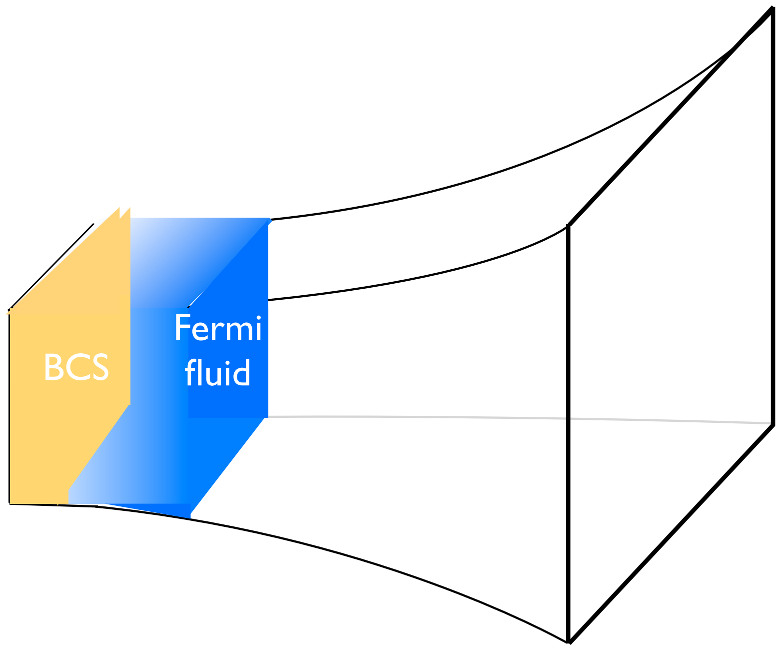
No dynamical scalar

- AdS Star with BCS Fermions



  - Expectation: Electron star with a superconducting core

- AdS Star with BCS Fermions



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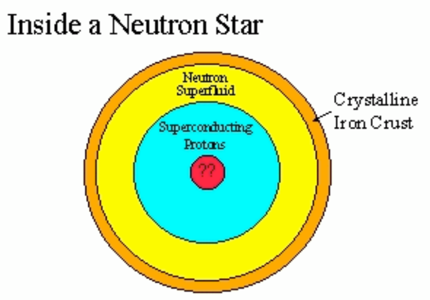
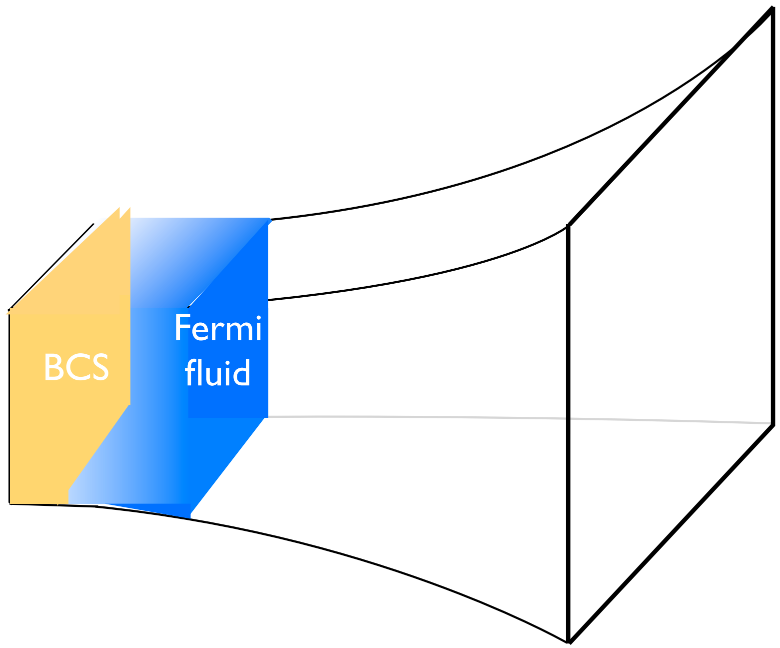
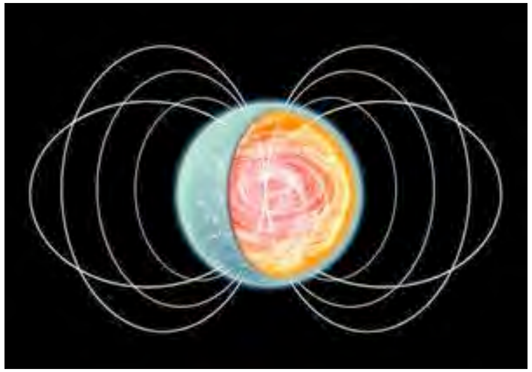


- AdS Star with BCS Fermions



  - Expectation: Electron star with a superconducting core



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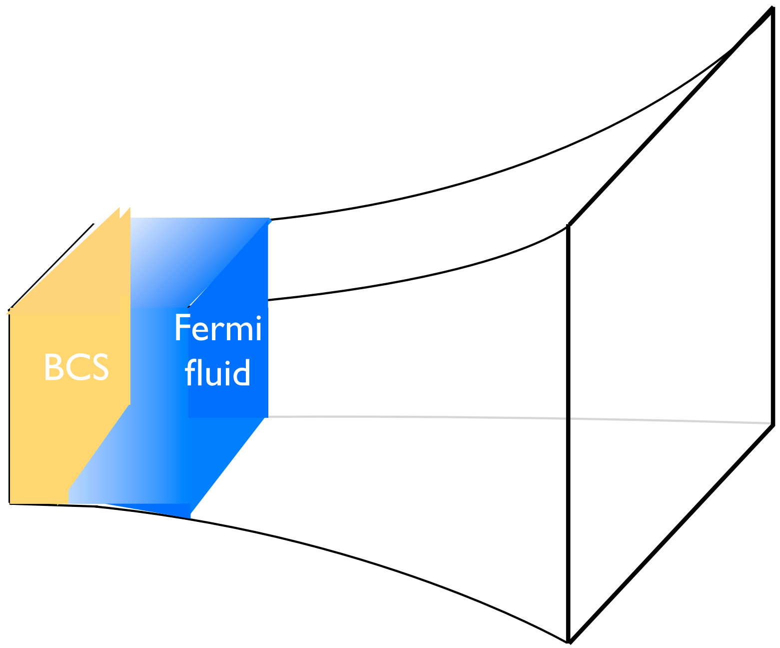

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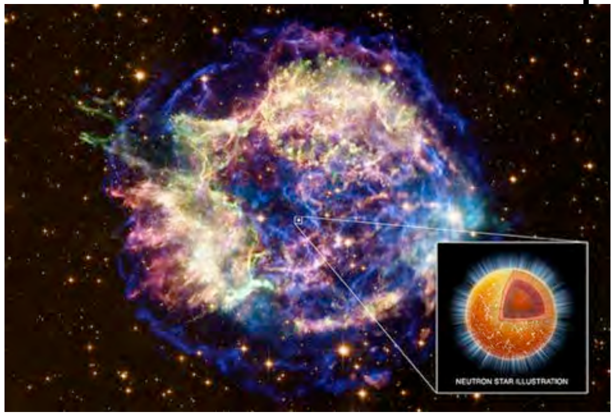
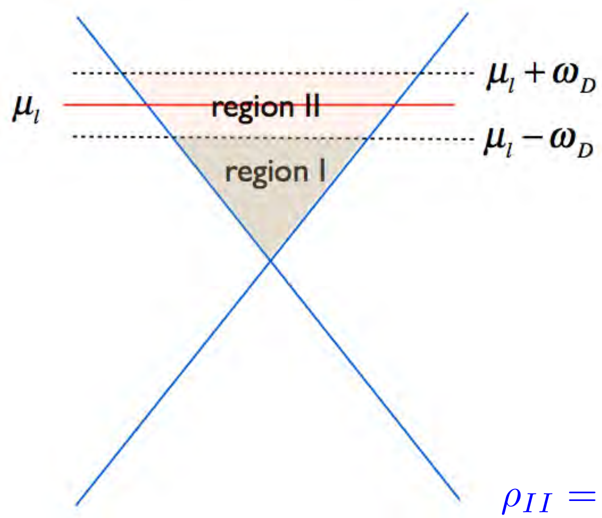


- Electron Star: fluid approx to fermions

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

  - Recall that the BCS interaction has a cut-off ω_D
  - Assume $\omega_D \ll \mu$



$$\rho_I = \frac{1}{\pi^2}\int_{m_\Psi}^{\mu-\omega_D} d\omega \omega^2 \sqrt{\omega^2 - m_\Psi^2}$$

$$p_I = \frac{1}{3\pi^2}\int_{m_\Psi}^{\mu-\omega_D} d\omega \omega \sqrt{\omega^2 - m_\Psi^2}$$

$$n_I = \frac{1}{3\pi^2}\int_{m_\Psi}^{\mu-\omega_D} d\omega \omega \sqrt{\omega^2 - m_\Psi^2}$$

$$\rho_{II} = \rho_{II}^{FL} + m_\phi^2\phi^2 + \frac{2}{\pi^2}\frac{\mu^3}{\sqrt{\mu^2 - m_\Psi^2}}|\eta_5|^2\phi^2 \ln\frac{\omega_D}{\eta_5\phi} + \ldots$$

$$p_{II} = p_{II}^{FL} - m_\phi^2\phi^2 + \frac{2}{\pi^2}\mu\sqrt{\mu^2 - m_\Psi^2}|\eta_5|^2\phi^2 \ln\frac{\omega_D}{\eta_5\phi} + \ldots$$

$$n_{II} = n_{II}^{FL} + \frac{2(2\mu^2 - m_\Psi^2)}{\pi^2\sqrt{\mu^2 - m_\Psi^2}}|\eta_5|^2\phi^2 \ln\frac{\omega_D}{\eta_5\phi} + \ldots$$

- Electron Star: fluid approx to fermions
  - There is now one extra equation:
  - The BCS gap equation: solution for $\eta_5\phi \ll \omega_D \ll \mu$

$$\Delta = 2\omega_D e^{-1/2\lambda\nu_0}$$

In the variables used here

$$\phi = \frac{1}{\eta_5}\omega_D e^{-m_\phi^2/(4\eta_5^2\nu_0)} \;, \quad \nu_0 = \frac{1}{2\pi^2}\mu\sqrt{\mu^2 - m_\Psi^2}$$

$$\ln\frac{\omega_D}{\eta_5\phi} = -\frac{m_\phi^2}{4\eta_5^2\nu_0}$$

- Electron Star: fluid approx to fermions

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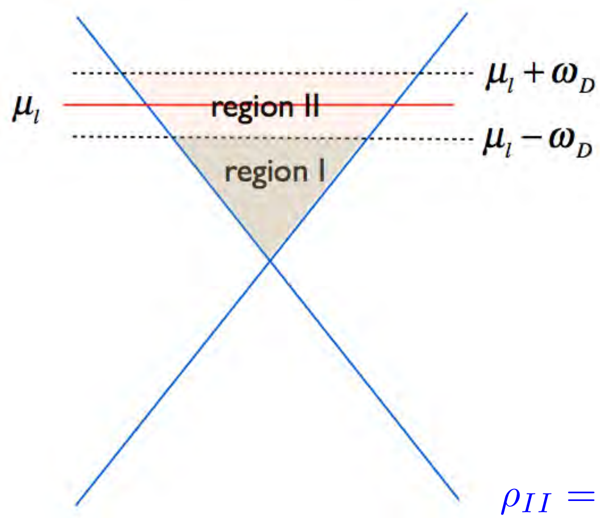

$$\ln \frac{\omega_D}{\eta_5 \phi} = -\frac{m_\phi^2}{4\eta_5^2 \nu_0}$$

$$\rho_I = \frac{1}{\pi^2} \int_{m_\Psi}^{\mu - \omega_D} d\omega \omega^2 \sqrt{\omega^2 - m_\Psi^2}$$

$$p_I = \frac{1}{3\pi^2} \int_{m_\Psi}^{\mu - \omega_D} d\omega \omega \sqrt{\omega^2 - m_\Psi^2}$$

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$$\rho_{II} = \rho_{II}^{FL} + m_\phi^2 \phi^2 + \frac{2}{\pi^2} \frac{\mu^3}{\sqrt{\mu^2 - m_\Psi^2}} |\eta_5|^2 \phi^2 \ln \frac{\omega_D}{\eta_5 \phi} + \ldots$$

$$p_{II} = p_{II}^{FL} - m_\phi^2 \phi^2 + \frac{2}{\pi^2} \mu \sqrt{\mu^2 - m_\Psi^2} |\eta_5|^2 \phi^2 \ln \frac{\omega_D}{\eta_5 \phi} + \ldots$$

$$n_{II} = n_{II}^{FL} + \frac{2(2\mu^2 - m_\Psi^2)}{\pi^2 \sqrt{\mu^2 - m_\Psi^2}} |\eta_5|^2 \phi^2 \ln \frac{\omega_D}{\eta_5 \phi} + \ldots$$

- Electron Star: fluid approx to fermions

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

  - Recall that the BCS interaction has a cut-off ω_D
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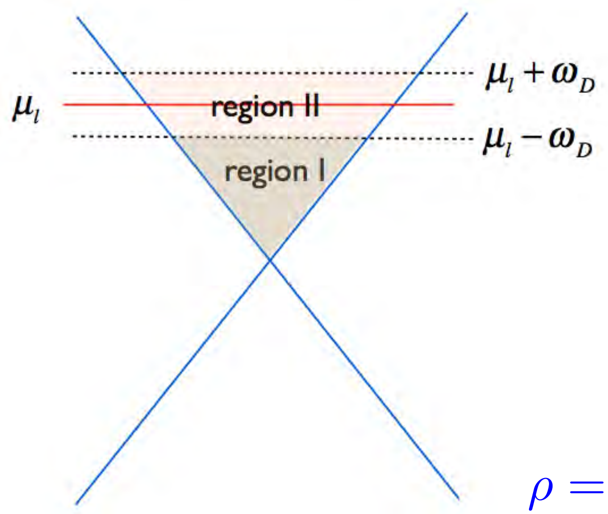
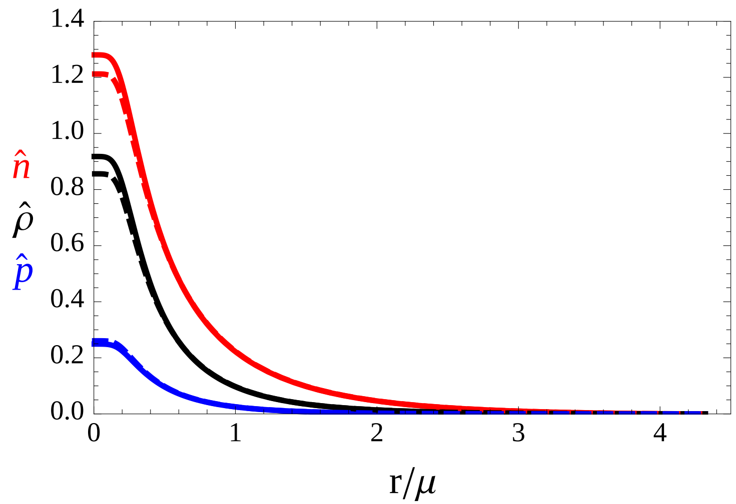


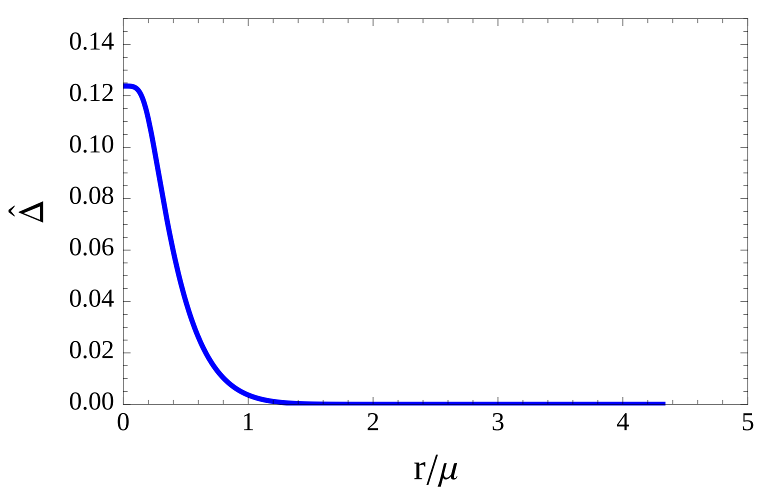
$$\rho = \rho_{FL} + \frac{2\mu^2 - m_\Psi^2}{(\mu^2 - m_\Psi^2)} m_\phi^2 \phi^2 + \ldots$$

$$p = p_{FL} + \ldots$$

$$n = n_{FL} + \frac{2\mu^2 - m_\Psi^2}{\mu(\mu^2 - m_\Psi^2)} m_\phi^2 \phi^2 + \ldots$$

The BCS Star



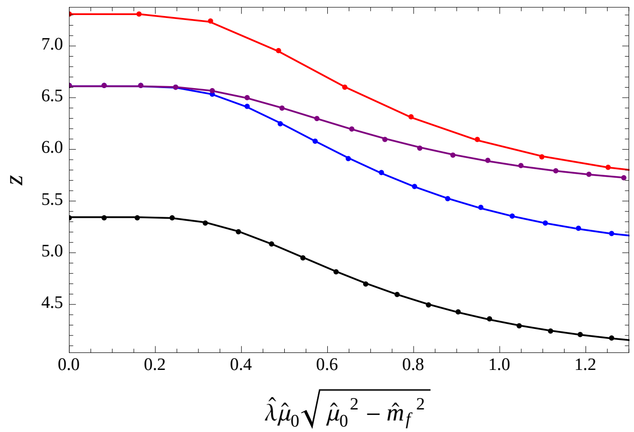


Dashed is Electron star solution

$$\hat{\Delta} = 2\eta_5\phi$$

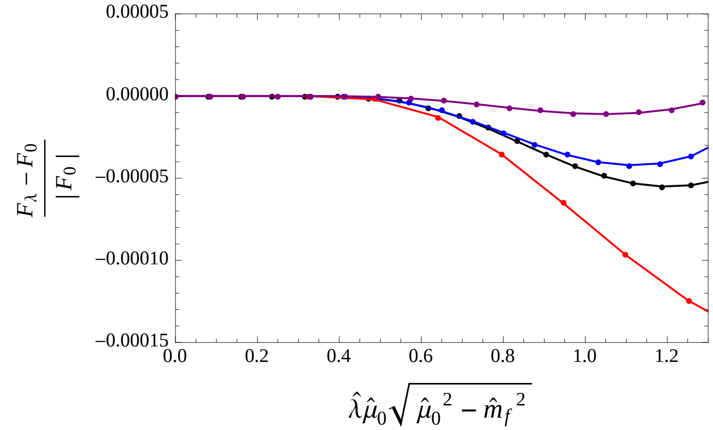
The BCS Star

- # Lifshitz IR

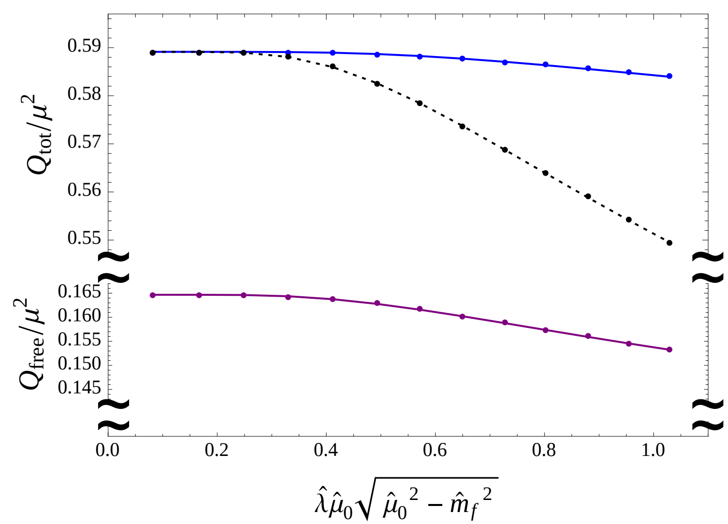


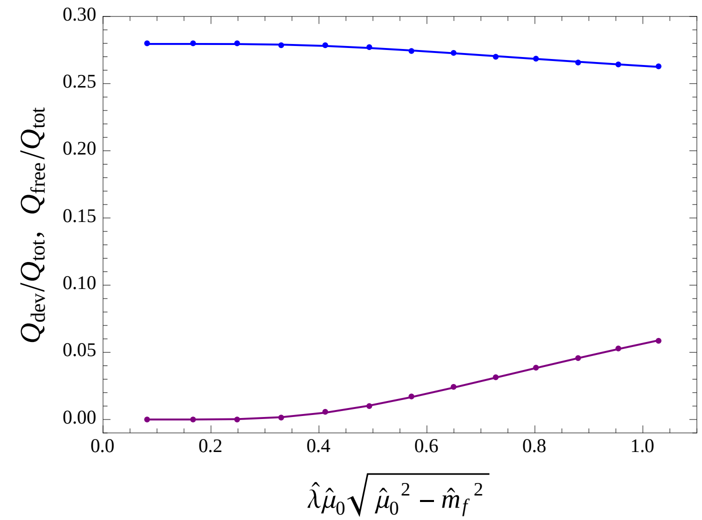
Outside of perturbative regime

$\eta_5\phi \ll \omega_D \ll \mu$



- # The Core





Q_{free} — Contribution to charge density from region I

Q_{total} — Total charge density

$Q_{dev} = \int_0^{r_s} \delta n$ — Additional charge density due to pairing

The BCS Star

- Phenomenology
  - Fermion spectral functions have a gap

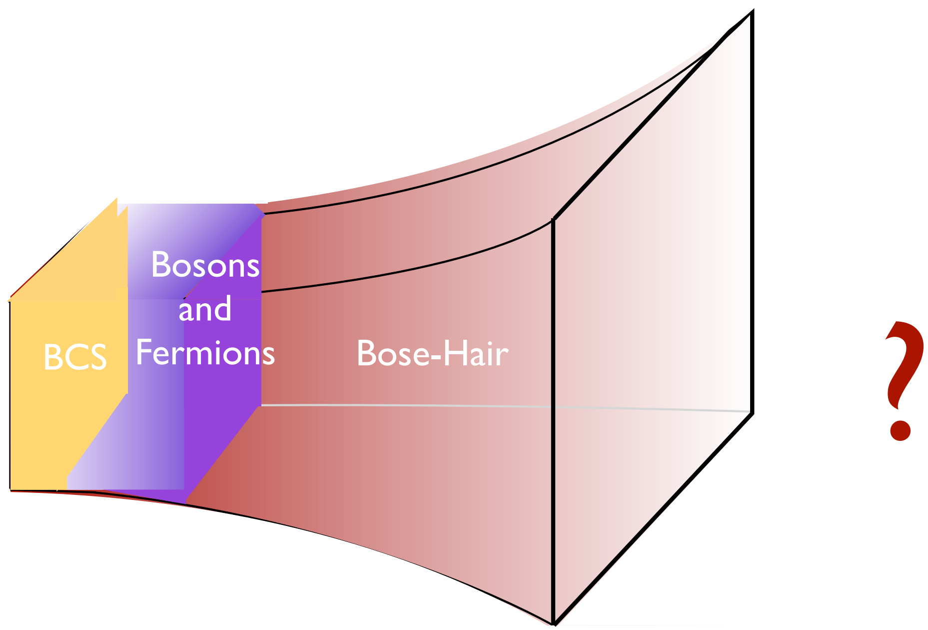
$$G_R = \langle \Psi^\dagger \Psi \rangle_R \qquad G_R^{-1} \sim \begin{pmatrix} \omega P_1 & Q_1 \\ Q_2 & \omega P_2 \end{pmatrix} \qquad Q_i \sim \hat{\Delta}$$

Gap below $\omega < \sqrt{Q_1 Q_2 / P_1 P_2}$

  - Conductivity has a (soft) gap

$$\mathrm{Re}\, \sigma \propto \delta(\omega) + \omega^2$$

- Adding a dynamical scalar?

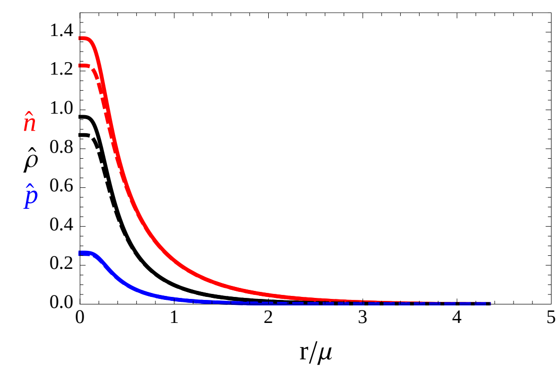


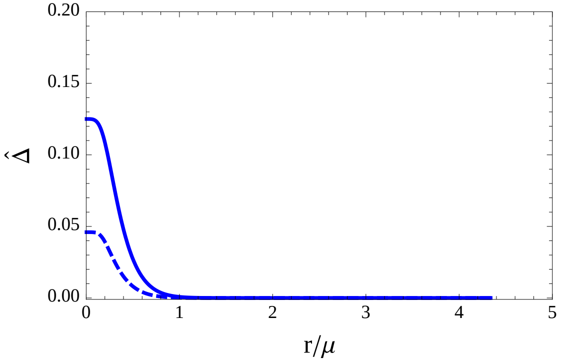
- Incompatible with the fluid approx

Cubrovic, Liu, Schalm, Sun, Zaanen

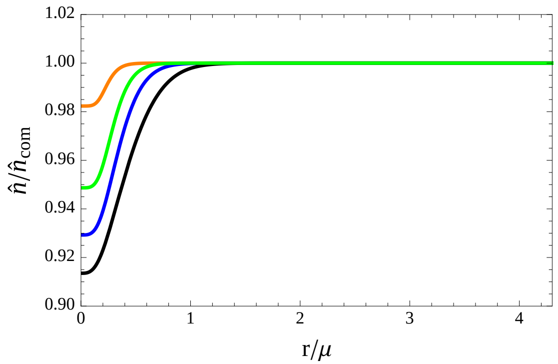
- Can take a scaling limit with $m_\phi^2 = \frac{1}{\kappa}\hat{m}_\phi^2$
- All kinetic terms decouple except $\Delta\mathcal{L} = 4q^2 A_\mu A^\mu \phi^2$

BCS-Stueckelberg Star





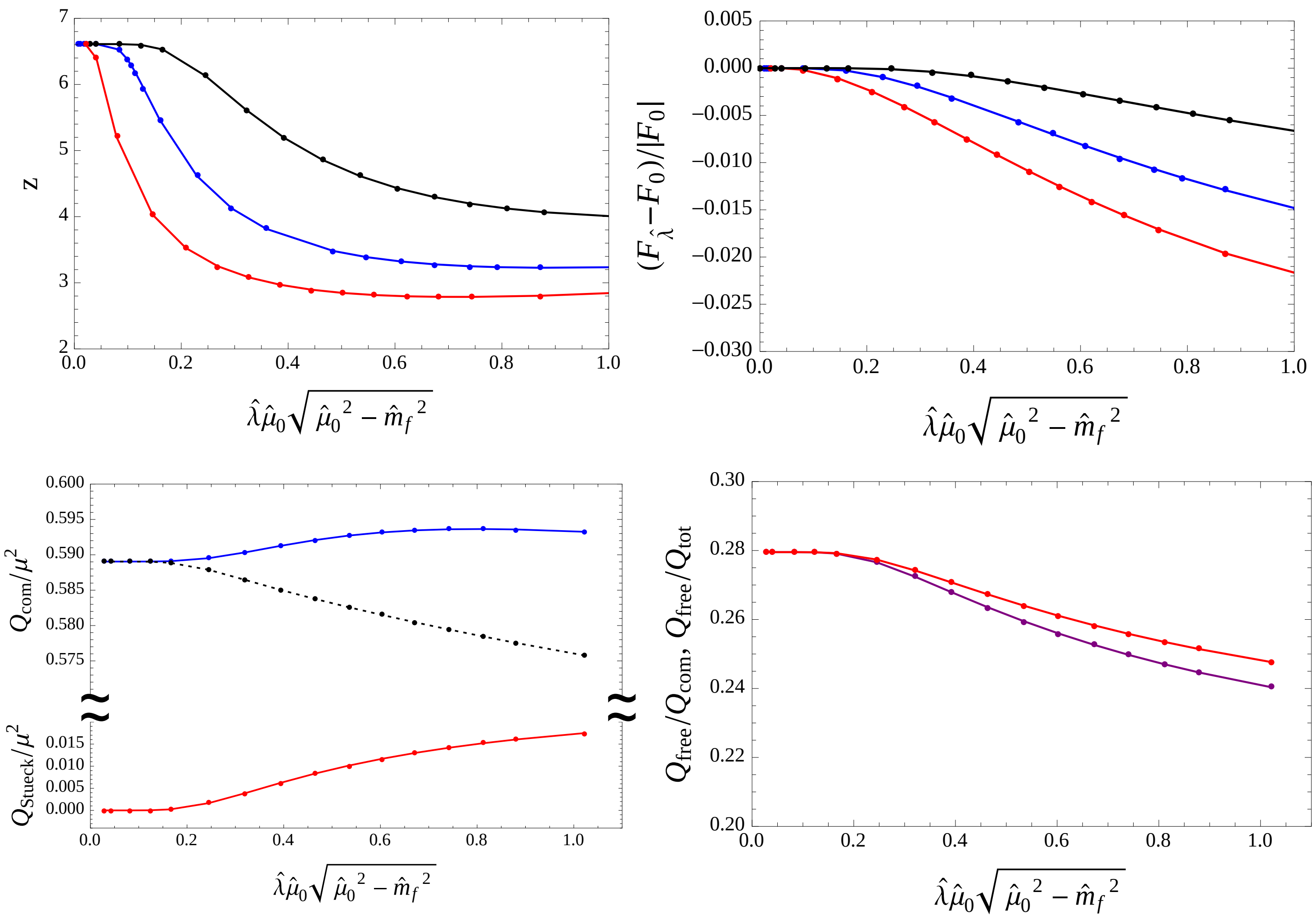
Dashed is BCS star



BCS star charge density/
BCS-Stueck charge density

Predominant change in the IR

BCS-Stueckelberg Star



Conclusions

- Holography checks off on generic IR behavior
  - BCS pairing driven instability and Cooper condensate controlled groundstate
  - Lifshitz IR with a soft conductivity gap
  - Essential development for realistic models
- Open directions
  - Theory: how to read off vev of double trace condensates
  - Experiment: Possible connection with BEC-BCS physics in cold atoms

Thank you.