Comments on entanglement: negativity at large c

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Introduction

Entanglement Entropy (EE) has attracted lots of attention.
Interesting for a variety of reasons, e.g.

- Non-local order parameter in QFT (topological EE)
- c-function characterizing RG flows
- Can be easily computed holographically
- Interesting window into black hole physics

Introduction

Other measurements of quantum entanglement include e.g. Renyi entropy characterized by an integer n. Entanglement entropy is obtained in the $n \to 1$ limit.

The focus of this talk will be another measure of entanglement: entanglement negativity.

Based on M. Kulaxizi and G. Policastro, A.P. arXiv:1407.2324 See also Cardy, Calabrese, Tonni; Rangamani, Rota

Outline

Entanglement entropy and entanglement negativity

Total Hilbert space $= \mathcal{H}$. $\mathcal{A}, \mathcal{B} \subset \mathcal{H}$. Reduced density matrix $\rho_{\mathcal{A}\cup\mathcal{B}} = \mathrm{tr}_{\mathcal{H}\cap(\mathcal{A}\cup\mathcal{B})}\, \rho$
Entanglement entropy $S = -tr \rho_{\mathcal{A}\cup\mathcal{B}} \log \rho_{\mathcal{A}\cup\mathcal{B}}$. Alternative measures of entanglement?

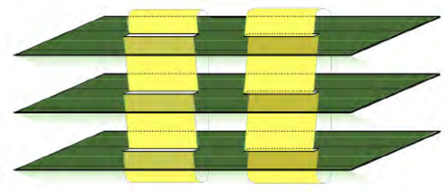
Can perform *partial transpose* w.r.t. $\mathcal{B}$, $[\rho^{T_B}_{\mathcal{A}\cup\mathcal{B}}]_{ab,a'b'} = [\rho_{\mathcal{A}\cup\mathcal{B}}]_{ab',a'b}$.

The resulting matrix $\rho^{T_B}_{\mathcal{A}\cup\mathcal{B}}$ is no longer a density matrix and some eigenvalues λ_i may be negative. Negativity is defined as $\mathcal{E} = \log \sum_i |\lambda_i|$.

Note that $\mathcal{E} \geq 0$. Moreover $\mathcal{E} = 0$ for a product (nonentangled) state.

Replica trick

To compute, use replica trick: $\mathcal{E} = \lim_{n_e \to 1} \ln \mathrm{tr}(\rho^{T_2})^{n_e}$, where n_e is even. (Cardy, Calabrese, Tonni)



Now consider a 1+1 dimensional CFT and two disjoint intervals: $A_1 = (z_1, z_2), \qquad B = (z_3, z_4), \qquad z_2 < z_3$

$$\mathrm{tr}(\rho_{A_2}^{T_B})^n = \langle \mathcal{T}_n(z_1)\bar{\mathcal{T}}_n(z_2)\bar{\mathcal{T}}_n(z_3)\mathcal{T}_n(z_4)\rangle$$

where $\mathcal{T}_n$, $\bar{\mathcal{T}}_n$ are the twist operators with conformal dimensions $h_{\mathcal{T}_n} = h_{\bar{\mathcal{T}}_n} = \frac{c}{24}\left(n - \frac{1}{n}\right) \equiv h\,.$

Limits

Negativity is a function of the cross-ratio:

$$x = \frac{(z_2 - z_1)(z_4 - z_3)}{(z_3 - z_1)(z_4 - z_2)}$$

In the limit of widely separated intervals $x \to 0$ all perturbative terms vanish.
In the opposite limit of two adjacent intervals $x \to 1$ the correlator is dominated by the $\bar{\mathcal{T}}_n(z_2)\bar{\mathcal{T}}_n(z_3)$ OPE and

$$\mathcal{E} \simeq -\frac{c}{4}\log(1-x)$$

Correlators at large c

Can write four-point function as

$$\langle \mathcal{O}_1(0)\mathcal{O}_2(x)\mathcal{O}_3(1)\mathcal{O}_4(\infty)\rangle = \sum_p a_p \mathcal{F}(c, h_p, h_i, x)\bar{\mathcal{F}}(c, \bar{h}_p, \bar{h}_i, \bar{x}) \,,$$

Can compute conformal block in the limit $c \to \infty$, h/c fixed.
$\mathcal{F}(c, h_p, h_i, x) \sim \exp\left[-\frac{c}{6} f\left(\frac{h_p}{c}, \frac{h_i}{c}, x\right)\right]$. where f is computed via the following procedure (monodromy problem).

Correlators at large c

Consider the following ODE

$$\psi''(z) + T(z)\psi(z) = 0$$

where
$T(z) = \sum_{i=1}^{i=4} \left(\frac{6h_i}{c(z-z_i)^2} - \frac{c_i}{z-z_i} \right)$, $T(z) \sim z^{-4}$ as $z \to \infty$.
This fixes all accessory parameters in terms of one, $c_2(x)$ and

$$T(z) = \frac{6h_1}{cz^2} + \frac{6h_2}{c(z-x)^2} + \frac{6h_3}{c(z-1)^2} + \frac{6(h_1+h_2+h_3-h_4)}{cz(1-z)} - \frac{c_2 x(1-x)}{z(z-x)(1-z)}$$

Now $c_2(x)$ is determined by imposing monodromy
$\mathrm{tr} M = -2\cos \pi\Lambda_p, \qquad h_p = \frac{c}{24}(1-\Lambda_p^2)$

Correlators at large c

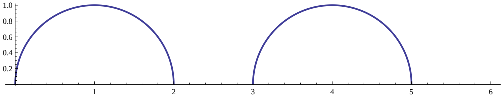
Conformal block $\mathcal{F}(c,h_p,h_i,x) \sim \exp\left[-\frac{c}{6}f\left(\frac{h_p}{c},\frac{h_i}{c},x\right)\right]$ is recovered by integrating $\frac{\partial f}{\partial x} = c_2(x)$

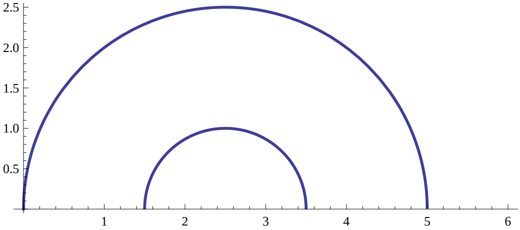
If there is a gap in the spectrum of operator dimensions, the four-point function in the vicinity of $x=0$ and $x=1$ is given by the conformal block which corresponds to the operator with the smallest h_p.

In case of entanglement entropy we need to compute $\rho^n \simeq \langle \mathcal{T}_n(0)\bar{\mathcal{T}}_n(x)\mathcal{T}_n(1)\bar{\mathcal{T}}_n(\infty)\rangle$ and then take the $n{\to}1$ limit $S = \lim_{n\to 1}\rho^n/(n-1)$. This correspond to $h_p=0$ and $\mathrm{tr}M = 2$.

Correlators at large c

The results are $S = c/3 \log x$ and $S = c/3 \log(1-x)$ for x close to 0 and 1 respectively. Precisely what holography predicts. (Hartman, Faulkner)

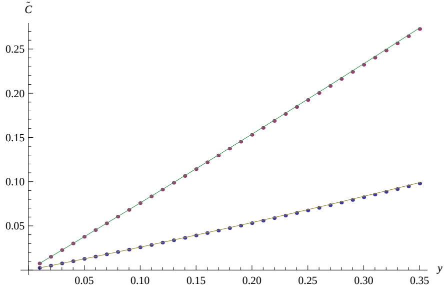




Negativity at large c

Now the intermediate operator is $\mathcal{T}_n^2$ whose dimension is $h = c/12(n/2 - 2/n) \to -c/8$. This corresponds to $\mathrm{tr} M = -2$

We can solve the monodromy problem (unfortunately, only numerically).



$$y(1-y)c_2(y) = -\frac{3}{4}\left(1 - \left(\frac{1}{2} \pm \frac{1}{4}\right) y + \dots\right)$$

Negativity at large c

We can integrate this $c_2(1-x)$ to obtain negativity as a function of x. Does not look like a simple function. If there is a holographic prescription, it is nontrivial.

THE END

Thank you!