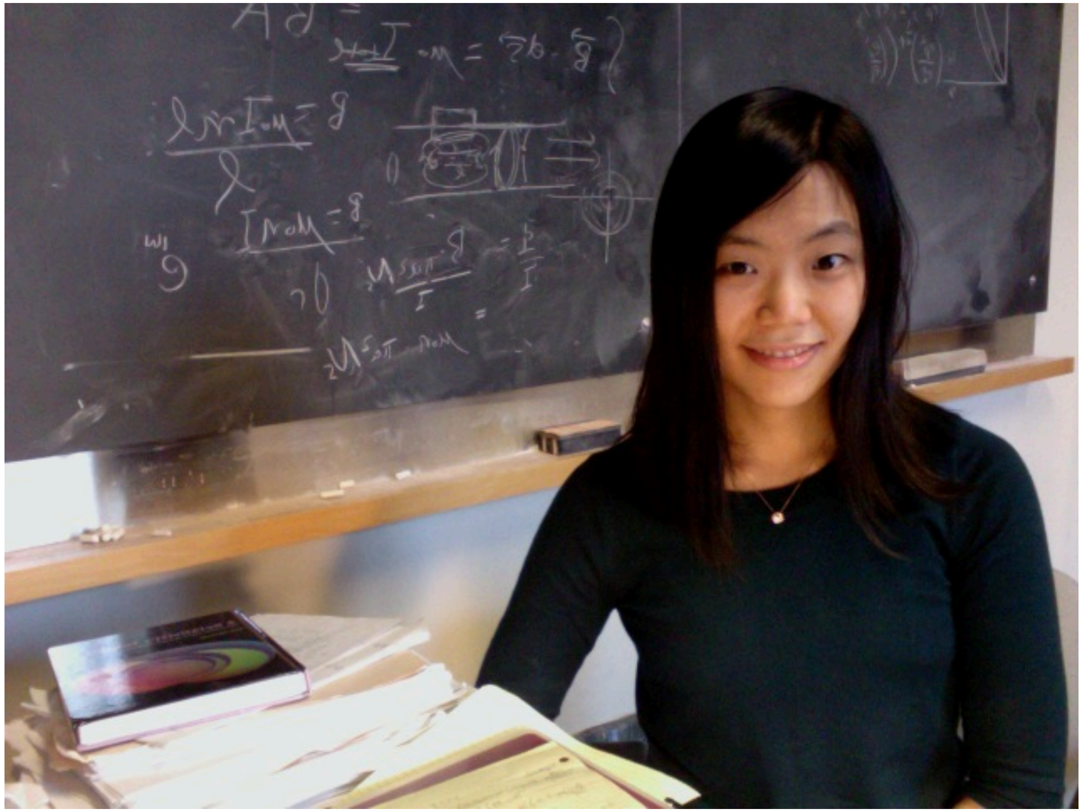
Entanglement Tsunami

Hong Liu



Josephine Suh

Mark Mezei

HL and Josephine Suh, 1305.7244, PRL 112, 011601 (2014)
HL and Josephine Suh, 1311.1200, PRD 89, 066012 (2014)
Casini, Hubeny, Maxfield, HL, Mezei, Suh, to appear

Quantum entanglement

A quantum system: divide into $A + B$

Hilbert space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

Wave function: $\Psi = \sum_n \psi_n(A) \otimes \chi_n(B)$

Simplest measure: entanglement entropy

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A$$

Entanglement and phases of Matter

Traditional (Landau) paradigm of phases:

Different orders characterized by different symmetries
Phase transitions: symmetry breaking

crystals, superconductors, magnets, ……

Such a classification not adequate:

gapped phases: FQH, spin liquids …

→ topological order

gapless systems: Non-Fermi liquids,
gapless spin liquids,
novel quantum critical points
…..

} Long range Quantum Entanglement (???)

In (2+1)-dimension: $S = \frac{L}{\epsilon} - \gamma + \cdots\cdots\cdots$

Short-range

long range entanglement

Gapped phase: γ provides a diagnostic of topological order.

Gapless systems:

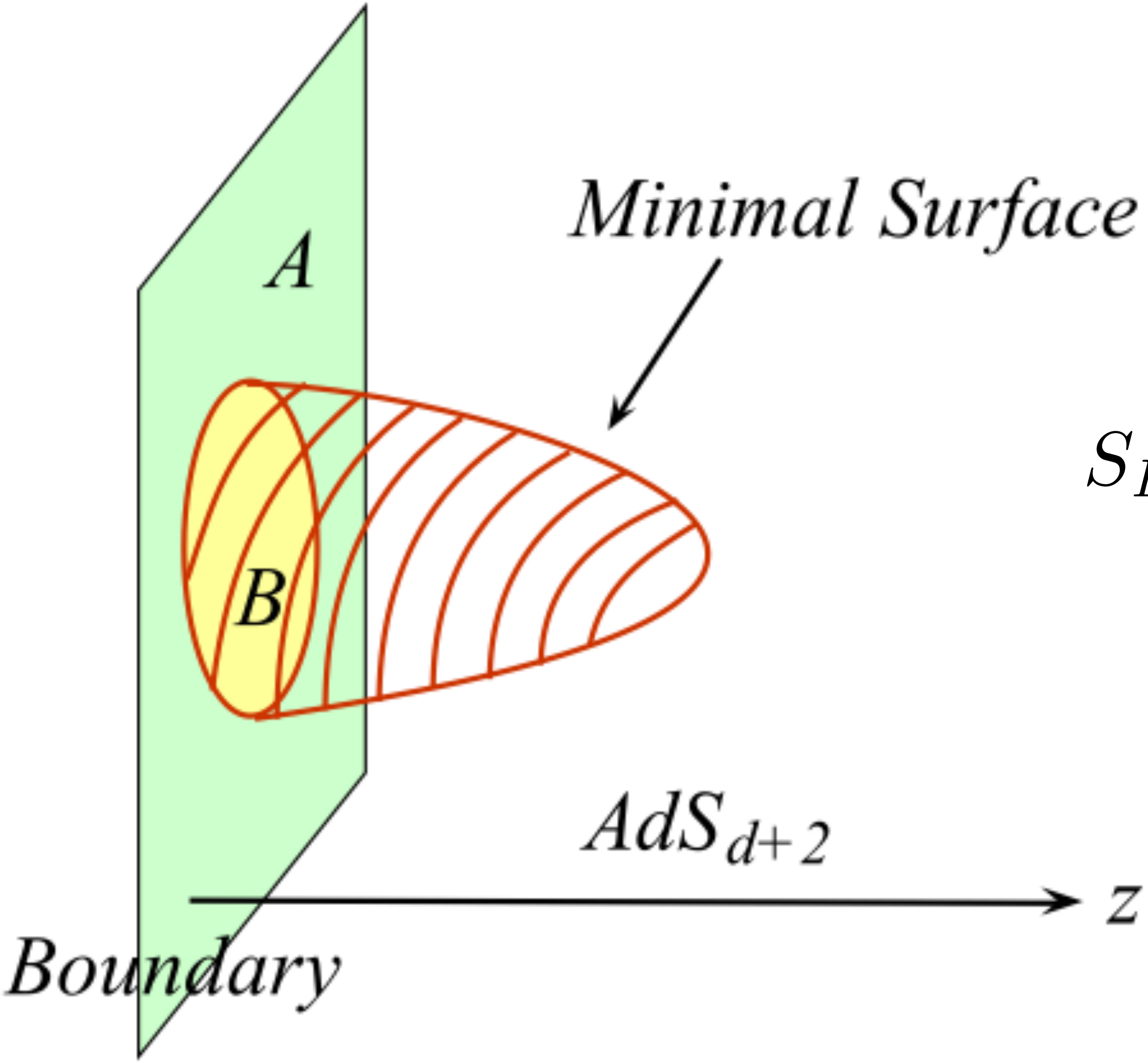
Lorentz invariant QFTs

γ provides a measure of number of degrees of freedom and decreases along RG

C-theorem or F-theorem

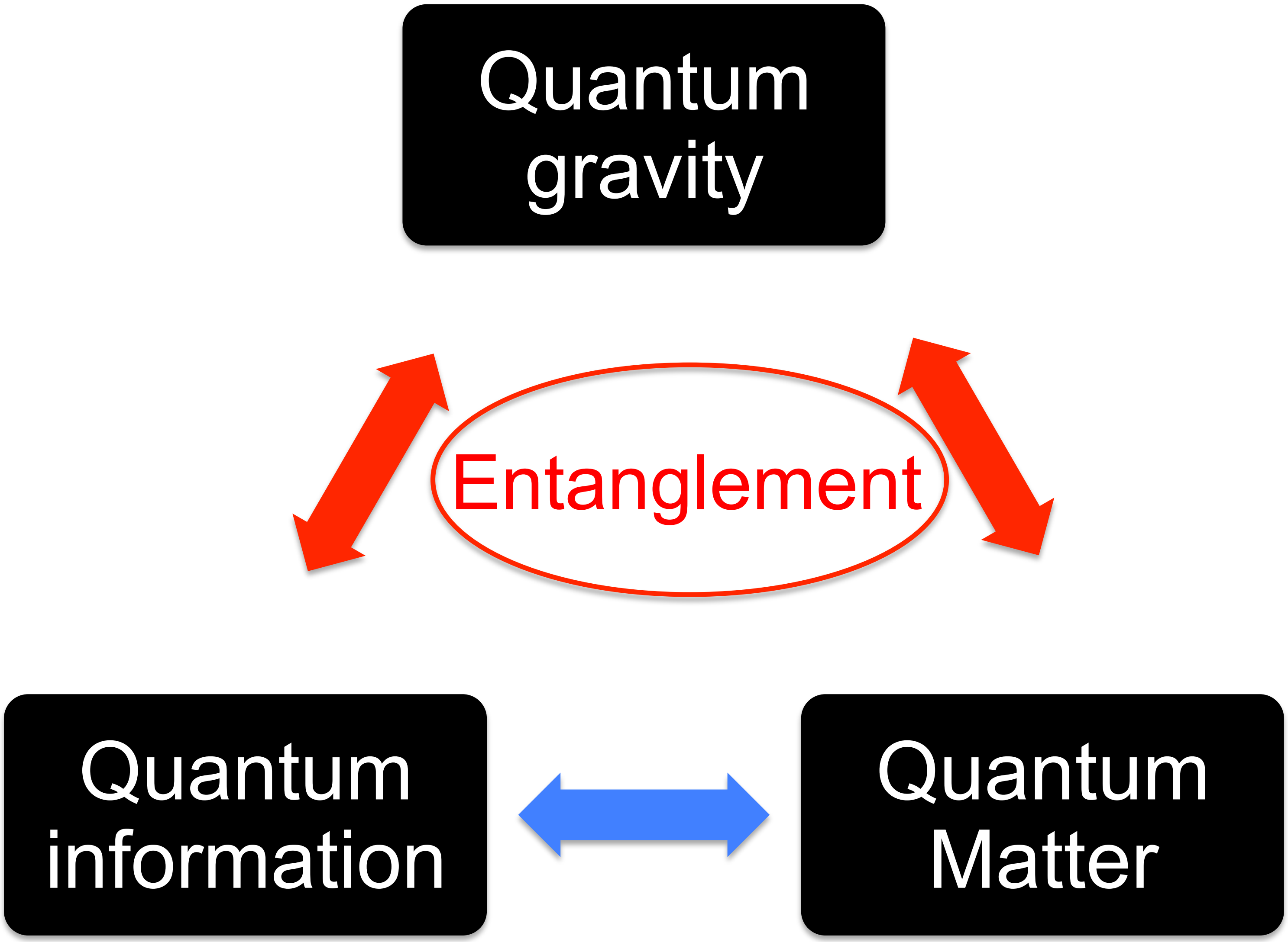
Geometry = Entanglement

Ryu and Takayanagi



$$S_B = \frac{\text{Area of } M}{4G_N}$$

A new paradigm ?



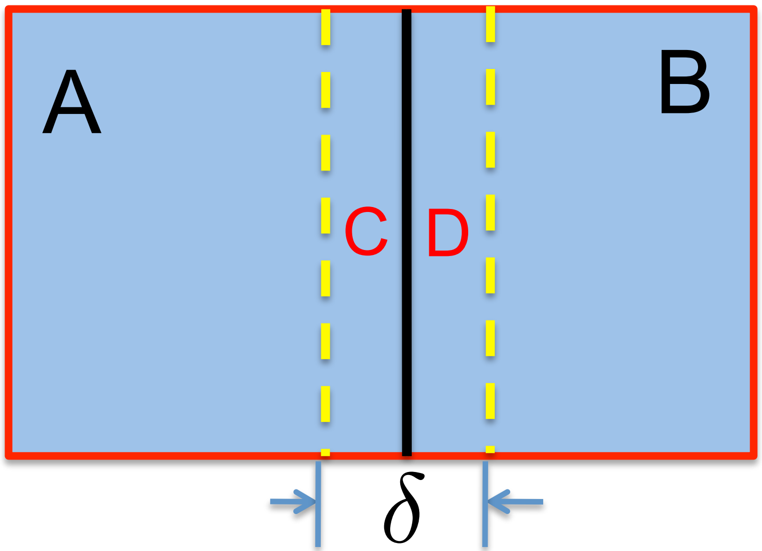
In this talk I discuss our recent exploration of evolution of quantum entanglement in equilibration processes.

How a system equilibrates:

foundation of quantum statistical physics, condensed matter, QCD, …….

- Entanglement: nonlocal probes of equilibration processes
- Equilibration provides a dynamical setting to study the generation of entanglement

Entanglement generation



$$\psi(t=0) = \psi_A \otimes \psi_B$$

$$\psi(t) = e^{-iHt}\psi(0)$$

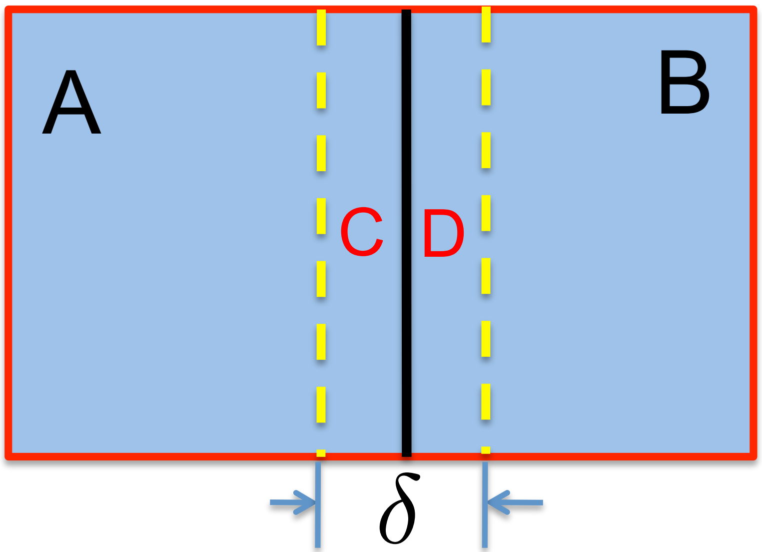
$$H = H_A + H_B + H_{AB}$$

How fast can entanglement be generated?

In most physical systems: Local Hamiltonian

$H_{AB} = H_{CD}$ δ : UV cutoff

Small incremental entangling conjecture/theorem



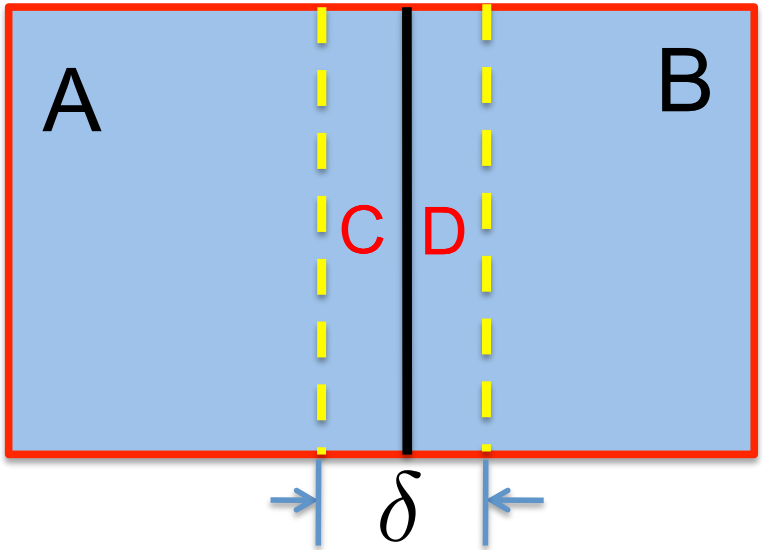
Dur, Vidal et al, Bravyi, Kitaev
Bennett et al, Van Acoleyen, Marien, Verstraete

$$H = H_A + H_B + H_{CD}$$

For spin systems:

$$\frac{dS_A}{dt} \leq c||H||\log d, \qquad d = \min(d_C, d_D)$$

d_C : dimension of Hilbert space of C



SIE:

$$\frac{dS_A}{dt} \leq c||H|| \log d$$

- Finite dimensional Hilbert space: not applicable in continuum limit.

- Gapped systems

For more general quantum systems ????

Neither ||H|| nor d can be precisely defined

How do we compare systems of different number of dof, different shapes, sizes of A etc ?

Equilibration processes in quantum field theories provide a good laboratory for studying such question.

A simple setup: global quenches

1. Start with a QFT in the ground state.

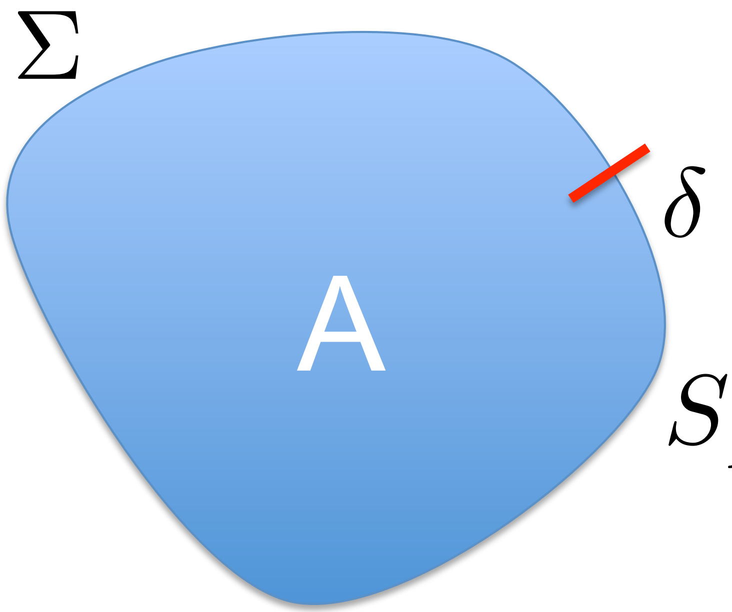
2. At t=0 in a very short time interval inject a uniform energy density
   - initial state homogeneous, isotropic, entanglement properties as vacuum

$S_A(t)?$

3. The system evolves to (thermal) equilibrium

The system is in a pure state throughout.

Entanglement in the vacuum



$$S_A = S_A^{\text{short}} + S_A^{\text{long}}$$

S_A^{short} : short-range entanglement near Σ, cutoff dependent

S_A^{long} : long range entanglement, insensitive to UV physics near Σ

Vacuum: $S_A^{\text{long}} = \text{const or } \log R$ R: characteristic size of A

Long ranged entangled d.o.f. are measure zero.

Entanglement in equilibrium state

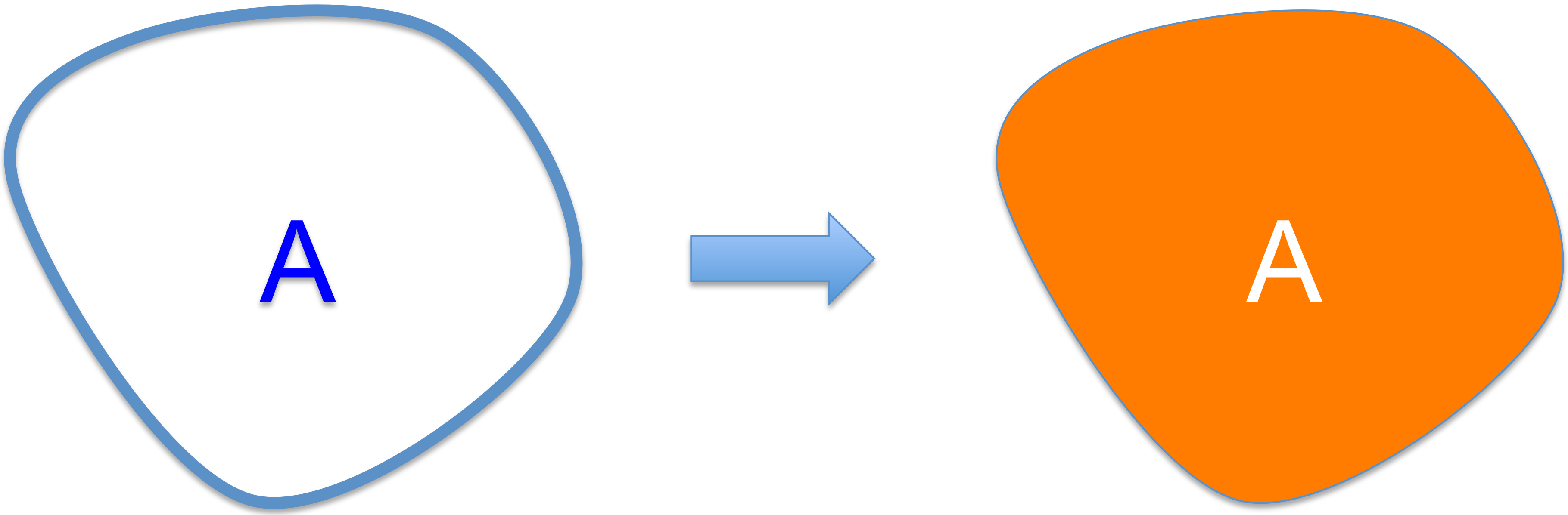
The system behaves macroscopically as a thermal state, with entanglement entropy disguised as thermal entropy:

$$S_A^{\mathrm{long,eq}} = s_{\mathrm{eq}} V_A$$

$$s_{\mathrm{eq}} : \text{ equilibrium entropy density}$$

V_A : volume of region A

Essentially all d.o.f. inside A becomes long ranged entangled with those outside A.



t=0
no entanglement

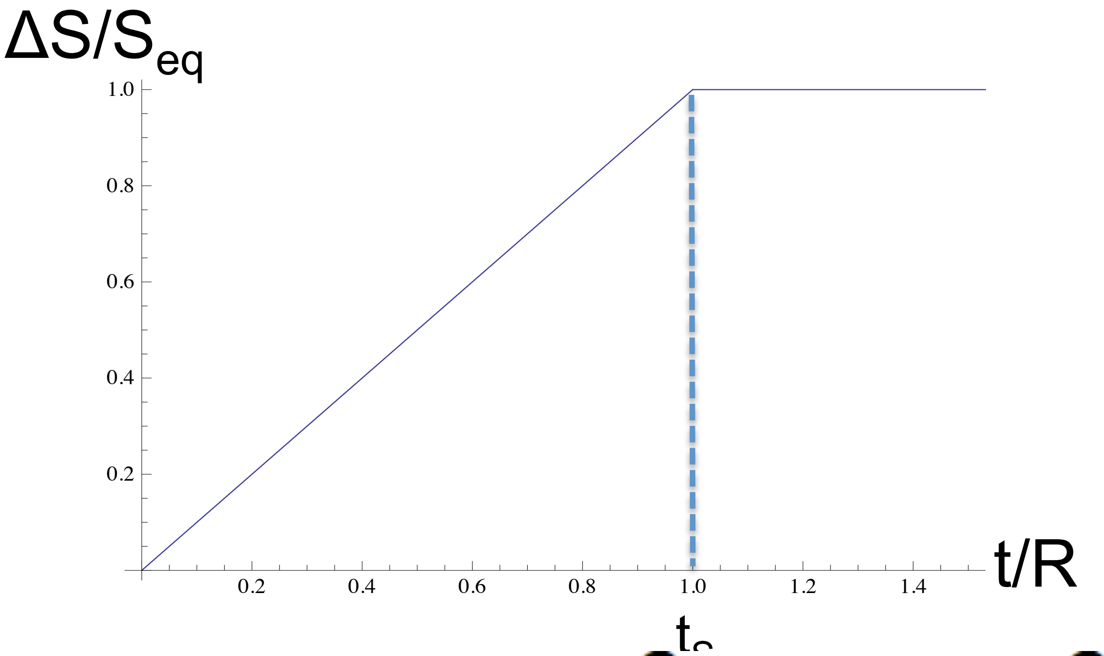
Equilibrium
all entangled

How? at what rate is entanglement generated?

$$\Delta S_A(t) = S_A(t) - S_A(t=0)$$

Previous results in (1+1)-d CFTs

Calabrese and Cardy

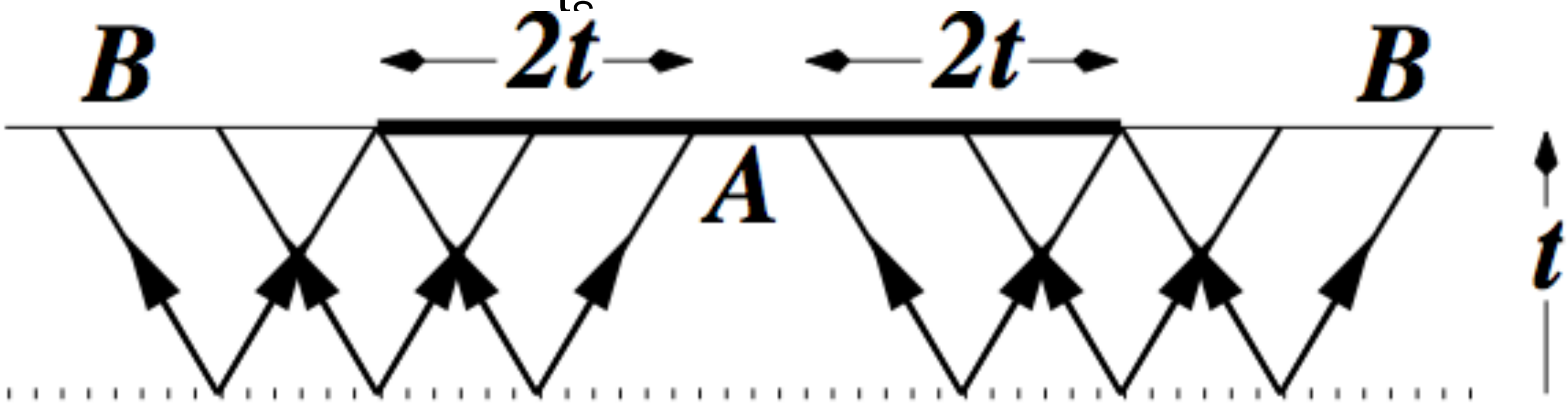


1. Linear growth with time

   (not too early and too late)

2. Slope = 1

3. Can be reproduced by free particles



Special techniques in one spatial dimension do not apply to higher dimensions:

- Equilibration processes: complicated non-equilibrium many-body dynamics, generally out of theoretical control.

- Entanglement entropy is notoriously difficult to calculate even for simple regions in the vacuum of a free theory, not to mention for general regions in interacting theories far from equilibrium.

String theory to the rescue!

Important earlier work:

Hubeny, Rangamani, Takayanagi: arXiv:0705.0016

Abajo-Arrastia, Aparicio and Lopez, arXiv:1006.4090

Albash and Johnson, arXiv:1008.3027

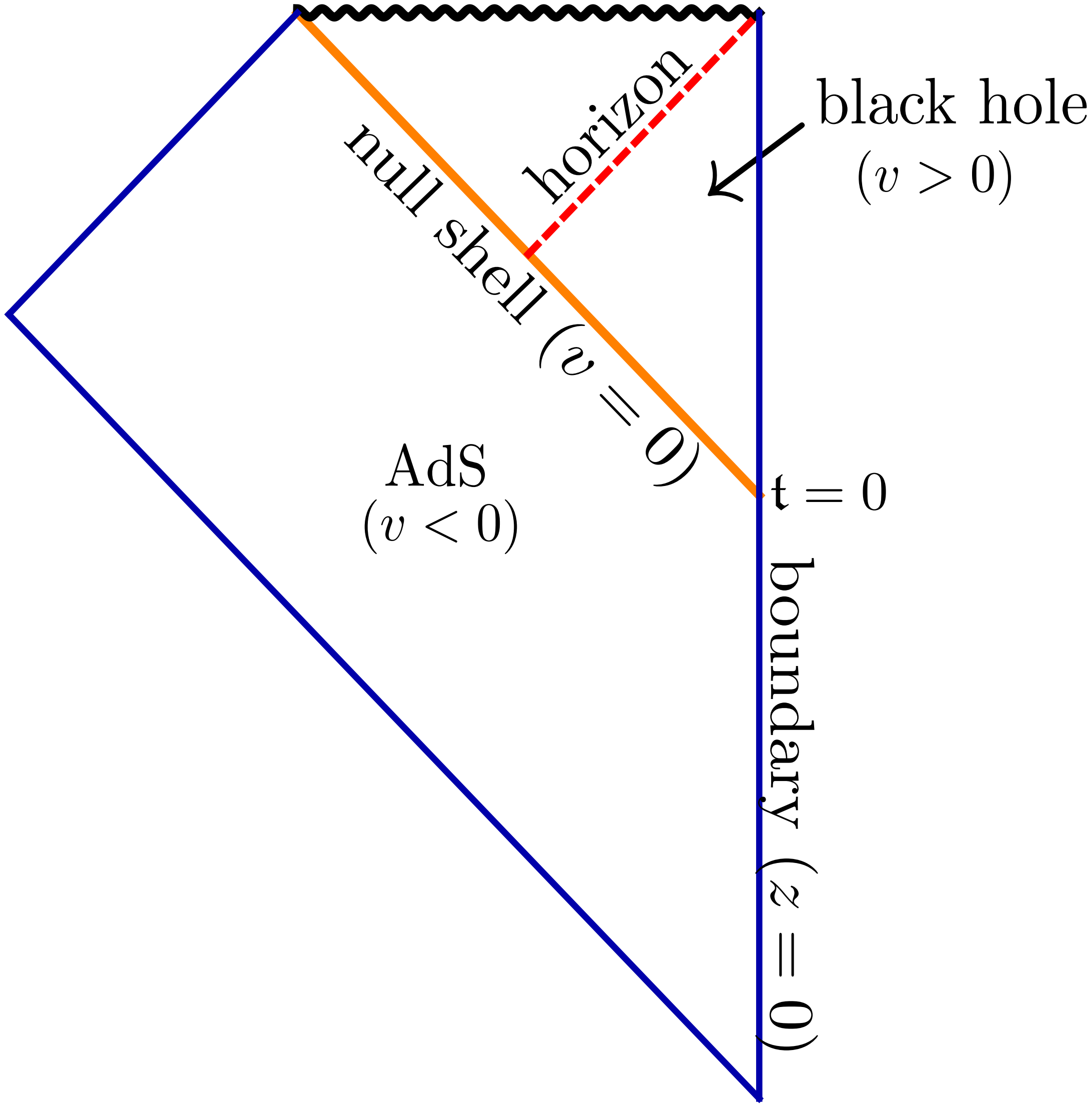
Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Muller and Schafer, Shigemori, Staessens arXiv:1012.4753, arXiv:1103.2683

Aparicio and Lopez, arXiv:1109.3571

Caceres and A. Kundu, arXiv:1205.2354

....

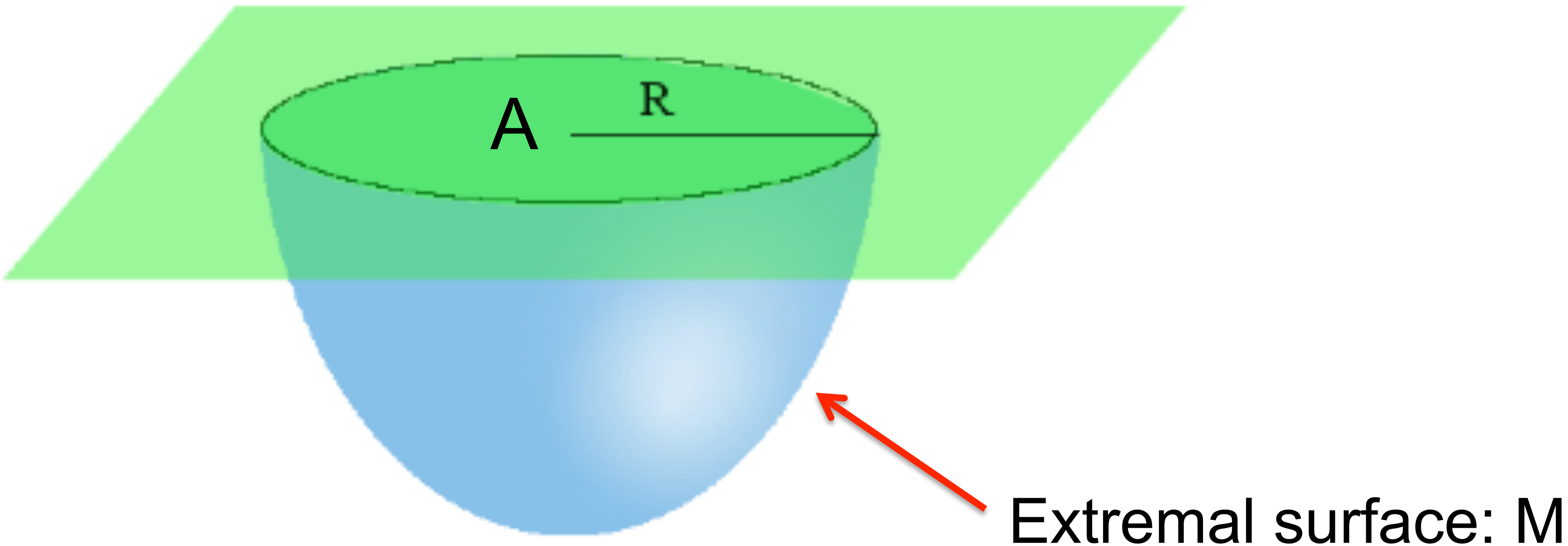
Holographic description of quench



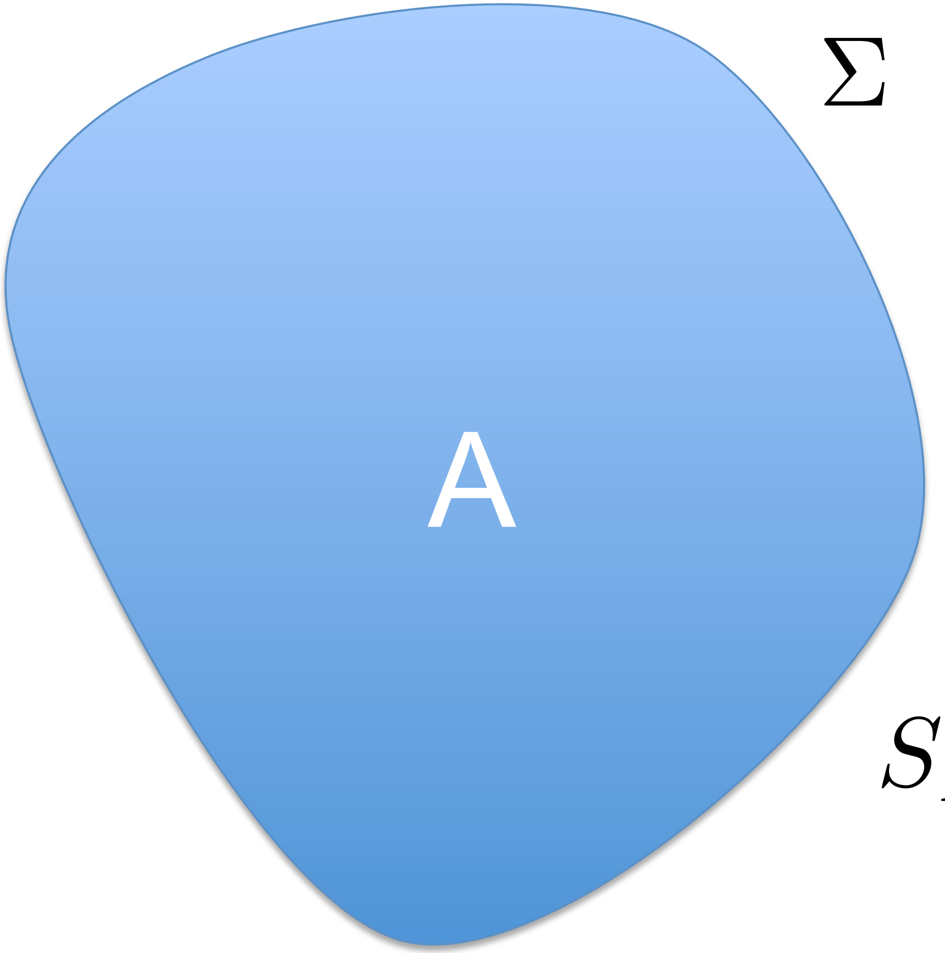
quench: thin shell collapse to form a black hole.

Holographic Entanglement entropy

Ryu, Takayanagi
Hubeny, Rangamani, Takayanagi



$$S_A = \frac{\text{Area of M}}{4\text{G}_\text{N}}$$



Area: A_Σ

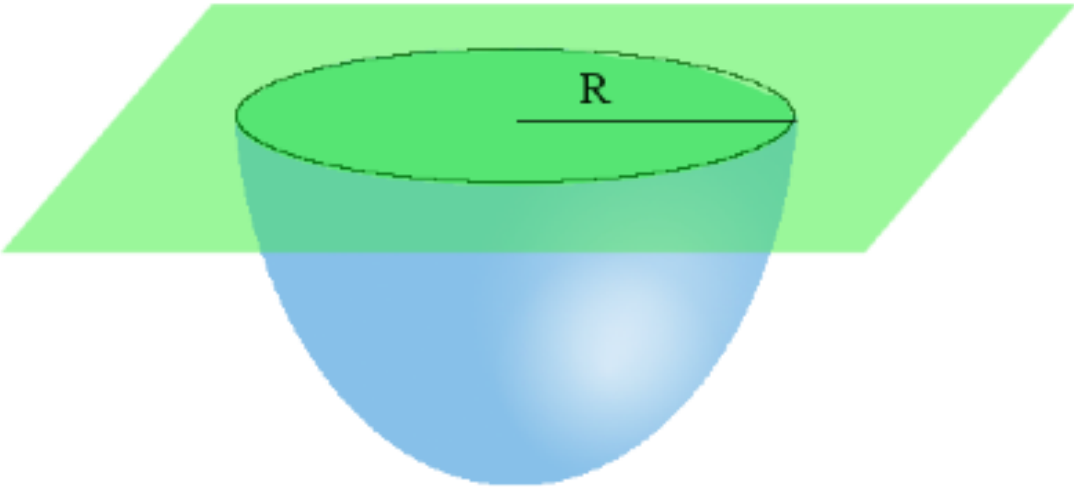
Volume: V_A

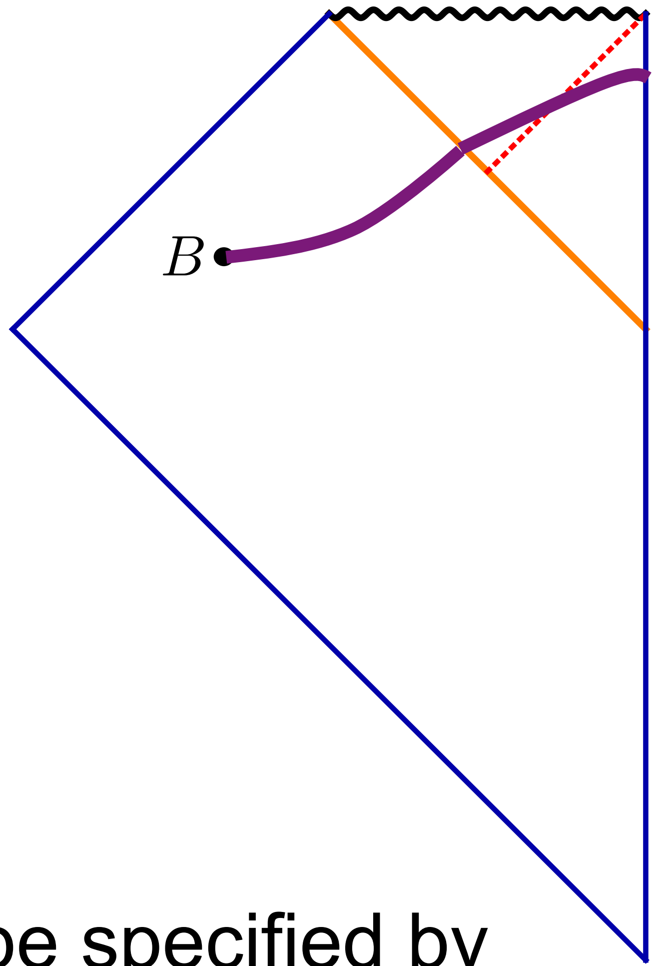
$S_A(t)$?

R: characteristic size of the region

Interested in long-distance physics: $R \to \infty$

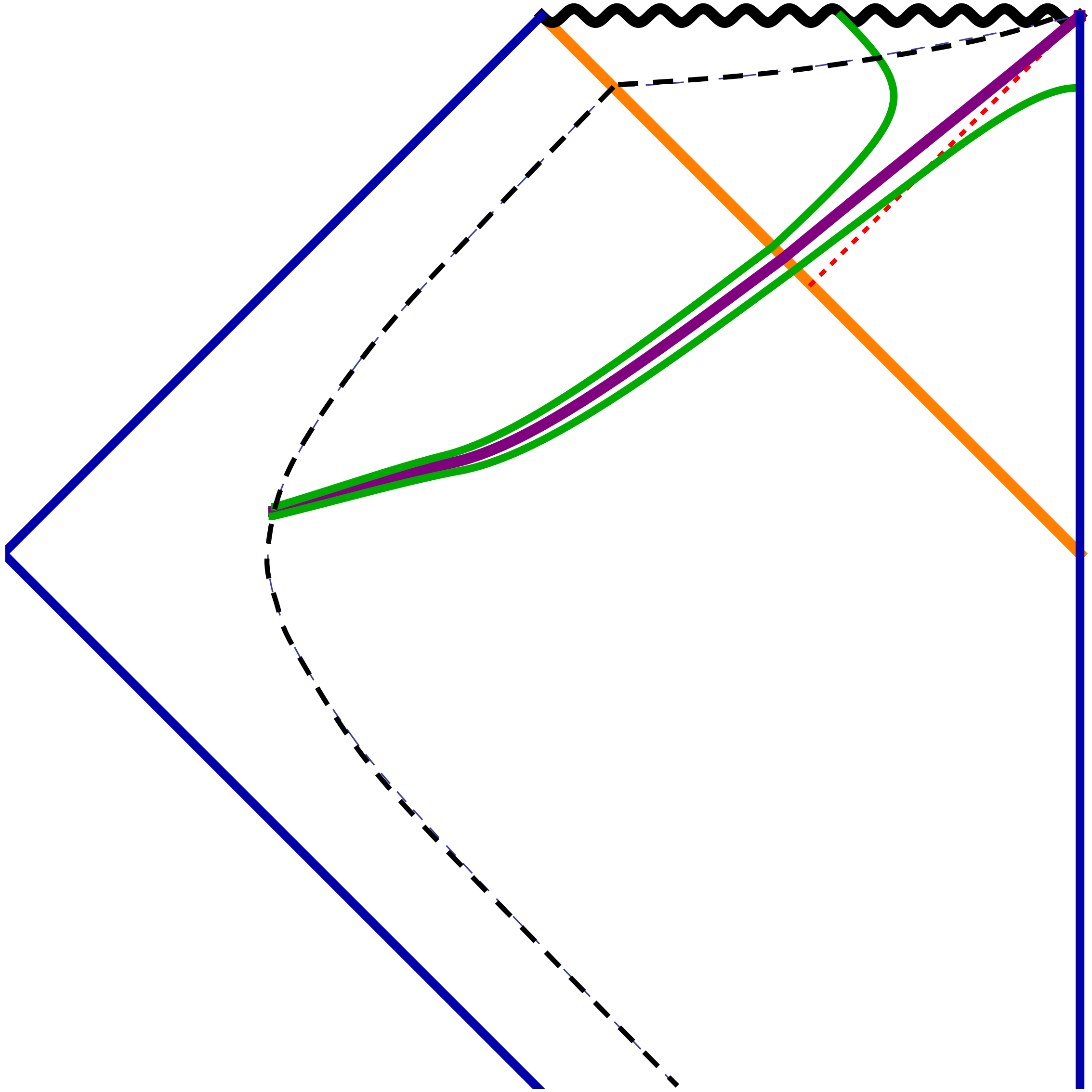
Gravity description





Each extremal surface can also be specified by conditions at the tip.

Large size and critical extremal surfaces

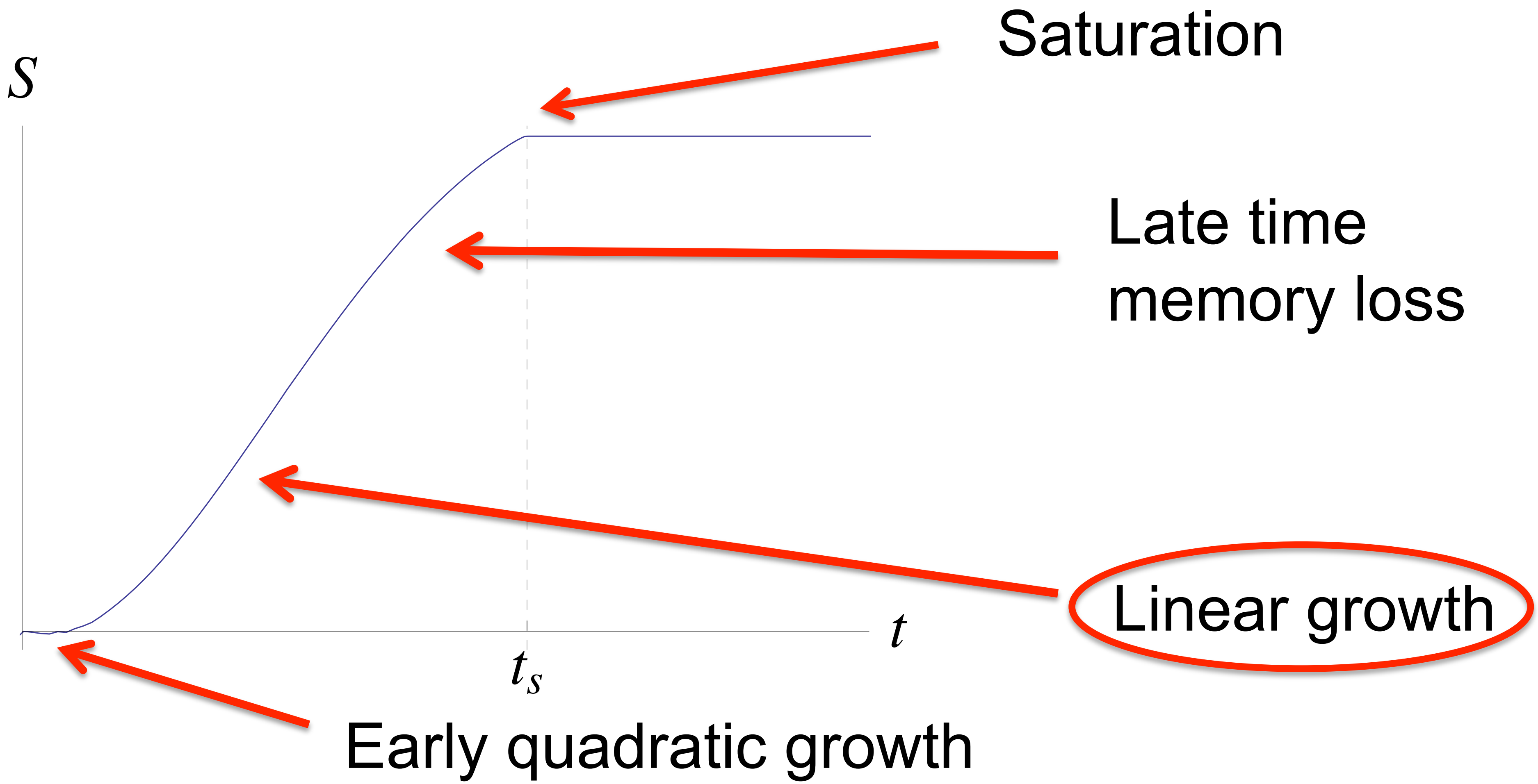


In general a rather complicated problem to determine time evolution of extremal surfaces

Critical extremal surfaces determine large R, large time behavior

Four scaling regimes in general dimensions

In the large size R limit: $R \gg 1/T$



Linear growth

For $R \gg t \gg 1/T$

See also
Hartman, Maldacena

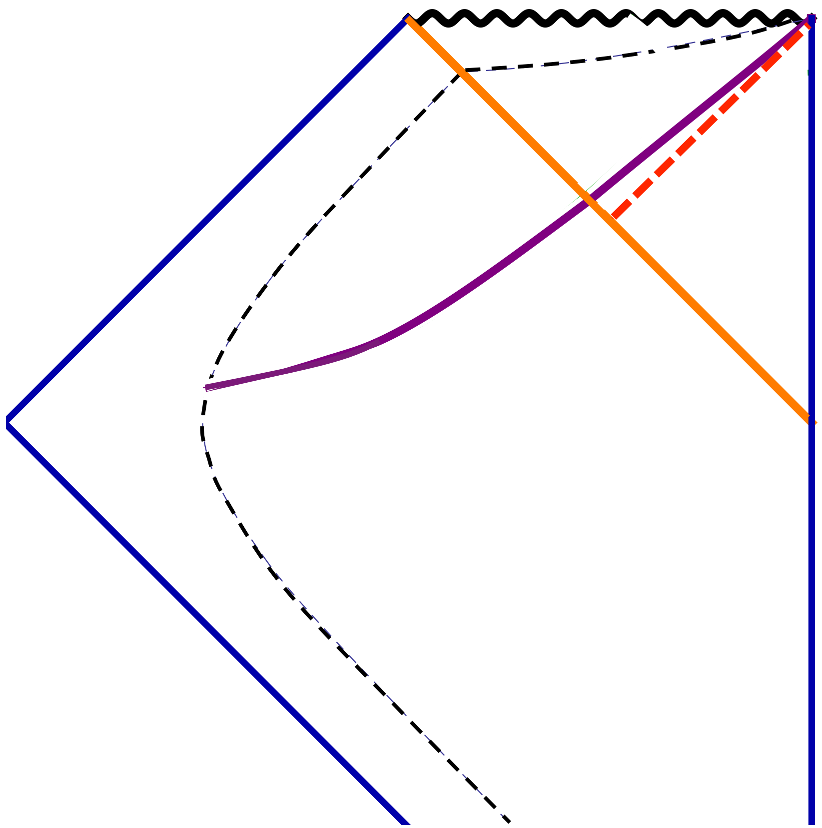
$$\Delta S_A(t) = v_E \, s_{\rm eq} \, A_\Sigma t + \cdots$$

$s_{\rm eq}$: Equilibrium entropy density

independent of shape, holographic theories under consideration, the nature of equilibrium state, also likely thermalization processes

v_E: dimensionless number characterizing final eq state.

Critical extremal surface for linear growth



The critical extremal surface runs along a constant radial slice inside the horizon

$$ds^2 = \frac{L^2}{z^2}\left(-h dt^2 + \frac{1}{f}dz^2 + d\vec{x}^2\right)$$

$$z_m : \ \text{minimum of} \ \frac{h(z)}{z^{2D}}$$

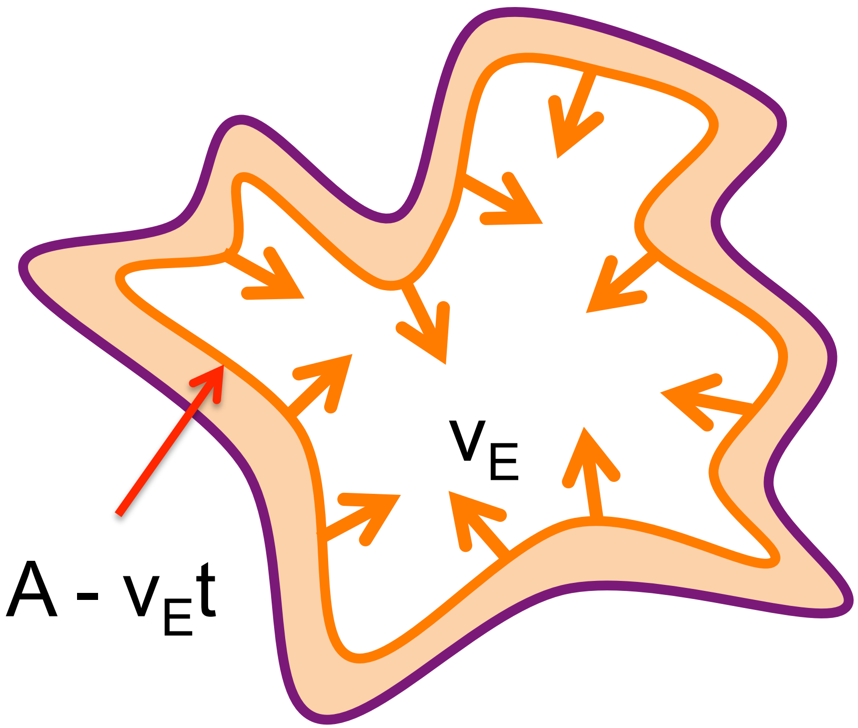
D: # of spatial dimensions

$$v_E = (z_h/z_m)^D \sqrt{-h(z_m)}$$

z_h : horizon size

Entanglement Tsunami

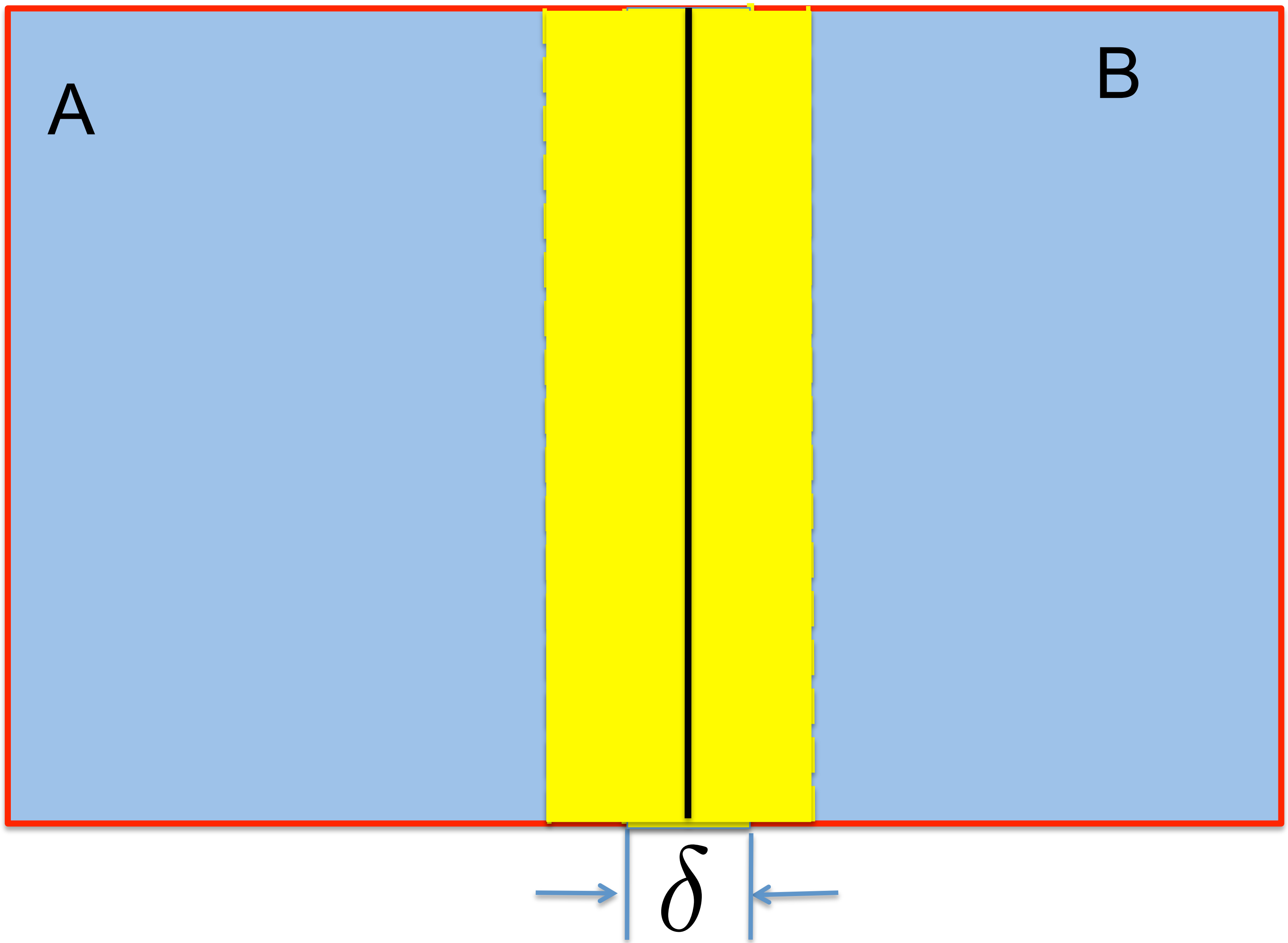
$$\Delta S_A(t) = v_E\, s_{\rm eq}\, A_\Sigma\, t = s_{\rm eq}\, (V_A - V_{A-v_E t})$$



suggests a picture of tsunami wave of entanglement, with a sharp wave front.

d.o.f. in the region covered by the wave is now entangled with those outside A

natural with evolution from a local Hamiltonian


A
B
δ

Tsunami velocity

$$\Delta S_A(t) = v_E \, s_{\rm eq} \, A_\Sigma t + \cdots$$

Neutral system (AdS Schwarzschild):

$$v_E^{(\rm S)} = \frac{(\eta-1)^{\frac{1}{2}(\eta-1)}}{\eta^{\frac{1}{2}\eta}} = \begin{cases} 1 & D=1 \\ \frac{\sqrt{3}}{2^{\frac{4}{3}}} = 0.687 & D=2 \\ \frac{\sqrt{2}}{3^{\frac{3}{4}}} = 0.620 & D=3 \\ \frac{1}{2} & D=\infty \end{cases}$$

$$\eta \equiv \frac{2D}{D+1}$$

Turning on chemical potential reduces v_E.

Upper bound on v_E ?

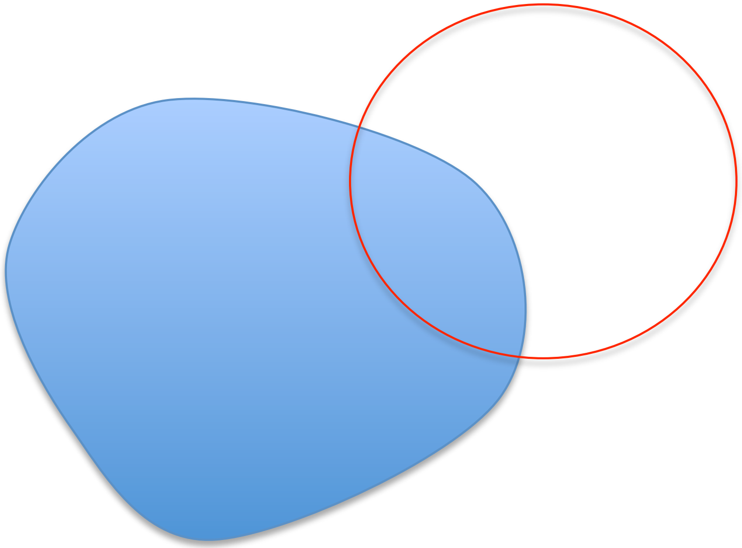
v_E should be constrained by causality.

In all gravity examples:

$$v_E \leq v_E^{(\mathrm{S})} = \frac{(\eta - 1)^{\frac{1}{2}(\eta-1)}}{\eta^{\frac{1}{2}\eta}} \qquad \eta \equiv \frac{2D}{D+1}$$

Null energy condition important

Comparing with free particle streaming



Assume:

- At t=0, there is a uniform density of "photons" with only local entanglement correlations.
- Entanglement spreads when photons propagate.

Leading to shape independent linear growth,

$$\Delta S_\Sigma(t) = v_E s_{\rm eq} A_\Sigma t + \cdots$$

For D=1:

$$v_{\text{streaming}} = v_{\text{CFT}} = v_{\text{gravity}} = 1$$

$$D \geq 2$$

$$v_{\text{streaming}} = \frac{\Gamma(\frac{D}{2})}{\sqrt{\pi}\Gamma(\frac{D+1}{2})} < \quad v_E^{(S)} \quad < 1$$

In strongly coupled systems, entanglement tsunami propagates faster than those from free particles traveling at speed of light !

$$D \to \infty : \; v_E^{(S)} \to \frac{1}{2}, \; v_{\text{streaming}} \to \sqrt{\frac{2}{\pi(D+1)}} \to 0$$

Bound on entanglement growth?

For any non-equilibrium processes:

$$\mathfrak{R}_A(t) \equiv \frac{1}{s_{\mathrm{eq}} A_\Sigma} \frac{dS_A}{dt}$$

dimensionless, can be compared among region A of different shapes, sizes, and systems of different number of d.o.f.

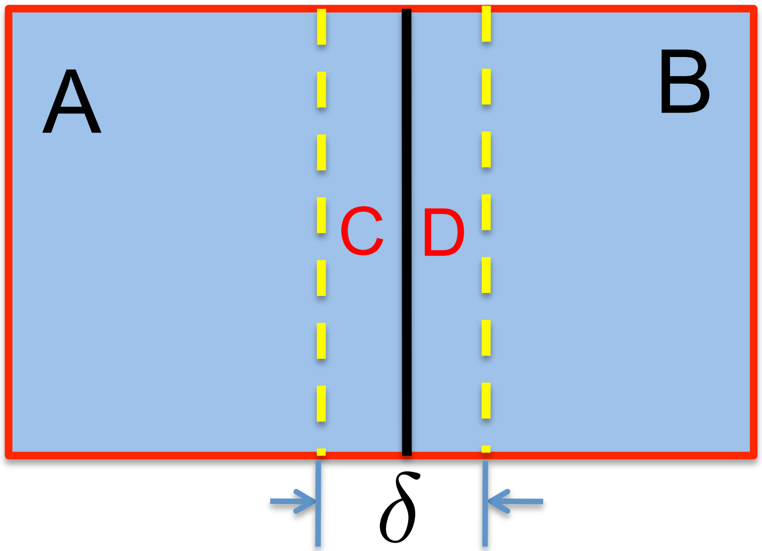
Indications from gravity: after local equilibration (t >>1/T)

$$\mathfrak{R}_A(t) \leq v_E^{(S)}$$

Comparing with small incremental entangling conjecture/theorem:

$$\frac{dS_A}{dt} \leq v_E^{(S)} \, s_{\rm eq} A_\Sigma$$

$$\frac{dS_A}{dt} \leq c||H|| \log d, \qquad d = \min(d_C, d_D)$$



δ

Future directions

- More examples:

  Both holographic and field theoretical

- Continuum limit of small incremental conjecture

- Implications for black hole physics

Thank You

Supplementary slides

Local equilibration scale

The system should locally equilibrate first at a time scale $\ell_{\rm eq}$ after which thermodynamics should apply locally.

Gravity side: horizon formation

Holographic systems: $\ell_{\rm eq} \sim \frac{1}{T} \ \ (\mu = 0)$

At $t \sim \ell_{\rm eq}$ nonlocal observables, such as $S_A(t)$ with $R \gg \ell_{\rm eq}$ may still be far from their equilibrium values.

Pre-local-equilibration evolution

For $t \ll \ell_{\rm eq}$

$$S(t) - S(0) = \frac{\pi}{D} \epsilon \, A_{\Sigma} \, t^2 + \cdots$$

ϵ : energy density A_{Σ}: area

independent of shape, independent of theories under consideration (for relativistic theories), and independent of the nature of final equilibrium state.

Saturation as a "phase transition"

Saturation: the tsunami covers the whole region:

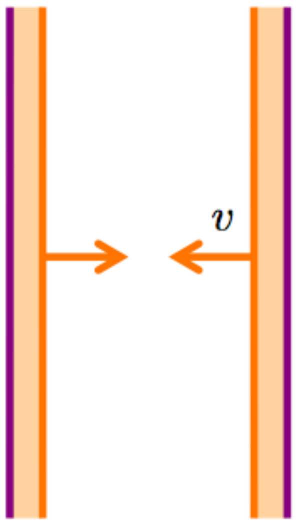
In the limit quench is sharp, there is a sharp saturation time $t_s(\Sigma)$.

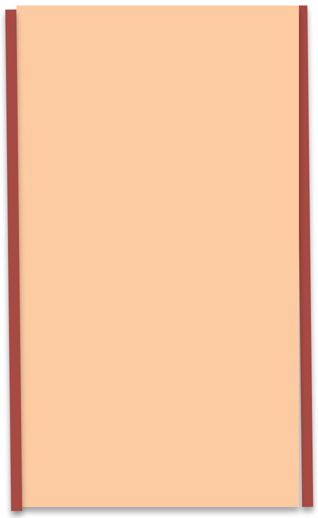
- Discontinuous saturation: first derivative of S(t) is discontinuous.

- Continuous saturations are characterized by critical behavior at saturation

$$S_\Sigma(t) - S_\Sigma^{(\mathrm{eq})} \propto -(t_s - t)^\gamma$$

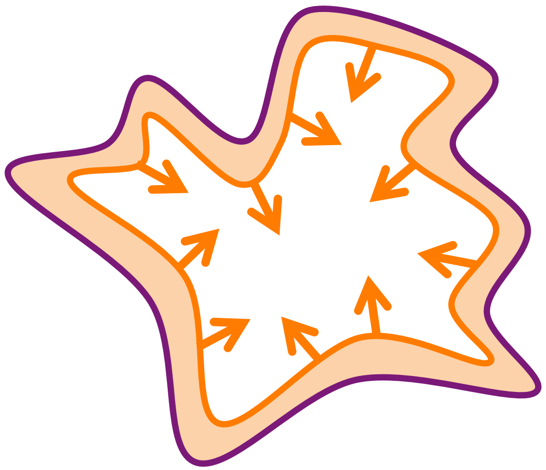
Saturation: simple geometries





strip: discontinuous, linear growth until saturation

$$\text{Strip}: \ t_s = \frac{R}{v_E}$$



Generic shape: curvature effects should be important, e.g. for a sphere, continuous saturation,

$$t_S = \frac{1}{c_{\text{sphere}}} R - \frac{D-1}{4\pi T} \log R + O(R^0)$$

Memory loss regime

For $t_s \gg t_s - t \gg \ell_{\rm eq}$ with Σ a sphere of radius R

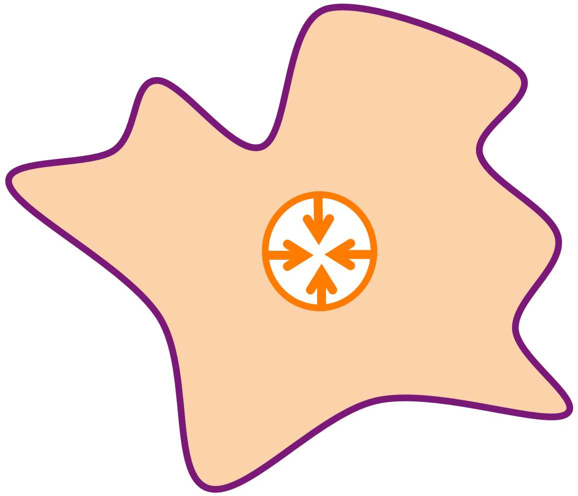
$$S(R,t) - S_{\rm eq}(R) = -s_{\rm eq}\lambda\left(t_s(R) - t\right)$$

λ : volume of region not yet entangled

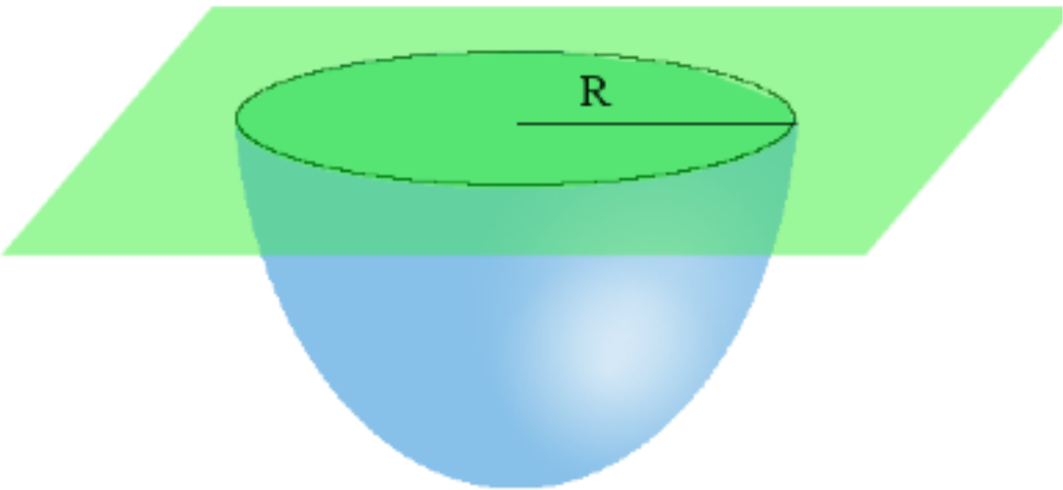
Independent of the size of the region: size information lost

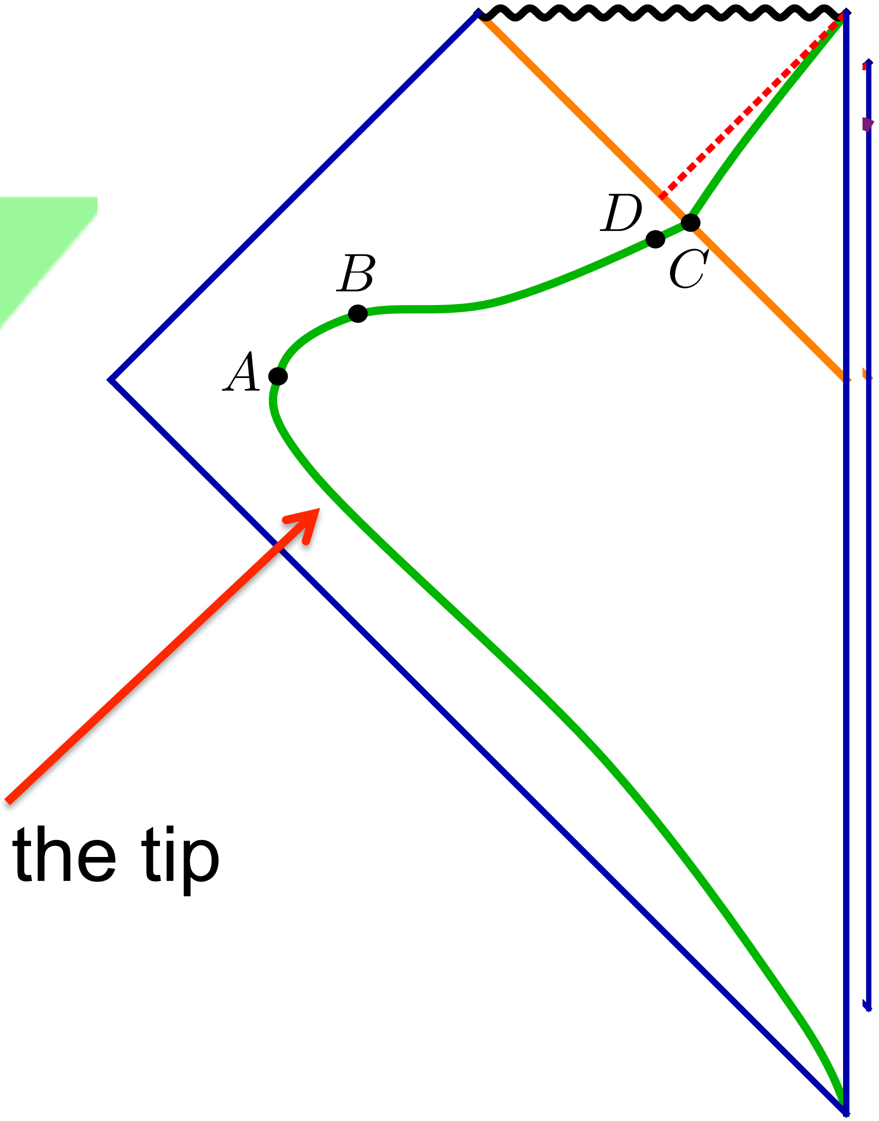
Speculation for generic shape :

Both shape and size will be forgotten at late times, flow to a "fixed point."



Gravity description





Trajectory of the tip
over time

Gravity description