Holographic Anyonic Superfluids

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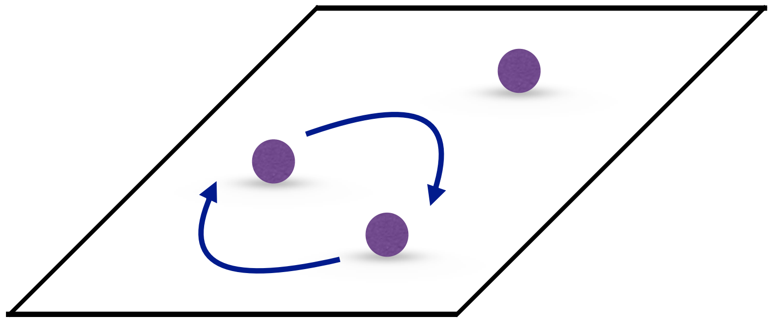
based on 1307.6336 and 1407.3794

Plan

- Anyons and $SL(2,\mathbb{Z})$
- Anyon Superfluids
- A Holographic Anyon Superfluid
- A Flowing Holographic Anyon Superfluid

Anyons

particles in 2+1 dim can have arbitrary statistics



$$|\psi_1\psi_2\rangle = e^{i\theta}|\psi_2\psi_1\rangle$$

- $\theta = 0$ bosons
- $\theta = \pi$ fermions
- $\theta = \pi\, p/q$ anyons

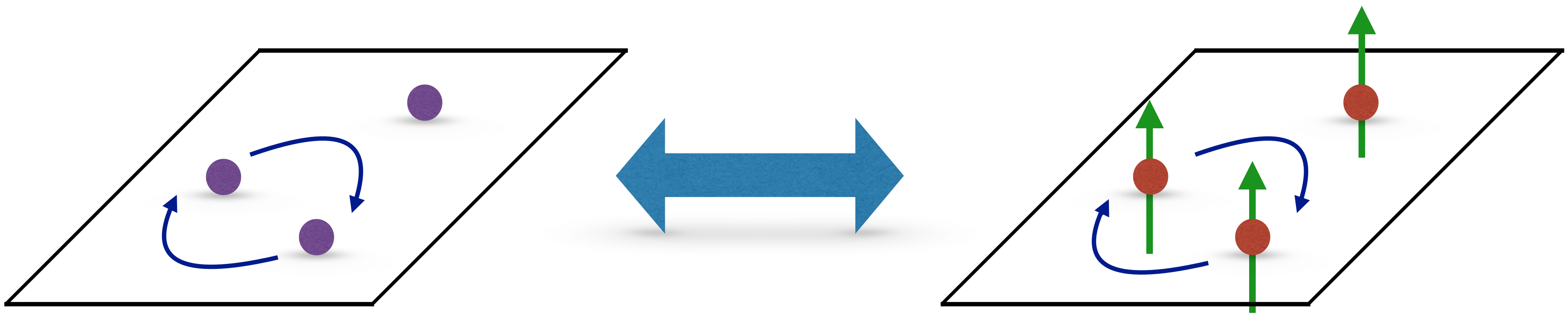
Leinaas, Myrheim

Wilczek

Alternate Description

charged particles with $\boldsymbol{n}$ magnetic fluxes attached

statistical phase $\theta \longleftrightarrow$ Aharonov-Bohm phase $\boldsymbol{\pi n}$



anyons

fermions with $\boldsymbol{n}$ fluxes

$$\theta_{stat} = \pi\,(1 - n)$$

$$\theta_{stat} + \theta_{AB} = \pi - \pi n$$

Flux attachment and $SL(2,\mathbb{Z})$

2+1 dim CFT

Witten
Burgess, Dolan

- $U(1)$ current - J
- external vector - $\mathcal{A}$
- define $\mathcal{B} = \frac{1}{2\pi} * d\mathcal{A}$

mapping to CFT'

- add Chern-Simons term for $\mathcal{A}$: $J' = J + \mathcal{B}$
- make $\mathcal{A}$ dynamical: $J' = \mathcal{B}$
- generate $SL(2,\mathbb{Z})$ $\left(\begin{array}{c} J' \\ \mathcal{B}' \end{array} \right) = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \left(\begin{array}{c} J \\ \mathcal{B} \end{array} \right)$

flux attachment: $\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$

Superfluids

- Flow without resistance
- For example:
  - Liquid ^{4}He, $T < 2.17$K
  - Holographic dual of hairy BH
- Spontaneously broken global symmetry

↓

massless mode

^{4}He fountain

Anyon Superfluids

Anyons in $B = 0$ ➡ Superfluid **Laughlin**

Start with:

- QH fluid of fermions, filling fraction ν
- Background E_x ➡ Hall current $J_y = \frac{\nu}{2\pi} E_x$

$SL(2,\mathbb{Z})$ with $\frac{d}{c} = \frac{\nu}{2\pi}$

$$J'_y = d\ J_y \neq 0 \quad \text{current}$$

$$B' = E'_x = 0 \quad \text{no sources}$$

$$\theta' = \pi\left(1 - \frac{1}{\nu}\right) \quad \text{anyons}$$

Superfluidity without symmetry breaking

Usually:

massless mode

spontaneous symmetry breaking

But, no symmetry breaking in this case

Superfluidity without symmetry breaking

Usually:

massless mode ⟷ spontaneous symmetry breaking

But, no symmetry breaking in this case

For anyons:

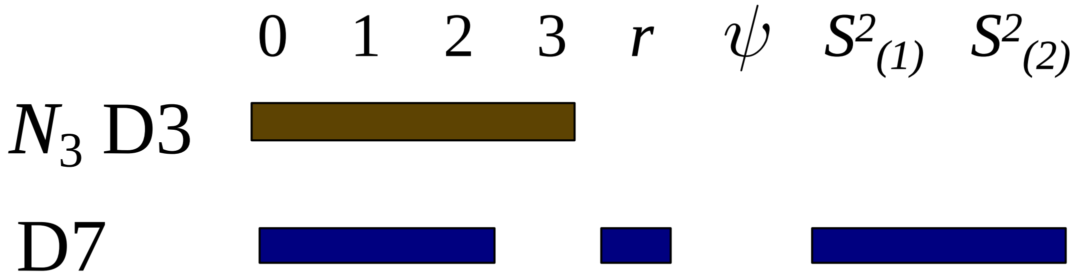
massless mode ⟷ spontaneous **fact** violation

$$[T_x, T_y] \neq 0$$

Chen, Wilczek, Witten, Halperin, Giddings

Holographic QH model: D3-D7′

Bergman, NJ, GL, ML



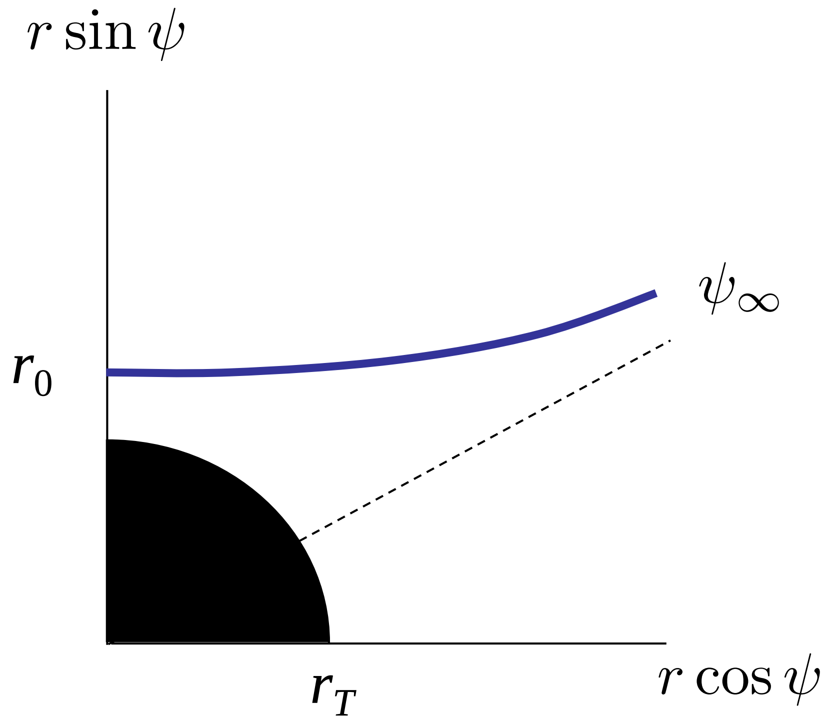
Probe D7:

- fermions on 2+1 dim defect
- wraps $S^2 \times S^2 \subset S^5$
- embedding $\psi(r)$
- ~~SUSY~~

Gauge Field:

$F_{\theta_1 \phi_1}$ - stabilization
F_{r0} - charge density $J_0 = D$
F_{12} - magnetic field B

Minkowski Embedding - QH state



- $$\nu = 2\pi \frac{D}{B} = 1 - \frac{2\psi_\infty}{\pi} + \frac{1}{4}\sin(4\psi_\infty)$$
- gapped
- $\sigma_{xx} = 0$ and $\sigma_{xy} = \dfrac{\nu}{2\pi}$

Alternative Quantization

Dirichlet conditions:

A fixed at boundary ➔ $\mathcal{B}$ fixed

Neumann conditions:

$\partial_r A$ fixed at boundary ➔ J fixed

$$\mathcal{B}' = J, \quad J' = -\mathcal{B}$$

General mixed conditions:

fix a linear combination of J and $\mathcal{B}$

implements $SL(2,\mathbb{Z})$

Fluctuations

Vary δJ' with fixed $\mathcal{B}'$

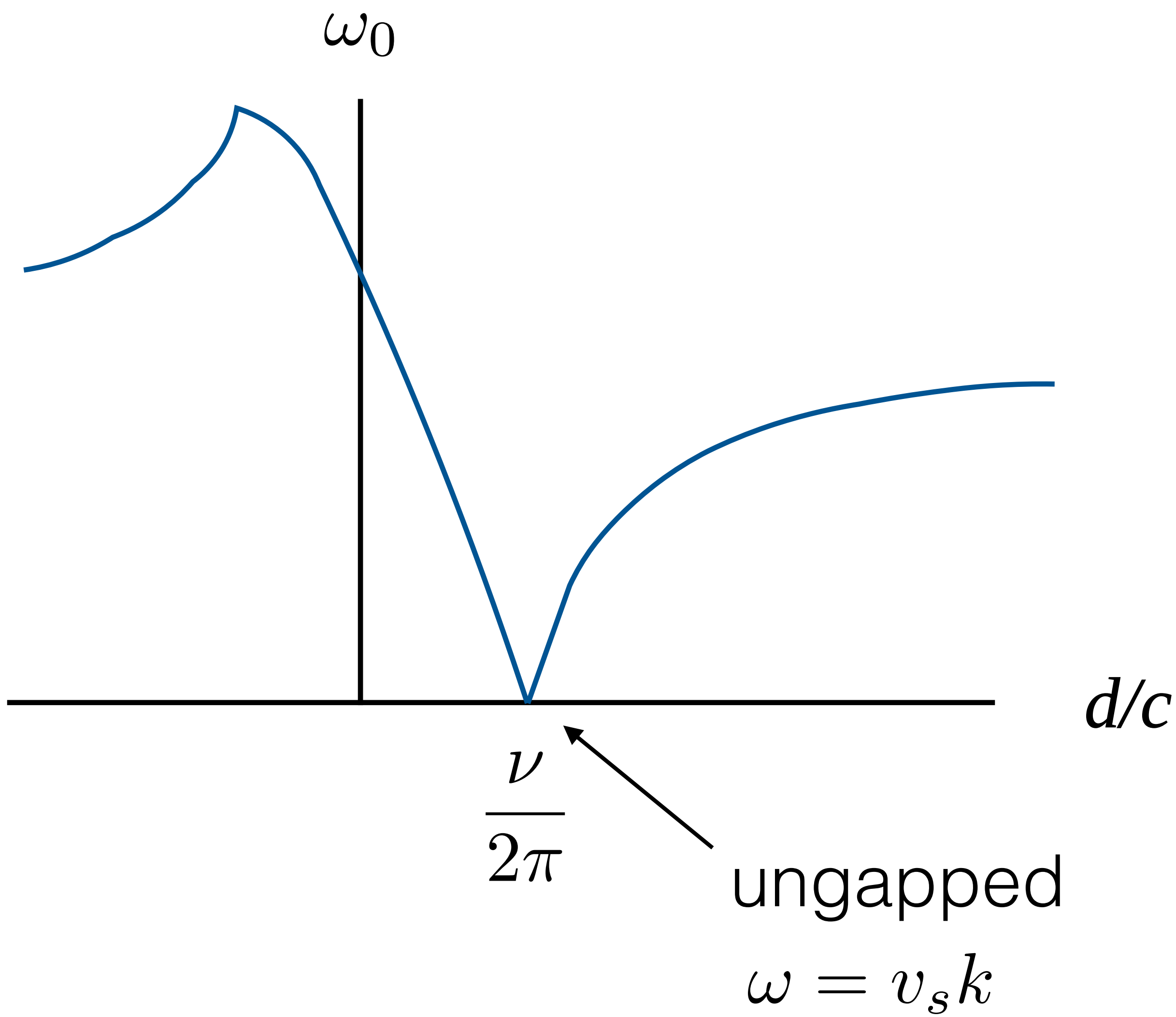
In original $SL(2,\mathbb{Z})$ variables:

$$0 = \delta B' = c\ \delta D + d\ \delta B$$

$$\frac{\delta D}{\delta B} = \frac{d}{c}$$

for $\frac{d}{c} = \frac{\nu}{2\pi}$ $\Rightarrow$ $\delta\nu = 0$

Mass of δJ' vs. d/c

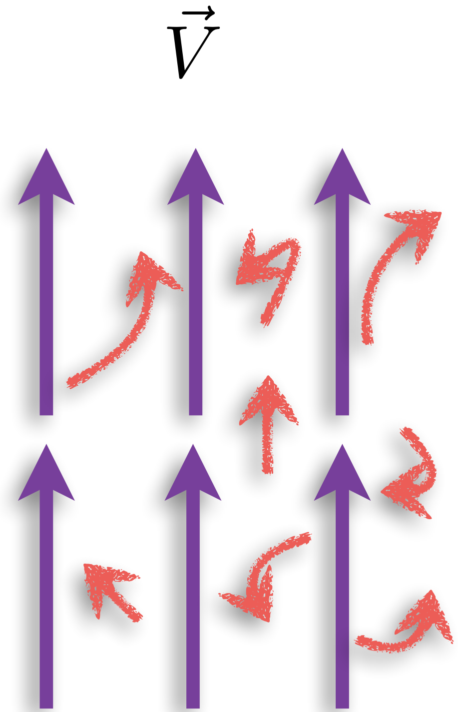


Superfluids can flow

Two component description:

- superfluid with velocity $\vec{V}$
- normal fluid

at low T, gas of phonons



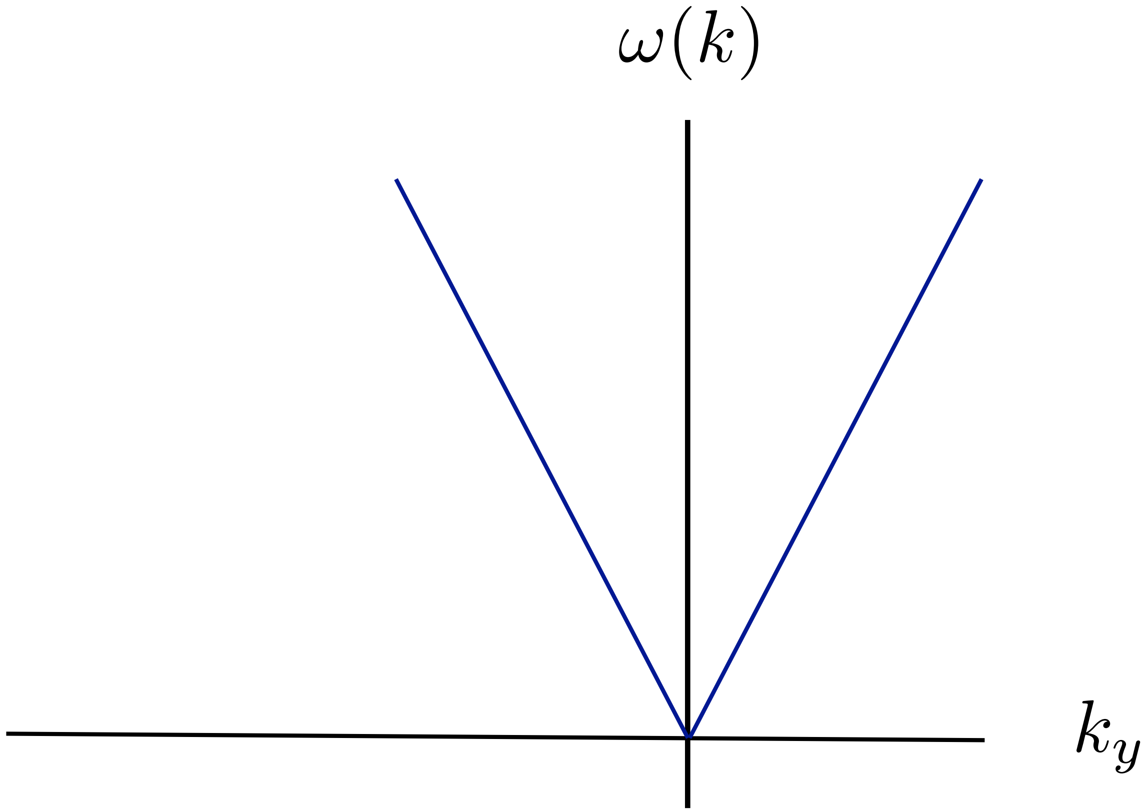
In holographic model:

Electric field E_x Superfluid velocity V_y

Phonon dispersion

$V_y = 0$

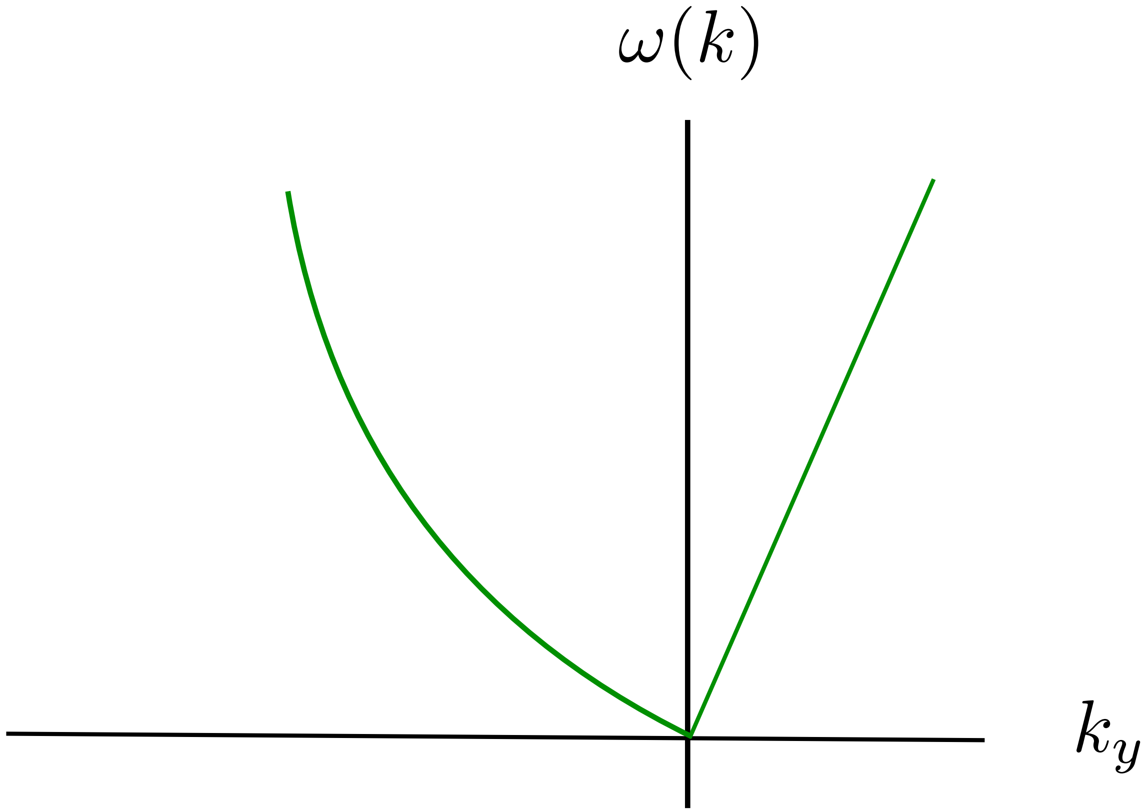
no superflow



Phonon dispersion

$$V_y > 0$$

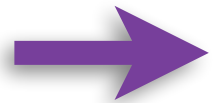
superflow

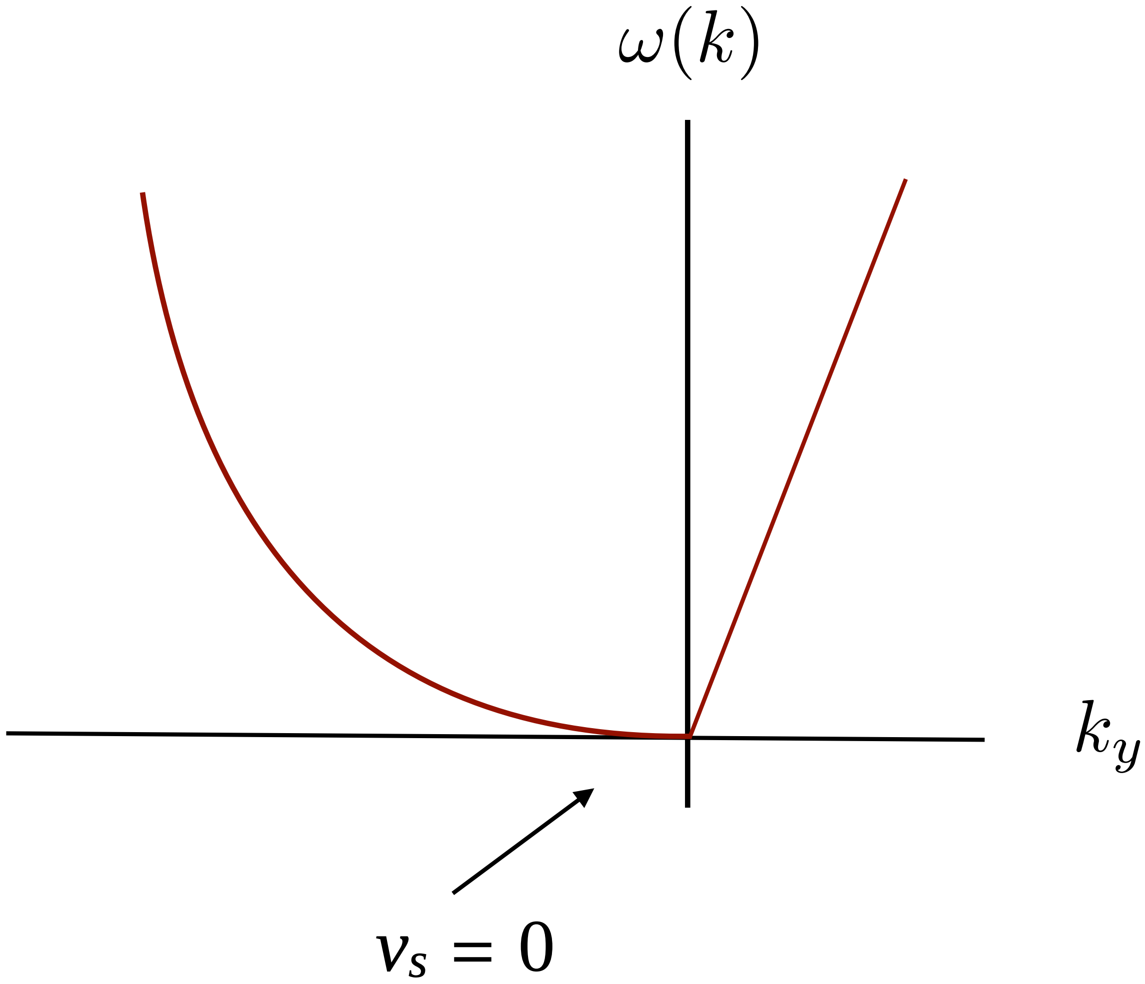


Phonon dispersion

$$V_y = V^{crit}$$

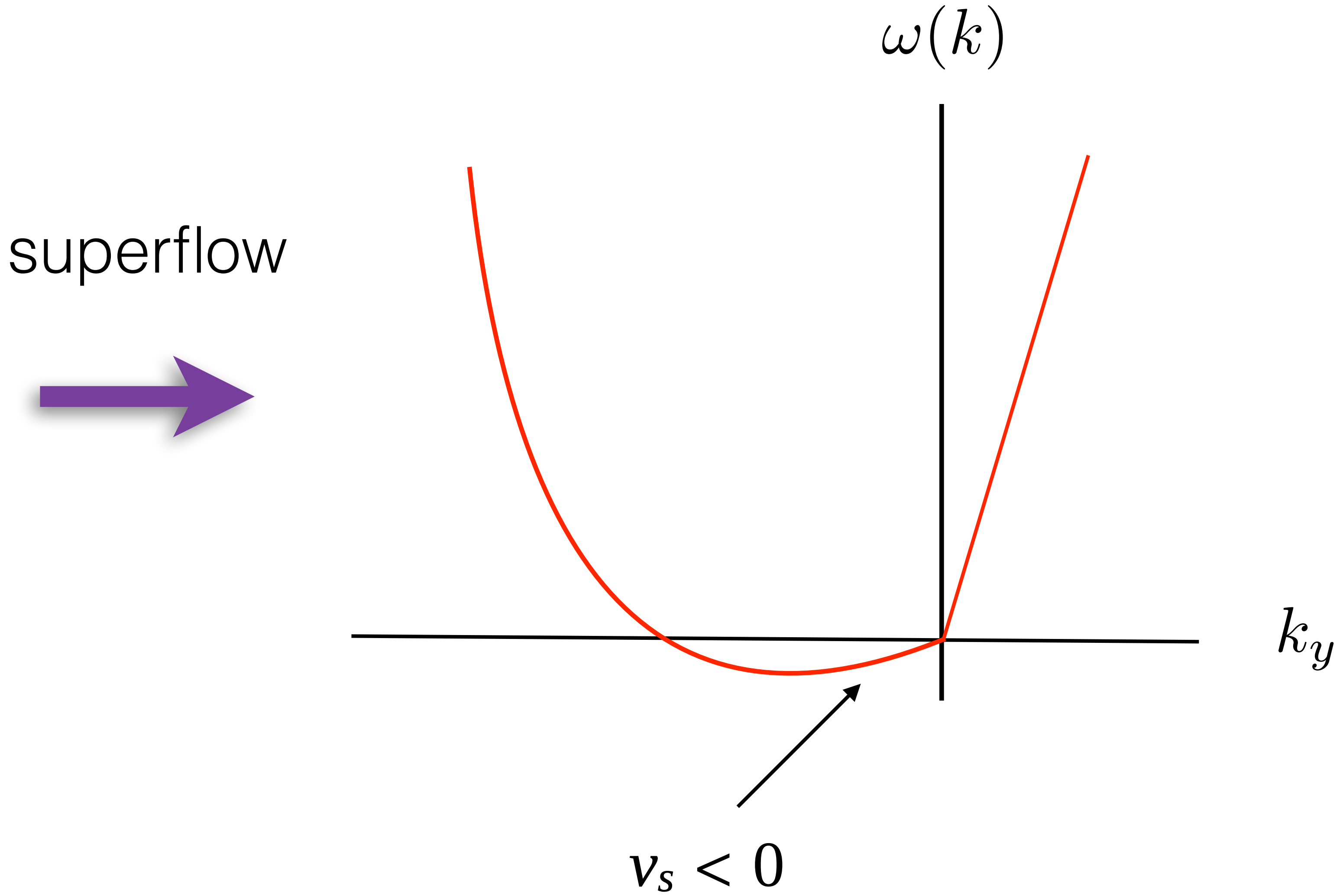
superflow





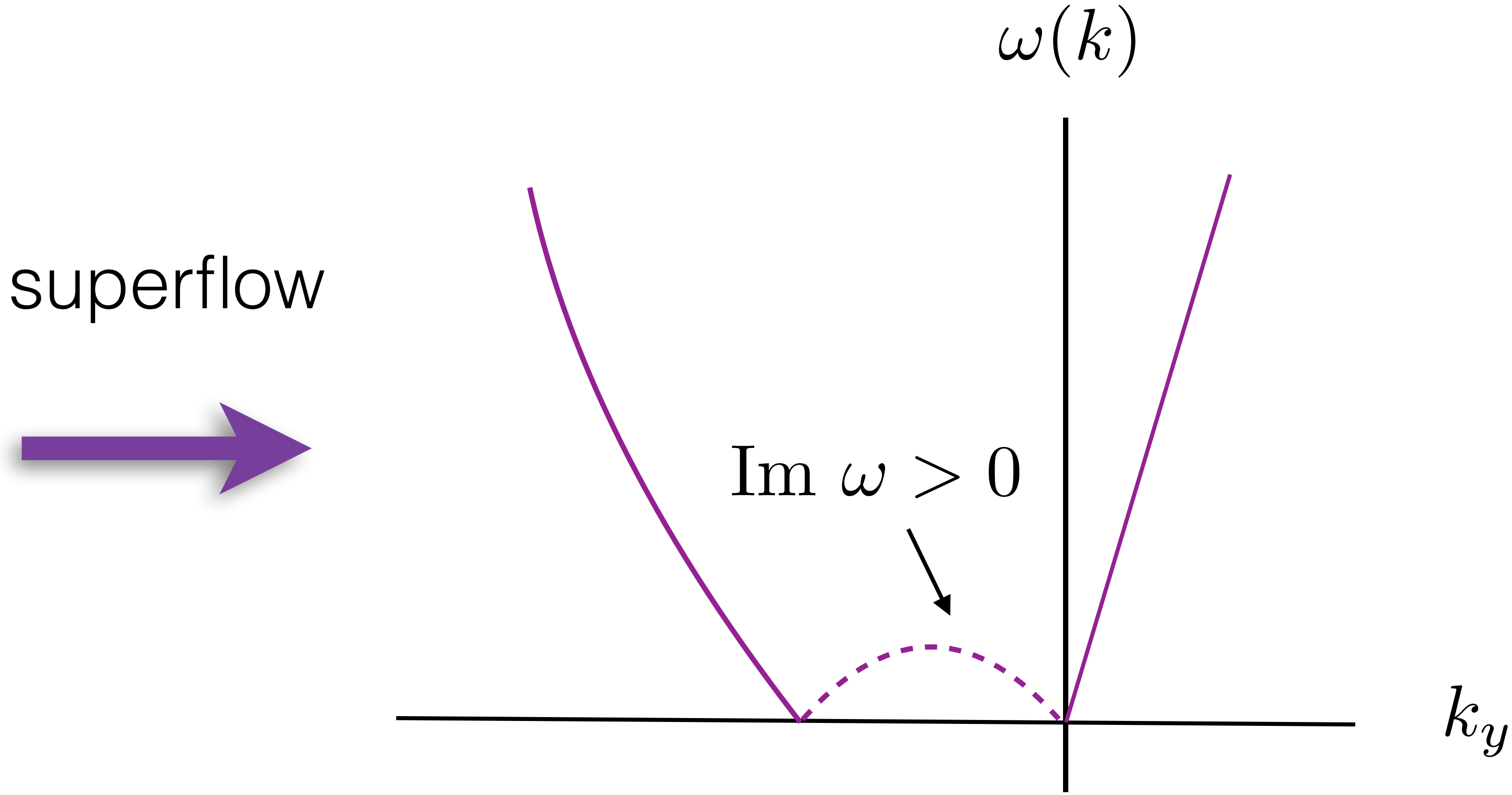
Phonon dispersion

$$V_y > V^{crit}$$

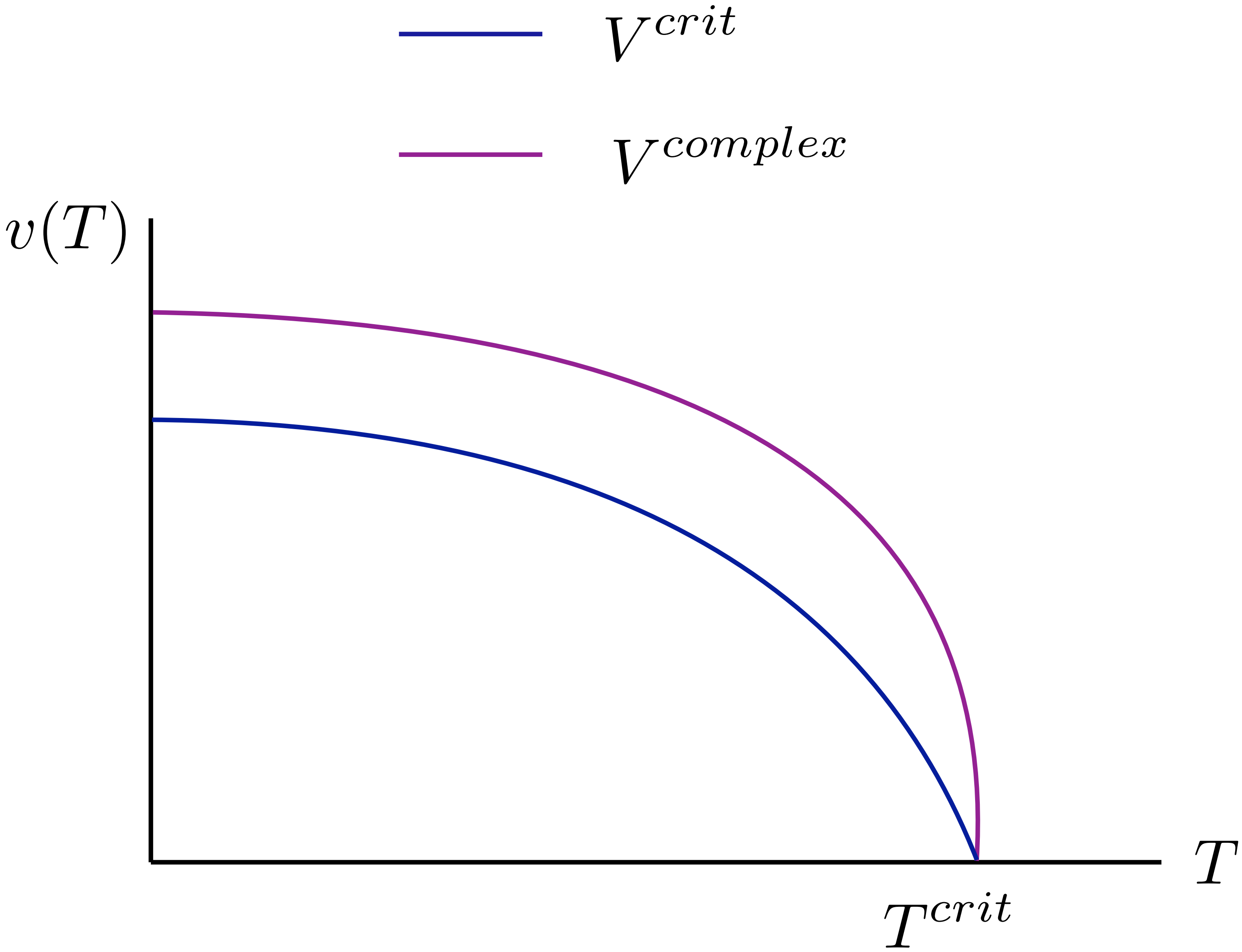


Phonon dispersion

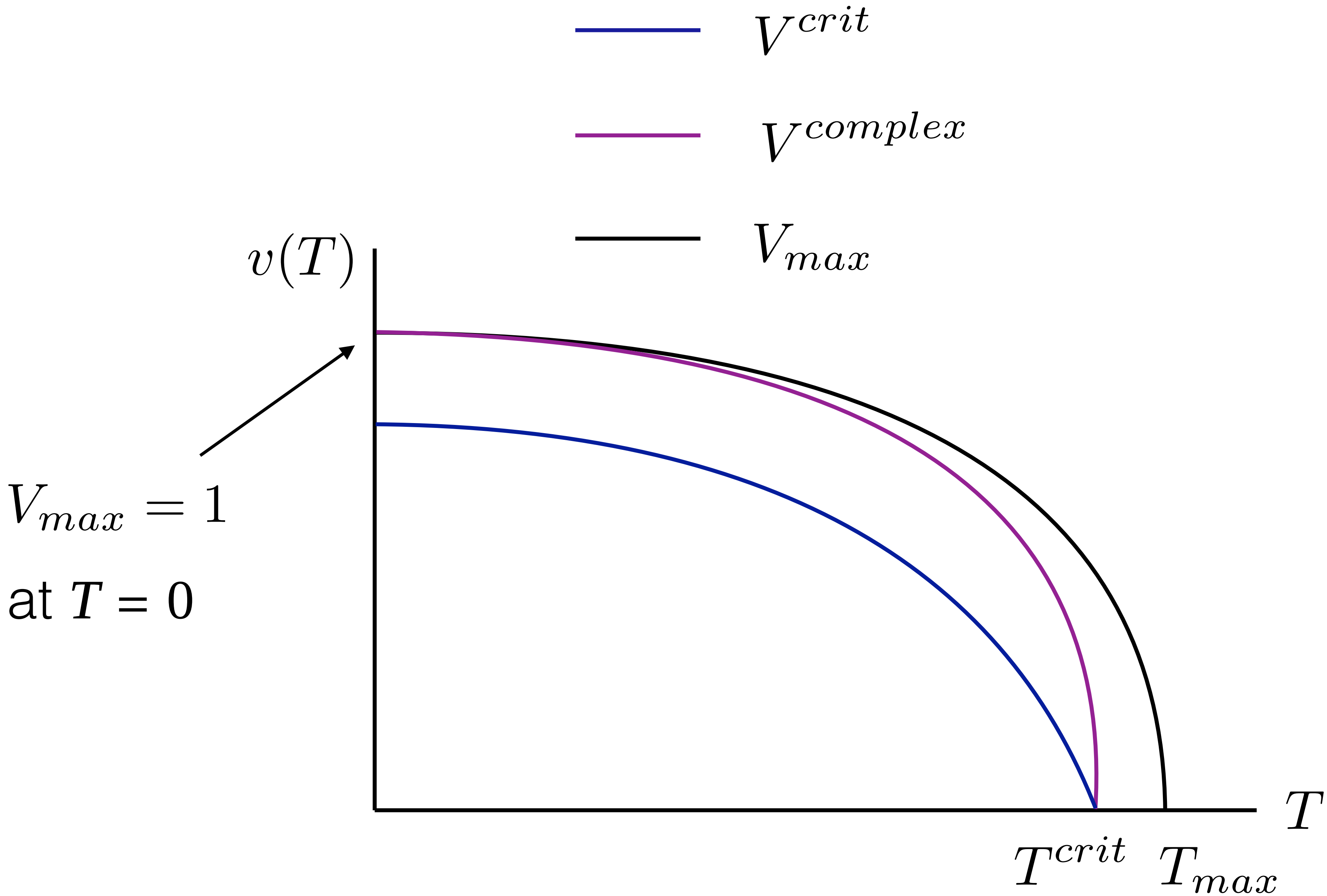
$$V_y > V^{complex} > V^{crit}$$



Velocity vs. Temperature



Velocity vs. Temperature



Summary

- Anyonic superfluid: unconventional superfluid
- Related to QH fluid by $SL(2,\mathbb{Z})$
- Holographic model of strongly-coupled anyon superflulid
    - $T \geq 0$
    - $V^{complex} > V^{crit}$
    - ground state for $V > V^{complex}$?