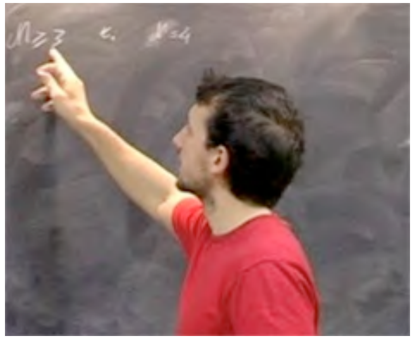
Optical properties of viscous electron liquids

Davide Forcella



Université Libre de Bruxelles

Jan Zaanen

Universiteit Leiden

Davide Valentinis

Université de Genève

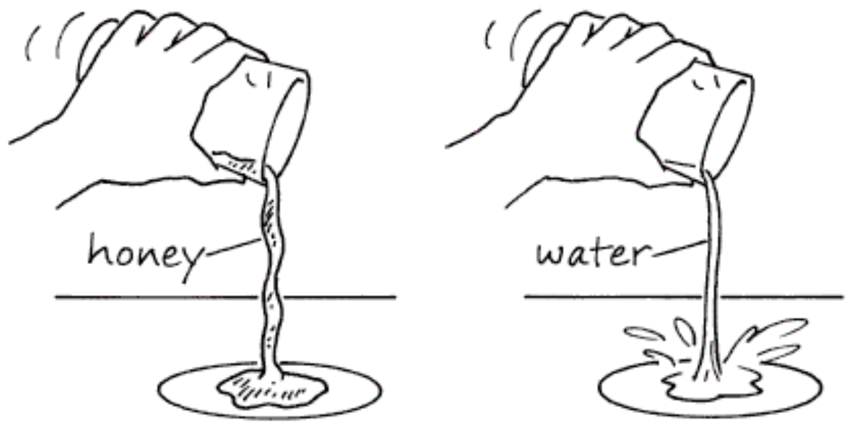
Dirk van der Marel

Université de Genève

Damien Stricker

Université de Genève

Viscosity: a reminder



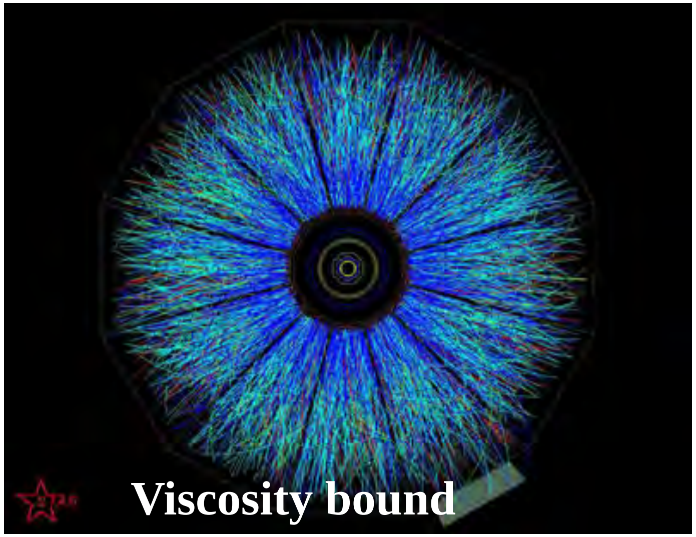
Navier-Stokes of incompressible $\left(\vec{\nabla}\cdot\vec{v}=0\right)$ fluids:

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right)=-\vec{\nabla}p+\eta\nabla^{2}\vec{v}+\vec{f}$$

Accelleration — Convection — Pressure gradient — Viscosity — Body forces

Viscosity: Areas of interest

Quark gluon plasma

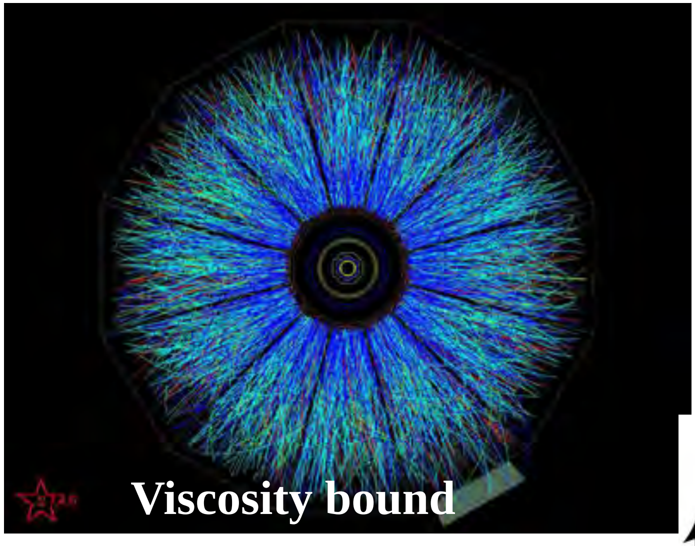


E. Shuryak, Prog. Part. Nucl. Phys. 53, 273 (2004).

A. Rebhan and D. Steineder, Phys. Rev. Lett. 108, 021601 (2012).

Viscosity: Areas of interest

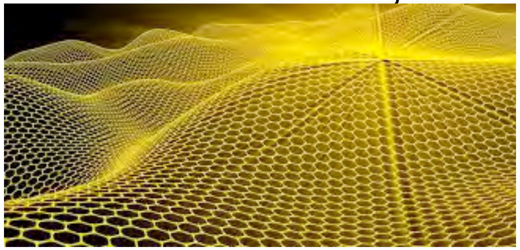
Quark gluon plasma



E. Shuryak, Prog. Part. Nucl. Phys. 53, 273 (2004).

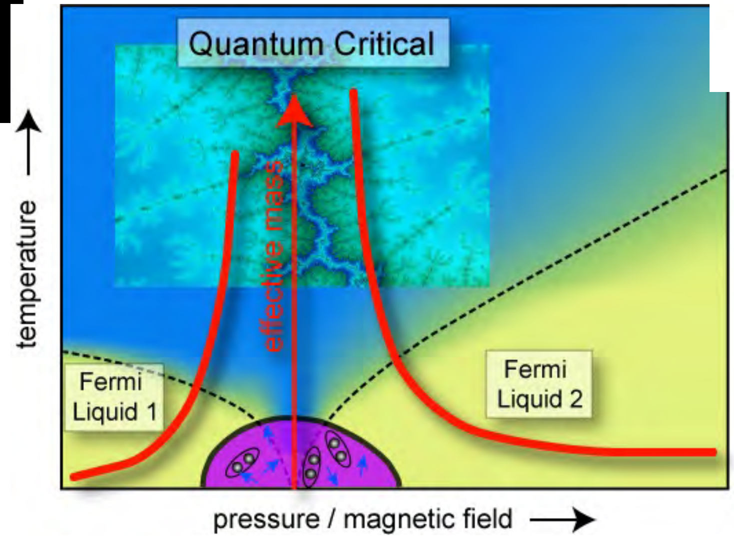
A. Rebhan and D. Steineder, Phys. Rev. Lett. 108, 021601 (2012).

Graphene: A Nearly Perfect Fluid,



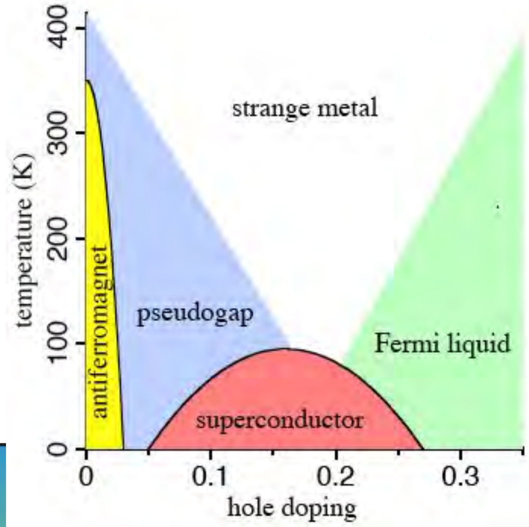
M. Müller et al, PRL 103, 025301 (2009)

Heavy fermions



J. Moreno and P. Coleman, Phys. Rev. B 53, R2995 (1996).

High T_c cuprates



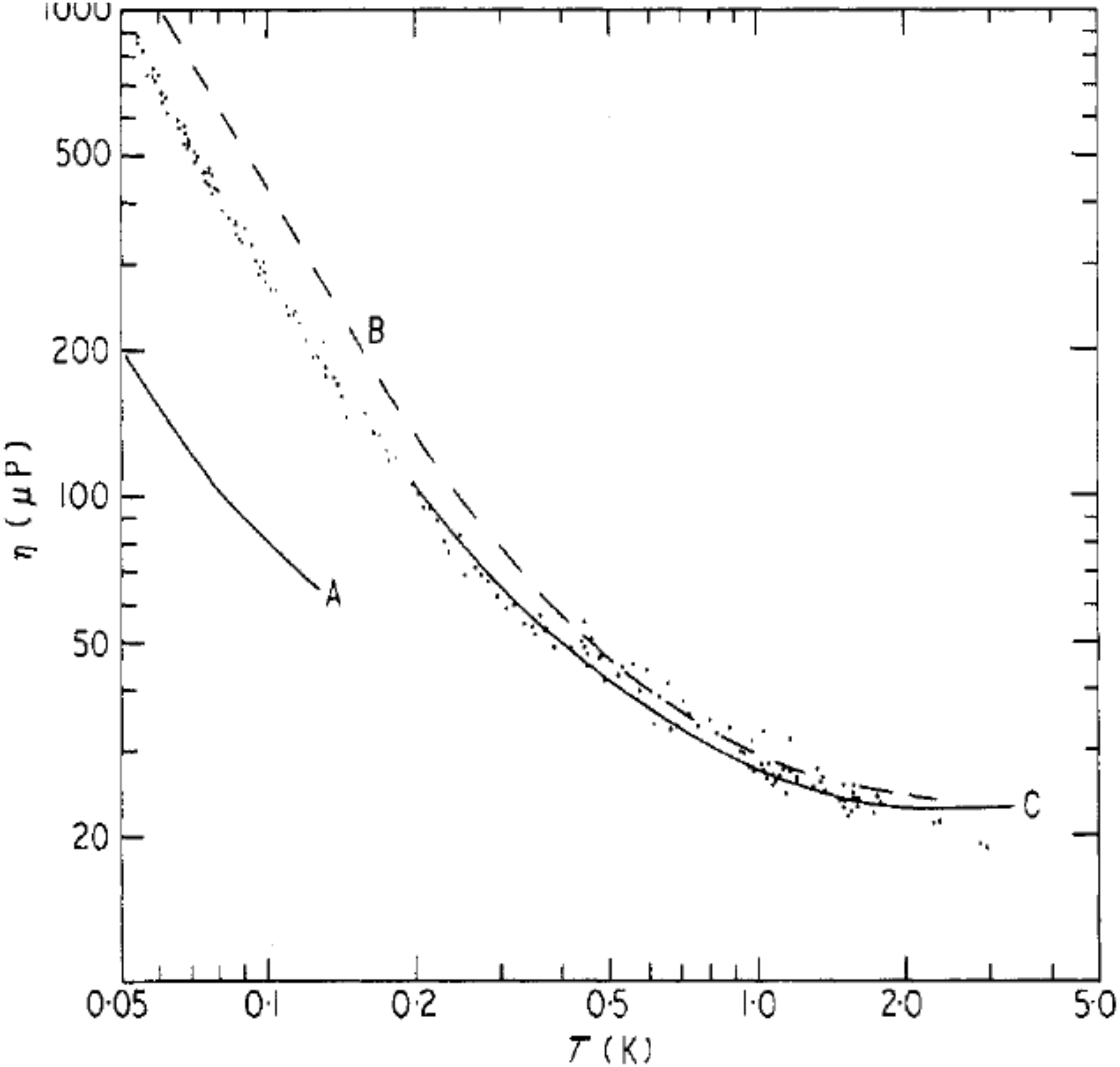
J. Zaanen, Nature 430, 512 (2004).

Viscosity: Areas of interest

But first: Liquid ^{3}He

A.A. Abrikosov and I.M. Khalatnikov,
Zh. Eksperim. Teor. Fiz. 33, 110 (1957)

Viscosity of ^3He



Black, Hall & Thompson, J Phys C 4, 129 (1971)

Propagation of sound in a neutral viscous liquid

$$\text{Navier-Stokes} \Rightarrow \left[-i\omega + \frac{\eta}{\rho} q^2\right] v = 0$$

1 mode for each frequency

$$q^2 = -i\frac{\rho}{\eta}\omega$$

Behaviour at finite frequency

Response to flow patterns that have existed in the past

Memory function: $\int_{-\infty}^{t} M_\eta(t'-t)\Delta\upsilon(t',r)\,dt'$

Frequency domain: $\eta(\omega)\Delta\upsilon(r)$

Generalized viscosity: $\eta(\omega)$

Reasonable approximation: $M_\eta(t)\tilde{\tau} = \eta(0)e^{-t/\tilde{\tau}}$

So that: $\eta(\omega) = \dfrac{\eta(0)}{1 - i\omega\tilde{\tau}}$

Viscosity According to Fermi Liquid theory

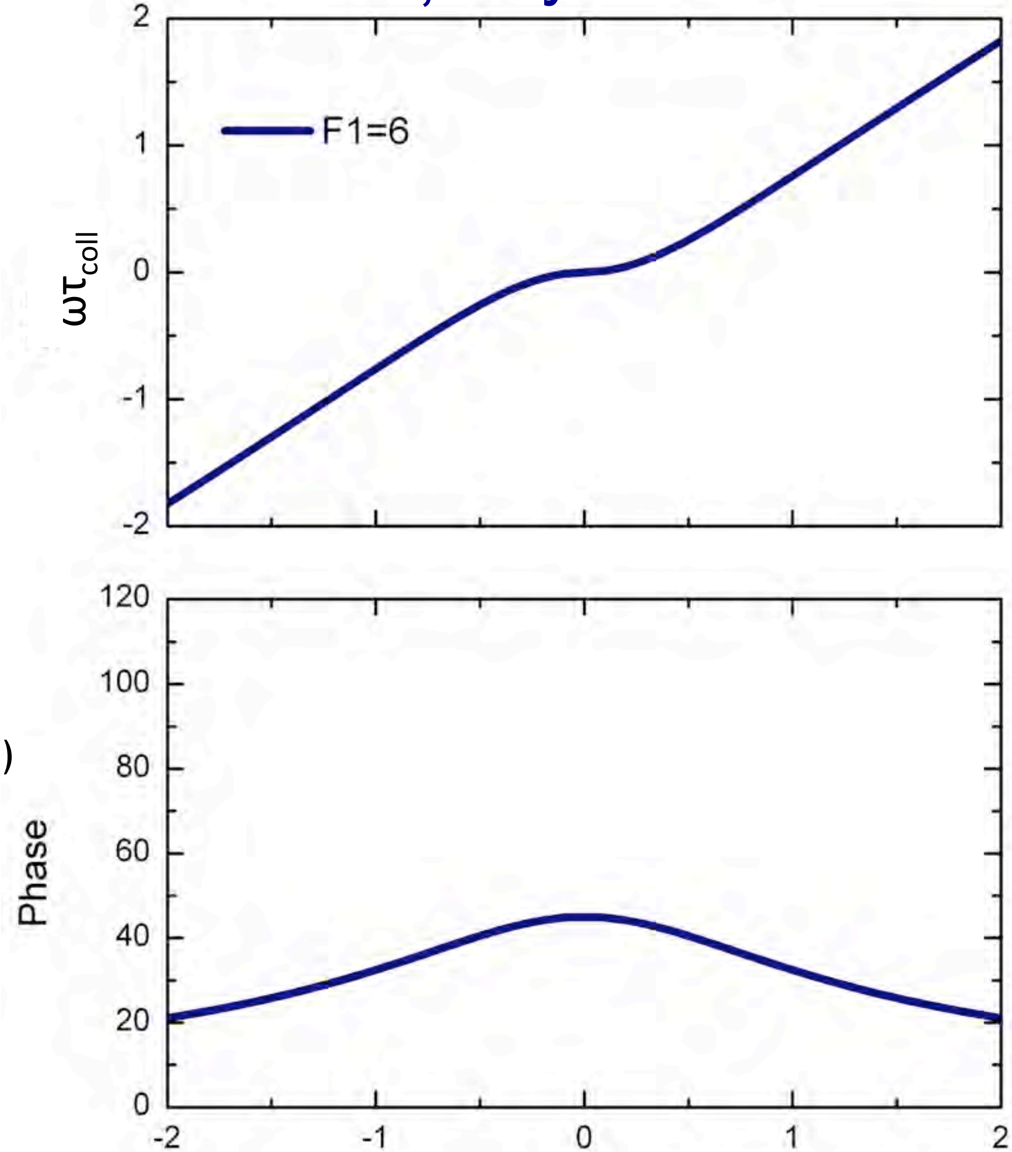
$$\left.\begin{aligned} \eta &\cong \frac{\rho \upsilon_F^2 \tau_{coll}(1+F_1/3)/5}{1-i\omega\tau_{coll}7\sqrt{F_1}/32} \\ \tau_{coll} &= \frac{8\hbar\left(1+F_1/3\right)T_F}{7\pi^3 F_1^2 \left(k_B T\right)^2} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} &\text{High T:} \quad \eta \cong \frac{a}{T^2} \text{ (hydrodynamic)} \\ &\text{Large } \omega: \quad \eta \cong i\frac{b}{\omega} + cT^2 \text{(collisionless)} \end{aligned}\right.$$

One of the Peculiar Consequences:
Transverse Sound !

A. A. Abrikosov and I. M. Khalatnikov, Zh. Eksperim. Teor. Fiz. 33, 110 (1957)

Transverse sound in neutral viscous Fermi liquid

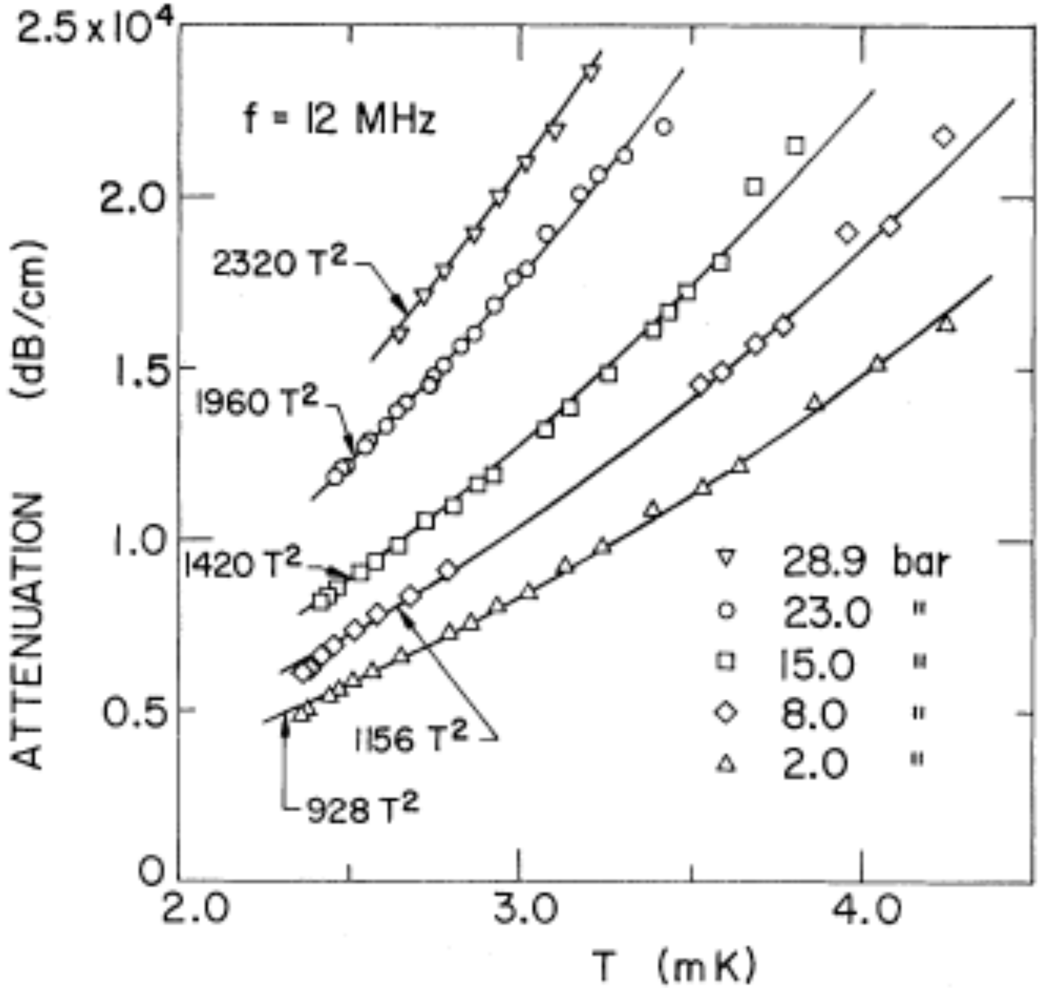
(Roach and Ketterson, Phys Rev Lett 36 (1976))



Arg q(ω)

$q\, v_F\, \tau_{coll}$

Observation of Transverse Zero Sound in Normal ^{3}He†



Viscosity and attenuation: $\mathrm{Im}\, q \cong \sqrt{\dfrac{\omega\rho}{2\eta}}$, the experiments confirm that $\lim_{T\to 0} \eta(T) \to \infty$

Pat R. Roach & J. B. Ketterson, Phys. Rev. Lett. 36 (1976)

Propagation of coupled transverse sound and EM field in viscous charged liquids

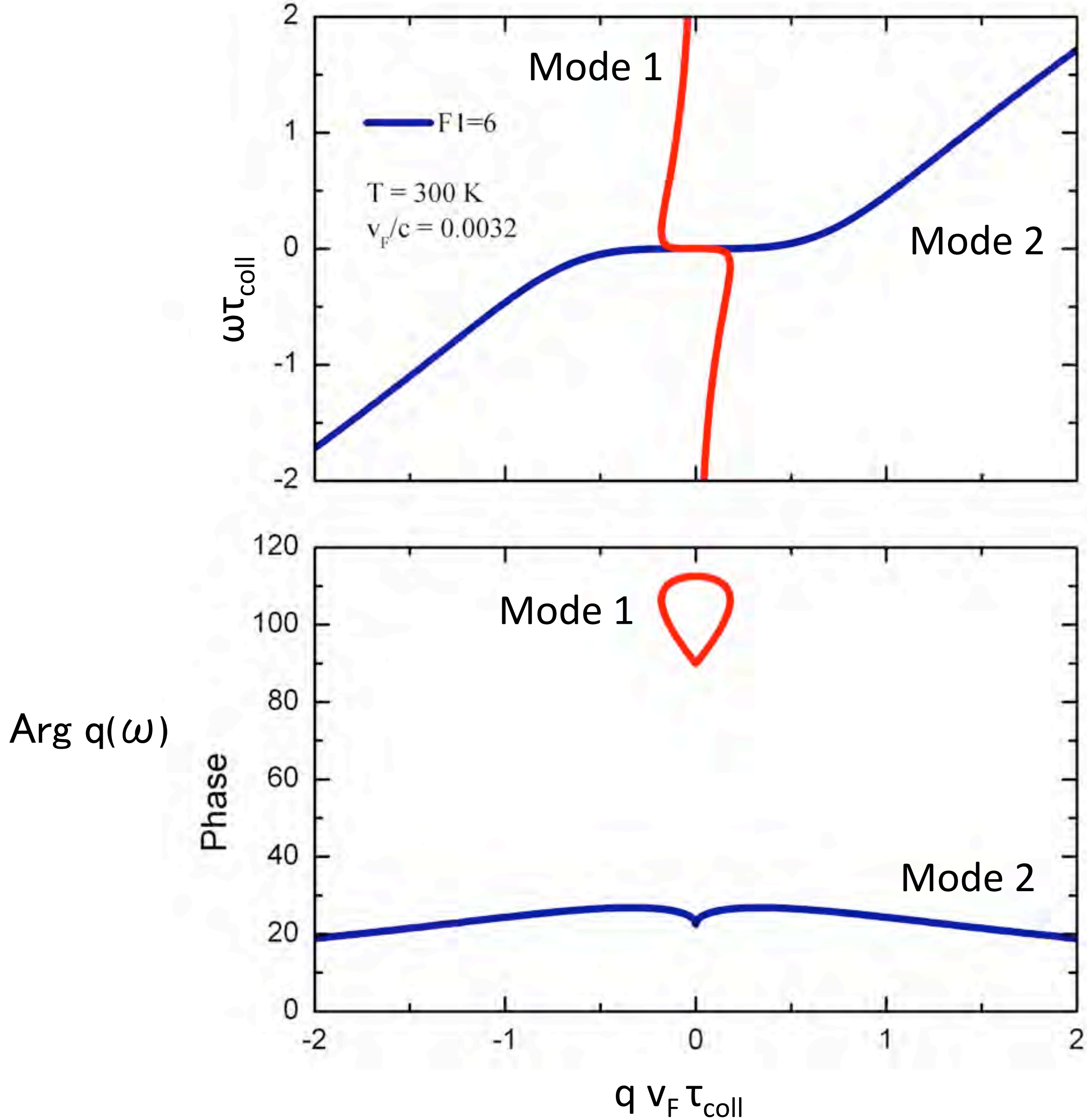
Maxwell equations $\Rightarrow \left[-c^2\partial_z^2+\partial_t^2\right]A=4\pi J$

Navier-Stokes+damping $\Rightarrow \left[\partial_t-\frac{\eta}{\rho}\partial_z^2+\frac{1}{\tau_K}\right]J=-\frac{(ne)^2}{\rho}\partial_t A$

Combine the two equations gives 2 modes for each frequency

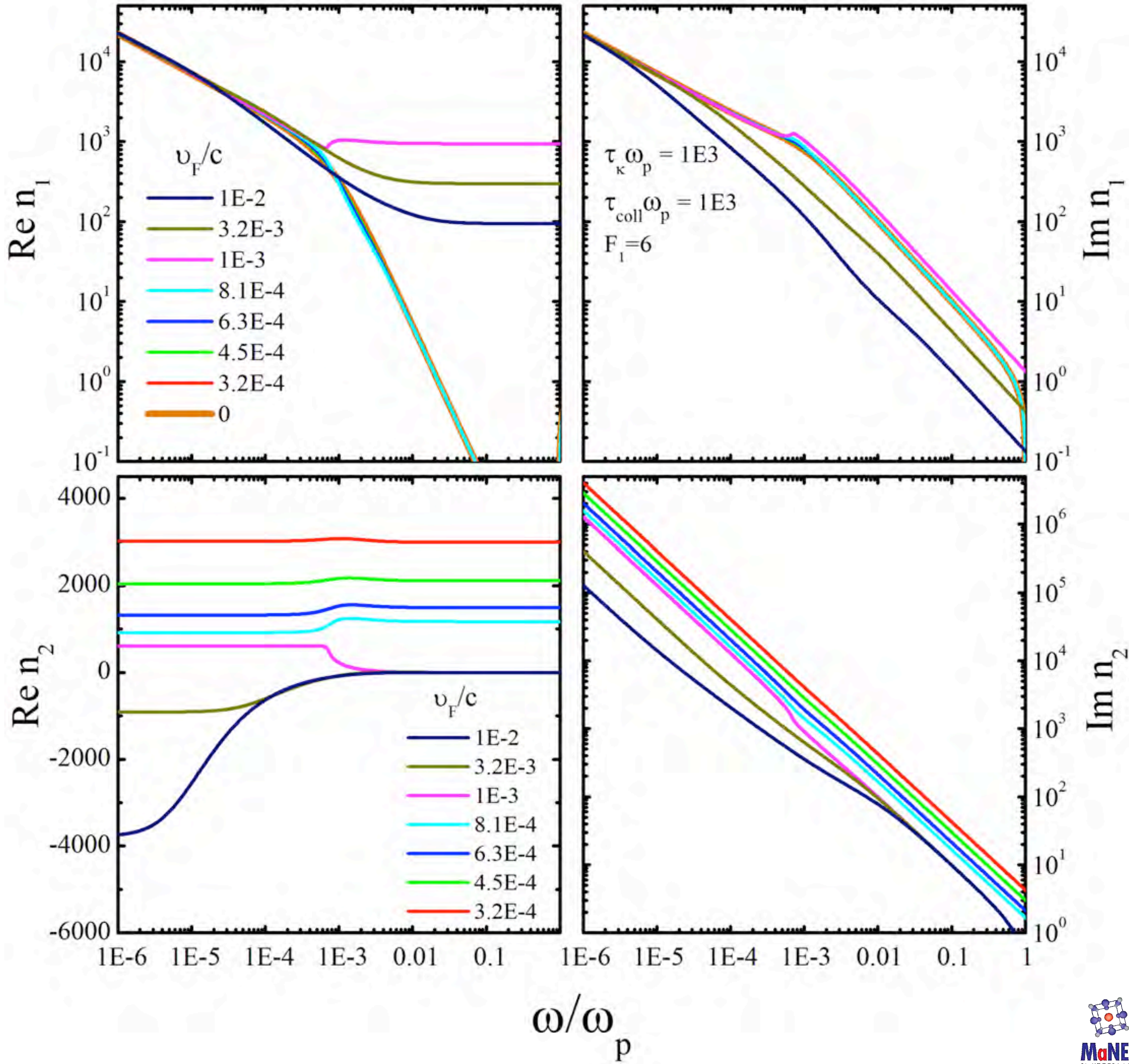
$$2\frac{q^2c^2}{\omega^2}=1-\frac{1-i\rho c^2\omega\tau_K}{\omega^2\eta\tau_K}\pm\sqrt{\left[1+\frac{1-i\rho c^2\omega\tau_K}{\omega^2\eta\tau_K}\right]^2+\frac{i4\rho c^2\omega_p^2}{\omega^3\eta}}$$

Transverse sound in a charged Fermi liquid @ 300 Kelvin



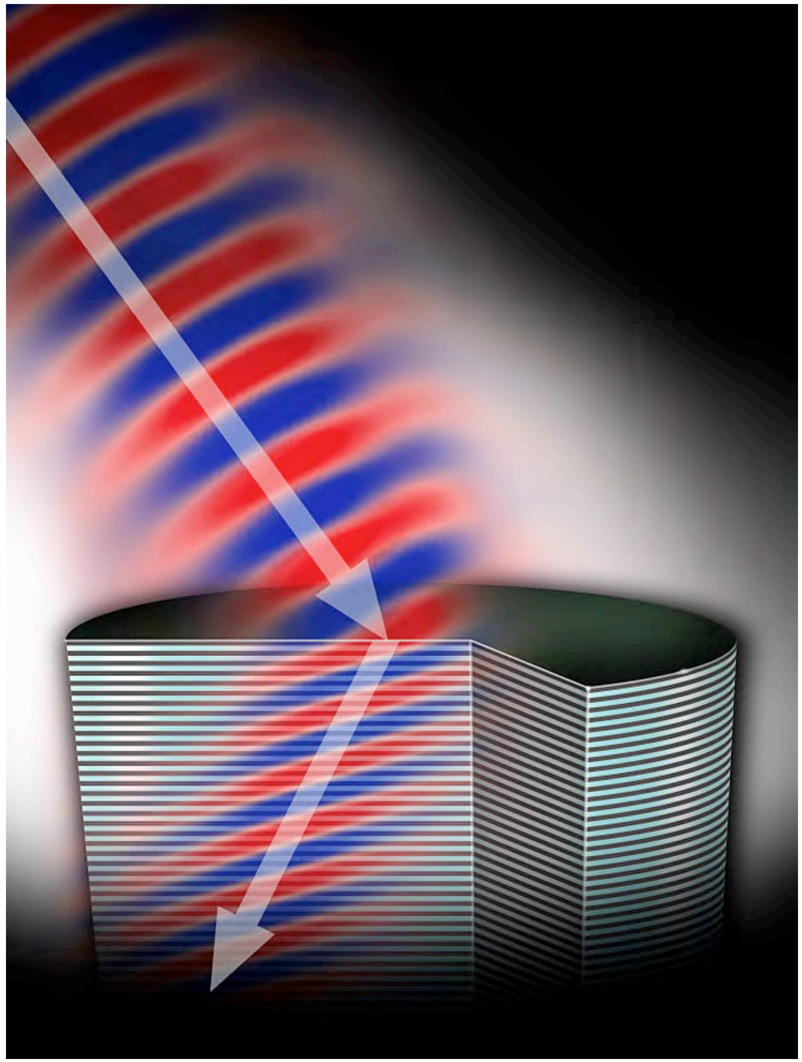
Two solutions for the index of refraction → Two "modes" for each frequency

$$n_j = \frac{q_j c}{\omega}$$



Negative Index of Refraction in the Quark Gluon Plasma

A. Amariti, D. Forcella, A. Mariotti and G. Policastro, JHEP 1104, 036 (2011).
A. Amariti, D. Forcella and A. Mariotti, JHEP 1301, 105 (2013).
A. Amariti, D. Forcella and A. Mariotti, arXiv:1010.1297.

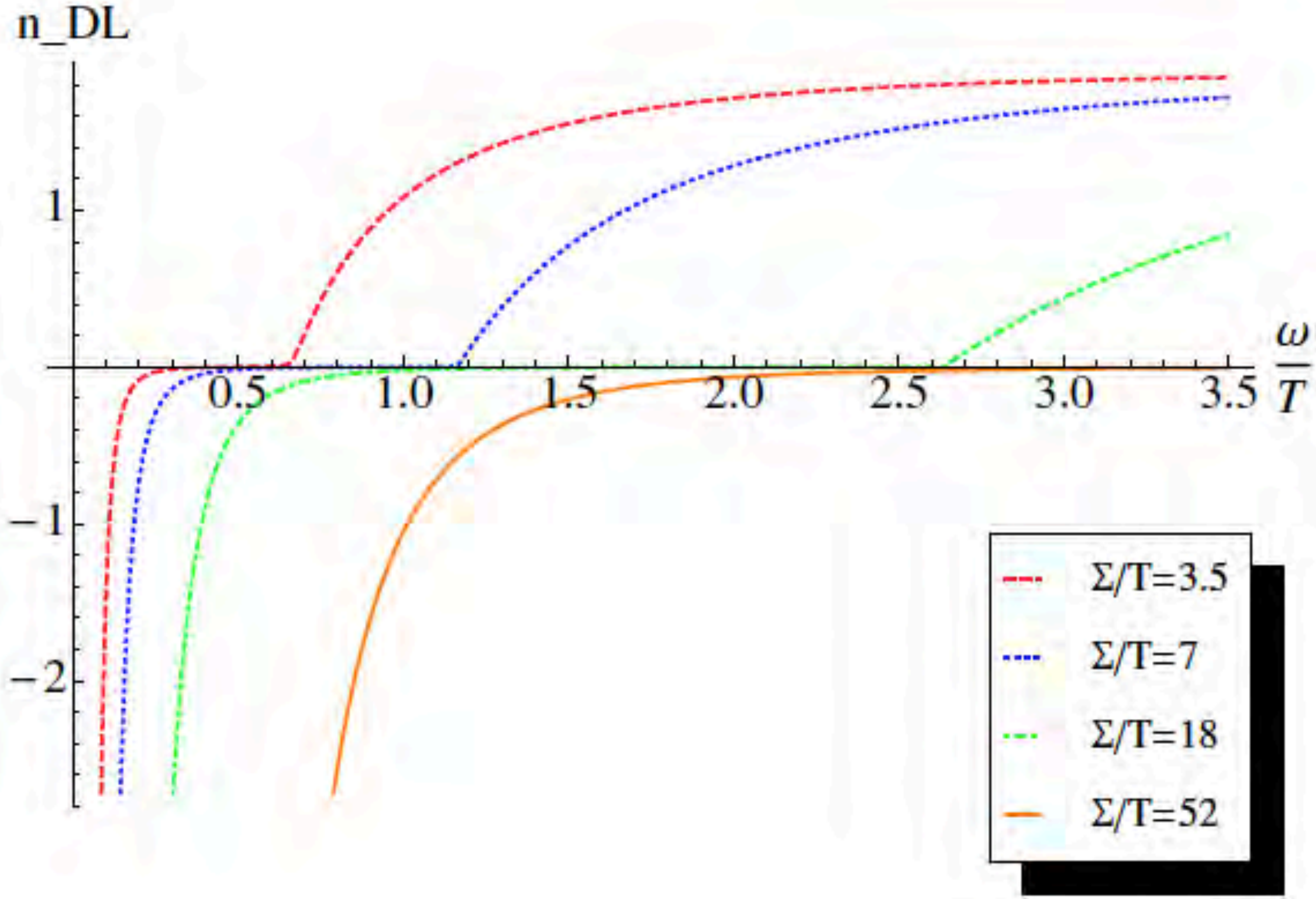


?

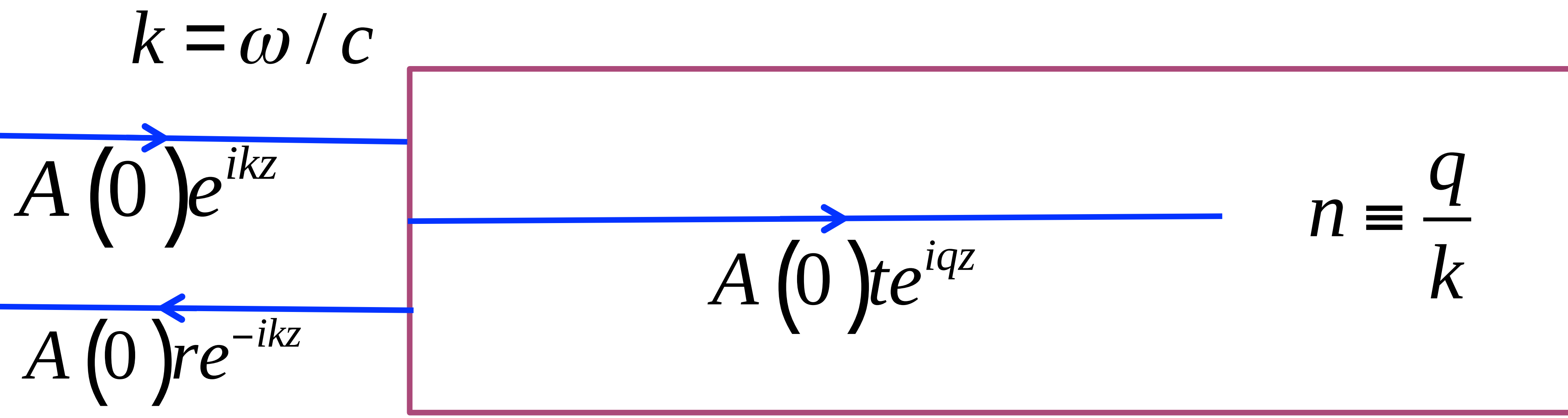
Holographic Optics and Negative Refractive Index

Amariti, D. Forcella, A. Mariotti and G. Policastro,

JHEP 1104, 036 (2011).



Reflection and transmission of EM waves at the boundary from vacuum to a non-viscous charged fluid



Constituant relations at the interface:

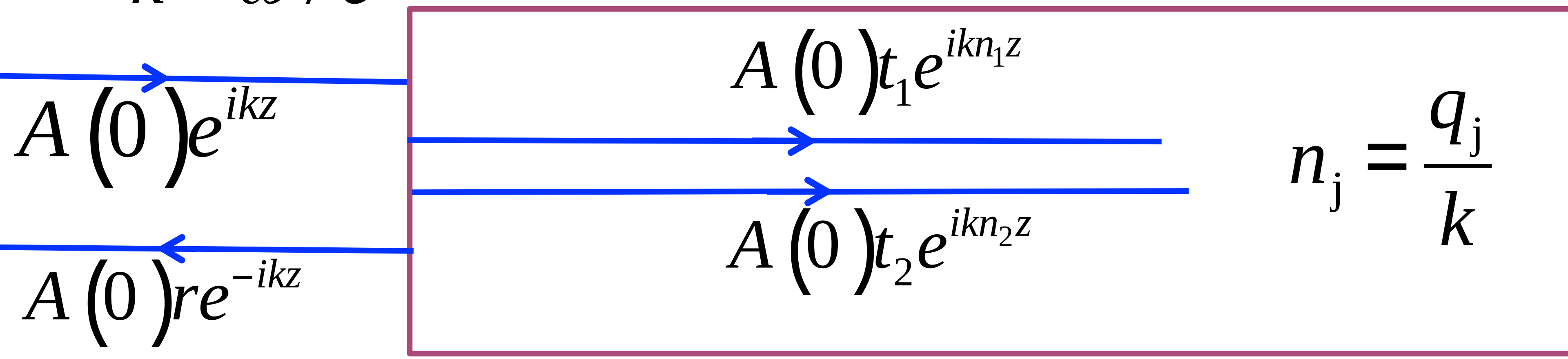
(1 From Maxwell) $A(0+\delta)-A(0-\delta)=0 \Rightarrow 1+r-t=0$

(2 From Maxwell) $\partial_z A(0+\delta)-\partial_z A(0-\delta)=0 \Rightarrow k-kr-qt=0$

$$\Rightarrow \left\{ t=\frac{2k}{q+k}=\frac{2}{1+n} \right.$$

Reflection and transmission of EM waves at the boundary from vacuum to a non-viscous charged fluid

$$k = \omega / c$$



Constituant relations at the interface:

(1 From Maxwell) $A(0+\delta) - A(0-\delta) = 0$

(2 From Maxwell) $\partial_z A(0+\delta) - \partial_z A(0-\delta) = 0$

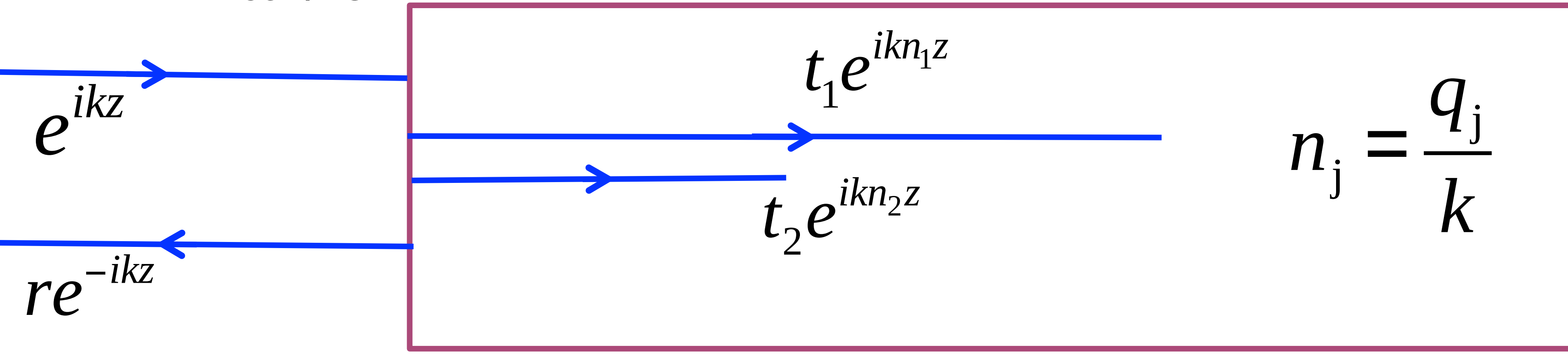
(3 From N&S) $\upsilon(0+\delta) - \lambda \partial_z \upsilon(0+\delta) = 0$

$$\Rightarrow \begin{cases} t_1 = \dfrac{2(n_1 - 1)}{(n_1+1)(n_2+n_1)} \dfrac{1 - i n_1 \lambda k}{1 + i(1 - n_1 - n_2)\lambda k} \\ r = t_1 + t_2 - 1 \end{cases}$$

$\lambda \equiv$ "slip length"

Reflection and transmission of EM waves at the boundary from vacuum to a viscous charged fluid

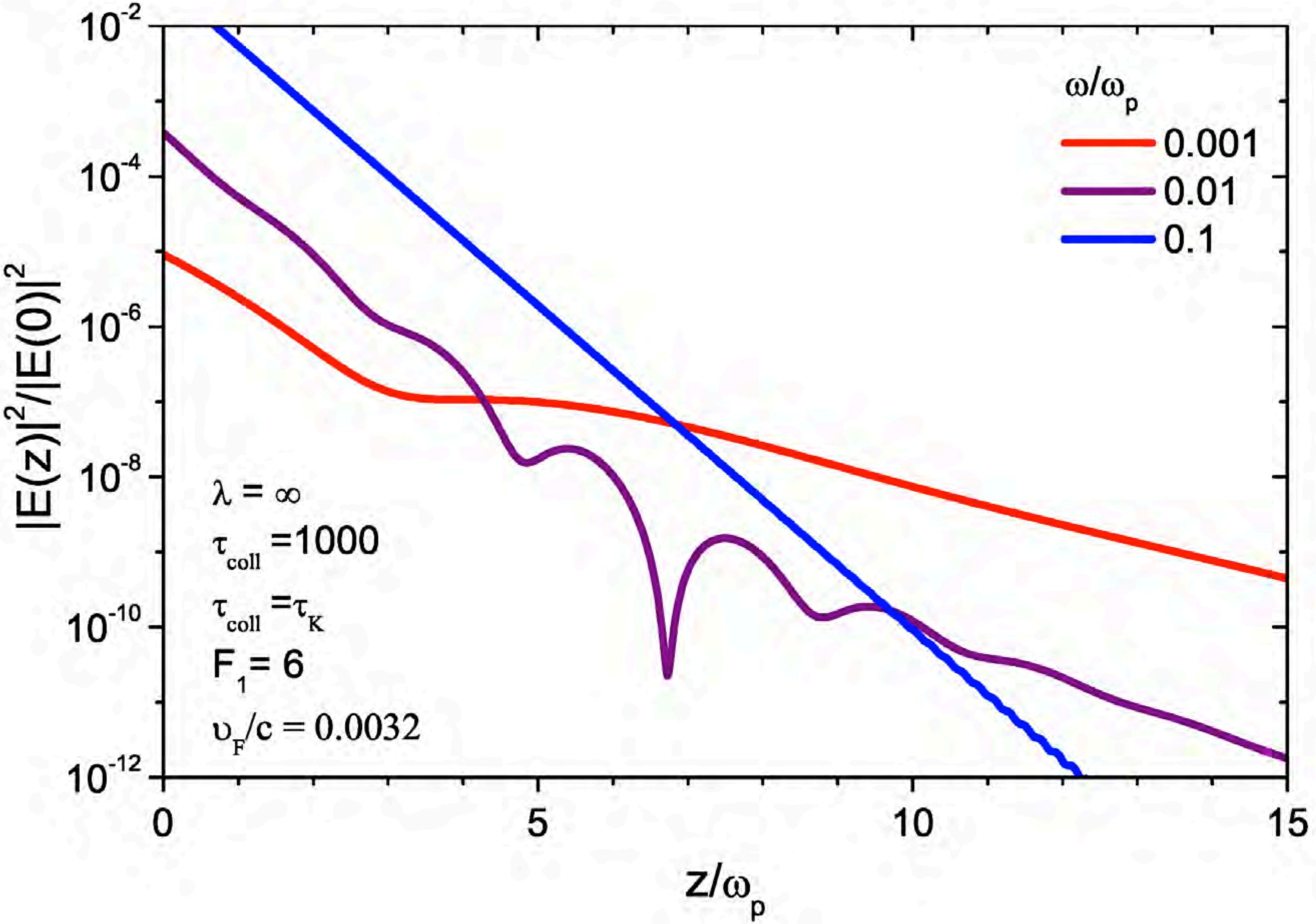
$$k = \omega / c$$



$$n_j = \frac{q_j}{k}$$

$$\left| A(\omega) / A(0) \right|^2 = \left| t_1 e^{ikn_1 z} + t_2 e^{ikn_2 z} \right|^2 = \left| t_1 e^{ikn_1 z} \right|^2 + \left| t_2 e^{ikn_2 z} \right|^2 + 2\,\mathrm{Re}\left(t_1 t_2 e^{i\omega (n_1 + n_2) z/c} \right)$$

Interference between the two modes inside the material: The Anomalous Skin Effect Beats !



Some Viscosity Numbers

Fermi Liquid theory

$$\frac{\eta(0)}{\rho c^2 \tau_{coll}} \cong \frac{\upsilon_F^2}{c^2}$$

Materials :

$\upsilon_F^2 / c^2 \approx 5 \cdot 10^{-5}$ (Al, C)

$\upsilon_F^2 / c^2 \approx 3 \cdot 10^{-12}$ (^{3}He)

Abrikosov & Khalatnikov,
Zh. Eksperim. Teor. Fiz. 33 (1957)

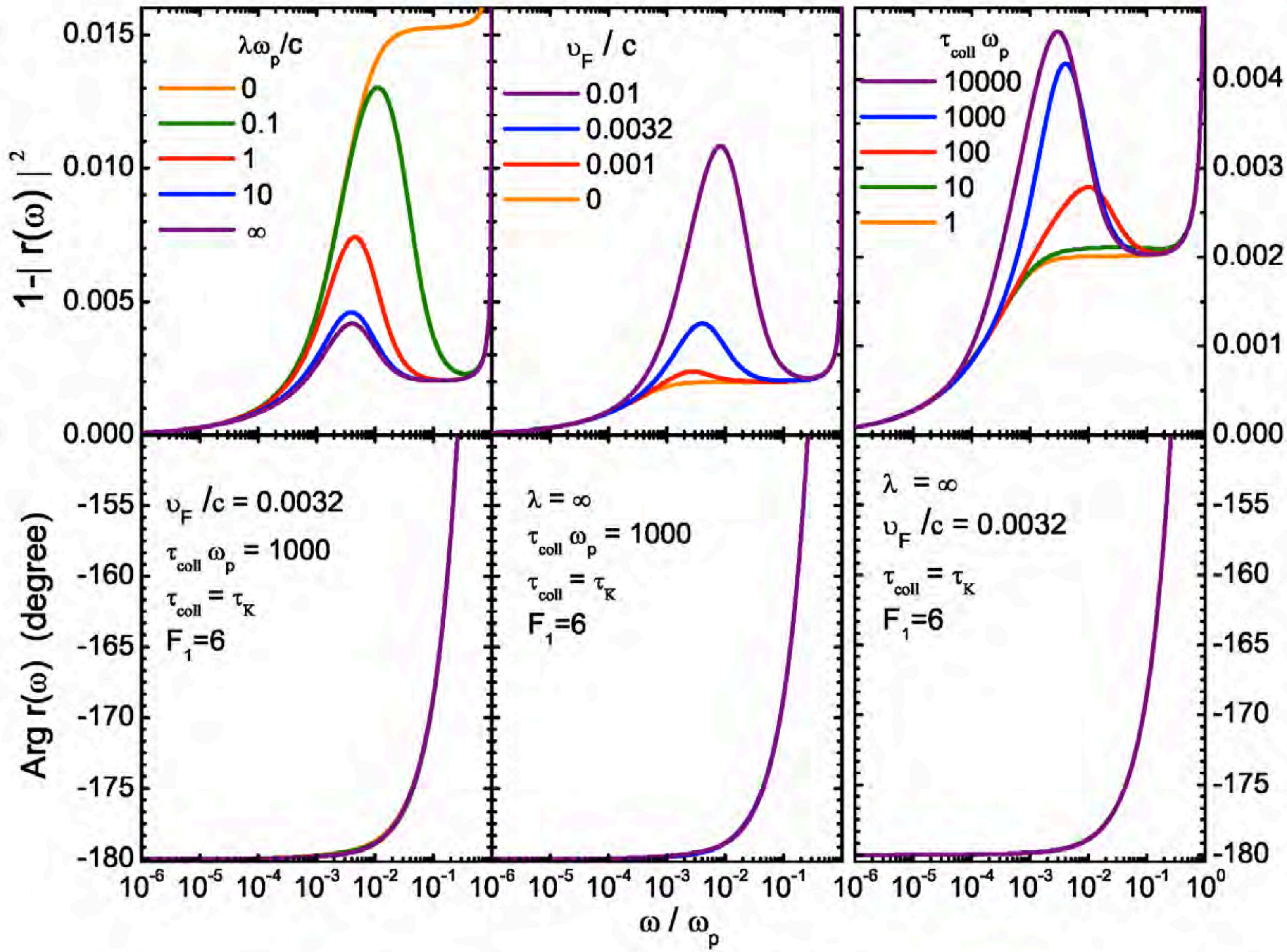
Quantum Critical

$$\frac{\eta(0)}{\rho c^2 \tau_{coll}} \cong \left(\frac{\lambda_e}{2 l_0}\right)^2 \left(\frac{s}{n k_B}\right)^2$$

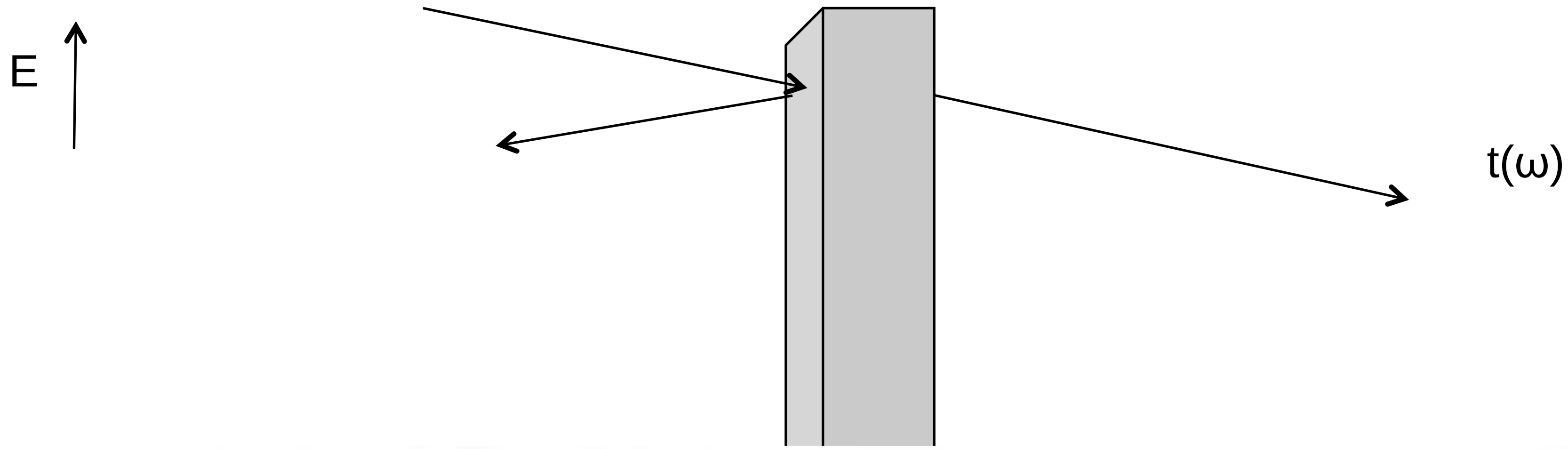
$$\frac{\eta(0)}{\rho c^2 \tau_{coll}} \leq 10^{-12}$$

Davison, Schalm & Zaanen,
arXiv:1311.2451 (2014)

Reflection and transmission of EM waves at the boundary from vacuum to a viscous charged fluid

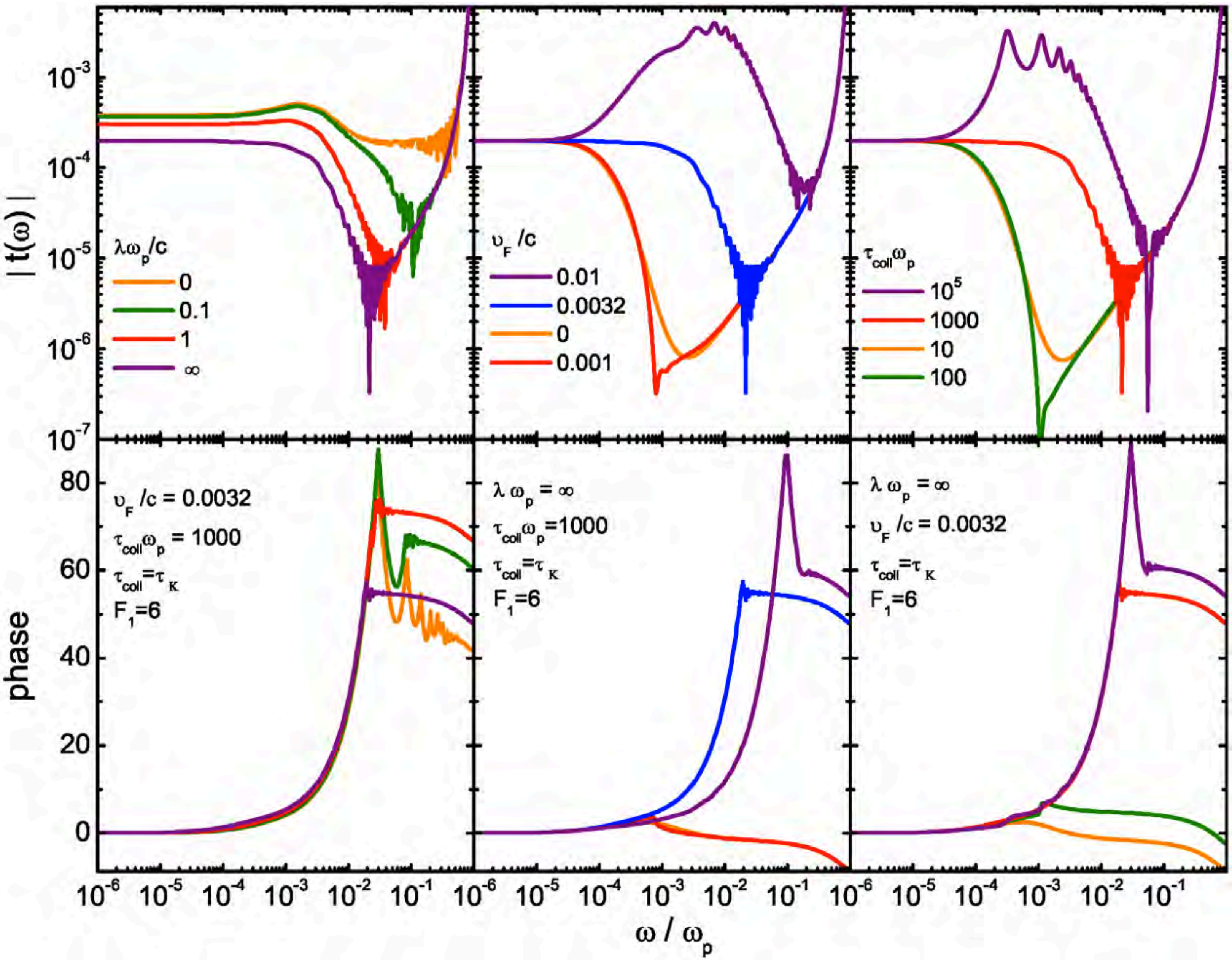


Transmission through a film



$$
\begin{aligned}
E(z)/E(0) &= e^{ikz} + r e^{-ikz} && (z < 0)\\
&= t_1 e^{in_1 kz} + \theta_1 e^{-in_1 kz} + t_2 e^{in_2 kz} + \theta_2 e^{-in_2 kz} && (0 < z < d)\\
&= t e^{ikz} && (z > d)
\end{aligned}
$$

Transmission through a film



Surface Impedance

The electromagnetic response of a metal, whether normal or superconducting, is described by a complex surface impedance:

$$Z_s = \frac{|E_\parallel|}{\int_0^\infty J(x)dx} = \frac{E_\parallel}{H_\parallel} = R_s + iX_s$$

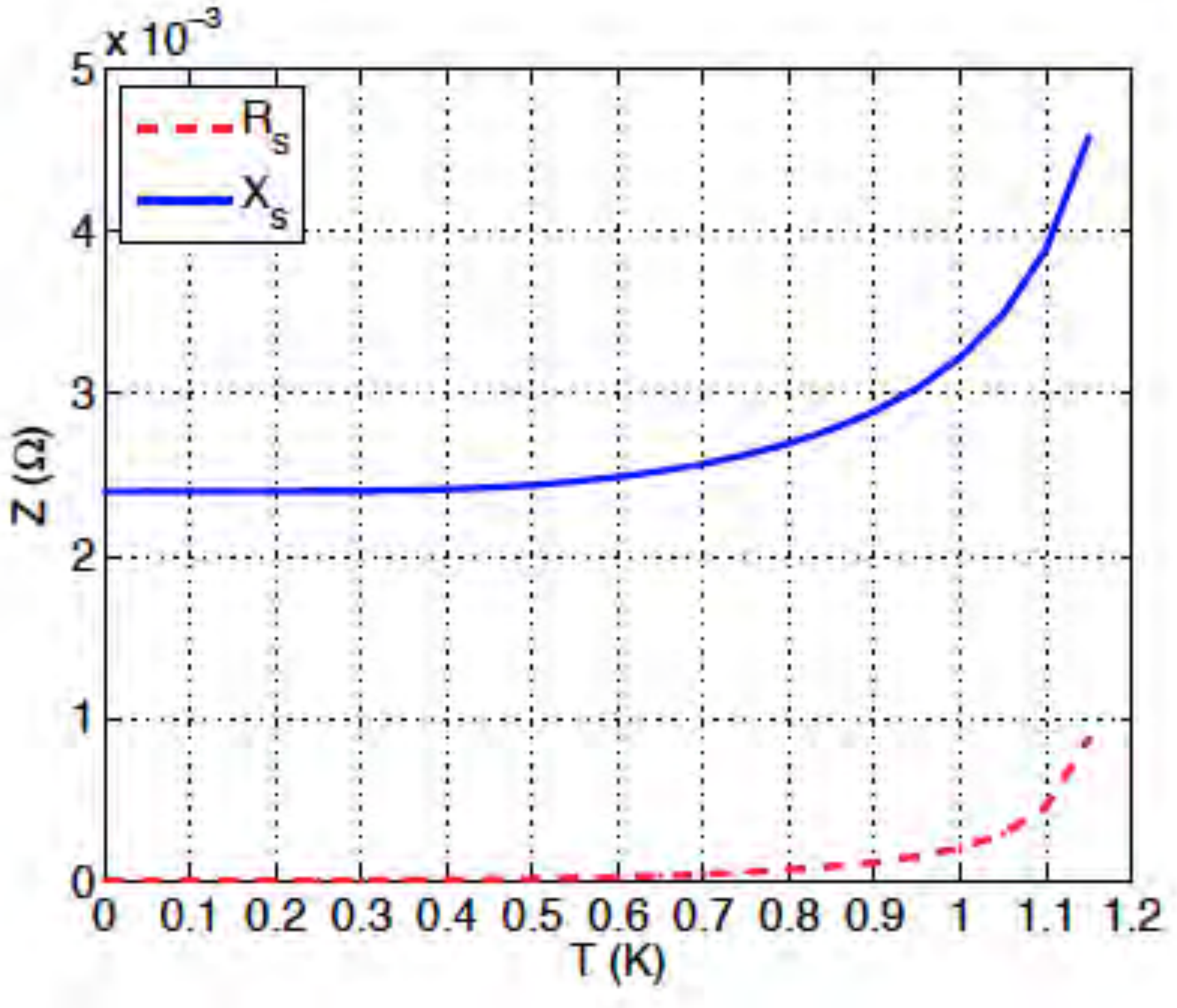
surface reactance

surface resistance

For a good conductor and $w < 10^{16}$ Hz $\frac{\partial D}{\partial t} \approx 0 \longrightarrow \nabla \times H = J$

The impedance of vacuum is: $Z_0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} \simeq 377\Omega$

Surface resistance **R**s and surface reactance **X**s of aluminum as a function of temperature.



The Physics of Superconducting Microwave Resonators.
Thesis by Jiansong Gao, Caltex (2008)

The theory of the anomalous skin effect in metals

G. E. H. Reuter and E. H. Sondheimer,

Proc. R. Soc. A 195, 336 (1948)

Franz Sondheimer

Under the combined action of the applied electromagnetic field and the collisions of the electrons with the lattice a steady state is set up, and the distribution function in the steady state is determined by the Boltzmann equation (Wilson 1936, pp. 124, 158)

$$\frac{\partial f}{\partial t} - \frac{2\pi\epsilon}{h}\left(\mathscr{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right).\mathrm{grad}_{\mathbf{k}} f + \mathbf{v}.\mathrm{grad}_{\mathbf{r}} f = -\frac{f-f_0}{\tau}, \qquad (4)$$

2·23. The current density $J(z)$ can now be calculated at once by means of the formula

$$J(z) = -2\epsilon\left(\frac{m}{h}\right)^3 \iiint v_x f \, dv_x \, dv_y \, dv_z.$$

$$Z = \frac{|E_{\parallel}|}{\int_0^{\infty} J(x)\,dx}$$

The theory of the anomalous skin effect in metals

G. E. H. Reuter and E. H. Sondheimer,
Proc. R. Soc. A 195, 336 (1948)

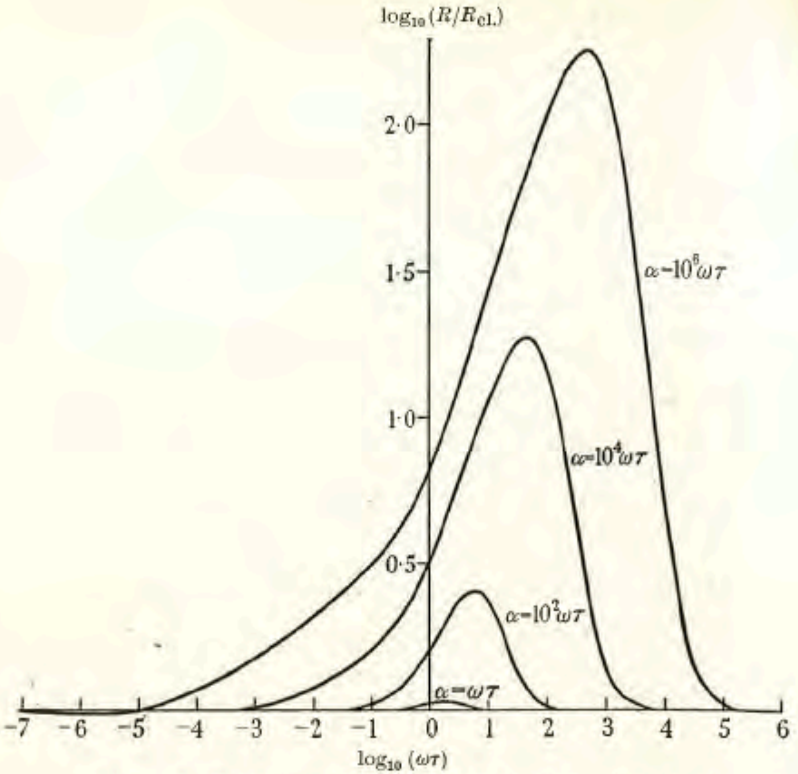


FIGURE 2. The frequency variation of the surface resistivity.

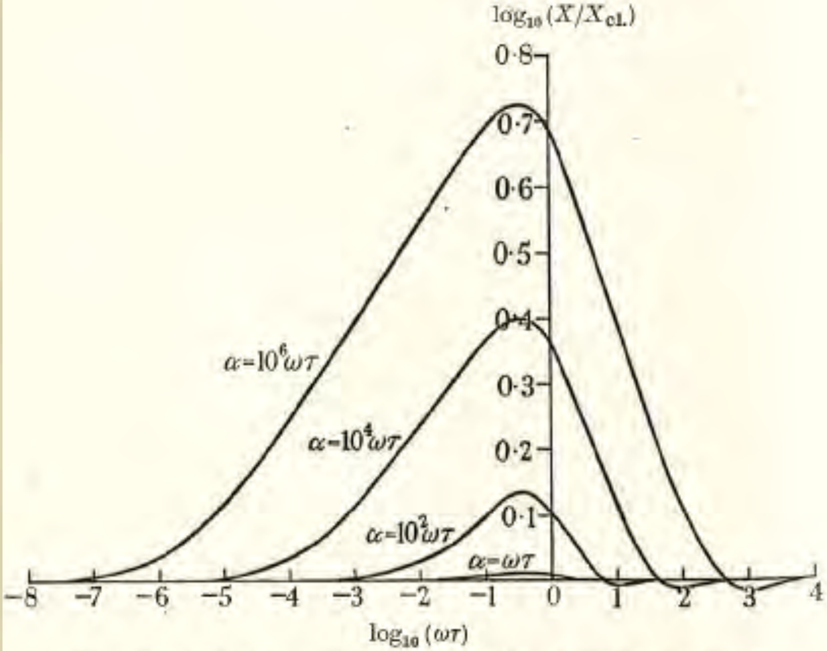
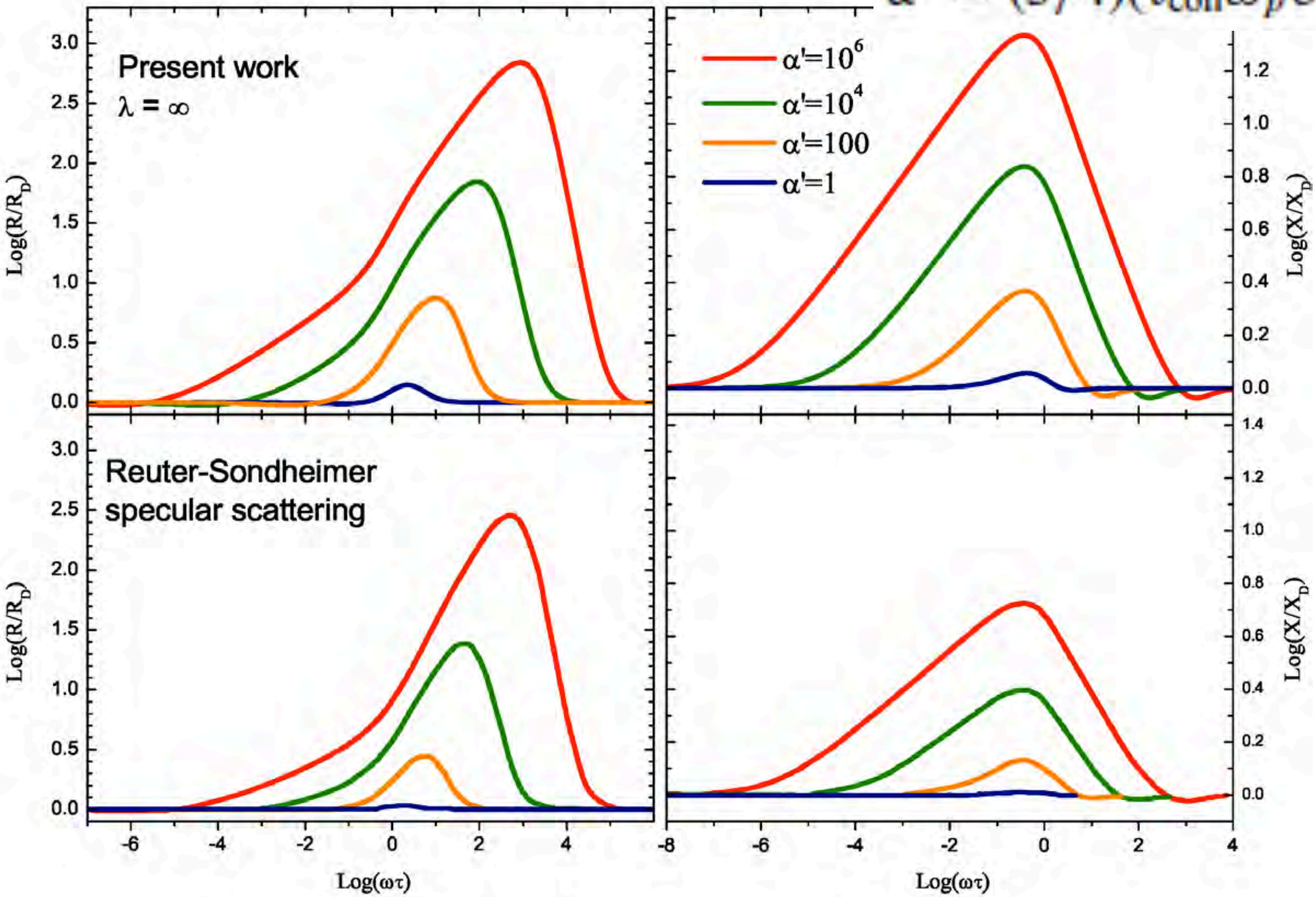


FIGURE 3. The frequency variation of the surface reactance.

Comparison of hydrodynamical approach (present work) and Reuter-Sondheimer model of anomalous skin effect (Proc.R.Soc. A195 (1948))

$$\frac{Z}{Z_0} = \frac{1-r}{1+r} \quad ; \quad \frac{Z_0}{Z_D} = \sqrt{1-\omega_p^2\omega^{-1}\left(\omega+i\tau_K^{-1}\right)^{-1}}$$

$$\alpha' = (3/4)(\tau_{coll}\omega_p v_F/c)^2$$



The anomalous skin effect and the reflectivity of metals
Hydronamical approach
C. W. Benthem and R. Kronig, *Physica 20. 293 (1954)*

"Kronig could arrive indirectly at an estimate of η ("internal friction") by comparing the formula for the stopping power derived by Kronig and Korringa with an expression for the same quantity obtained by Kramers"

$$\eta = \frac{3}{8} n\hbar \Rightarrow \nu = \frac{3}{8}\frac{\hbar}{m}$$

Ralph de Laer Kronig

On the possible influence of electron interaction on the reflectivity of metals

C. W. Benthum, *Appl. Sci. Research B 1, 275 (1959)*

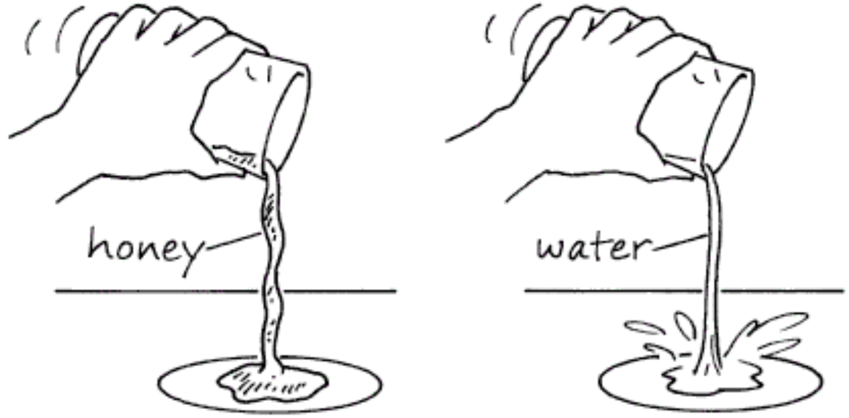
"It seems, therefore, that the effect … wil be too small to measure"

Holography and hydrodynamics: diffusion on stretched horizons
P. Kovtun, D.T. Son and A.O. Starinets
JHEP 10 (2003) 064

$$\eta >> \hbar \frac{s}{k_B}$$

Coefficient of "Internal Friction"
deduced from expressions for the stopping power
R. Kronig
Physica 15 (1949) 667.

$$\eta = \frac{3}{8} \hbar n$$



What is more viscous ?

Strongly interacting

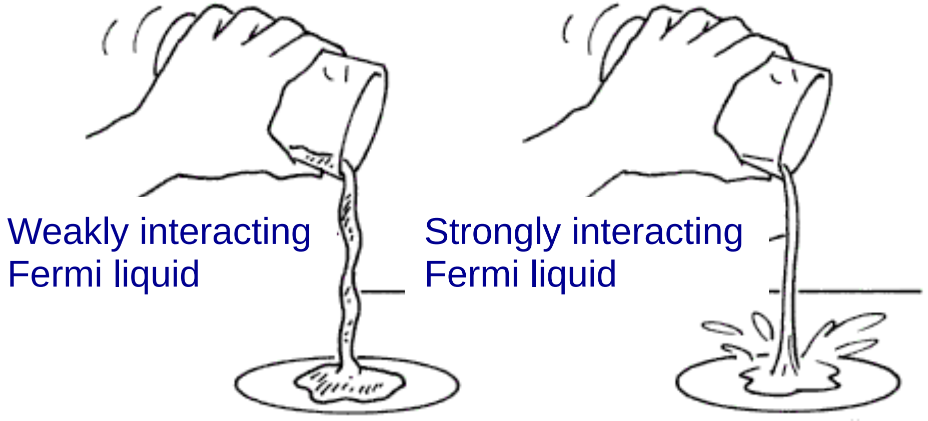
Fermi liquids

(^3He, Sr_2RuO_4, UPt_3,
High T_c superconductors…)

Weakly interacting

Fermi liquids

(Aluminium, silver…)

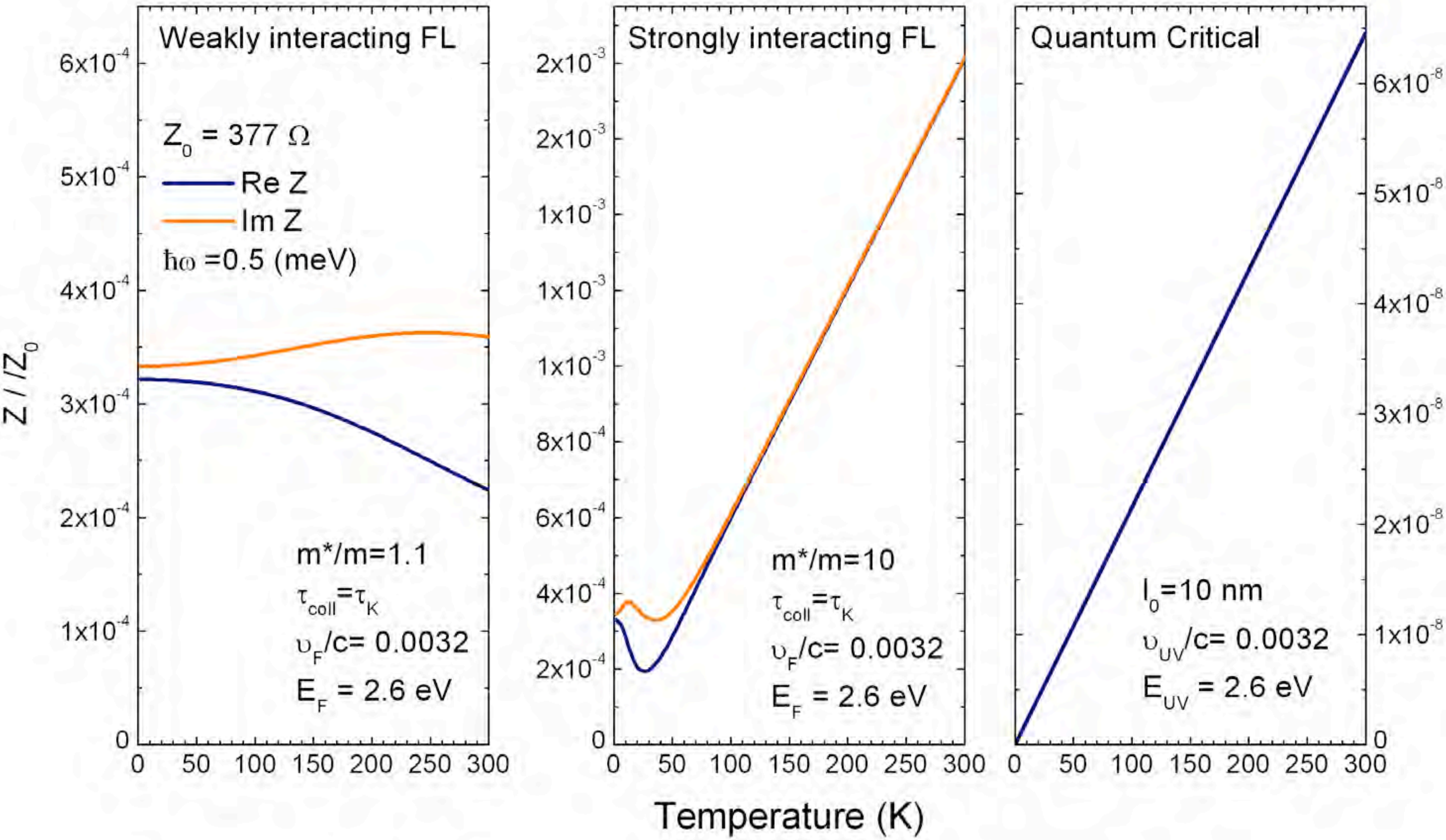


Weakly interacting

Fermions

(Aluminium, silver…)

Surface Impedance of correlated electrons



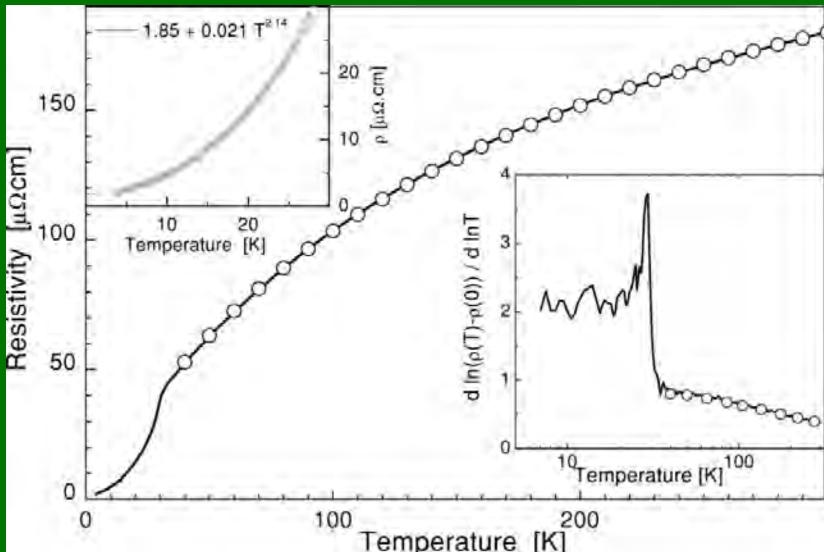
Conclusions Part I

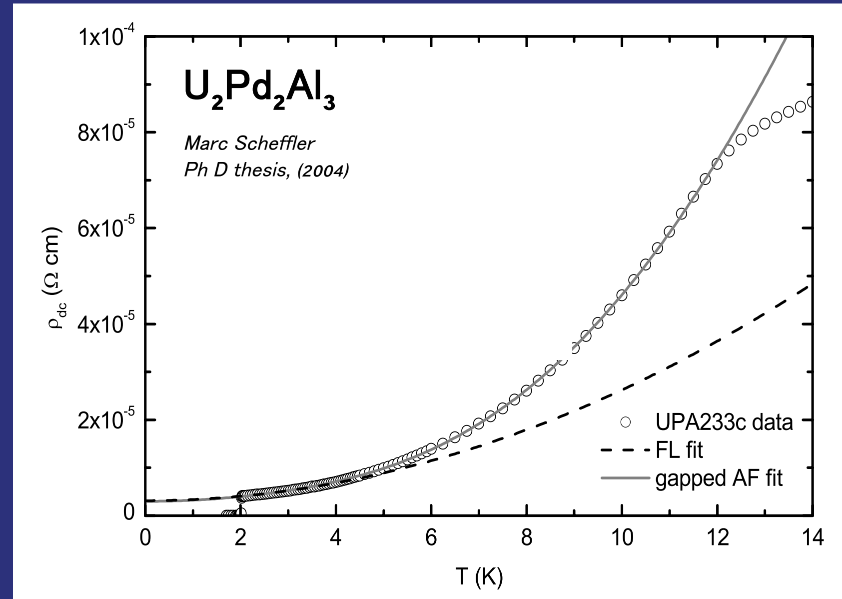
- Viscosity : another manifestation of correlated electron behaviour
- Viscosity of a Fermi liquid grows as $1/T^2$
- Electron liquids, plasmas etc: System of viscous liquid coupled to EM-field
- Consequence 1: Absorbtion peak at $\omega \tau = 1$
- Consequence 2: Two modes (instead of one) inside the plasma
- Consequence 3: EM-field oscillations due to interference of the two modes
- Strongly interacting Fermi liquid → low viscosity

Further reading: Phys. Rev. B 90, 035143 (2014)

MnSi

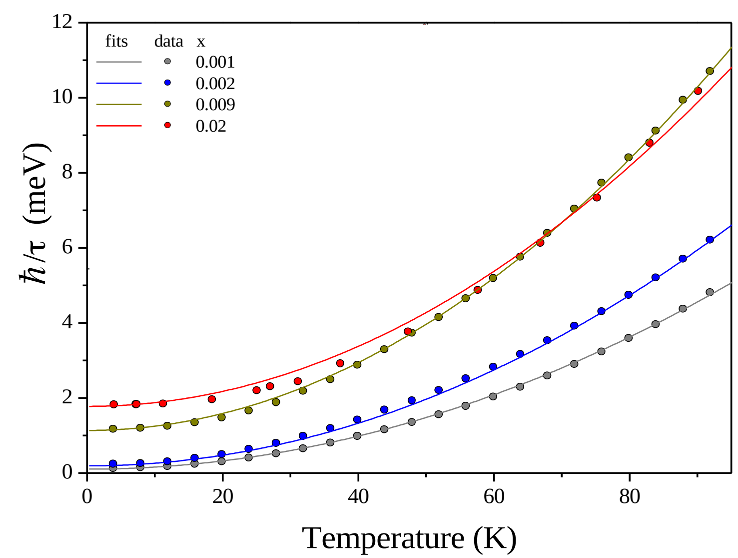
FP Mena et al PRB 67 (2003)





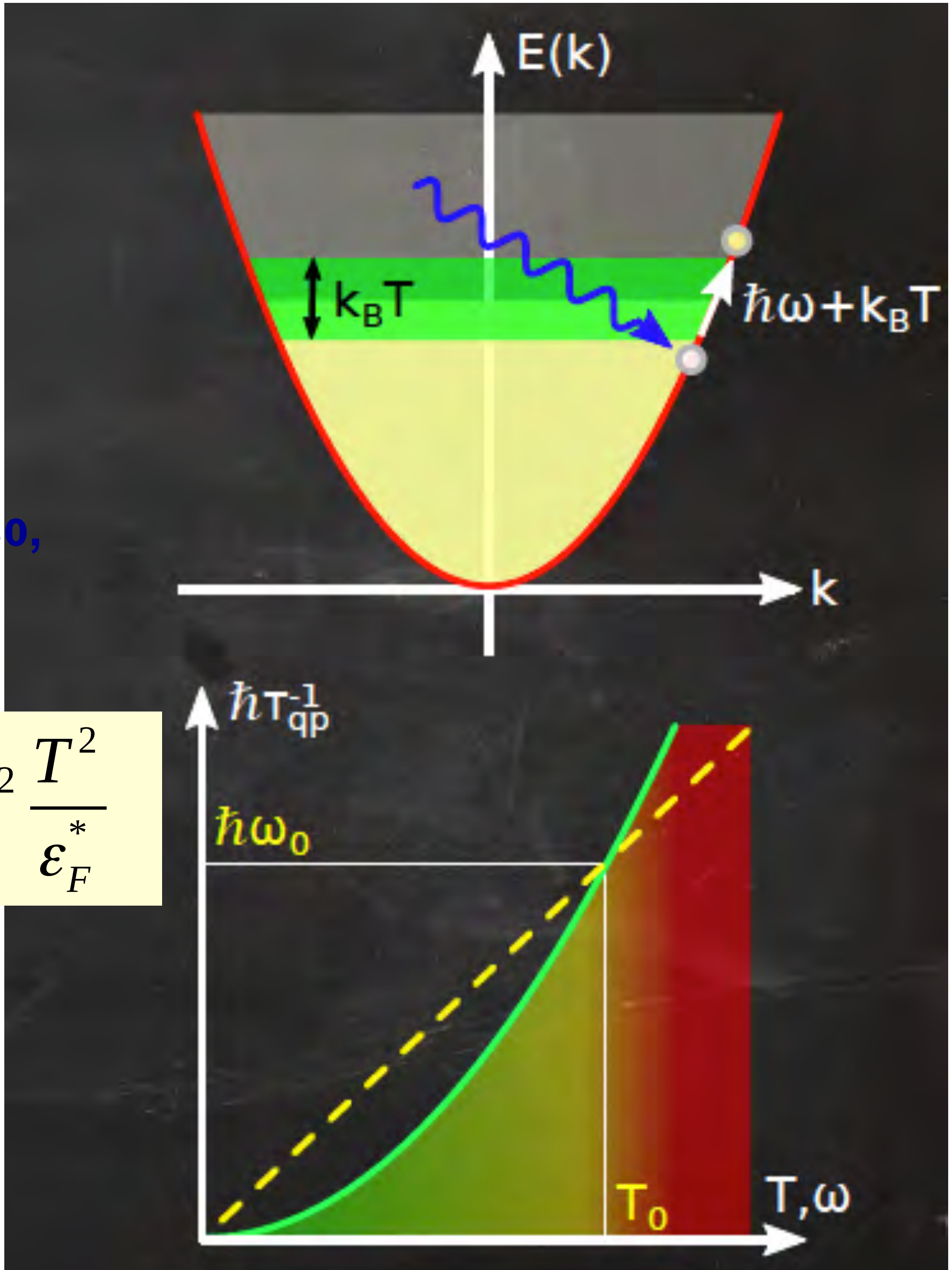
$SrTi_{1-x}Nb_xO_3$

DvdM, I. I. Mazin, J.L.M. van Mechelen
PRB **84**, 205111 (2011)



Landau-Fermi Liquids

L. Landau
Zh. Eksp. Teor. Fiz. **30**, **1058 (1956)**



$$\frac{1}{\tau} \approx \lambda^2 \frac{T^2}{\varepsilon_F^*}$$

Optical conductivity and optical constants

Zero viscosity: Single solution of $qc/\omega \equiv n(\omega)$

Maxwell: $\left[q^2c^2 - \omega^2\right]A(\omega) = 4\pi J(\omega)$

$$\Rightarrow \frac{J(\omega)}{A(\omega)} = \frac{\omega^2}{4\pi}\left\{\left[n(\omega)\right]^2 - 1\right\}$$

$$\sigma(\omega) \equiv \frac{J(\omega)}{i\omega A(\omega)} = \frac{\omega}{4\pi i}\left\{\left[n(\omega)\right]^2 - 1\right\}$$

Zero viscosity.

Optical conductivity of non-interacting charges.

Momentum relaxation time = τ_K

$$\sigma(\omega) = \frac{ne^2/m}{1/\tau_K - i\omega}$$

Generalization to interacting quasi-particles

$$\sigma(\omega,T) = \frac{ine^2/m}{\hbar\omega + M(\omega)} = \frac{ne^2/m}{1/\tau(\omega) - i\omega m^*(\omega)/m}$$

W Götze & P Wölfle, PRB 6, 1226 (1972)
JW Allen & JC Mikkelsen, PRB 15, 2952 (1977)

Generalization to interacting quasi-particles

$$\sigma(\omega,T) = \frac{ne^2/m}{1/\tau(\omega) - i\omega m^*(\omega)/m}$$

Straightforward inversion of the experimental data:

$$\frac{1}{\tau(\omega)} = \mathrm{Re}\frac{ne^2/m}{\sigma(\omega)}$$

W Götze & P Wölfle, PRB 6, 1226 (1972)
JW Allen & JC Mikkelsen, PRB 15, 2952 (1977)

Fermi liquid

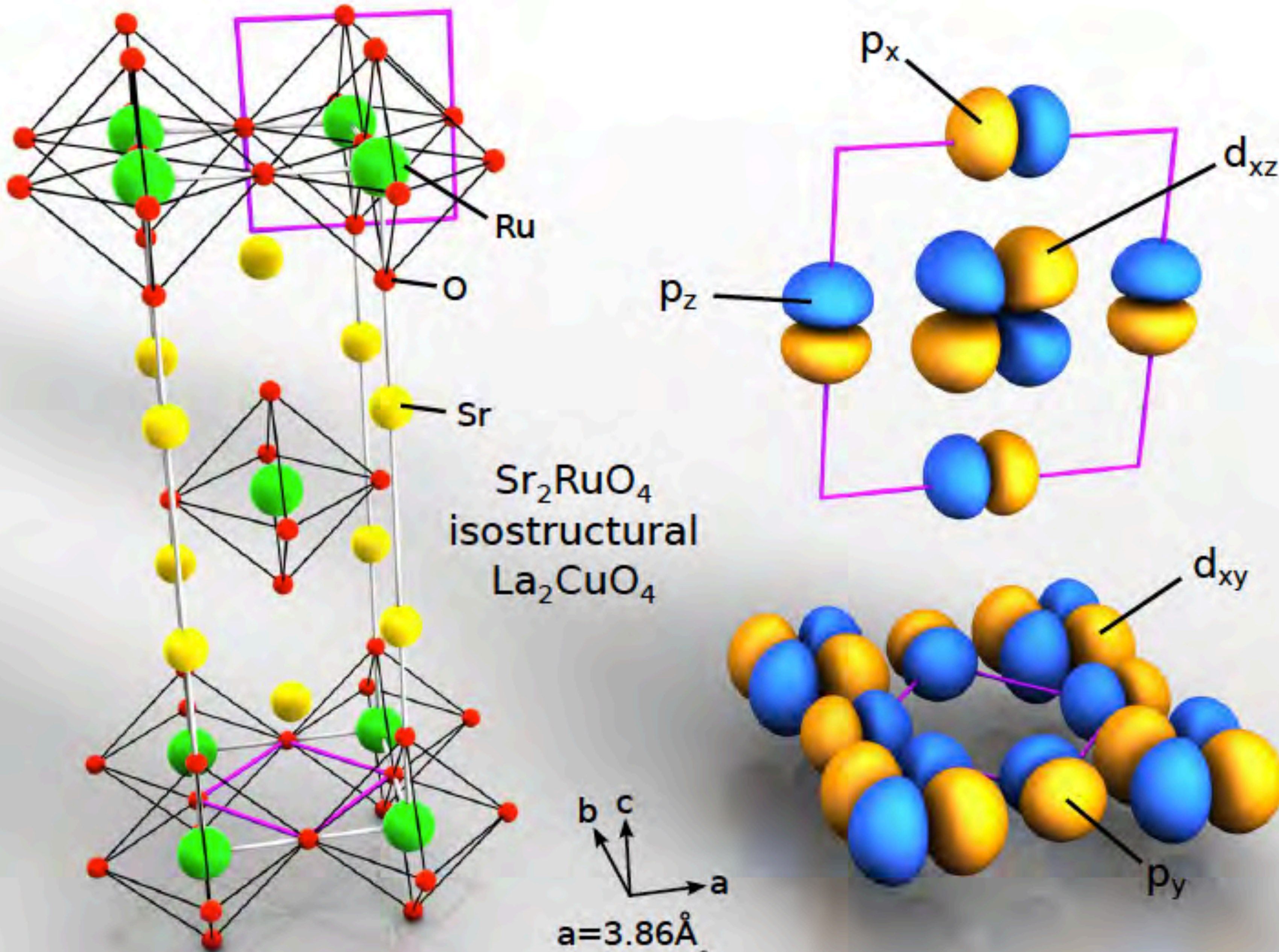
Single particle life time: $\tau_{sp}(\varepsilon,T) \propto \left[\varepsilon^2 + \pi^2 \left(k_B T\right)^2\right]^{-1}$

Optical relaxation rate: $$\begin{cases} 1/\tau_{opt}(\omega,T) \propto (\hbar\omega)^2 + (p\pi k_B T)^2 \\ p = 2 \end{cases}$$

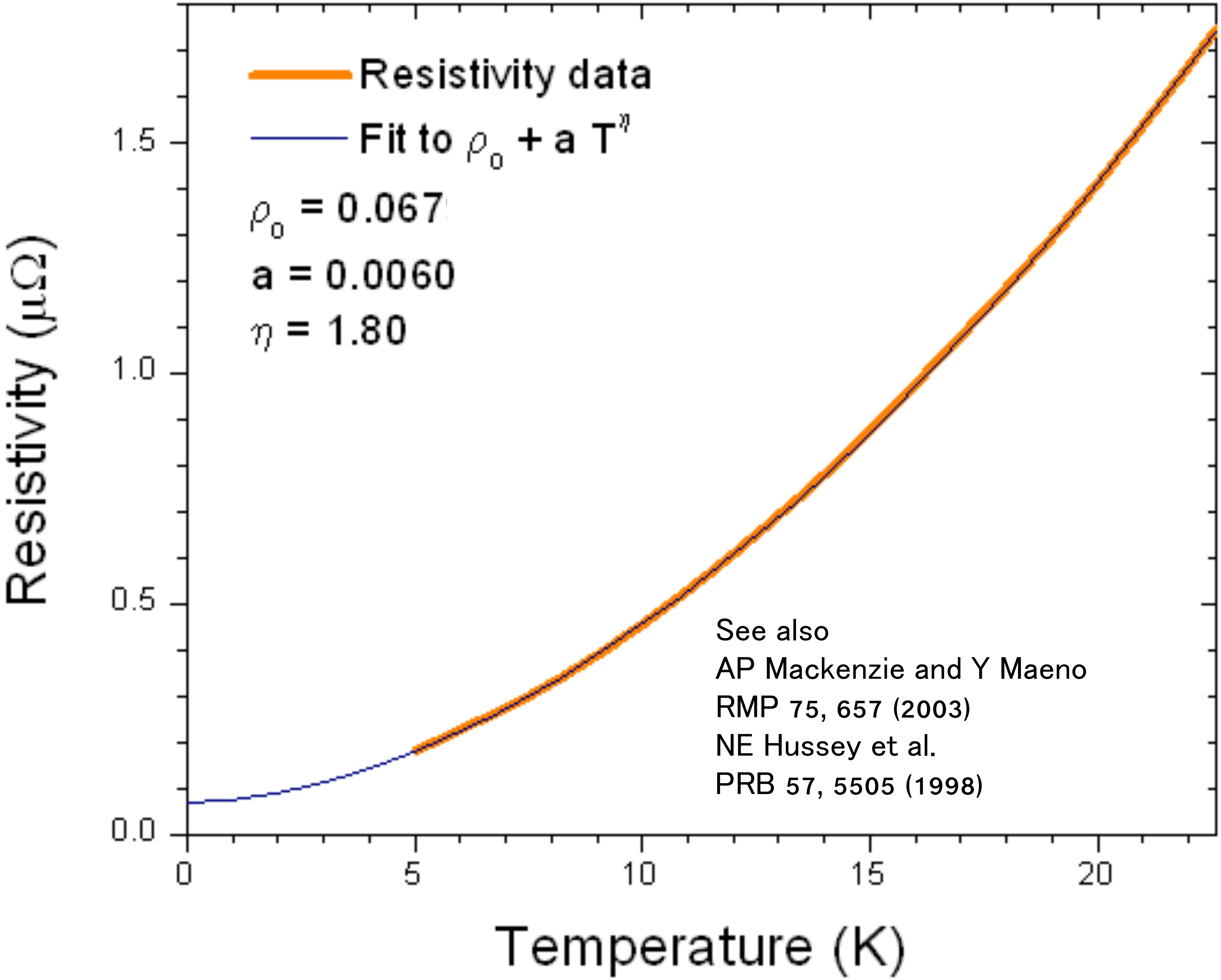
R. N. Gurzhi, Sov. Phys. JETP 35, 673 (1959)

D. L. Maslov & A. V. Chubukov, PRB 86, 155137 (2012)

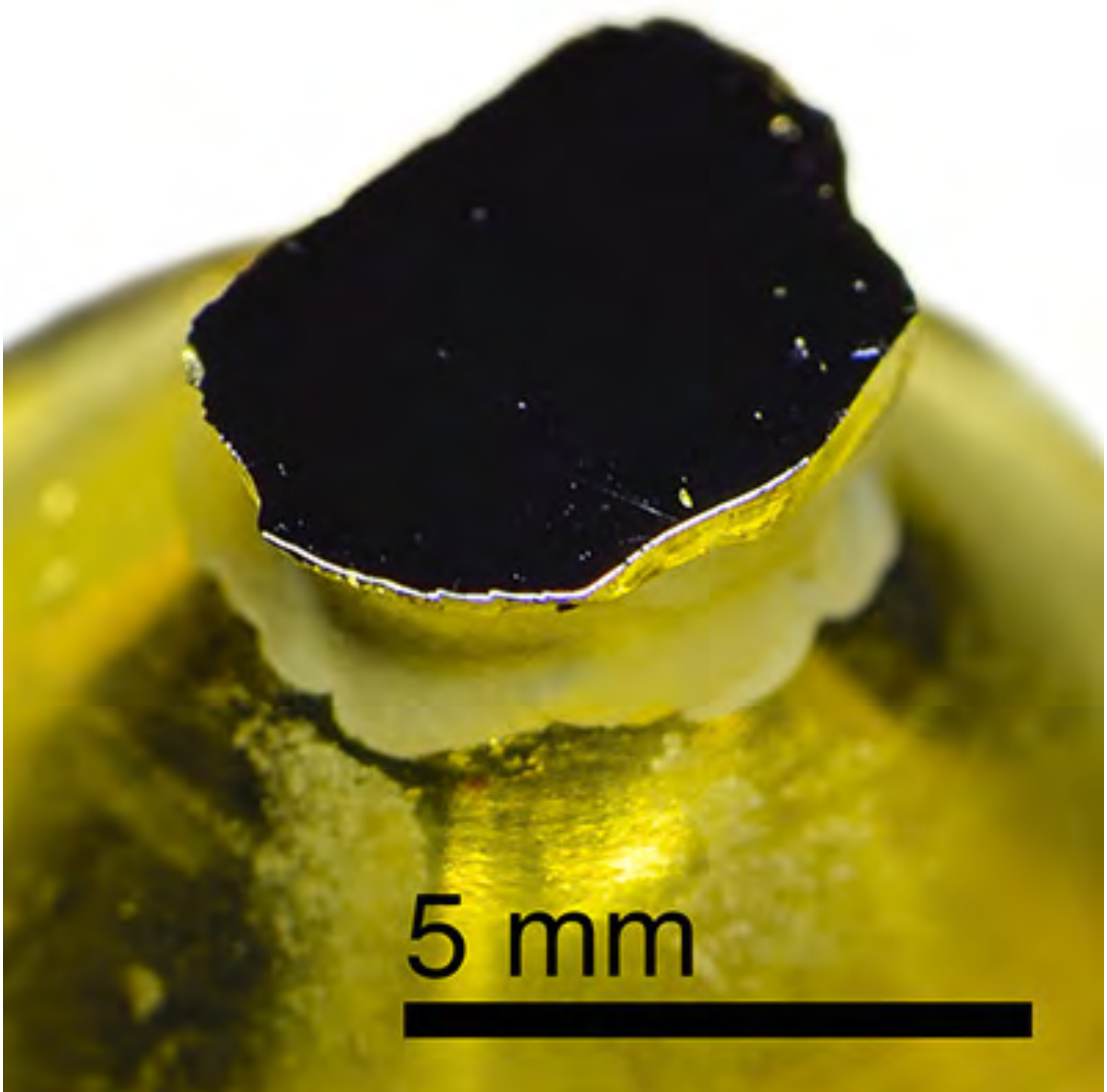
C. Berthod *et al*, PRB 87, 115109 (2013)



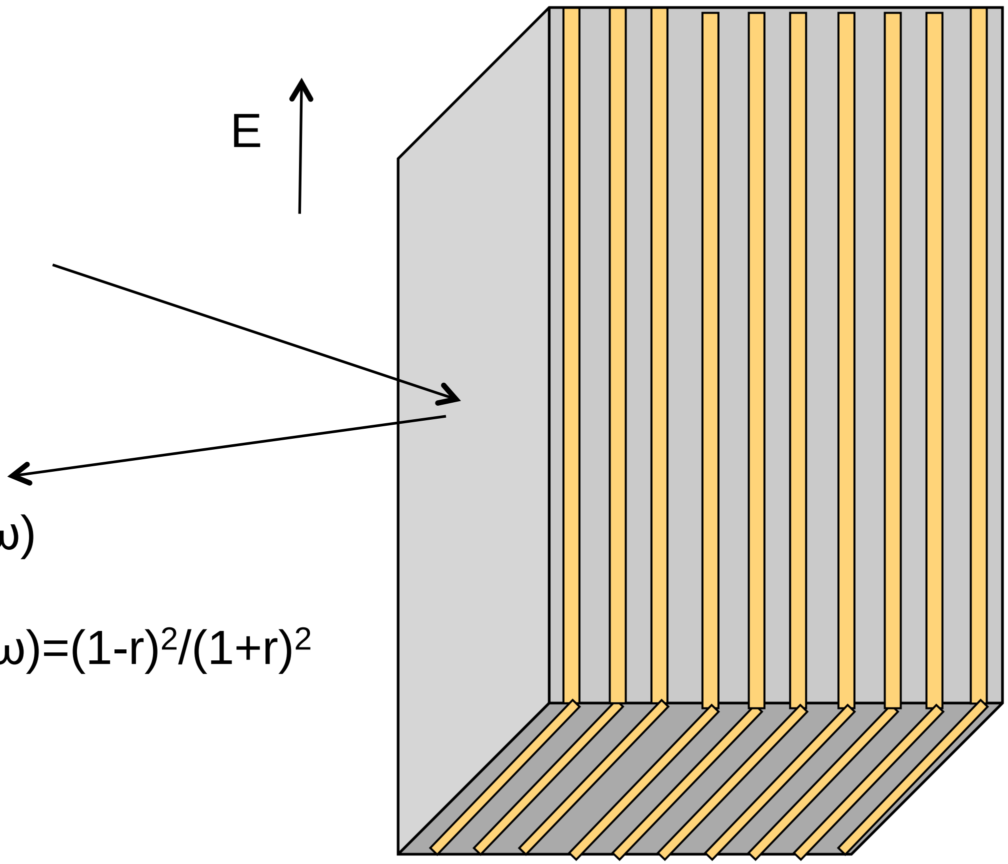
Ru
O
Sr
Sr_2RuO_4
isostructural
La_2CuO_4
b
c
a
a=3.86Å
c=12.72Å
p_x
d_{xz}
p_z
d_{xy}
p_y



See also
AP Mackenzie and Y Maeno
RMP 75, 657 (2003)
NE Hussey et al.
PRB 57, 5505 (1998)


5 mm

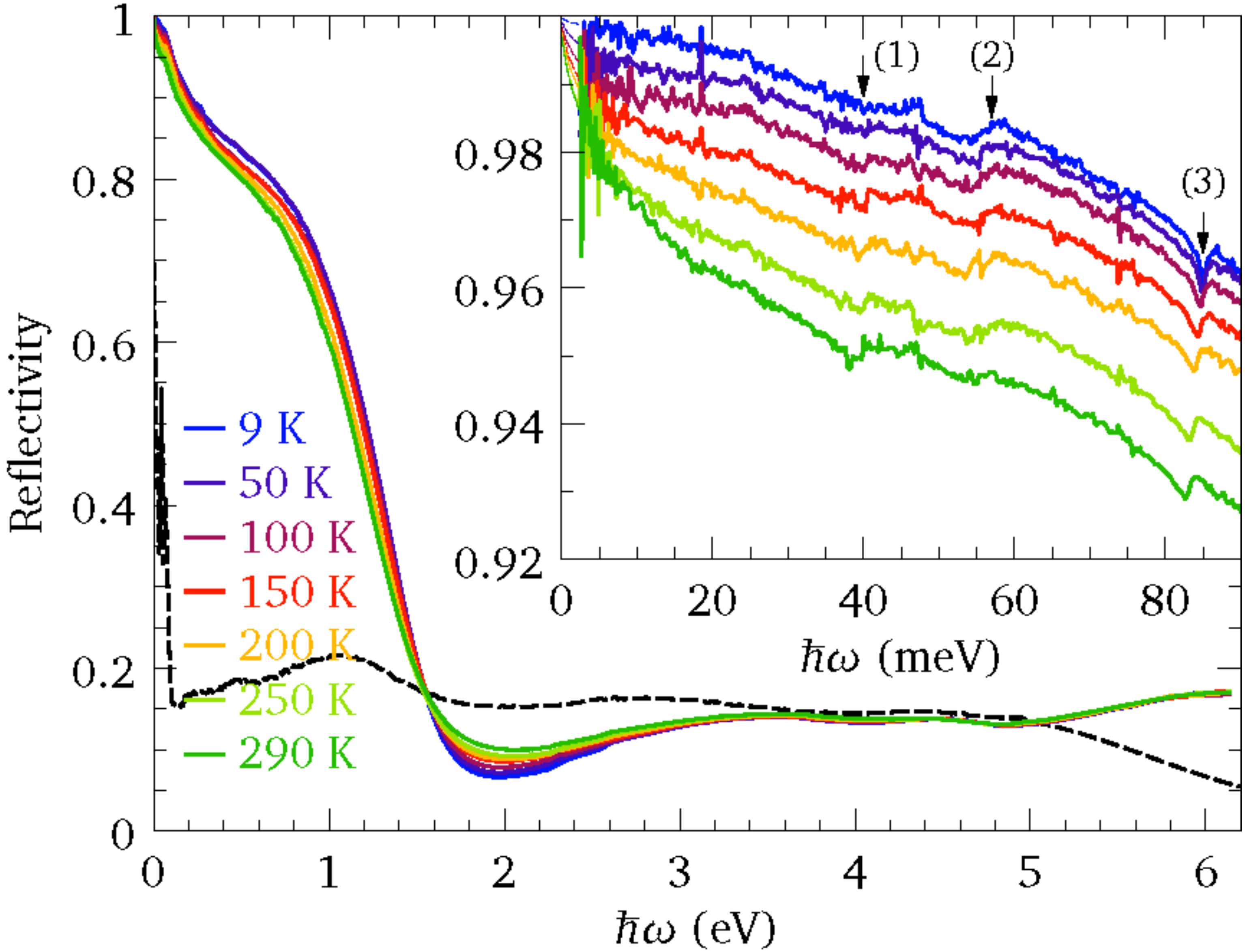
Measuring Reflectance of Sr_2RuO_4



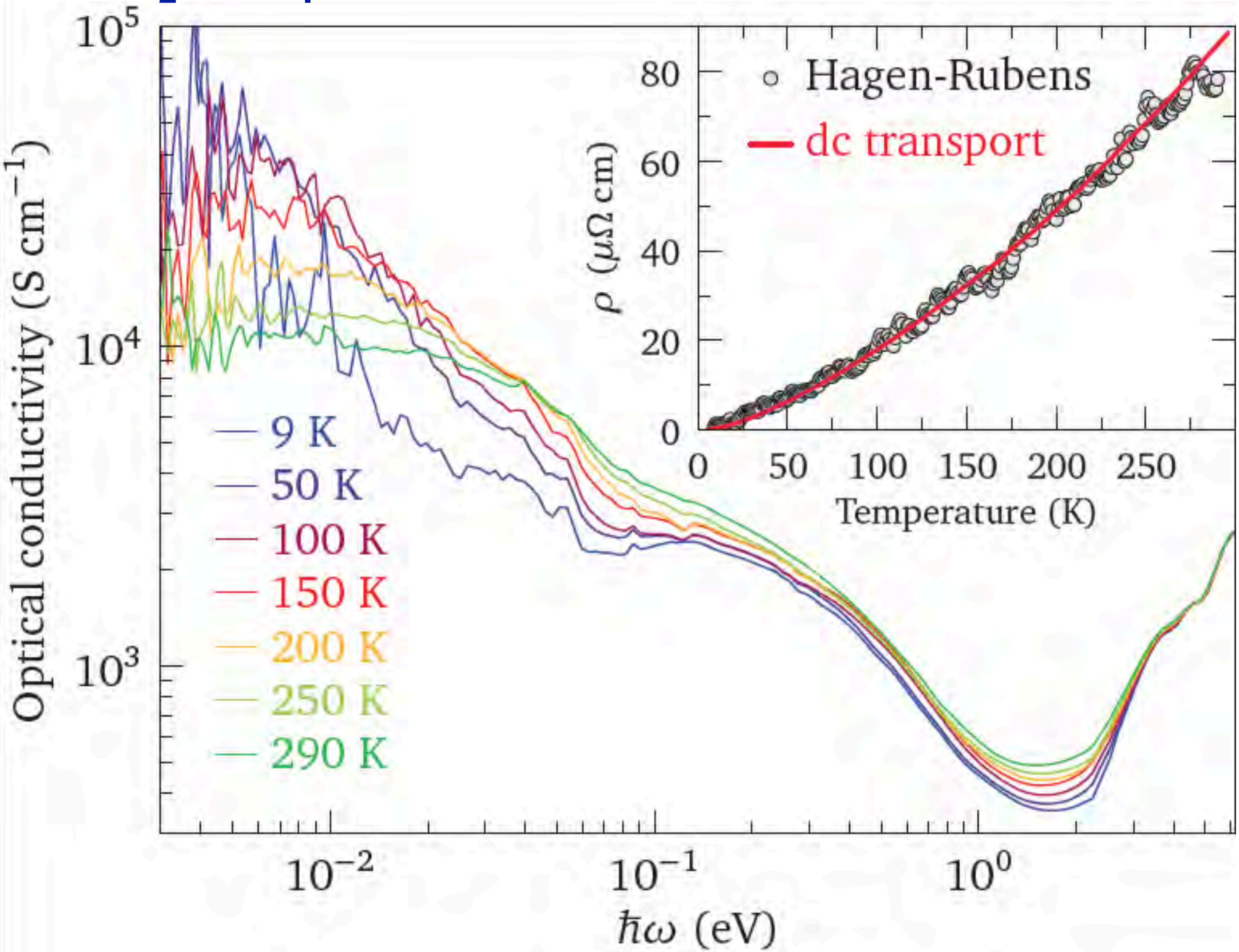
$r(\omega)$

$\varepsilon(\omega)=(1-r)^2/(1+r)^2$

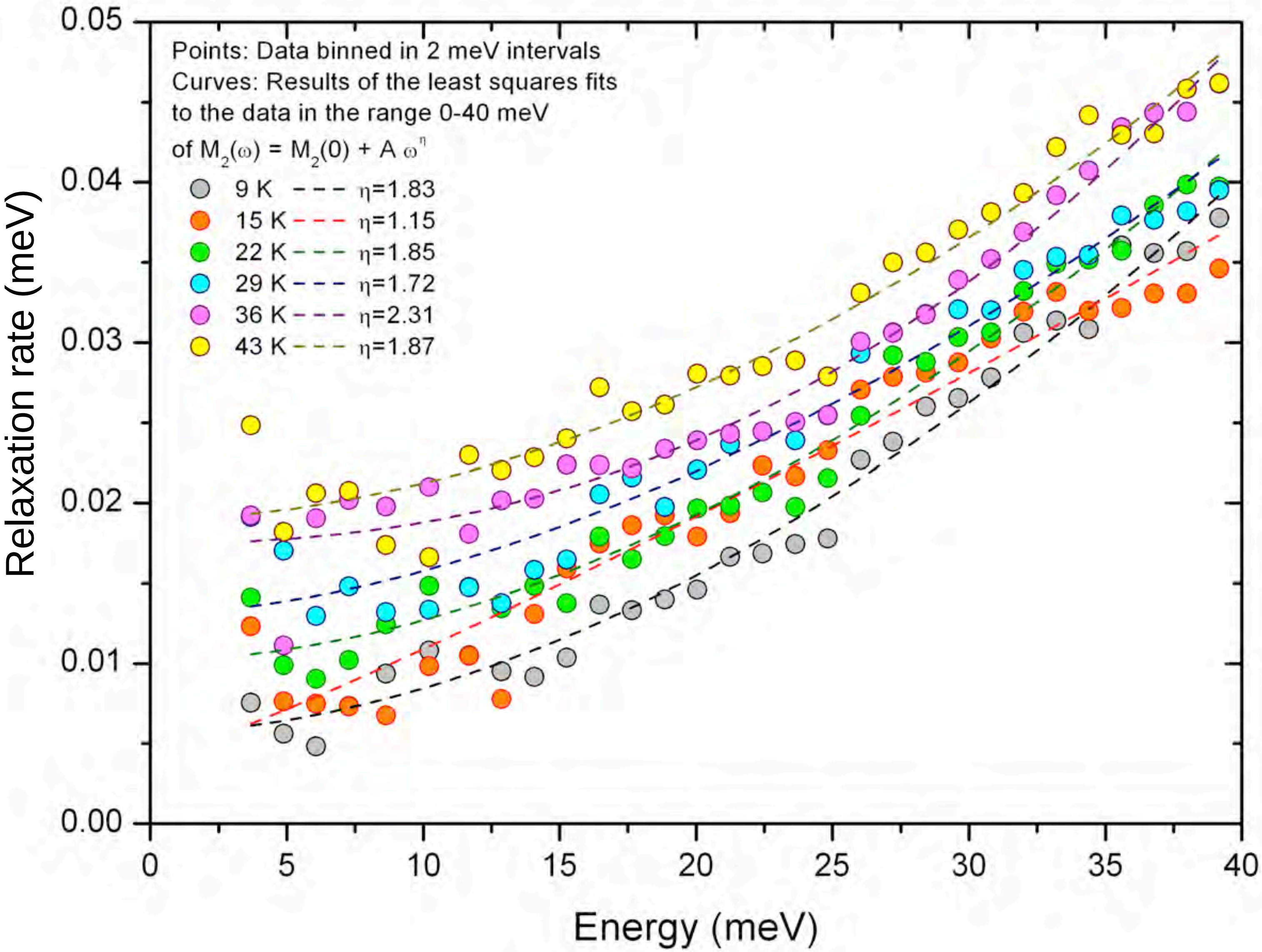
Ab-plane Reflectivity of Sr_2RuO_4



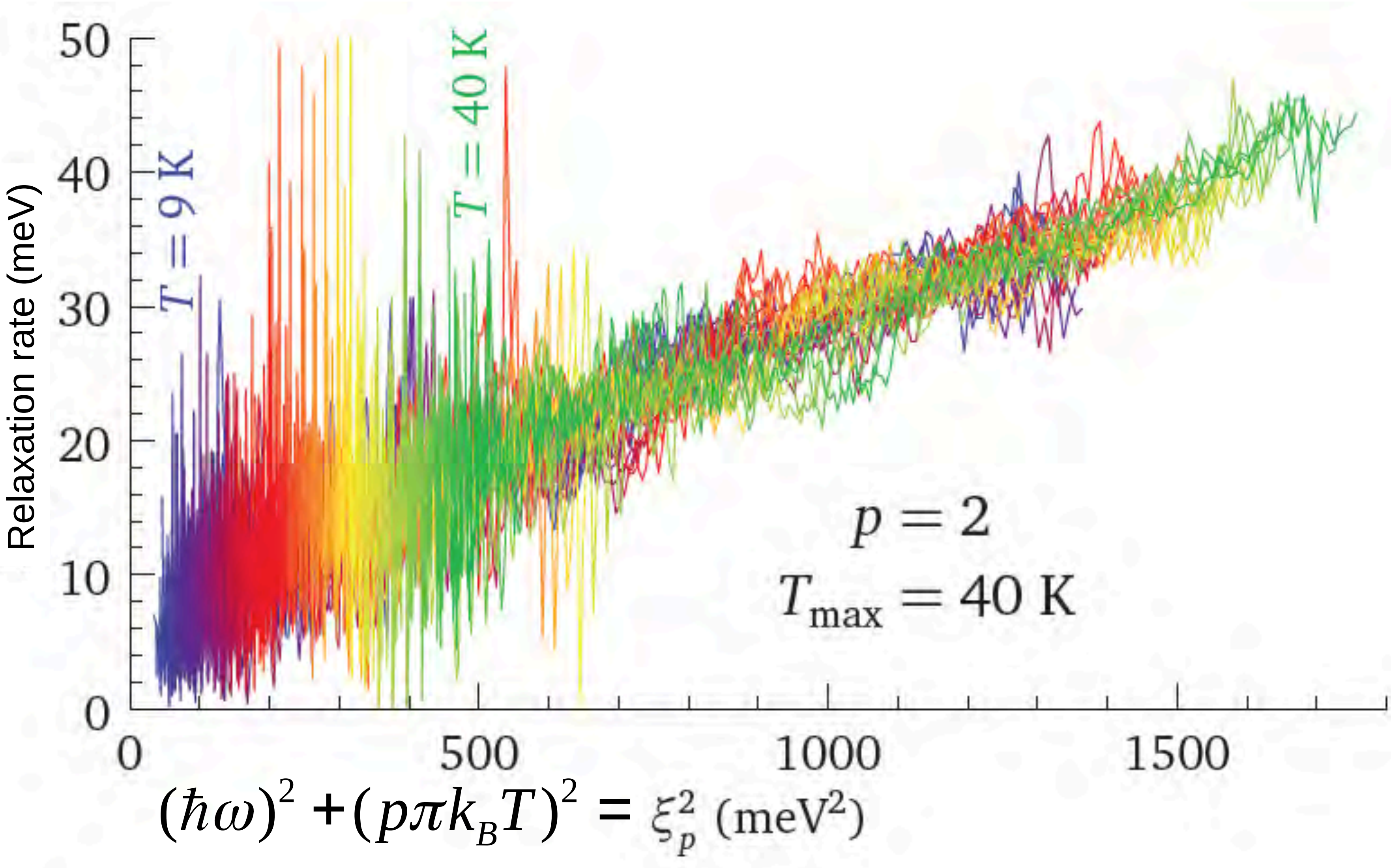
Sr_2RuO_4: Optical conductivity



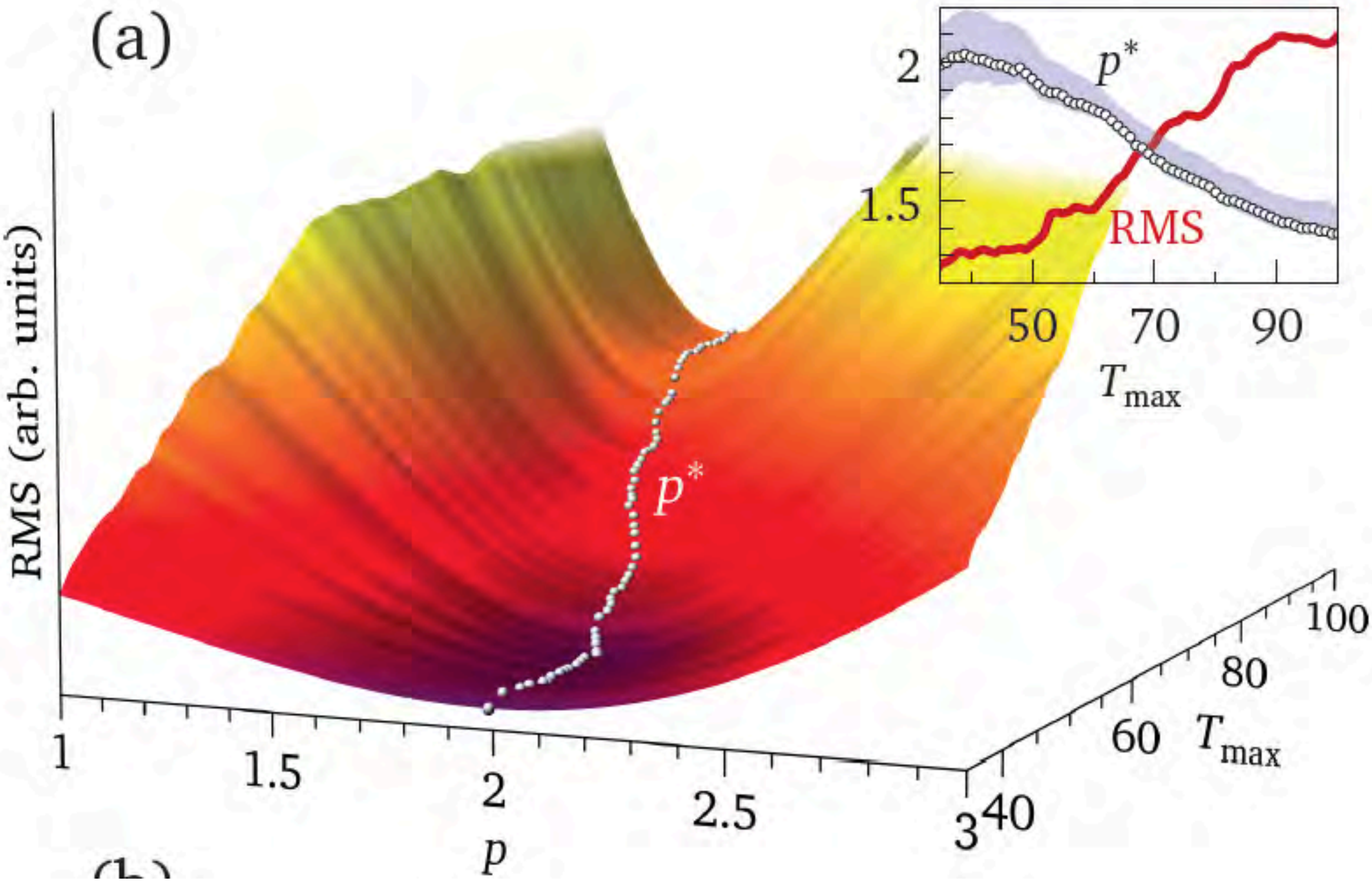
Sr_2RuO_4: Energy dependend Relaxation rate



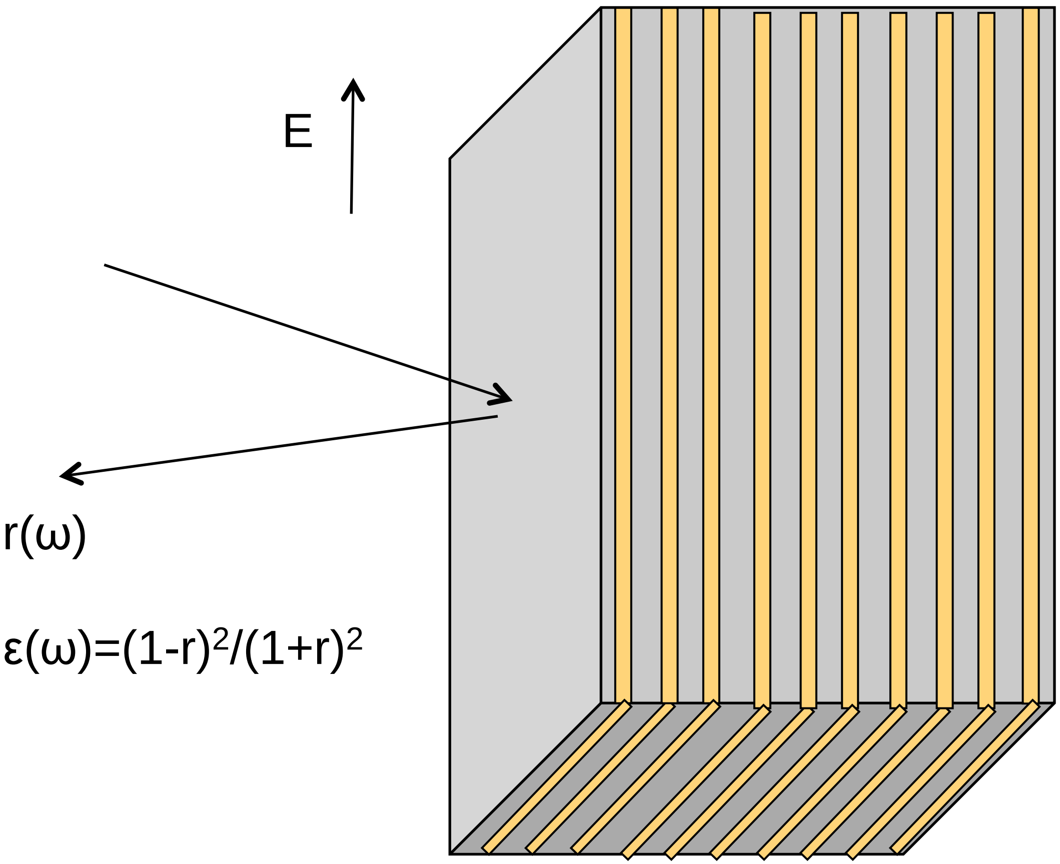
Sr_2RuO_4: Scaling collapse

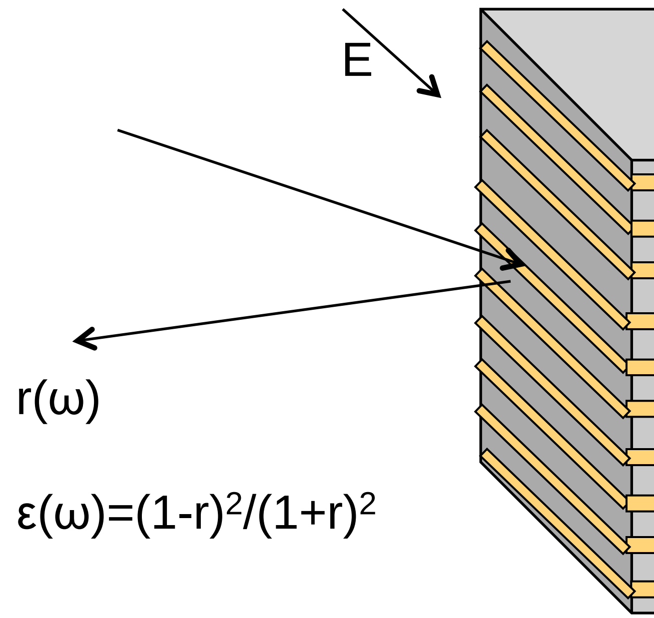


Sr_2RuO_4: Scaling collapse

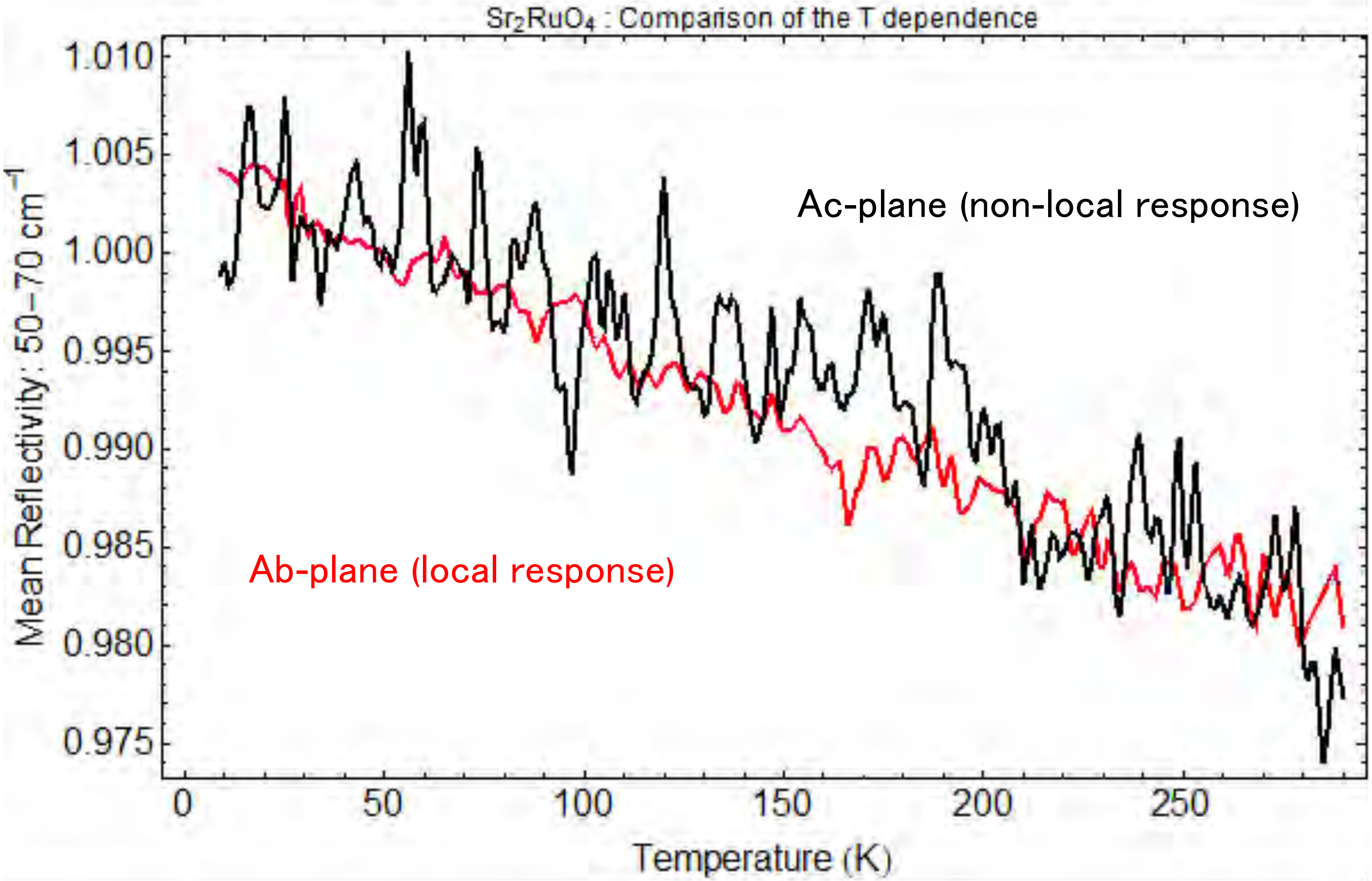


Experimental test on Sr_2RuO_4

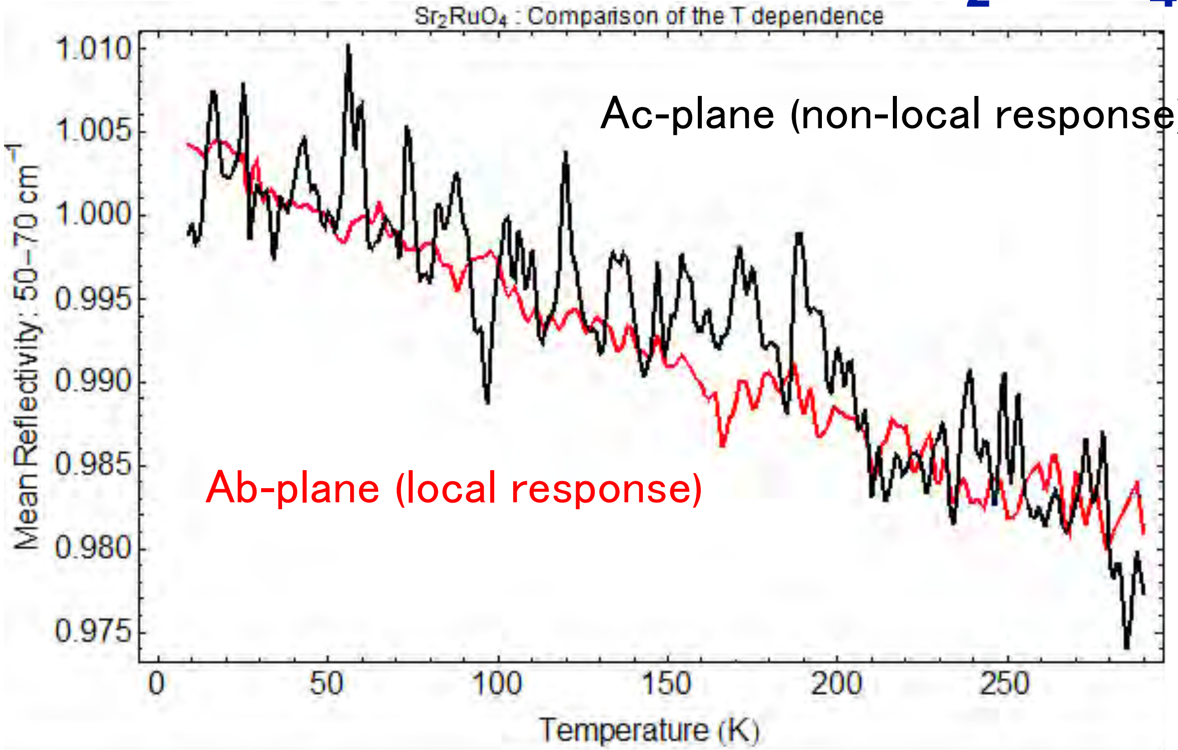




Experimental test on Sr_2RuO_4



Experimental test on Sr_2RuO_4



The overlap of both signals with and without viscosity implies an upper limit for η / τ

$$\Rightarrow \frac{\eta}{\tau} < 10^{-7} \rho c^2$$

Conclusions Part II

- Sr_2RuO_4: A strongly interacting Fermi liquid
- Lifetime grows as $1/T^2$ and as $1/\omega^2$
- Universal scaling of the optical momentum relaxation rate: $1/\tau = A\{(\hbar\omega)^2 + (2\pi k_B T)^2\}$

Further reading: Phys. Rev. Lett. 113, 087404 (2014)

- Viscosity parameter $\eta / \tau < 10^{-7} \rho c^2$

http://www.m2s-2015.ch

Materials and Mechanisms of Superconductivity 2015

The International Conference M2S HTSC 2015 will take place from Sunday, August 23 until Friday, August 28 in Geneva, Switzerland.

Location :
Geneva International Conference Center

$$\left.\begin{array}{l}\mathbf{H}\neq\mathbf{B};\mathbf{D}\neq\mathbf{E}\\ \text{magnetic permeability: } \mathbf{H}=\mu^{-1}(\omega)\mathbf{B}\\ \text{dielectric permittivity: } \mathbf{D}=\varepsilon(\omega)\mathbf{E}\end{array}\right\}\Rightarrow q^2=\varepsilon(\omega)\mu(\omega)\frac{\omega^2}{c^2}$$

Gauge transformation

$$\left.\begin{array}{l}\tilde{\mathbf{H}}=\mathbf{H}+\partial\mathbf{N}/\partial t\\ \Rightarrow\tilde{\mu}^{-1}(\omega)=\mu^{-1}(\omega)-i\omega\mathbf{N}/\mathbf{B}\\ \tilde{\mathbf{D}}=\mathbf{D}+c\nabla\times\mathbf{N}\\ \Rightarrow\tilde{\varepsilon}(\omega)=\varepsilon(\omega)+ic\mathbf{q}\times\mathbf{N}/\mathbf{E}\\ \text{Take: } \mathbf{N}\equiv\frac{i}{\omega}\left(1-\mu^{-1}(\omega)\right)\mathbf{B}\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\tilde{\mathbf{H}}=\mathbf{B}\\ \Rightarrow\tilde{\mu}=1\\ \tilde{\mathbf{D}}=\mathbf{D}+\left(1-\mu^{-1}(\omega)\right)\frac{q^2c^2}{\omega^2}\mathbf{E}\\ \Rightarrow\tilde{\varepsilon}(q,\omega)=\varepsilon(\omega)+\left(1-\mu^{-1}(\omega)\right)\frac{q^2c^2}{\omega^2}\end{array}\right\}\Rightarrow q^2=\tilde{\varepsilon}(q,\omega)\frac{\omega^2}{c^2}$$