A Holographic Model of the Kondo Effect (Part 1)

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Quantum Field Theory, String Theory, and Condensed Matter Physics
Kolymbari, Greece
September 1, 2014
Credits

Based on 1310.3271

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Outline:

- The Kondo Effect
- The CFT Approach
- Top-Down Holographic Model
- Bottom-Up Holographic Model
- Summary and Outlook
The Kondo Effect

The screening of a magnetic moment by conduction electrons at low temperatures.
\( \vec{\mu} \propto g \vec{S} \)
The Kondo Hamiltonian

\[ H_K = \sum_{k, \sigma} \varepsilon(k) c_{k\sigma}^\dagger c_{k\sigma} + g_K \vec{S} \cdot \sum_{k\sigma k' \sigma'} c_{k\sigma}^\dagger \frac{1}{2} \vec{\tau}_{\sigma \sigma'} c_{k' \sigma'} \]

\[ c_{k\sigma}^\dagger, c_{k\sigma} \]

\[ \sigma = \uparrow, \downarrow \]

\[ c_{k\sigma} \rightarrow e^{i\alpha} c_{k\sigma} \]

\[ \varepsilon(k) = \frac{k^2}{2m} - \varepsilon_F \]
The Kondo Hamiltonian

\[ H_K = \sum_{k, \sigma} \varepsilon(k) c_{k\sigma}^{\dagger} c_{k\sigma} + g_K \vec{S} \cdot \sum_{k\sigma k' \sigma'} c_{k\sigma}^{\dagger} \frac{1}{2} \vec{\tau}_{\sigma\sigma'} c_{k'\sigma'} \]

- \( \vec{S} \): Spin of magnetic moment
- \( \vec{\tau} \): Pauli matrices
- \( g_K \): Kondo coupling
Running of the Coupling

\[ \beta g_K \propto -g_K^2 + \mathcal{O}(g_K^3) \]

Asymptotic freedom!

The Kondo Temperature

\[ T_K \sim \Lambda_{\text{QCD}} \]
Running of the Coupling

\[ \beta g_K \propto -g_K^2 + \mathcal{O}(g_K^3) \]

At low energy, the coupling diverges!

\[ g_K \rightarrow \infty \]

The Kondo Problem

What is the ground state?
Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

Fermi liquid description (Nozières 1975)

Bethe Ansatz/Integrability
(Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)

Large-N expansion
(Anderson, Read, Newns, Doniach, Coleman, ... 1970-80s)

Quantum Monte Carlo
(Hirsch, Fye, Gubernatis, Scalapino,... 1980s)

Conformal Field Theory (CFT)
(Affleck and Ludwig 1990s)
UV

Fermi liquid + decoupled spin

IR

size $\propto 1/T_K$

“Kondo screening cloud”
On average, the NET spin of the Kondo screening cloud equals that of a single electron.

\[ \text{size} \propto \frac{1}{T_K} \]
The Kondo screening cloud spin binds with the impurity spin. The anti-symmetric singlet of $SU(2)$ is given by:

$$\frac{1}{\sqrt{2}} \left( | \uparrow i \downarrow e \rangle - | \downarrow i \uparrow e \rangle \right)$$

This is known as the "Kondo singlet".
Fermi liquid + decoupled spin

Fermi liquid
+ NO magnetic moment
+ electrons EXCLUDED from impurity location
Kondo Effect in Many Systems

Alloys

Cu, Ag, Au, Mg, Zn, ... doped with Cr, Fe, Mo, Mn, Re, Os, ...

Quantum dots


**Generalizations**

Enhance the spin group

\[ SU(2) \rightarrow SU(N) \]

Representation of impurity spin

\[ s_{\text{imp}} = 1/2 \quad \longrightarrow \quad R_{\text{imp}} \]

Multiple “channels” or “flavors”

\[ c \rightarrow c^{\alpha} \quad \alpha = 1, \ldots, k \]

\[ U(1) \times SU(k) \]
Generalizations

Kondo model specified by

\[ N, k, R_{\text{imp}} \]

Apply the techniques mentioned above...

IR fixed point: NOT always a fermi liquid

“Non-Fermi liquids”
Open Problems

Entanglement Entropy

Affleck, Laflorenic, Sørensen 0906.1809

Quantum Quenches

Latta et al. 1102.3982

Multiple Impurities

Kondo:

\[ \vec{S}_i \cdot \vec{S}_j \]

Form singlets with each other

Form singlets with electrons

Competition between these can produce a quantum phase transition
Open Problems

Entanglement Entropy
Affleck, Laflorencie, Sørensen 0906.1809

Quantum Quenches
Latta et al. 1102.3982

Multiple Impurities

Heavy fermion compounds

Kondo lattice
Open Problems

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Quantum Quenches

Multiple Impurities

Let's try AdS/CFT!

Heavy fermion compounds

Kondo lattice

FIG. 1: Quantum critical points in heavy fermion metals. a: AF ordering temperature \( T_N \) vs. Au concentration \( x \) for CeCu\(_6\)−\( x \)Au\( x \) (Ref. 7), showing a doping induced QCP. b: Suppression of the magnetic ordering in YbRh\(_2\)Si\(_2\) by a magnetic field. Also shown is the evolution of the exponent \( \alpha \) in \( \Delta \rho \equiv [\rho(T) - \rho_0] \propto T^\alpha \), within the temperature-field phase diagram of YbRh\(_2\)Si\(_2\) (Ref. 9). Blue and orange regions mark \( \alpha = 2 \) and 1, respectively. c: Linear temperature dependence of the electrical resistivity for Ge-doped YbRh\(_2\)Si\(_2\) over three decades of temperature (Ref. 9), demonstrating the robustness of the non-Fermi liquid behavior in the quantum critical regime. d: Temperature vs. pressure phase diagram for CePd\(_2\)Si\(_2\), illustrating the emergence of a superconducting phase centered around the QCP. The Néel- (\( T_N \)) and superconducting order temperatures (\( T_c \)) are indicated by closed and open symbols, respectively.
GOAL

Find a holographic description of the Kondo Effect
Find a holographic description of the Kondo Effect for Single Impurity ONLY.
Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

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CFT Approach to the Kondo Effect

Affleck and Ludwig 1990s

Reduction to one spatial dimension

Kondo interaction preserves spherical symmetry

\[
g_K \delta^3(\vec{x}) \vec{S} \cdot c^\dagger(\vec{x}) \frac{\vec{r}}{2} c(\vec{x})
\]

restrict to s-wave

restrict to momenta near \( k_F \)

\[
c(\vec{x}) \approx \frac{1}{r} \left[ e^{-ik_F r} \psi_L(r) - e^{+ik_F r} \psi_R(r) \right]
\]
Consider the limit $\lambda \to \infty$ for convenience, in spatial dimension.

The strong coupling fixed point is the same as the weak coupling fixed point except for an exchange to enter the site-0, since that would destroy the singlet state, costing an energy.

In terms of left-moving description of the single-electron wave-function. Note that at zero Kondo coupling, the parity even single particle configuration: one electron at the site 0 forms a singlet with the impurity:

$$\langle \uparrow \downarrow \rangle = 0: \lambda \to \infty$$

What is the behaviour of the parity even channel corresponds to a ferromagnetic Kondo model? The simple assumption is that

$$\sum \phi_i = 1$$

On the other hand, at parity odd ones are unaffected.

Thus we simply form free electron Bloch states with the boundary condition

$$\psi_i(0) = 0, \text{ where } \psi_i = \sin \frac{ki}{\pi}.$$

The ground state of the interaction term will be the following configuration:

$$\psi_L(-\lambda) \equiv \psi_R(\lambda)$$

FIG. 4. Reflecting the left-movers to the negative axis.
CFT Approach to the Kondo Effect

\[ H_K = \frac{v_F}{2\pi} \int_{-\infty}^{+\infty} dr \left[ \psi_L^\dagger i\partial_r \psi_L + \delta(r) \tilde{g}_K \vec{S} \cdot \psi_L^\dagger \vec{\tau} \psi_L \right] \]

\[ \tilde{g}_K \equiv \frac{k_F^2}{2\pi^2 v_F} \times g_K \]

RELATIVISTIC chiral fermions

\[ v_F = \text{“speed of light”} \]

chiral CFT!
Spin $SU(N)$

$k \geq 1$

$J = \psi_L^\dagger \psi_L$

$U(1)$

$\vec{J} = \psi_L^\dagger \vec{t} \psi_L$

$SU(N)$

$J^A = \psi_L^\dagger t^A \psi_L$

$SU(k)$
\[ z \equiv \tau + ir \]

\[ J^A(z) = \sum_{n \in \mathbb{Z}} z^{-n-1} J^A_n \]

\[ [J^A_n, J^B_m] = i f^{ABC} J^C_{n+m} + N \frac{n}{2} \delta^{AB} \delta_{n,-m} \]

\[ SU(k)_N \text{ Kac-Moody Algebra} \]

N counts net number of chiral fermions
CFT Approach to the Kondo Effect

\[ H_K = \frac{v_F}{2\pi} \int_{-\infty}^{+\infty} dr \left[ \psi_L^{\dagger} i \partial_r \psi_L + \delta(r) \tilde{g}_K \vec{S} \cdot \psi_L^{\dagger} \vec{\tau} \psi_L \right] \]

Full symmetry:

\[(1 + 1)d\] chiral conformal symmetry

\[ SU(N)_k \times SU(k)_N \times U(1)_{kN} \]
CFT Approach to the Kondo Effect

\[ H_K = \frac{v_F}{2\pi} \int_{-\infty}^{+\infty} dr \left[ \psi_L^\dagger i \partial_r \psi_L + \delta(r) \tilde{g}_K \vec{S} \cdot \psi_L^\dagger \vec{\tau} \psi_L \right] \]

\[ J = \psi_L^\dagger \psi_L \quad U(1) \]

\[ \vec{J} = \psi_L^\dagger \vec{\tau} \psi_L \quad SU(N) \]

\[ J^A = \psi_L^\dagger t^A \psi_L \quad SU(k) \]

Kondo coupling: \( \vec{S} \cdot \vec{J} \) marginal
Eigenstates are representations of the Kac-Moody algebra.

Determine how representations re-arrange between UV and IR.

\[ R_{\text{highest weight}}^{UV} \otimes R_{\text{imp}} = R_{\text{highest weight}}^{IR} \]

\[ SU(N)_k \times SU(k)_N \times U(1)_{Nk} \]
CFT Approach to the Kondo Effect

**Take-Away Messages**

Central role of the Kac-Moody Algebra

Kondo coupling: $\vec{S} \cdot \vec{J}$
Outline:

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• The CFT Approach
• Top-Down Holographic Model
• Bottom-Up Holographic Model
• Summary and Outlook
GOAL

Find a holographic description of the Kondo Effect
STRATEGY

Follow the CFT approach

Reproduce the Symmetries

Reproduce the Kondo coupling $\vec{S} \cdot \vec{J}$
What classical action do we write on the gravity side of the correspondence?
How do we describe holographically...

1. The chiral fermions?
2. The impurity?
3. The Kondo coupling?
### Top-Down Model

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- 3-7 and 7-3
- 3-5 and 5-3
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Open strings
### Top-Down Model

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CFT with holographic dual
### Top-Down Model

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Decouple
## Top-Down Model

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(1+1)-dimensional chiral fermions
Top-Down Model

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3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

the impurity
## Top-Down Model

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Kondo interaction
The D3-branes

| $N_c$ | D3 | X | X | X | X | X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

3-3 strings

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$\int_{S^5} F_5 \propto N_c$

$F_5 = dC_4$

Type IIB Supergravity

$AdS_5 \times S^5$
The D3-branes

\[
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N_c & D3 & X & X & X & X & & & & & \\
\end{array}
\]

3-3 strings

\[
\mathcal{N} = 4 \text{ SYM} \\
N_c \rightarrow \infty \\
\lambda \rightarrow \infty
\]

\[
\text{Type IIB Supergravity } \quad AdS_5 \times S^5
\]

Our Kondo model will have TWO coupling constants

\'
t Hooft and Kondo
Anti-de Sitter Space

\[ ds^2 = \frac{dr^2}{r^2} + r^2 (-dt^2 + dx^2 + dy^2 + dz^2) \]

Figure 1: The two slicings of AdS\(_5\). The horizontal axis is the direction \( x \) transverse to the brane and the vertical axis is the radial direction of AdS interpolating from the boundary (solid line) to the horizon (dashed line). The figure on the left shows lines of constant \( \rho \) while the figure on the right shows lines of constant \( r \).
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3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Decouple
Probe Limit

\[ N_7/N_c \to 0 \text{ and } N_5/N_c \to 0 \]

\( U(N_7) \times U(N_5) \) becomes a global symmetry

Total symmetry:

\[ SU(N_c) \times U(N_7) \times U(N_5) \]

(plus R-symmetry)
## Top-Down Model

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3-5 and 5-3

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(1+1)-dimensional chiral fermions
The D7-branes

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Skenderis, Taylor hep-th/0204054
Harvey and Royston 0709.1482, 0804.2854
Buchbinder, Gomis, Passerini 0710.5170

(1+1)-dimensional chiral fermions $\psi_L$

$$SU(N_c) \times U(N_7) \times U(N_5)$$

$N_c$ $\overline{N}_7$ singlet
The D7-branes

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(1+1)-dimensional chiral fermions \( \psi_L \)

Kac-Moody algebra

\[ SU(N_c)_{N_7} \times SU(N_7)_{N_c} \times U(1)_{N_cN_7} \]
The D7-branes

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Differences from Kondo

(1+1)-dimensional chiral fermions $\psi_L$

Do not come from reduction from (3+1) dimensions

Genuinely relativistic
The D7-branes

(1+1)-dimensional chiral fermions $\psi_L$

Differences from Kondo

$SU(N_c)$ is gauged!

$\vec{J} = \psi_L^\dagger \vec{\tau} \psi_L$
The D7-branes

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$SU(N_c)$ is gauged!

Harvey and Royston 0709.1482, 0804.2854
Buchbinder, Gomis, Passerini 0710.5170
In the probe limit, the gauge anomaly is suppressed...

\[ SU(N_c)_{N_7} \rightarrow SU(N_c) \]

...but the global anomalies are not.

\[ SU(N_7)_{N_c} \times U(1)_{N_cN_7} \rightarrow SU(N_7)_{N_c} \times U(1)_{N_cN_7} \]
\[ \mathcal{N} = 4 \text{ SYM} \]
\[ N_c \rightarrow \infty \]
\[ \lambda \rightarrow \infty \]

Probe \( \psi^L \)

\[ \text{Type IIB Supergravity} \]
\[ AdS_5 \times S^5 \]

Probe D7-branes
\[ AdS_3 \times S^5 \]
\[ \mathcal{N} = 4 \text{ SYM} \]
\[ N_c \to \infty \]
\[ \lambda \to \infty \]

Type IIB Supergravity

\[ AdS_5 \times S^5 \]

Probe D7-branes

\[ AdS_3 \times S^5 \]

\[ ds^2 = \frac{dr^2}{r^2} + r^2 \left( -dt^2 + dx^2 + dy^2 + dz^2 \right) + ds^2_{S^5} \]
\[ \mathcal{N} = 4 \text{ SYM} \]

\[
N_c \rightarrow \infty \\
\lambda \rightarrow \infty
\]

\[ \text{Type IIB Supergravity} \]

\[ AdS_5 \times S^5 \]

\[ \text{Probe D7-branes} \]

\[ AdS_3 \times S^5 \]

\[ \psi_L \]

\[ U(N_7) \text{ Current } J \]

\[ U(N_7) \text{ Gauge field } A \]
Kac-Moody Algebra Chern-Simons Gauge Field

Gukov, Martinec, Moore, Strominger  
hep-th/0403225

Kraus and Larsen  
hep-th/0607138
Probe D7-branes along $\text{AdS}_3 \times S^5$

\[ S_{D7} = \frac{1}{2} T_{D7} (2\pi \alpha')^2 \int P[C_4] \wedge \text{tr} F \wedge F + \ldots \]

\[ = -\frac{1}{2} T_{D7} (2\pi \alpha')^2 \int P[F_5] \wedge \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) + \ldots \]

\[ = -\frac{N_c}{4\pi} \int_{\text{AdS}_3} \text{tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) + \ldots \]

$U(\mathcal{N}_7)_{N_c}$ Chern-Simons gauge field
Chern-Simons Gauge Field in $\text{AdS}_3$

The chiral fermions:
### Top-Down Model

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The impurity
The D5-branes

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Skenderis, Taylor hep-th/0204054
Camino, Paredes, Ramallo hep-th/0104082
Gomis and Passerini hep-th/0604007

(0+1)-dimensional fermions $\chi$

$SU(N_c) \times U(N_7) \times U(N_5)$

$N_c$ singlet $\overline{N_5}$
The D5-branes

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$SU(N_c)$ is “spin”

$\vec{S} = \chi^\dagger \vec{\tau} \chi$

“slave fermions”

“Abrikosov pseudo-fermions”

Abrikosov, *Physics* 2, p.5 (1965)
$N_5 = 1$

Integrate out $\chi$

\[
\text{Det } (\mathcal{D}) = \text{Tr}_R P \exp \left[ i \int dt A_t \right]
\]

\[
R = \begin{array}{ccc}
\vdots \\
\end{array}
\]

\[
Q = \chi \dagger \chi
\]

$U(N_5) = U(1) \text{ charge}$
\( \mathcal{N} = 4 \) SYM

\( N_c \to \infty \)

\( \lambda \to \infty \)

\[
\begin{align*}
\text{Type IIB Supergravity} & \quad \text{Probe D5-branes} \\
\text{AdS}_5 \times S^5 & \quad \text{AdS}_2 \times S^4
\end{align*}
\]
\[ N = 4 \text{ SYM} \]
\[ N_c \rightarrow \infty \]
\[ \lambda \rightarrow \infty \]

Probe $\chi$

Type IIB Supergravity
\[ AdS_5 \times S^5 \]

Probe D5-branes
\[ AdS_2 \times S^4 \]

\[ ds^2 = \frac{dr^2}{r^2} + r^2 (-dt^2 + dx^2 + dy^2 + dz^2) + ds^2_{S^5} \]
$N = 4$ SYM

$N_c \rightarrow \infty$

$\lambda \rightarrow \infty$

Probe $\chi$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe D5-branes

$AdS_2 \times S^4$

$U(N_5)$ Current $J$

$U(N_5)$ Gauge field $A$

Electric flux
Probe D5-brane along $AdS_2 \times S^4$

Camino, Paredes, Ramallo hep-th/0104082

Dissolve $Q$ strings into the D5-brane

$AdS_2$ electric field $f_{rt}$

$$\sqrt{-g} f^{tr} \bigg|_{\partial AdS_2} = Q = \chi^\dagger \chi$$
Answer #2

The impurity:

Yang-Mills Gauge Field in $AdS_2$
## Top-Down Model

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Kondo interaction
The Kondo Interaction

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Complex scalar!

$$SU(N_c) \times U(N_7) \times U(N_5)$$

singlet $\overline{N}_7 \quad N_5$

$$\mathcal{O} \equiv \psi_L^\dagger \chi$$
The Kondo Interaction

\[ SU(N_c) \text{ is "spin"} \]

\[ \vec{J} = \psi_L^\dagger \vec{\tau} \psi_L \quad \vec{S} = \chi^\dagger \vec{\tau} \chi \]

\[ \vec{S} \cdot \vec{J} = \chi^\dagger \vec{\tau} \chi \cdot \psi_L^\dagger \vec{\tau} \psi_L \]

\[ \vec{\tau}_{ij} \cdot \vec{\tau}_{kl} = \delta_{il} \delta_{jk} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \]

\[ \vec{S} \cdot \vec{J} = |\psi_L^\dagger \chi|^2 + \mathcal{O}(1/N_c) \]

“double trace”
\( N = 4 \) SYM

\( N_c \to \infty \)

\( \lambda \to \infty \)

Type IIB Supergravity

\( AdS_5 \times S^5 \)

Probe D7-branes

\( AdS_3 \times S^5 \)

Probe D5-branes

\( AdS_2 \times S^4 \)

Bi-fundamental scalar

\( AdS_2 \times S^4 \)

\[ \mathcal{O} \equiv \psi^\dagger_L \chi \]
Answer #3

The Kondo interaction:

Bi-fundamental scalar in $AdS_2$
Top-Down Model

\[ \mathcal{X} \]

\[ r = \infty \]

\[ r = 0 \]

\[ N_c \int_{\text{AdS}_3} A \wedge F \]

\[ \int_{\text{AdS}_2} \text{tr} f^2 \]

\[ \int_{\text{AdS}_2} |D\Phi|^2 + V(\Phi^\dagger \Phi) \]

\[ D\Phi = \partial\Phi + iA\Phi - ia\Phi \]
Top-Down Model

$N_c \int_{AdS_3} A \wedge F$

$\int_{AdS_2} \text{tr} f^2$

$\int_{AdS_2} |D\Phi|^2 + V(\Phi^\dagger \Phi)$

$r = \infty$

$r = 0$

What is $V(\Phi^\dagger \Phi)$?
We don't know.
$N_c \int_{AdS_3} A \wedge F$

$\int_{AdS_2} \text{tr} f^2$

$\int_{AdS_2} |D\Phi|^2 + V(\Phi^\dagger \Phi)$

Switch to bottom-up model!
Outline:

• The Kondo Effect
• The CFT Approach
• Top-Down Holographic Model
• Bottom-Up Holographic Model
• Summary and Outlook
\[ N_c \int_{AdS_3} A \wedge F \quad \int_{AdS_2} \text{tr} f^2 \quad \int_{AdS_2} |D\Phi|^2 + V(\Phi^\dagger \Phi) \]

Bottom-Up Model
We pick $V(\Phi^\dagger \Phi)$.
Bottom-Up Model

\[ N_c \int_{AdS_3} A \wedge F \]

\[ \int_{AdS_2} \text{tr} f^2 \]

\[ \int_{AdS_2} |D\Phi|^2 + V(\Phi^\dagger \Phi) \]

\[ V(\Phi^\dagger \Phi) = m^2 \Phi^\dagger \Phi \]
Boundary Conditions

We choose $m^2 = \text{Breitenlohner-Freedman bound}$

$$\Phi(r) = \tilde{c} r^{-1/2} + c r^{-1/2} \log r + \ldots$$

Our double-trace (Kondo) coupling:

$$c = \tilde{g} K \tilde{C}$$

Witten hep-th/0112258
Berkooz, Sever, Shomer hep-th/0112264

$$\sqrt{-g} f^{rt} \bigg|_{\partial \text{AdS}_2} = Q$$
A holographic superconductor in $AdS_2$
Phase Transition

$T > T_c$
\[ \sqrt{-g} f^{tr} \big|_{\partial \text{AdS}_2} \neq 0 \quad \phi (r) = 0 \]
\[ \langle \psi_L^+ \chi \rangle = 0 \]

$T < T_c$
\[ \sqrt{-g} f^{tr} \big|_{\partial \text{AdS}_2} \neq 0 \quad \phi (r) \neq 0 \]
\[ \langle \psi_L^+ \chi \rangle \neq 0 \]

Superconductivity???
The large-N Kondo effect!

Phase Transition

\[ T > T_c \]

\[
\sqrt{-gf^{tr}} \bigg|_{\partial AdS_2} \neq 0 \quad \phi(r) = 0
\]

\[
\langle \psi_L^\dagger \chi \rangle = 0
\]

\[ T < T_c \]

\[
\sqrt{-gf^{tr}} \bigg|_{\partial AdS_2} \neq 0 \quad \phi(r) \neq 0
\]

\[
\langle \psi_L^\dagger \chi \rangle \neq 0
\]

The large-N Kondo effect!
Solutions of the Kondo Problem

- Numerical RG (Wilson 1975)
- Fermi liquid description (Nozières 1975)
- Bethe Ansatz/Integrability (Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)
- Large-N expansion (Anderson, Read, Newns, Doniach, Coleman, ... 1970-80s)
- Quantum Monte Carlo (Hirsch, Fye, Gubernatis, Scalapino, ... 1980s)
- Conformal Field Theory (CFT) (Affleck and Ludwig 1990s)
Phase Transition

\[ T > T_c \]
\[ \sqrt{-g} f^{tr} \bigg|_{\partial \text{AdS}_2} \neq 0 \quad \phi(r) = 0 \]
\[ \langle \psi_L^\dagger \chi \rangle = 0 \]

\[ T < T_c \]
\[ \sqrt{-g} f^{tr} \bigg|_{\partial \text{AdS}_2} \neq 0 \quad \phi(r) \neq 0 \]
\[ \langle \psi_L^\dagger \chi \rangle \neq 0 \]

\[ T_c \propto T_K \]
Phase Transition

\[ T > T_c \]
\[ \sqrt{-gf^{tr}} \bigg|_{\partial AdS_2} \neq 0 \quad \phi(r) = 0 \]
\[ \langle \psi_L^\dagger \chi \rangle = 0 \]

\[ T < T_c \]
\[ \sqrt{-gf^{tr}} \bigg|_{\partial AdS_2} \neq 0 \quad \phi(r) \neq 0 \]
\[ \langle \psi_L^\dagger \chi \rangle \neq 0 \]

Represents the formation of the Kondo singlet
The phase transition is an ARTIFACT of the large-N limit!
The actual Kondo effect is a crossover.
Outline:

- The Kondo Effect
- The CFT Approach
- Top-Down Holographic Model
- Bottom-Up Holographic Model
- Summary and Outlook
What is the holographic dual of the Kondo effect?

Holographic superconductor in $AdS_2$

coupled as a defect
to a Chern-Simons gauge field in $AdS_3$
Outlook

- Entropy? Heat Capacity? Resistivity?
- Multi-channel?
- Other impurity representations?
- Entanglement entropy?
- Quantum Quenches?
- Multiple Impurities?
Thank You.