A Holographic Model of the Kondo Effect (Part 1)

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Quantum Field Theory, String Theory, and Condensed Matter Physics
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Credits

Based on 1310.3271

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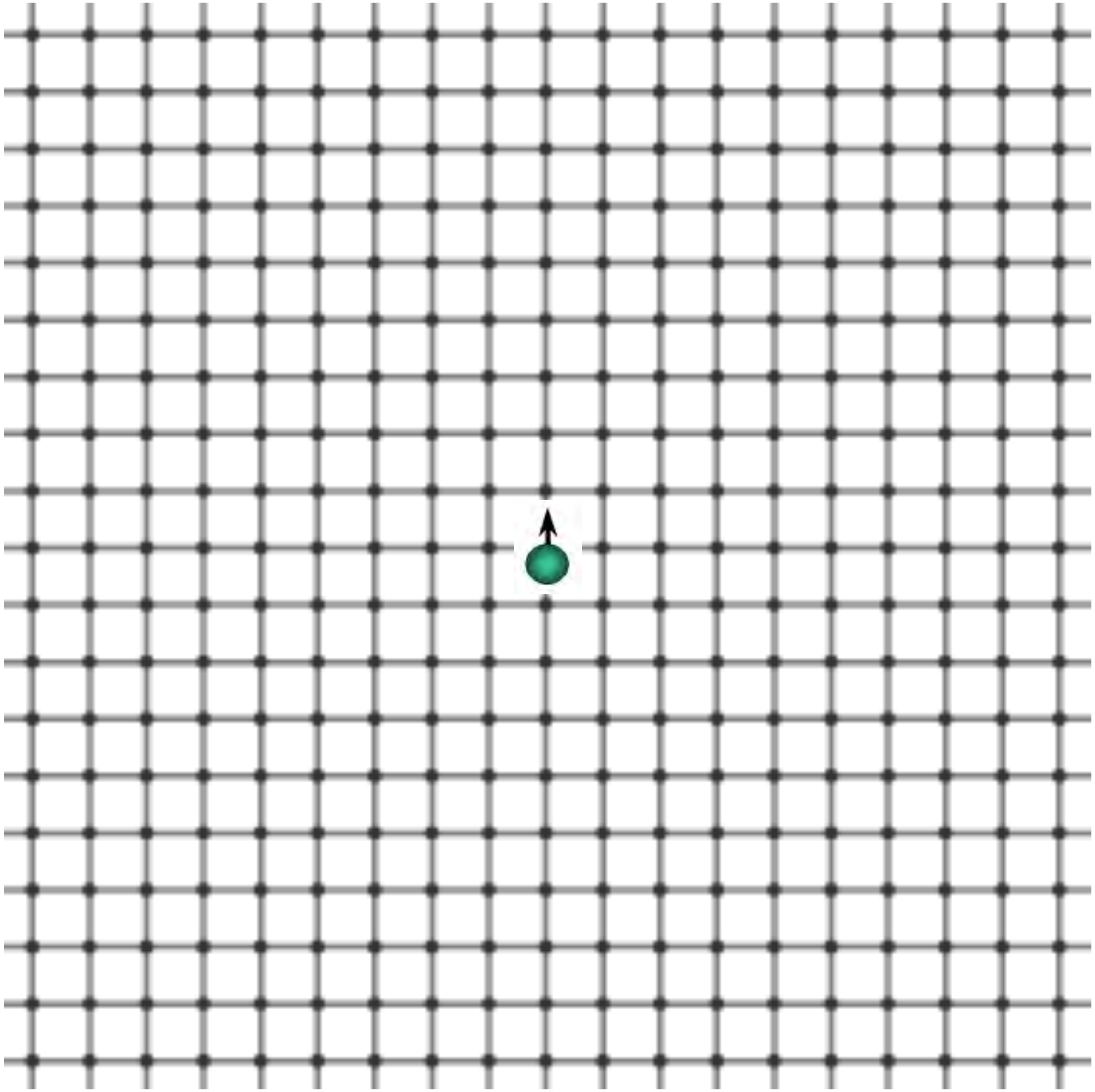
Outline:

- The Kondo Effect
- The CFT Approach
- Top-Down Holographic Model
- Bottom-Up Holographic Model
- Summary and Outlook

The Kondo Effect

The screening
of a magnetic moment
by conduction electrons
at low temperatures



$$\vec{\mu} \propto g\vec{S}$$

The Kondo Hamiltonian

$$H_K = \sum_{k,\sigma} \varepsilon(k)\, c^\dagger_{k\sigma} c_{k\sigma} + g_{_K} \vec{S} \cdot \sum_{k\sigma k'\sigma'} c^\dagger_{k\sigma} \frac{1}{2} \vec{\tau}_{\sigma\sigma'} c_{k'\sigma'}$$

$$c^\dagger_{k\sigma}\, , c_{k\sigma}$$

Conduction electrons

$$\sigma = \uparrow, \downarrow$$

Spin $SU(2)$

$$c_{k\sigma} \longrightarrow e^{i\alpha} c_{k\sigma}$$

Charge $U(1)$

$$\varepsilon(k) = \frac{k^2}{2m} - \varepsilon_F$$

Dispersion relation

The Kondo Hamiltonian

$$H_K = \sum_{k,\sigma} \varepsilon(k)\, c^{\dagger}_{k\sigma} c_{k\sigma} + g_K \vec{S} \cdot \sum_{k\sigma k'\sigma'} c^{\dagger}_{k\sigma} \frac{1}{2} \vec{\tau}_{\sigma\sigma'} c_{k'\sigma'}$$

$\vec{S}$ — Spin of magnetic moment

$\vec{\tau}$ — Pauli matrices

g_K — Kondo coupling

Running of the Coupling

$$\beta_{g_K} \propto -g_K^2 + \mathcal{O}(g_K^3)$$

Asymptotic freedom!

UV

$$g_K \longrightarrow 0$$

The Kondo Temperature

$$T_K \sim \Lambda_{\mathrm{QCD}}$$

Running of the Coupling

$$\beta_{g_K} \propto -g_K^2 + \mathcal{O}(g_K^3)$$

At low energy, the coupling diverges!

IR

$$g_K \longrightarrow \infty$$

The Kondo Problem

What is the ground state?

Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

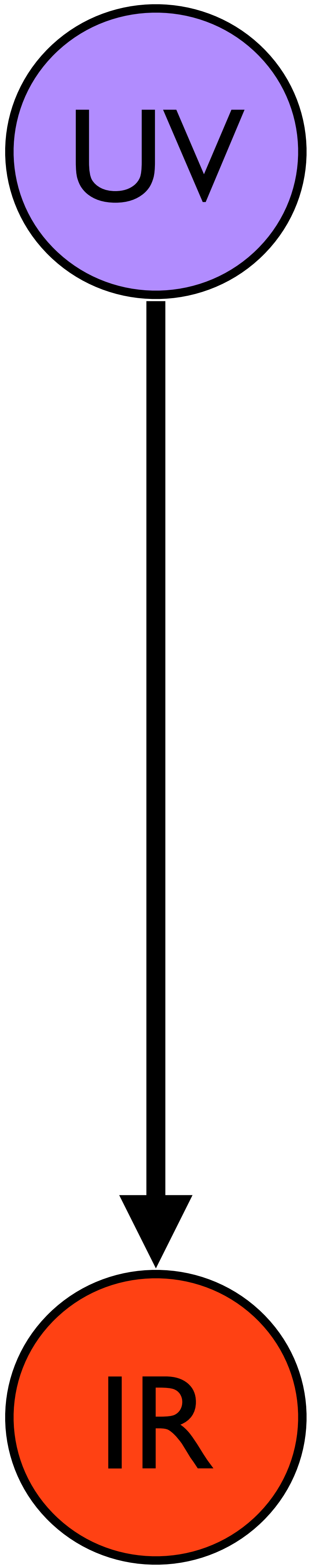
Fermi liquid description (Nozières 1975)

Bethe Ansatz/Integrability
(Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)

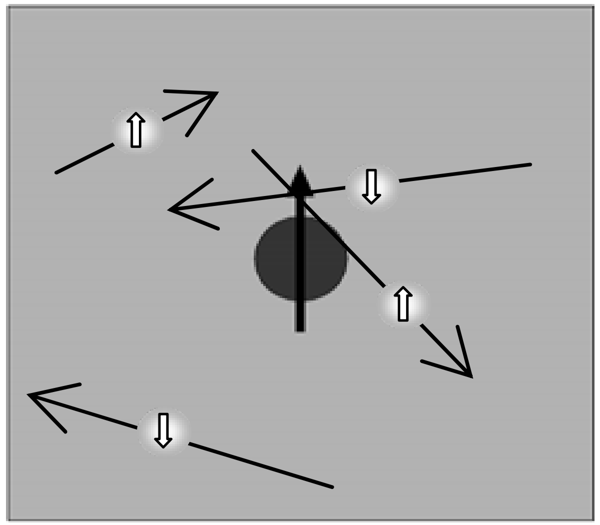
Large-N expansion
(Anderson, Read, Newns, Doniach, Coleman, ...1970-80s)

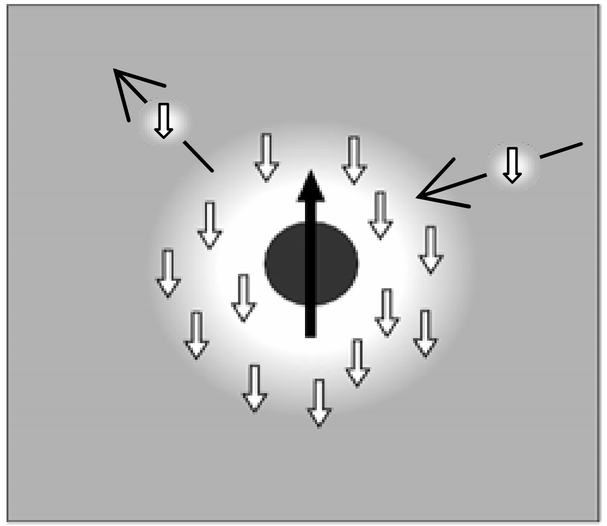
Quantum Monte Carlo
(Hirsch, Fye, Gubernatis, Scalapino,... 1980s)

Conformal Field Theory (CFT)
(Affleck and Ludwig 1990s)



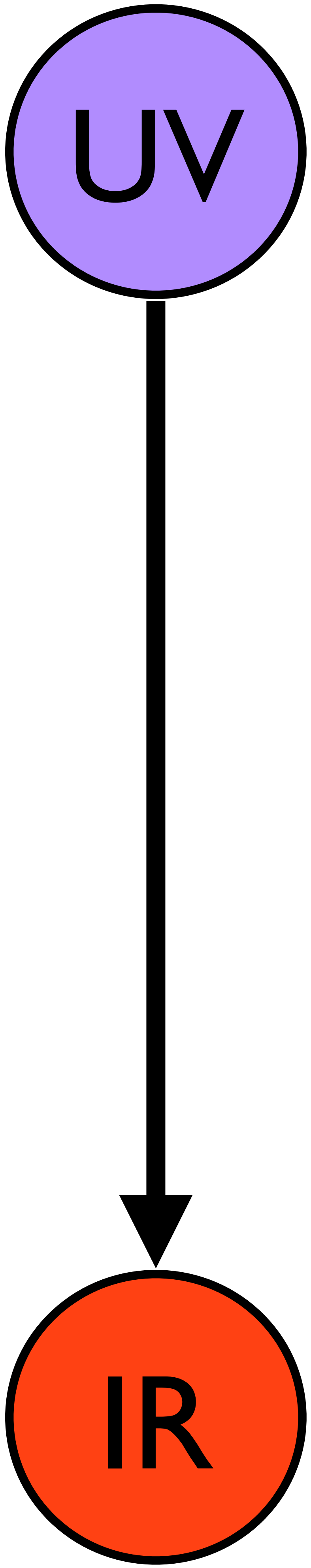
Fermi liquid + decoupled spin



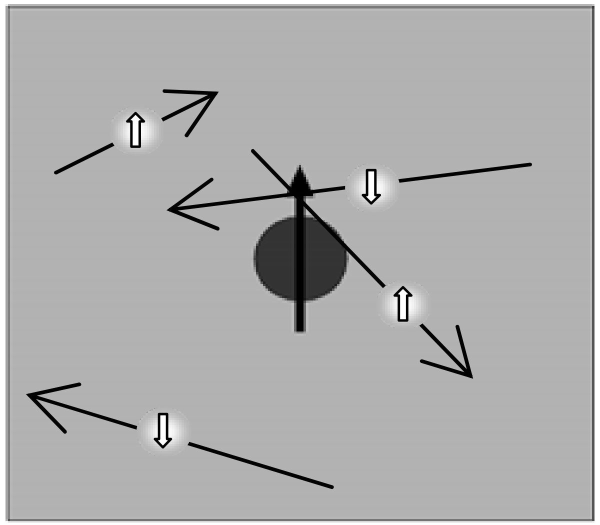


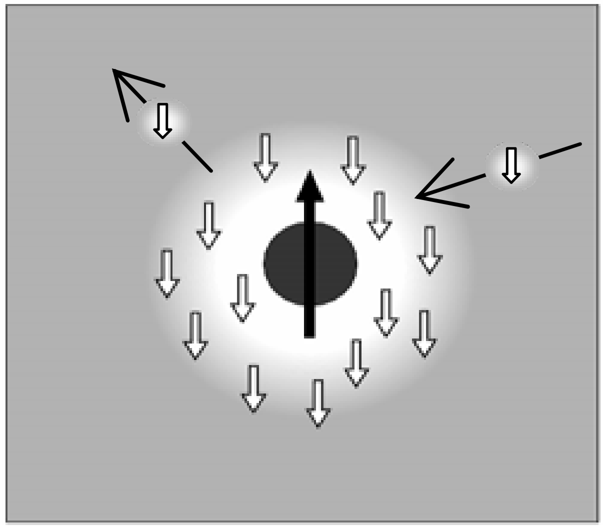
size $\propto 1/T_K$

"Kondo screening cloud"



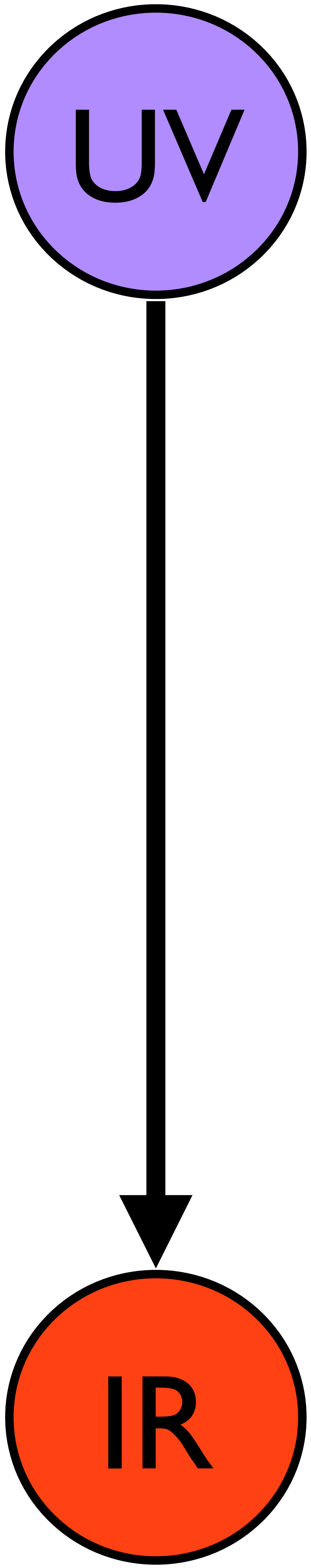
Fermi liquid + decoupled spin





size $\propto 1/T_K$

On average, the NET spin
of the Kondo screening cloud
equals that of a single electron



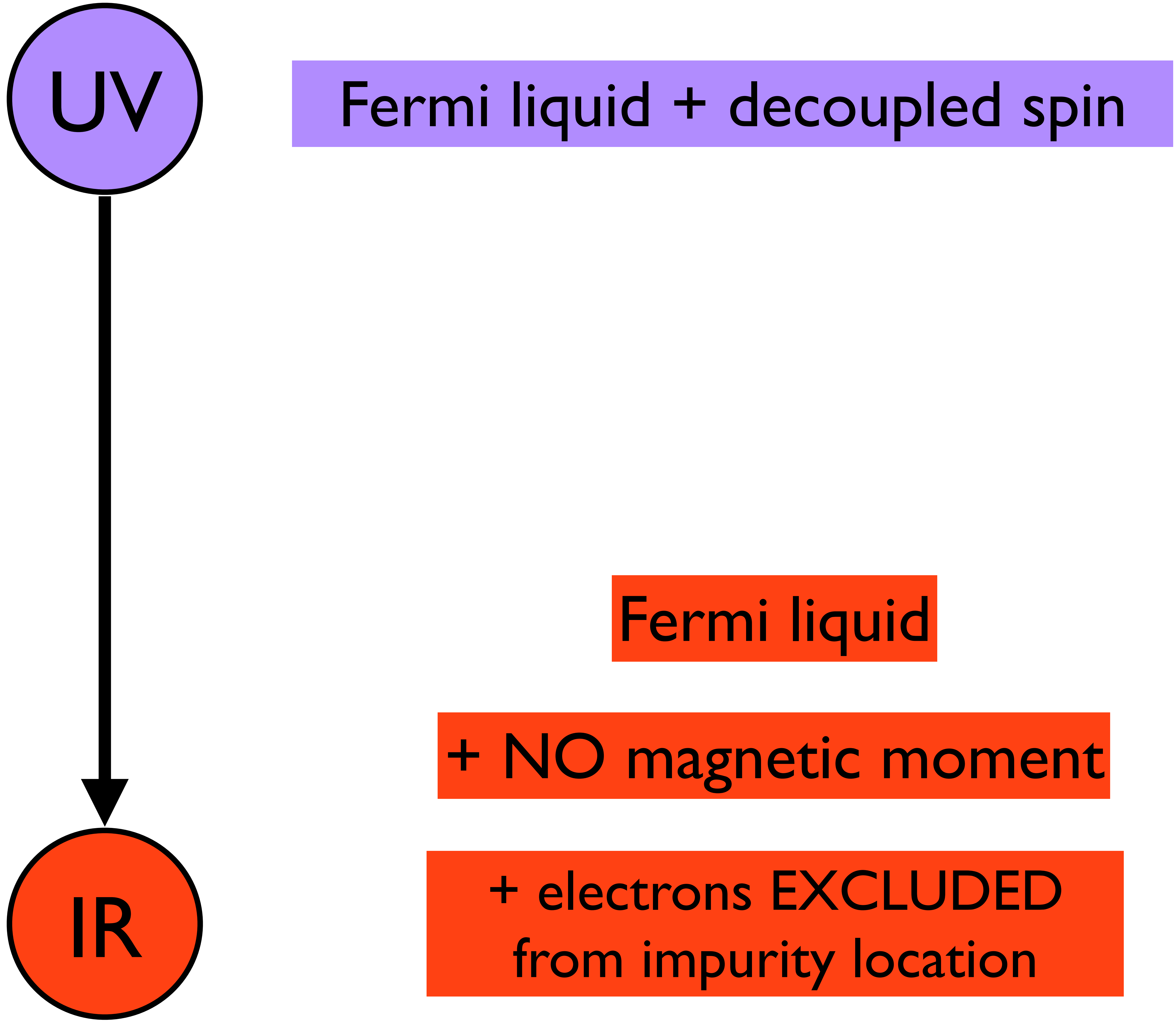
Fermi liquid + decoupled spin

The Kondo screening cloud spin binds with the impurity spin

Anti-symmetric singlet of $SU(2)$

$$\frac{1}{\sqrt{2}}\left(|\uparrow_i \downarrow_e\rangle - |\downarrow_i \uparrow_e\rangle\right)$$

“Kondo singlet”

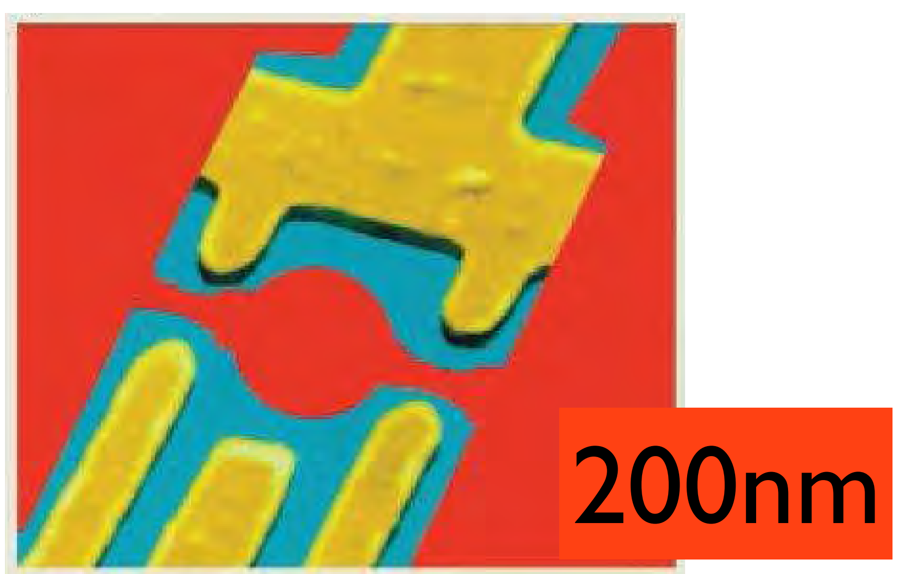

UV
Fermi liquid + decoupled spin
Fermi liquid
+ NO magnetic moment
+ electrons EXCLUDED from impurity location
IR

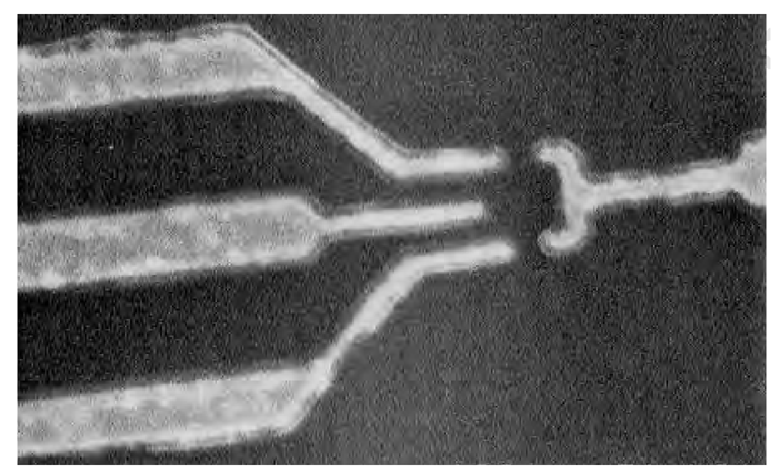
Kondo Effect in Many Systems

Alloys

Cu, Ag, Au, Mg, Zn, ... doped with Cr, Fe, Mo, Mn, Re, Os, ...

Quantum dots





Goldhaber-Gordon, et al., **Nature** 391 (1998), 156-159.

Cronenwett, et al., **Science** 281 (1998), no. 5376, 540-544.

Generalizations

Enhance the spin group

$$SU(2) \to SU(N)$$

Representation of impurity spin

$$s_{\rm imp} = 1/2 \quad \longrightarrow \quad R_{\rm imp}$$

Multiple "channels" or "flavors"

$$c \to c^{\alpha} \qquad \alpha = 1, \ldots, k$$

$$U(1) \times SU(k)$$

Generalizations

Kondo model specified by

$$N, \; k, \; R_{\text{imp}}$$

Apply the techniques mentioned above...

IR fixed point:

NOT always a fermi liquid

"Non-Fermi liquids"

Open Problems

Entanglement Entropy

Affleck, Laflorencie, Sørensen 0906.1809

Quantum Quenches

Latta et al. 1102.3982

Multiple Impurities

Kondo:	Form singlets with electrons
$\vec{S}_i \cdot \vec{S}_j$	Form singlets with each other

Competition between these can produce a

QUANTUM PHASE TRANSITION

Open Problems

Entanglement Entropy

Affleck, Laflorencie, Sørensen 0906.1809

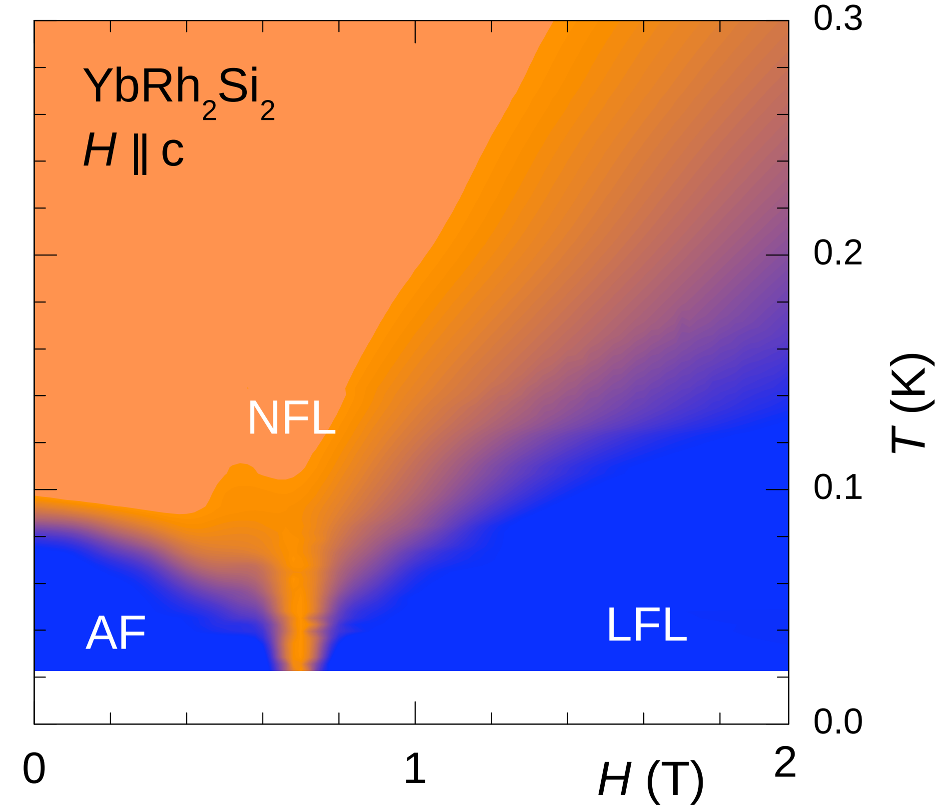
Quantum Quenches

Latta et al. 1102.3982

Multiple Impurities

Heavy fermion compounds

Kondo lattice



Open Problems

Entanglement Entropy

Affleck, Laflorencie, Sørensen 0906.1809

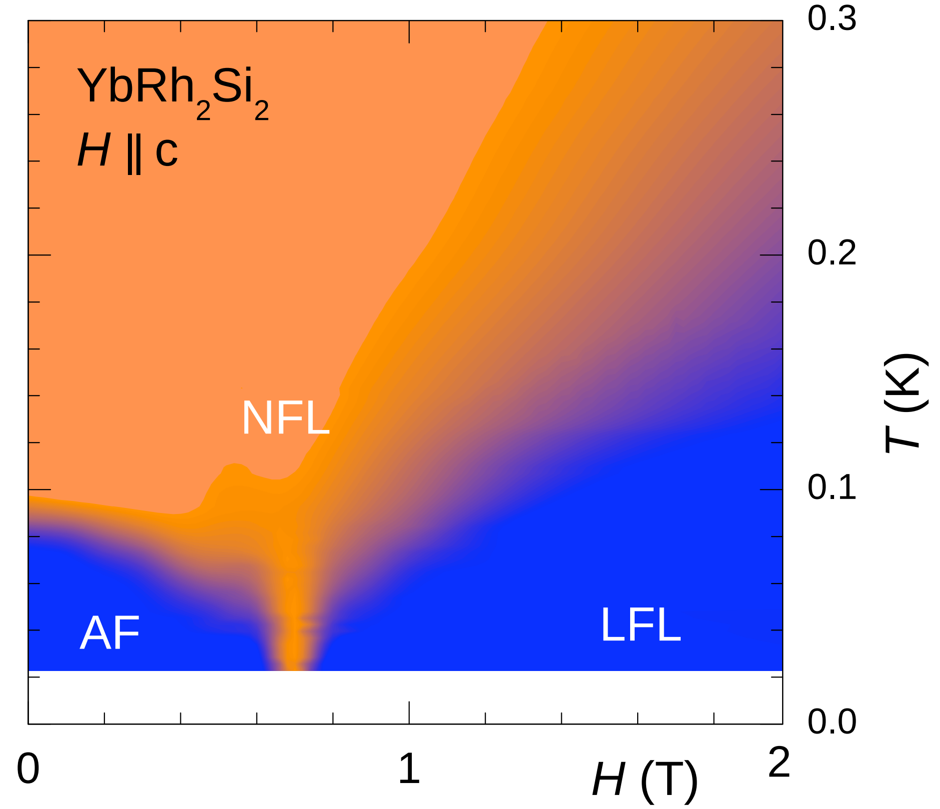
Quantum Quenches

Let's try AdS/CFT!

Multiple impurities

Heavy fermion compounds

Kondo lattice



GOAL

Find a holographic description
of the
Kondo Effect

GOAL

Find a holographic description

Single Impurity ONLY

Kondo Effect

Solutions of the Kondo Problem

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CFT Approach to the Kondo Effect

Affleck and Ludwig 1990s

Reduction to one spatial dimension

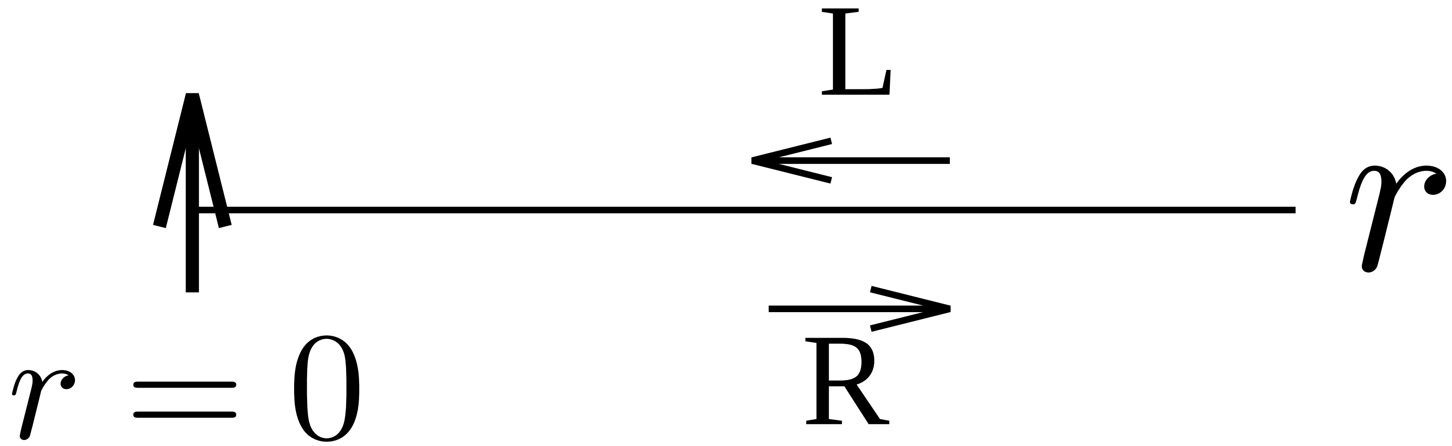
Kondo interaction preserves spherical symmetry

$$g_K \delta^3(\vec{x}) \vec{S} \cdot c^\dagger(\vec{x}) \frac{\vec{\tau}}{2} c(\vec{x})$$

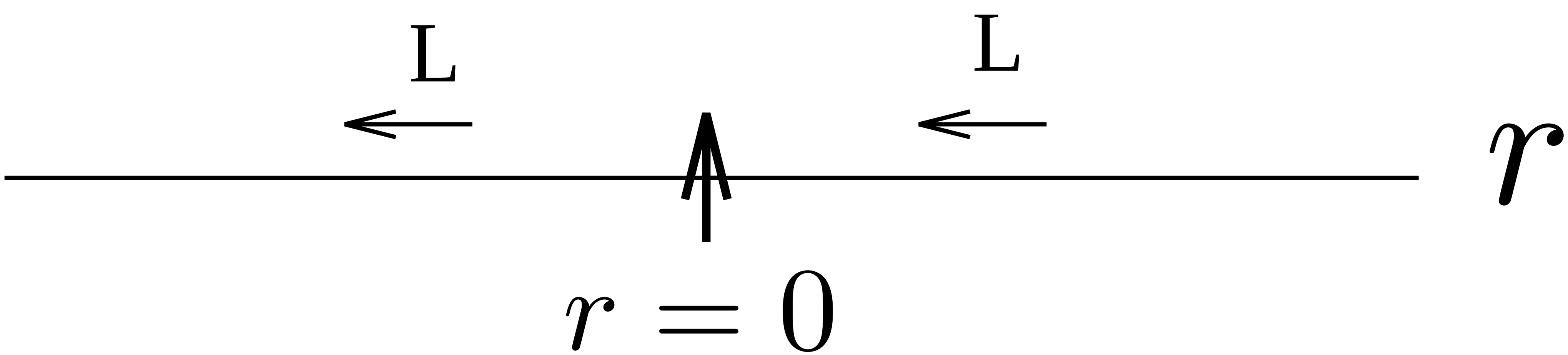
restrict to s-wave

restrict to momenta near k_F

$$c(\vec{x}) \approx \frac{1}{r} \left[e^{-ik_F r} \psi_L(r) - e^{+ik_F r} \psi_R(r) \right]$$



$$\psi_L(-r) \equiv \psi_R(+r)$$



CFT Approach to the Kondo Effect

$$H_K = \frac{v_F}{2\pi}\int_{-\infty}^{+\infty} dr \left[\psi_L^\dagger i\partial_r \psi_L + \delta(r)\,\tilde{g}_K\,\vec{S}\cdot\psi_L^\dagger \vec{\tau}\,\psi_L\right]$$

$$\tilde{g}_K \equiv \frac{k_F^2}{2\pi^2 v_F} \times g_K$$

RELATIVISTIC chiral fermions

v_F = "speed of light"

chiral CFT!

Spin $SU(N)$ | $k \geq 1$

$$J = \psi_L^\dagger \psi_L \qquad U(1)$$

$$\vec{J} = \psi_L^\dagger \, \vec{\tau} \, \psi_L \qquad SU(N)$$

$$J^A = \psi_L^\dagger \, t^A \, \psi_L \qquad SU(k)$$

$$z \equiv \tau + ir$$

$$J^A(z) = \sum_{n\in\mathbb{Z}} z^{-n-1} J_n^A$$

$$[J_n^A, J_m^B] = if^{ABC} J_{n+m}^C + N \frac{n}{2} \delta^{AB} \delta_{n,-m}$$

$SU(k)_N$ Kac-Moody Algebra

N counts net number of chiral fermions

CFT Approach to the Kondo Effect

$$H_K = \frac{v_F}{2\pi}\int_{-\infty}^{+\infty} dr \left[\psi_L^\dagger i\partial_r \psi_L + \delta(r)\,\tilde{g}_K\,\vec{S}\cdot\psi_L^\dagger \vec{\tau}\,\psi_L\right]$$

Full symmetry:

$(1+1)d$ chiral conformal symmetry

$$SU(N)_k \times SU(k)_N \times U(1)_{kN}$$

CFT Approach to the Kondo Effect

$$H_K = \frac{v_F}{2\pi}\int_{-\infty}^{+\infty} dr \left[\psi_L^\dagger i\partial_r \psi_L + \delta(r)\,\tilde{g}_K\,\vec{S}\cdot\psi_L^\dagger \vec{\tau}\,\psi_L\right]$$

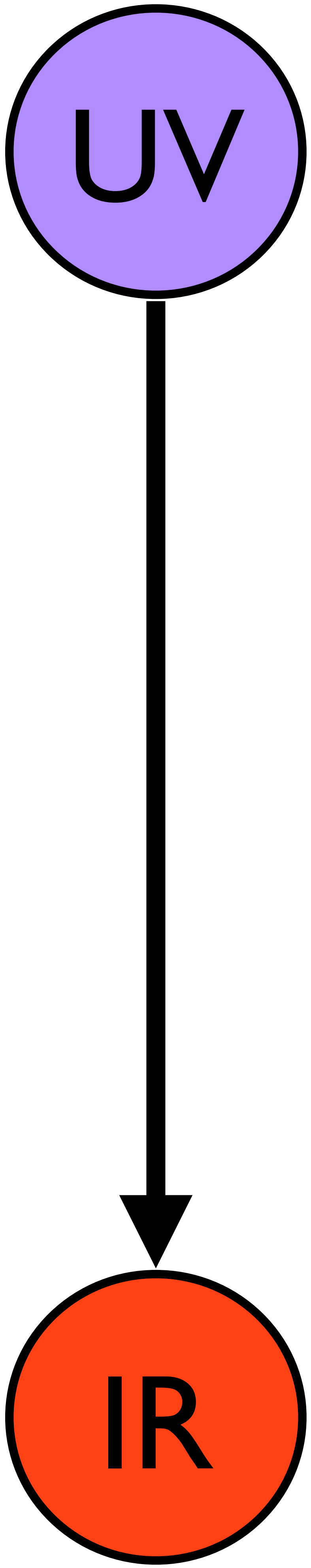
$$J = \psi_L^\dagger \psi_L \qquad U(1)$$

$$\vec{J} = \psi_L^\dagger\,\vec{\tau}\,\psi_L \qquad SU(N)$$

$$J^A = \psi_L^\dagger\, t^A\, \psi_L \qquad SU(k)$$

Kondo coupling: $\vec{S}\cdot\vec{J}$

marginal



$$SU(N)_k \times SU(k)_N \times U(1)_{Nk}$$

Eigenstates are representations of the Kac-Moody algebra

Determine how representations re-arrange between UV and IR

$$R^{UV}_{\text{highest weight}} \otimes R_{\text{imp}} = R^{IR}_{\text{highest weight}}$$

$$SU(N)_k \times SU(k)_N \times U(1)_{Nk}$$

CFT Approach to the Kondo Effect

Take-Away Messages

Central role of the Kac-Moody Algebra

Kondo coupling: $\vec{S} \cdot \vec{J}$

Outline:

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- Top-Down Holographic Model
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- Summary and Outlook

GOAL

Find a holographic description
of the
Kondo Effect

STRATEGY

Follow the CFT approach

Reproduce the Symmetries

Reproduce the Kondo coupling $\vec{S} \cdot \vec{J}$

What classical action do we write
on the gravity side of the correspondence?

How do we describe holographically...

① The chiral fermions?

② The impurity?

③ The Kondo coupling?

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Open strings

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

CFT with holographic dual

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Decouple

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

(1+1)-dimensional chiral fermions

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

the impurity

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Kondo interaction

The D3-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						

3-3 strings

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

$$\int_{S^5} F_5 \propto N_c \qquad F_5 = dC_4$$

The D3-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						

3-3 strings

$\mathcal{N} = 4$ SYM
$N_c \to \infty$
$\lambda \to \infty$

$=$

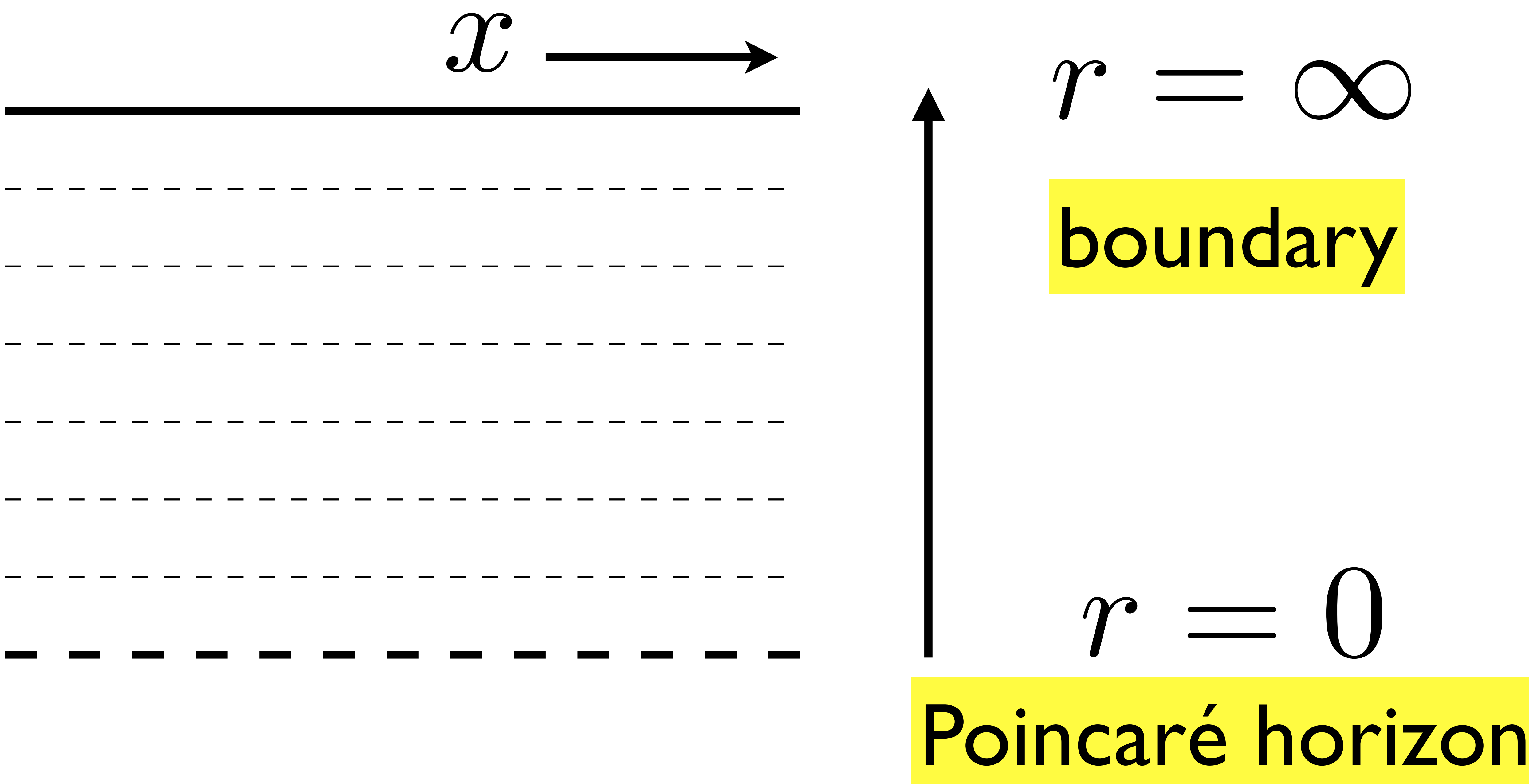
Type IIB Supergravity
$AdS_5 \times S^5$

Our Kondo model will have TWO coupling constants

't Hooft and Kondo

Anti-de Sitter Space

$$ds^2 = \frac{dr^2}{r^2} + r^2\left(-dt^2 + dx^2 + dy^2 + dz^2\right)$$



Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Decouple

Probe Limit

$$N_7/N_c \to 0 \;\text{ and }\; N_5/N_c \to 0$$

$U(N_7) \times U(N_5)$ becomes a global symmetry

Total symmetry:

$$\underbrace{SU(N_c)}_{\text{gauged}} \times \underbrace{U(N_7) \times U(N_5)}_{\text{global}}$$

(plus R-symmetry)

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

(1+1)-dimensional chiral fermions

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

Skenderis, Taylor hep-th/0204054

Harvey and Royston 0709.1482, 0804.2854

Buchbinder, Gomis, Passerini 0710.5170

(1+1)-dimensional chiral fermions ψ_L

$$SU(N_c) \times U(N_7) \times U(N_5)$$

$$N_c \qquad \overline{N}_7 \qquad \text{singlet}$$

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

Skenderis, Taylor hep-th/0204054

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Buchbinder, Gomis, Passerini 0710.5170

(1+1)-dimensional chiral fermions ψ_L

Kac-Moody algebra

$$SU(N_c)_{N_7} \times SU(N_7)_{N_c} \times U(1)_{N_c N_7}$$

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

Differences from Kondo

(1+1)-dimensional chiral fermions ψ_L

Do not come from reduction from (3+1) dimensions

Genuinely relativistic

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

(1+1)-dimensional chiral fermions ψ_L

Differences from Kondo

$SU(N_c)$ is gauged!

$$\vec{J} = \psi_L^\dagger \vec{\tau} \psi_L$$

The D7-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X

$SU(N_c)$ is gauged!

Gauge Anomaly!

Harvey and Royston 0709.1482, 0804.2854

Buchbinder, Gomis, Passerini 0710.5170

Probe Limit

$$N_7/N_c \to 0$$

In the probe limit, the gauge anomaly is suppressed...

$$SU(N_c)_{N_7} \to SU(N_c)$$

... but the global anomalies are not.

$$SU(N_7)_{N_c} \times U(1)_{N_c N_7} \to SU(N_7)_{N_c} \times U(1)_{N_c N_7}$$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe ψ_L

$=$

Probe D7-branes

$AdS_3 \times S^5$

$\mathcal{N} = 4$ SYM

$$N_c \to \infty$$

$$\lambda \to \infty$$

$=$

Type IIB Supergravity

$$AdS_5 \times S^5$$

Probe ψ_L

$=$

Probe D7-branes

$$AdS_3 \times S^5$$

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(-dt^2 + dx^2 + dy^2 + dz^2\right) + ds^2_{S^5}$$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

=

Type IIB Supergravity

$AdS_5 \times S^5$

Probe ψ_L

=

Probe D7-branes

$AdS_3 \times S^5$

$U(N_7)$ Current J

=

$U(N_7)$ Gauge field A

Current J $=$ Gauge field A

Kac-Moody Algebra $=$ Chern-Simons Gauge Field

rank and level of algebra $=$ rank and level of gauge field

Gukov, Martinec, Moore, Strominger
hep-th/0403225

Kraus and Larsen
hep-th/0607138

Probe D7-branes along $AdS_3 \times S^5$

$$S_{D7} = +\frac{1}{2}T_{D7}(2\pi\alpha')^2 \int P[C_4] \wedge \mathrm{tr}\, F \wedge F + \ldots$$

$$= -\frac{1}{2}T_{D7}(2\pi\alpha')^2 \int P[F_5] \wedge \mathrm{tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right) + \ldots$$

$$= -\frac{N_c}{4\pi}\int_{AdS_3} \mathrm{tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right) + \ldots$$

$U(N_7)_{N_c}$ Chern-Simons gauge field

Answer #1

The chiral fermions:

Chern-Simons Gauge Field in AdS_3

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3 ← the impurity

7-5 and 5-7

The D5-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_5 D5	X				X	X	X	X	X	

Skenderis, Taylor hep-th/0204054

Camino, Paredes, Ramallo hep-th/0104082

Gomis and Passerini hep-th/0604007

(0+1)-dimensional fermions χ

$$SU(N_c) \times U(N_7) \times U(N_5)$$

$$N_c \qquad \text{singlet} \qquad \overline{N}_5$$

The D5-branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_5 D5	X				X	X	X	X	X	

$SU(N_c)$ is "spin"

$$\vec{S} = \chi^\dagger \vec{\tau} \chi$$

"slave fermions"

"Abrikosov pseudo-fermions"

Abrikosov, **Physics** 2, p.5 (1965)

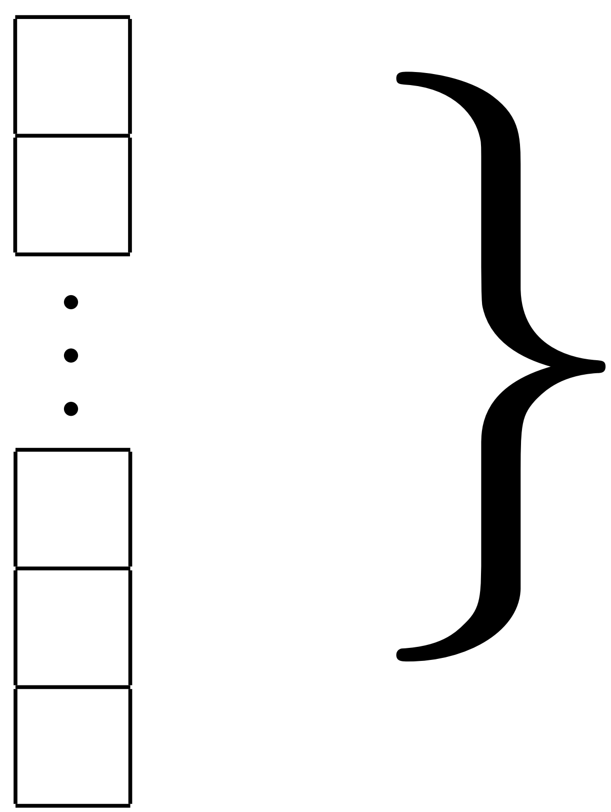
Gomis and Passerini hep-th/0604007

$$N_5 = 1$$

Integrate out χ

$$\mathrm{Det}\,(\not{D}) = \mathrm{Tr}_R P \exp\left[i \int dt\, A_t \right]$$

$$R =$$



$$Q = \chi^\dagger \chi$$

$U(N_5) = U(1)$ charge

$\mathcal{N} = 4$ SYM
$N_c \to \infty$
$\lambda \to \infty$

$=$

Type IIB Supergravity
$AdS_5 \times S^5$

Probe χ

$=$

Probe D5-branes
$AdS_2 \times S^4$

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

$=$

Type IIB Supergravity

$AdS_5 \times S^5$

Probe χ

$=$

Probe D5-branes

$AdS_2 \times S^4$

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(-dt^2 + dx^2 + dy^2 + dz^2\right) + ds^2_{S^5}$$

$\mathcal{N}=4$ SYM $N_c \to \infty$ $\lambda \to \infty$	=	Type IIB Supergravity $AdS_5 \times S^5$
Probe χ	=	Probe D5-branes $AdS_2 \times S^4$
$U(N_5)$ Current J	=	$U(N_5)$ Gauge field a
Q	=	Electric flux

Probe D5-brane along $AdS_2 \times S^4$

Camino, Paredes, Ramallo hep-th/0104082

Dissolve Q strings into the D5-brane

AdS_2 electric field f_{rt}

$$\sqrt{-g} f^{tr}\big|_{\partial AdS_2} = Q = \chi^\dagger \chi$$

Answer #2

The impurity:

Yang-Mills Gauge Field in AdS_2

Top-Down Model

	0	1	2	3	4	5	6	7	8	9
N_c D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

3-3 and 5-5 and 7-7

3-7 and 7-3

3-5 and 5-3

7-5 and 5-7

Kondo interaction

The Kondo Interaction

	0	1	2	3	4	5	6	7	8	9
N_5 D5	X				X	X	X	X	X	
N_7 D7	X	X			X	X	X	X	X	X

Complex scalar!

$$SU(N_c) \times U(N_7) \times U(N_5)$$

singlet $\qquad \overline{N}_7 \qquad N_5$

$$\mathcal{O} \equiv \psi_L^\dagger \chi$$

The Kondo Interaction

$SU(N_c)$ is "spin"

$$\vec{J} = \psi_L^\dagger \vec{\tau}\, \psi_L$$

$$\vec{S} = \chi^\dagger \vec{\tau}\, \chi$$

$$\vec{S} \cdot \vec{J} = \chi^\dagger \vec{\tau} \chi \cdot \psi_L^\dagger \vec{\tau} \psi_L$$

$$\vec{\tau}_{ij} \cdot \vec{\tau}_{kl} = \delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}$$

$$\vec{S} \cdot \vec{J} = |\psi_L^\dagger \chi|^2 + \mathcal{O}(1/N_c)$$

"double trace"

$\mathcal{N} = 4$ SYM

$N_c \to \infty$

$\lambda \to \infty$

=

Type IIB Supergravity

$AdS_5 \times S^5$

Probe ψ_L

=

Probe D7-branes

$AdS_3 \times S^5$

Probe χ

=

Probe D5-branes

$AdS_2 \times S^4$

$\mathcal{O} \equiv \psi_L^\dagger \chi$

=

Bi-fundamental scalar

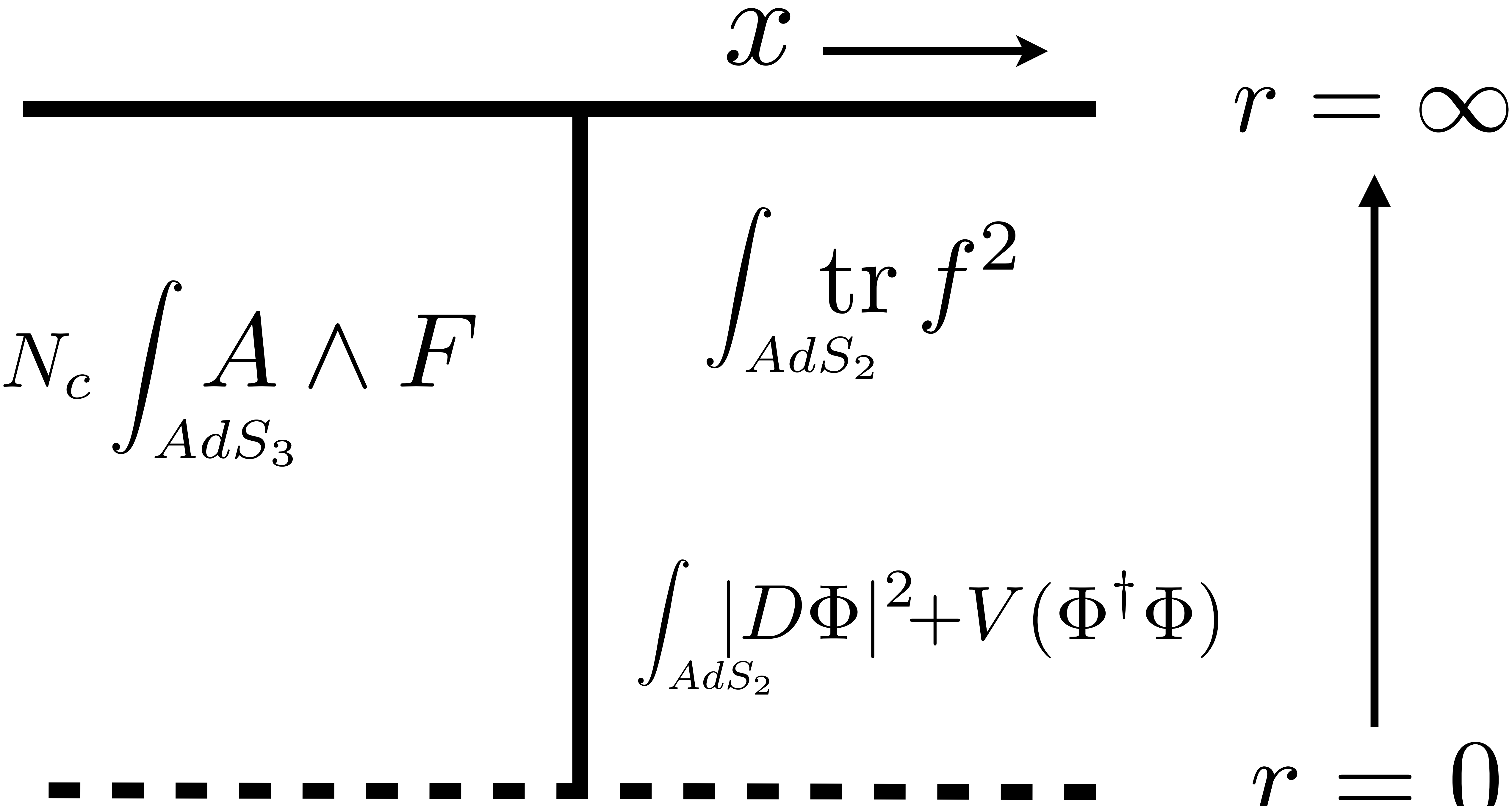
$AdS_2 \times S^4$

Answer #3

The Kondo interaction:

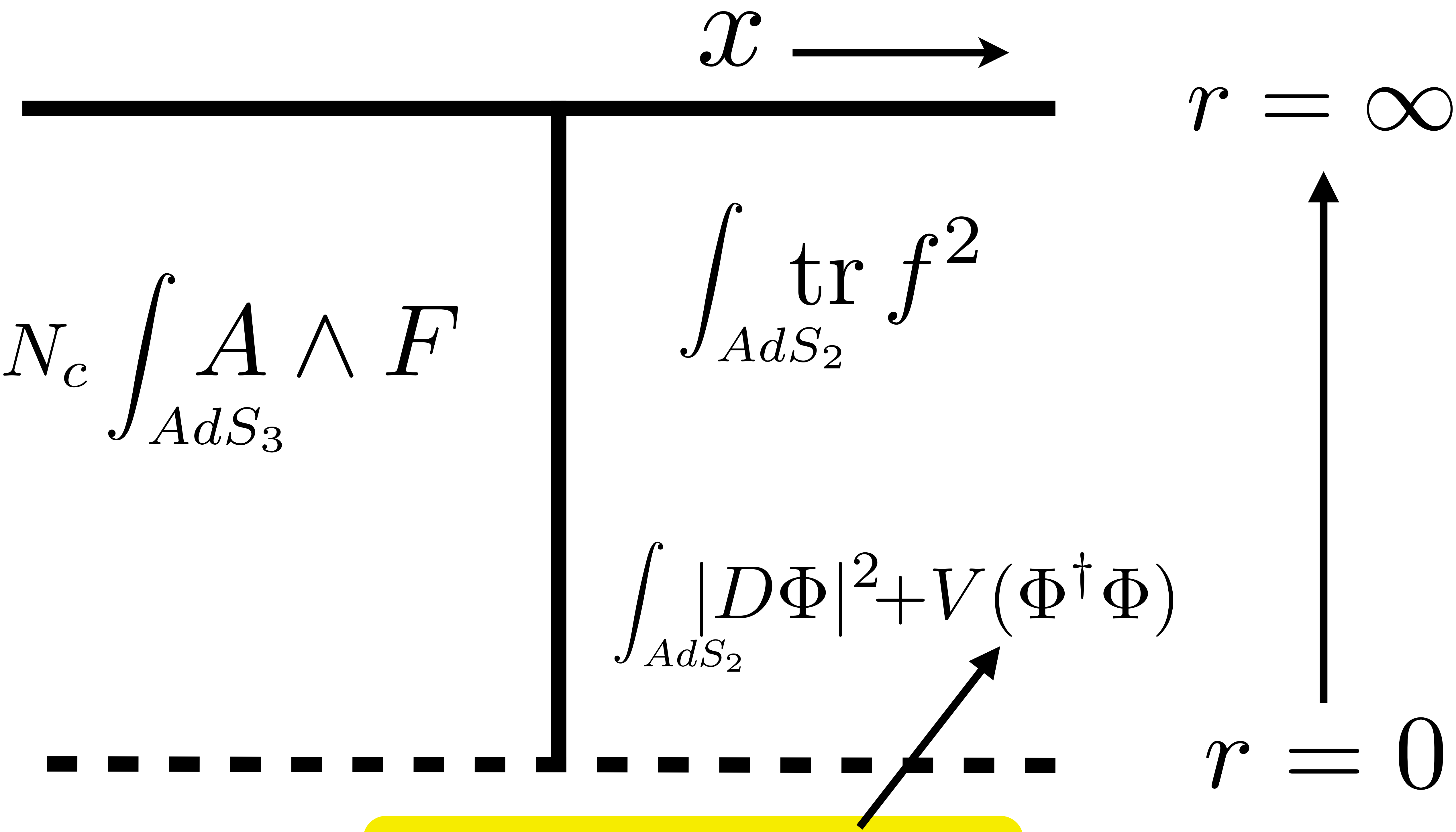
Bi-fundamental scalar in AdS_2

Top-Down Model



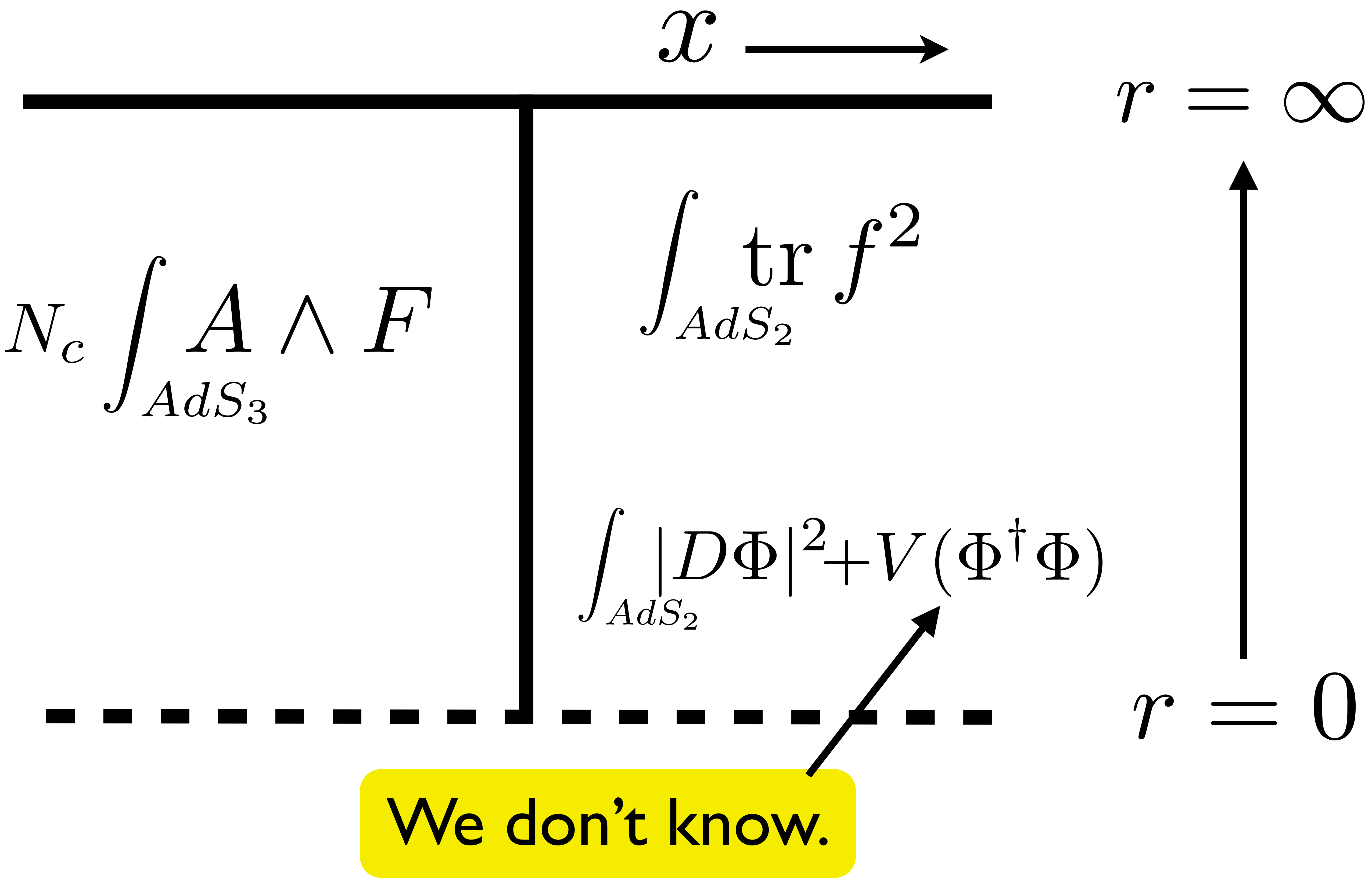
$$D\Phi = \partial\Phi + iA\Phi - ia\Phi$$

Top-Down Model

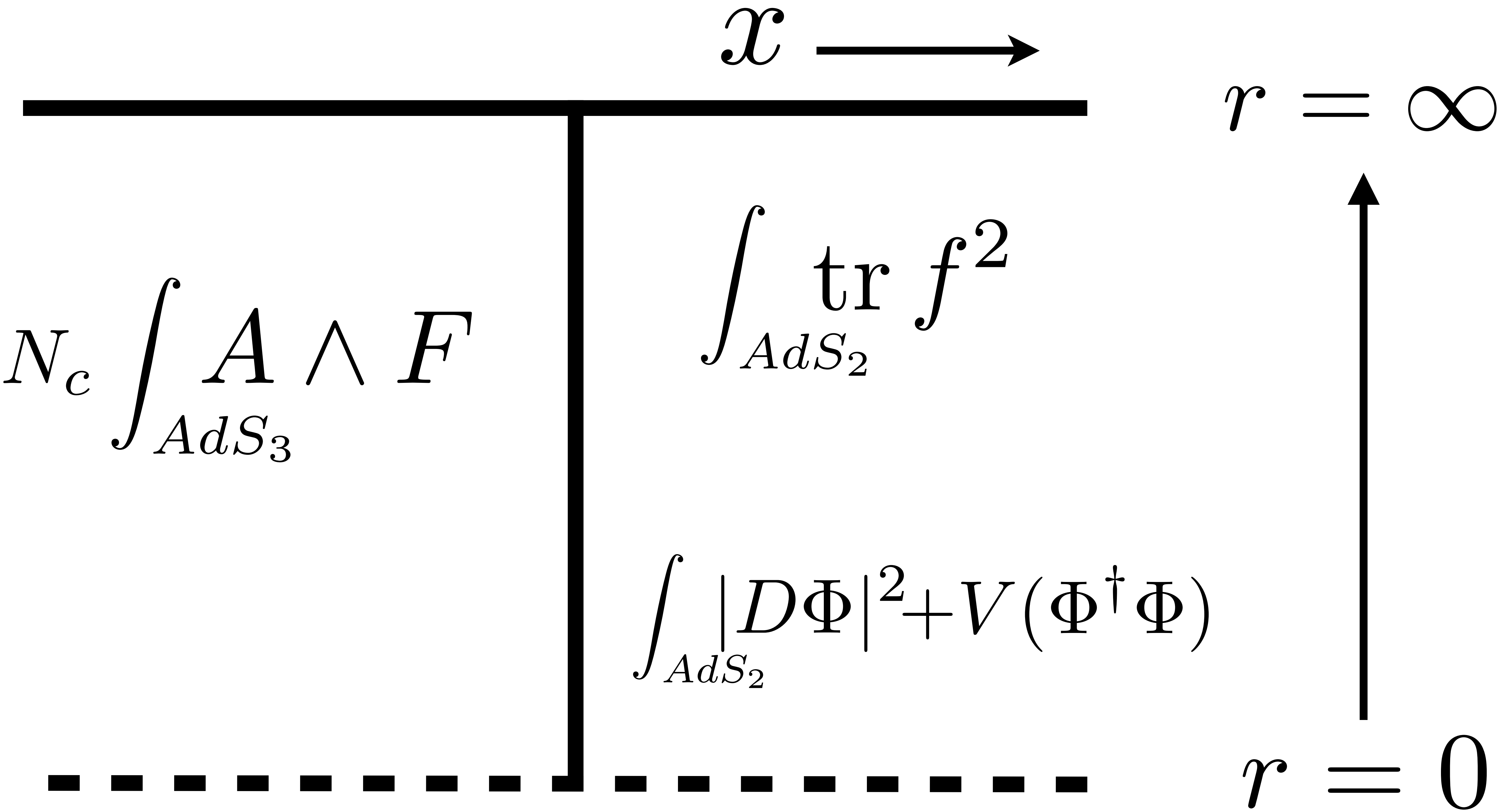


What is $V(\Phi^\dagger \Phi)$?

Top-Down Model



Top-Down Model

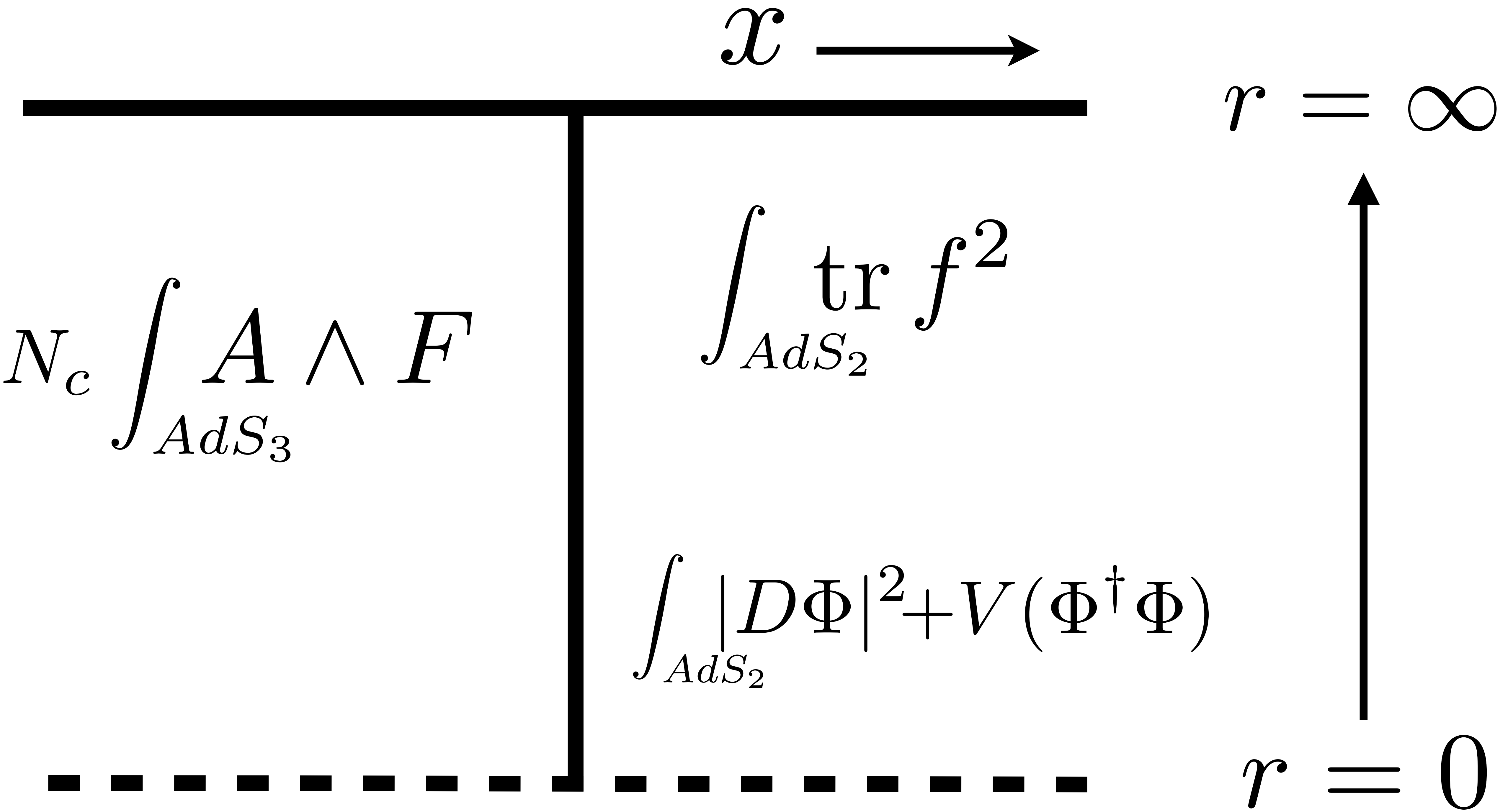


Switch to bottom-up model!

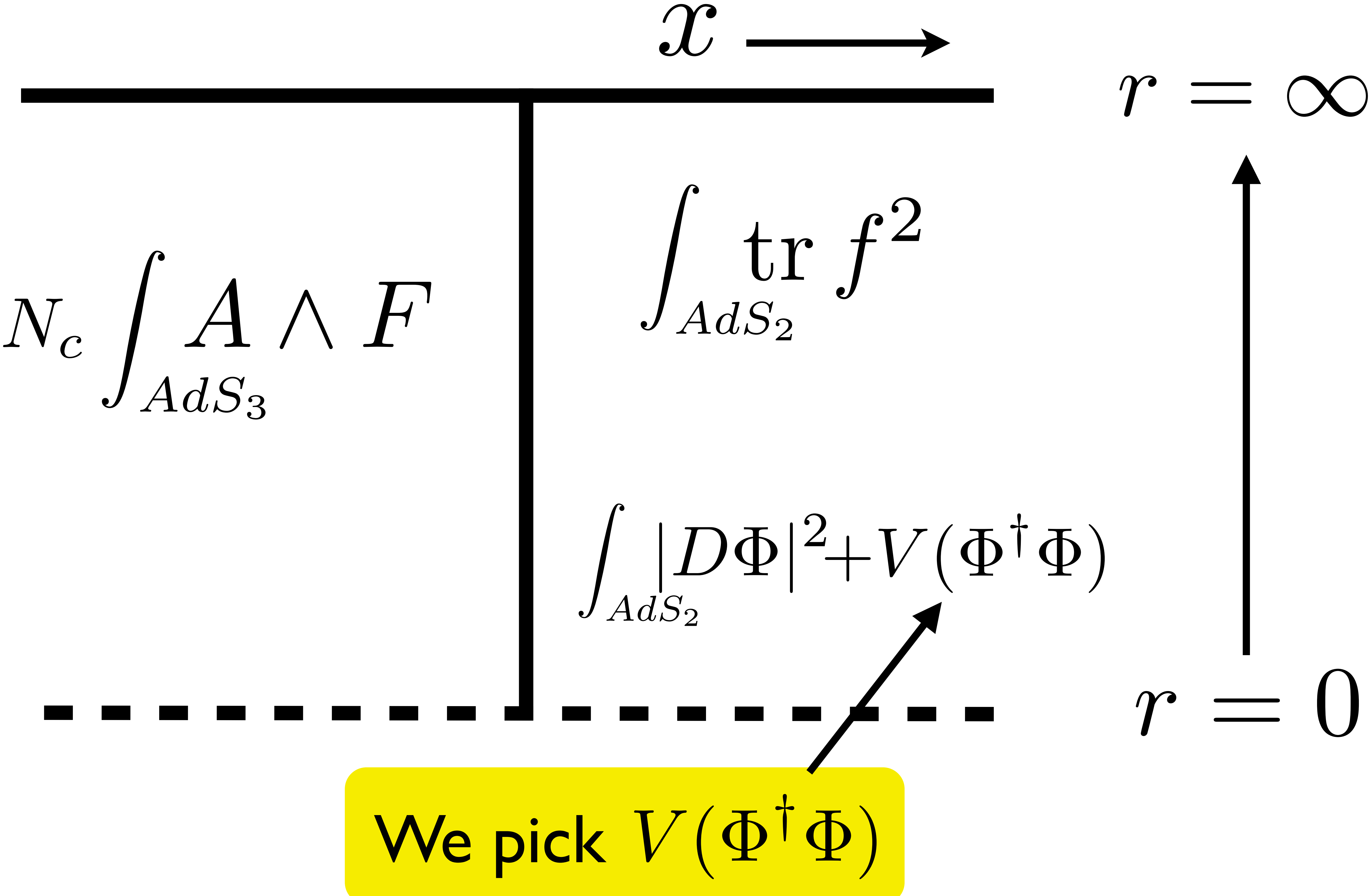
Outline:

- The Kondo Effect
- The CFT Approach
- Top-Down Holographic Model
- Bottom-Up Holographic Model
- Summary and Outlook

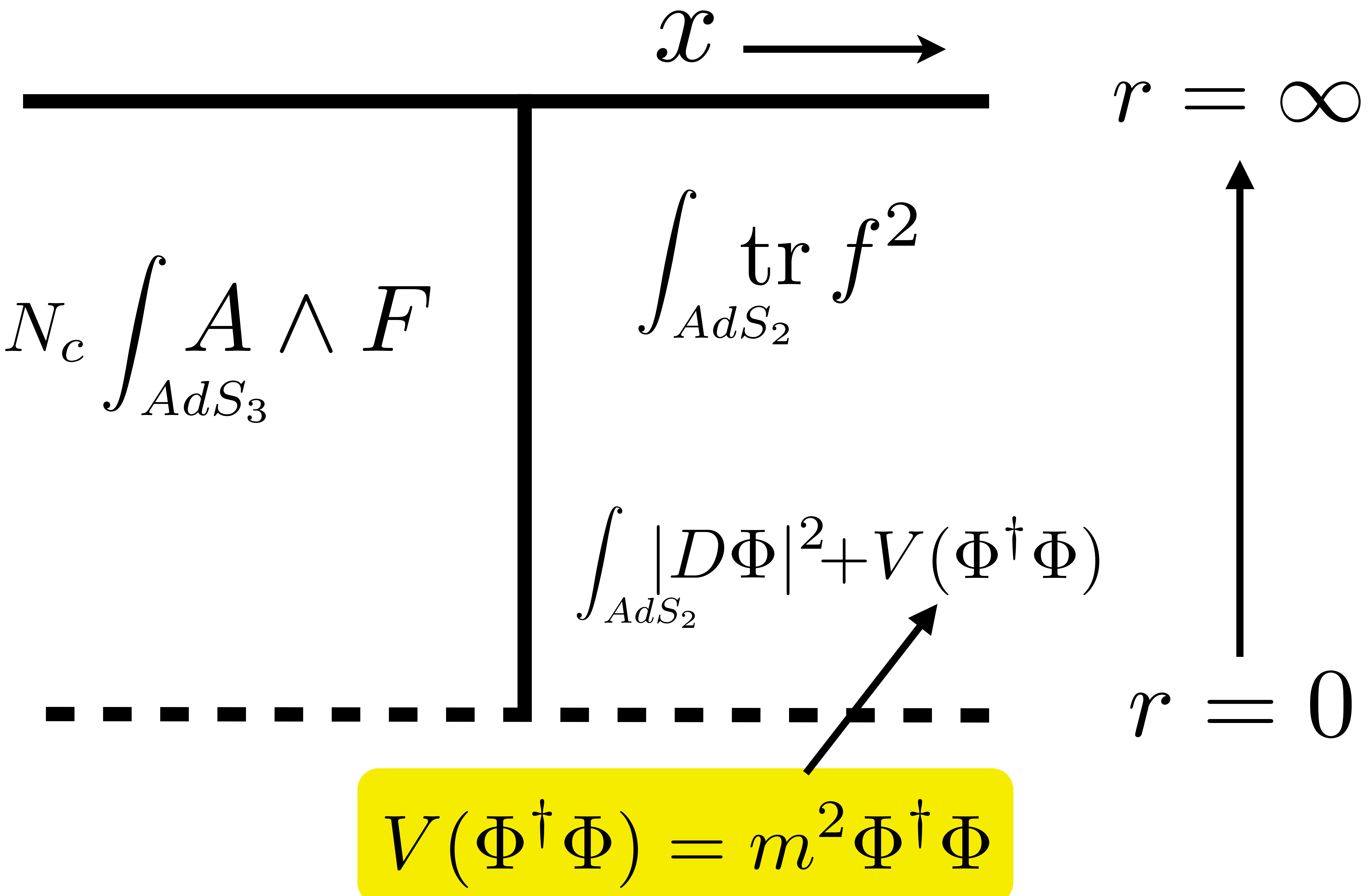
Bottom-Up Model



Bottom-Up Model



Bottom-Up Model



Boundary Conditions

We choose $m^2 =$ Breitenlohner-Freedman bound

$$\Phi(r) = \tilde{c}\, r^{-1/2} + c\, r^{-1/2} \log r + \ldots$$

Our double-trace (Kondo) coupling:

$$c = \tilde{g}_K\, \tilde{c}$$

Witten hep-th/0112258

Berkooz, Sever, Shomer hep-th/0112264

$$\sqrt{-g} f^{rt} \Big|_{\partial AdS_2} = Q$$

Phase Transition

$T > T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi\left(r\right) = 0$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$T < T_c$ $\quad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi(r) \neq 0$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

A holographic superconductor in AdS_2

Phase Transition

$$T > T_c \qquad \sqrt{-g} f^{tr} \Big|_{\partial AdS_2} \neq 0 \qquad \phi\left(r\right) = 0$$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$$T < T_c \qquad \sqrt{-g} f^{tr} \Big|_{\partial AdS_2} \neq 0 \qquad \phi(r) \neq 0$$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

Superconductivity???

Phase Transition

$$T > T_c \qquad \sqrt{-g} f^{tr}\Big|_{\partial AdS_2} \neq 0 \qquad \phi\left(r\right) = 0$$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$$T < T_c \qquad \sqrt{-g} f^{tr}\Big|_{\partial AdS_2} \neq 0 \qquad \phi(r) \neq 0$$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

The large-N Kondo effect!

Solutions of the Kondo Problem

Numerical RG (Wilson 1975)

Fermi liquid description (Nozières 1975)

Bethe Ansatz/Integrability
(Andrei, Wiegmann, Tsvelick, Destri, ... 1980s)

Large-N expansion
(Anderson, Read, Newns, Doniach, Coleman, ...1970-80s)

Quantum Monte Carlo
(Hirsch, Fye, Gubernatis, Scalapino,... 1980s)

Conformal Field Theory (CFT)
(Affleck and Ludwig 1990s)

Phase Transition

$$T > T_c \qquad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi\left(r\right) = 0$$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$$T < T_c \qquad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi(r) \neq 0$$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

$$T_c \propto T_K$$

Phase Transition

$T > T_c$

$$\sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi(r) = 0$$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$T < T_c$

$$\sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi(r) \neq 0$$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

Represents the formation of the Kondo singlet

Phase Transition

$$T > T_c \qquad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi(r) = 0$$

$$\langle \psi_L^\dagger \chi \rangle = 0$$

$$T < T_c \qquad \sqrt{-g} f^{tr}\big|_{\partial AdS_2} \neq 0 \qquad \phi(r) \neq 0$$

$$\langle \psi_L^\dagger \chi \rangle \neq 0$$

The phase transition is an ARTIFACT of the large-N limit!

The actual Kondo effect is a crossover

Outline:

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Summary

What is the holographic dual of the Kondo effect?

Holographic superconductor in AdS_2

coupled as a defect

to a Chern-Simons gauge field in AdS_3

Outlook

- Entropy? Heat Capacity? Resistivity?
- Multi-channel?
- Other impurity representations?
- Entanglement entropy?
- Quantum Quenches?
- Multiple Impurities?

Thank You.