(Non)renormalization of anomalous conductivities

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Anomalous transport in a chiral plasma

- A relativistic chiral plasma e.g QCD with $T \gg m_{u,d}$ or Weyl semimetal with velocity $\vec{u}(x)$ with non-trivial

  magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$ and vorticity $\vec{\omega} = \langle \vec{\nabla} \times \vec{u} \rangle$.

- As a result of the chiral anomaly

$$\partial_\mu J^{5\mu} = a_1 \, F \wedge F + a_2 \, R \wedge R + a_3 \, \mathrm{Tr}(G \wedge G) \,,$$

  Anomalous electric currents are produced:

$$\vec{J} = \sigma_B \vec{B} + \sigma_V \vec{w} \,.$$

  with $\sigma_B \sim a_1, \; a_3$ and $\sigma_V \sim a_2$.

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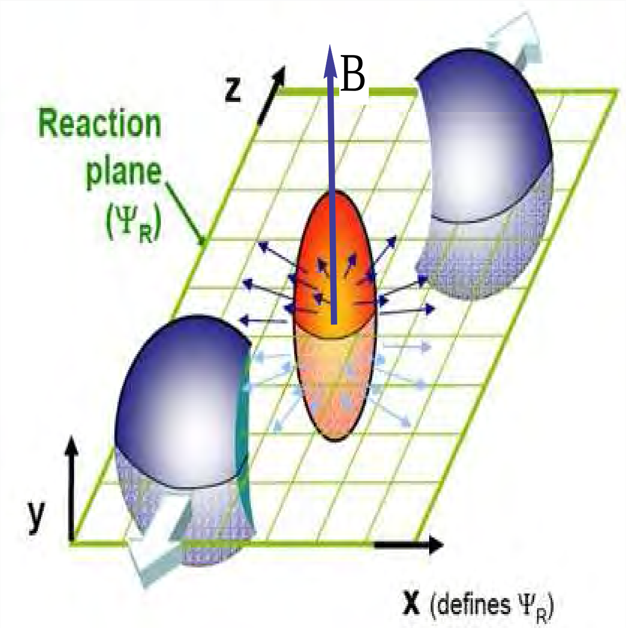
  with $\sigma_B \sim a_1, \; a_3$ and $\sigma_V \sim a_2$.

- Coefficients a_1, a_2 , a_3 are one-loop exact Adler, Bardeen, '69
- Do σ_B and σ_V receive radiative corrections or not?
- We answer this question with AdS/CFT.

Not just an academic question:



- A time dependent magnetic field $eB \approx 5 - 15 \times m_\pi^2$ RHIC (LHC).
- Similarly $\vec{w} \neq 0$ because of the conservation of angular momentum.
- Anomalous electric currents J have observable effects in *charged hadron production*.
- The Chiral magnetic and the chiral vortical effects.

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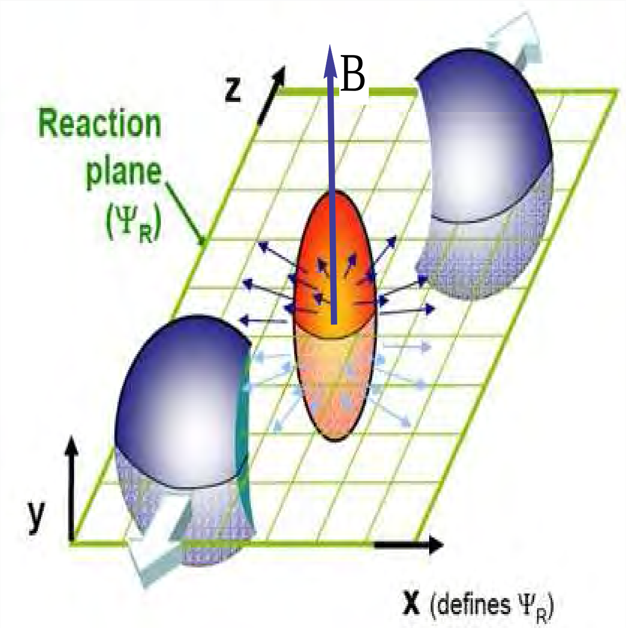

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- Electric currents generated in Weyl semimetals with observables effects.

  Kharzeev and Yee '12

Chiral Magnetic Current in QCD

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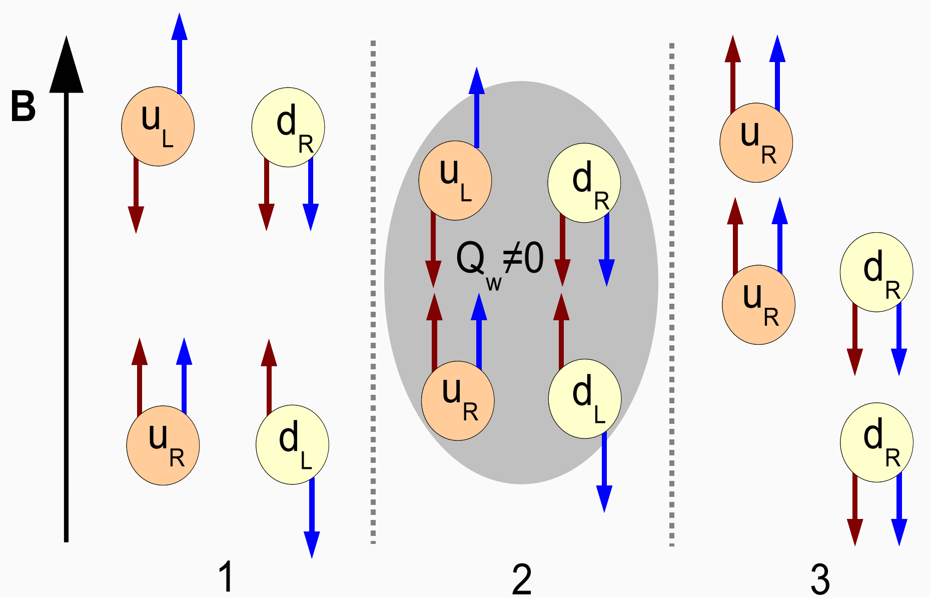
- Under B spin degeneracy of quarks lifted due $H \sim -q\vec{s} \cdot \vec{B}$:
- Anomalous chirality: $Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4x\, \mathrm{Tr}(G \wedge G)$



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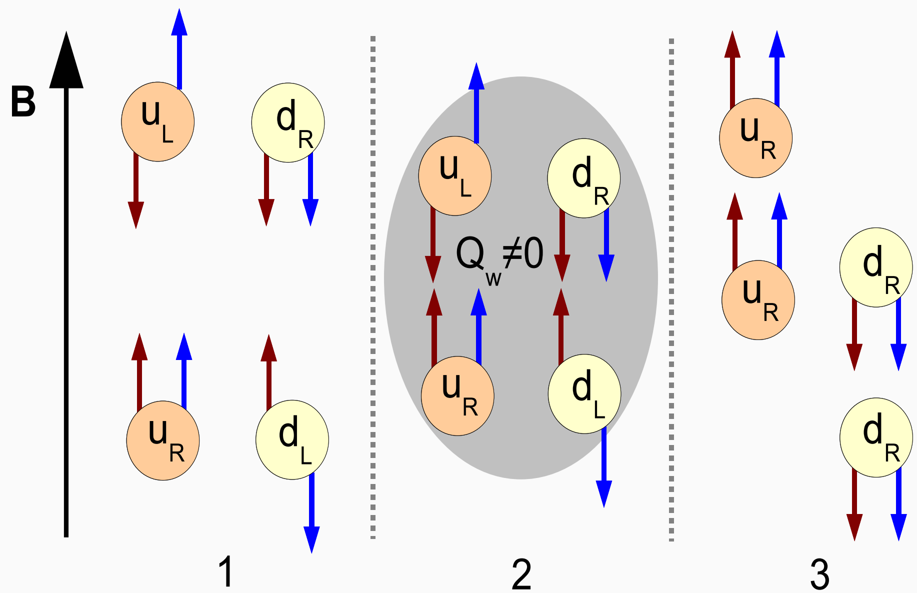

- Macroscopic manifestation of the chiral anomaly
- Anomalous magnetohydrodynamics: $\vec{J} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$

  Kharzeev et al ’07
- μ_5 encodes the imbalance $N_L \neq N_R$

First consider no dynamical gluons: $a_3 = 0$

Hydrodynamic description

A plasma with velocity u^μ, energy ϵ, pressure P, charge density n, axial charge density n_5, chemical potential μ, axial chemical potential μ_5, magnetic field B^μ and vorticity ω^μ and no gluonic anomaly $a_3 = 0$ (no gluonic contribution):

- Constitutive relations:

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} \\ J^\mu &= n\, u^\mu + \nu^\mu \,, \qquad \nu^\mu = \sigma_B B^\mu + \sigma_V \omega^\mu \\ J^{5\mu} &= n_5\, u^\mu + \nu_5^\mu \,, \qquad \nu_5^\mu = \sigma_{B,5} B^\mu + \sigma_{V,5}\omega^\mu \end{aligned}$$

  with $\tau^{\mu\nu}$ and ν^μ the anomalous contributions.

- Equations of motion:

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\alpha} J_\alpha \,, \qquad \partial_\mu J^\mu = 0 \\ \partial_\mu J^{5\mu} &= a_1\, \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} + a_2\, \epsilon^{\mu\nu\alpha\beta} R^\kappa_{\tau\mu\nu} R^\tau_{\kappa\alpha\beta} \,. \end{aligned}$$

Anomalous conductivities

- Require positivity of the entropy current: $\partial_\mu(su^\mu) \geq 0$:

  Son, Surowka '09

  $$\sigma_B = a_1\,\mu_5\,, \qquad \sigma_{B,5} = a_1\,\mu$$
  $$\sigma_V = a_1\,\mu\mu_5\,, \qquad \sigma_{V,5} = \frac{a_1}{2}(\mu^2+\mu_5^2) + CT^2$$

- C is due to the mixed axial-gravitational anomaly, thus $C=0$ when $a_2 = 0$
- Value of C is undetermined. Neimann, Oz '09

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- Value of C is undetermined. Neimann, Oz '09
- σ_B, $\sigma_{B,5}$ and σ_V is unrenormalized
- $\sigma_{V,5}$ may or may not be renormalized depending on C.

Field theory arguments

Kubo formulae:

- $\vec{B}$ and $\vec{\omega}$ in the x-direction
- Electric current in the x-direction:

$$\sigma_B = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^x J^z \rangle \,,$$

$$\sigma_V = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^x T^{0z} \rangle \,,$$

- Axial current in the x-direction:

$$\sigma_{B,5} = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^{5^x} J^z \rangle \,,$$

$$\sigma_{V,5} = \lim_{k_y \to 0} \frac{i}{k_y} \langle J^{5^x} T^{0z} \rangle \,.$$

- Use these formulae in the field theory and holographic calculations.

- Anomalous two-point functions ’s can be related to the anomalous three point functions Γ^{VVA}.
- Γ^{VVA} is strongly constrained by the vector and axial Ward identities
- Anomalous conductivities fixed completely up to C.
- Complete agreement with hydrodynamics

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- Effective field theory on the cone Jensen et al ’13:
- Generic anomalous 4D theory on a cone $\times R^2$
- Construct an EFT action $\Rightarrow$ demand continuity as the deficit angle vanishes:
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- Assumpstions may break down in theories with phase transitions $\Rightarrow$ desirable to check in holography

Holographic approach Landsteiner et al '11

- Let's illustrate the calculation in the conformal plasma:
- First we ignore dynamical glue i.e. set $a_3 = 0$.
- The action:

$$S = \frac{1}{16\pi G}\int_M \sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F^2\right] + a_1\, A\wedge F\wedge F + a_2\, A\wedge R\wedge R + \cdots$$

- Solution: AdS-RN blackhole with gauge field A

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- Fluctuate $\Phi^I_k(r) = \Big(A_x(r),\, h^x_t(r),\, A_z(r),\, h^z_t(r)\Big)$, with $k = k_y$.
- Calculate the two-pfs $G_{IJ}(k)$, in the limit $k \to 0$

$$\sigma_B = \frac{\mu}{4\pi^2}, \qquad \sigma_V = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}\,.$$

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- Confirms the generic form derived in FT and hydro above!
- Fixes $C = \frac{1}{24}$ and agrees with the EFT result $C = -8\pi^2 a_2$!

An example with phase transition

U.G., A. Jansen '14

- Want to check validity of the EFT arguments in a theory with conf/deconf. transition
- In flat space this requires an intrinsic scale "Λ_{QCD}"
- Break conformality by $\langle\mathcal{O}\rangle \neq 0 \Rightarrow$ non-trivial bulk scalar Φ
- $S = -\frac{1}{16\pi G}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\nabla\Phi)^2 - V(\Phi) - Z(\Phi)F^2\right) + \cdots$

  Gao, Zhang '06
- $V(\Phi) = -\frac{3}{(2+\alpha^2)^2}\left\{4\alpha^2(\alpha^2-1)e^{-\frac{8\Phi}{3\alpha}} + 4(4-\alpha^2)e^{\frac{4\alpha\Phi}{3}} + 24\alpha^2 e^{-\frac{2(2-\alpha^2)\Phi}{3\alpha}}\right\},$

  $Z(\Phi) = e^{-\frac{4}{3}\alpha\Phi}\,.$
- For $\alpha = 0$ reduces to conformal plasma.
- Expand V near minimum $\Phi = 0 \Rightarrow m^2 = -\frac{32}{3}\,.$
- Deformation of $\mathcal{N} = 4$ $\langle\mathcal{O}\rangle$ with $\Delta_{\mathcal{O}} = 2$ regardless of α
- Analytic, dilatonic and charged, asymptotically AdS BH

- Thermodynamics: U.G., A. Jansen '14
  - Corresponding thermal gas obtained analytically
  - Hawking-Page transition between BH and TG at finite T_c only for $\alpha = 2$:
  - $\Delta G = M - \mu\, Q - TS \approx -\frac{2\pi^3 V_3}{3G}\, T_c^3\, (T - T_c), \qquad T \to T_c$

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$$\begin{aligned}
\langle J^x J^z \rangle &= -\frac{i\kappa k \rho_h}{\sqrt{2}\pi G}\sqrt{1-\xi}\, v \left(1 - v^2\right)^{\xi}\,, \\
\langle J^x {T_t}^z \rangle &= \frac{i\kappa k v^2 \rho_h^2}{2\pi G}(1-\xi)\left(1 - v^2\right)^{2\xi} \\
&\quad + \frac{2ik\lambda\rho_h^2}{\pi G}\left(1 - v^2\right)^{2\xi - 1}\left((\xi - 1)v^2 + 1\right)^2 \text{, etc.}
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with ρ_h horizon location, $v \sim \Lambda_{QCD}$, $\xi = \frac{\alpha^2 - 1}{\alpha^2 + 2}$

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  - Yet, when expressed in $T = T(\rho_h, v, \alpha)$ and $\mu = \mu(\rho_h, v, \alpha)$:

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- A non-trivial check on the EFT and hydro arguments

Now consider dynamical gluons: $a_3 \neq 0$

Anomalous conductivities with glue

- In QCD-like theories
  $\partial_\mu J^{5\mu} = a_1 \, \mathrm{Tr}(F \wedge F) + a_3 \, \mathrm{Tr}(G \wedge G) + a_2 \, R \wedge R \,,$
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- Non of the non-renormalization arguments above apply when $a_3 \neq 0$
- Direct FT calculation $\Rightarrow$ $\sigma_{V,5}$ receives perturbative corrections from dynamical glue loops Golkar, Son '12; Hou et al '12
- Lattice-QCD: both σ_B and $\sigma_{V,5}$ receive huge corrections Yamamoto '12, Braguta et al. '13
- Hydro arguments above do not apply $\Rightarrow$ need hydro d.o.f. for $\mathrm{Tr}(G \cdot \tilde{G})$
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- Nor does the EFT argument!
- Can we find an alternative approach through holography?

Holography with dynamical glue

- How to compute σ_B, σ_V at strong coupling?
- First: how to realize $a_3 \neq 0$ situation in holography?

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  Klebanov, Ouyang, Witten '02; Casero, Kiritsis, Paredes '07
- WZ term for the flavor and gauge branes:

$$S_{tot} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\Big(R - \frac{4}{3}(\partial\phi)^2 - \frac{1}{4}Z_1(\Phi)F^2 - V(\phi)\Big)$$

$$+ \int d^5x\Big(a_1 A\wedge F\wedge F + a_2 A\wedge R\wedge R\Big),$$

$$-\int d^5x\sqrt{-g}\Big(\frac{Z_0(\Phi)}{2}(dC_0 - A)^2\Big) + \int d^4x\sqrt{-h}a_3\, C_0\, \mathrm{Tr}(G\wedge G)$$

- The axion $C_0 \Leftrightarrow \mathrm{Tr}\, G\wedge G$
- Generates the correct anomaly including a_3 by $A\to A + d\epsilon, C_0 \to C_0 + \epsilon$

Application to anomalous conductivities

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- Now apply the same procedure to compute $\langle J_x J_z \rangle$, $\langle T_{0x} J_z \rangle$, starting from the new action, including the axion term
- Two important differences:
  1. Gauge field gets a mass:
     $\partial_N \left(\sqrt{-g} Z_1(\Phi) F^{MN} \right) = \sqrt{-g}\, Z_0(\Phi)\, A^M$.
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- These terms change the entire calculation and generate the radiative corrections when $\alpha_3 \neq 0$ at strong coupling.
- The holographic mechanism to produce the dynamical gluon contribution to the anomalous conductivities!

- Summary
  - Reviewed EFT and hydro arguments for one-loop excactness of anomalous conductivities when $a_3 = 0$
  - Reproduced this in a non-trivial holographic background with a conf/deconf transition
  - No EFT or hydro arguments for $a_3 \neq 0 \Rightarrow$ expect radiative corrections
  - Shown how to reproduce the radiative corrections

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  - Shown how to reproduce the radiative corrections
- Outlook
  - Compute these corrections in a precise setting
  - Study what happens to the conductivities in the low T confined phase, particularly at the transition.
  - No non-renormalization argument for the full correlators $\langle J_x J_z(\omega, k)\rangle \Rightarrow$ study them in holography

THANK YOU !