Holographic Lattices, Metals and Insulators

Jerome Gauntlett

1311.3292, 1401.5077,
1406.4742, 1409.xxxx

Aristomenis Donos

Holographic tools provide a powerful framework for investigating strongly coupled systems, in a large N limit, using weakly coupled theories of gravity

Make contact with real systems?

Greatly enriched our understanding of holography and of black holes in AdS spacetime

Examples

- Superconducting phases - with s,p and d-wave order
- Spatially modulated phases - stripes, helices, checkerboards,...
- New ground states - Lifshitz, Schrodinger, hyperscaling violating, ...

Metal - Insulator transition

Dramatic reorganisation of degrees of freedom

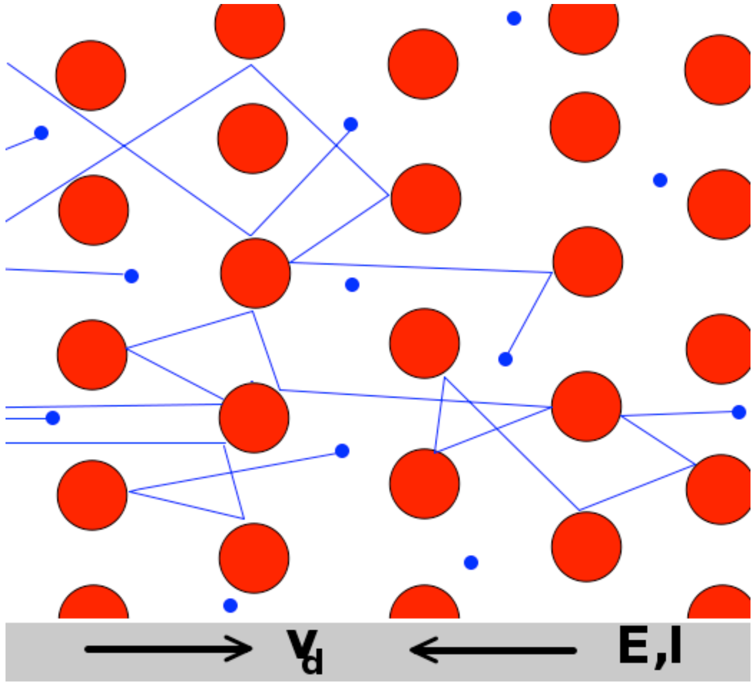
Furthermore, seen in strongly coupled context in Nature

How can we realise them holographically?

[Hartnoll, Donos]

Drude Model of transport in a metal

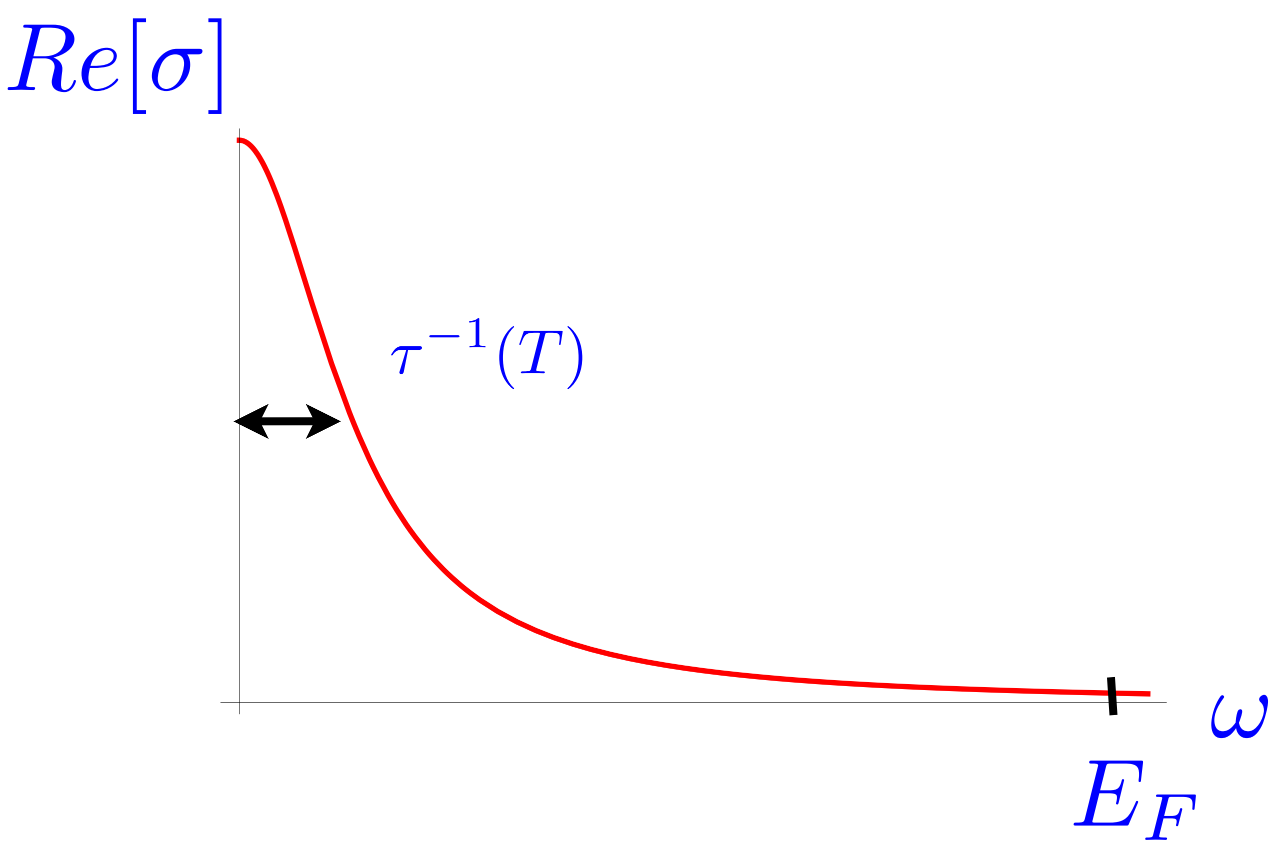
e.g. quasi-particles and no interactions

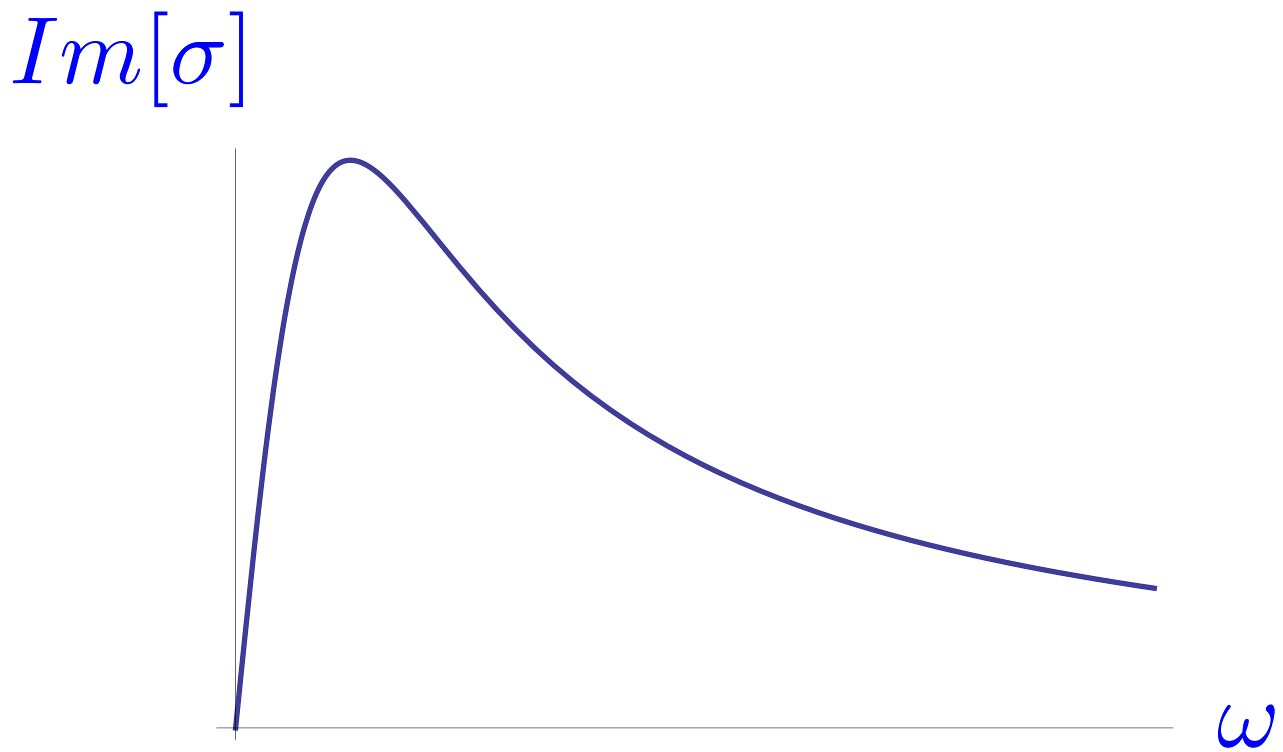


$$J(\omega) = \sigma(\omega)E(\omega)$$

$$\sigma(\omega) = \frac{\sigma_{DC}}{1 - i\omega\tau}$$

$$\sigma_{DC} = \frac{nq^2\tau}{m}$$





"Coherent" or "good" metal

$\tau \to \infty \qquad \sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$

- Drude physics doesn't require quasi-particles

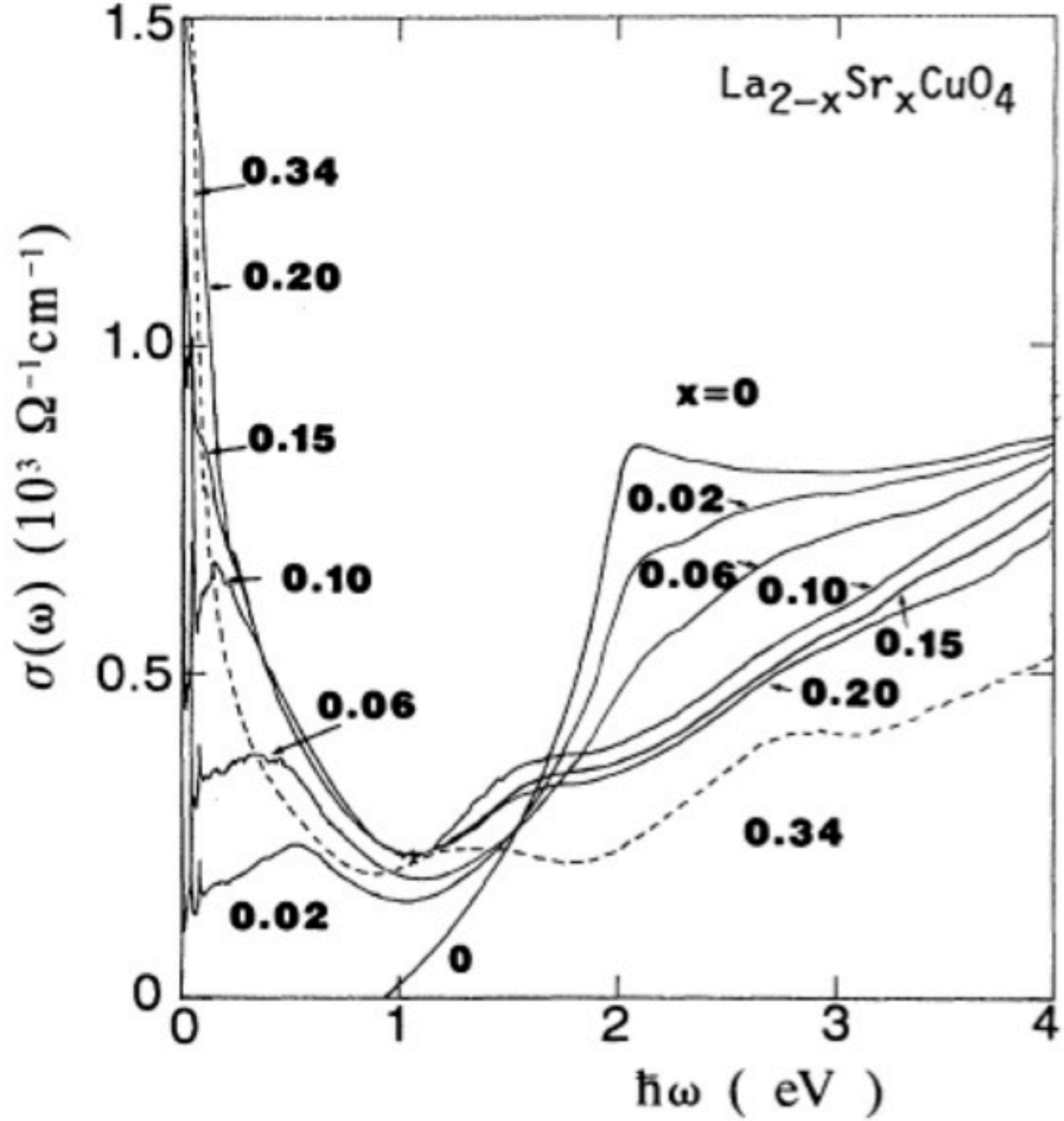
  Arises when momentum is nearly conserved

  In some situations can be studied perturbatively using "memory matrix" formalism [Hartnoll,Hofman]

- There are also "incoherent" metals without Drude peaks

- Insulators with $\sigma_{DC} = 0$

Holographically we will realise coherent, incoherent metals, insulators and transitions between them.



Interaction driven and strongly coupled

Holographic CFTs at finite charge density

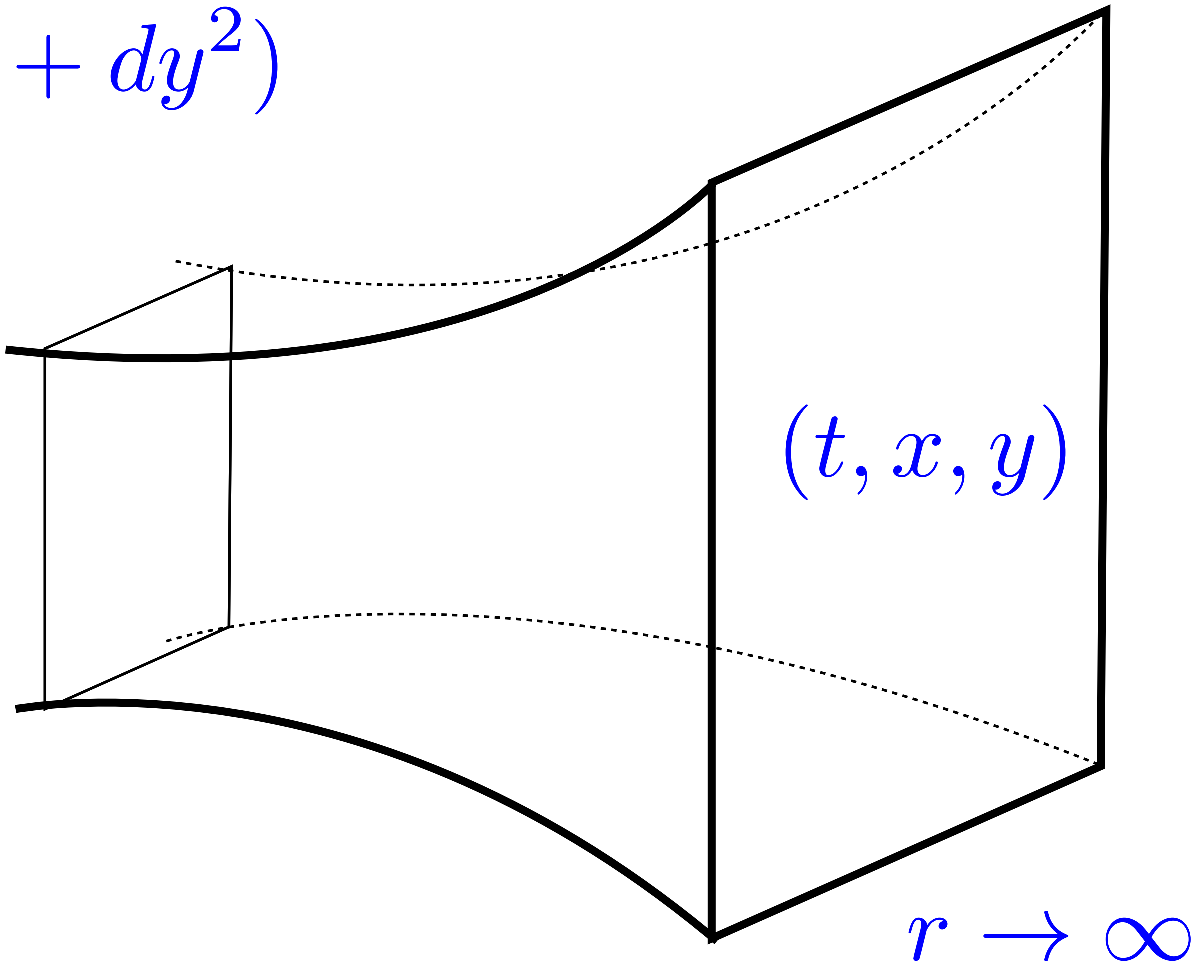
Focus on d=3 CFT and consider D=4 Einstein-Maxwell theory:

$$S = \int d^4x \sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 + \dots\right]$$

Admits AdS_4 vacuum solution $\leftrightarrow$ d=3 CFT with global U(1)

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + r^2(dx^2 + dy^2)$$

$$A = 0$$



Holographic dictionary

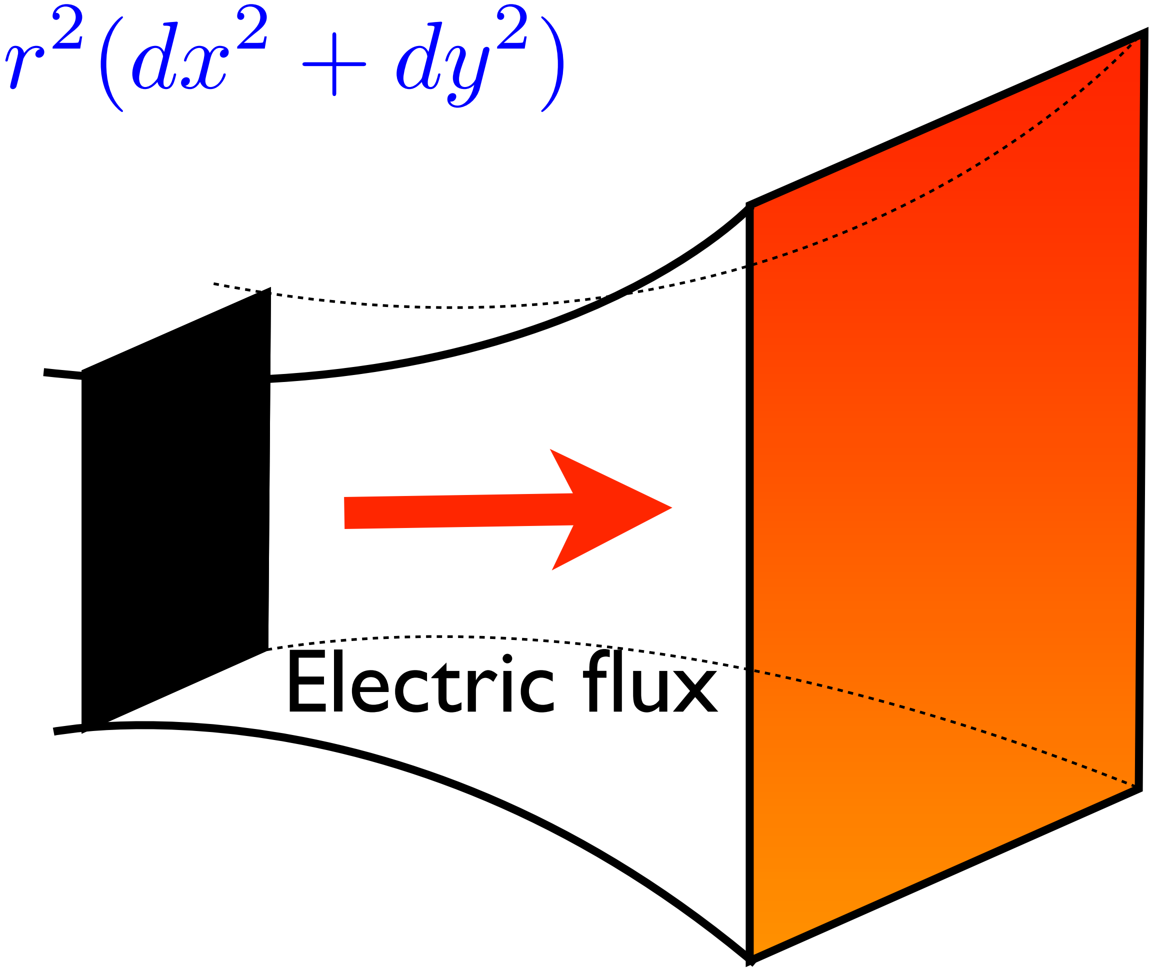
$$g_{\mu\nu} \leftrightarrow T^{ab}$$

$$A_\mu \leftrightarrow J^a$$

CFT at finite T and chemical potential μ - described by the electrically charged AdS-Reissner-Nordstrom black hole

$$ds^2 = -U dt^2 + \frac{dr^2}{U} + r^2(dx^2 + dy^2)$$

$$A_t = \mu(1 - \frac{r_+}{r})$$



d=3 CFT

μ T

IR: $r \to r_+$

UV: $r \to \infty$

Black hole horizon topology $\mathbb{R}^2$ and temp T ← AdS_4

At T=0 AdS-RN black hole interpolates between

IR		UV
$AdS_2 \times \mathbb{R}^2$	$\longleftarrow$	AdS_4

Interpretation: at T=0 a locally quantum critical ground state appears

The AdS-RN black hole describes holographic matter at finite charge density that is translationally invariant $\Rightarrow$ momentum conserved

Electrical conductivity calculation

$$\delta A_x = e^{-i\omega t} a_x(r)$$

$$\sigma(\omega) = -i \frac{G_{J_x J_x}(\omega)}{\omega}$$

$$\delta g_{tx} = e^{-i\omega t} h_{tx}(r)$$

Electrical conductivity calculation

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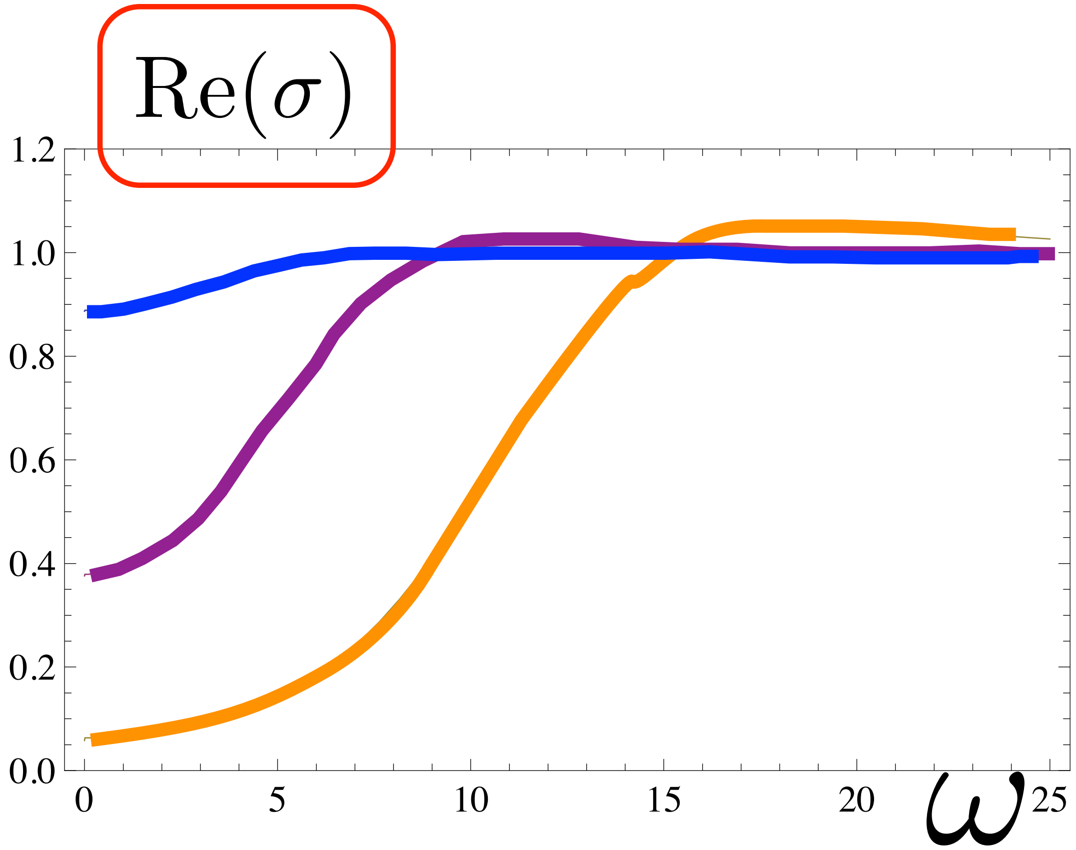
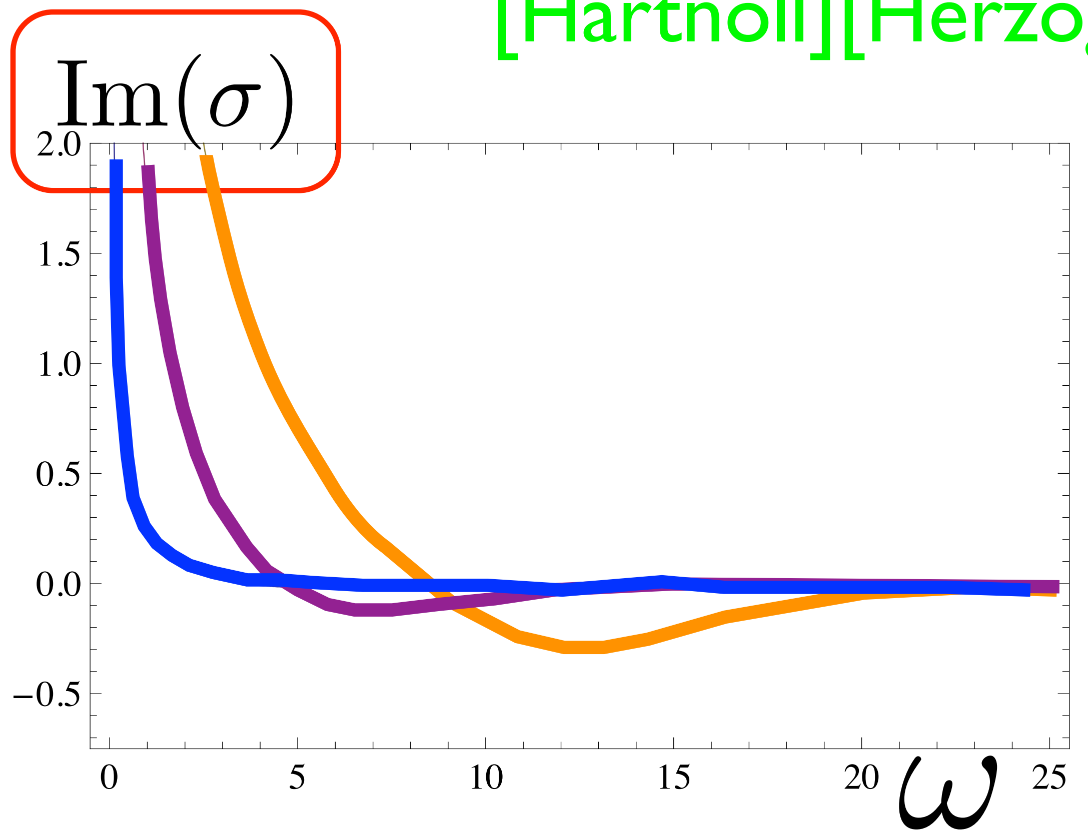
[Hartnoll][Herzog]





Electrical conductivity calculation

$$\delta A_x = e^{-i\omega t} a_x(r)$$

$$\sigma(\omega) = -i \frac{G_{J_x J_x}(\omega)}{\omega}$$

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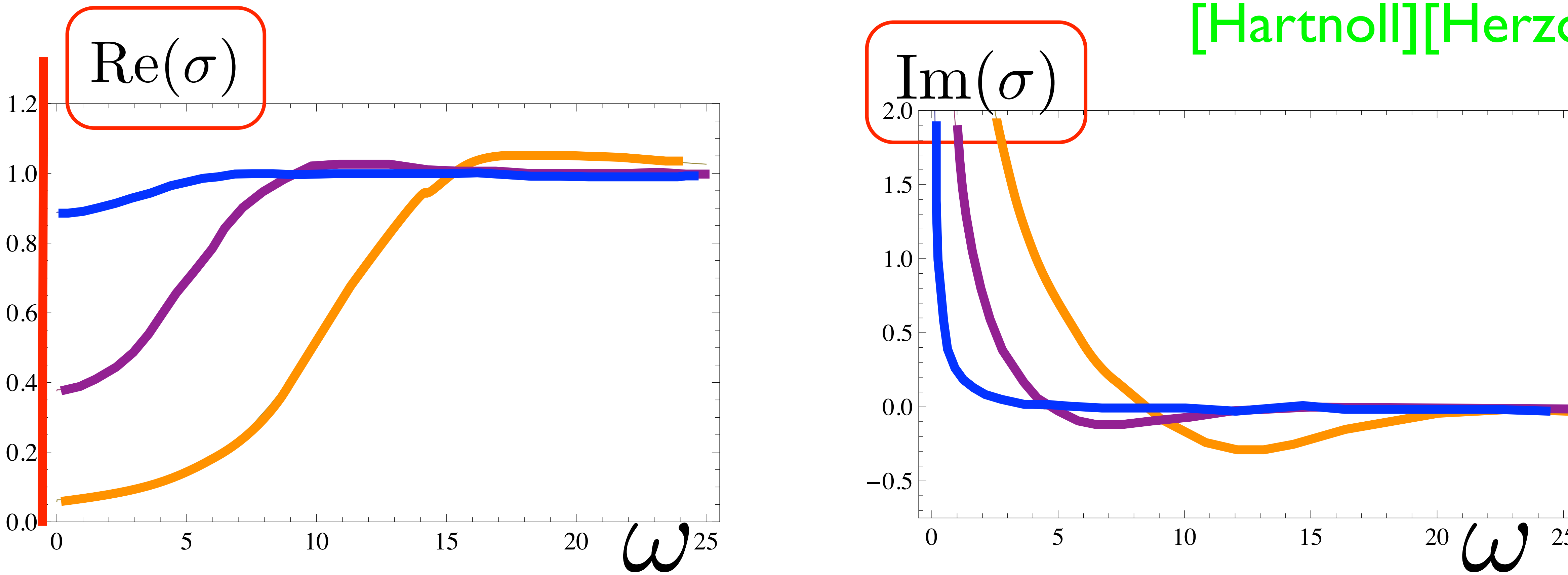
[Hartnoll][Herzog]



More precisely $\sigma(\omega) \sim \delta(\omega) + \frac{i}{\omega}$ near $\omega \sim 0$

Infinite DC conductivity arises because translation invariance implies there is no momentum dissipation: Drude physics

Holographic Lattices and metals

To realise more realistic metals and/or insulators we want to construct charged black holes that explicitly break translations using a deformation of the CFT

E.g. Spatially dependent $\mu(x)$

At the AdS_4 boundary, impose:

$$A_t(r,x) = \mu(x) + \frac{q(x)}{r} + \ldots$$

E.g. Couple a D=4 bulk scalar field ϕ , dual to operator $\mathcal{O}$ in the CFT with dimension Δ . Deform CFT by $\mathcal{O}(x)$:

$$\phi(r,x) \sim \frac{\lambda(x)}{r^{3-\Delta}} + \ldots$$

A few examples of periodic, monochromatic lattices in one spatial dimension have been studied [Horowitz, Santos, Tong]

$$\mu(x) = \mu_0 + A\cos(kx)$$

(Case when $\mu_0 = 0$ [Chesler, Lucas, Sachdev])

$$\lambda(x) = \lambda\cos(kx)$$

Need to solve PDEs to get these D=4 black holes

Q-lattices: simplified construction with ODEs. Find some agreement and some differences with [Horowitz, Santos, Tong] as well as many new results

Also: revisited $\mu(x)$ case - see talk by Donos

Plan

- Holographic Q-lattices - solve ODEs

  (D=5 helical lattices [Donos,Hartnoll][Donos,Gouteraux,Kiritsis])

- Calculation of thermoelectric DC conductivity σ_{DC} , α_{DC} , $\bar{\kappa}_{DC}$ in terms of black hole horizon data

  Analogous to $\eta = \frac{s}{4\pi}$ [Policastro,Kovtun,Son,Starinets]

  (For σ_{DC} c.f. [Iqbal,Liu][Davison][Blake,Tong,Vegh][Andrade,Withers])

  Find some interesting general results eg a bound on $\bar{L} \equiv \bar{\kappa}/(\sigma T)$

- Q-lattices can give coherent metals, incoherent metals and insulators and transitions between them.

Holographic Q-lattices

- Illustrative D=4 model

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

- Choose Φ, V, Z so that we have an AdS_4 vacuum and that AdS-RN is a solution at $\phi = 0$

- Particularly interested in cases where χ is periodic.

  eg if it is the phase of a complex scalar field $\varphi = \phi e^{i\chi}$ with $\Phi = \phi^2$

  Analysis also covers cases when χ is not periodic e.g. [Azeneyagi,Takayanagi,Li][Mateos,Trancanelli][Andrade,Withers]

- The model has a gauge $U(1)$ and a global $U(1)$ symmetry

  Exploit the global bulk symmetry to break translations

Ansatz for fields

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx^2 + e^{2V_2} dy^2$$

$$A = a(r) dt$$

$$\chi = k x_1, \qquad \phi = \phi(r)$$

UV expansion:

$$U = r^2 + \ldots, \qquad e^{2V_1} = r^2 + \ldots \qquad e^{2V_2} = r^2 + \ldots$$

$$a = \mu + \frac{q}{r} \ldots, \qquad \phi = \frac{\lambda}{r^{3-\Delta}} + \ldots$$

IR expansion: regular black hole horizon

Homogeneous and anisotropic and periodic holographic lattices

UV data: $T/\mu \qquad \lambda/\mu^{3-\Delta} \qquad k/\mu$

- Reminiscent of Coleman's construction of Q-balls

- Various generalisations possible by allowing for more general global symmetries e.g. we can have two global U(1)s and two fields χ_i allowing for breaking more translations

$$\chi_1 = k_1 x \qquad \chi_2 = k_2 y$$

Isotropic if $k_1 = k_2$

Analytic result for DC in terms of horizon data

Apply electric fields and thermal gradients and find linear response

Generalised Ohm/Fourier Law:

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

J^a Electric current

$Q^a = T^{ta} - \mu J^a$ Heat current

For Q-lattice black holes the DC matrices $\sigma, \alpha, \bar{\alpha}, \bar{\kappa}$ diagonal

- Calculating σ and $\bar{\alpha}$

Switch on constant electric field perturbation

$$A_x = -Et + \delta a_x(r)$$

supplemented with $\delta g_{tx}(r)$ $\delta g_{rx}(r)$ $\delta\chi(r)$

Gauge equation of motion:

$$\nabla_\mu(Z(\phi)F^{\mu\nu}) = 0 \quad \Rightarrow \quad \partial_r(\sqrt{-g}Z(\phi)F^{rx}) = 0$$

$$J = -e^{V_2 - V_1}Z(\phi)U\delta a_x' + qe^{-2V_1}\delta g_{tx}$$

Use Einstein equations and regularity at the black hole horizon to relate J and E to get σ

- Calculating σ and $\bar{\alpha}$

Switch on constant electric field perturbation

$$A_x = -Et + \delta a_x(r)$$

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$$\boxed{J = -e^{V_2 - V_1} Z(\phi) U \delta a'_x + q e^{-2V_1} \delta g_{tx}}$$

Use Einstein equations and regularity at the black hole horizon to relate J and E to get σ

Perturbed metric has a timelike Killing vector k^μ

$$G^{\mu\nu} = \nabla^\mu k^\nu + \ldots \qquad \Rightarrow \qquad \nabla_\mu G^{\mu\nu} = -\frac{V}{2}k^\mu$$

Similar steps then relate Q and E to get $\bar{\alpha}$

- Calculating α and $\bar{\kappa}$

  Consider a source for electric and heat currents

$$g_{tx} = t\delta f_2(r) + \delta g_{tx}(r)$$

$$A_x = t\delta f_1(r) + \delta a_x(r)$$

  Similar steps, with a subtlety that there is both a static and a linear in time-dependent heat current

  Static piece: conductivity

  Time dependent piece: static susceptibility

$$G_{QQ}(\omega = 0) = T^{xx}$$

  Note: $G_{QJ}(\omega = 0) = G_{JJ}(\omega = 0) = 0$

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

$$ds^2 = -U dt^2 + U^{-1} dr^2 + e^{2V_1} dx^2 + e^{2V_2} dy^2$$

$$\chi = k x_1 \qquad A = a dt$$

$$\alpha_{DC} = \bar{\alpha}_{DC} = -\left[\frac{4\pi q}{k^2\Phi(\phi)}\right]_{r=r_+} \qquad \bar{\kappa}_{DC} = \left[\frac{4\pi s T}{k^2\Phi(\phi)}\right]_{r=r_+}$$

$$\sigma_{DC} = \left[e^{-V_1+V_2} Z(\phi) + \frac{q^2 e^{-V_1-V_2}}{k^2\Phi(\phi)}\right]_{r=r_+}$$

"Pair evolution" term. Given by $(J/E)_{Q=0} \equiv \sigma - \alpha^2\bar{\kappa}^{-1}T$

Second term "Dissipation" term

Different ground states can be dominated by first or second term

- Some general results

  Define thermal conductivity at zero current

  $$\kappa = \bar{\kappa} - \alpha\bar{\alpha}T/\sigma$$

  For dissipation dominated T=0 ground states κ and $\bar{\kappa}$ can have different low temperature scaling (n.b. $\kappa = \bar{\kappa}$ for FL)

- $$\bar{L} \equiv \frac{\bar{\kappa}}{\sigma T} \leq \frac{s^2}{q^2}$$

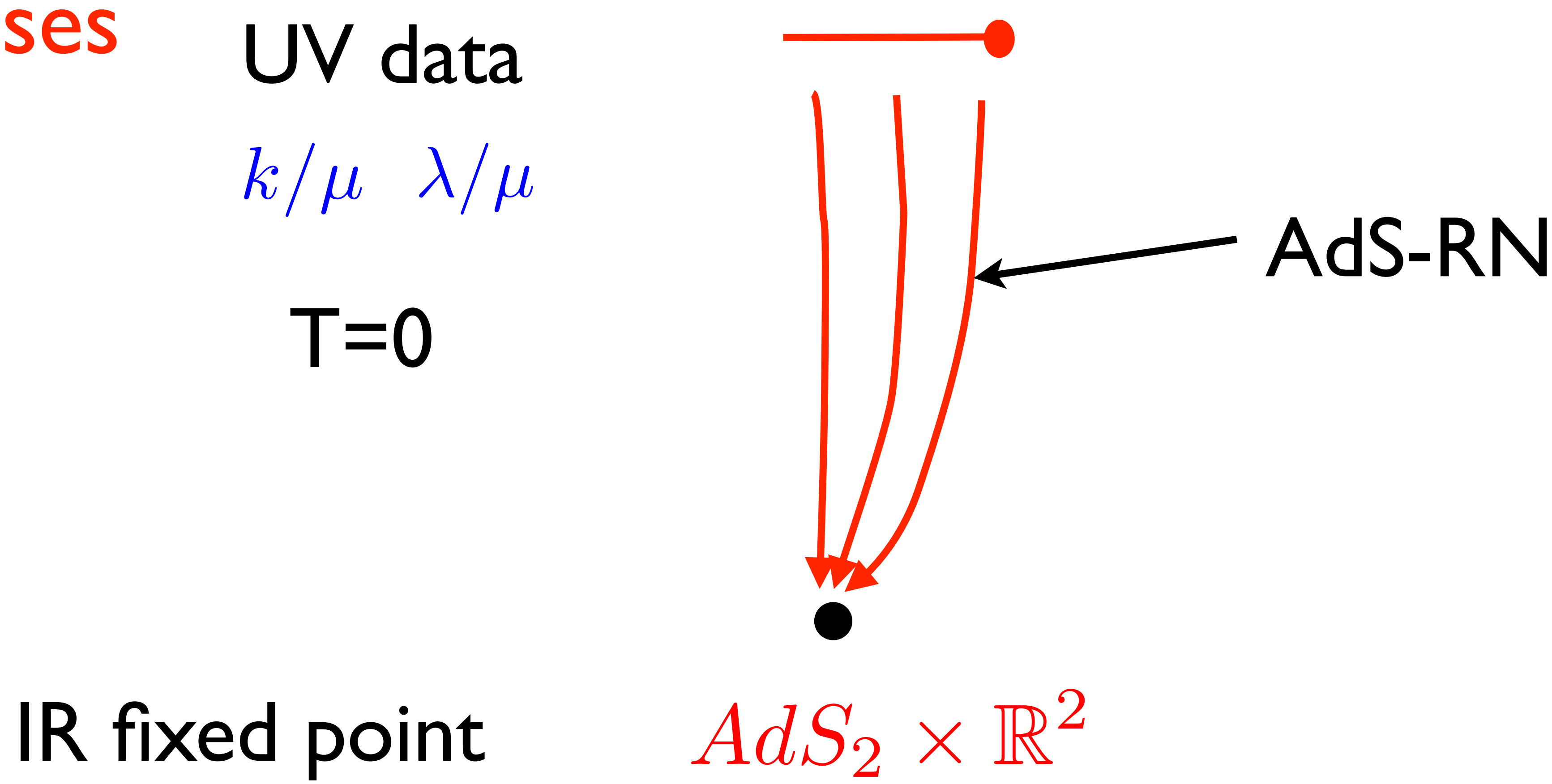
Bound is saturated for dissipation dominated systems

c.f. Wiedemann-Franz Law.

Complementary result using memory matrix [Mahajan,Barkeshli,Hartnoll]

- $$\frac{\bar{\kappa}}{\alpha} = -\frac{Ts}{q}$$

Coherent metal phases



At T=0 the black holes approach $AdS_2 \times \mathbb{R}^2$ in the IR

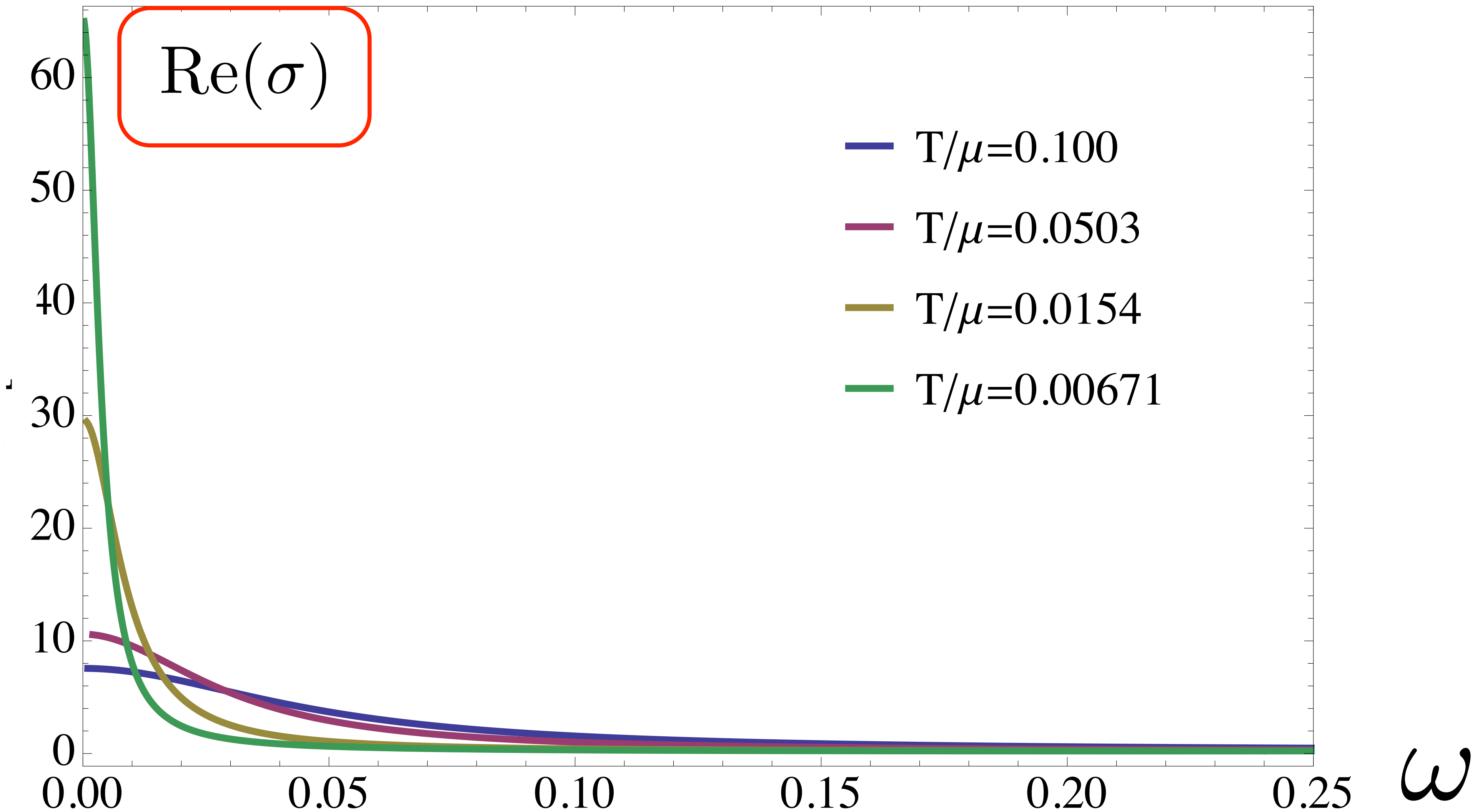
perturbed by irrelevant operator with $\Delta(k_{IR}) \geq 1$ [Hartnoll, Hoffman]

Note: k_{IR} depends on RG flow

Low T DC conductivity is dissipation dominated: $\sigma \sim T^{2-2\Delta(k_{IR})}$

Always have $\kappa \sim T$ but $\bar{\kappa} \sim T^{3-2\Delta(k_{IR})}$ and $\bar{\kappa} \to 0, \infty$

Drude peaks at finite T

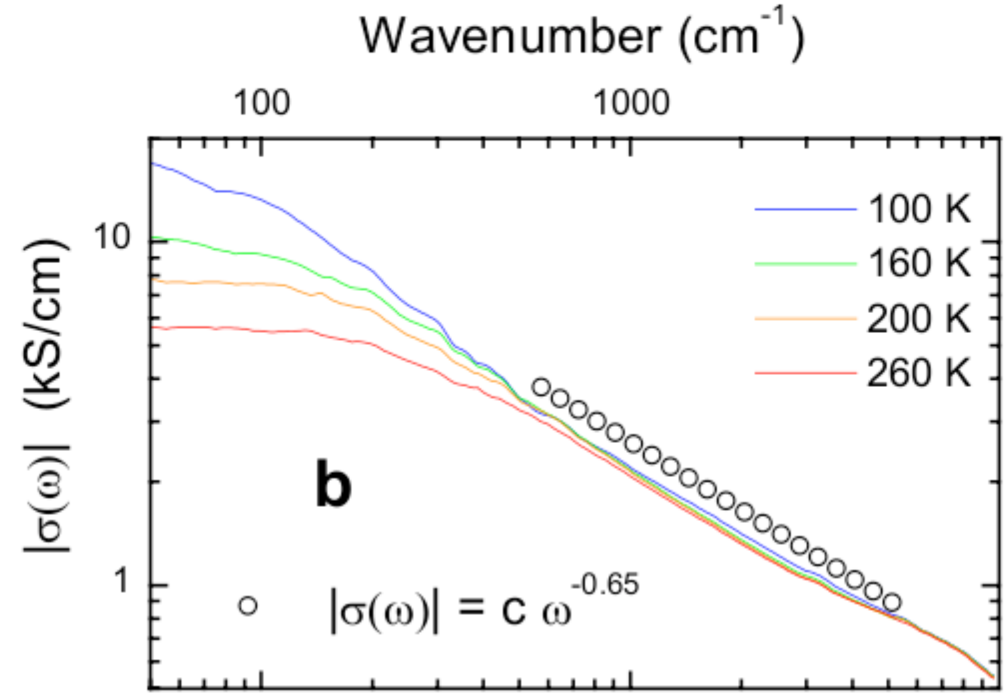


Similar to what was seen for different lattices in [Horowitz,Santos,Tong]

Intermediate scaling?

[Horowitz,Santos,Tong]

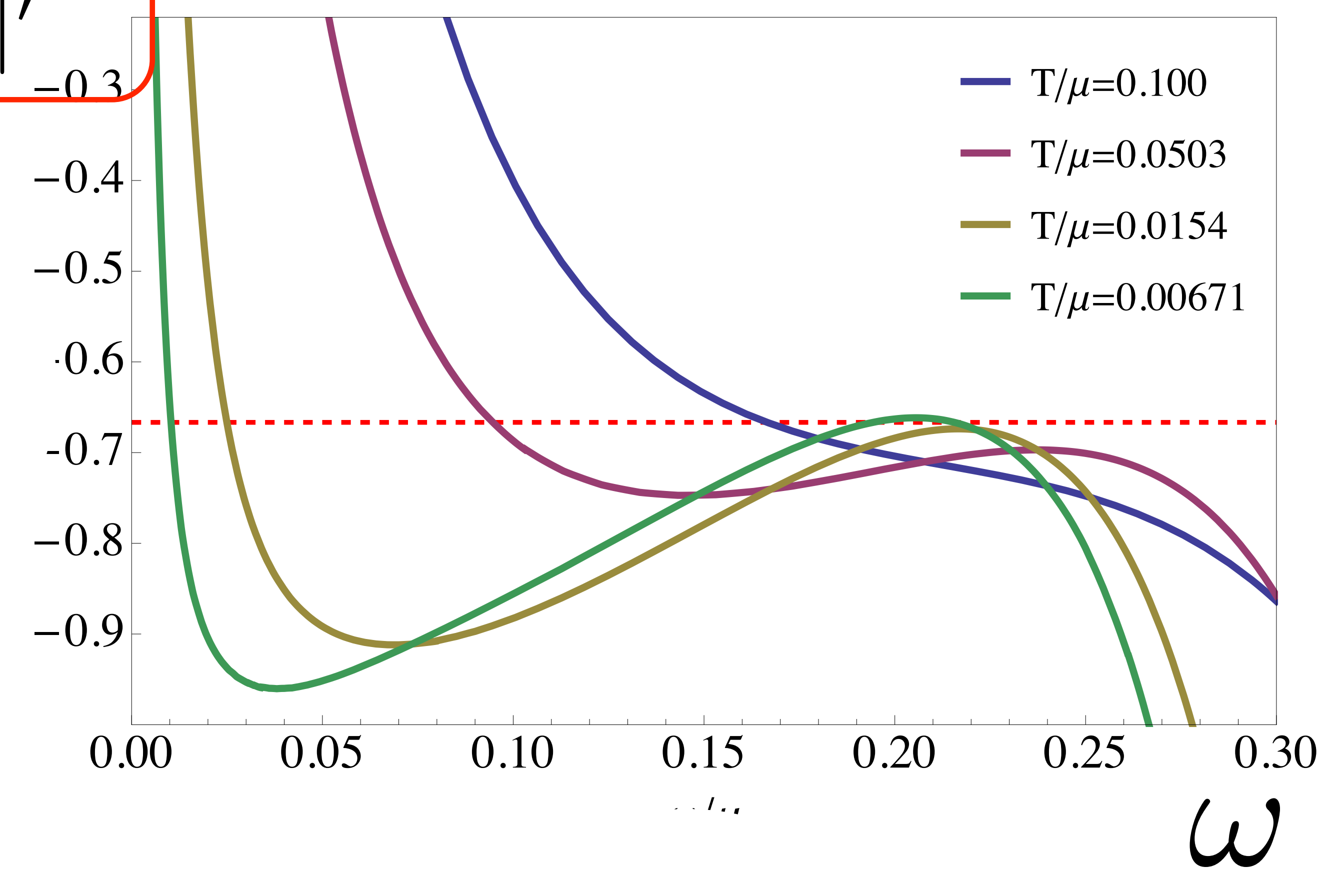
$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$



Reminiscent of cuprates

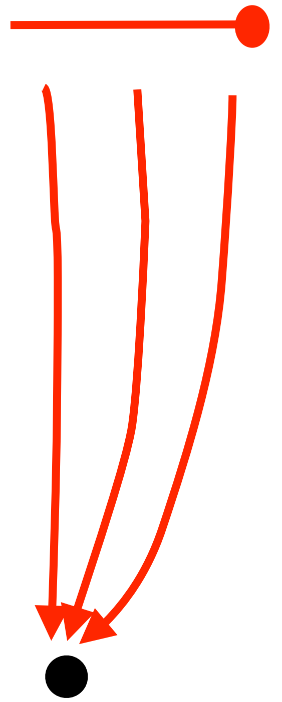
$$1 + \omega\frac{|\sigma|''}{|\sigma|'}$$

Do not see this scaling for the Q-lattice:



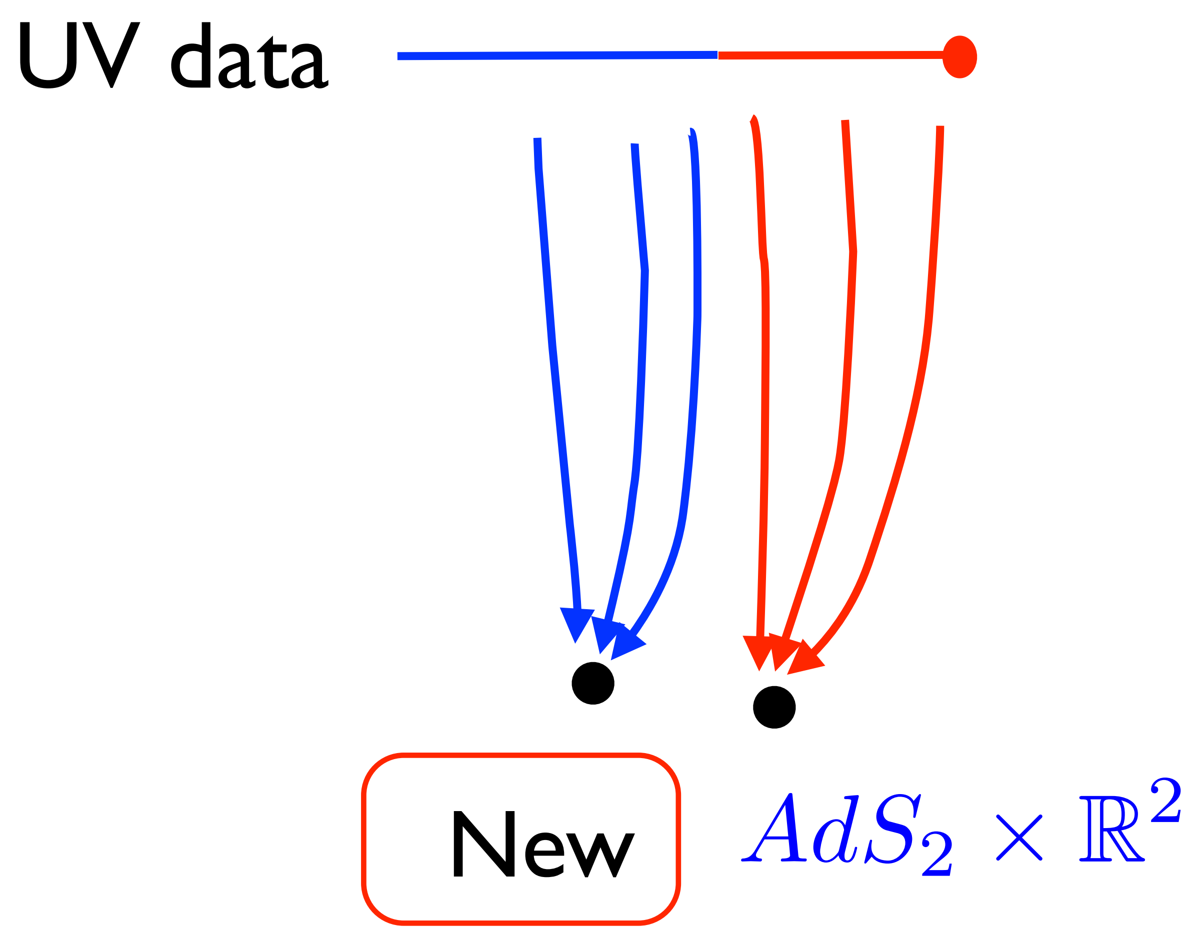
Insulating phases

UV data

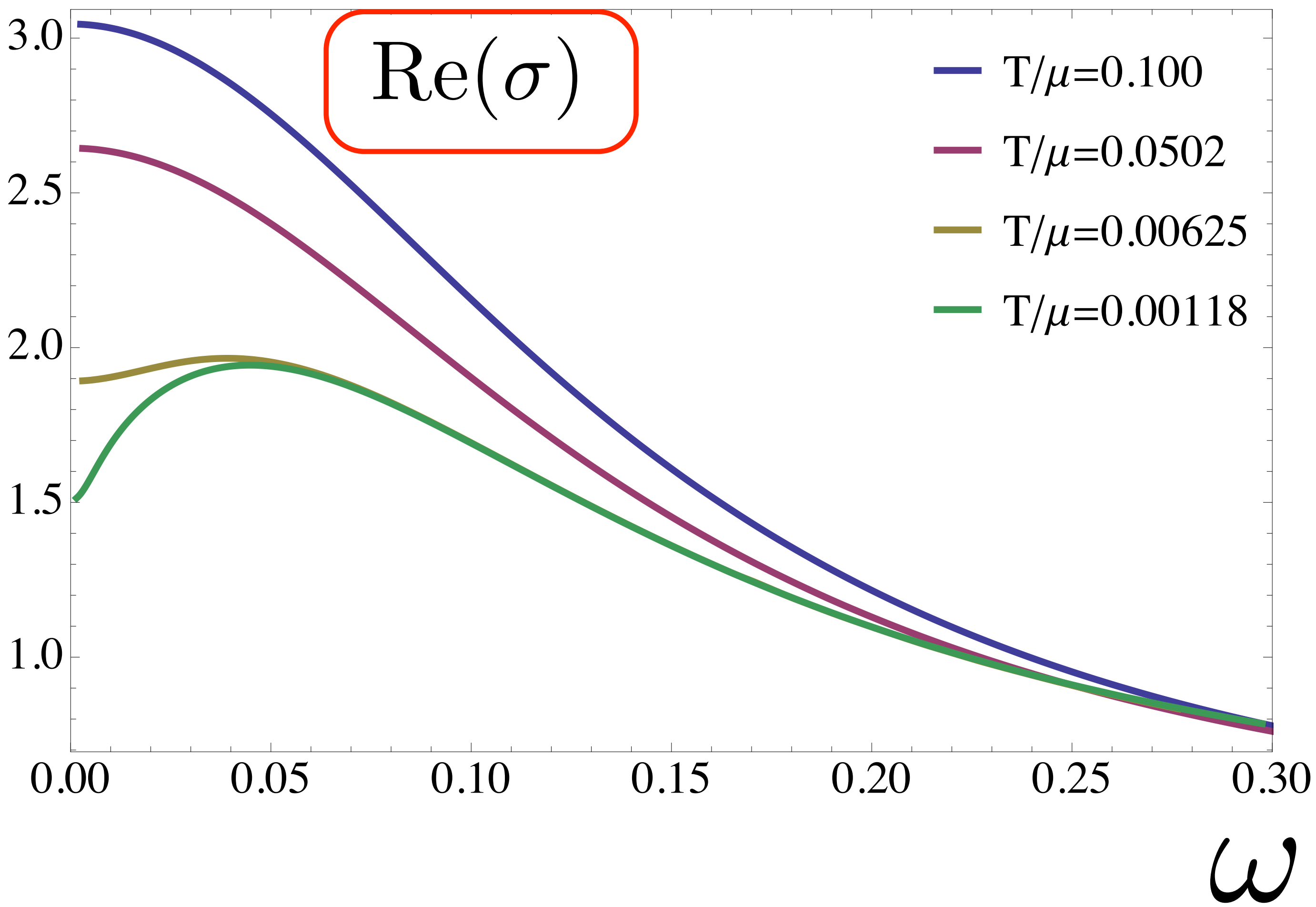


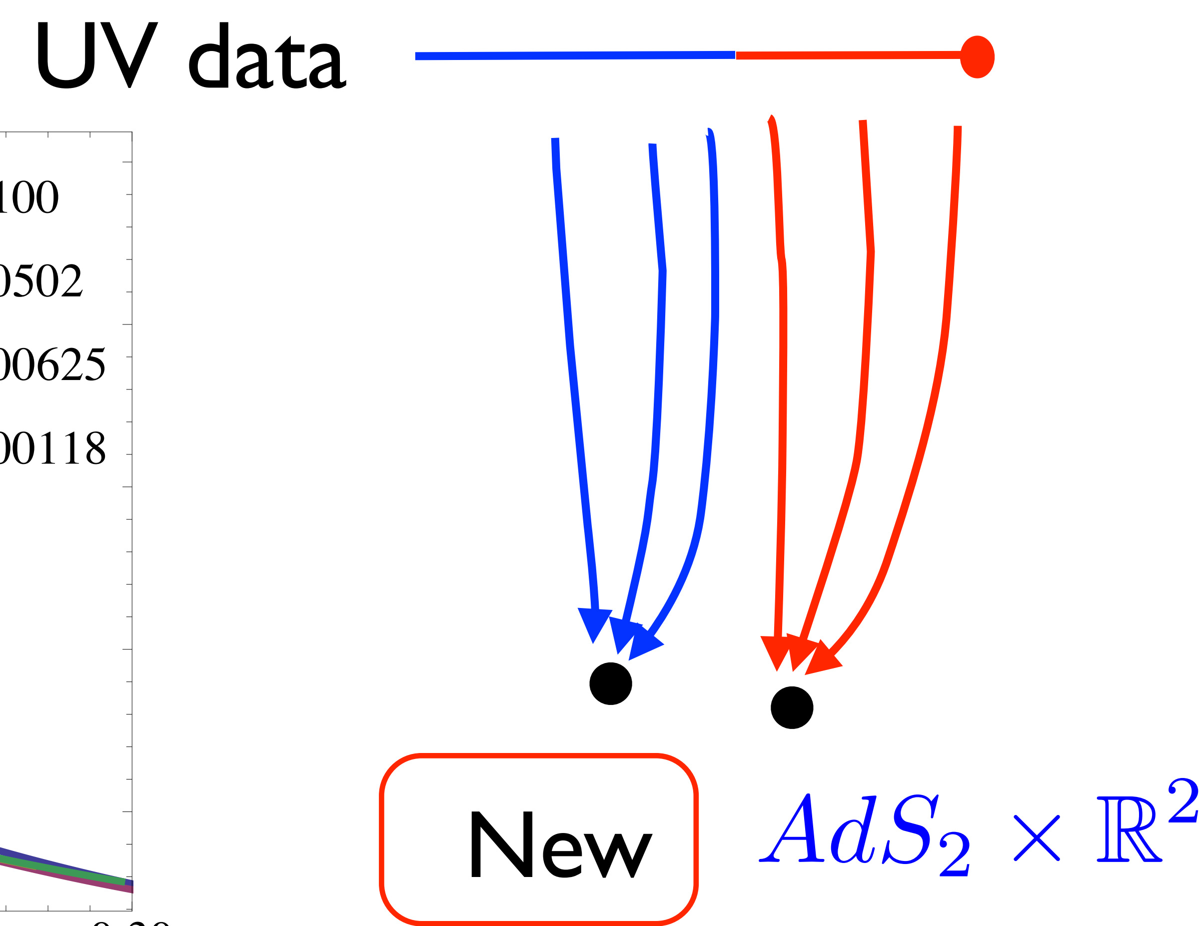
$AdS_2 \times \mathbb{R}^2$

Insulating phases



Insulating phases





Appearance of a mid-frequency hump.
Spectral weight is being transferred, consistent with sum rule

What are the T=0 insulating ground states??

Focus on specific models (see also [Gouteraux])

New Insulating and Metallic ground states - Anisotropic

$$\mathcal{L} = R - \frac{1}{2}\left[(\partial\phi)^2 + \Phi(\phi)(\partial\chi)^2\right] + V(\phi) - \frac{Z(\phi)}{4}F^2$$

Focus on models and T=0 ground states which are solutions with $\phi \to \infty$ as $r \to 0$

and $$\mathcal{L} \to R - \frac{3}{2}\left[(\partial\phi)^2 + e^{2\phi}(\partial\chi)^2\right] + e^{\phi} - \frac{e^{\gamma\phi}}{4}F^2$$

IR "fixed point" solutions

$$ds^2 \sim -r^u dt^2 + r^{-u} dr^2 + r^{v_1} dx^2 + r^{v_2} dy^2$$

$$e^{\phi} \sim r^{-\phi_0} \qquad A \sim r^a dt \qquad \chi = kx$$

with exponents fixed by γ

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$$e^{\phi} \sim r^{-\phi_0} \qquad A \sim r^a dt \qquad \chi = kx$$

with exponents fixed by γ

- Calculate AC conductivity

Obtained using a matching argument [Faulkner,Liu,McGreevy,Vegh] with ground state correlators at T=0. Valid: $T << \omega << \mu$

$$\sigma_{AC} \sim \omega^{c(\gamma)}$$

- Calculate DC conductivity using analytic formula

For $T << \mu$ the scaling is obtained from the IR fixed point solutions

$$\sigma_{DC} \sim T^{b(\gamma)}$$

In these models we have $b = c$
(as we have for the $AdS_2 \times \mathbb{R}^2$ coherent metals)

$$\sigma_{DC} \sim T^{b(\gamma)}$$

$$T << \mu$$

$$\sigma_{AC} \sim \omega^{c(\gamma)}$$

$$T << \omega << \mu$$

$b = c > 0$ Have new type of insulating ground states

$b = c < 0$ Have new type of incoherent metallic ground states not associated with Drude physics

$b = c = 0$ Novel metallic ground states with finite conductivity at T=0

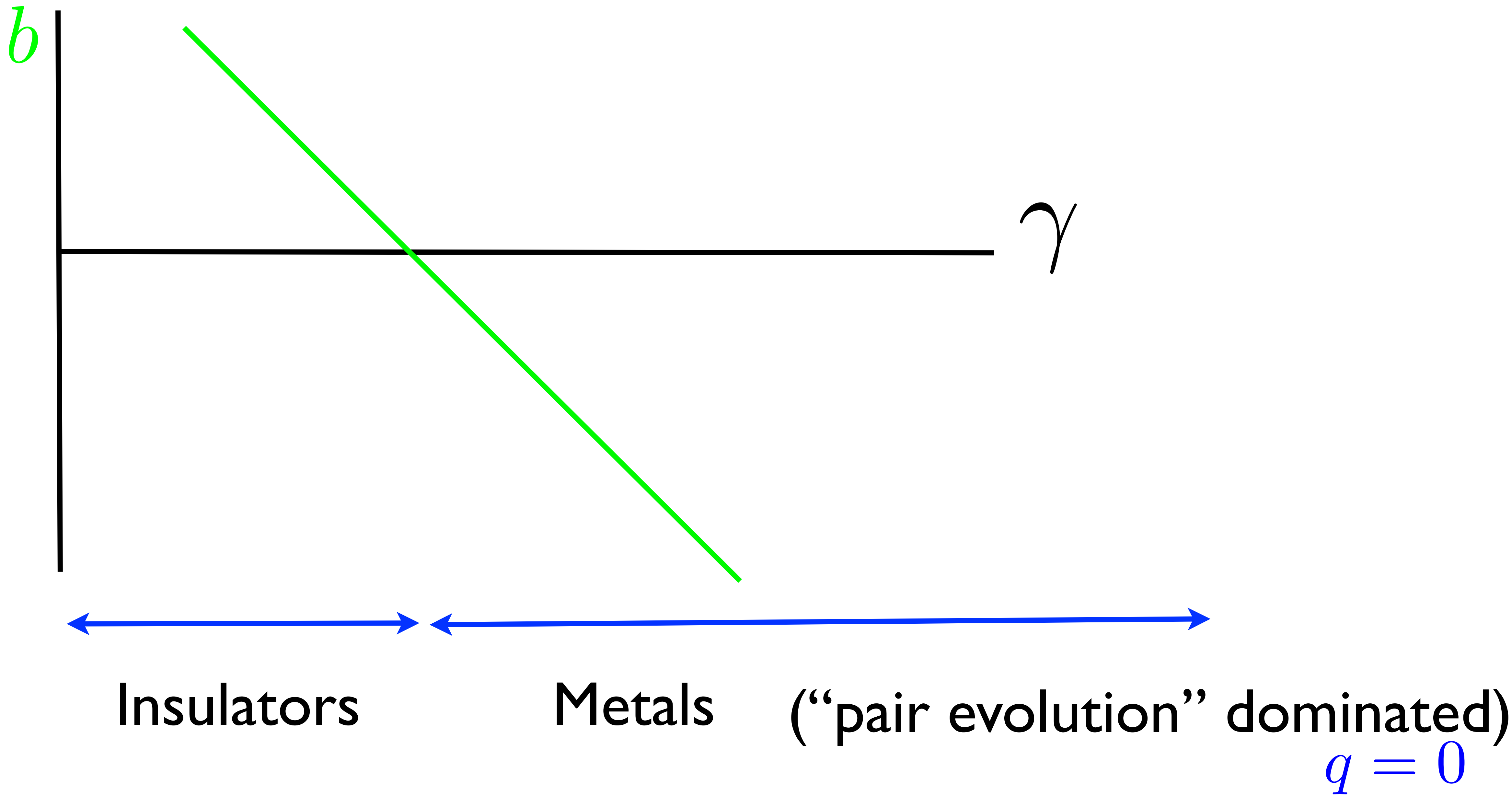
Metallic ground states are all thermal insulators: $\kappa, \bar{\kappa} \to 0$

Both terms in σ_{DC} scale in the same way

New Insulating and Metallic ground states - Isotropic

Models with $\chi_1 = kx$ and $\chi_2 = ky$

$\sigma_{DC} \sim T^b$

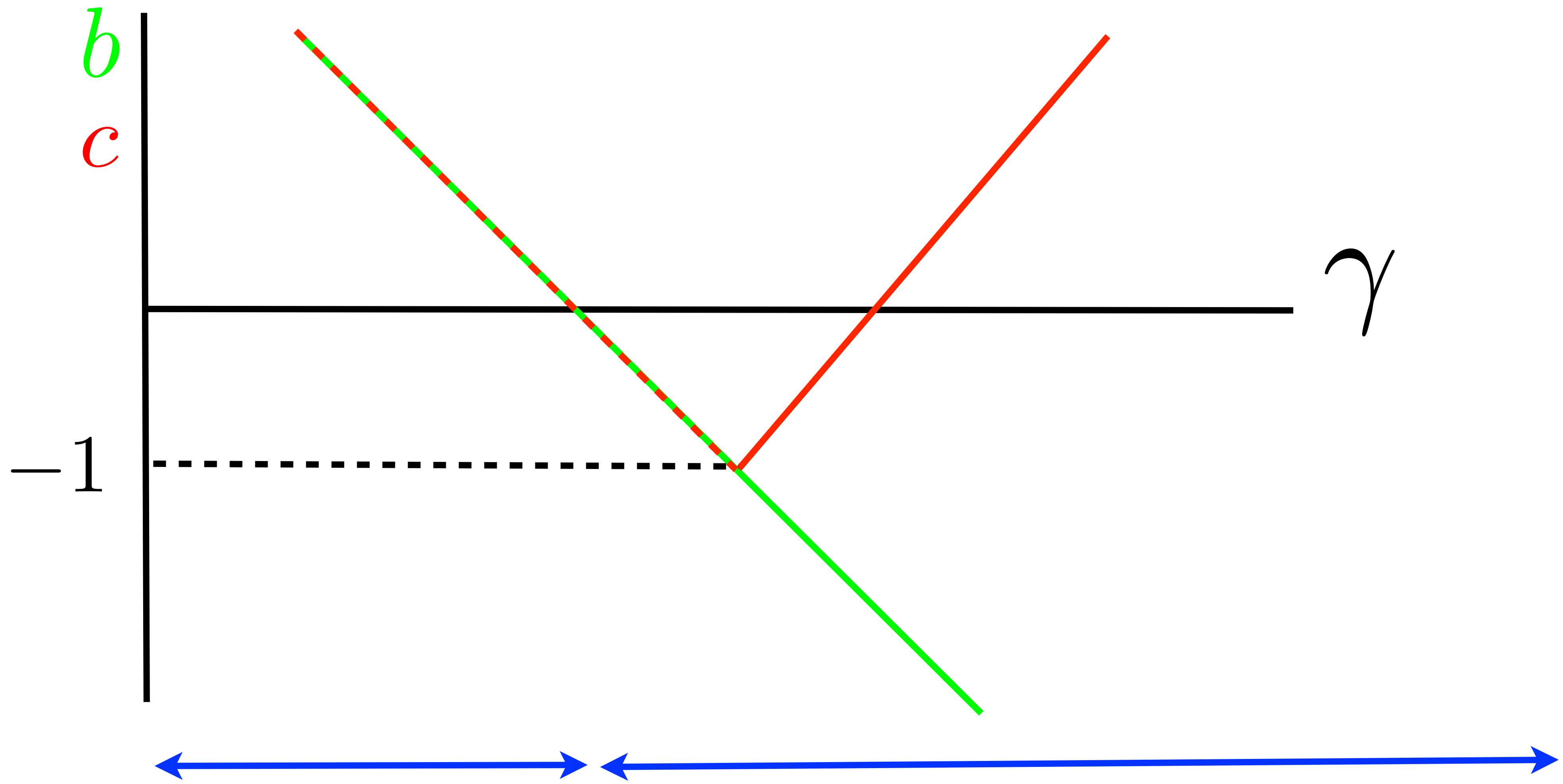


New Insulating and Metallic ground states - Isotropic

Models with $\chi_1 = kx$ and $\chi_2 = ky$

$\sigma_{DC} \sim T^b$ $\sigma_{AC} \sim \omega^c$

All have $\kappa, \bar{\kappa} \to 0$



Insulators Metals ("pair evolution" dominated)

$q = 0$

Reappearance of sharp peaks not related to the charge density and Drude physics

Summary

- Holographic Q-lattices are simple and illuminating
- Analytic result for DC conductivity in terms of horizon data.

  Bound on $\bar{L} \equiv \bar{\kappa}/(\sigma T)$
- Coherent metallic phases with Drude peaks
- No intermediate 2/3 scaling in AC conductivity

  Absent in another recent example [Taylor, Woodhead]
- Also find novel metallic $(\kappa \to 0)$ and insulating ground states

  Can be "pair evolution" dominated metal ground states with $q = 0$

  Metal-Insulator and Metal-Metal transitions

- Lattices are a good way to look for new holographic ground states

Alternatives

- Construct them directly
- Find the ground states of holographic phases that spontaneously break symmetries

Key results for Inhomogeneous lattices $\mu(x)$

Donos Talk @ 7:30 pm Friday!

- Analytic results for DC conductivity
  - Bound on $\bar{L} \equiv \bar{\kappa}/(\sigma T)$
  - High temperature behaviour

$$\sigma \to 1 + \frac{(\int \mu)^2}{\int \mu^2 - (\int \mu)^2} \geq 1$$

Like Mott-Ioffe-Regel bound?

- No intermediate scaling for $|\sigma(\omega)|$

- As $T \to 0$ black holes approach $AdS_2 \times \mathbb{R}^2$ in the IR

No exotic "floppy" ground states seen by [Hartnoll,Santos]