Renormalization group flows of Yang-Mills theories with charged fundamental matter

Javier Tarrío

with A. Faedo, A. Kundu and D. Mateos

Quantum Field Theory, String Theory and Condensed Matter Physics

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Motivation

We are interested in studying the effects of Yang-Mills theories with fundamental charge density at strong coupling.

$\Rightarrow$ Can get insight on the phase diagram of QCD at low temperatures.

$\Rightarrow$ For a top-down approach one usually starts instead with $\mathcal{N} = 4$SYM with additional degrees of freedom.

$\Rightarrow$ But UV behavior not under a reasonable control!

Motivation

We are interested in studying the effects of Yang-Mills theories with fundamental charge density at strong coupling, for generic number of dimensions.

$\Rightarrow$ Allowing to change the number of dimensions may provide intuition and give context to the $d = 4$ case.

$\Rightarrow$ For condensed matter the cases with $d = 2, 3$ are of intrinsic interest, as well.

$\Rightarrow$ In this talk I will focus in $d = 2 + 1$ YM theories.

Reminder: holographic dual to d=3 SYM [Itzhaki et al. '98]

$\Rightarrow$ Stack of N_c D2-branes in flat space

$$e^{\phi(u)} \leftrightarrow \mathrm{tr}F^2 \ ,$$
$$G_{MN}(u, S^6) \leftrightarrow T^{\mu\nu}, \ \mathrm{tr}F^4 \ .$$

$\Rightarrow$ With N_c given by the flux along S^6

$$F_6 = 5\, L^5\, \omega_6 \quad \Rightarrow \quad \frac{1}{2\kappa^2}\int_{S^6} F_6 = T_2\, N_c \ .$$

$\Rightarrow$ And the radial coordinate a geometrization of the energy scale

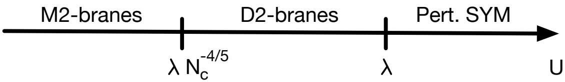
$$u \equiv U\, \ell_s^2 \ .$$

Reminder: holographic dual to d=3 SYM [Itzhaki et al. '98]

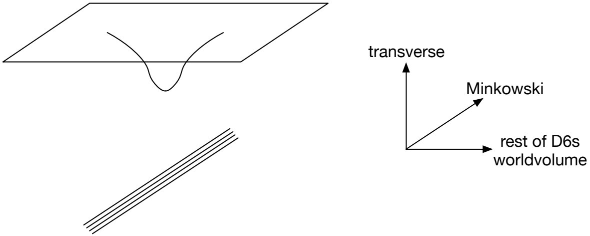
⇒ Solution can be expressed in terms of a dimensionless effective coupling

$$g_{eff}^2 \sim \frac{\lambda}{U}$$

⇒ This determines the validity of the gravity description



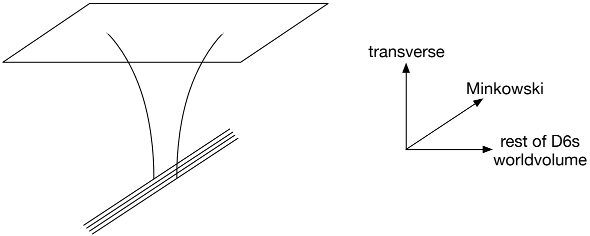
Our setup [Cherkis & Hashimoto '02] [Faedo, Mateos & JT, to appear]



⇒ Restricting to SUGRA fields means considering mostly adjoint matter, we need stringy sources for fundamental matter.

⇒ An extra set of D6-branes gives us the degrees of freedom to describe matter in this representation. [Karch & Katz '02]

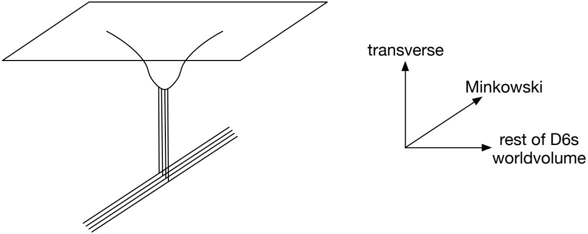
Our setup [Work in progress]



⇒ A finite charge density is dual to strings dissolved in the D6-branes. The brane bends towards the origin.

⇒ The throat can be interpreted as a collection of strings pulling down the D6-branes. [Kobayashi et al. '06]

Our setup



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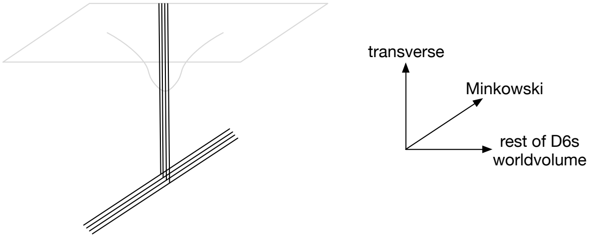


$\Rightarrow$ One approximation: Non-dynamic quarks $\to$ only strings in the holographic description.

$\Rightarrow$ The action is given by SUGRA with a stringy source term

$$S_{total} = S_{IIA} - \underbrace{\frac{n_q}{2\pi\ell_s^2}\int\left(\sqrt{-G_{tt}\,G_{rr}}\,\mathrm{d}t\wedge\mathrm{d}r - B_2\right)\wedge\Xi_8}_{S_{strings}}\ .$$

Our setup

⇒ The presence of strings necessarily sources dissolved baryonic branes supergravity fields

$$\mathrm{d}\left(e^{-2\phi} * H_3\right) + F_2 \wedge F_6 - \frac{1}{2} F_4 \wedge F_4 = \underbrace{-\frac{2\kappa_{10}^2}{2\pi\ell_s^2}\, n_q\, \Xi_8}_{sources} \,,$$

with $F_2 = -\frac{q}{L}\,\mathrm{d}x^1 \wedge \mathrm{d}x^2$.

⇒ At a given energy scale, U, the effective charge density is

$$n_q^{eff} \equiv q \left(\frac{L}{u}\right)^4 \sim \frac{n_q}{N_c^2}\frac{\lambda^2}{U^4} \ .$$

UV geometry

⇒ For $q = n_q = 0$ the solution is that of a stack of D2-branes, dual to (S)YM theories [Itzhaki et al. '98]

⇒ For finite charge density the UV is governed by that geometry, but subleading corrections exist

$$e^{\phi} = \left(\frac{u}{L}\right)^{-\frac{5}{4}} \left[1 - \alpha_\phi \underbrace{q \left(\frac{L}{u}\right)^4}_{n_q^{eff}} + \underset{\langle \mathrm{tr} F^2 \rangle}{\underset{\downarrow}{v_\phi}} \left(\frac{L}{u}\right)^5 + \mathcal{O}\left(\frac{L}{u}\right)^8\right] .$$

IR geometry

$\Rightarrow$ There is an exact solution with a HV-Lif metric with $z = 5$ and $\theta = 1$

$$t \to \lambda^5 \, t \ , \qquad x^i \to \lambda \, x^i \ , \qquad \mathrm{d}s^2 \to \lambda \, \mathrm{d}s^2 \ ,$$

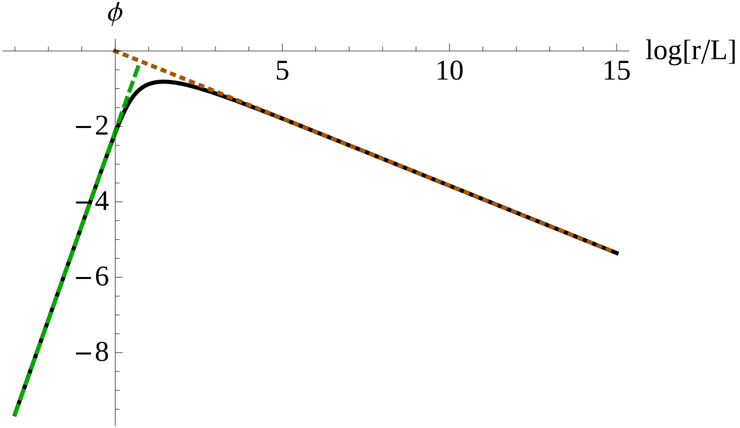
notice in particular that

$$p - \theta = 1 \ .$$

$\Rightarrow$ Also running scalars

$$\underset{\mathrm{tr}F^2}{\overset{e^{\phi}}{\downarrow}} \sim \left(\frac{r}{q\,L}\right)^{\frac{5}{2}} \qquad \underset{\mathrm{tr}F^4}{\overset{e^{\eta}}{\downarrow}} \sim \left(\frac{r}{q\,L}\right)^{-\frac{1}{8}} \ .$$

RG flow in d=2+1 (S)YM



RG flow in d=2+1 (S)YM

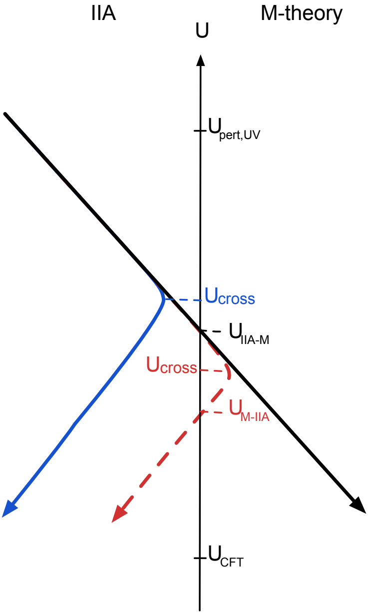
⇒ For fixed λ, different scales appear depending on n_q/N_c^2.

⇒ Crossover scale at

$$U_{cross} \sim \lambda^{\frac{1}{2}} \left(\frac{n_q}{N_c^2}\right)^{\frac{1}{4}} \quad .$$

⇒ Critical density for the need of an M-theory description

$$\left(\frac{n_q}{N_c^2}\right)_{crit} \sim \lambda^2\, N_c^{-\frac{16}{5}} \quad .$$



A note: changing the number of dimensions

$\Rightarrow$ There are RG flows driving the theory to an IR characterized by dynamical and hyperscaling-violating exponents for different dimensionalities ($p = 3$ studied in [Kumar '12])

p	1	2	3	4	5
z	13/3	5	7	∞	-1
θ	2/3	1	0	$-z$	10

$\Rightarrow$ The $p = 4$ case can be understood as the $z \to \infty$ limit with $z/\theta = -1$. see [Hartnoll & Shaghoulian '12]

$$\mathrm{d}s^2 = \left(\frac{r}{L}\right)^{\frac{1}{2}} \left(-\frac{r^2}{L^2} f(r)\mathrm{d}t^2 + \mathrm{d}x_4^2 + \#^2\, q^{-\frac{1}{2}}\, \frac{L^2}{r^2} \frac{\mathrm{d}r^2}{f(r)}\right) \,.$$

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$\Rightarrow$ Uplift to M-theory gives an $AdS_7 \to AdS_3 \times \mathbb{R}^4$ DW solution.

$$\mathrm{d}s^2 = -\frac{r^2}{\mathcal{L}^2} f(r)\mathrm{d}t^2 + \frac{r^2}{\mathcal{L}^2} d\psi^2 + \frac{\mathcal{L}^2}{r^2}\frac{\mathrm{d}r^2}{f(r)} + \mathrm{d}x_4^2 + \frac{3}{2}\mathcal{L}^2\mathrm{d}\Omega_4^2 \ ,$$

$$F_4 = \frac{\sqrt{2}}{3^{1/4}}\frac{1}{\mathcal{L}}\left[\mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}x^4 + \frac{3}{2}\mathcal{L}^4\omega_4\right] \ .$$

compare to [D'Hoker & Kraus '09]

Basic thermodynamics

⇒ At low temperatures

$$T \ll \lambda^{\frac{1}{4}} \left(\frac{n_q}{N_c^2} \right)^{\frac{3}{8}}$$

only the HV-Lifshitz geometry matters (but UV time!) and

$$s \sim N_c^2 \left(\frac{n_q}{N_c^2} \right)^{\frac{4}{5}} (\lambda\, T)^{\frac{1}{5}} \ .$$

⇒ Free energy obtained from the first law.

⇒ At large temperatures usual D2-brane thermodynamics

$$s \sim N_c^2\, \lambda^{-\frac{1}{3}}\, T^{\frac{7}{3}} \ .$$

Conclusions

⇒ Supergravity with strings gives an accurate description of $d \leq 6$ (S)YM with an external charge density, $n_q \sim N_c^2$, at strong coupling and large N_c.

⇒ Supersymmetry and the global symmetry group do not play an important role in these constructions. [Faedo, Fraser & Kumar, '13]

⇒ The IR is described by a theory with dynamical and hyperscaling violating exponents.

p	1	2	3	4	5
z	$13/3$	5	7	∞	-1
θ	$2/3$	1	0	$-z$	10

Outlook

$\Rightarrow$ Next step: understand how this picture changes when the charge is dynamic. [Work in progress]

$\Rightarrow$ Dynamic flavor requires additional degrees of freedom in the supergravity description, however the HV-Lif solution described here still exists in the IR.

$\Rightarrow$ Different RG flows parameterized by a single parameter

$$\gamma \equiv \frac{n_q}{N_c^2} \frac{1}{\lambda^2} \left(\frac{N_c}{N_f}\right)^4 \ .$$

Thank you

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