Mott gaps, unparticles and Fermi Arcs

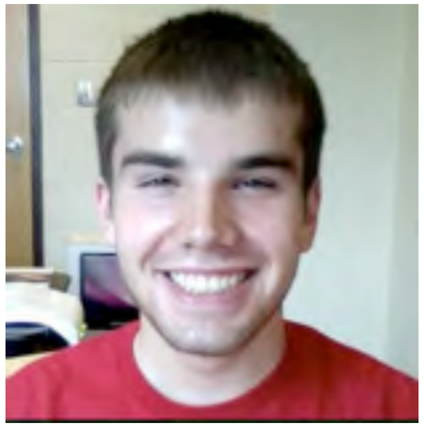
Thanks to: NSF, EFRC (DOE)

Kiaran Dave

Charlie Kane

Brandon Langley

J. A. Hutasoit



Garrett Vanacore

PRL, 110, 090403 (2013)

PRB, 88, 115129 (2013)

PRD, 90, 044022 (2014)

propagators

$$G \propto \langle T\psi(0)\psi^\dagger(t)\rangle$$

propagators

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Fermi liquids

$$\frac{1}{p^2}$$

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$$\frac{1}{p^2}$$

poles at p=0

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what about

$$(p^2)^{d_U - d/2}$$

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vanishing propagator!
(zeros)

gapped systems

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$$d_U > d/2$$

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$$\dim[\psi] = d_U$$

vanishing propagator!
(zeros)

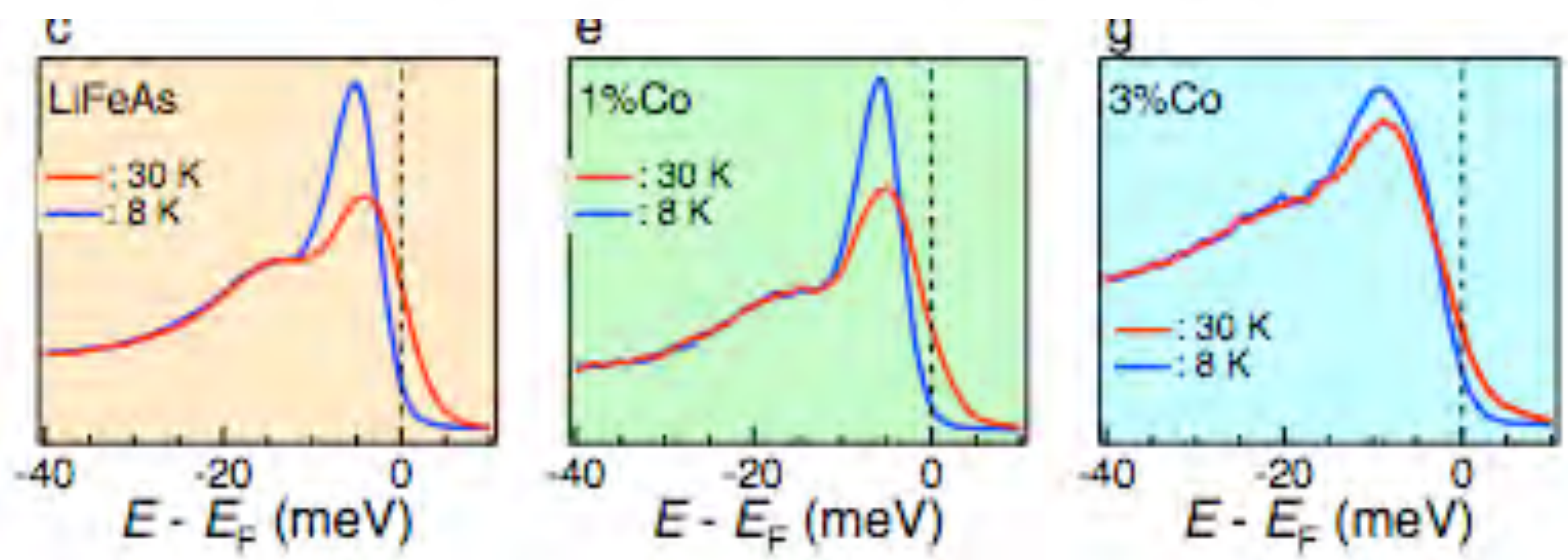
gapped systems

changing scaling dimension

are such propagators important?

Observation of strong electron pairing on bands without Fermi surfaces in $LiFe_{1-x}Co_xAs$

H. Miao[1], T. Qian[1,*], X. Shi[1], P. Richard[1,2], T. K. Kim[3], M. Hoesch[3], L. Y. Xing[1], X. –C. Wang[1], C. –Q. Jin[1,2], J. –P. Hu[1,2,4] and H. Ding[1,2,*]

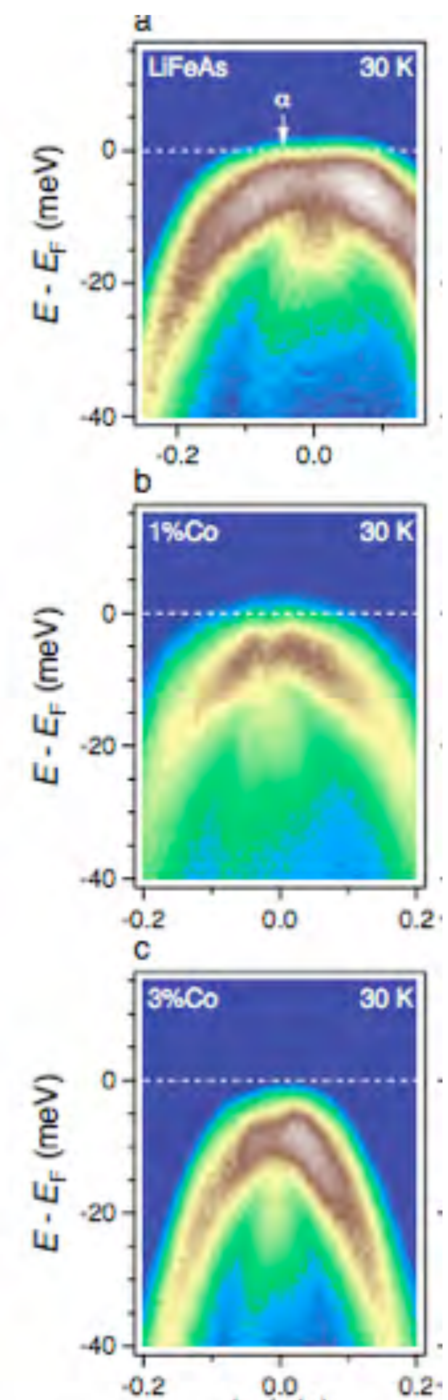


gapped degrees of freedom=> beyond particles (BCS)

$LiFe_{1-x}Co_xAs$

$LiFe_{1-x}Co_xAs$

H. Ding
1406.0983



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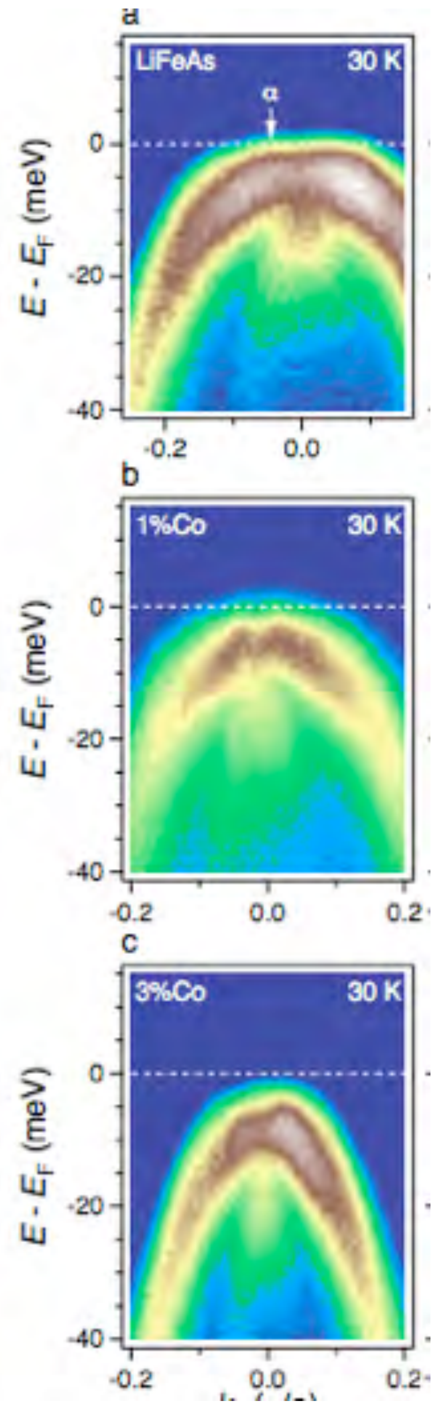
H. Ding
1406.0983

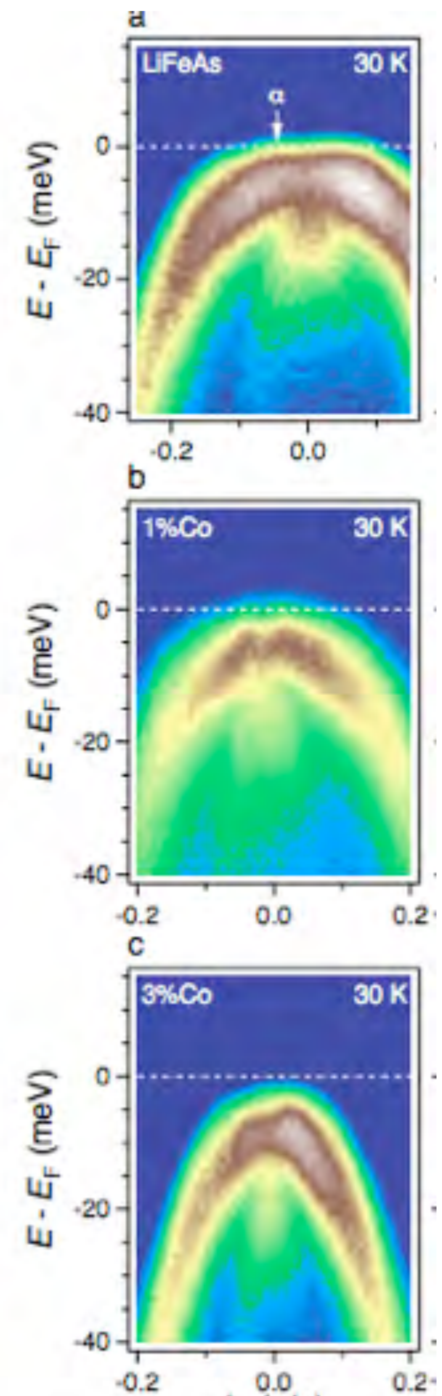


$> \Delta_{SC}$

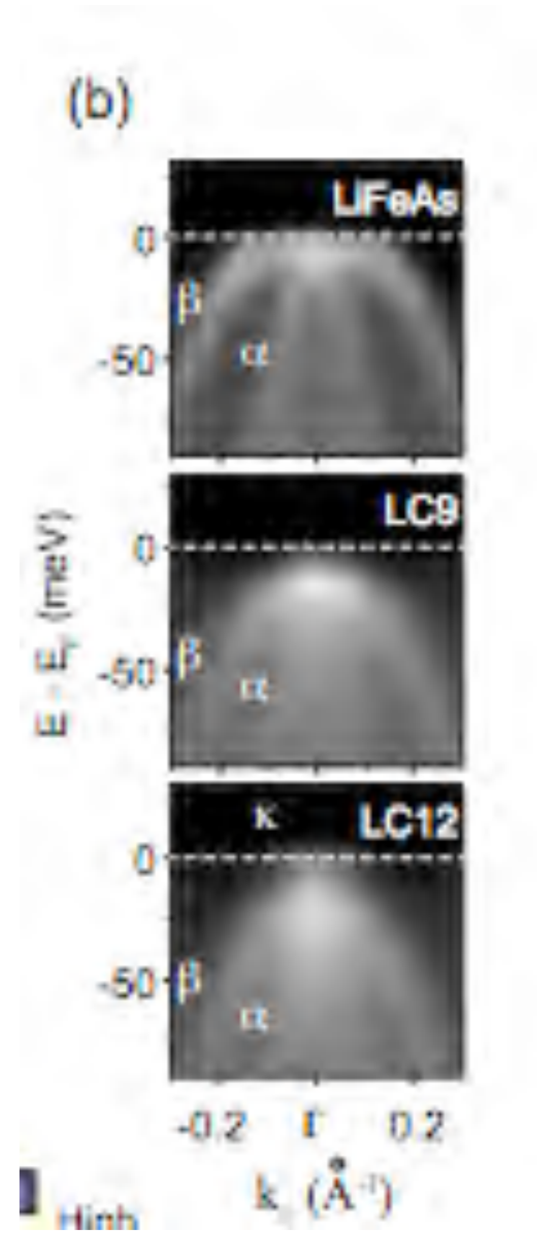
$LiFe_{1-x}Co_xAs$

H. Ding
1406.0983

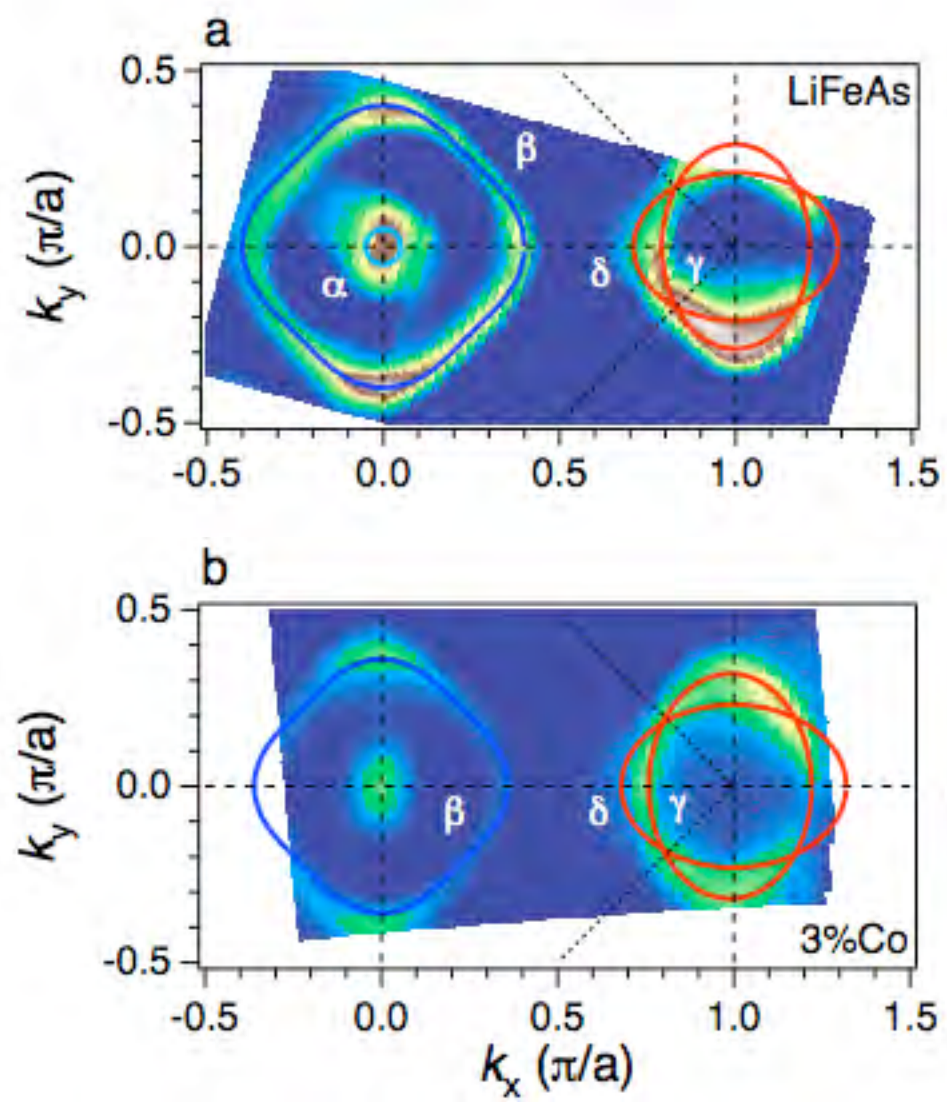
Z. R. Ye



$> \Delta_{SC}$



compute FS volume



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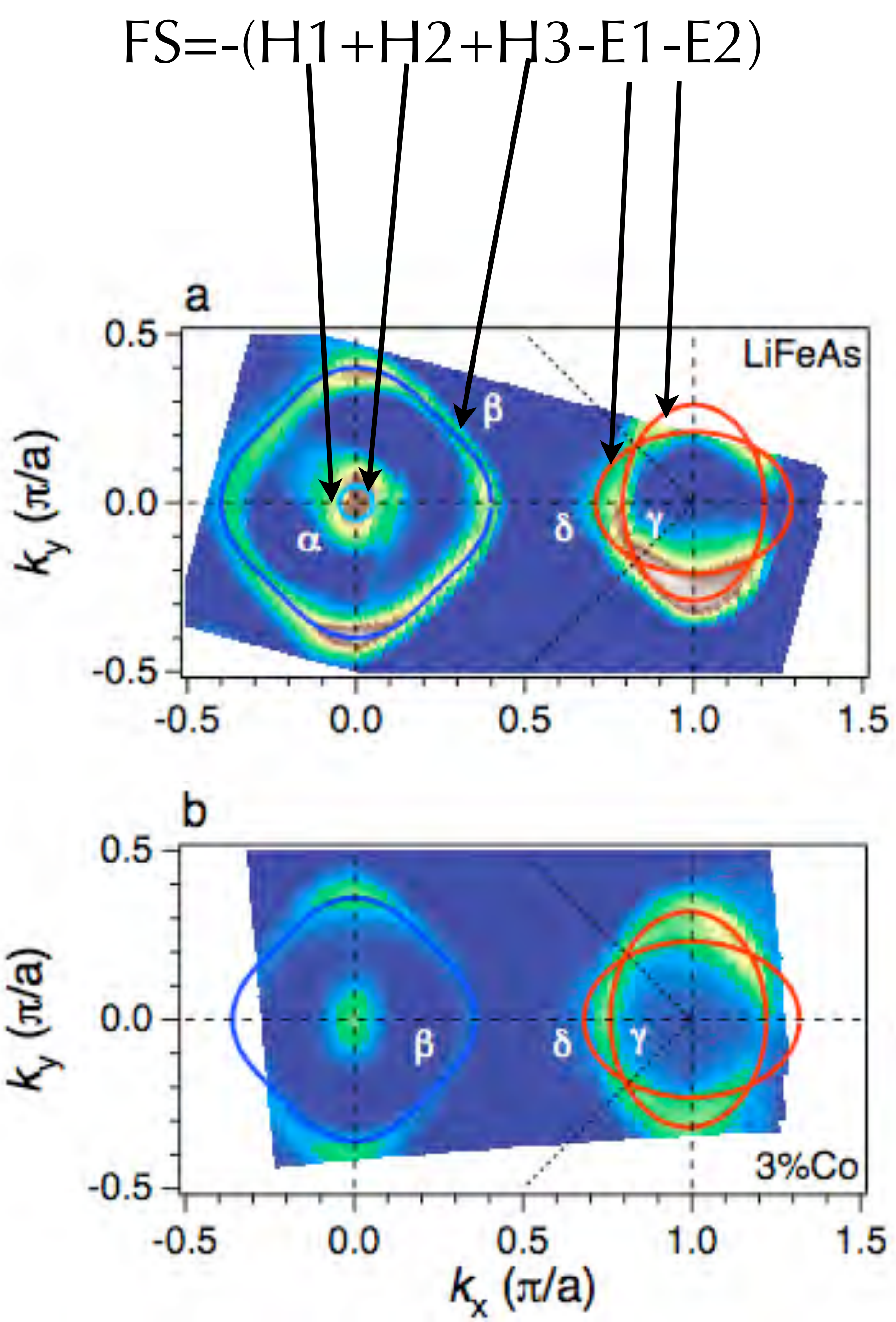

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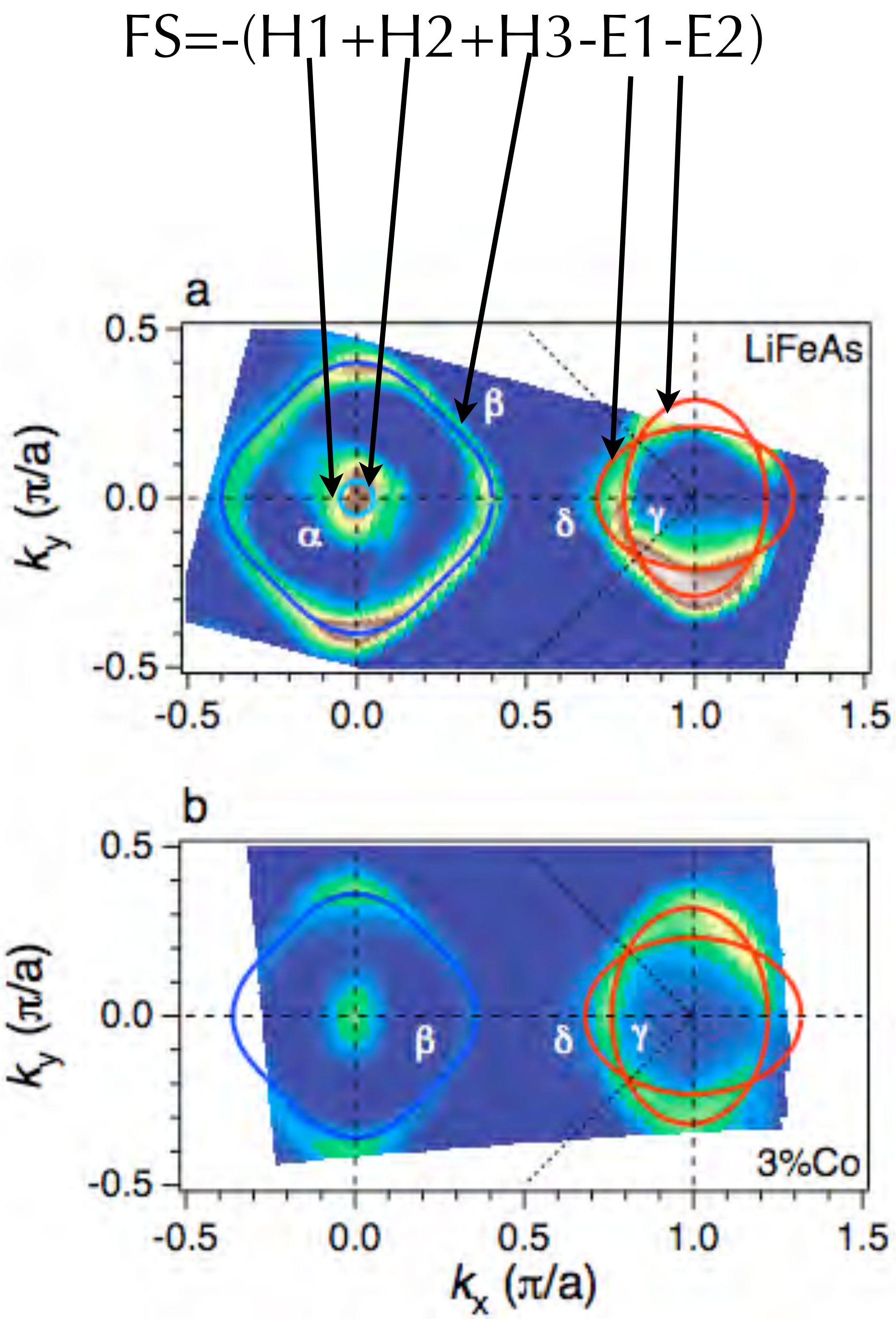
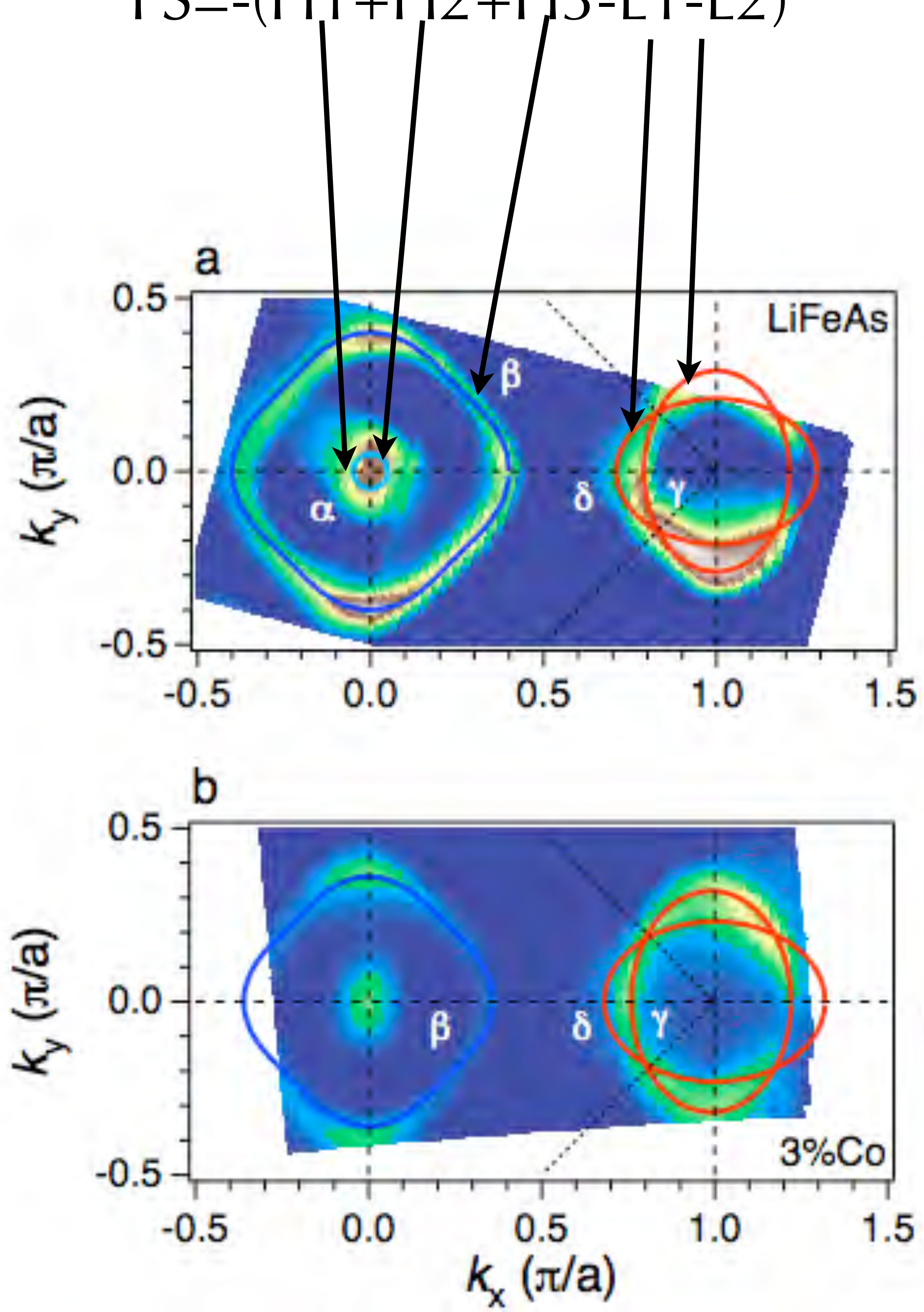


FS(b)-FS(a)=0.18

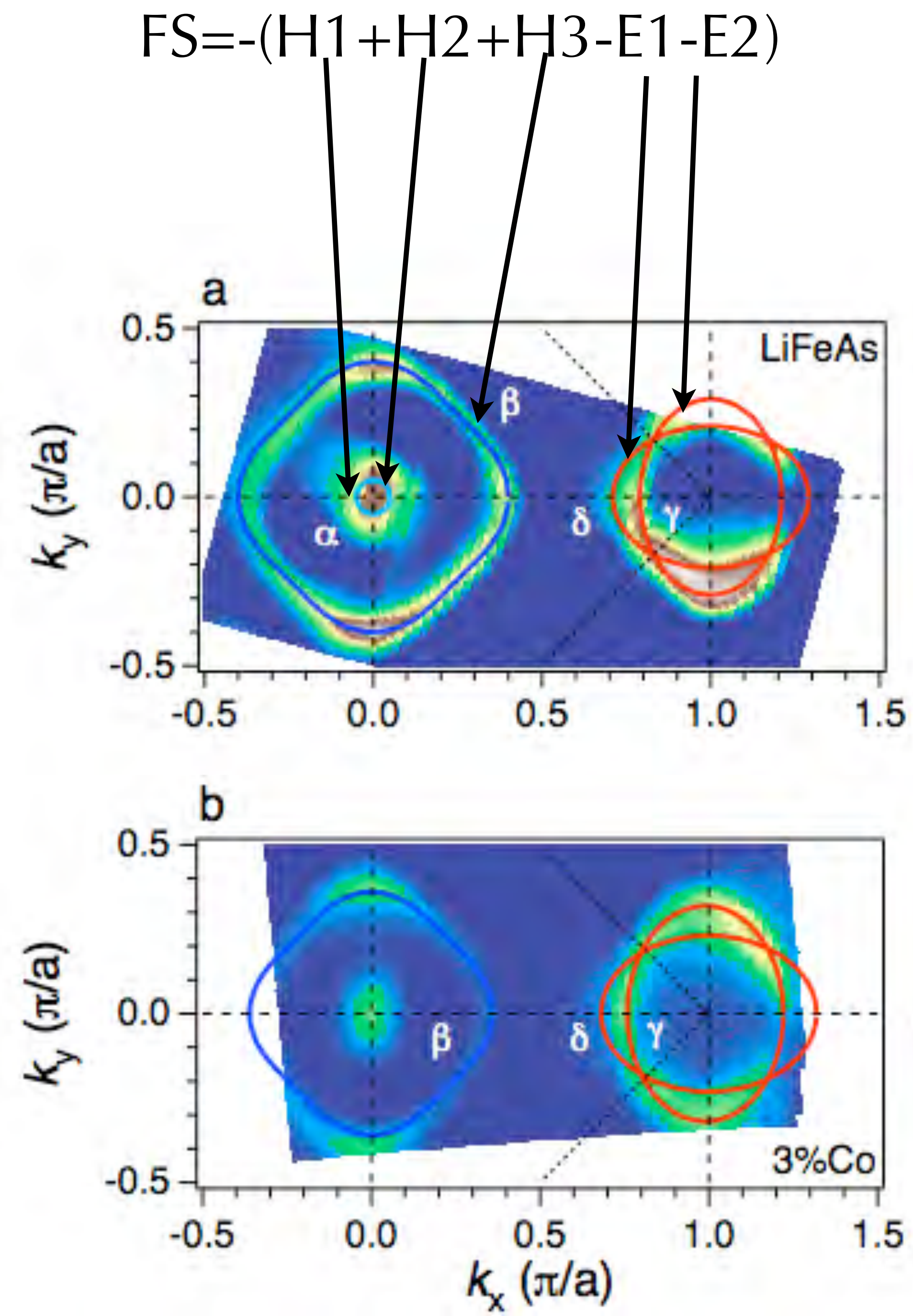
compute FS volume

FS=-(H1+H2+H3-E1-E2)



FS(b)-FS(a)=0.18 ≈ 0.03

compute FS volume



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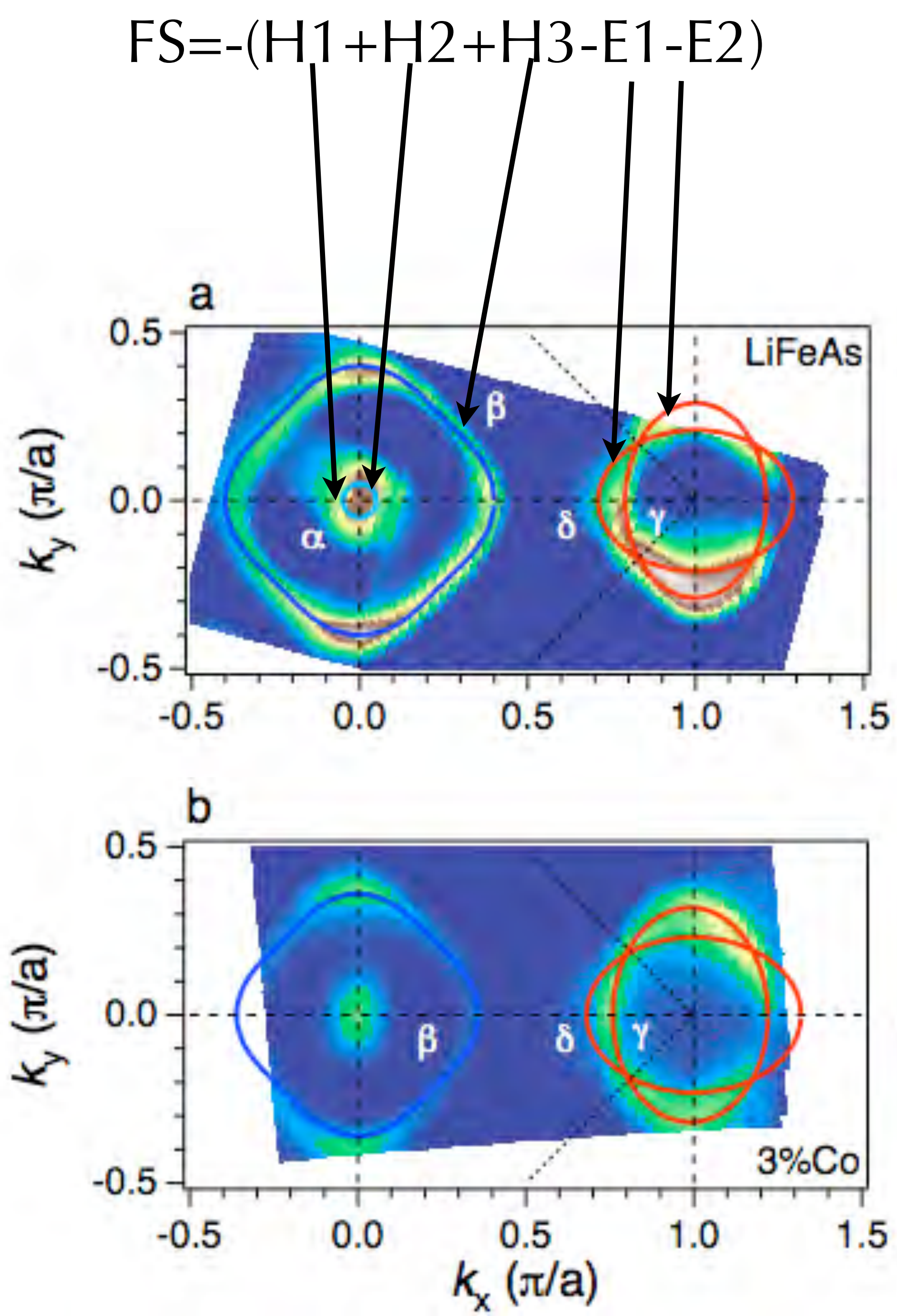
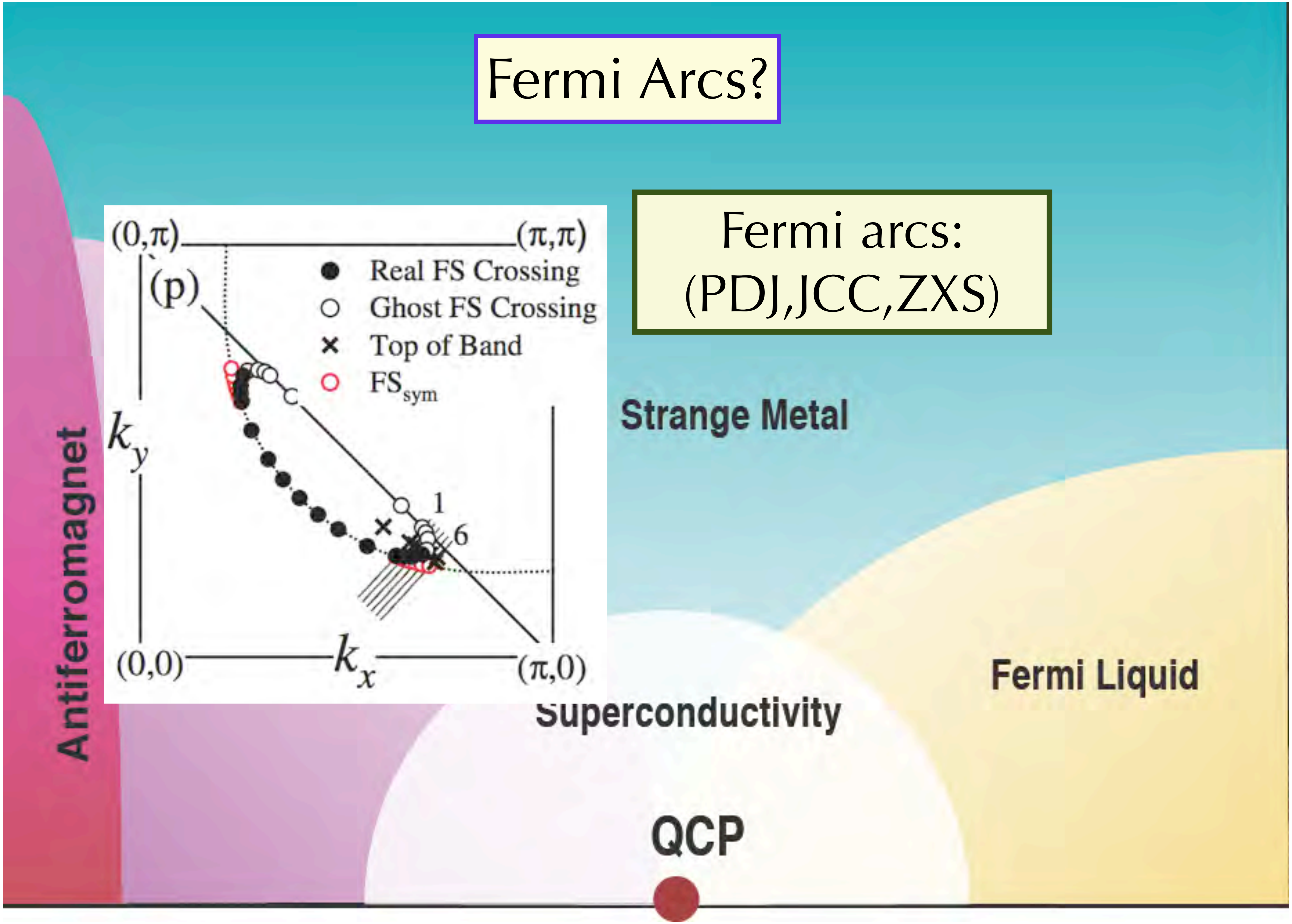


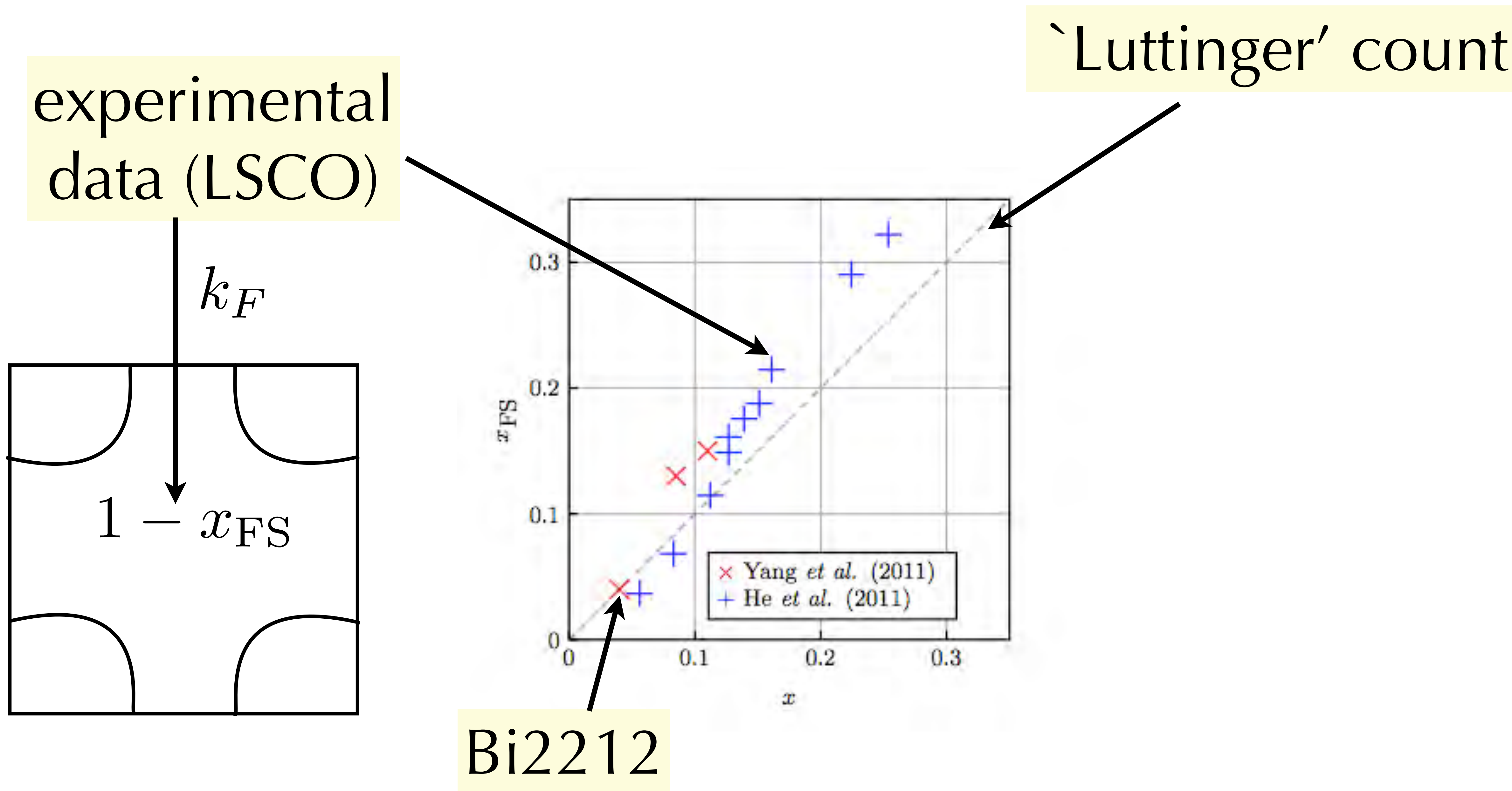
FS(b)-FS(a)=0.1[illegible] ≈ [illegible]03

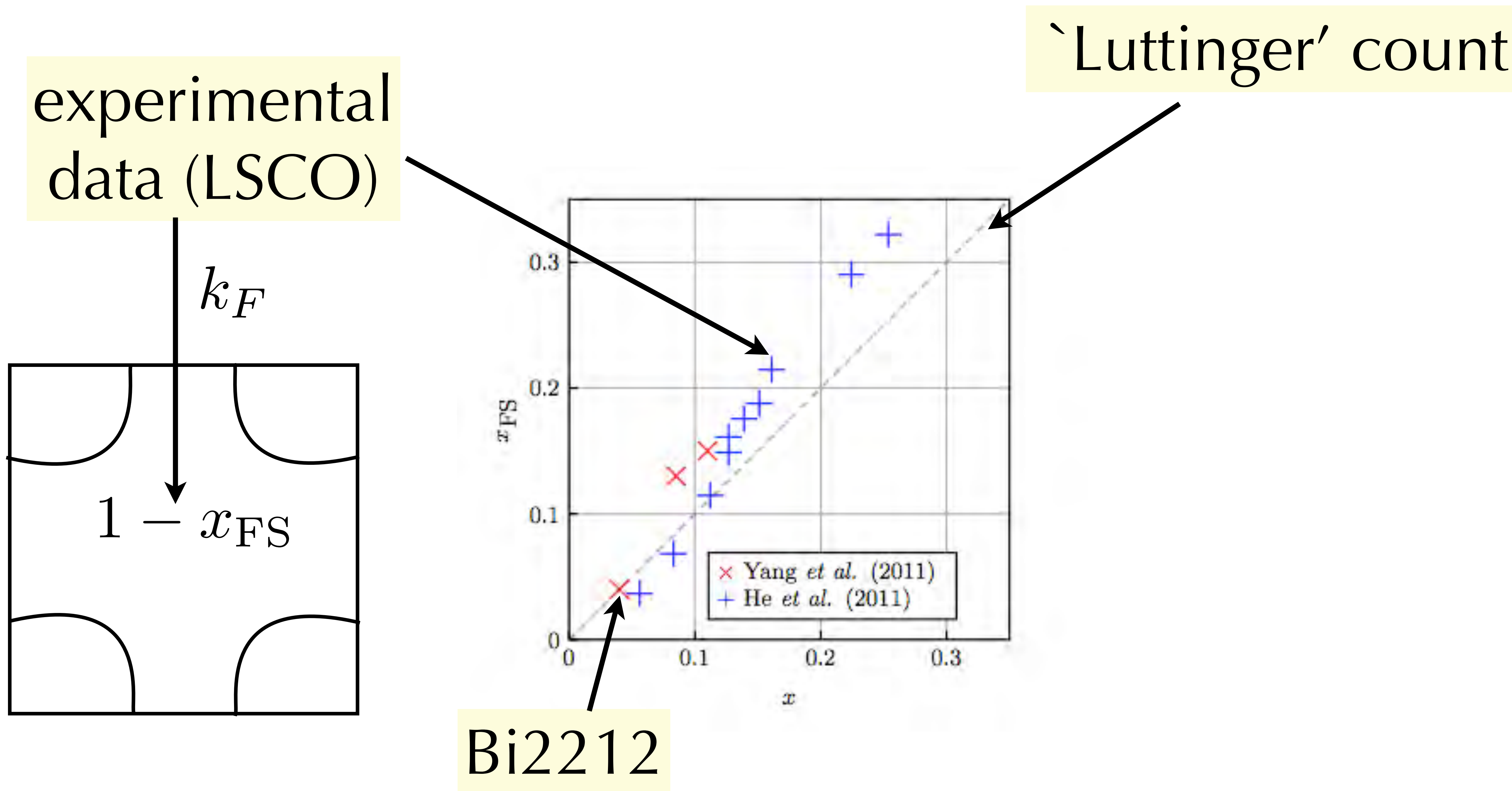
off by a factor of 6

what's the extra stuff?

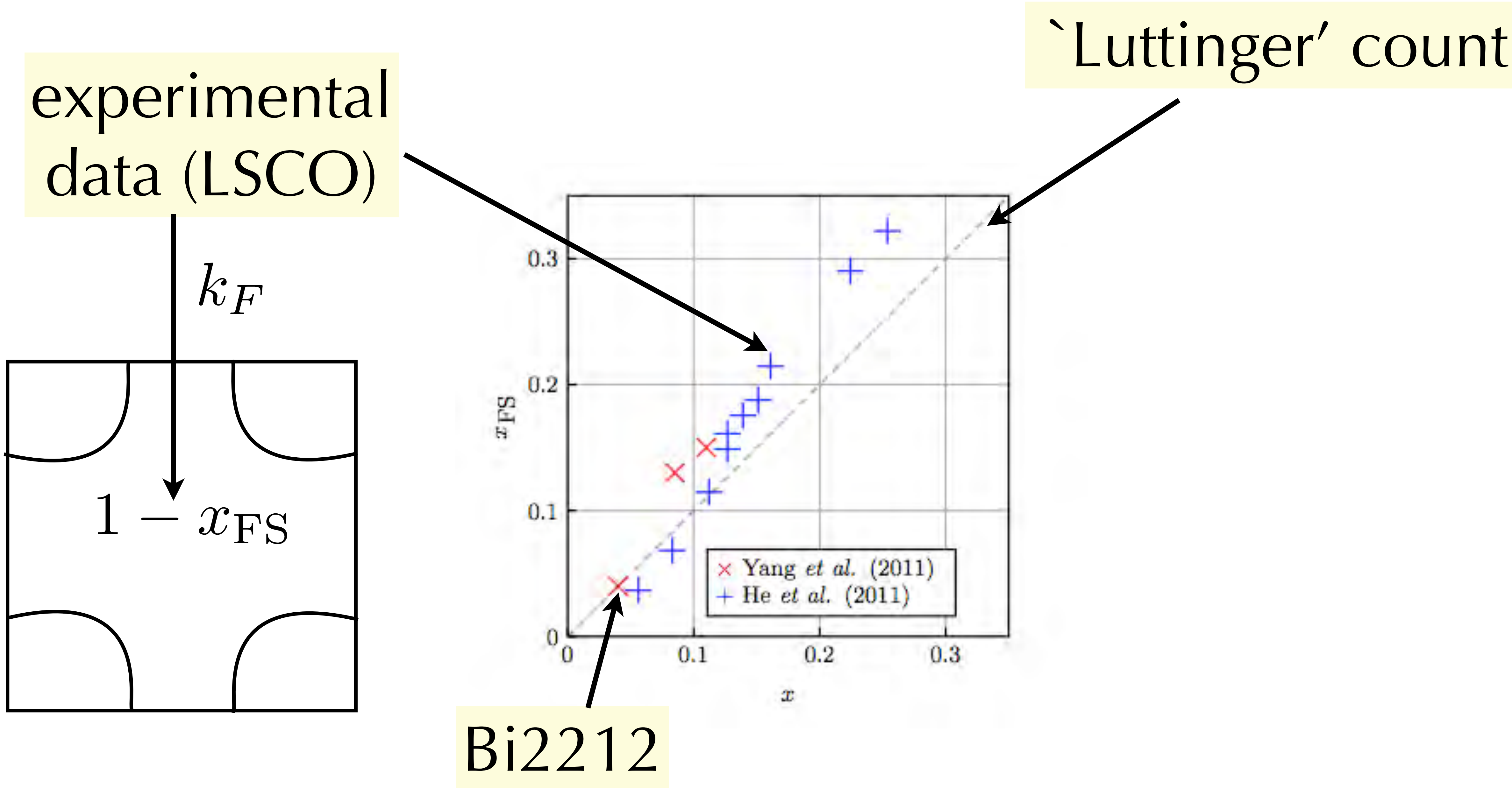


Fermi Arcs?
Fermi arcs:
(PDJ,JCC,ZXS)
(0,π)
(π,π)
(p)
Real FS Crossing
Ghost FS Crossing
Top of Band
FS sym
1
6
k y
(0,0)
k x
(π,0)
Strange Metal
Antiferromagnet
Superconductivity
Fermi Liquid
QCP


experimental data (LSCO)
`Luttinger' count
k_F
$1 - x_{\rm FS}$
0.3
0.2
0.1
0
$x_{\rm FS}$
0.1
0.2
0.3
x
× Yang *et al.* (2011)
+ He *et al.* (2011)
Bi2212



$$x_{\mathrm{FS}} \neq x$$



$$x_{\rm FS} \neq x$$

Luttinger pole-count fails

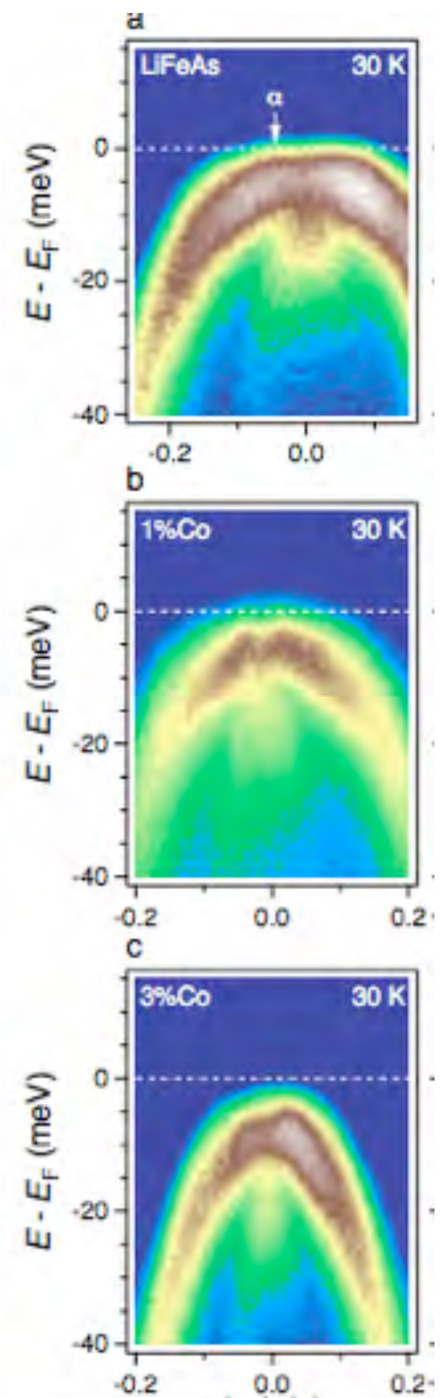
What happens when Luttinger's theorem Fails?

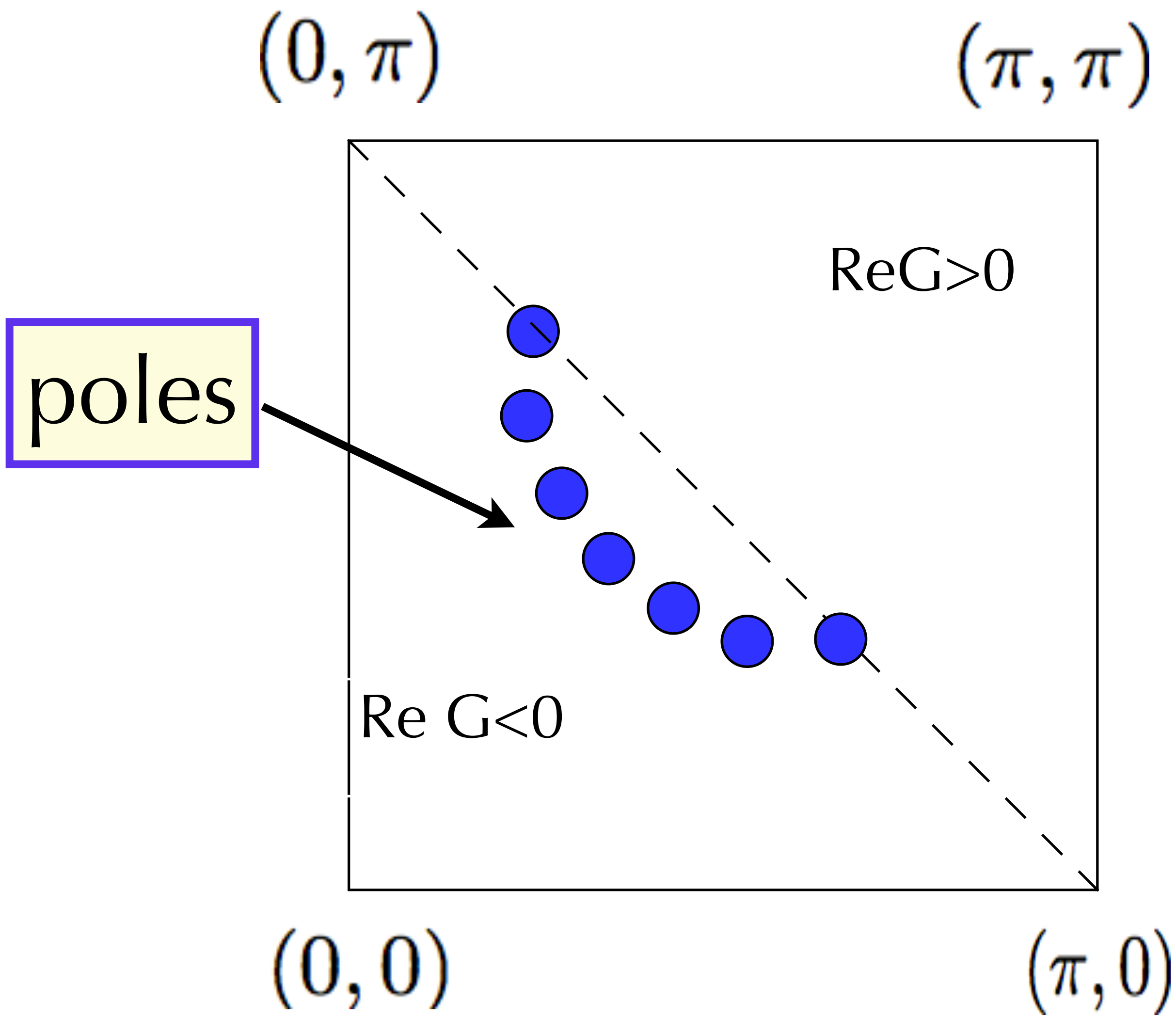
What happens when Luttinger's theorem Fails?

unparticles

what do Fermi arcs and LiFeAs have in common?

H. Ding
1406.0983





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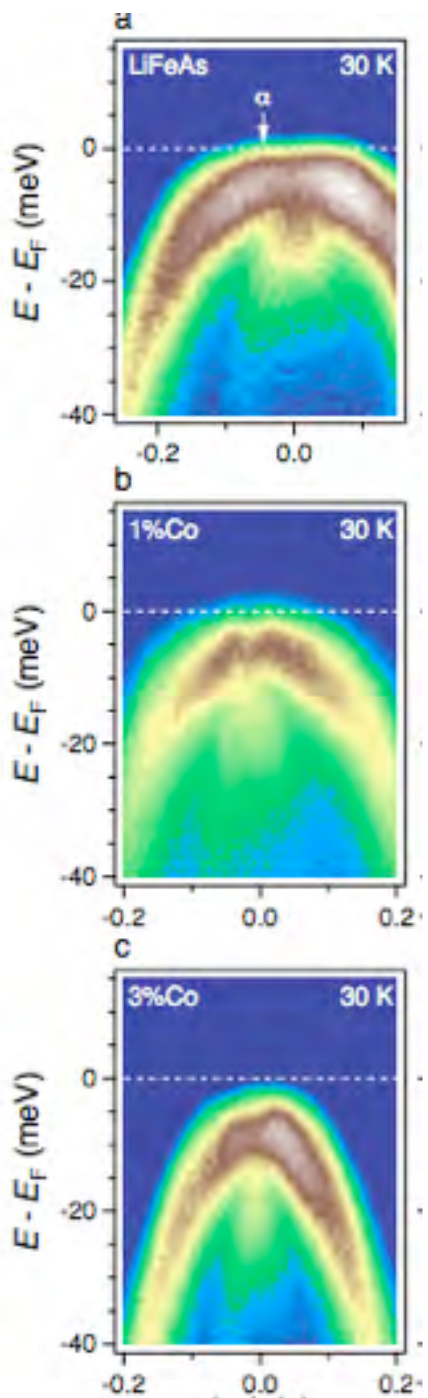
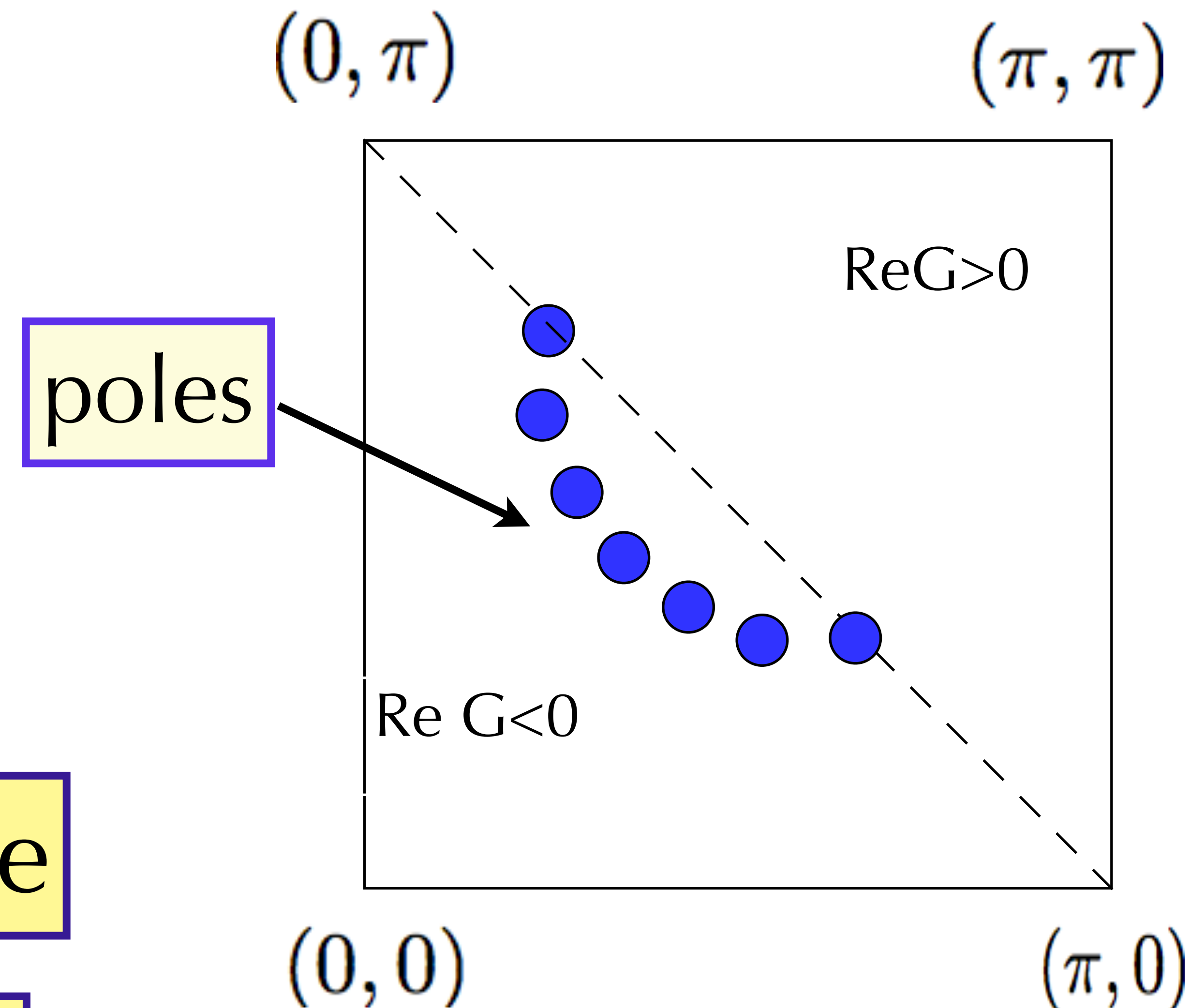


gapped state

vanishing propagator



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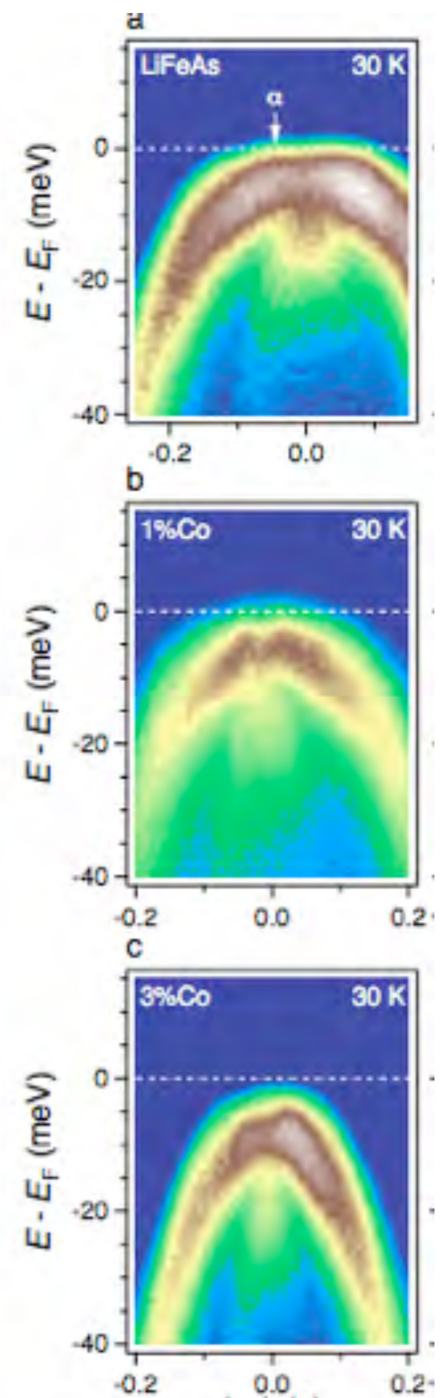

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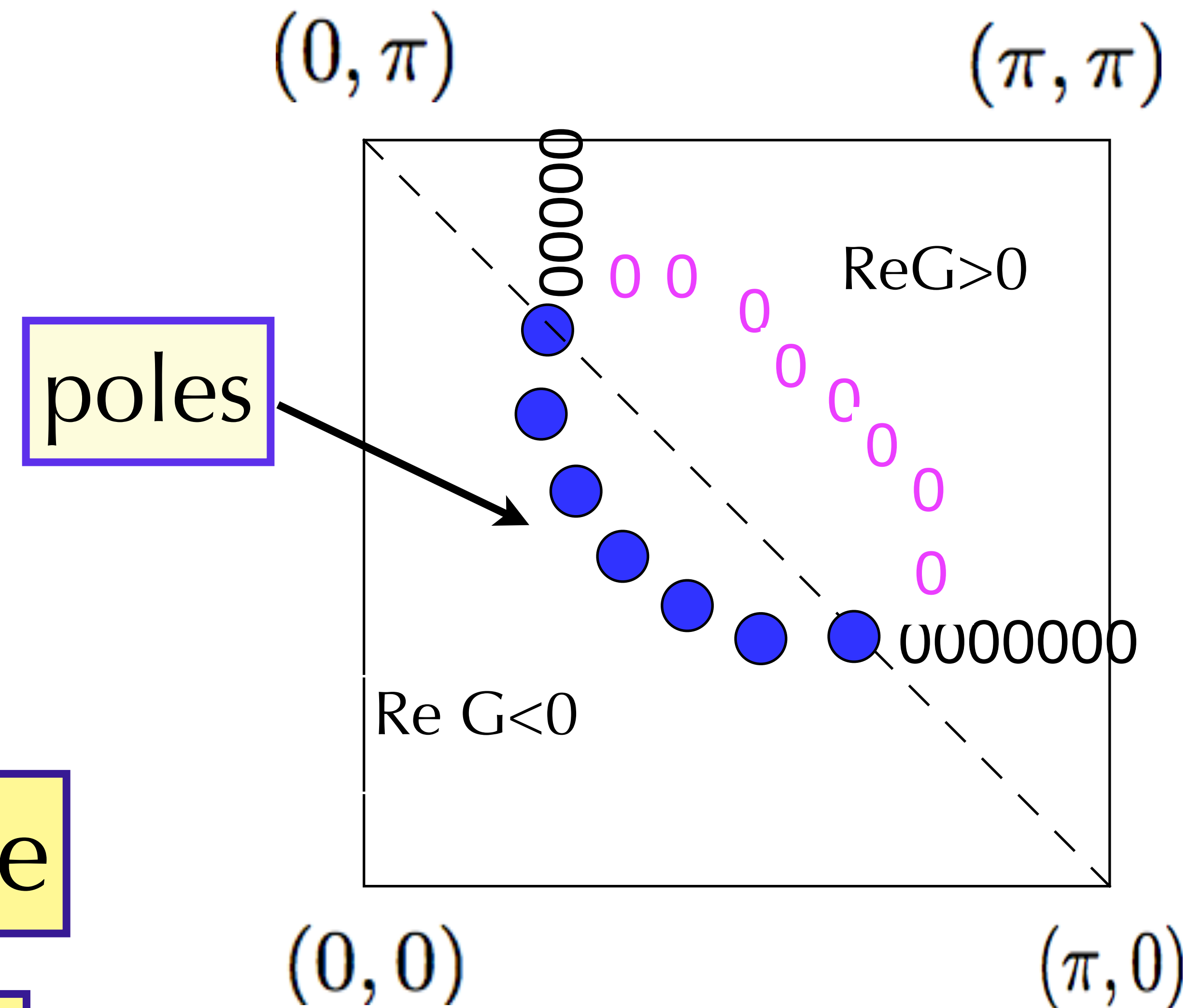
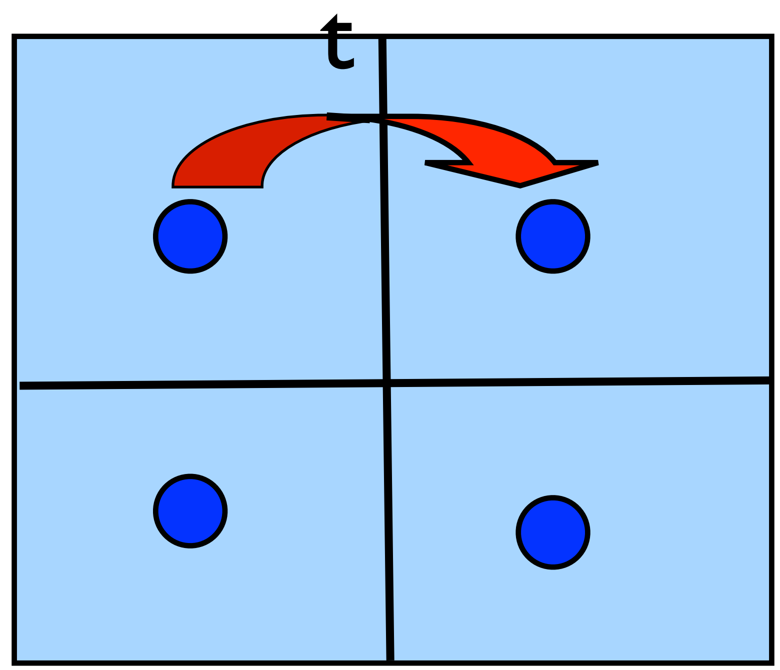


Mott mechanism (not Slater)

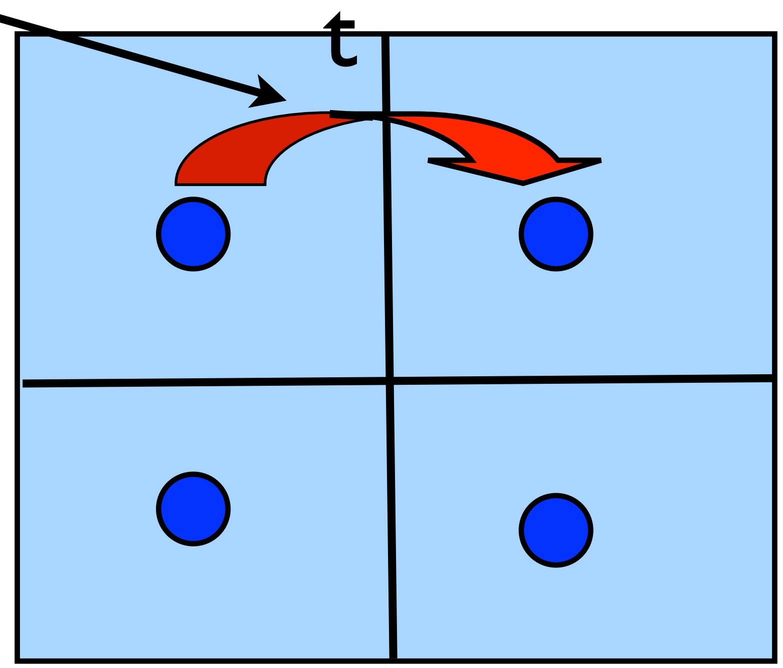


Sir Neville

Mott mechanism (not Slater)

NiO insulates d^8 ?

perhaps this costs energy



Sir Neville

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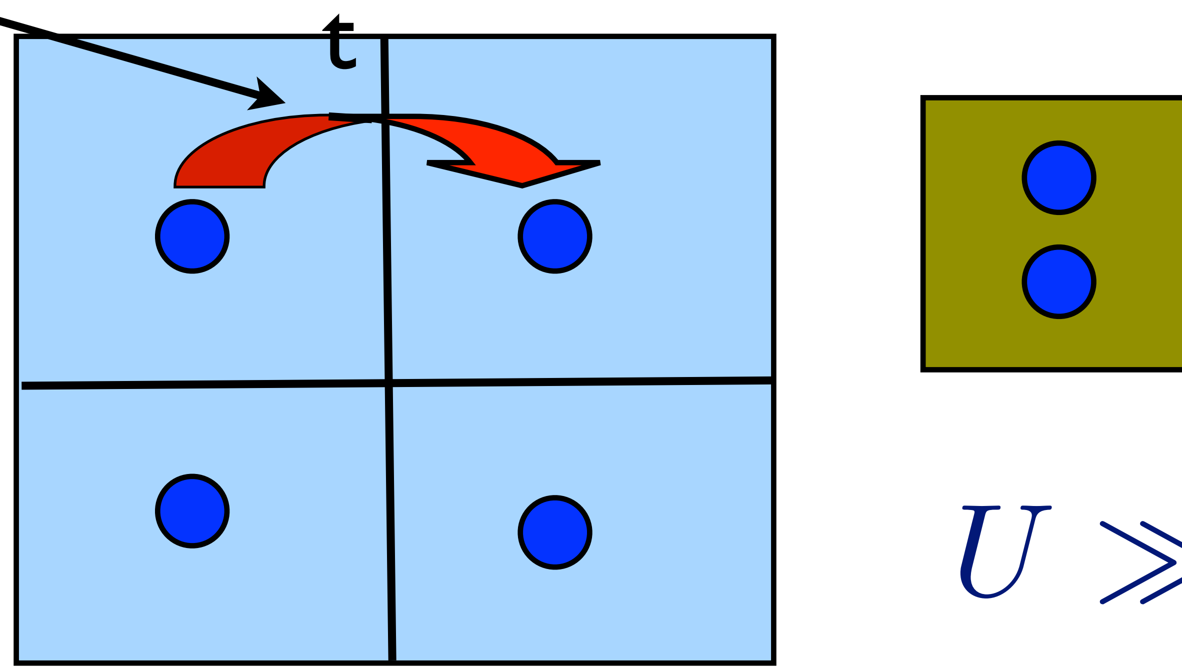
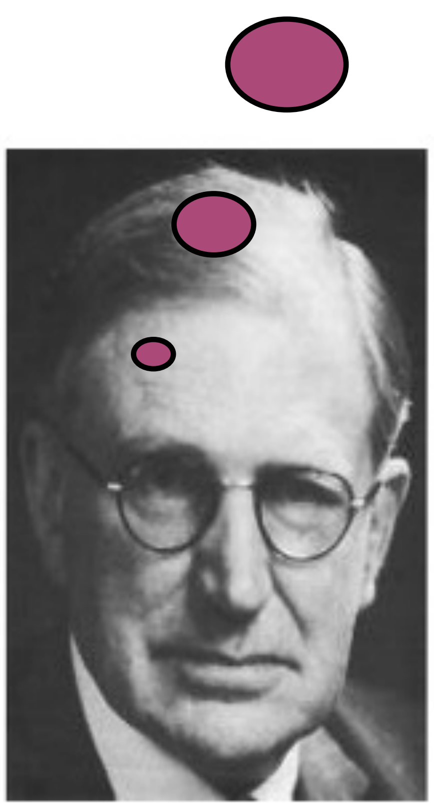


$$U \gg t$$



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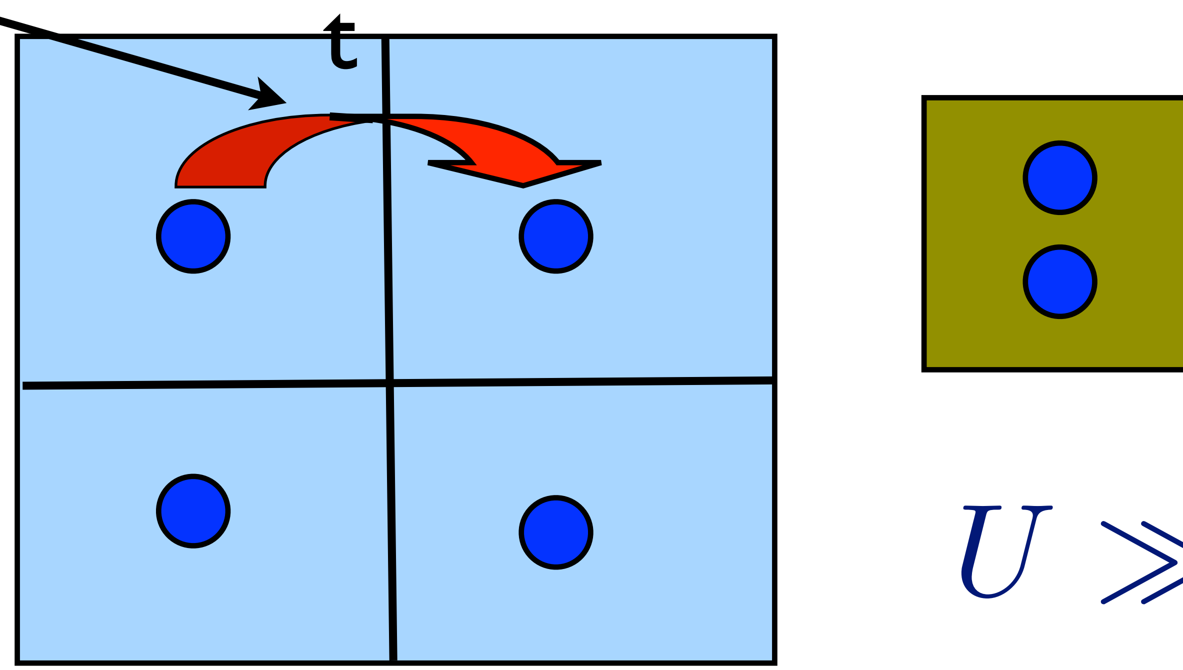

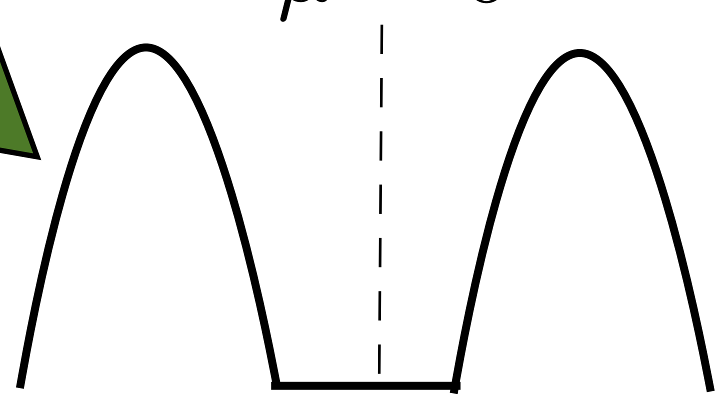

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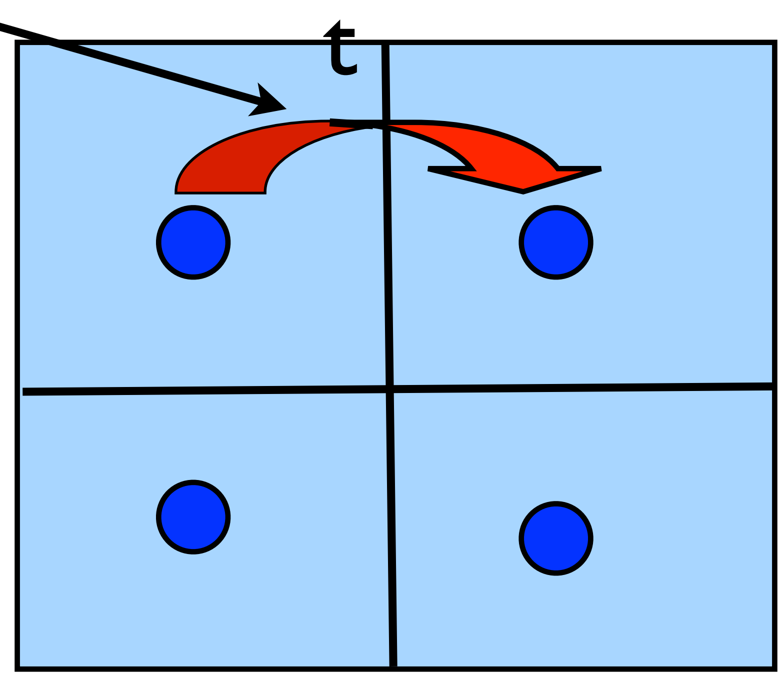

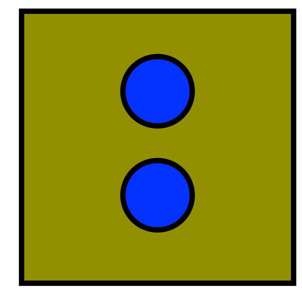

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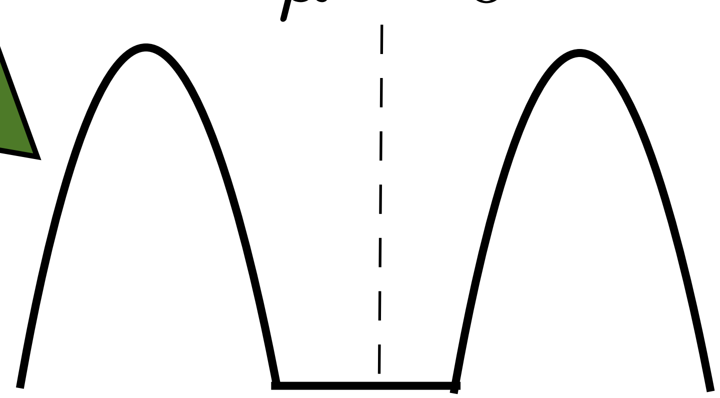


no change in
size of
Brillouin zone

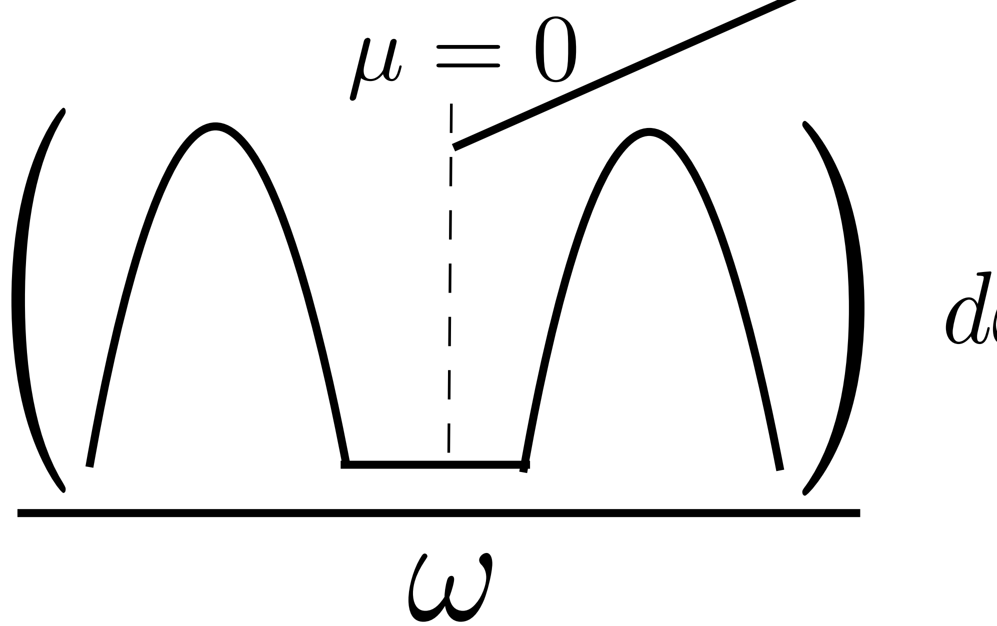
Sir Neville

Mott Problem

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty}$$

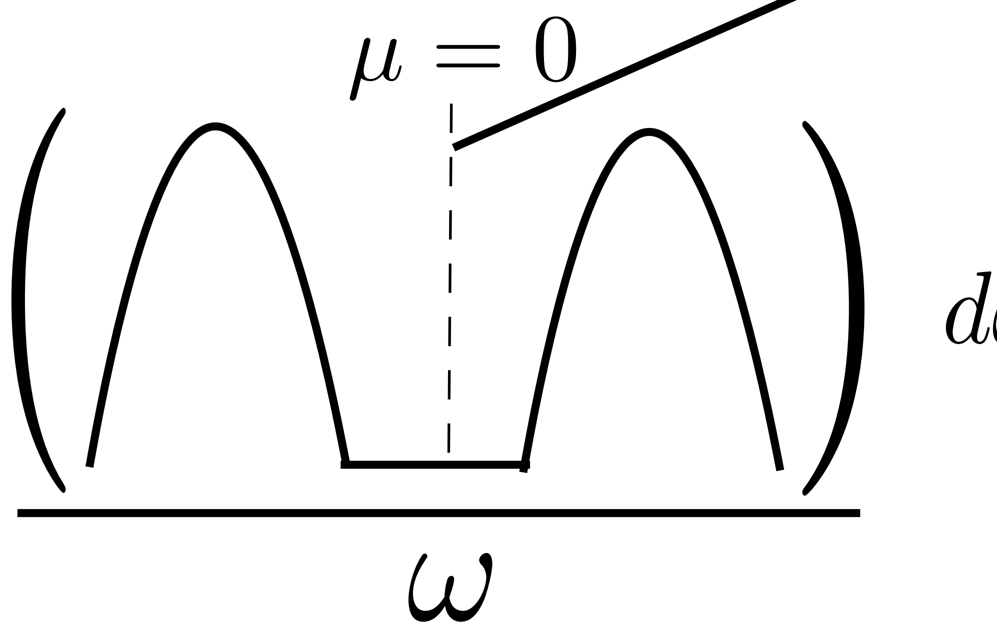


$$d\omega$$

Kramers
-Kronig

Mott Problem

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \left(\quad \right) d\omega$$



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Kramers
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= below gap+above gap

Mott Problem

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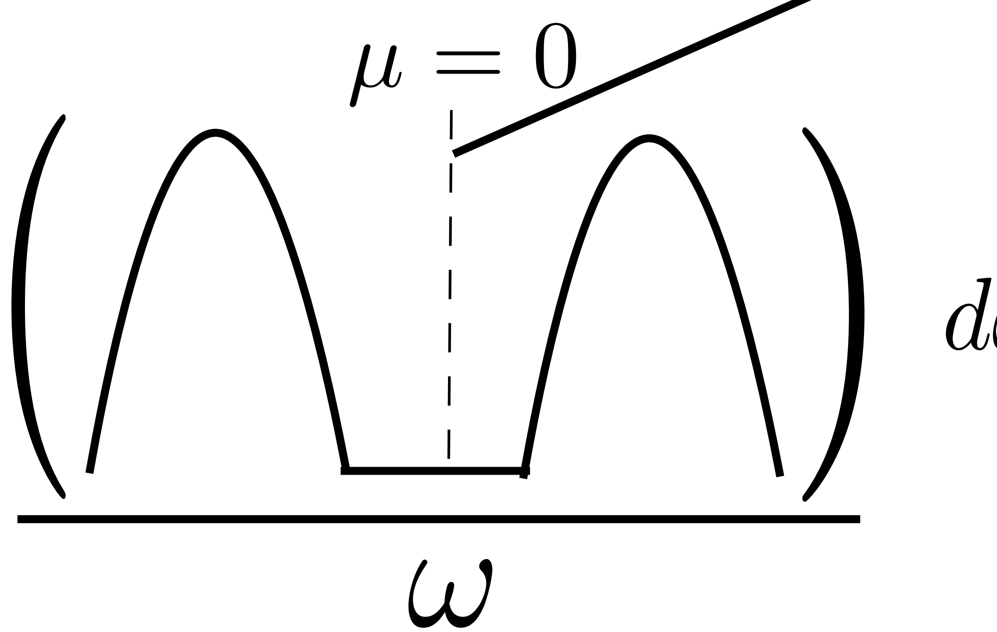
$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty}$$



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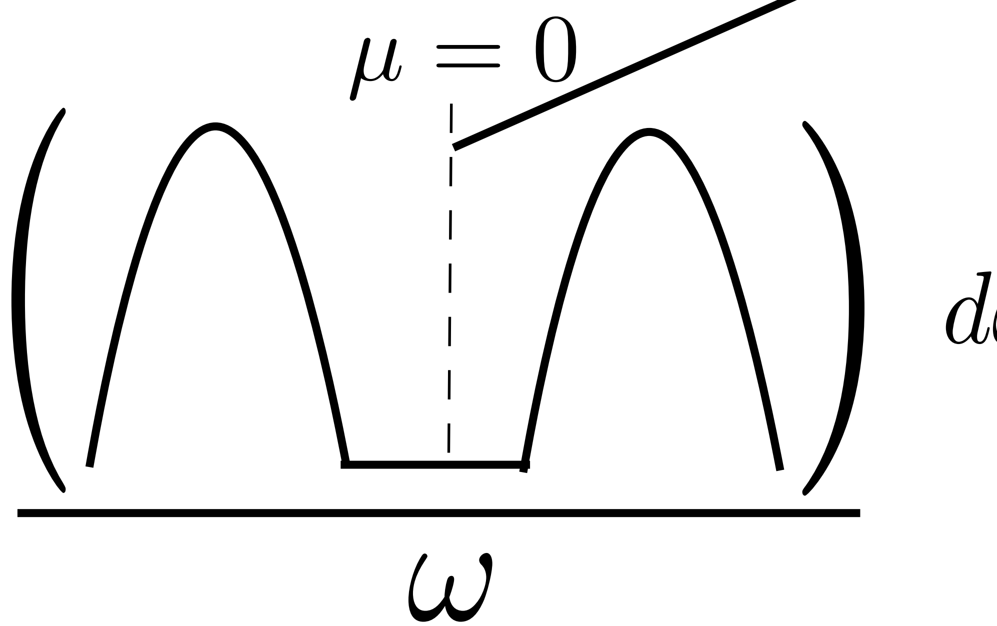


$d\omega$

Kramers -Kronig

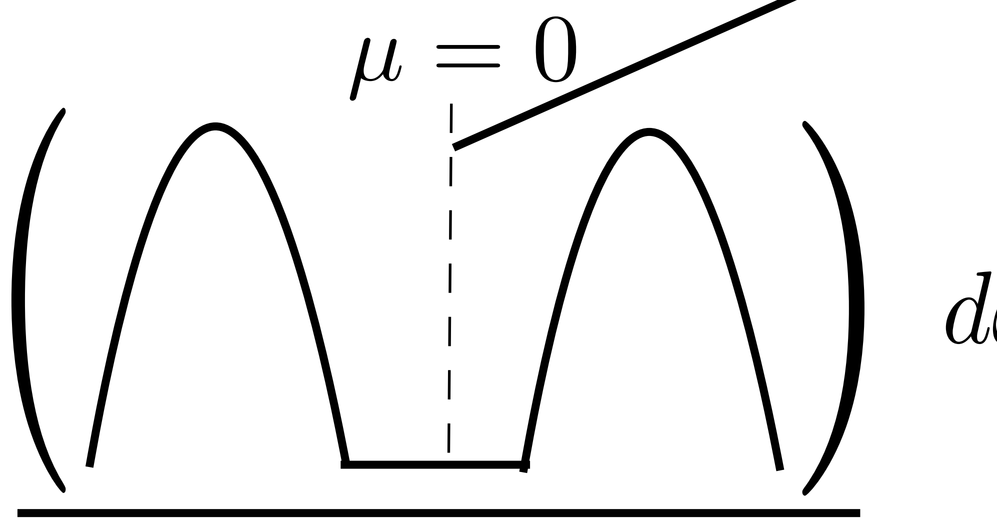
= below gap+above gap = 0

$$\mathrm{Det}G(\mathbf{k}, \omega = \mathbf{0}) = \mathbf{0}$$ (single band)

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty} \frac{(\ldots)}{\omega}\, d\omega$$



Kramers -Kronig

= below gap+above gap = 0

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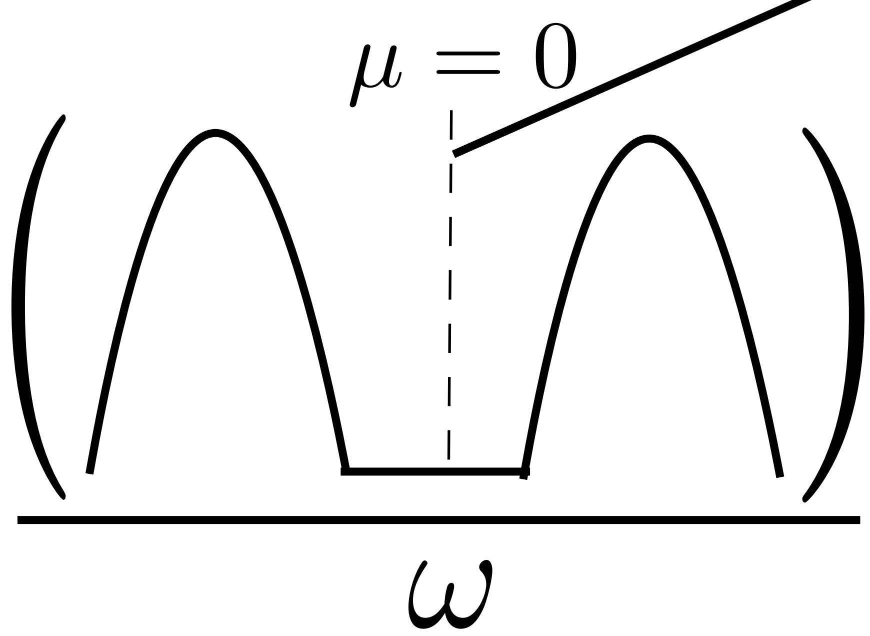
DetReG(0,p)=0 Mottness

Mott Problem

Im G=0

$$\mathrm{Re}G(0,p) = \int_{-\infty}^{\infty}$$



$$d\omega$$

Kramers -Kronig

= below gap+above gap = 0

$\mathrm{Det}G(\mathbf{k},\omega=\mathbf{0})=\mathbf{0}$ (single band)

DetReG(0,p)=0 Mottness not true in MF theories

Luttinger theorem: singularities of $\ln G$

$$n = \frac{2i}{(2\pi)^{d+1}} \int d^d\mathbf{p} \int_{-\infty}^{0} d\xi \ln \frac{G^R(\xi, \mathbf{p})}{G_R^*(\xi, \mathbf{p})}$$

Luttinger theorem: singularities of $\ln G$

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$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

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poles+zeros
(all sign changes)

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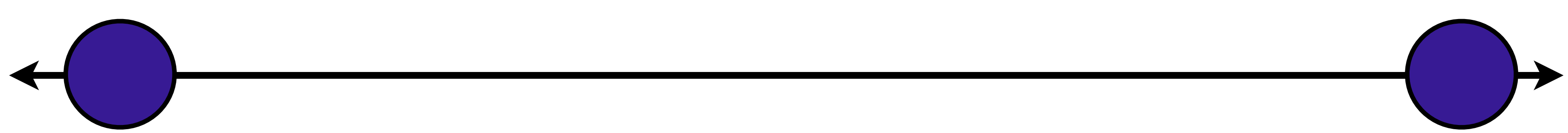

Fermi Liquids

Mott Insulators

PHYSICAL REVIEW B **68**, 085113 (2003)

Some consequences of the Luttinger theorem: The Luttinger surfaces in non-Fermi liquids and Mott insulators

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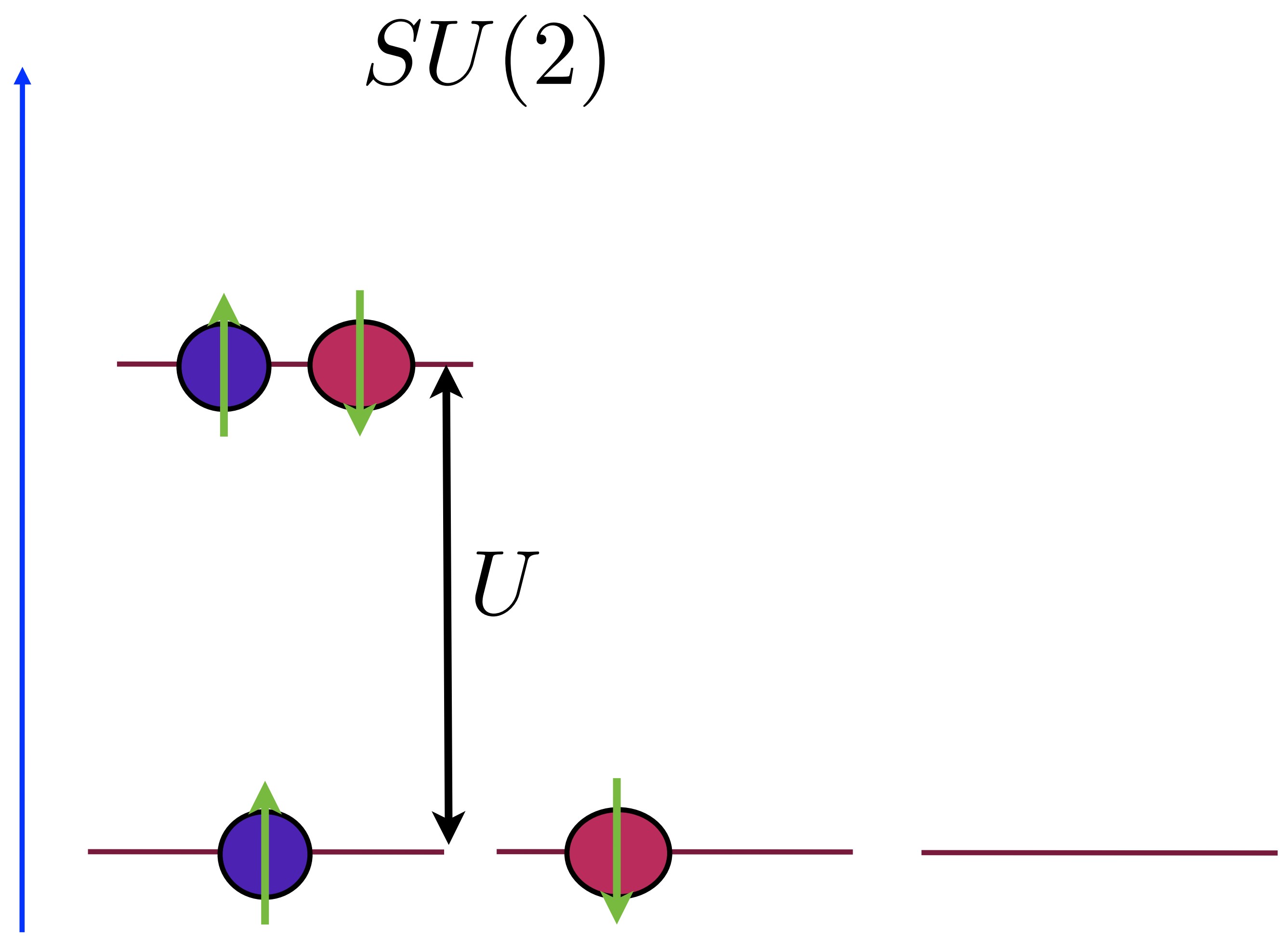
(Received 30 January 2003; published 27 August 2003)

The proof just presented is good for any state of our system: FL, NFL, or MI [in other words, either of poles or zeros can be used to change the sign of G_r in Eq. (1)]. The only way to incapacitate the Luttinger theorem in form (1) is to assume that the limit $T \to 0$ is discontinuous. Actually, one has to require that the whole line $T=0$ is a line of phase transitions.

also, pw anderson, tm Rice, Tsvelik,etc.

Is this famous theorem from 1960 correct?

simple problem: n=1



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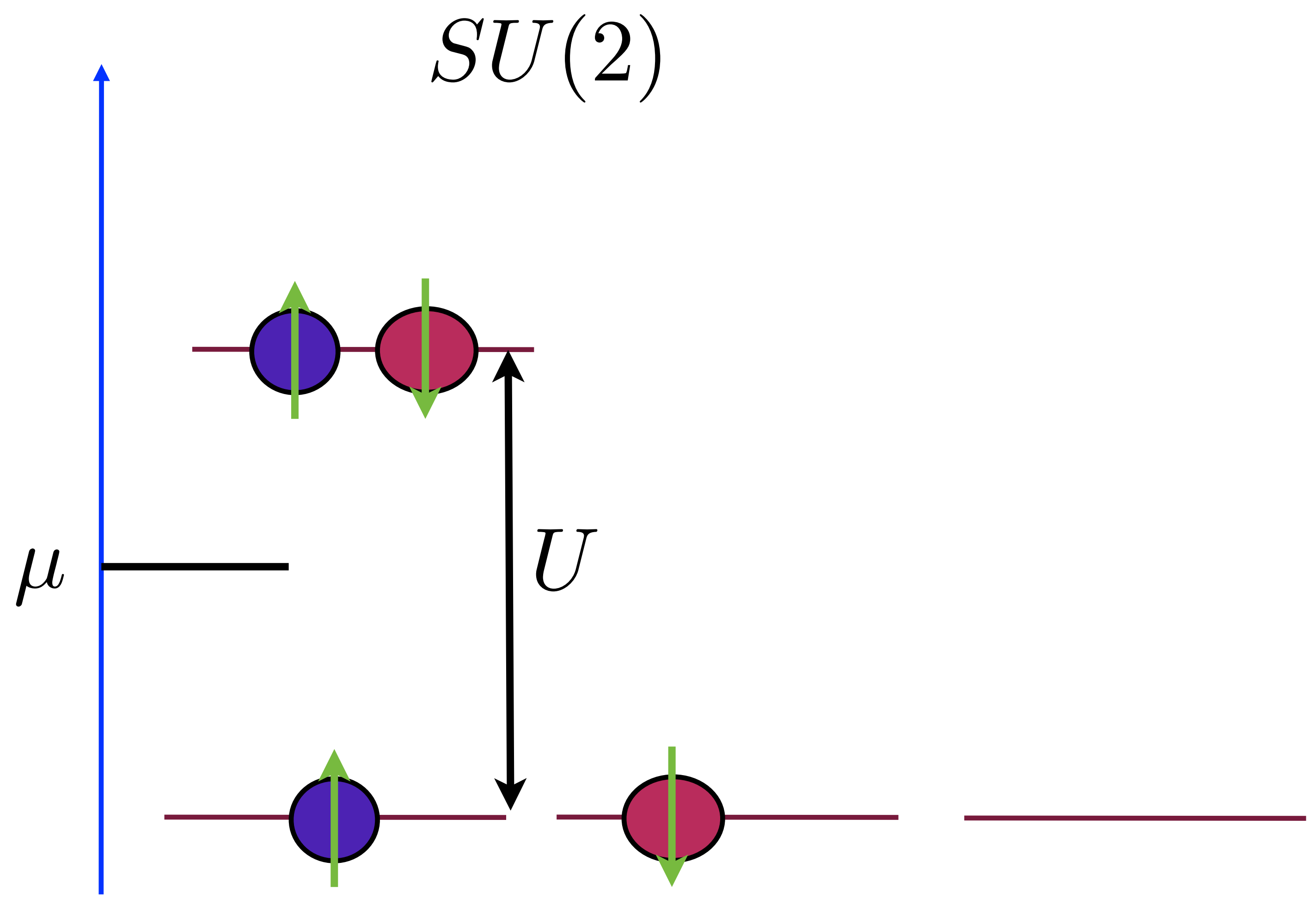

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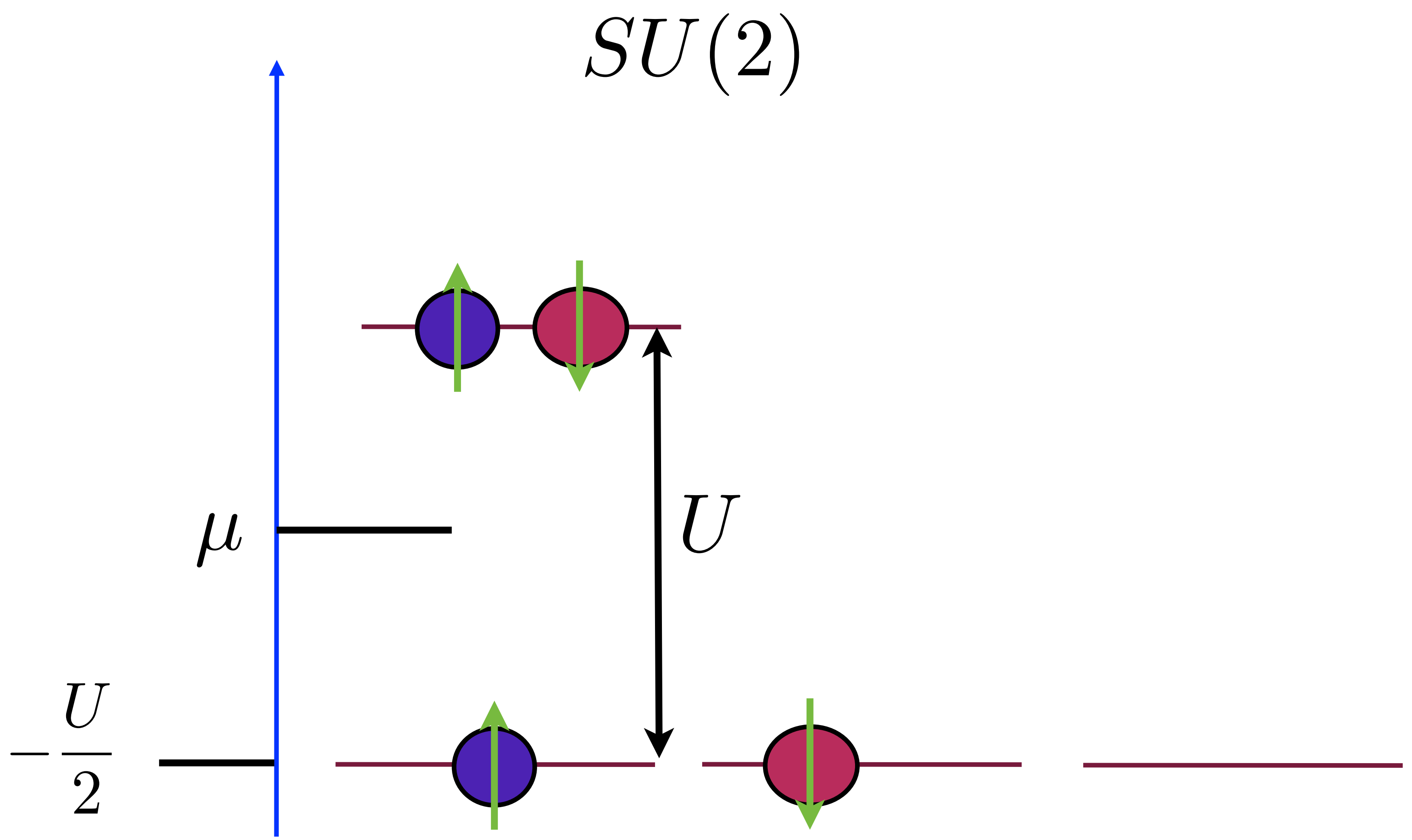

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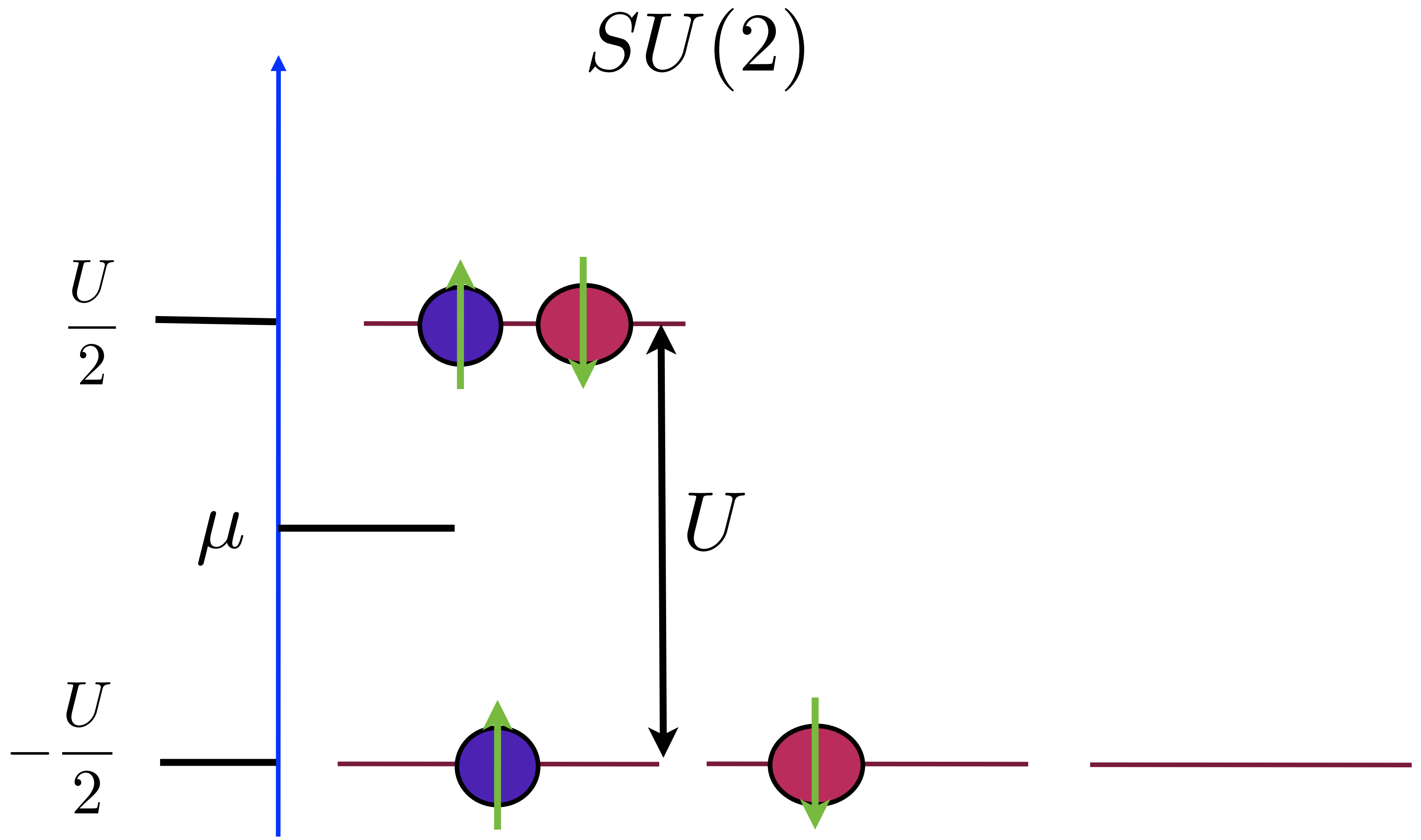
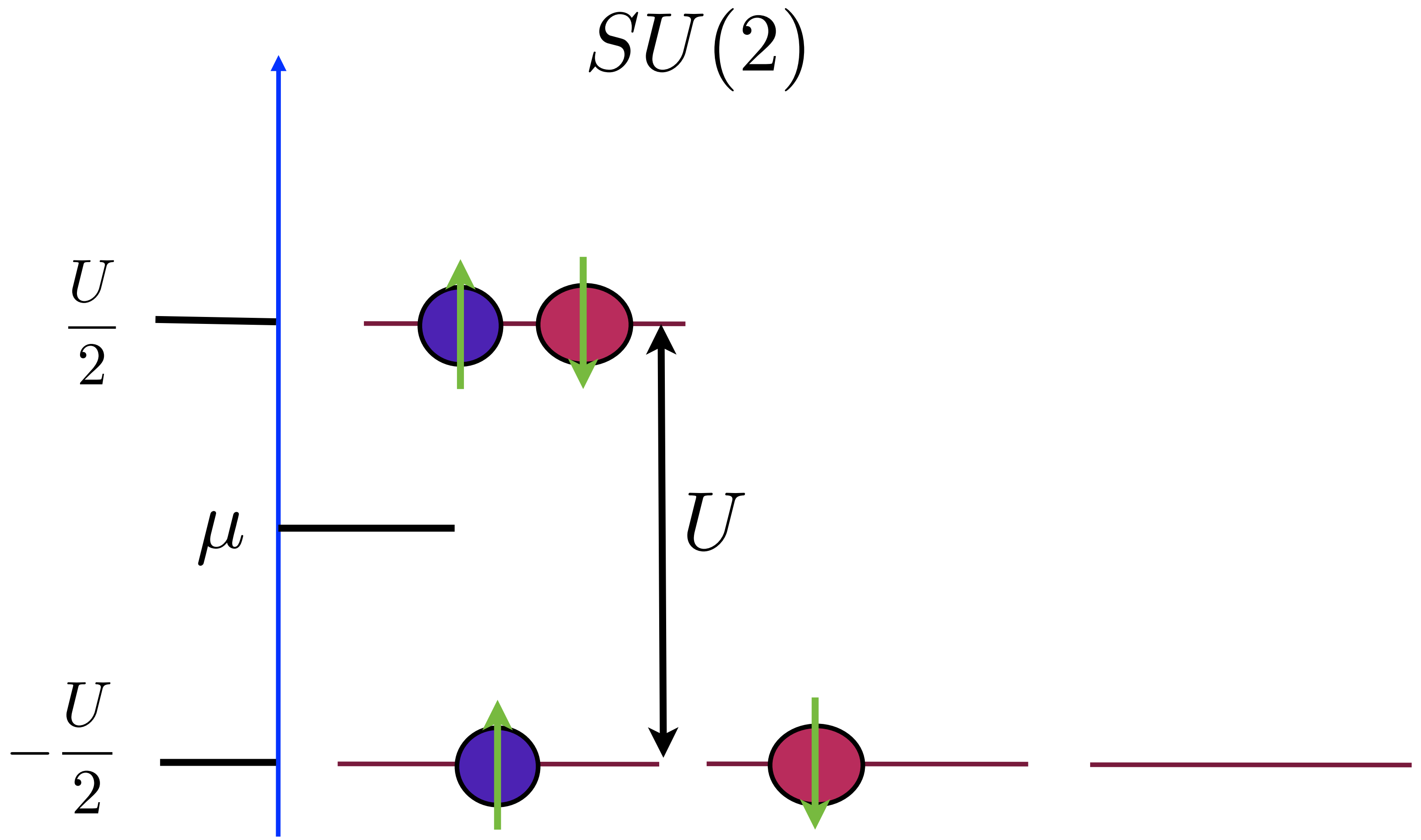
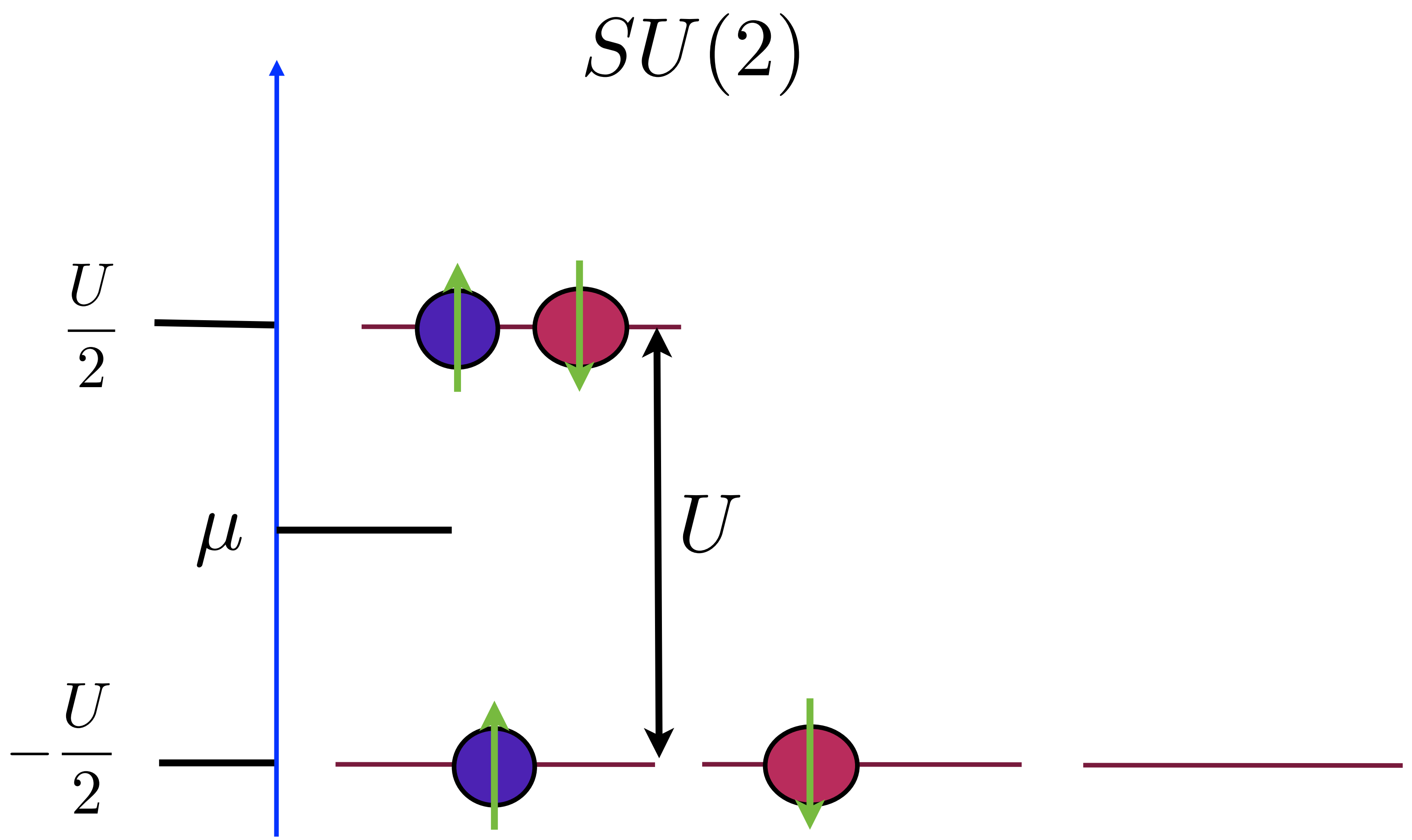


simple problem: n=1



$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2}$$

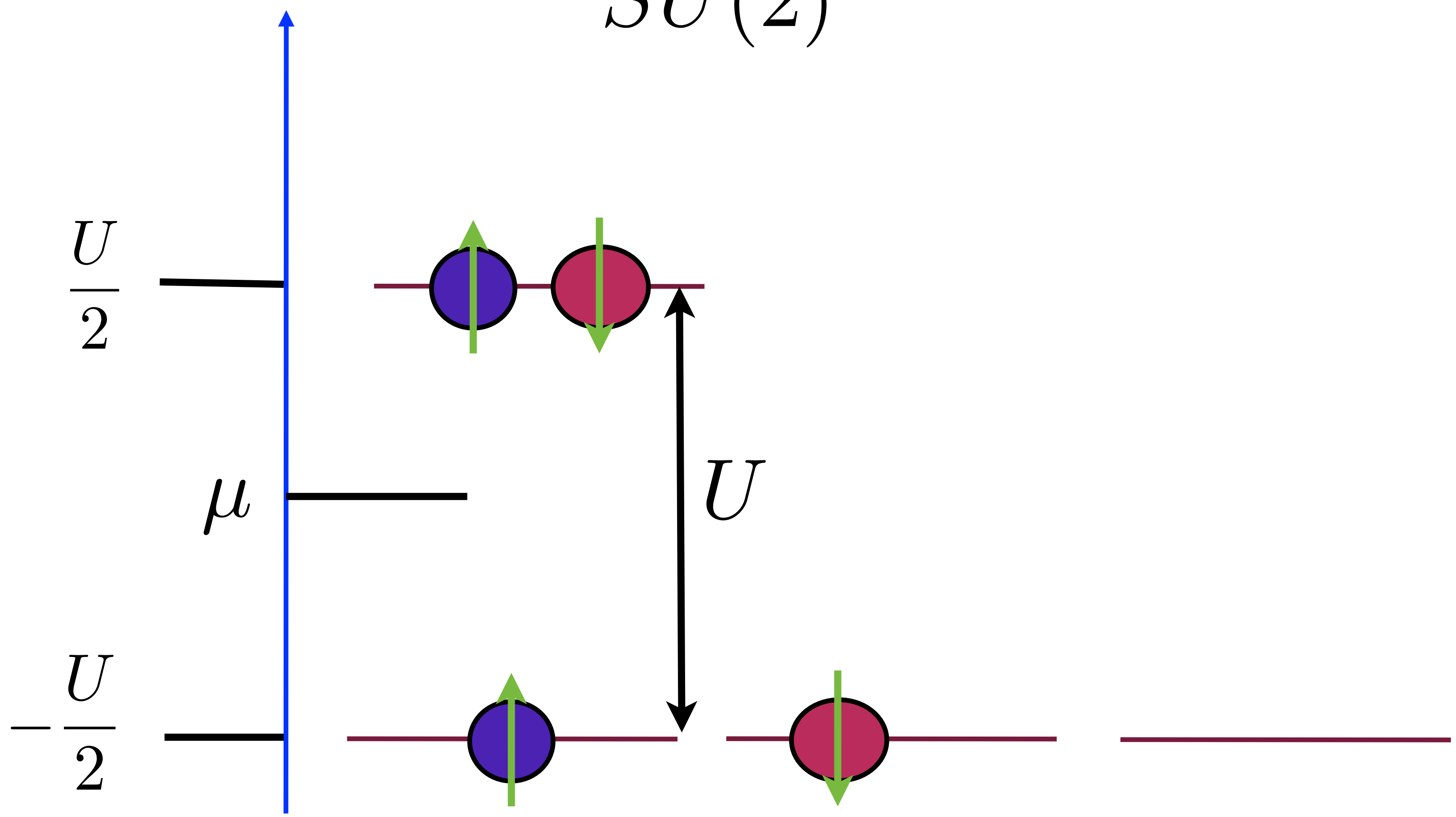
simple problem: n=1



$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

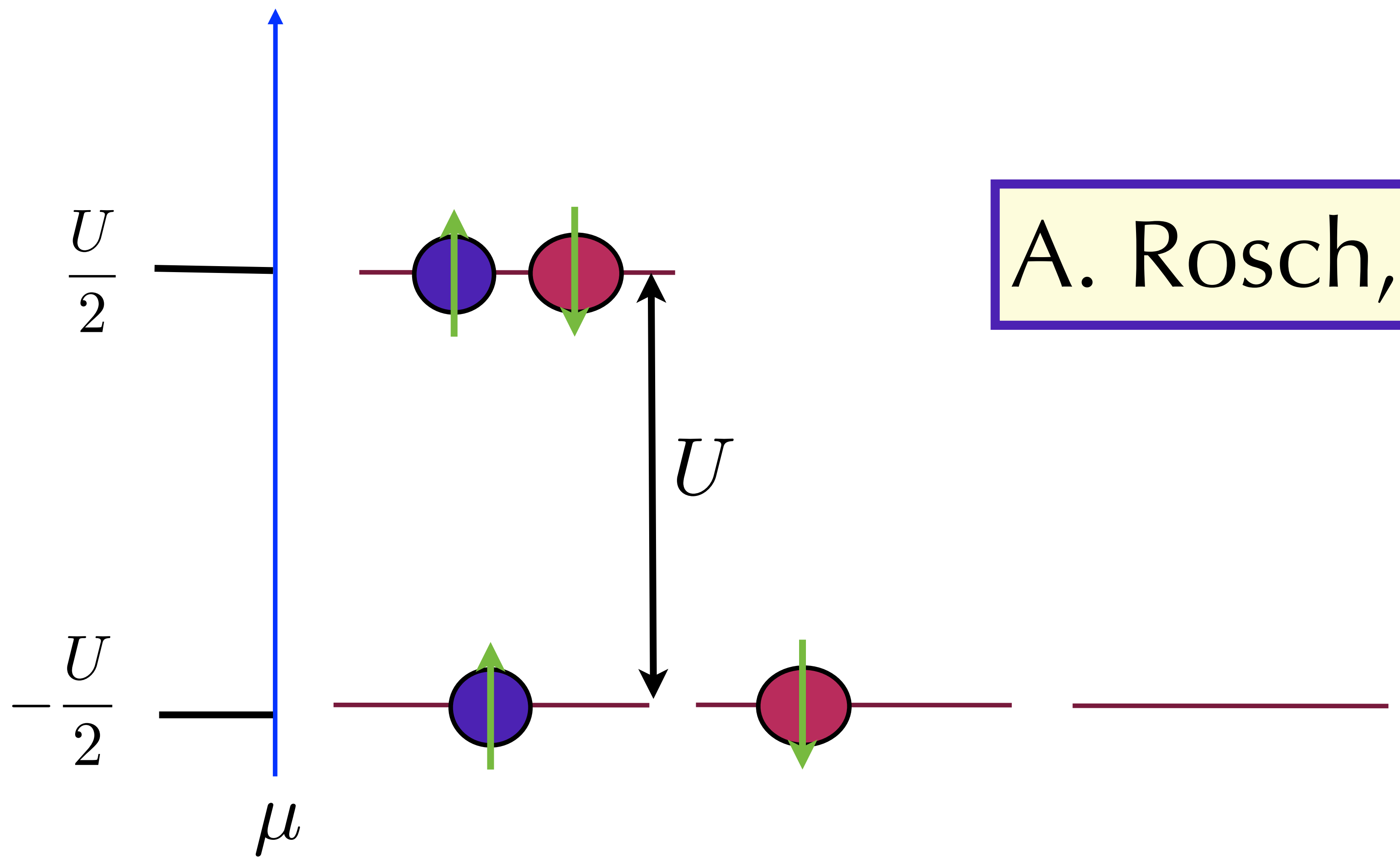
simple problem: n=1

$SU(2)$



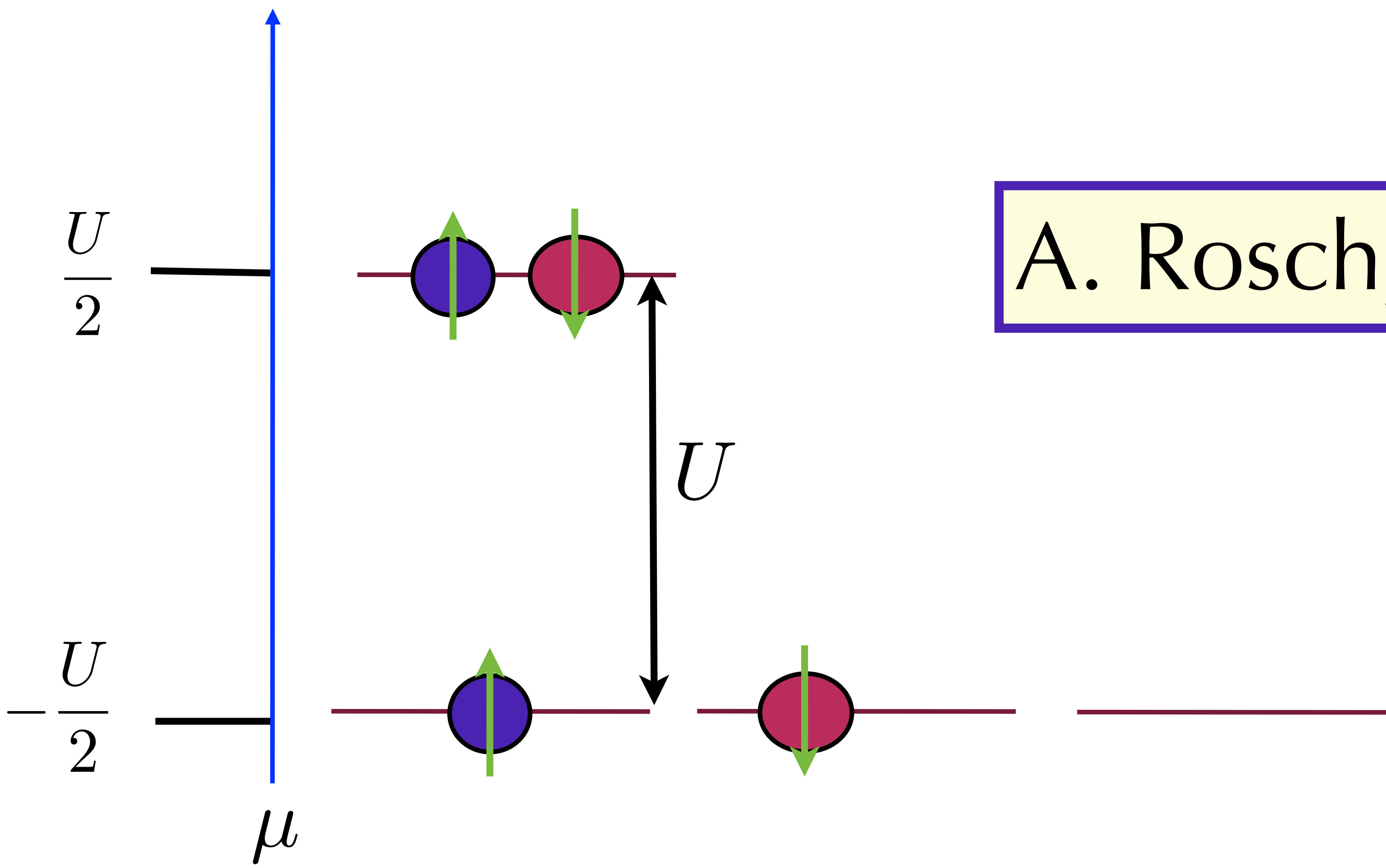
$$G = \frac{1}{\omega + U/2} + \frac{1}{\omega - U/2} = 0 \quad \text{if} \quad \omega = 0$$

$$n = 2\theta(0) = 1$$



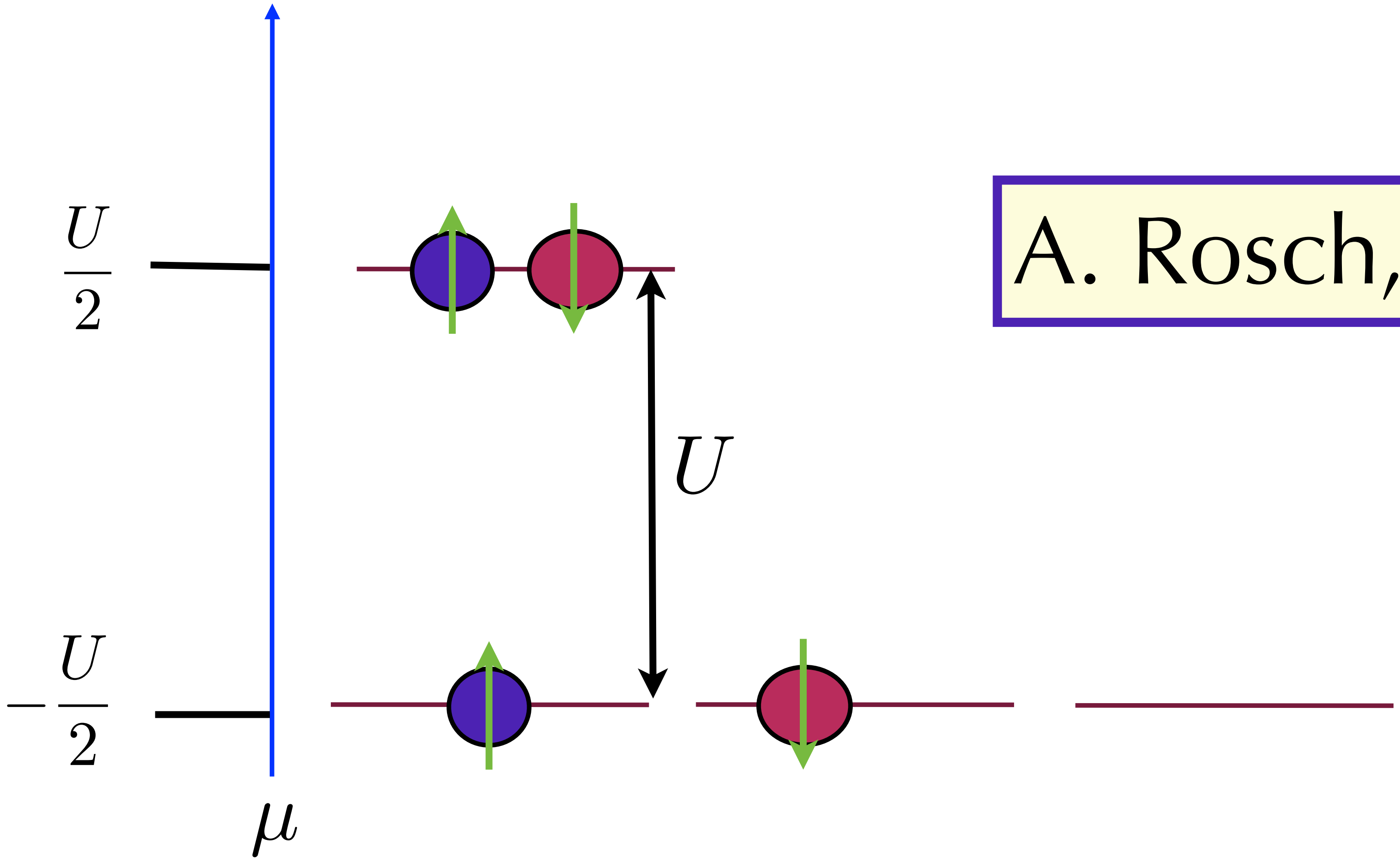
A. Rosch, 2007

$$G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



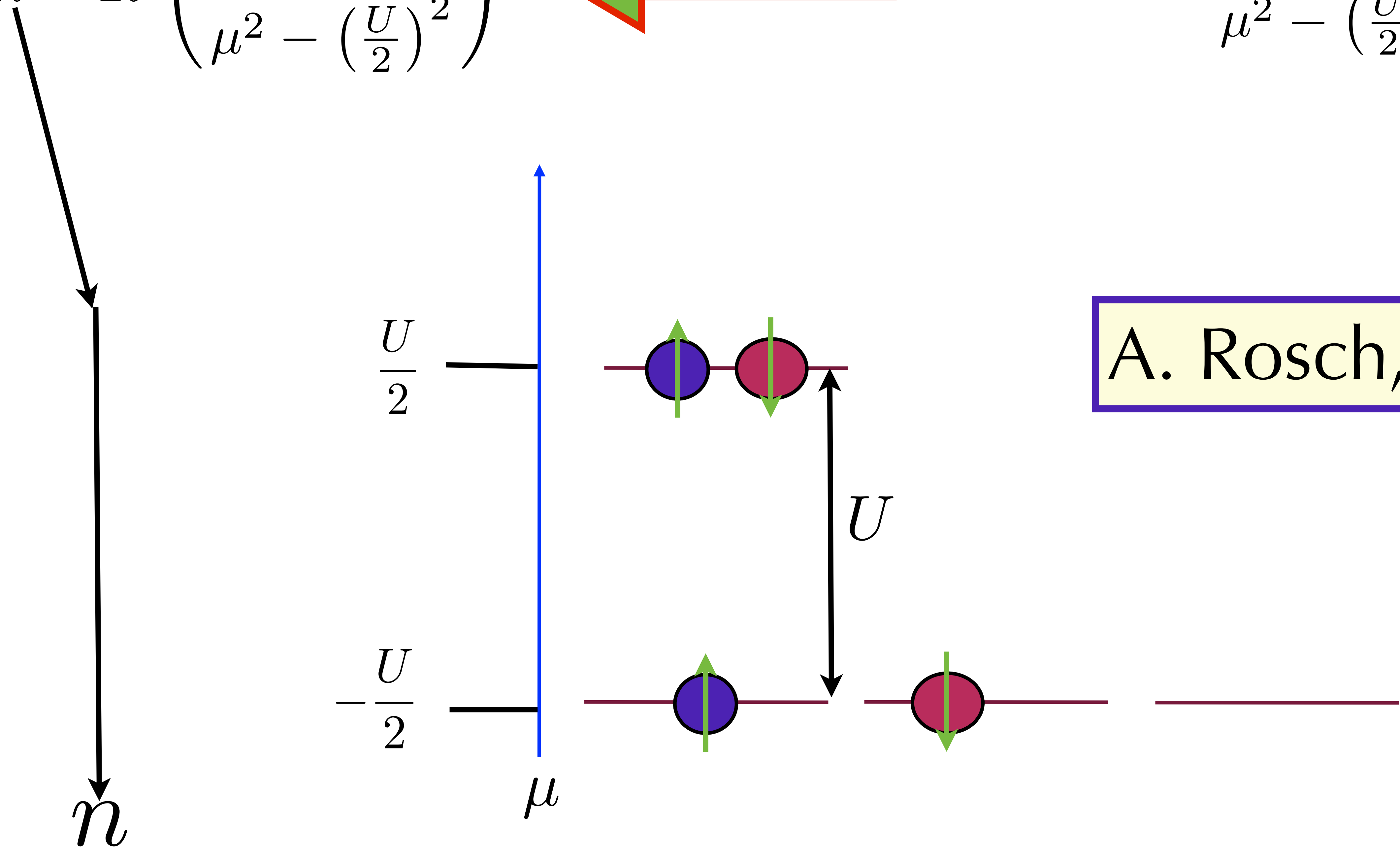
A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \Longleftarrow G(\omega = 0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



A. Rosch, 2007

$$n = 2\theta\left(\frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}\right) \longleftarrow G(\omega=0) = \frac{2\mu}{\mu^2 - \left(\frac{U}{2}\right)^2}$$



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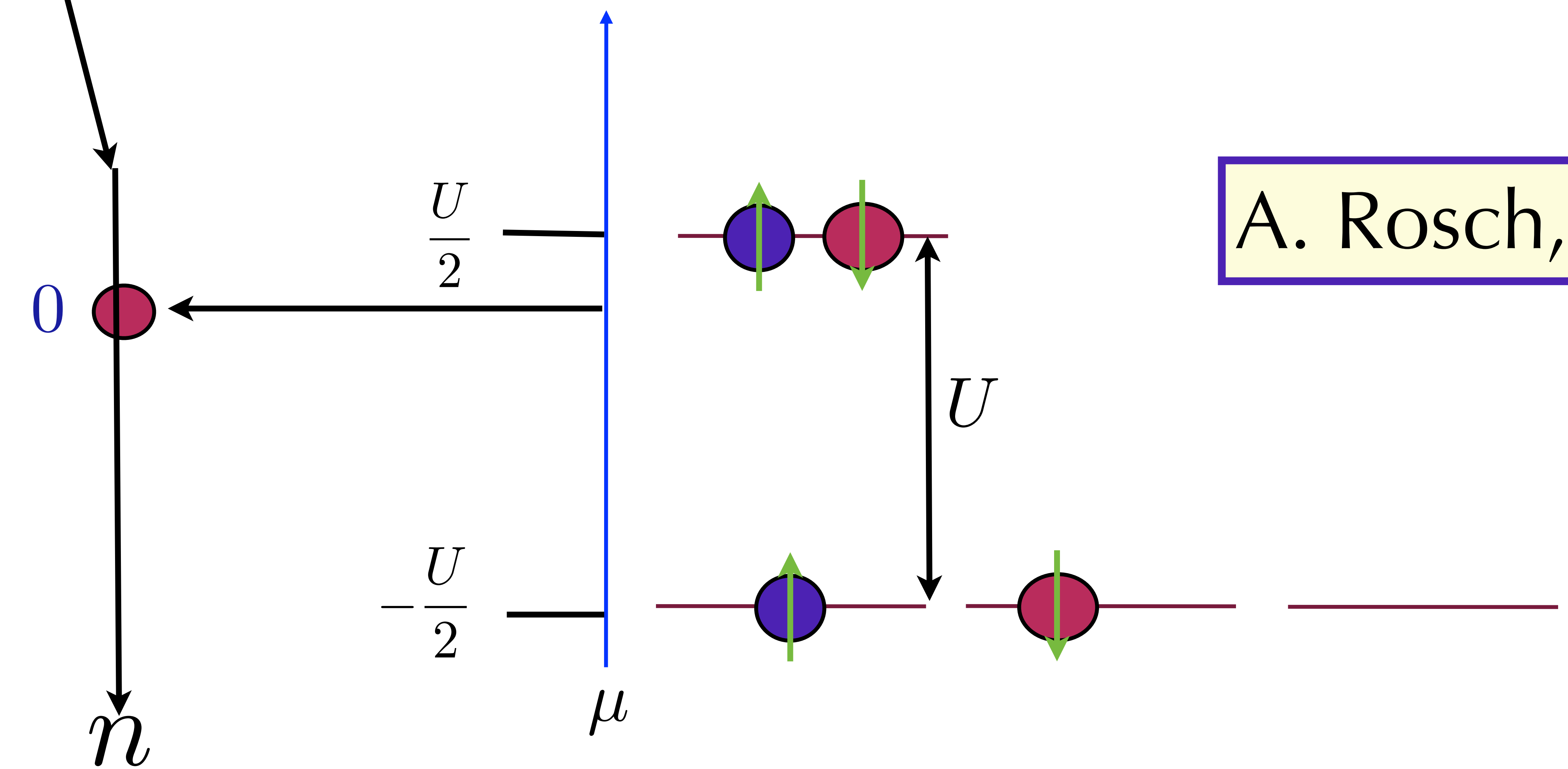

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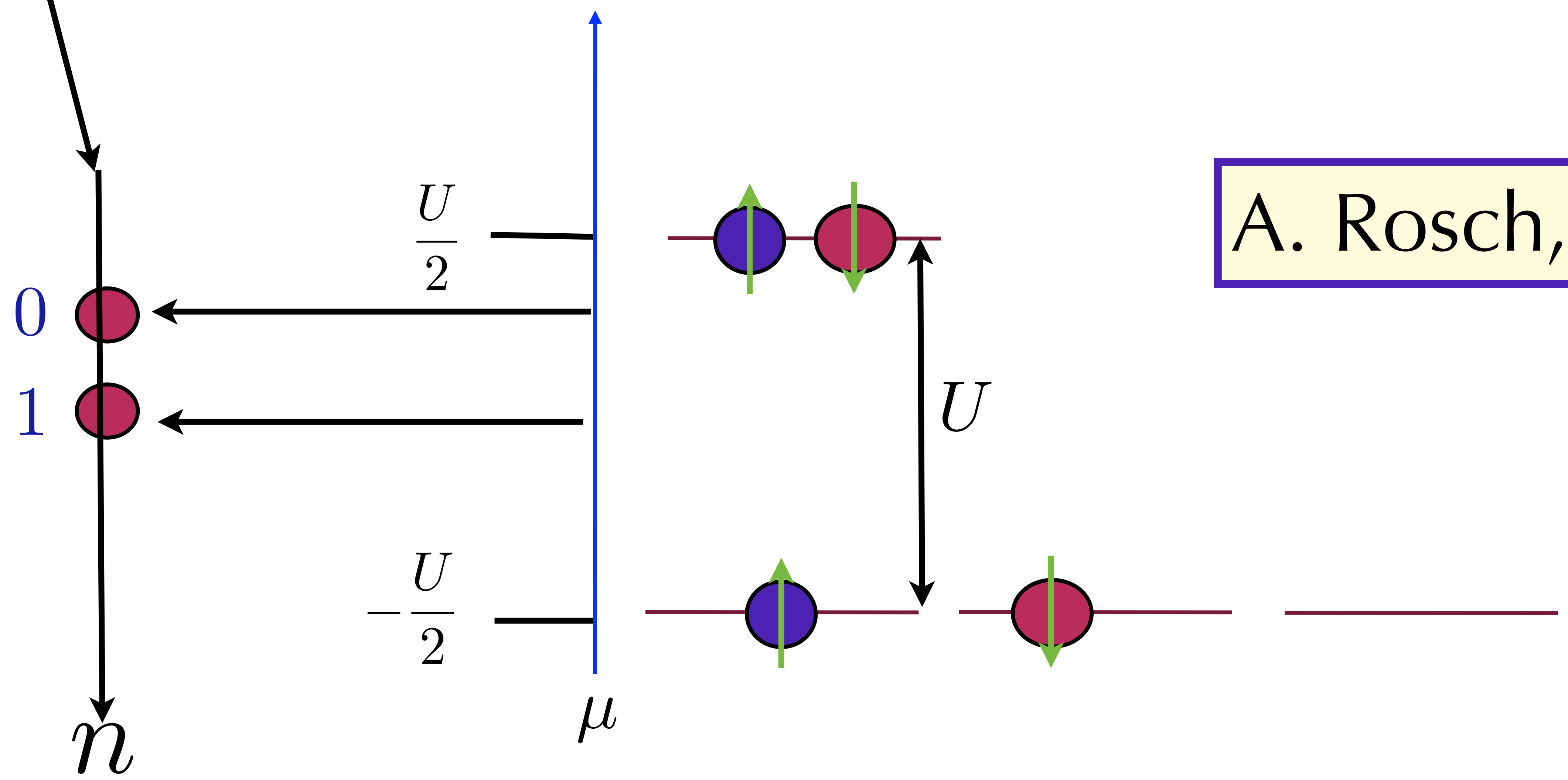

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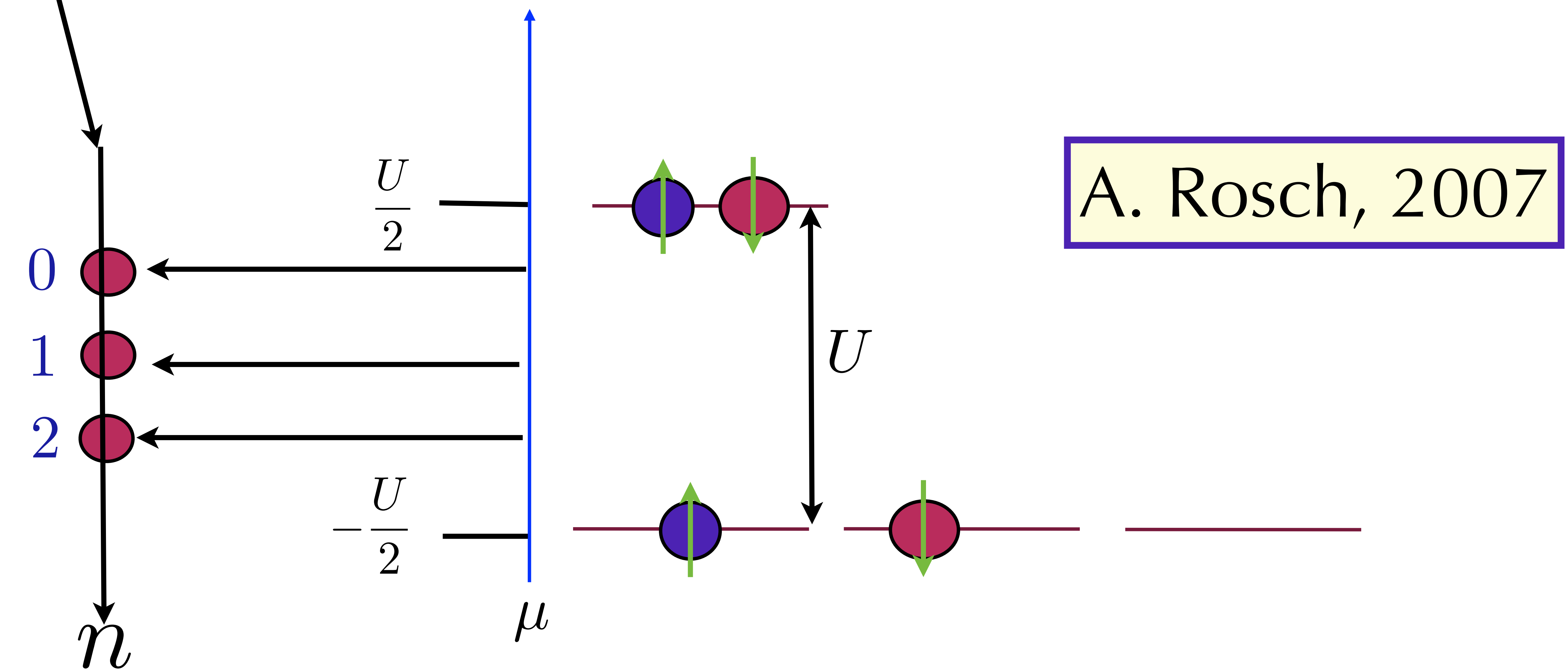

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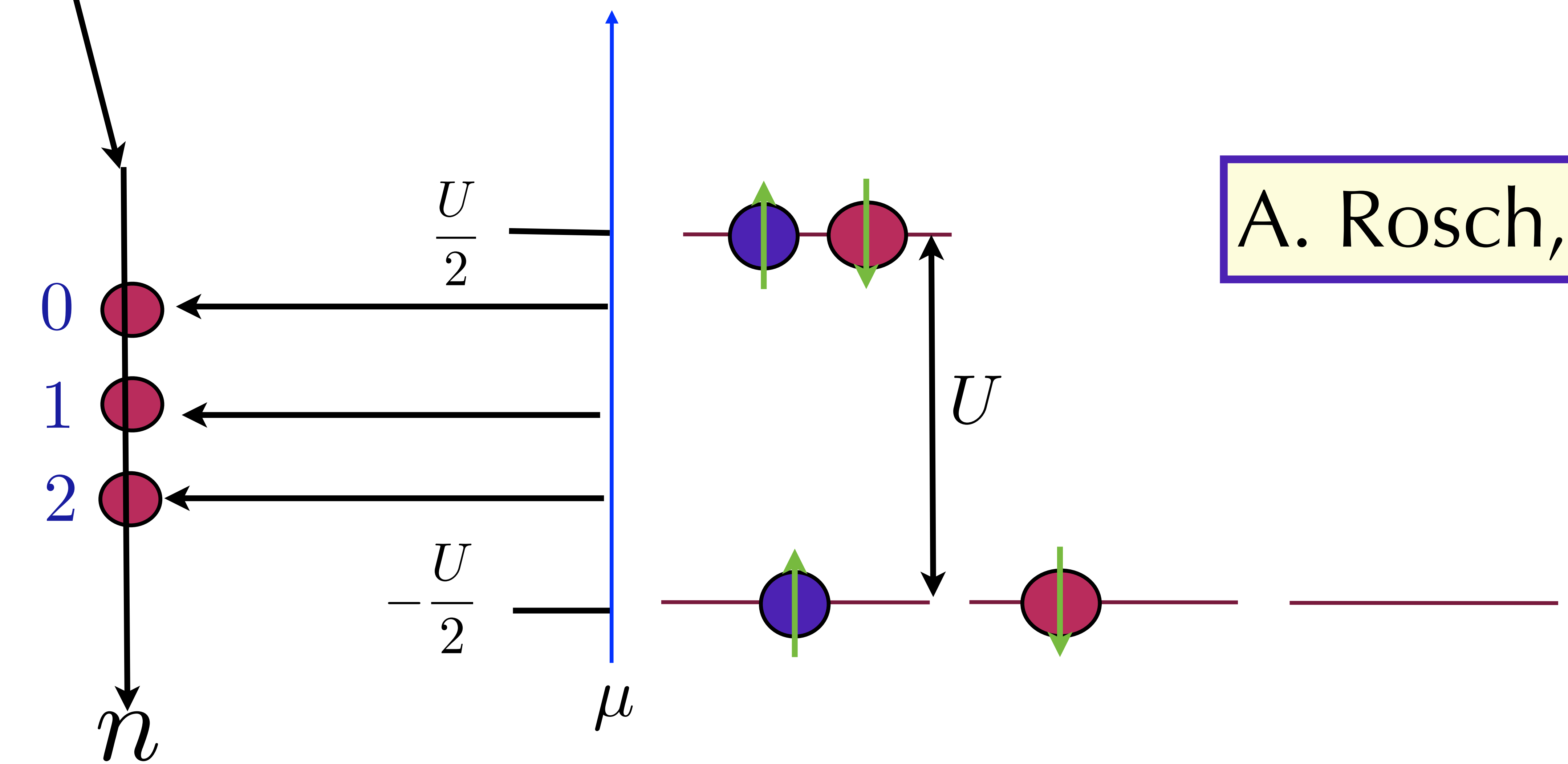


A. Rosch, 2007

no conservation law

fix chemical potential

$$\lim_{T\to 0} \mu(T)$$

fix chemical potential

$$\lim_{T\to 0} \mu(T)$$

n=1

fix chemical potential

$$\lim_{T\to 0}\mu(T)$$

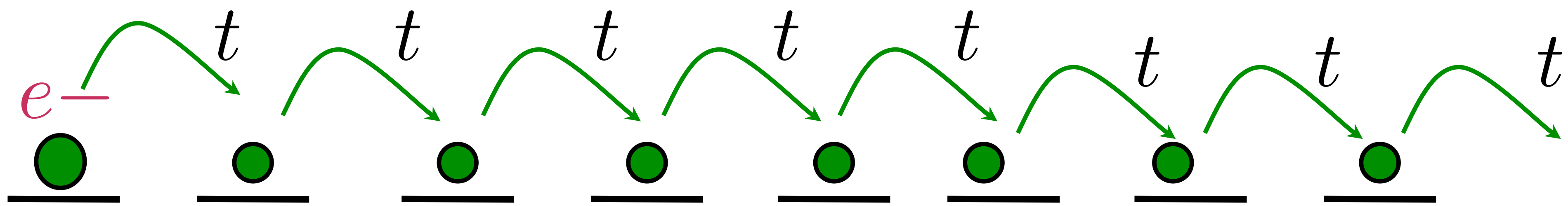
n=1

does this fix all the problems?

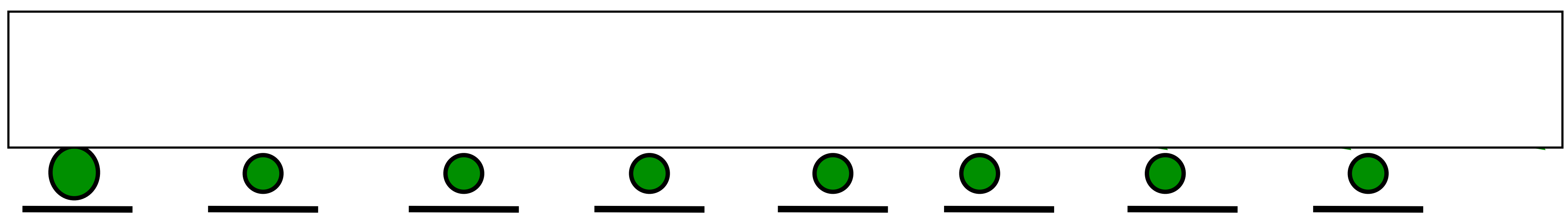
No

A model with zeros but Luttinger fails

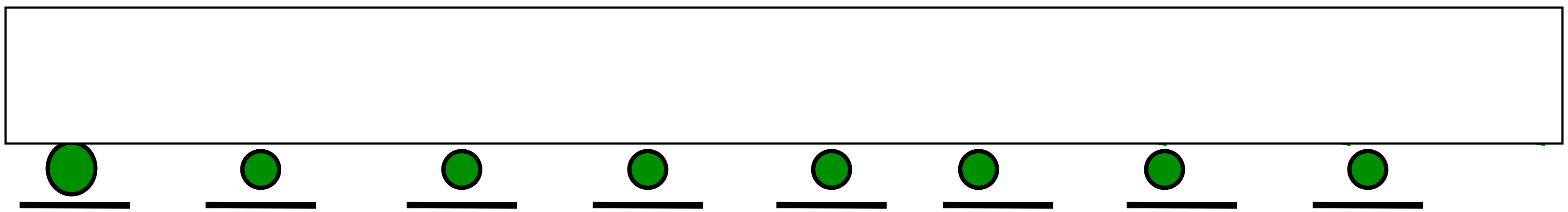
A model with zeros but Luttinger fails



A model with zeros but Luttinger fails

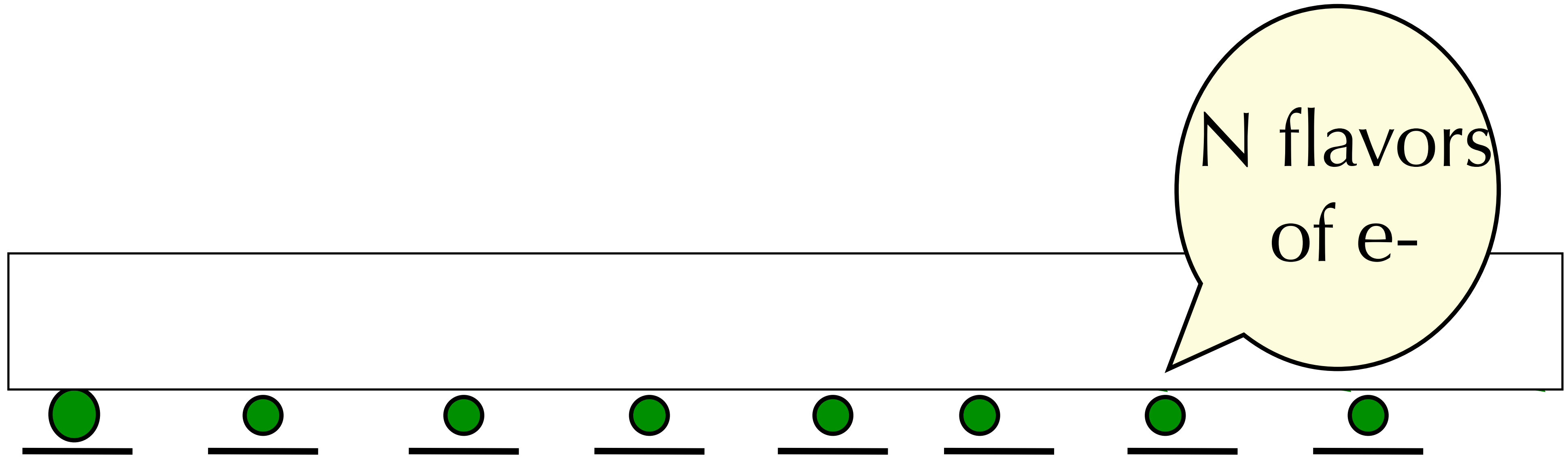


A model with zeros but Luttinger fails



no hopping=> no propagation (zeros)

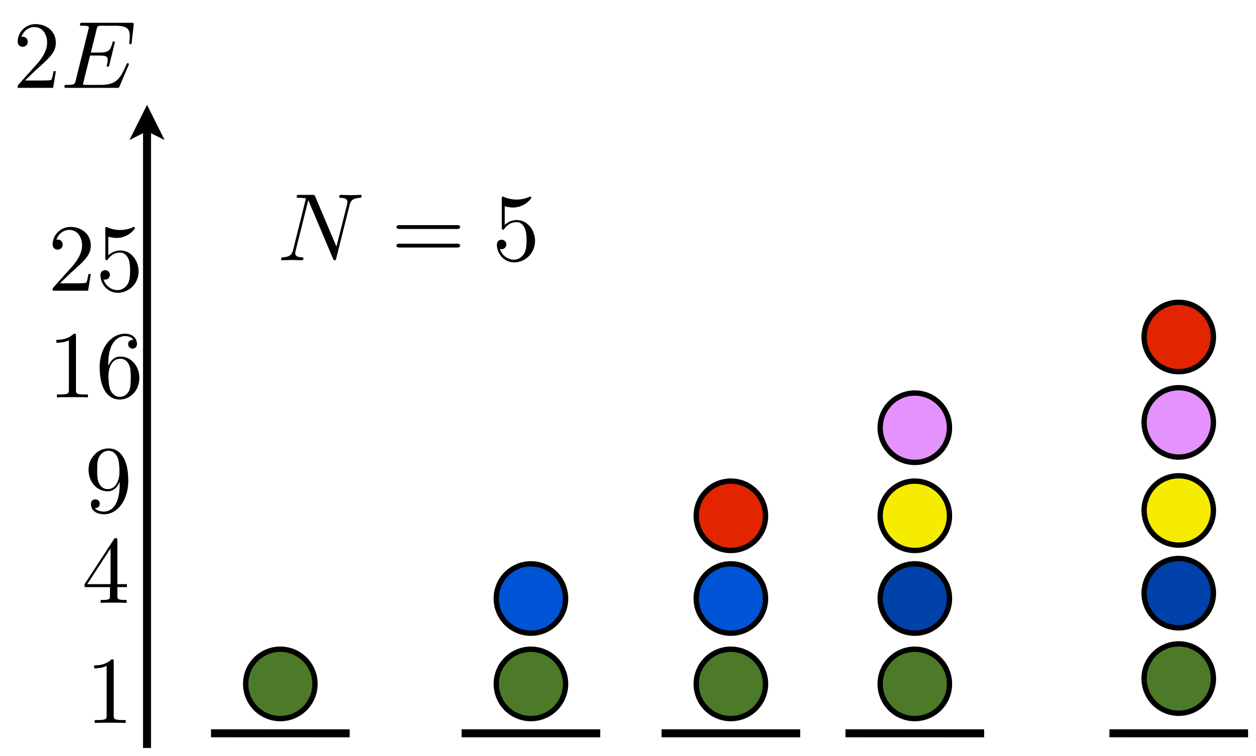
A model with zeros but Luttinger fails



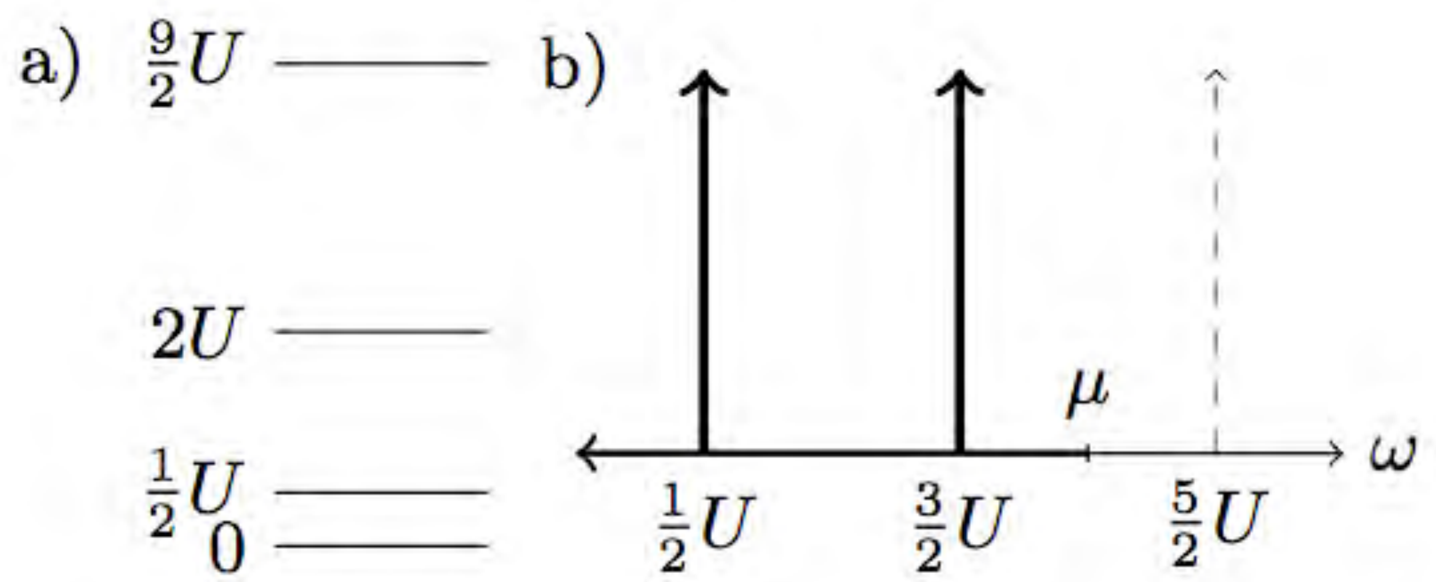
no hopping=> no propagation (zeros)

$$SU(N)$$

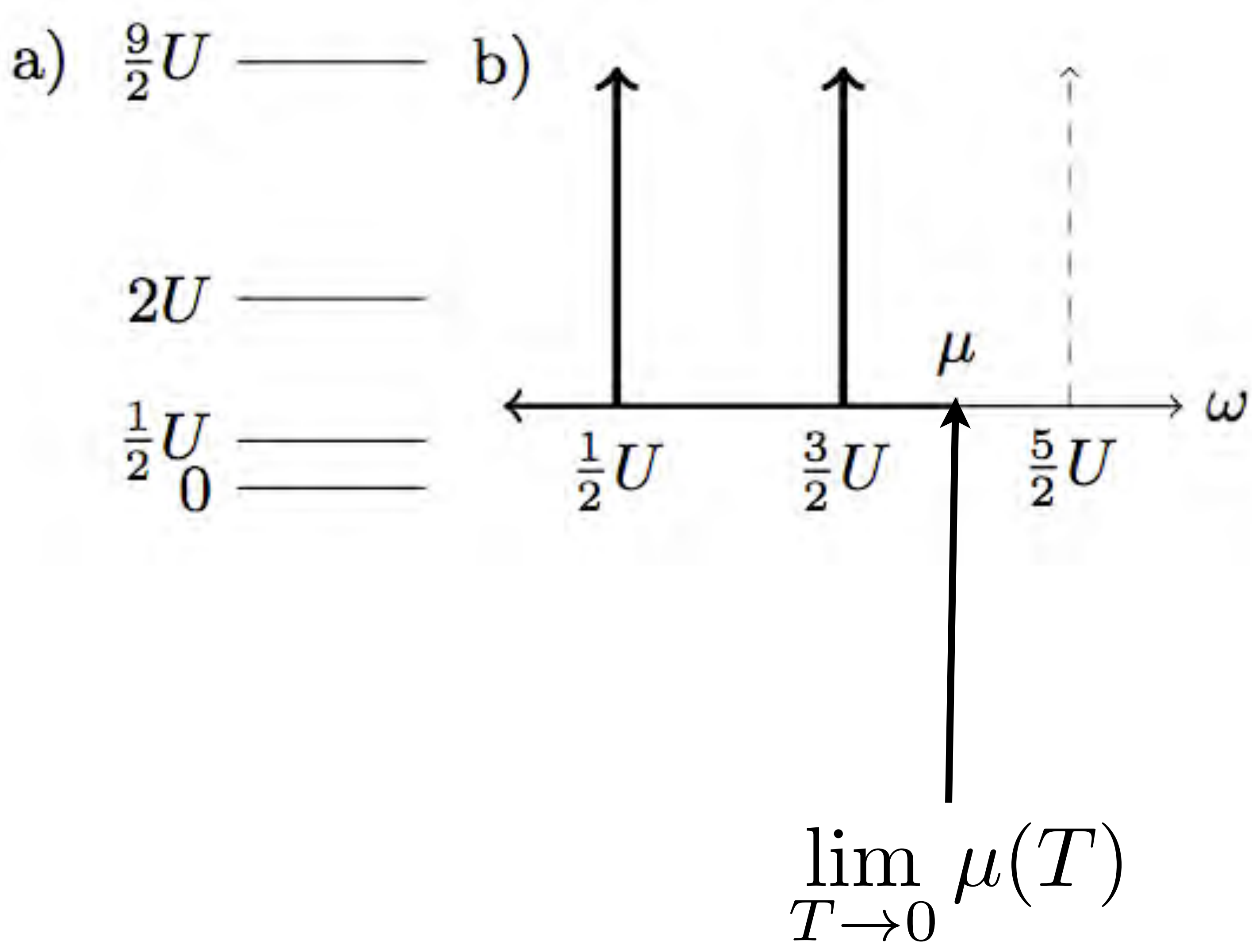
$$H = \frac{U}{2}(n_1 + \cdots n_N)^2$$



no particle-hole symmetry



no particle-hole symmetry



$$G_{\alpha\beta}(\omega = 0) = \frac{\delta_{\alpha\beta}}{K(n+1) - K(n)} \left(\frac{2n - N}{N} \right)$$

$$G_{\alpha\beta}(\omega = 0) = \underbrace{\frac{\delta_{\alpha\beta}}{K(n+1) - K(n)}}_{>0} \left(\frac{2n - N}{N} \right)$$

Luttinger's theorem

$$n = N\Theta(2n - N)$$

Luttinger's theorem

$$n = N\underbrace{\Theta(2n - N)}_{0,1,1/2}$$

Luttinger's theorem

$$n = N\Theta(\underbrace{2n - N}_{0,1,1/2})$$

$$n = 2$$

$$N = 3$$

Luttinger's theorem

$$n = N\Theta(\underbrace{2n - N}_{0,1,1/2})$$

$$n = 2$$

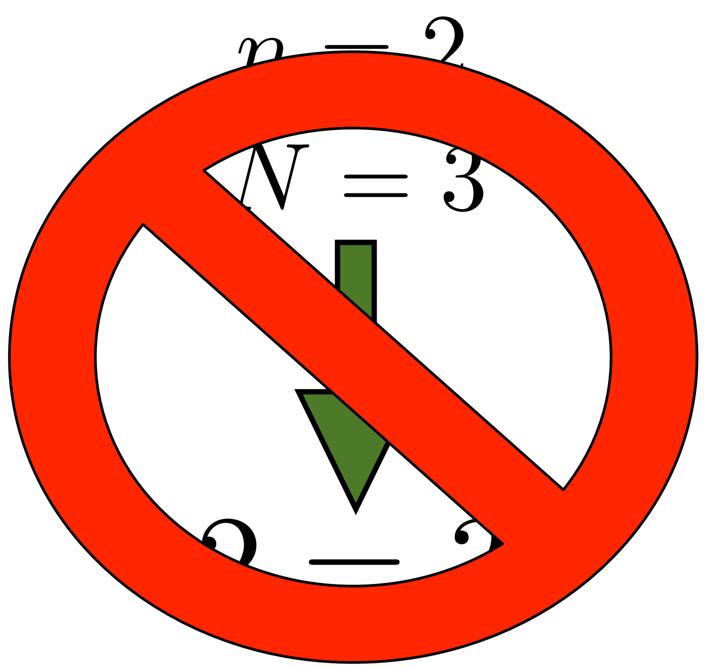
$$N = 3$$

$$\Downarrow$$

$$2 = 3$$

Luttinger's theorem

$$n = N\Theta(\underbrace{2n - N}_{0,\,1,\,1/2})$$

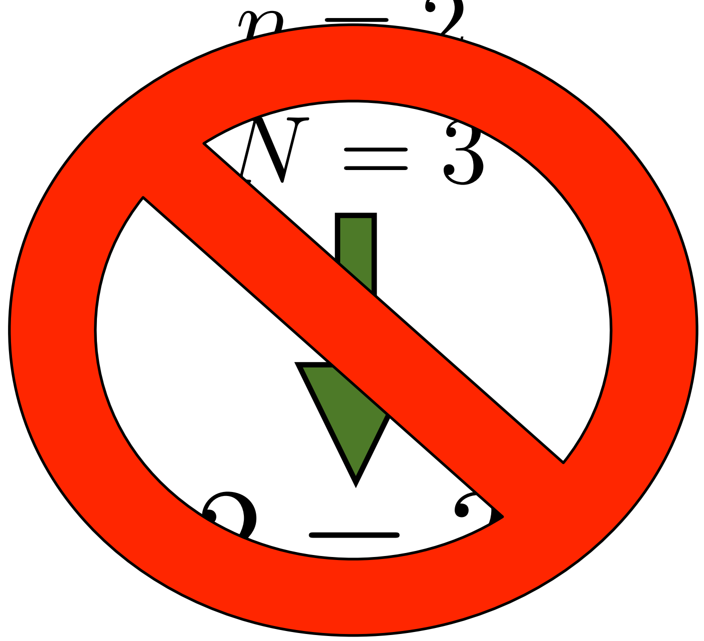


Luttinger's theorem

$$n = N\Theta(\underbrace{2n - N}_{0,\,1,\,1/2})$$

$0, 1, 1/2$

even

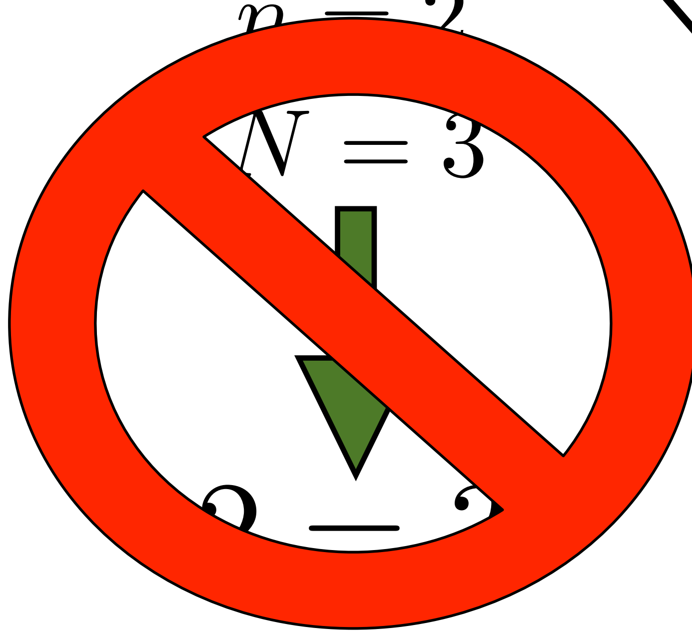


Luttinger's theorem

$$n = N\Theta(2n - N)$$

$0, 1, 1/2$

even



odd

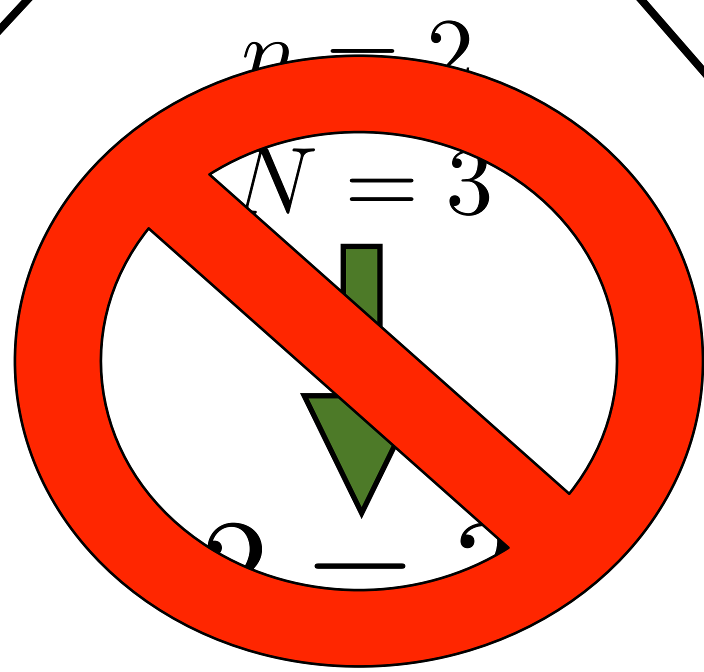
Luttinger's theorem

$$n = N\Theta(2n - N)$$

$\underbrace{\Theta(2n-N)}$: $0, 1, 1/2$

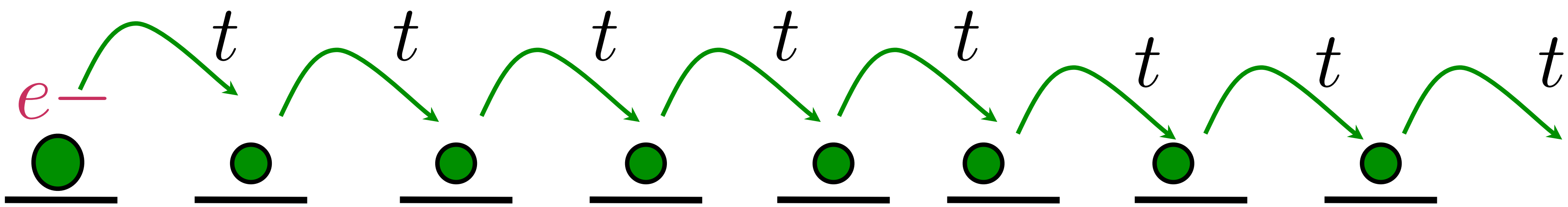
even

odd



no solution

does the degeneracy matter?



$$t = 0^{+}$$

Problem

$G=0$

Problem

G=0

$$G = \frac{1}{E - \varepsilon_p - \Sigma}$$

$\Sigma \to \infty$

inherent problem

inherent problem

$$\delta I[G] = \int d\omega \Sigma \delta G$$

inherent problem

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$\Big\downarrow$ if $\Sigma \to \infty$

inherent problem

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$$\text{if } \Sigma \to \infty$$

integral does not exist

inherent problem

$$\delta I[G] = \int d\omega \Sigma \delta G$$

if $\Sigma \to \infty$

integral does not exist

inherent problem

$$\delta I[G] = \int d\omega \Sigma \delta G$$

$\text{if } \Sigma \rightarrow \infty$

integral does not exist

Luttinger's Theorem

inherent problem

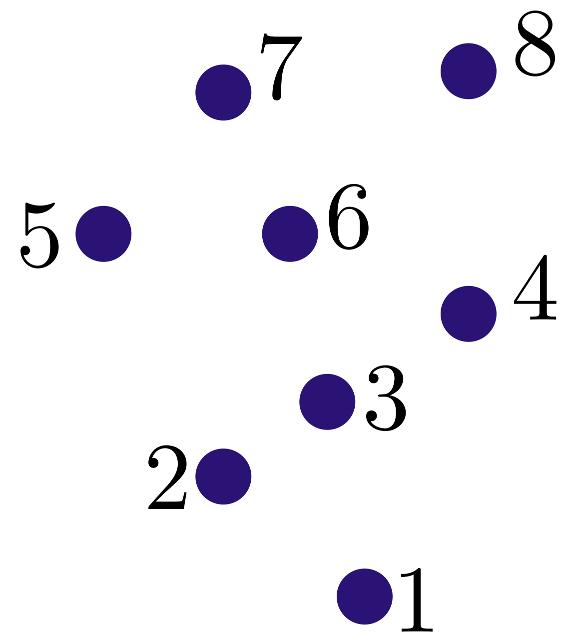
$$\delta I[G] = \int d\omega \Sigma \delta G$$

↓ if $\Sigma \to \infty$

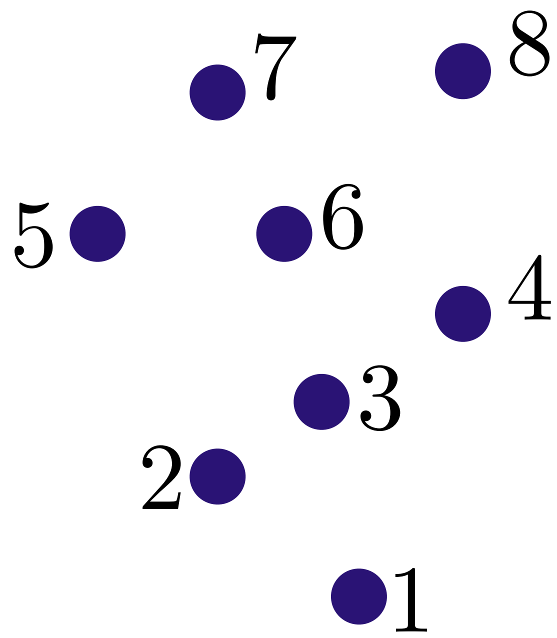
integral does not exist

↓

how to count particles?



how to count particles?



some charged stuff
has no particle interpretation

what is the extra stuff?

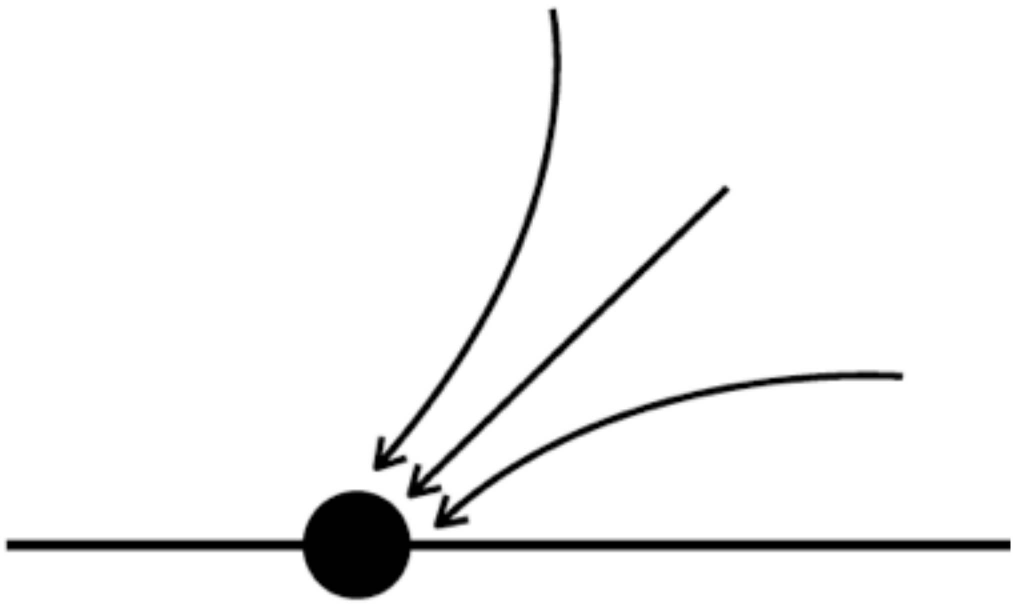
propagators

$$G \propto \langle T\psi(0)\psi^\dagger(t)\rangle$$

$$(p^2)^{d_U - d/2}$$

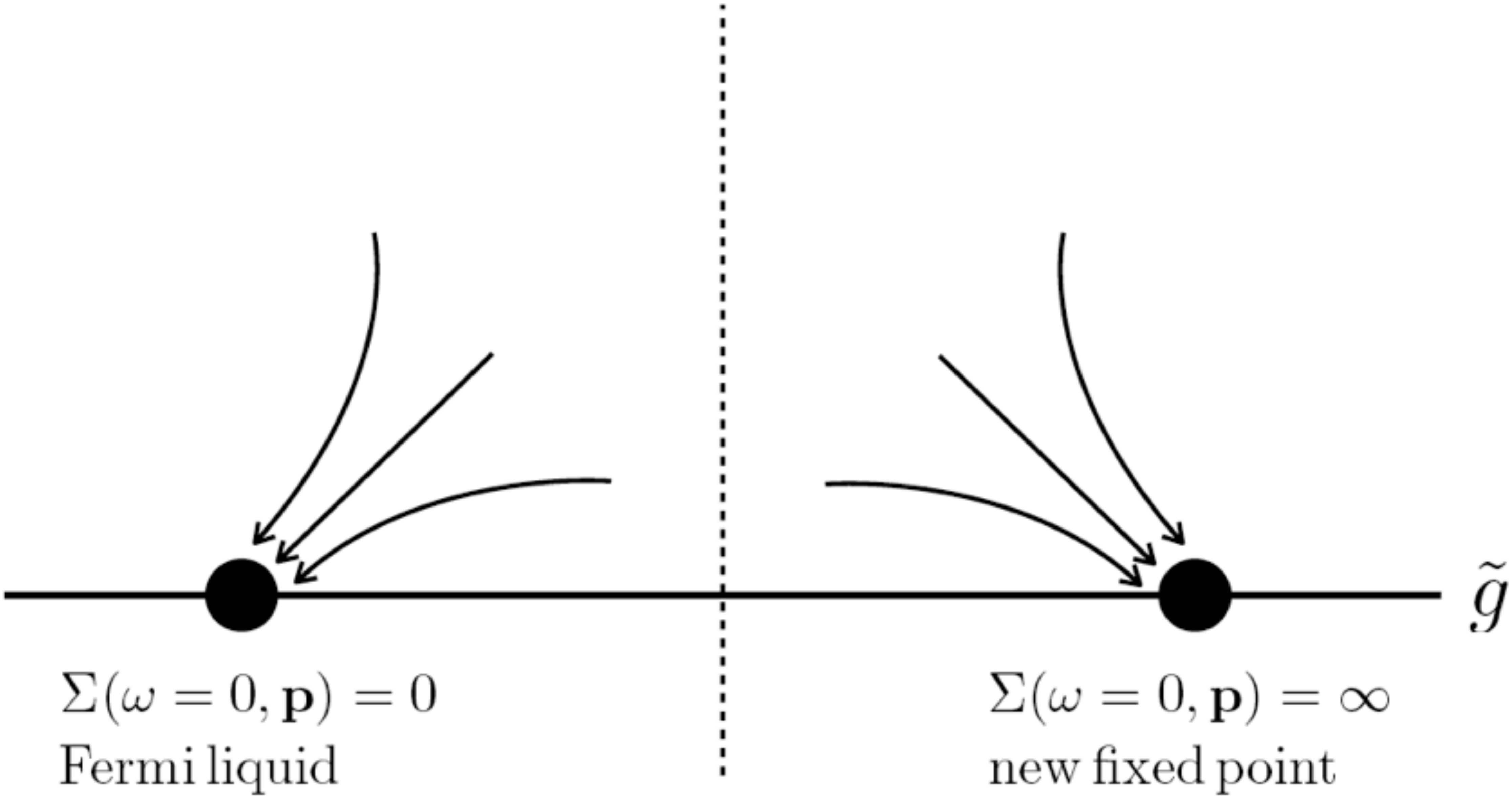
$$\dim[\psi] = d_U$$

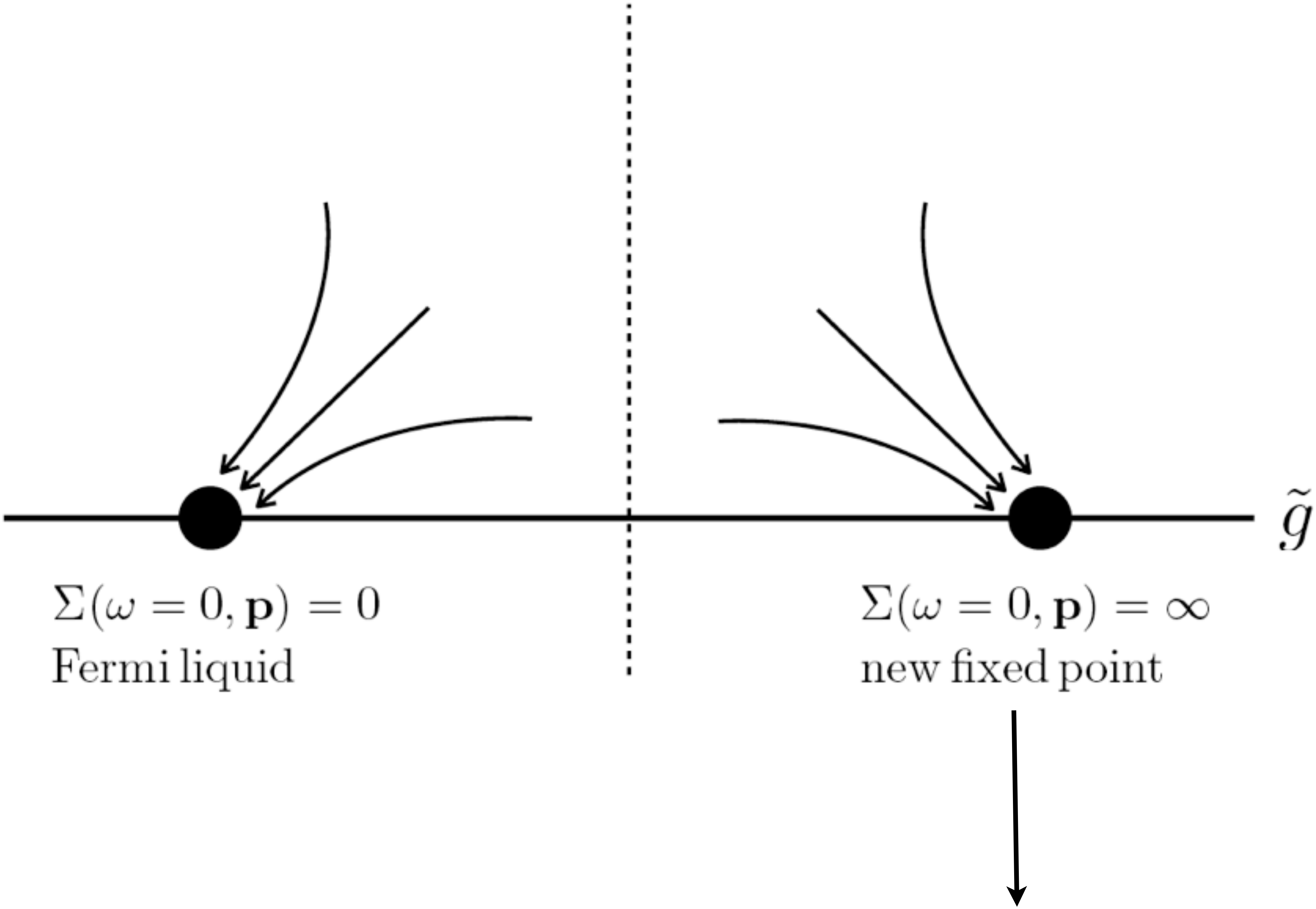
how can such large anomalous dimensions be generated?



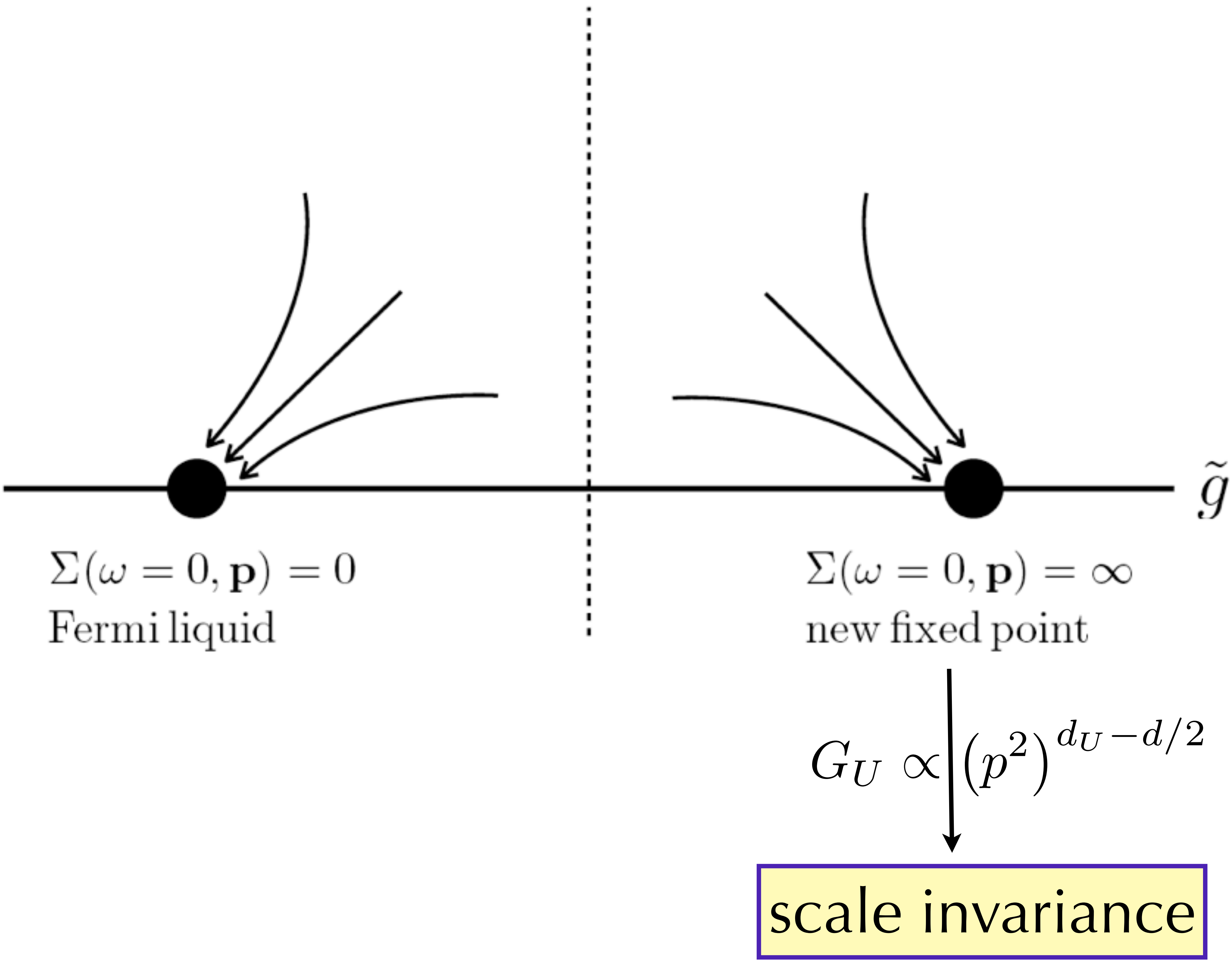
$\Sigma(\omega = 0, \mathbf{p}) = 0$

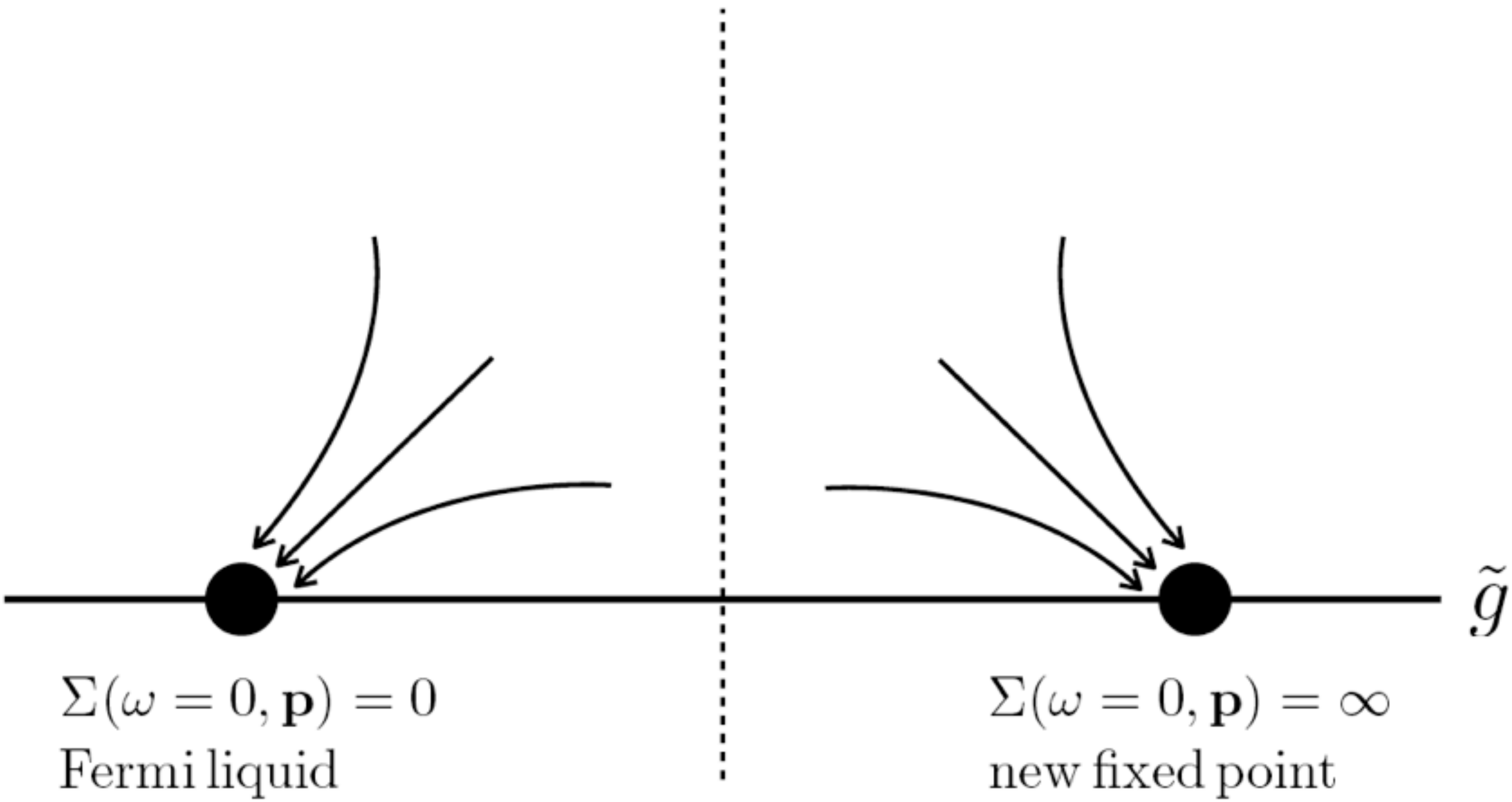
Fermi liquid


$\tilde{g}$
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid
$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point



scale invariance

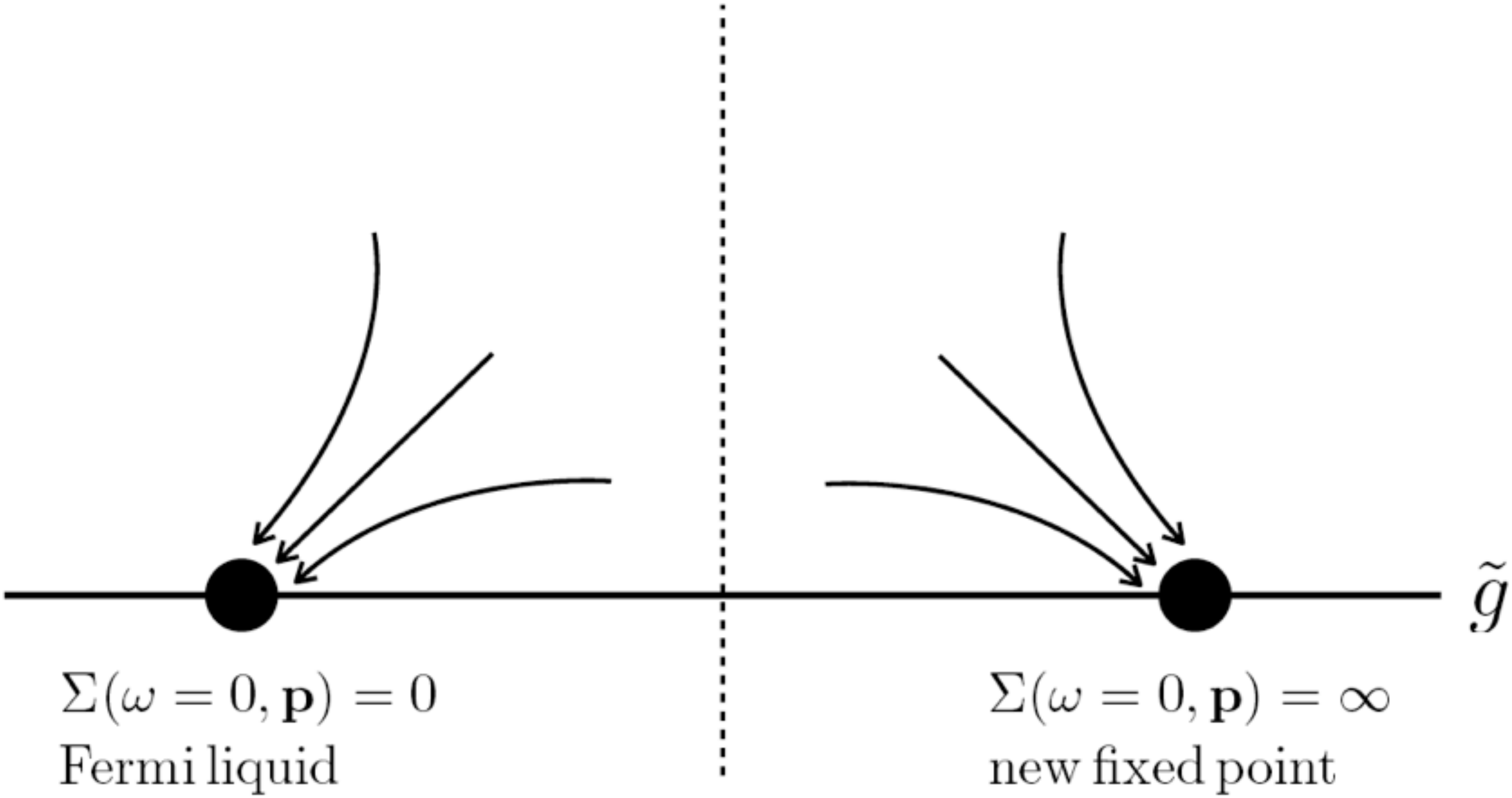

$\tilde{g}$
$\Sigma(\omega = 0, \mathbf{p}) = 0$
Fermi liquid
$\Sigma(\omega = 0, \mathbf{p}) = \infty$
new fixed point
$G_U \propto (p^2)^{d_U - d/2}$
scale invariance



$$G_U \propto (p^2)^{d_U - d/2}$$

scale invariance

$$d_U > d/2$$



$$G_U \propto \left(p^2\right)^{d_U - d/2}$$

unparticles

scale invariance

$$d_U > d/2$$

unparticles

unparticles

↓

no well-defined mass
(all possible mass, energy
incoherent stuff)

unparticles

↓

no well-defined mass
(all possible mass, energy
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$$\mathcal{L}_{\rm eff} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

unparticles

no well-defined mass
(all possible mass, energy
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$$\mathcal{L}_{\text{eff}} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

but $m \propto 1/L$

unparticles

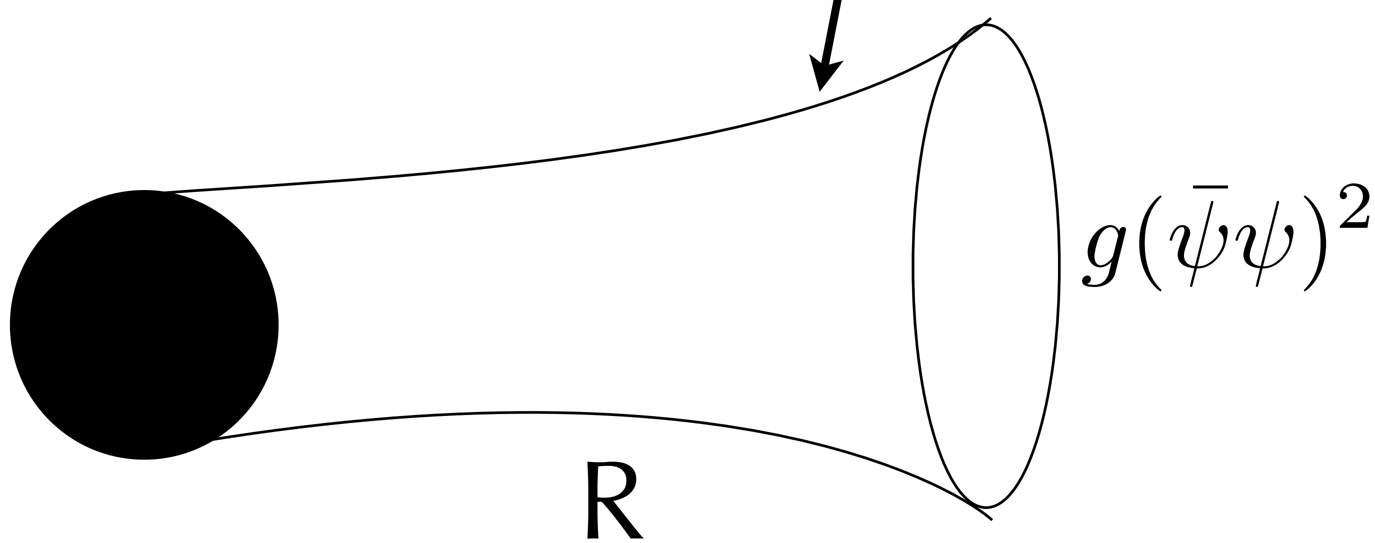
no well-defined mass
(all possible mass, energy
incoherent stuff)

$$\mathcal{L}_{\rm eff} = \int_0^\infty \mathcal{L}(x, m^2) dm^2$$

but $m \propto 1/L$ → hidden extra dimension

replace coupled theory with geometry

scale-invariant anti de-Sitter



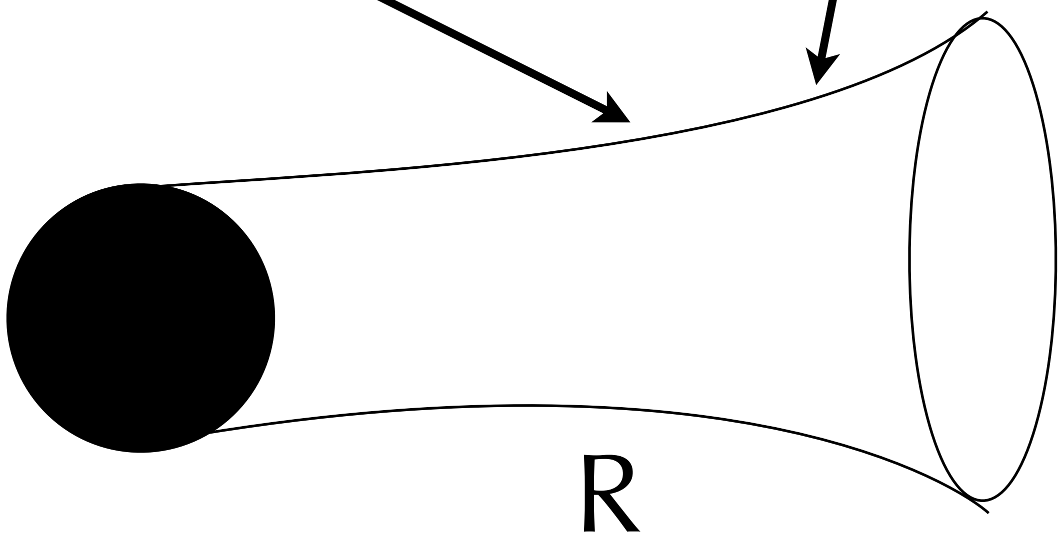
$$R \propto \sqrt{g}$$

d_U

replace coupled theory with geometry

mass integration

scale-invariant anti de-Sitter



$g(\bar{\psi}\psi)^2$

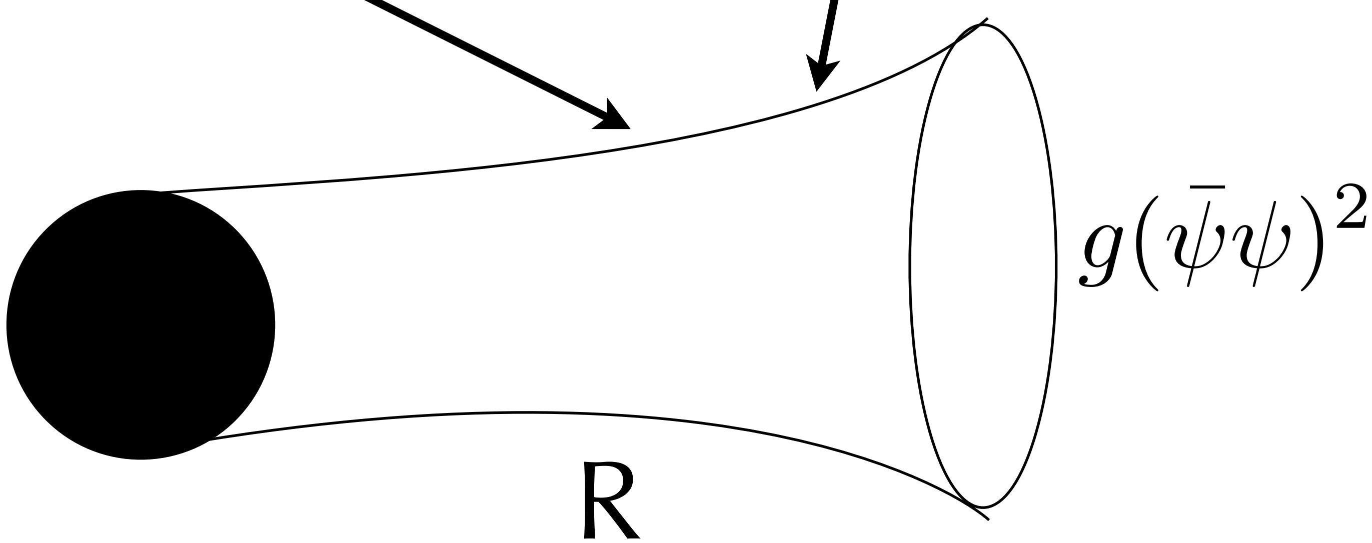
$R \propto \sqrt{g}$

d_U

replace coupled theory
with geometry

mass
integration

scale-invariant
anti de-Sitter



$$R \propto \sqrt{g}$$

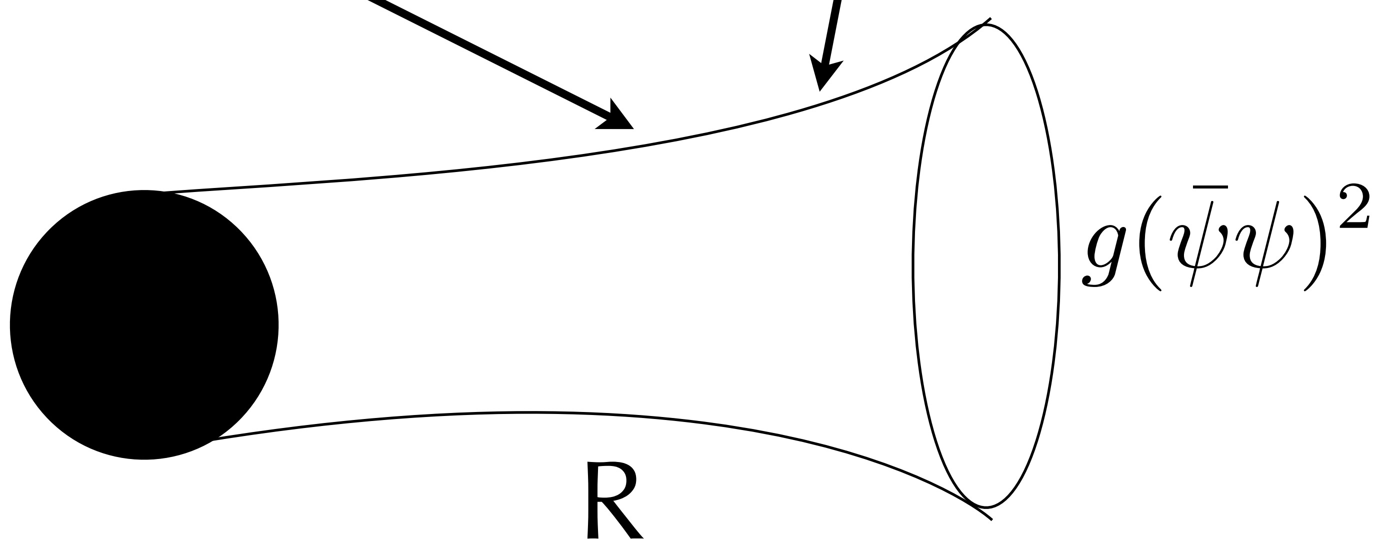
cannot describe systems at g=0!

d_U

replace coupled theory with geometry

mass integration

scale-invariant anti de-Sitter



$$R \propto \sqrt{g}$$

cannot describe systems at g=0!

can we use this construction to fix d_U?

$$\mathcal{L} = \quad \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big)$$

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\Big)\, dm^2$$

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m) \partial_\mu \phi(x,m) + m^2 \phi^2(x,m) \Big) \, dm^2$$

theory with all possible mass!

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big)\, dm^2$$

theory with all possible mass!

$$\phi \to \phi(x, m^2/\Lambda^2)$$
$$x \to x/\Lambda$$
$$m^2/\Lambda^2 \to m^2$$

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

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$$\phi \to \phi(x, m^2/\Lambda^2)$$
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$$\mathcal{L} \to \Lambda^4 \mathcal{L}$$

scale invariance is restored!!

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) dm^2$$

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not particles

unparticles

$$\mathcal{L} = \int_0^{\infty} \left(\partial^\mu \phi(x,m) \partial_\mu \phi(x,m) + m^2 \phi^2(x,m) \right) dm^2$$

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not particles

propagator

$$\left(\int_0^\infty dm^2 m^{2(d_U - d/2)} \frac{i}{p^2 - m^2 + i\epsilon}\right)^{-1} \propto p^{2(d_U - d/2)}$$

fixing d_U

fixing d_U

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m) \partial_\mu \phi(x,m) + m^2 \phi^2(x,m)\right) m^{2\delta} dm^2$$

fixing d_U

$$\mathcal{L} = \int_0^\infty \big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\big)\, m^{2\delta} dm^2$$

$$m = z^{-1}$$

fixing d_U

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$m^2_{\mathrm{AdS}} = \frac{d_U(d_U - d)}{R^2}$$

fixing d_U

$$\mathcal{L} = \int_0^\infty \left(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\right) m^{2\delta} dm^2$$

$$m = z^{-1}$$

$$\frac{1}{R^2} = \frac{d_U(d_U - d)}{R^2}$$

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$$m = z^{-1}$$

$$\frac{1}{R^2} = \frac{d_U(d_U - d)}{R^2}$$

$$\mathcal{L} = \int_0^\infty dz\, \frac{2R^2}{z^{5+2\delta}} \left[\frac{1}{2}\frac{z^2}{R^2}\eta^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{\phi^2}{2R^2}\right]$$

fixing d_U

$$\mathcal{L} = \int_0^\infty \Big(\partial^\mu \phi(x,m)\partial_\mu \phi(x,m) + m^2\phi^2(x,m)\Big) m^{2\delta} dm^2$$

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can be absorbed with AdS metric

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\,dz\,\sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$\sqrt{-g} = (R/z)^{5+2\delta}$$

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\,dz\,\sqrt{-g}\left(\partial_a\Phi\partial^a\Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

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$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

unparticle lives in

$$d = 4 + 2\delta \qquad \delta \leq 0$$

action on $AdS_{5+2\delta}$

$$S = \frac{1}{2}\int d^{4+2\delta}x\, dz\, \sqrt{-g}\left(\partial_a \Phi \partial^a \Phi + \frac{\Phi^2}{R^2}\right)$$

$$ds^2 = \frac{L^2}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right) \qquad \sqrt{-g} = (R/z)^{5+2\delta}$$

unparticle lives in

$$d = 4 + 2\delta \qquad \delta \leq 0$$

generating functional for unparticles

Claim: $Z_{\mathrm{QFT}} = e^{-S_{\mathrm{ADS}}^{\mathrm{on-shell}}(\phi(\phi_{\partial \mathrm{ADS}=\mathrm{J}_{\mathcal{O}}}))}$

$$S = \frac{1}{2}\int d^d x\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

Claim: $Z_{\rm QFT} = e^{-S^{\rm on-shell}_{\rm ADS}(\phi(\phi_{\partial {\rm ADS}={\rm J}_{\mathcal{O}}}))}$

$$S \;=\; \frac{1}{2}\int d^dx\, g^{zz}\sqrt{-g}\,\Phi(z,x)\partial_z\Phi(z,x)\Big|_{z=\epsilon}$$

$$\langle \Phi_U(x)\Phi_U(x')\rangle = \frac{1}{|x-x'|^{2d_U}}$$

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$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

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$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

scaling dimension is fixed

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

$$G_U(0) = 0$$

unparticle (AdS) propagator has zeros!

$$G_U(p) \propto p^{2(d_U - d/2)}$$

$$d_U = \frac{d}{2} + \frac{\sqrt{d^2+4}}{2} > \frac{d}{2}$$

$$G_U(0) = 0$$

top-down construction

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

[1] *Theoretical Physics Group, Blackett Laboratory, Imperial College, London SW7 2AZ, U.K.*
[2] *D.A.M.T.P. University of Cambridge, Cambridge, CB3 0WA, U.K.*

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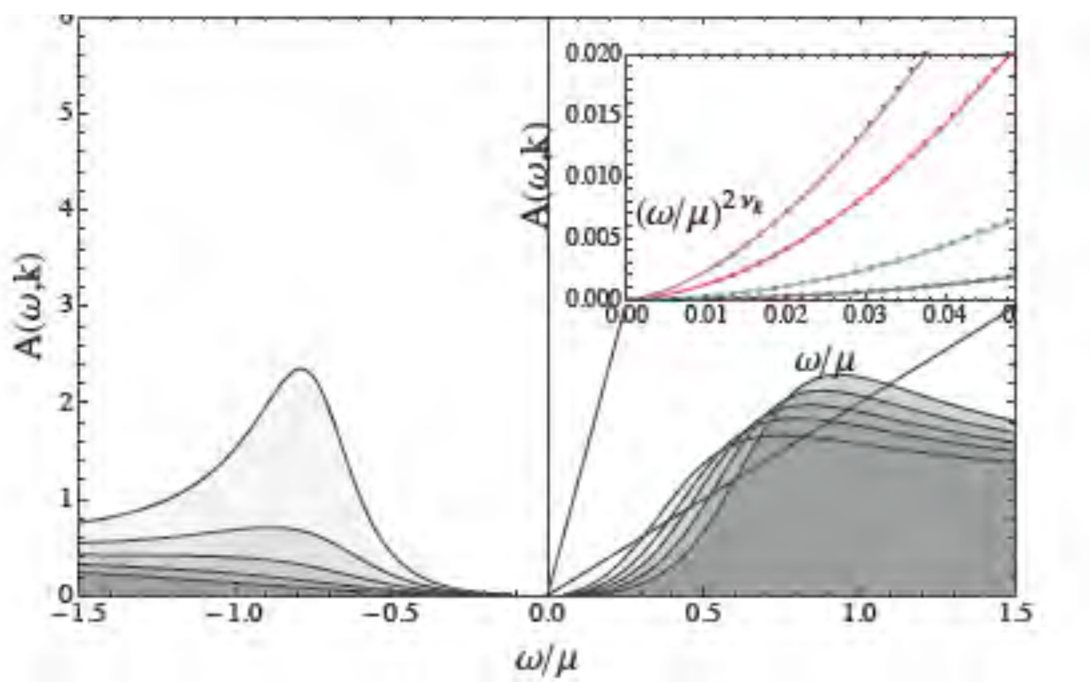

also Gubser, et al.

top-down construction

Universal fermionic spectral functions from string theory

Jerome P. Gauntlett,[1] Julian Sonner,[1,2] and Daniel Waldram[1]

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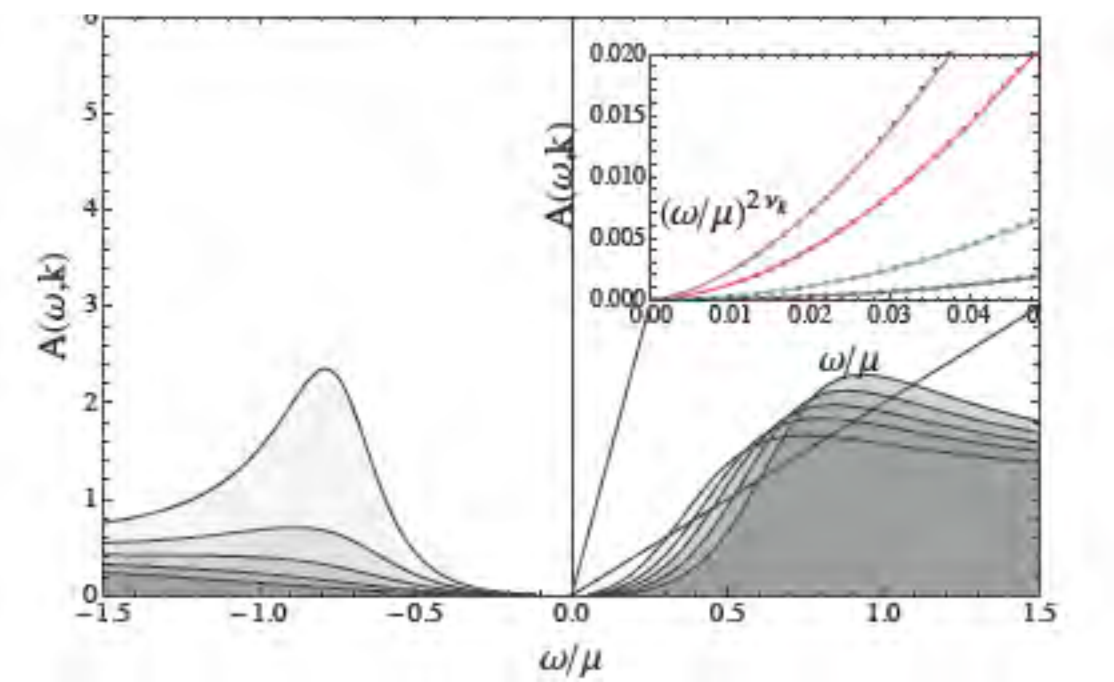

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$$(\not{D} - m - \frac{i}{2}F^{\mu\nu}\Gamma_{\mu\nu})\psi_\rho + iF\mu\nu\Gamma_\mu\Gamma_\rho\psi_\nu = 0$$

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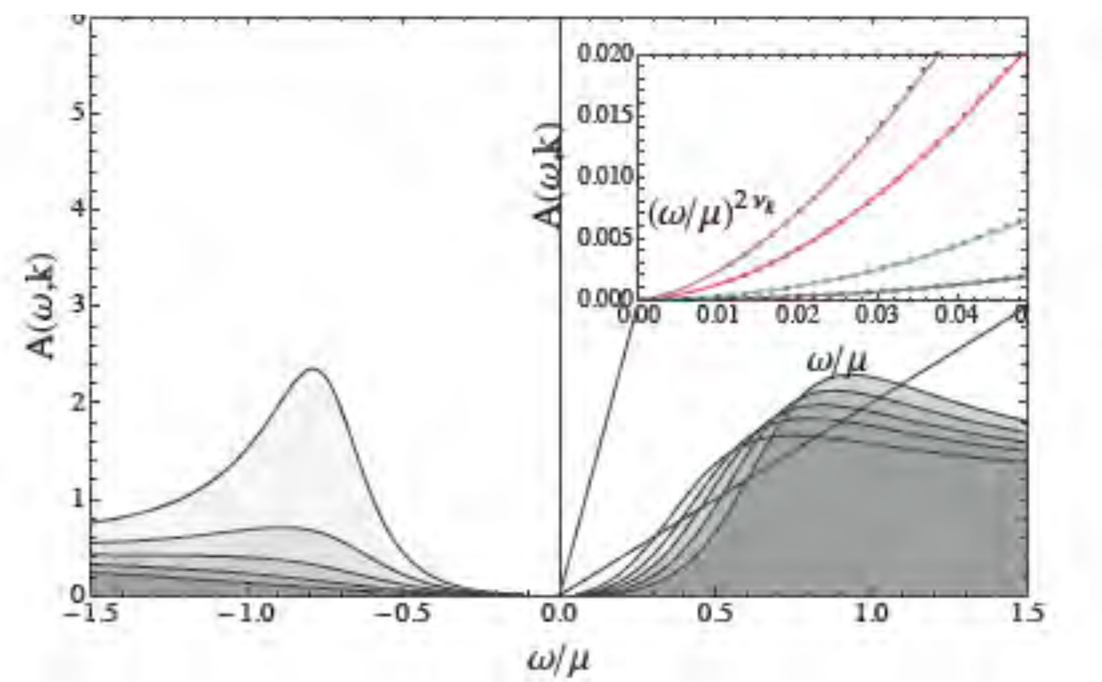

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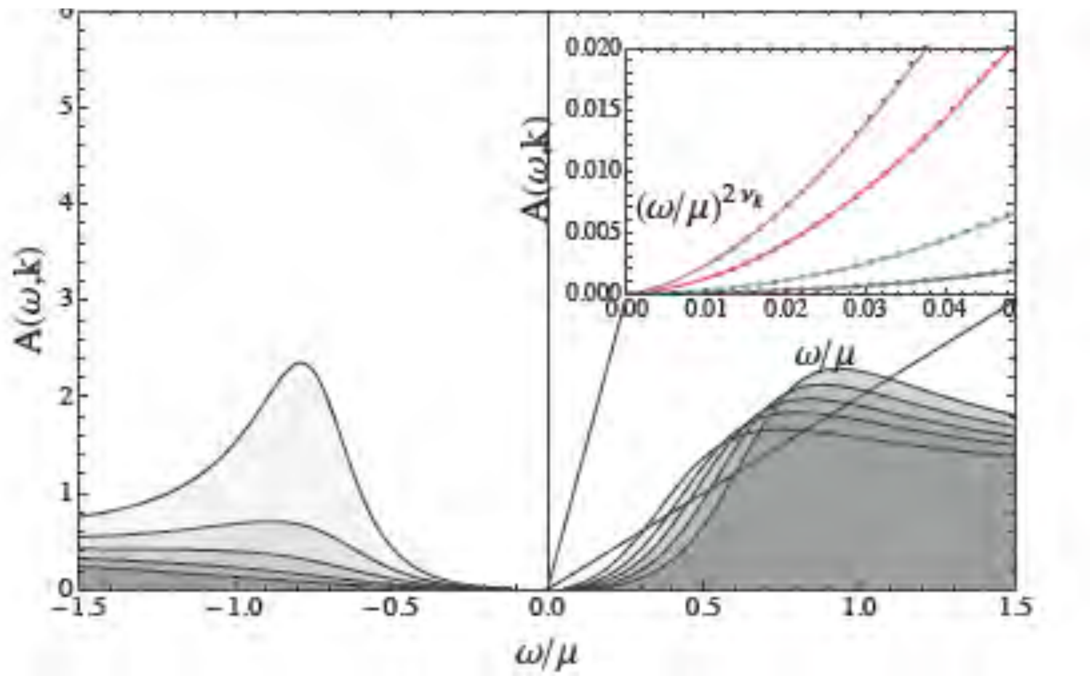

fixed by supersymmetry

also Gubser, et al.

$$(\not{D} - m - \frac{i}{2} F^{\mu\nu}\Gamma_{\mu\nu})\psi_\rho + iF\mu\nu\Gamma_\mu\Gamma_\rho\psi_\nu = 0$$

what about bottom-up constructions?

$$S_{\text{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g}\, i\bar{\psi}(\Gamma^M D_M - m + \cdots)\psi$$

$$S_{\mathrm{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g}\, i \bar{\psi} (\Gamma^M D_M - m + \cdots) \psi$$

what is hidden here?

$$S_{\rm probe}(\psi,\bar\psi) = \int d^dx\sqrt{-g}\,i\bar\psi(\Gamma^M D_M - m + \cdots)\psi$$

what is hidden here?

consider $\sqrt{-g}\,i\bar\psi(\not{D} - m - ipF_{\mu\nu}\Gamma^{\mu\nu})\psi$

$$S_{\mathrm{probe}}(\psi, \bar{\psi}) = \int d^d x \sqrt{-g}\, i\bar{\psi}(\Gamma^M D_M - m + \cdots)\psi$$

what is hidden here?

one possibility

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what is hidden here?

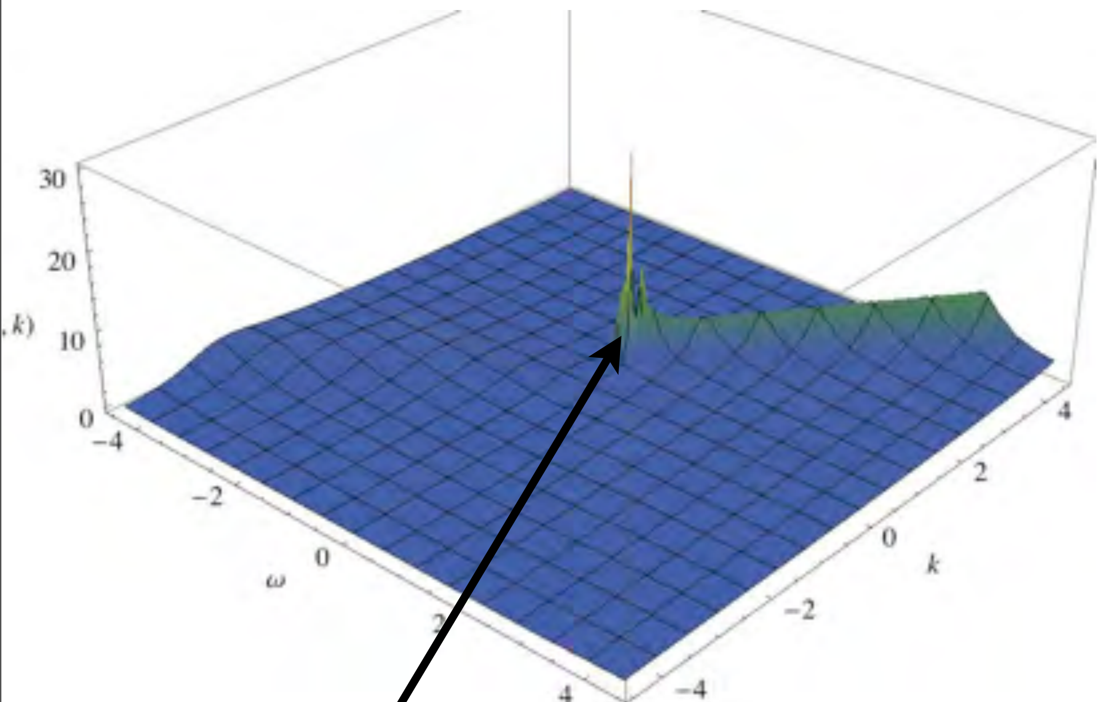
one possibility

consider $\sqrt{-g}\, i \bar{\psi} (\not{D} - m - i p F_{\mu\nu} \Gamma^{\mu\nu}) \psi$

what happens at the boundary?

How is the spectrum modified?

P=0

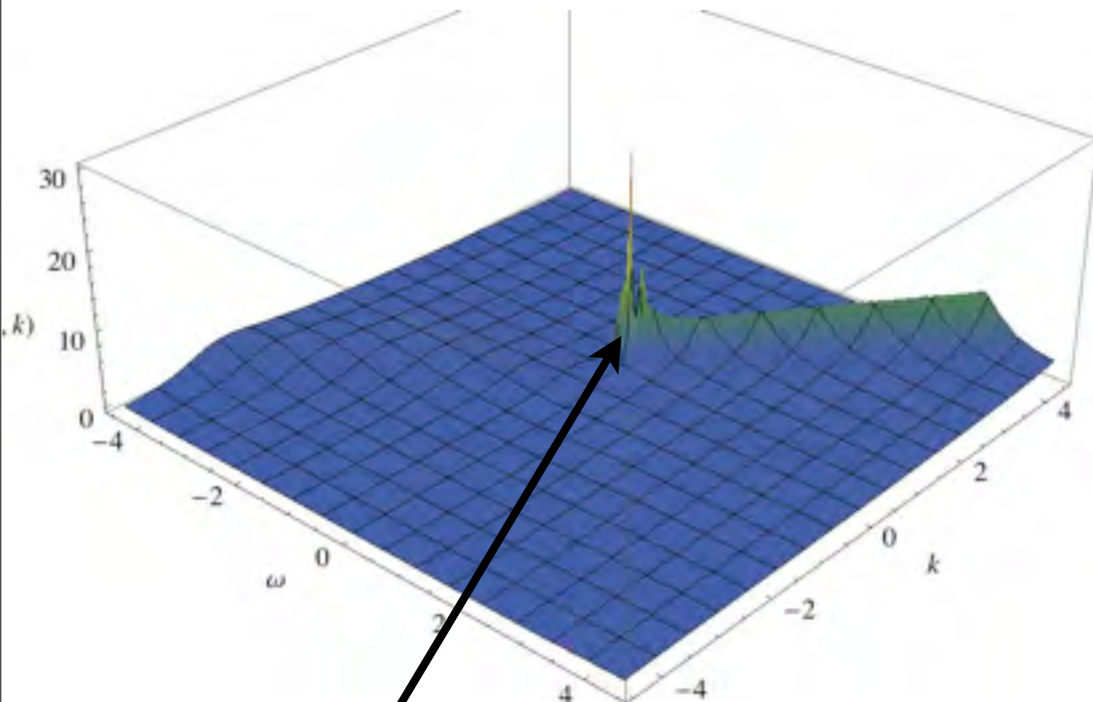


Fermi surface peak

How is the spectrum modified?

P

P=0



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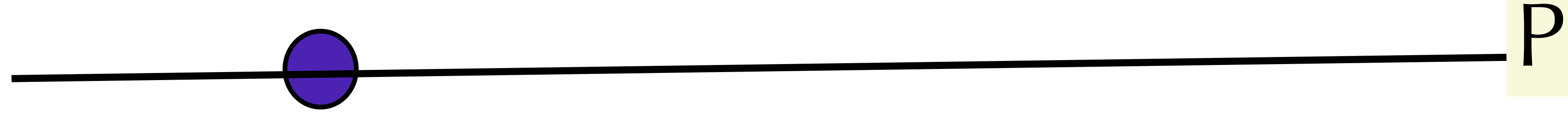
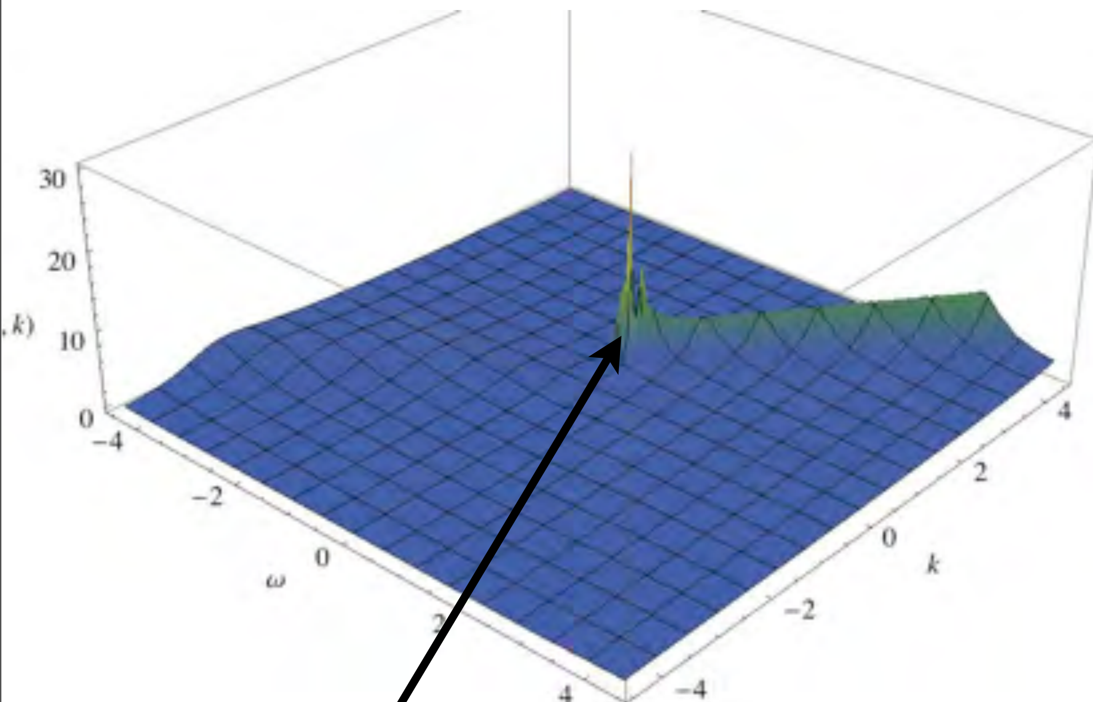


P=0



Fermi surface peak

$$-1.54 < p < -0.53$$

$$1 > \nu_{k_F} > 1/2$$

$$\Re\omega \propto k - k_F$$

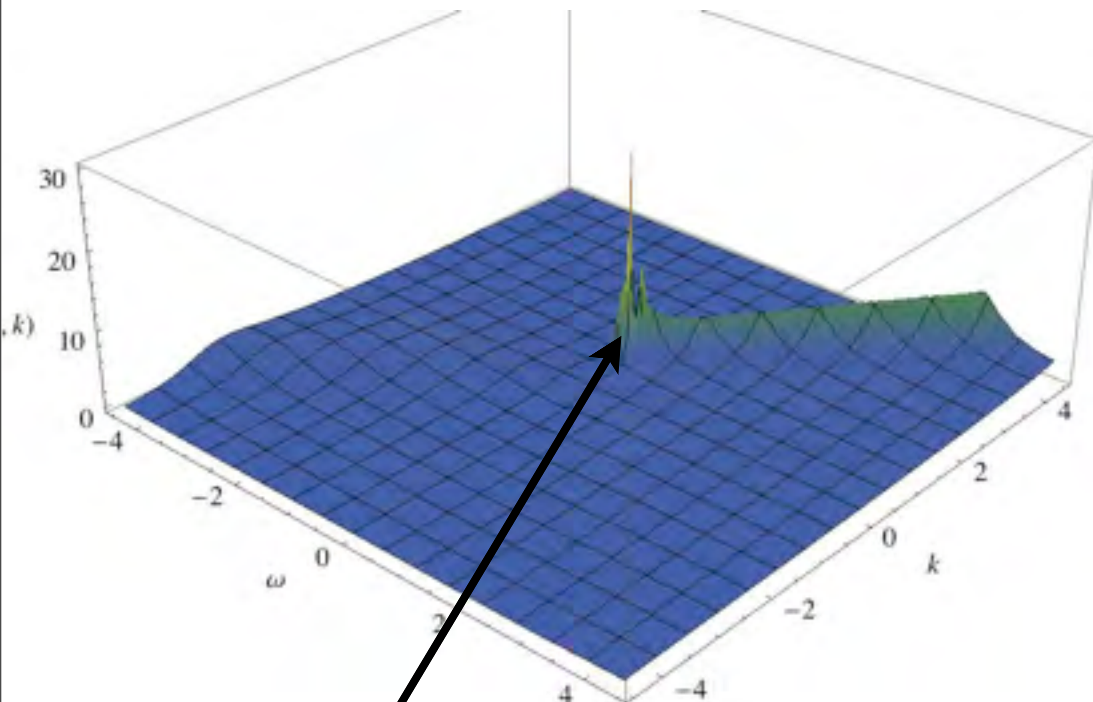
$$\Im\omega \propto (k - k_F)^{2\nu_{k_F}}$$

`Fermi Liquid'

How is the spectrum modified?

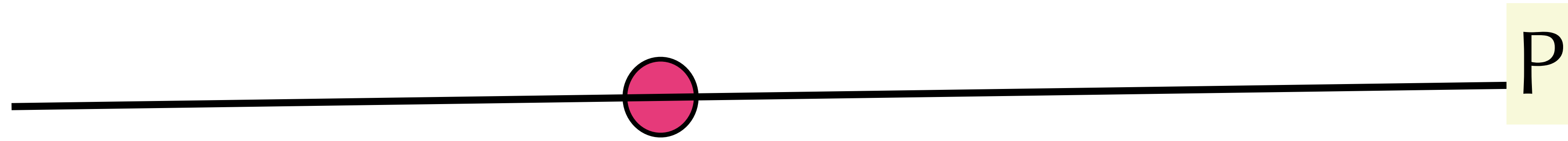
P

P=0

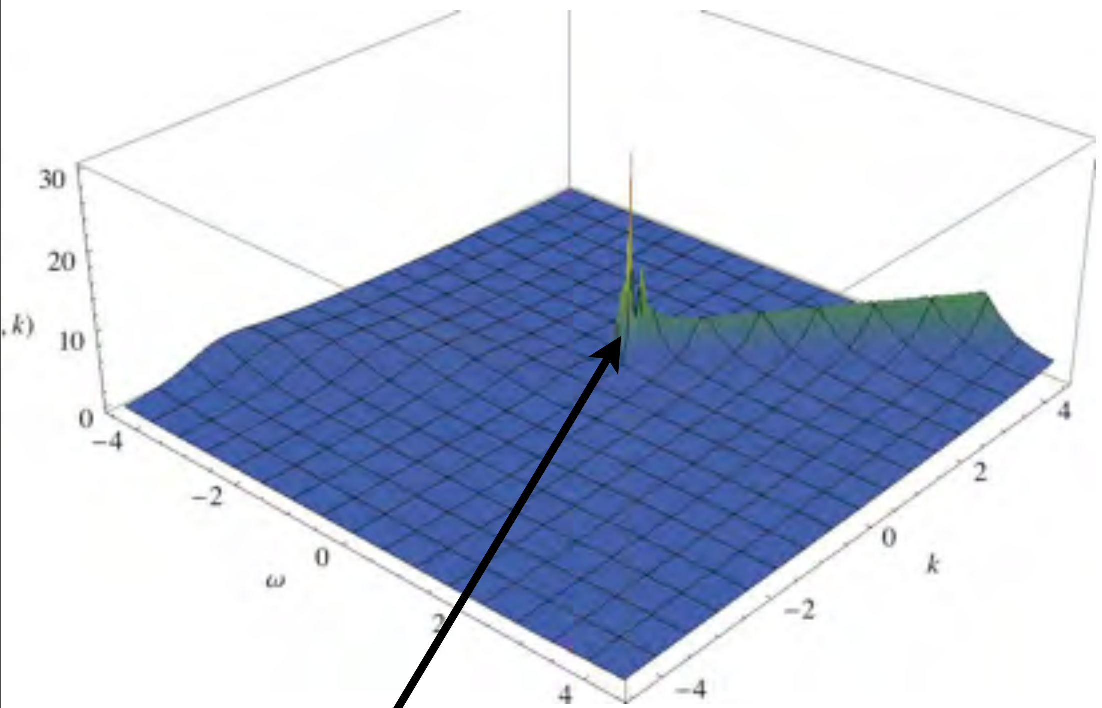


Fermi surface peak

How is the spectrum modified?



P=0



$$p = -0.53$$

$$\nu_{k_F} = 1/2$$

MFL

$$-0.53 < p < 1/\sqrt{6}$$

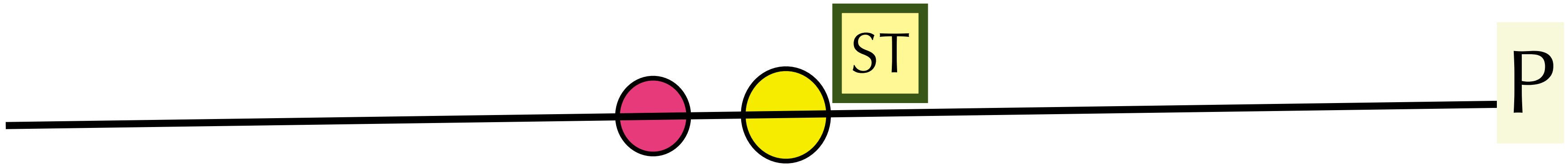
$$1/2 > \nu_{k_F} > 0$$

$$\Re\omega = \Im\omega \propto (k - k_F)^{1/(2\nu_{k_F})}$$

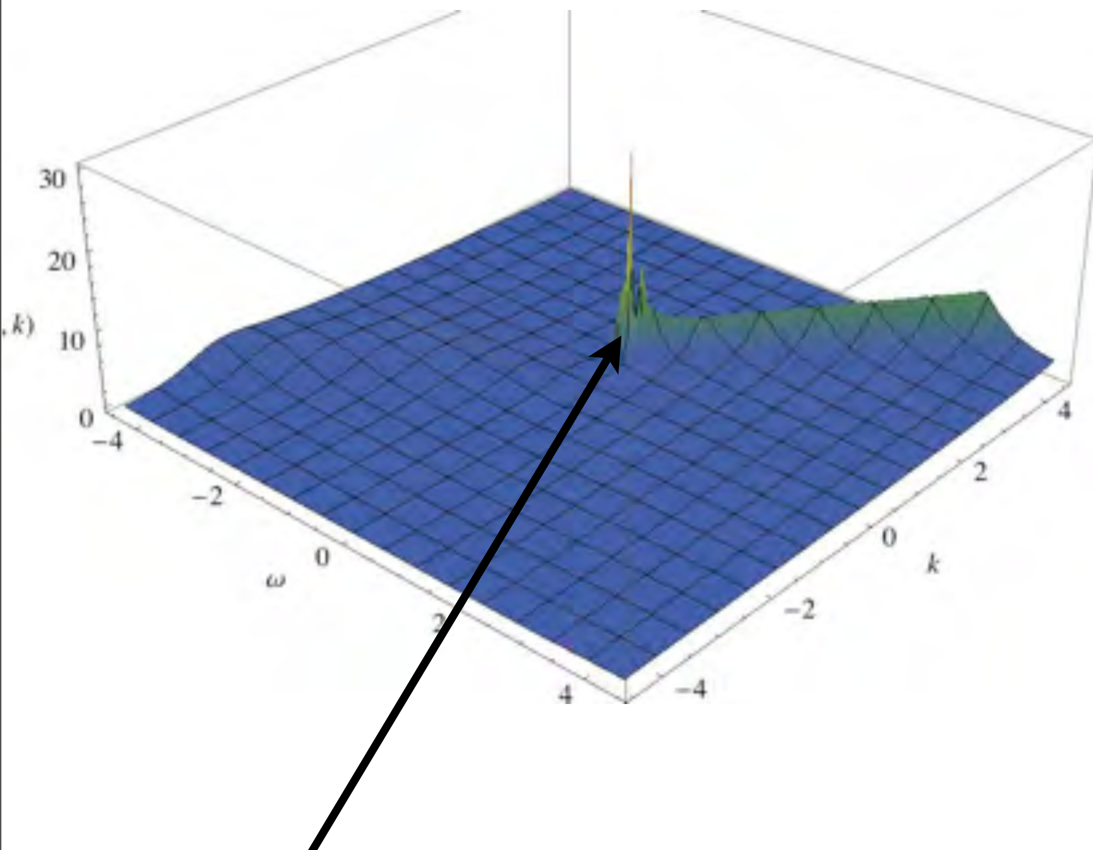
NFL

Fermi surface peak

How is the spectrum modified?



P=0



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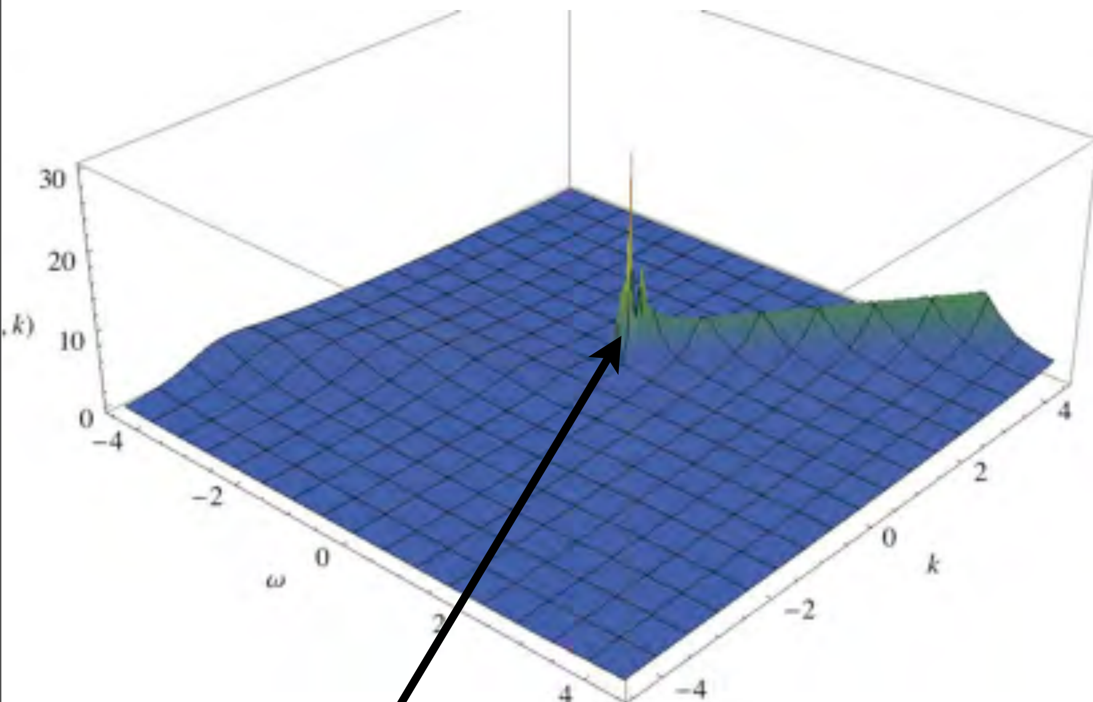
NFL

How is the spectrum modified?

ST

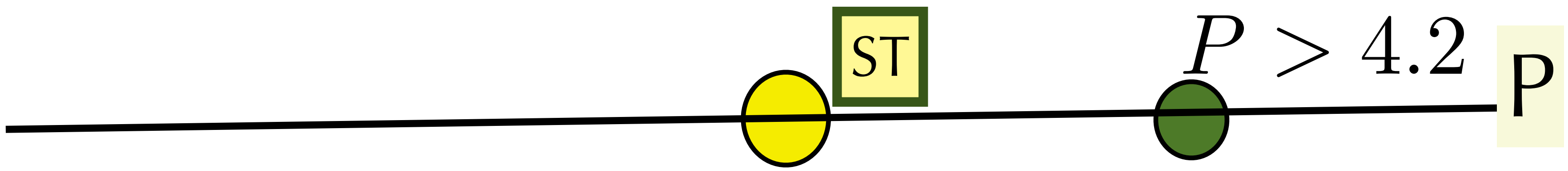
P

P=0

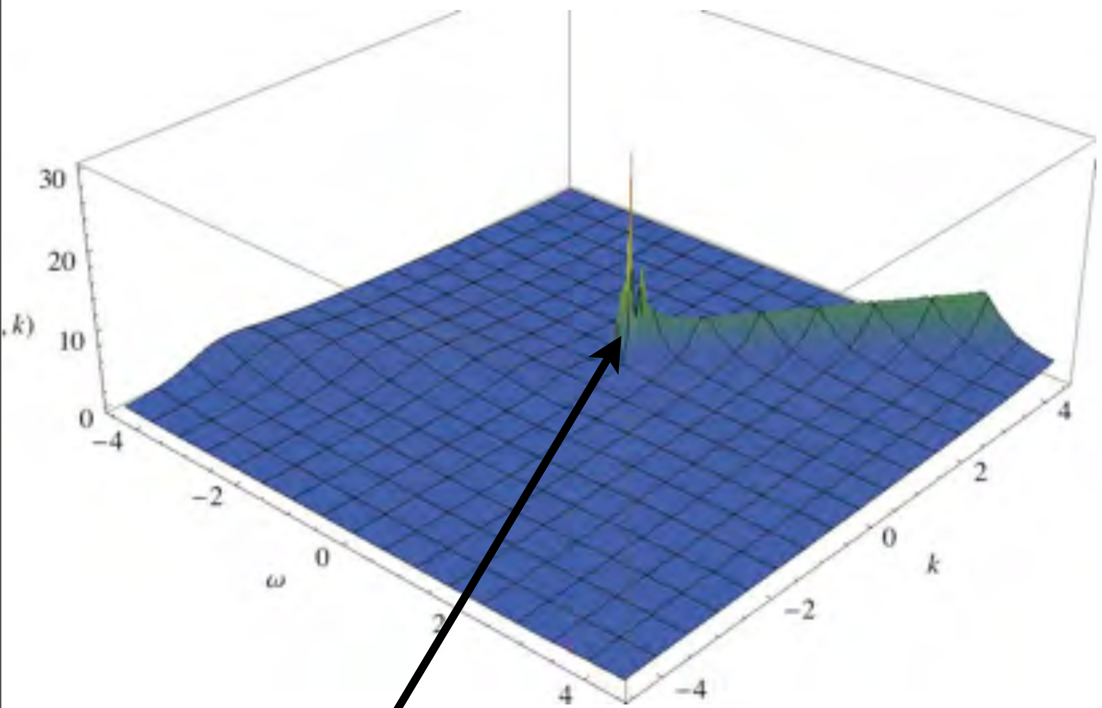


Fermi surface peak

How is the spectrum modified?



P=0



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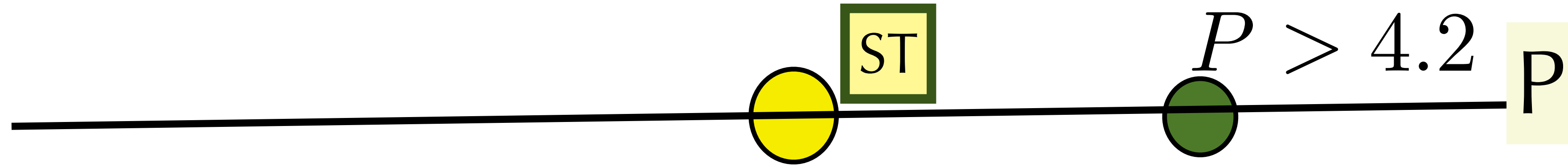
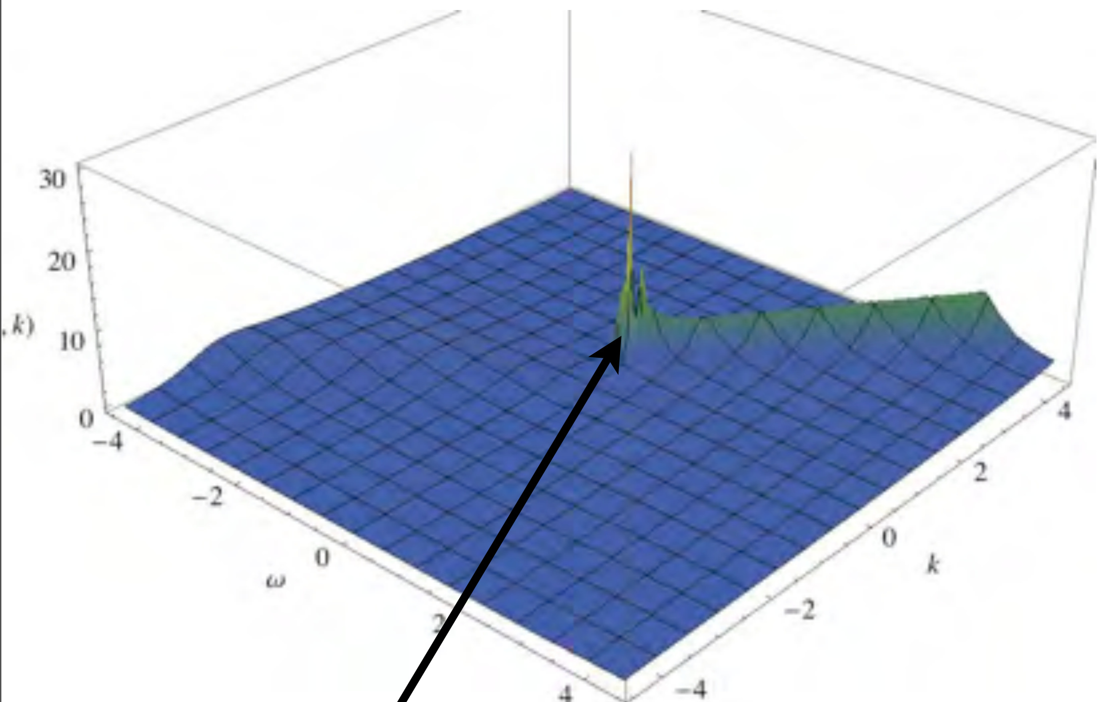
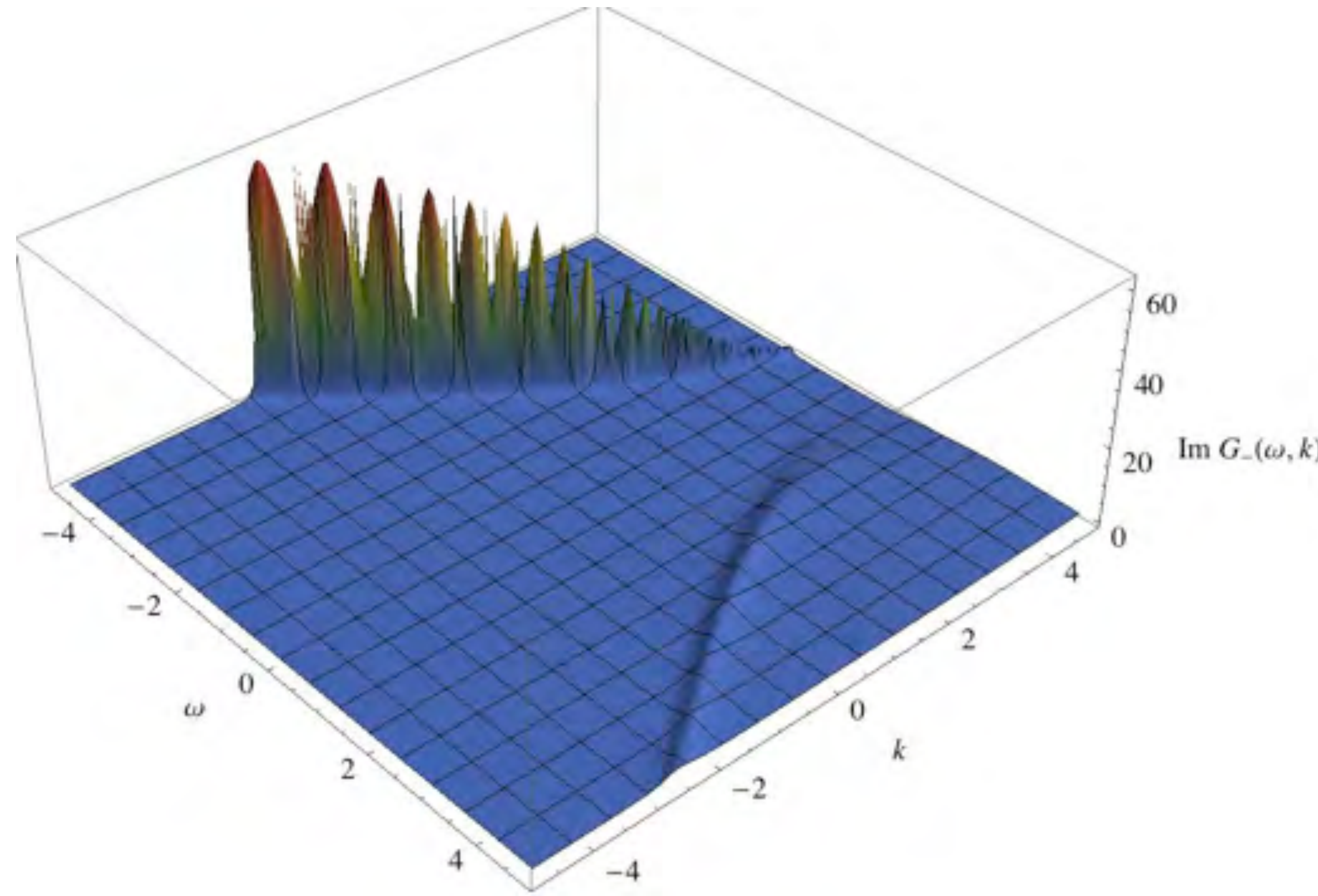


P=0



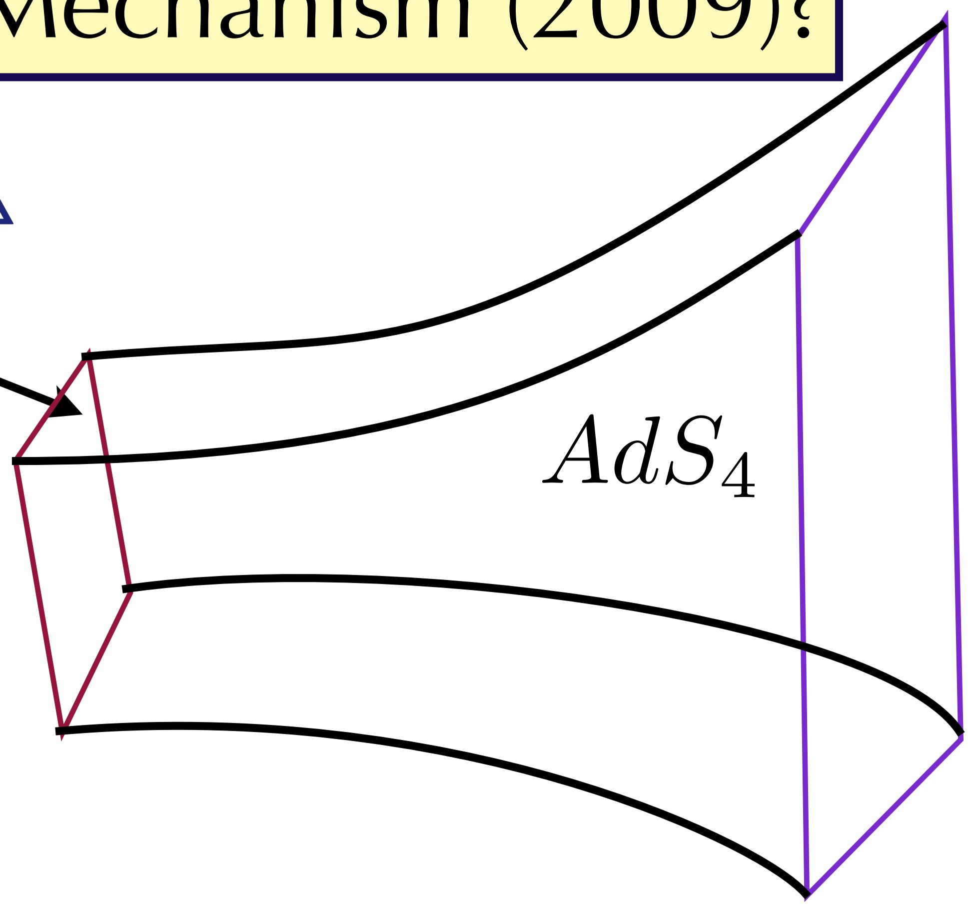


Fermi surface peak

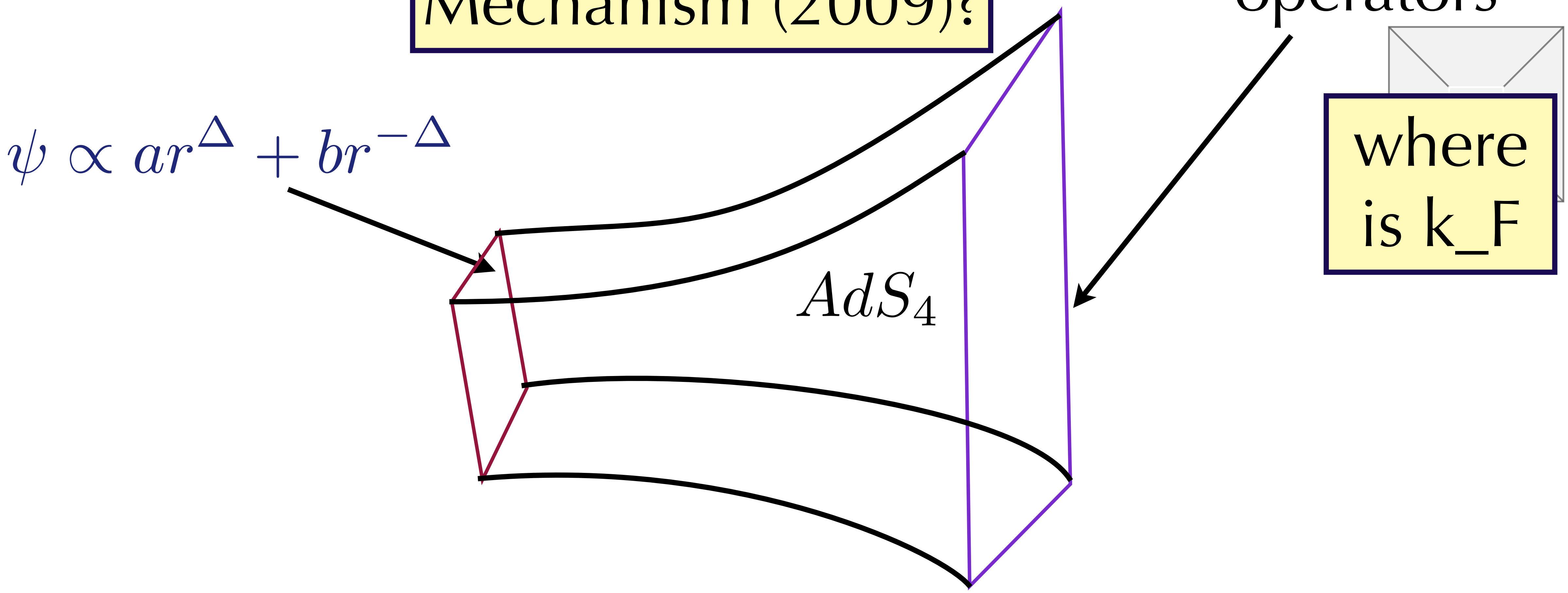
Edalati,Leigh, PP PRL, 106 (2011)

Mechanism (2009)?

$$\psi \propto a r^{\Delta} + b r^{-\Delta}$$



Mechanism (2009)?



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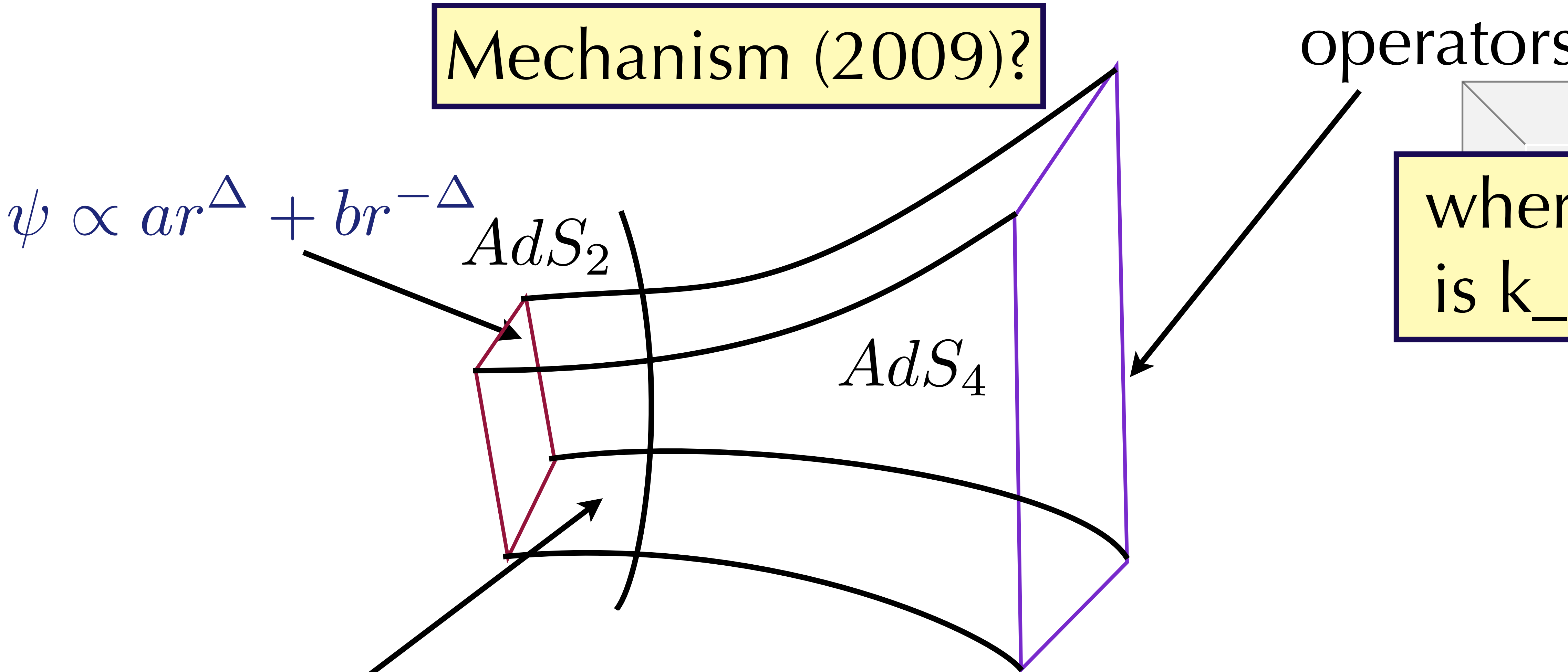
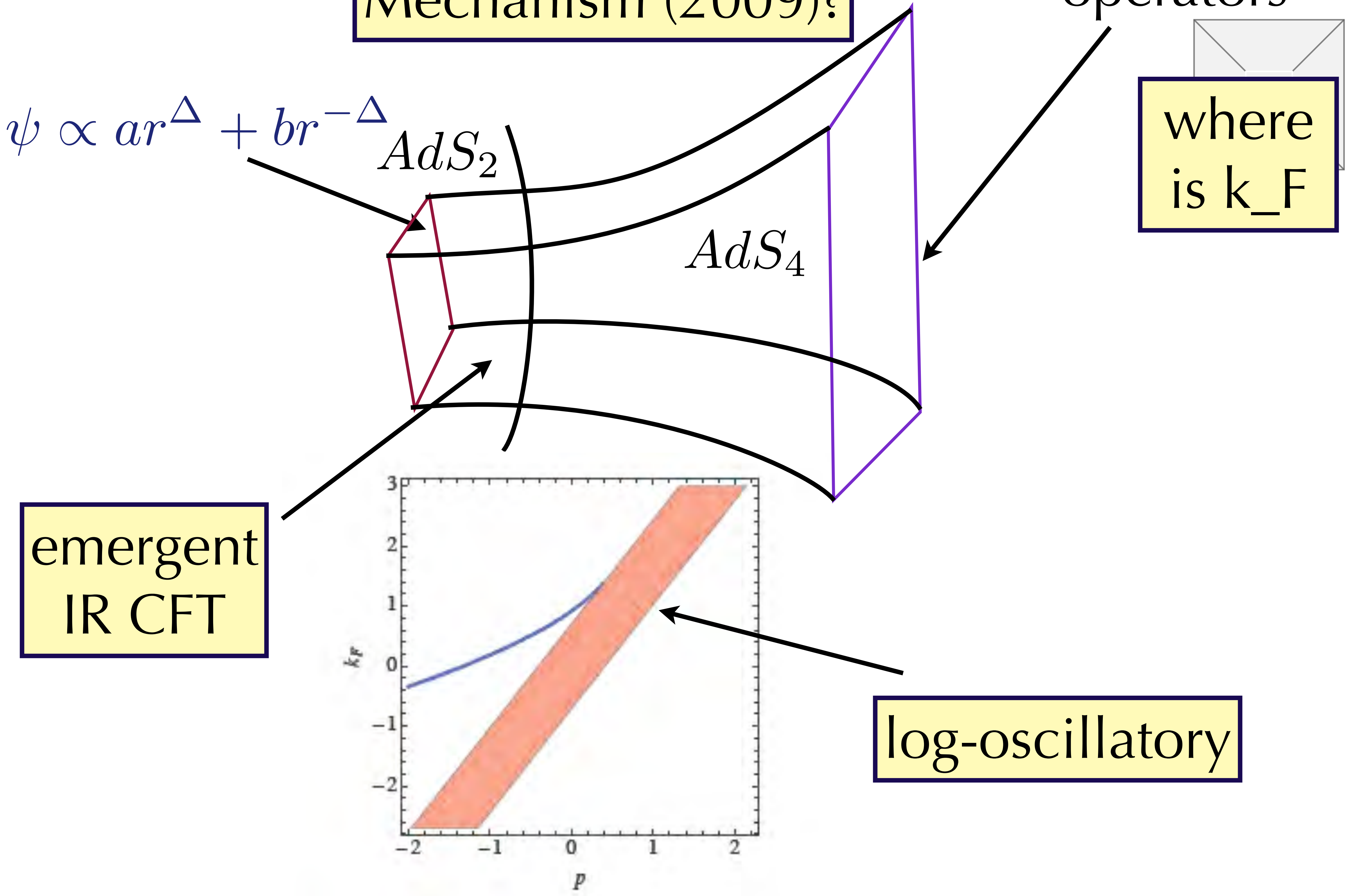


where is k_F

emergent IR CFT

Mechanism (2009)?

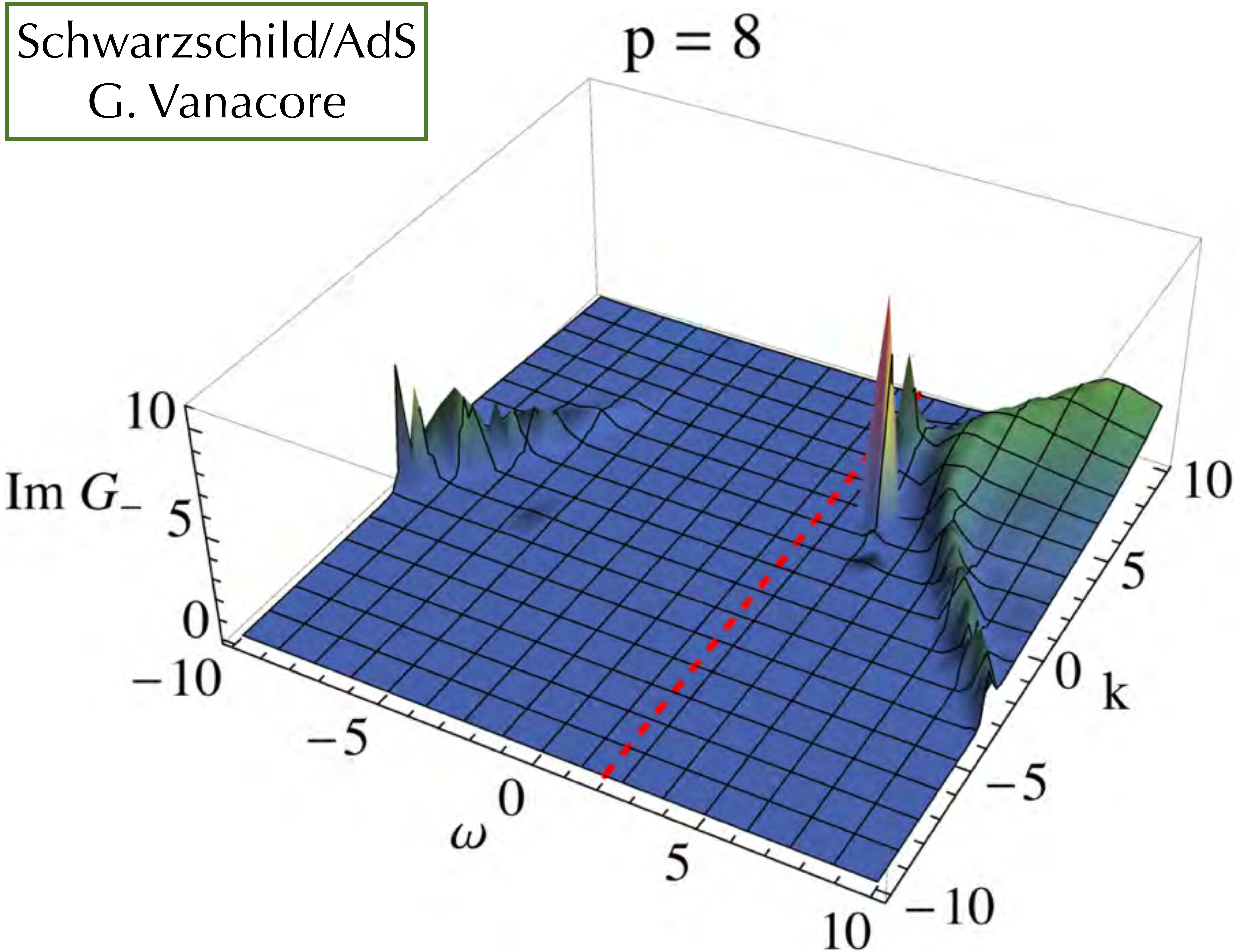


k_F moves into log-oscillatory region: IR $\mathcal{O}_{\pm}$ acquires a complex dimension

Is the log-oscillatory region necessary?

No

Schwarzschild/AdS
G. Vanacore



chiral symmetry and Pauli term

chiral symmetry and Pauli term

$$\psi \to e^{i\alpha\Gamma_5}\psi$$

chiral symmetry and Pauli term

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X breaks chiral symmetry if

$$\{\Gamma_5, X\} \neq 0$$

chiral symmetry and Pauli term

$$\psi \rightarrow e^{i\alpha\Gamma_5}\psi$$

X breaks chiral symmetry if

$$\{\Gamma_5, X\} \neq 0$$

$$\Downarrow \quad \{\Gamma_5, \Gamma_{\mu\nu}F^{\mu\nu}\} \neq 0$$

Pauli term breaks chiral symmetry

chiral symmetry and Pauli term

$$\psi \to e^{i\alpha\Gamma_5}\psi$$

X breaks chiral symmetry if

$$\{\Gamma_5, X\} \neq 0$$

↓ $\{\Gamma_5, \Gamma_{\mu\nu}F^{\mu\nu}\} \neq 0$

Pauli term breaks chiral symmetry

↓

helicity on the boundary (scaling dimension)

+k and -k have different scaling dimensions

+k and -k have different scaling dimensions

hidden duality

Flow equations

$$u^2\sqrt{f(u)}\partial_u \xi_\pm = -2(mL)u\xi_\pm + \left[v_-(u) \mp k\right] + \left[v_+(u) \pm k\right]\xi_\pm^2,$$

$$v_\pm(u) = \frac{1}{\sqrt{f(u)}}\left[\omega + Qq(1-u^{2-d})\right] \pm Qpu^{2-d}.$$

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$$\xi_\pm \to \zeta_\pm \equiv 1/\xi_\pm$$

$$p \to -p$$

$$k \to -k$$

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Green functions are -inverses of one another!!

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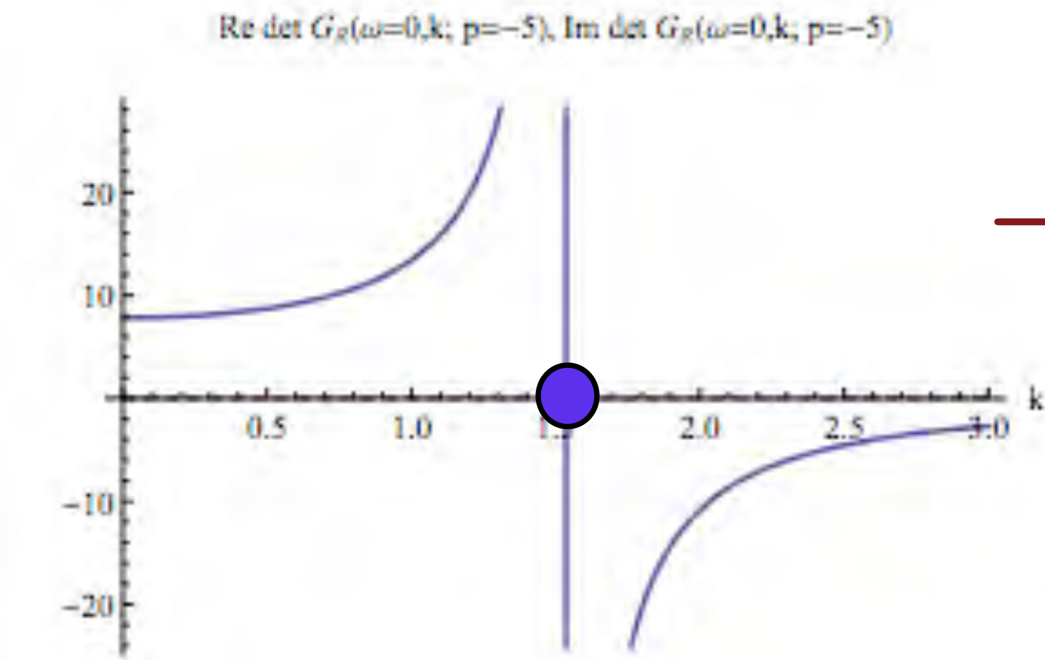
Green functions are -inverses of one another!!

$$\mathrm{Det}\mathrm{G}_\mathrm{R}(\omega, k; m, p) = \frac{1}{\mathrm{Det}\mathrm{G}_\mathrm{R}(\omega, k; -m, -p)}$$

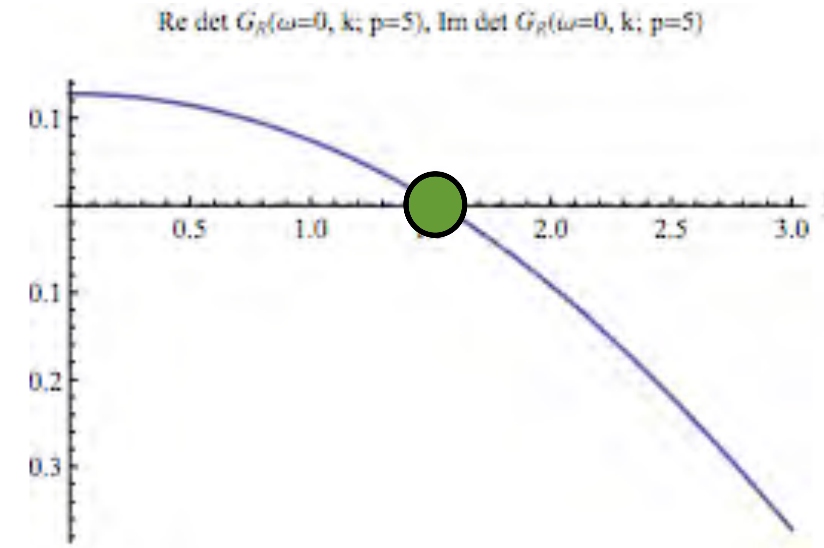
Reissner-Nordstrom/AdS

hep-th: 1404.4010



$$-p \to p$$

poles $\to$ zeros



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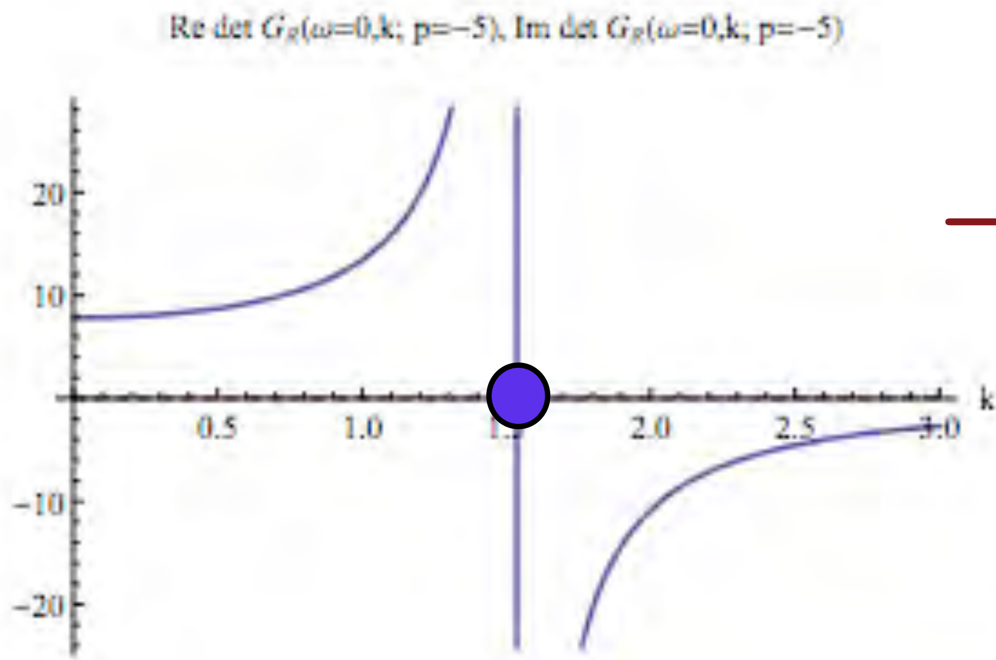

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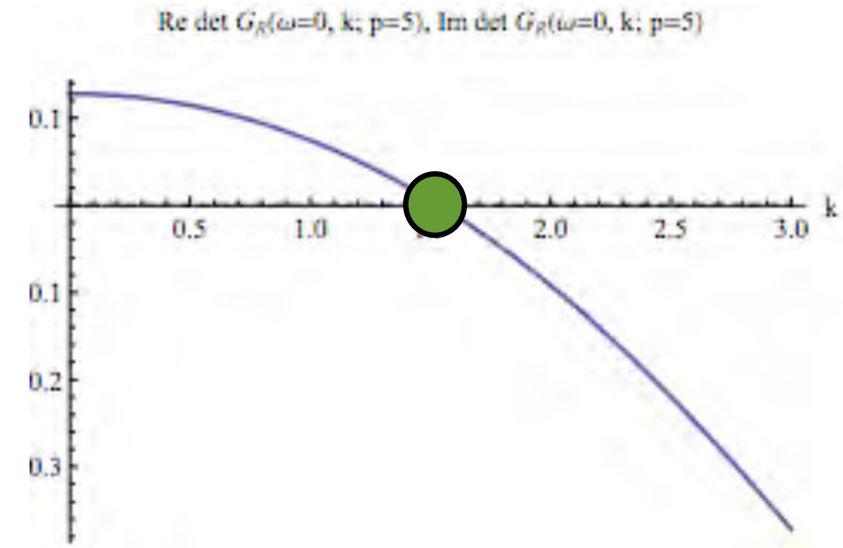


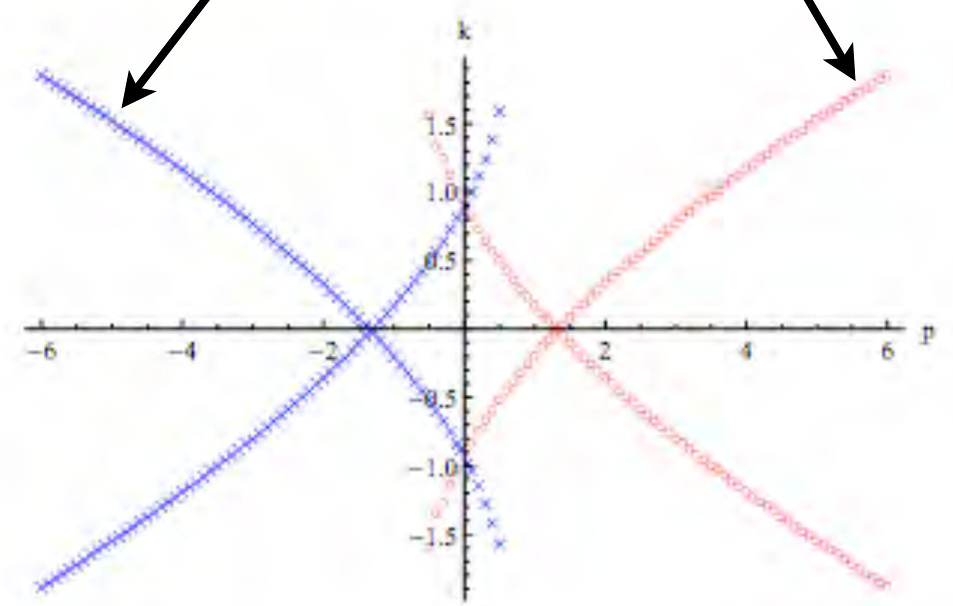
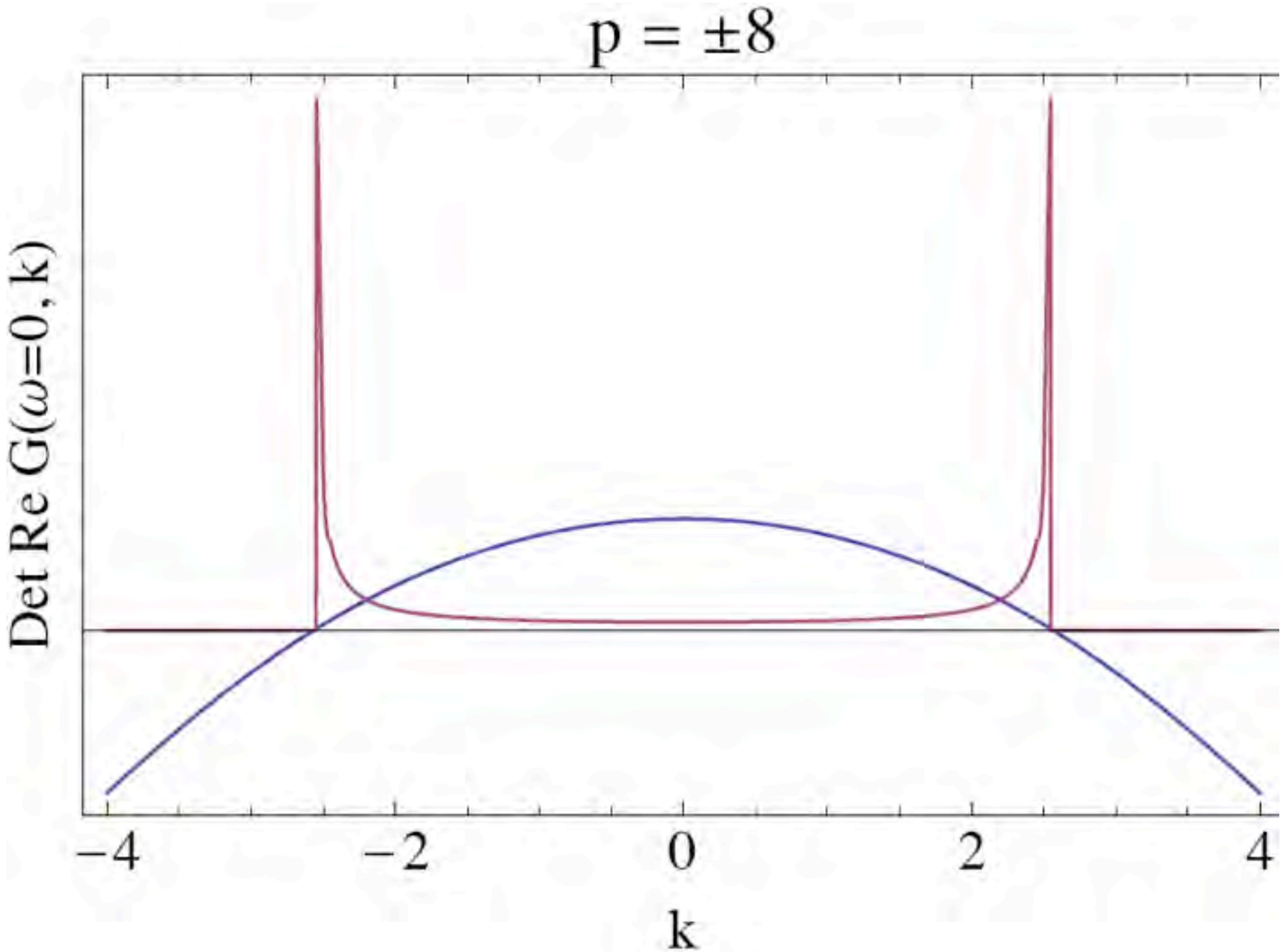


FIG. 4. Poles at $k = k_F$ (blue lines) and zeroes at $k = k_L$ (red lines) *vs.* p with $q = 1$. Notice the symmetry under $k \to -k$, and the duality of poles and zeroes under $p \to -p$.

Schwarzschild/AdS



General Result

Parameter choices	$G_{\pm}(\omega, k; m, p)$	$\mathrm{DetG_R}(\omega, k; m, p)$
$k \leftrightarrow -k$	$G_{\mp}(\omega, -k; m, p)$	—
$m = 0$ $p = 0$	$\frac{-1}{G_{\pm}(\omega,-k)}$	1
$m \neq 0$ $p \neq 0$	$\frac{-1}{G_{\pm}(\omega,-k;-m,-p)}$	$\frac{1}{\mathrm{DetG_R}(\omega,k;-m,-p)}$

Gap is due to zeros not vanishing of Z!

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Gap is due to zeros not vanishing of Z!

Mott problem

Fermi arcs?

Fermi arcs?

consider

$$ip\Gamma_{\mu\nu}F^{\mu\nu} \to ip\Gamma\Gamma_{\mu\nu}F^{\mu\nu}$$

Fermi arcs?

consider

$$ip\Gamma_{\mu\nu}F^{\mu\nu} \rightarrow ip\Gamma\Gamma_{\mu\nu}F^{\mu\nu}$$

$$\Gamma = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

Fermi arcs?

consider

$$ip\Gamma_{\mu\nu}F^{\mu\nu} \to ip\Gamma\Gamma_{\mu\nu}F^{\mu\nu}$$

$$\Gamma = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

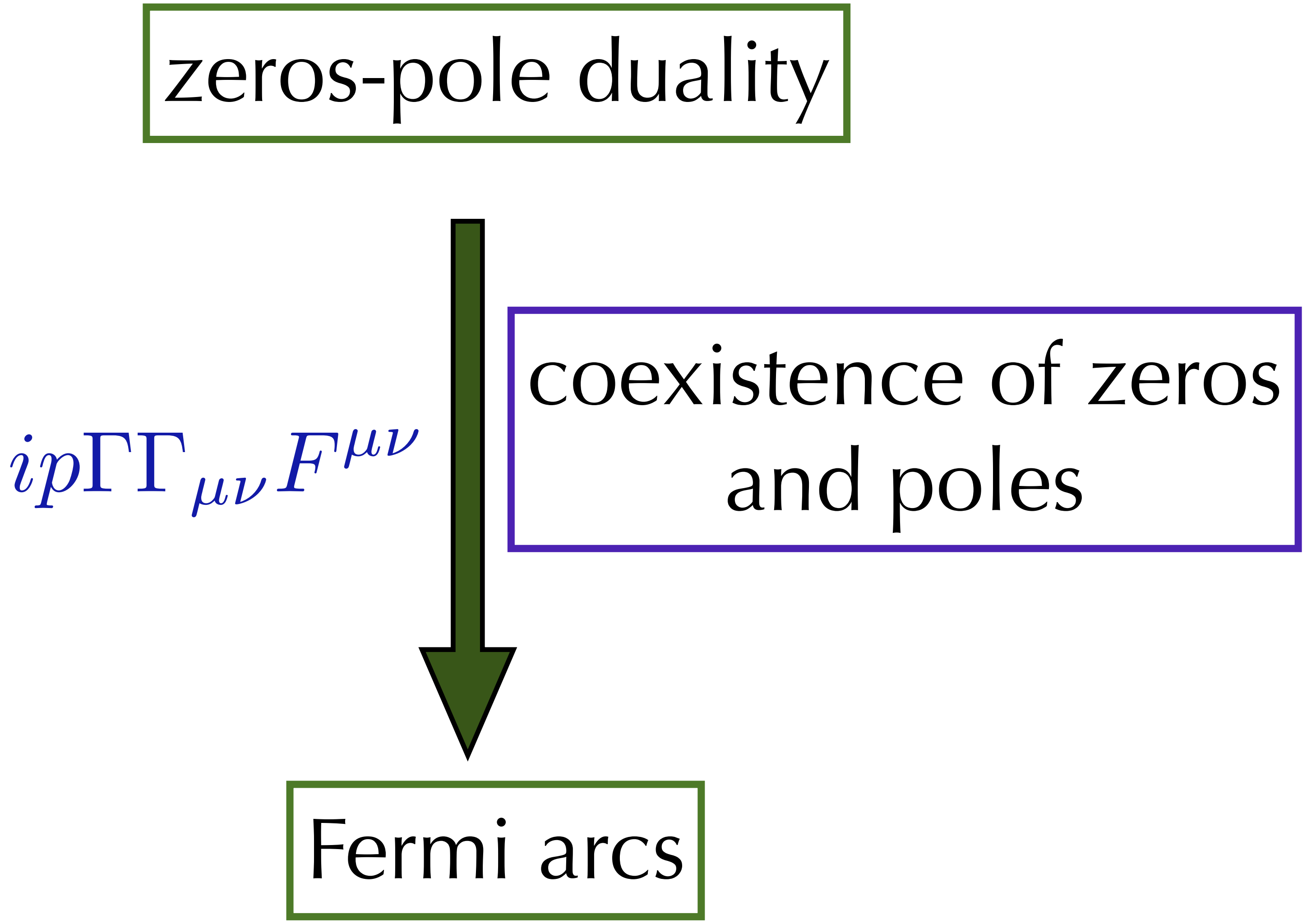
-k and +k have different sign for the Pauli term!!

zeros-pole duality

zeros-pole duality

$$ip\Gamma\Gamma_{\mu\nu}F^{\mu\nu}$$

coexistence of zeros and poles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Fermi arcs

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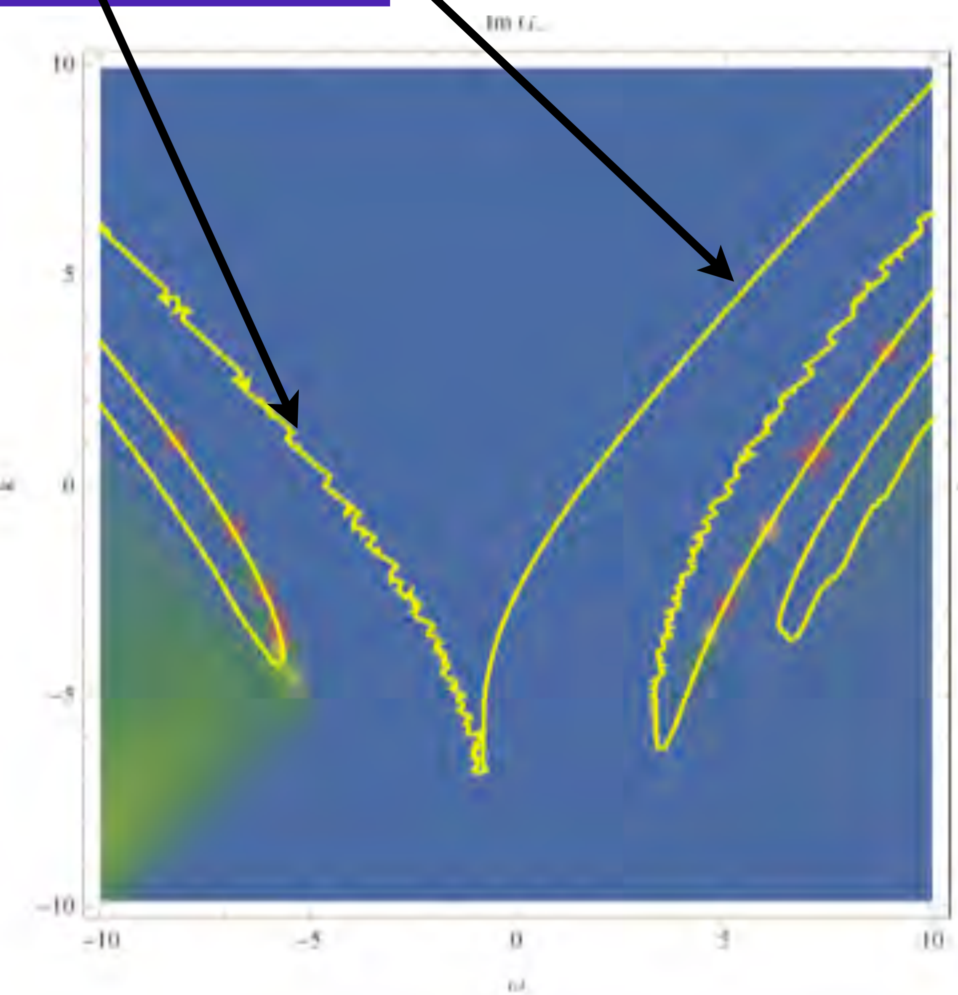

zeros-pole duality

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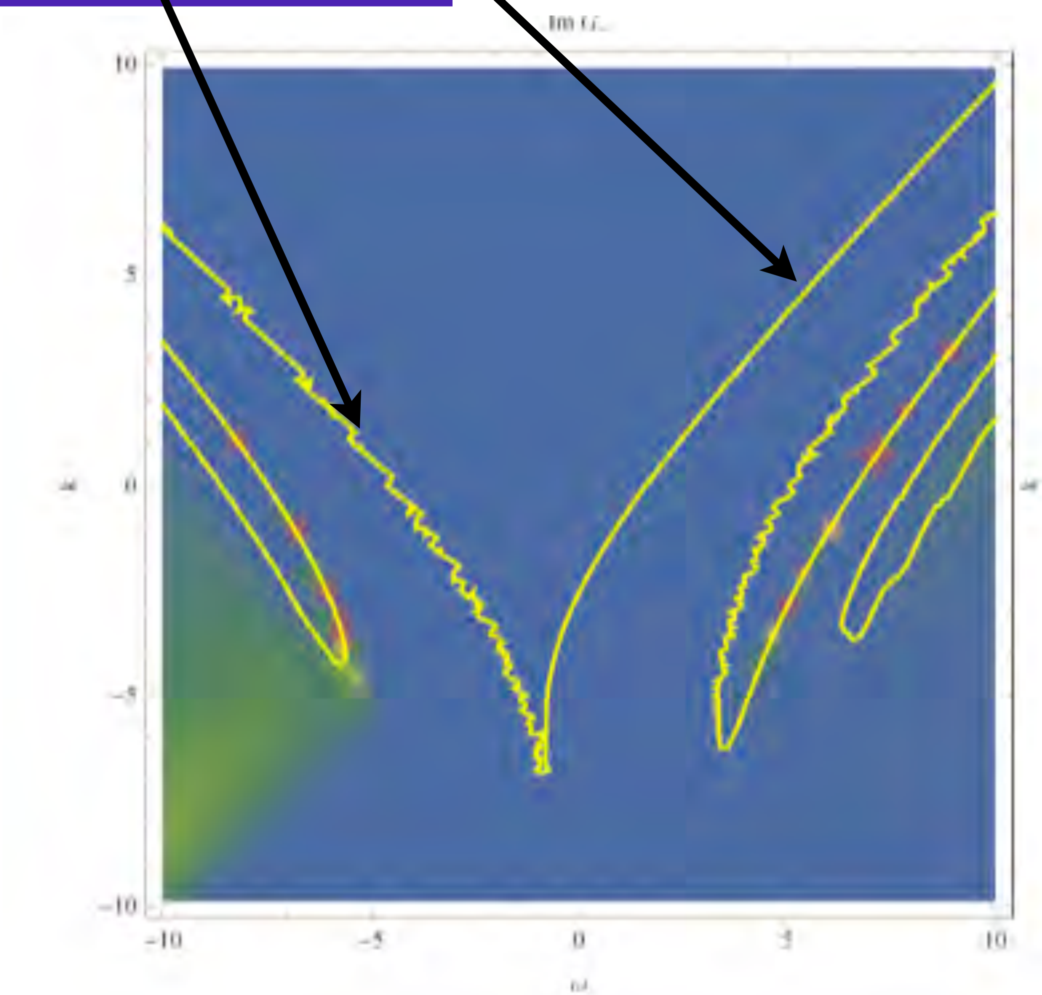
coexistence of zeros and poles



Fermi arcs

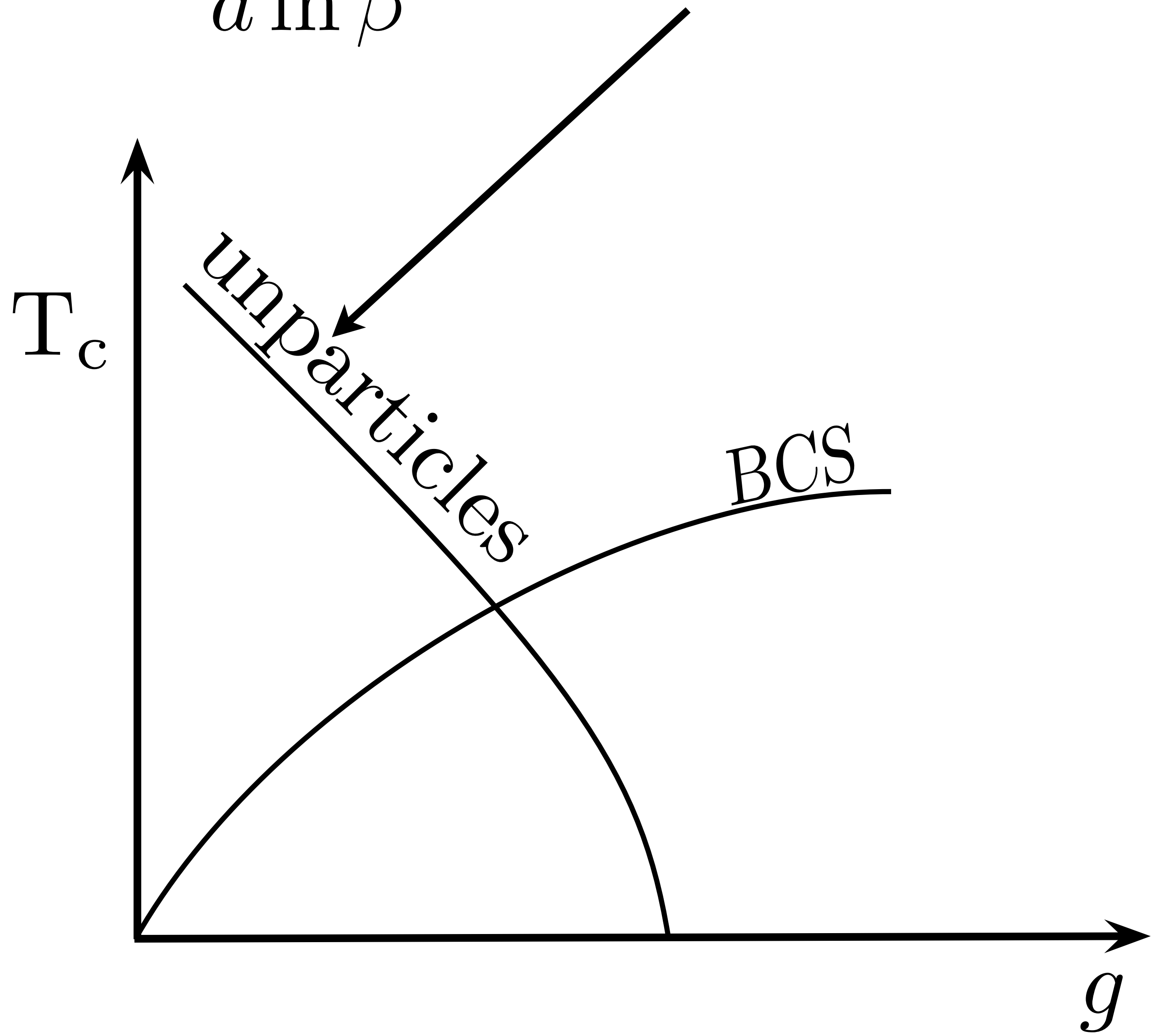
effective spin-orbit

Superconducting Instability with unparticles

ladder approximation

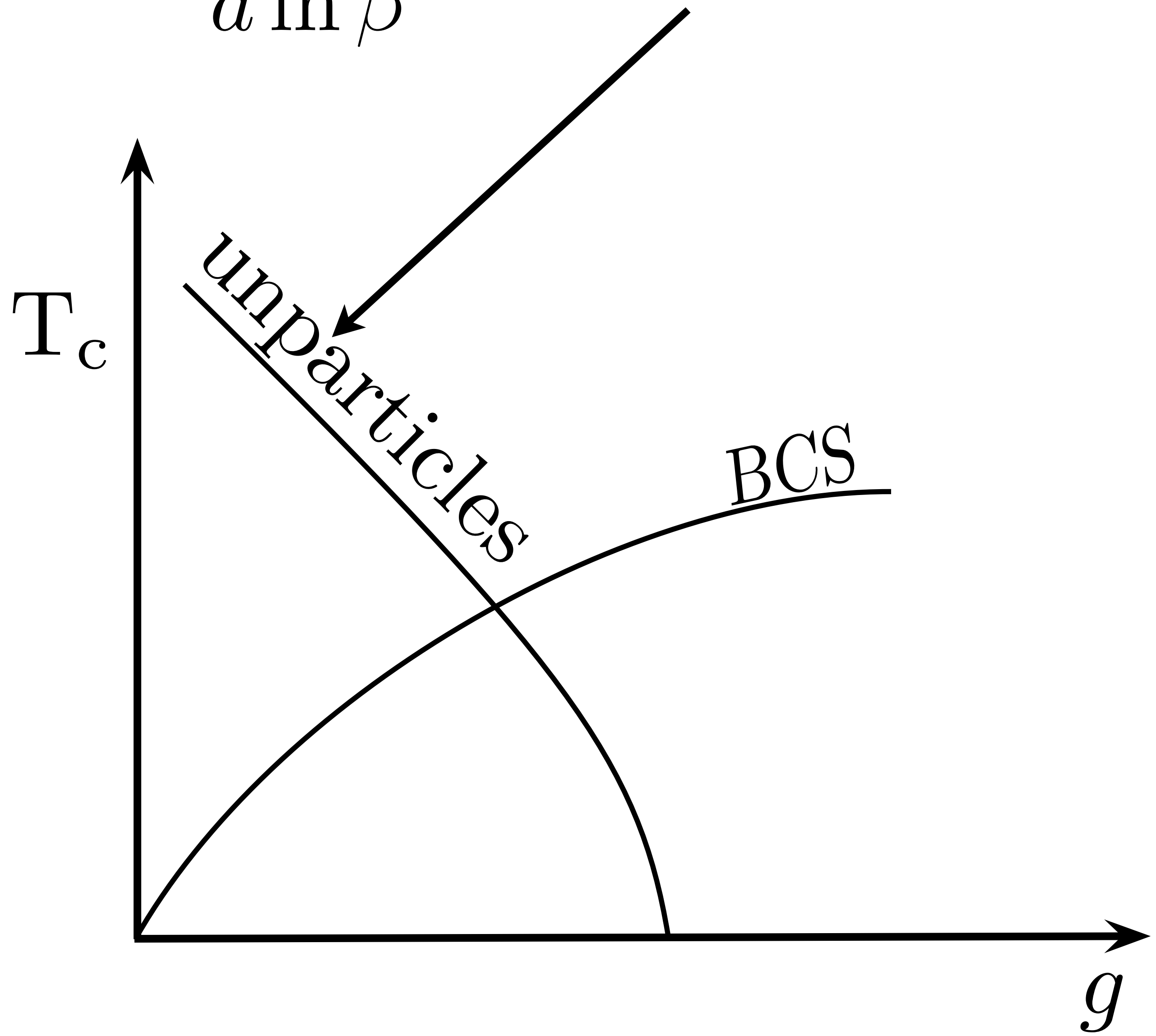
$$1 = \lambda T \sum_{n\vec{k}} |w_{n\vec{k}}|^2 G_U\left(\omega_n, \vec{k}\right) G_U\left(-\omega_n, -\vec{k}\right),$$

$$\frac{d\ln g}{d\ln\beta} = 4d_U - d > 0$$



tendency towards pairing (any instability which establishes a gap)

$$\frac{d\ln g}{d\ln\beta} \quad = \quad 4d_U - d > 0$$



tendency towards pairing (any instability which establishes a gap)

see also, dresden group, 1407.8492

prediction: algebraic pairing susceptibility

$$G\left(\Lambda k, i\Lambda\omega_n\right) \quad = \quad \Lambda^{2d_U - D} G\left(k, i\omega_n\right).$$

$$\chi\left(0, i\Omega\right) \quad = \quad \frac{T}{N}\sum_{n,k} G\left(-k, -i\omega_n\right) G\left(k, i\omega_n + i\Omega\right)$$

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see also J. Zaanen

zeros-poles duality (Fermi arcs)
chiral symmetry breaking??