UNDERSTANDING THE DYNAMICS OF THE CHIRAL VORTICAL EFFECT

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based on:
arXiv:1312.1204
K. Landsteiner, E. Megías, F. P-B

European Union
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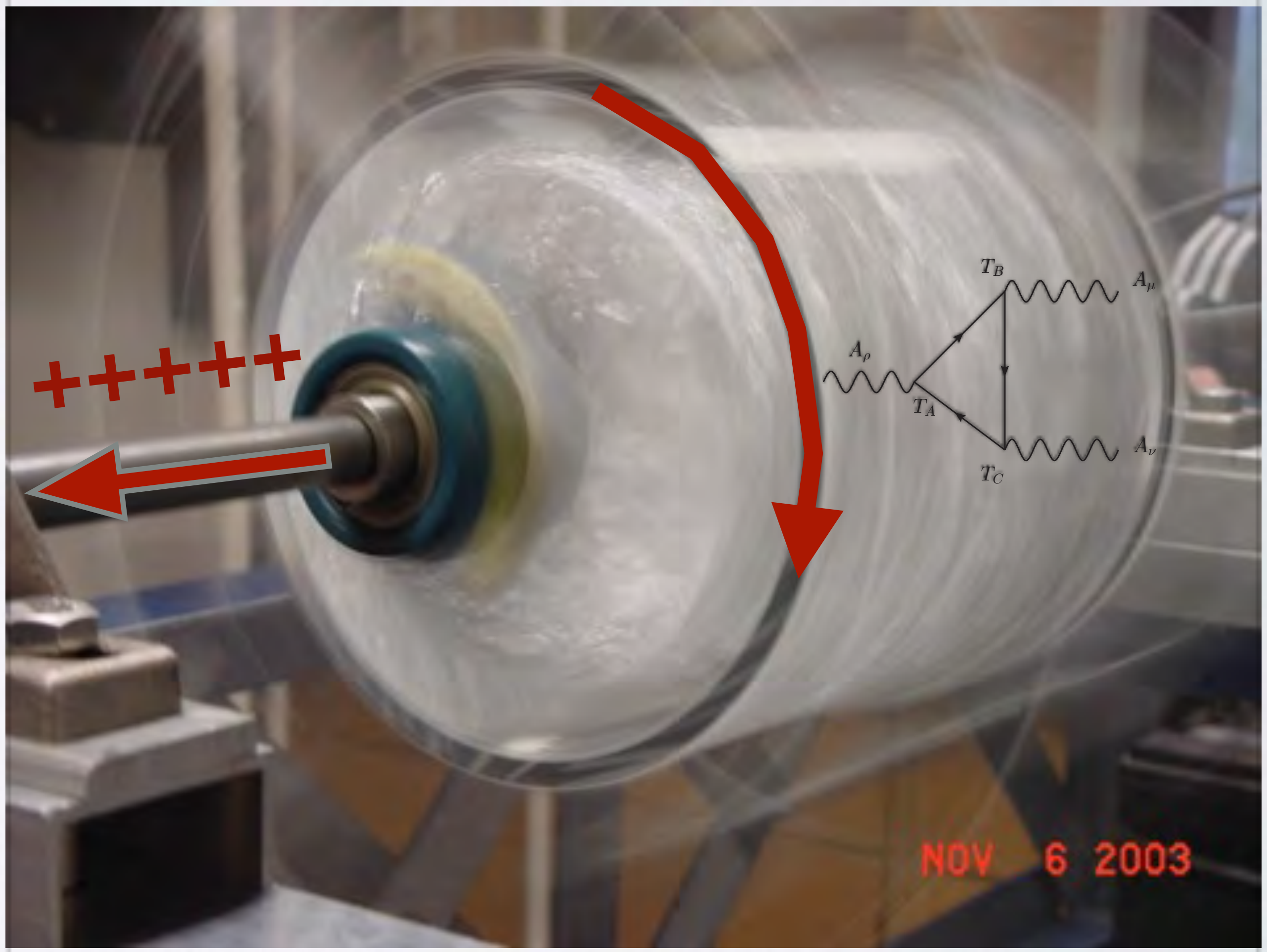
OUTLINE

- the chiral magnetic effect (CME) and quantum anomalies
  - time evolution
- the chiral vortical effect (CVE)
  - holographic model
  - time evolution
    - consistency checks
- physical implications

MOTIVATION

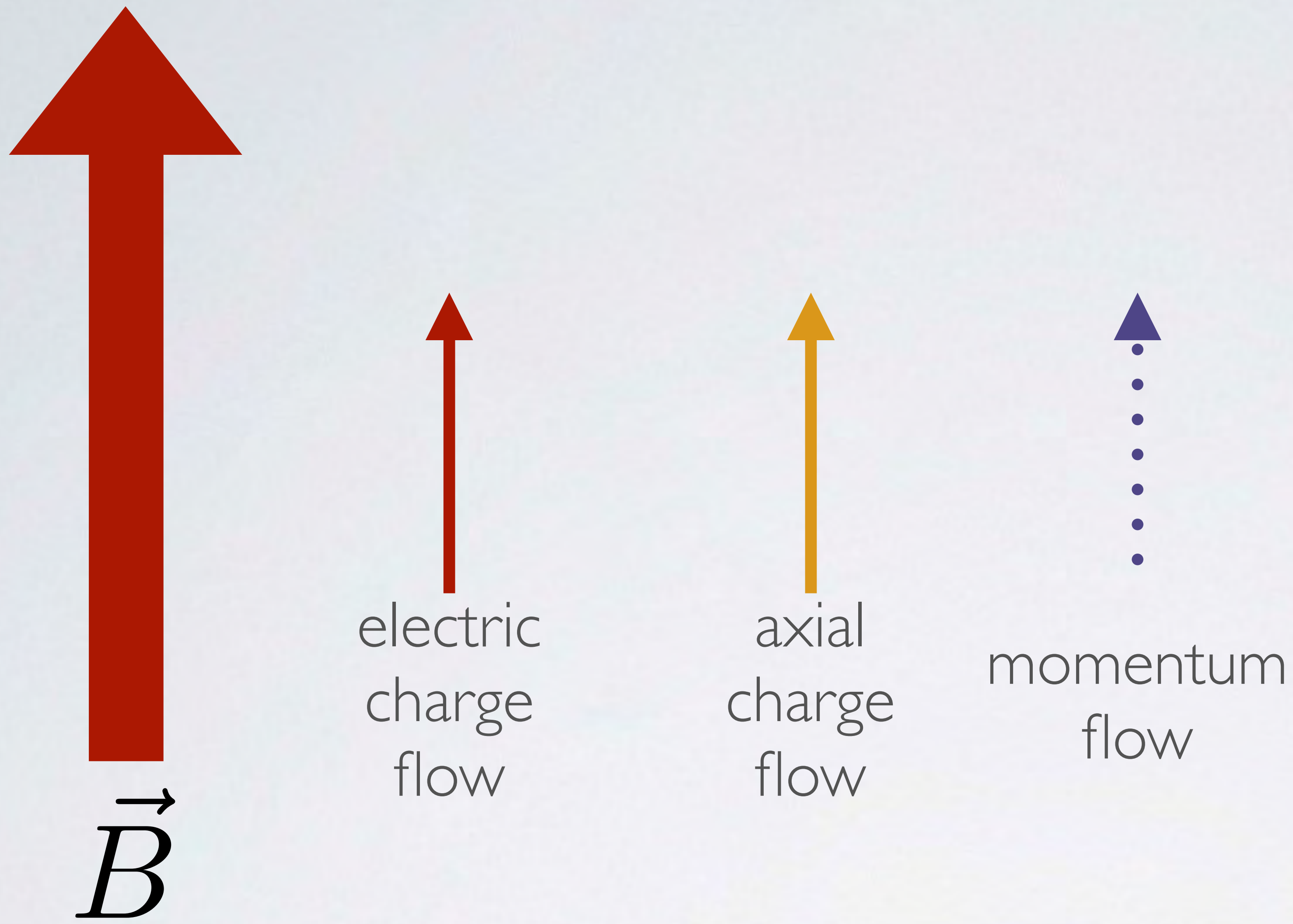
- quantum anomalies are interesting
- mixed gravitational anomaly could be measure for the first time
- non dissipative transport
- the non dissipative nature is not destroyed at high temperature (could be powered in some cases)
- macroscopical effect of a purely quantum property
- origin of the charge separation observed in the QGP???
- possibly realized in Weyl semi-metals
- non renormalization theorems
- …



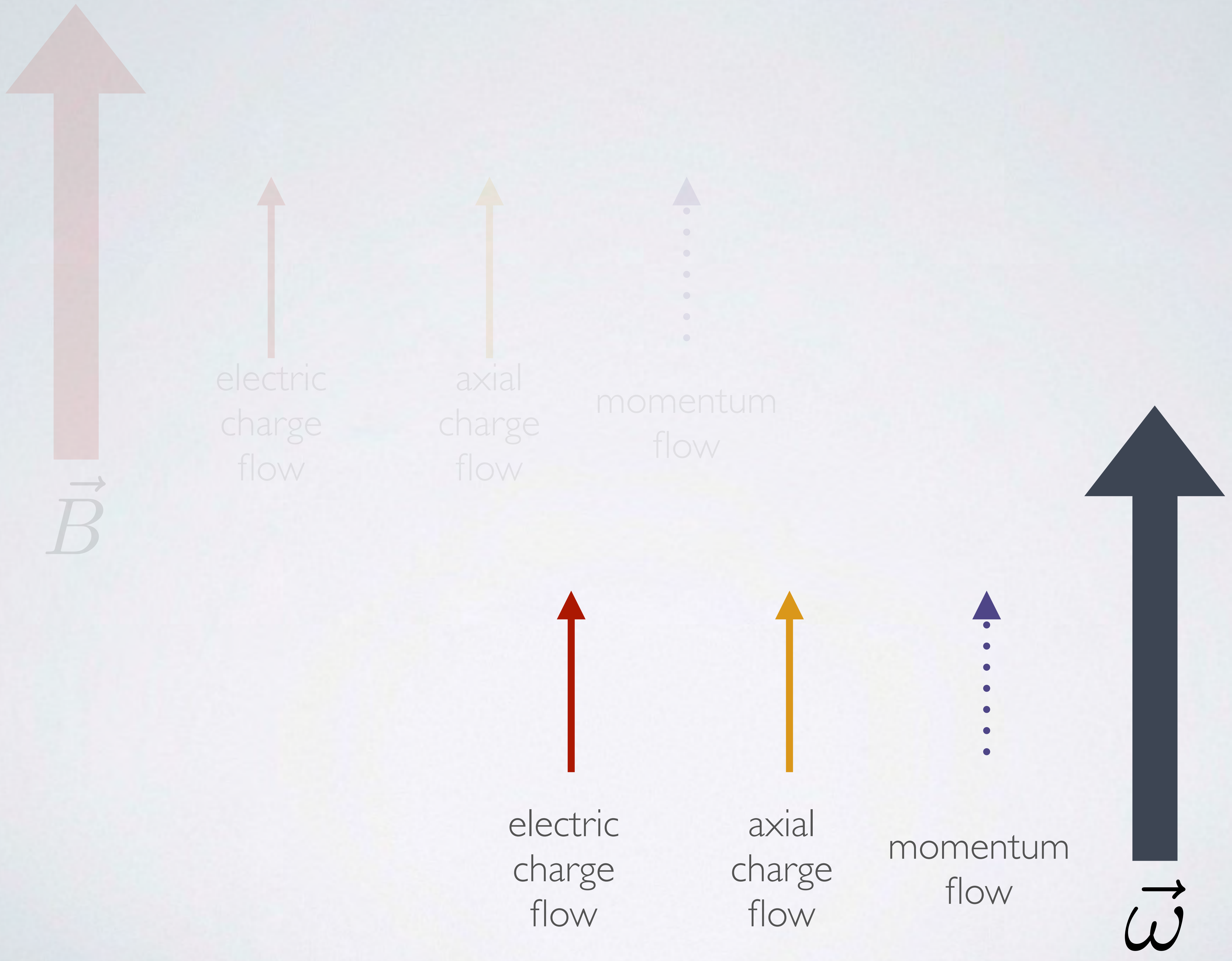
ANOMALIES

what do we know about quantum anomalies?

- In a field theory global symmetries can be violated at the quantum level
- Theories with massless fermions have axial and vector global symmetries
- It is not possible to preserve in the QFT (even dimensions) both symmetries
- In (3+1)d there are two types of axial anomalies



$\vec{B}$
electric charge flow
axial charge flow
momentum flow



$\vec{B}$
electric charge flow
axial charge flow
momentum flow
electric charge flow
axial charge flow
momentum flow
$\vec{\omega}$

Kubo formulas

$$j_\varepsilon^i = T^{ti}$$

$$\vec{j}_a = \begin{pmatrix} \vec{j}_e \\ \vec{j}_5 \\ \vec{j}_\varepsilon \end{pmatrix}$$

- $\vec{j}_e$ ← charge transport
- $\vec{j}_5$ ← chirality transport
- $\vec{j}_\varepsilon$ ← energy transport

Kubo formulas

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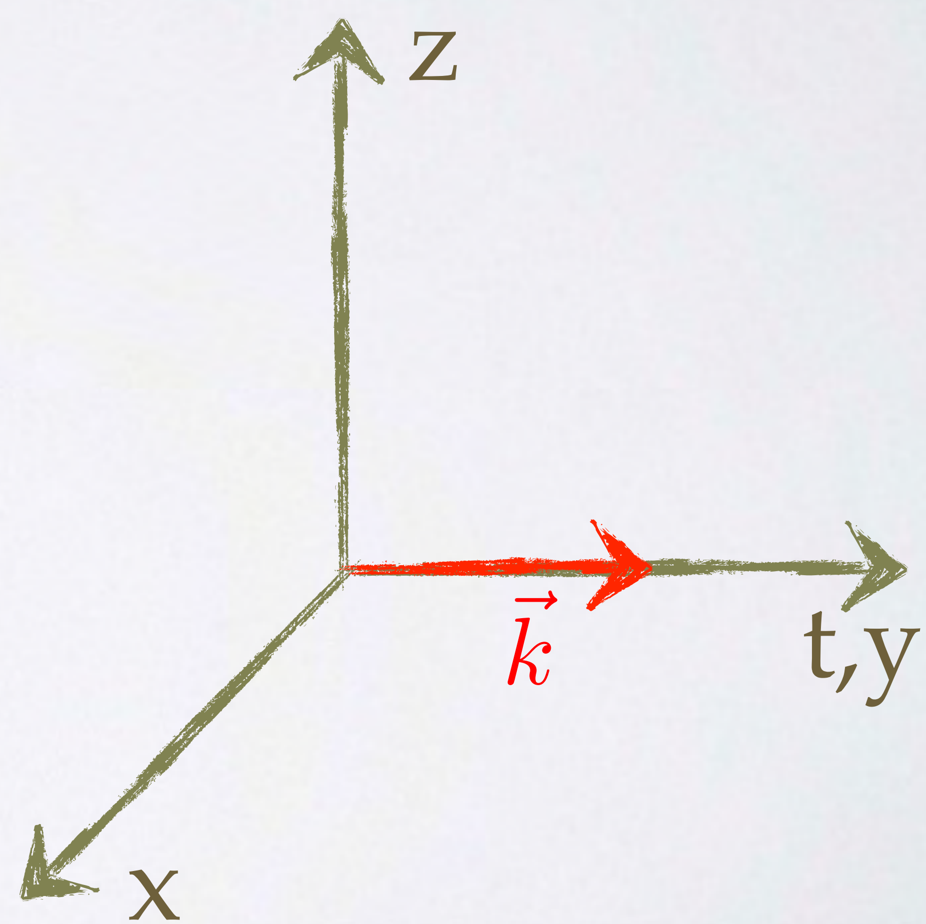
- $\vec{j}_e$ ← charge transport
- $\vec{j}_5$ ← chirality transport
- $\vec{j}_\varepsilon$ ← energy transport

$$\vec{j}_a = \sigma_a^B \vec{k} \times \vec{A}$$

$$\sigma_a^B = \lim_{q_z \to 0} \frac{i}{q_z} G^R_{j_a^x j^y}(0, q_z)$$



Kubo formulas for anomaly induced transport

$$\vec{J} = \sigma^V \nabla \times \vec{v}$$

[Amado, Landsteiner & F. P-B, JHEP.1105 (2011)]

$$ds^2 = (\eta_{\mu\nu} + \epsilon h_{\mu\nu})dx^\mu dx^\nu$$

$$u^\mu = (1, \vec{0}) \longrightarrow u_\mu = g_{t\mu}$$

$$A_i^g \equiv g_{ti}$$

$$\vec{J} = \sigma^V \nabla \times \vec{A}^g$$

$$\sigma^V = - \lim_{q_z \to 0} \frac{i}{q_z} G^R_{j^x\, T^{ty}}(0, q_z)$$

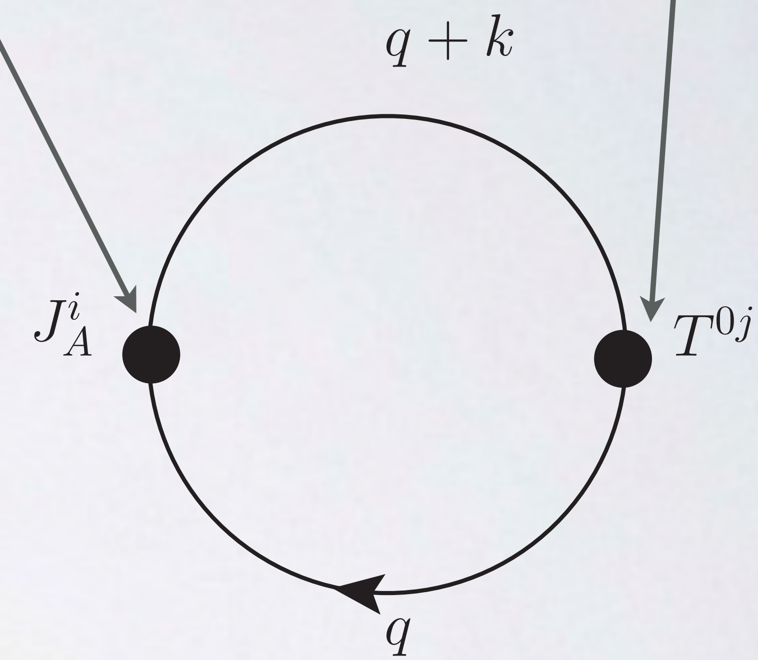
free fermions
&
anomaly induce transport

$$T^{0i} = \frac{i}{4}\bar{\psi}(\gamma^0\partial^i + \gamma^i\partial^0)\mathcal{P}_+\psi$$

$$\vec{j}_c = \bar{\psi}\, T_c \vec{\gamma}\mathcal{P}_+\psi$$

$$\mu^f = \sum_a q_a^f \mu_a \,, \quad H_a = q_a^f \delta_g^f$$

$$\sigma = \lim_{k\to 0} \frac{1}{k}$$

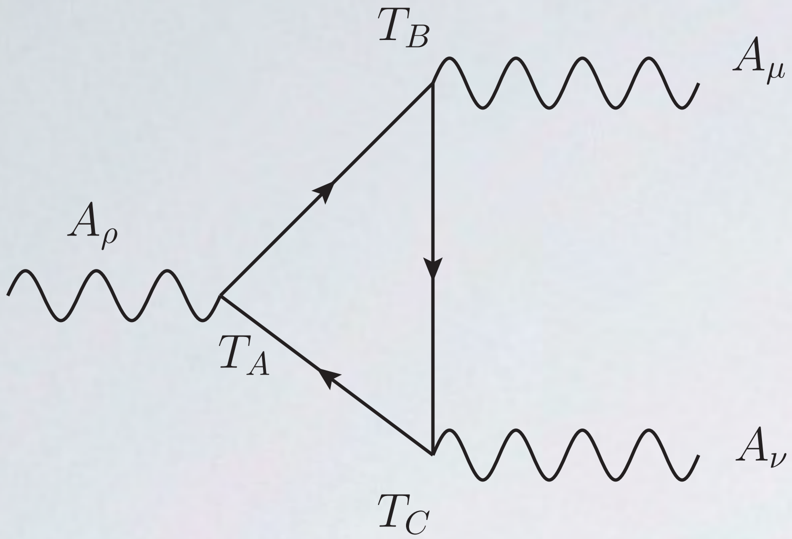


$$\sigma_{ab}^B = \frac{1}{4\pi^2}\sum_c \mathrm{Tr}(T_a\{T_b, H_c\})\mu_c$$

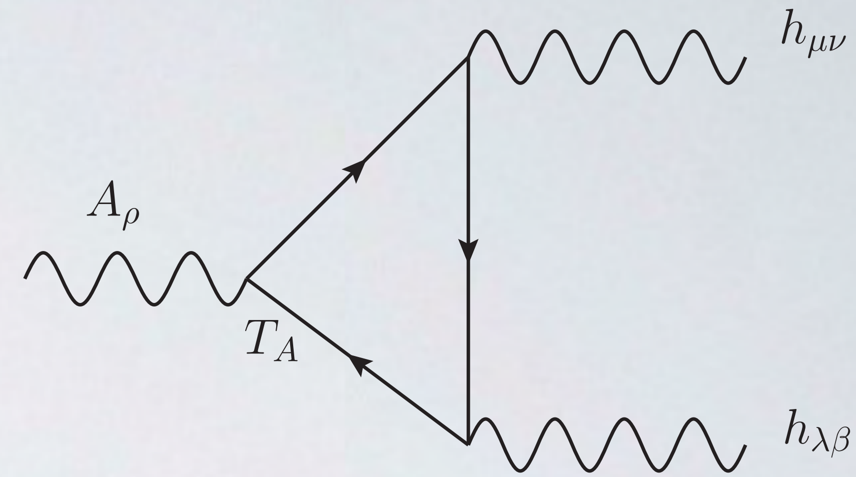
[Kharzeev & Warringa, PRD.80 (2009)]

$$\sigma_a^V = \frac{1}{16\pi^2}\left[\sum_{b,c} \mathrm{Tr}(T_a\{H_b, H_c\})\mu_b\mu_c + \frac{2\pi^2}{3}T^2\mathrm{Tr}(T_a)\right]$$

[Landsteiner, Megías & F. P-B, PRL. 107 (2011)]



$c_{abc} \sim \mathrm{Tr}[T_a\{T_b, T_c\}]$



$b_a \sim \mathrm{Tr}[T_a]$

in the case of interest for QCD

$U(1)_V \times U(1)_A$

$$\nabla_\mu j^\mu = 0$$

$$\nabla_\mu j_5^\mu = \frac{1}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\lambda} R^\alpha{}_{\beta\mu\nu} R^\beta{}_{\alpha\rho\lambda}$$

Solutions for U(1)$_V$xU(1)$_A$

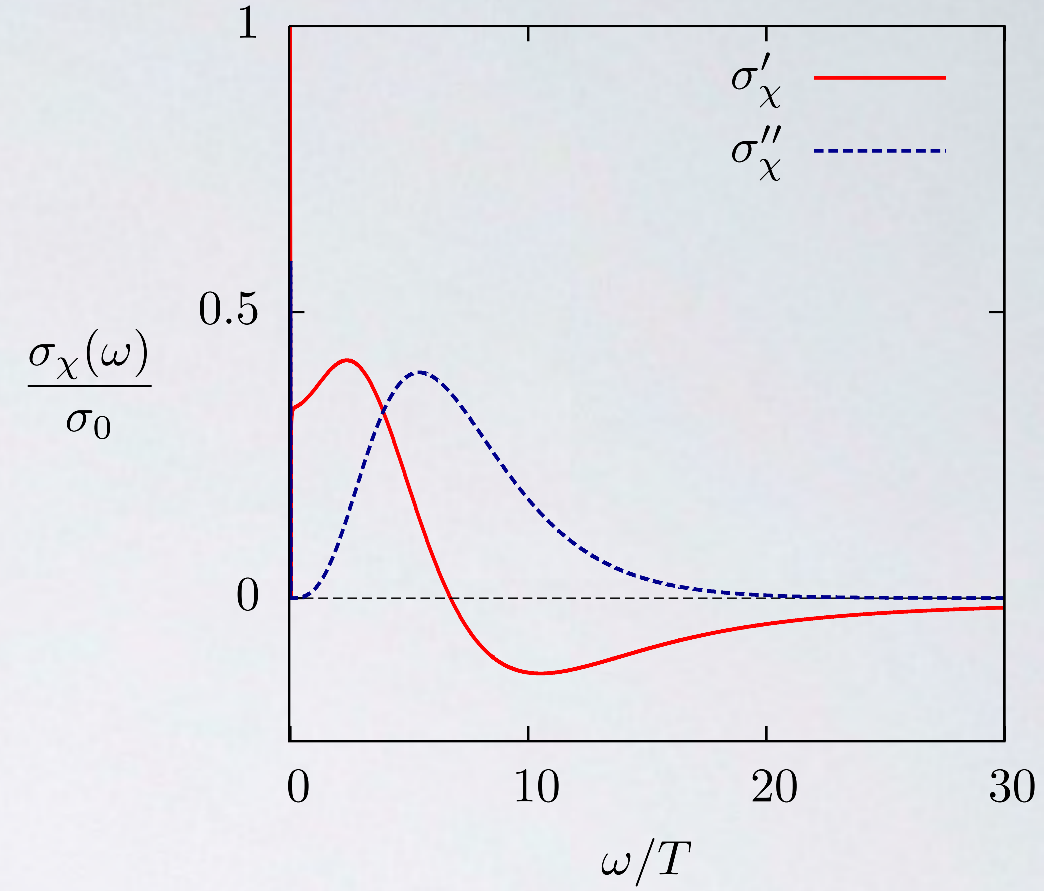
$$\sigma^{B}_{A,(0)} = \frac{1}{2\pi^2}\begin{cases} \mu_5 \\ \mu \\ \mu\mu_5 \end{cases} \quad , \qquad A = e, 5, \epsilon$$

$$\sigma^{B_5}_{A,(0)} = \frac{1}{2\pi^2}\begin{cases} \mu \\ \mu_5 \\ \frac{\mu^2+\mu_5^2}{2} + \frac{\pi^2 T^2}{6} \end{cases}$$

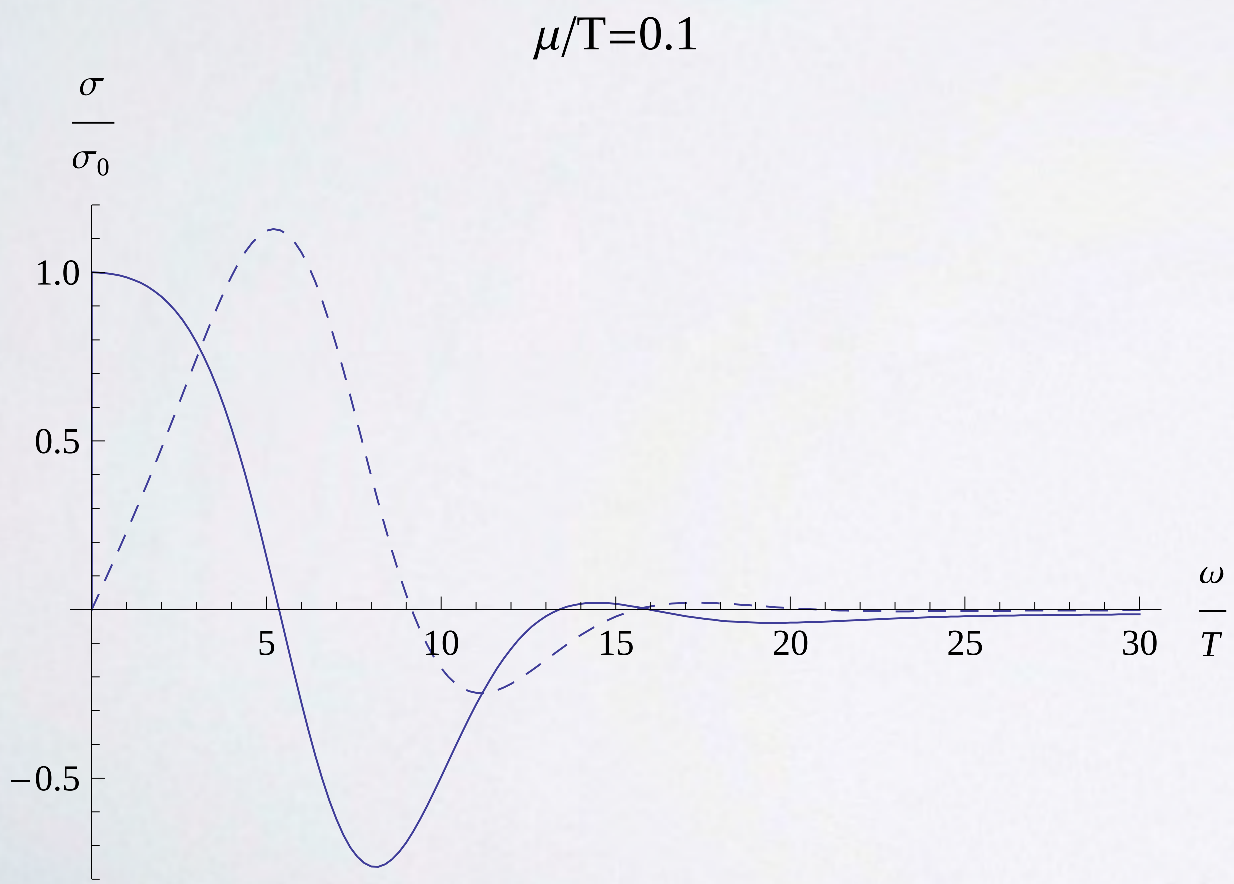
free fermions
conductivities

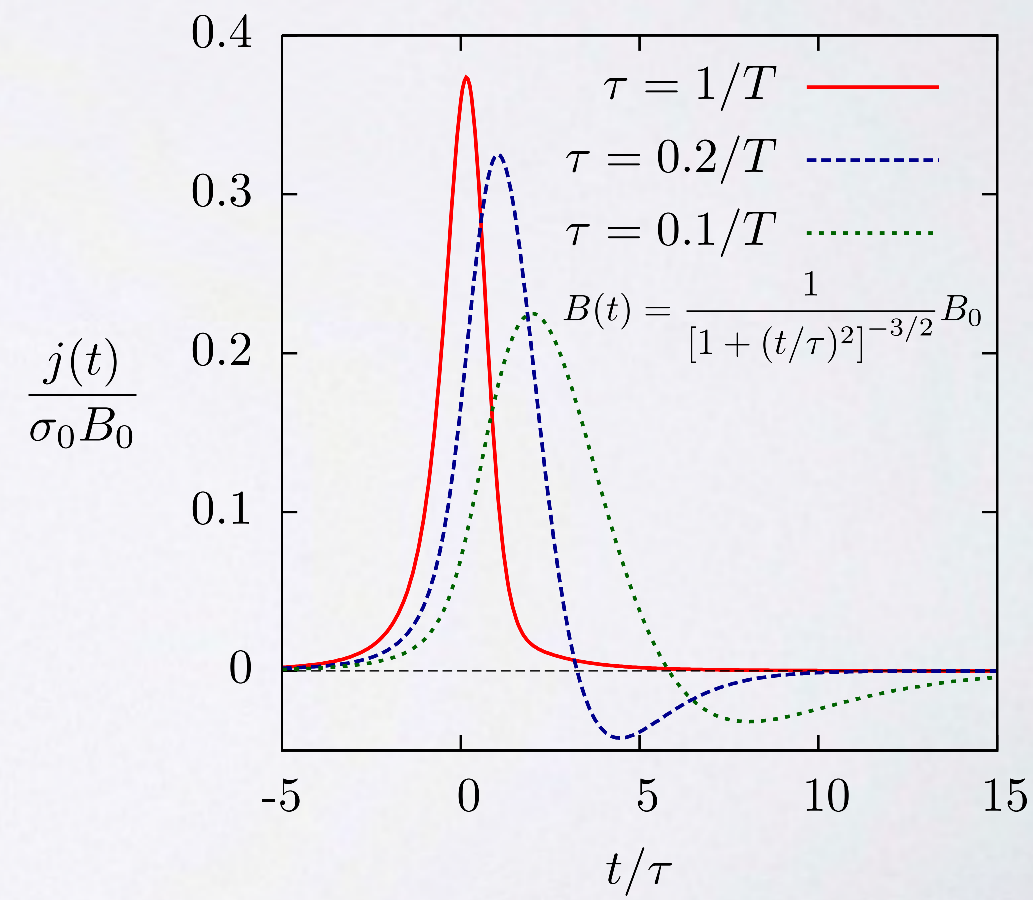
[Kharzeev & Warringa, arXiv:0907.5007]

free fermions
magnetic
conductivity



holographic
magnetic
conductivity





[Ho-Ung Yee, arXiv:0908.4189]

Solutions for $U(1)_V \times U(1)_A$

$$\sigma^V_{A,(0)} = \frac{1}{2\pi^2} \begin{cases} \mu\mu_5 \\ \frac{\mu^2+\mu_5^2}{2} + \frac{\pi^2 T^2}{6} \\ \mu_5 \left(\frac{\mu^2+\mu_5^2}{3} + \frac{\pi^2 T^2}{3} \right) \end{cases}$$

[Vilenkin, Phys. Rev. D20, 1807 (1979)]

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[Vilenkin, Phys. Rev. D20, 1807 (1979)]

$$\sigma^V(\omega) = \sigma^V_0\left(\delta_{\omega,0} + i\omega\delta(\omega)\right)$$

[Landsteiner, Megías & P-B. 1312.1204]

free fermions
conductivities

holographic model

- we need to build a model with a U(1)VxU(1)A global symmetry
- the vector current must be conserved
- the axial current must be anomalous with the mixed gauge-gravitational anomaly included

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + \frac{12}{L^2} - \frac{1}{4}\left(F_{MN}F^{MN} + F^{(5)}_{MN}F^{(5)MN}\right)\right]$$

$A_M(r,x)$ is a vector gauge field in the bulk → the boundary value corresponds to a background vector source $A_\mu(\infty,x)$

$A^{(5)}_M(r,x)$ is an axial gauge field in the bulk → the boundary value corresponds to a background axial source $A^{(5)}_\mu(\infty,x)$

$g_{MN}(r,x)$ is the bulk metric → the boundary background metric $h_{\mu\nu}(\infty,x)$

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$$S_{ano} = \int d^5x\sqrt{-g}\left[\epsilon^{MNPQR}A^{(5)}_M\left(\frac{\kappa}{3}F^{(5)}_{NP}F^{(5)}_{QR} + \kappa F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)\right]$$

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$$\delta_{\xi_5}(S + S_{ano} + S_{bound}) \propto \int_\partial d^4x\sqrt{-h}\xi_5\epsilon^{\mu\nu\rho\beta}\left(\frac{\kappa}{3}F^{(5)}_{\mu\nu}F^{(5)}_{\rho\beta} + \kappa F_{\mu\nu}F_{\rho\beta} + \lambda R^\alpha{}_{\delta\mu\nu}R^\delta{}_{\alpha\rho\beta}\right)$$

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$$\nabla_\mu j^\mu_5 = \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{384\pi^2}\epsilon^{\mu\nu\rho\lambda}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}$$

$$ds^2 = \frac{r^2}{L^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2 f(r)}dr^2$$

$$A = \left(\beta - \frac{\mu\, r_{\rm H}^2}{r^2}\right) dt$$

$$A^{(5)} = \left(\gamma - \frac{\mu_5\, r_{\rm H}^2}{r^2}\right) dt$$

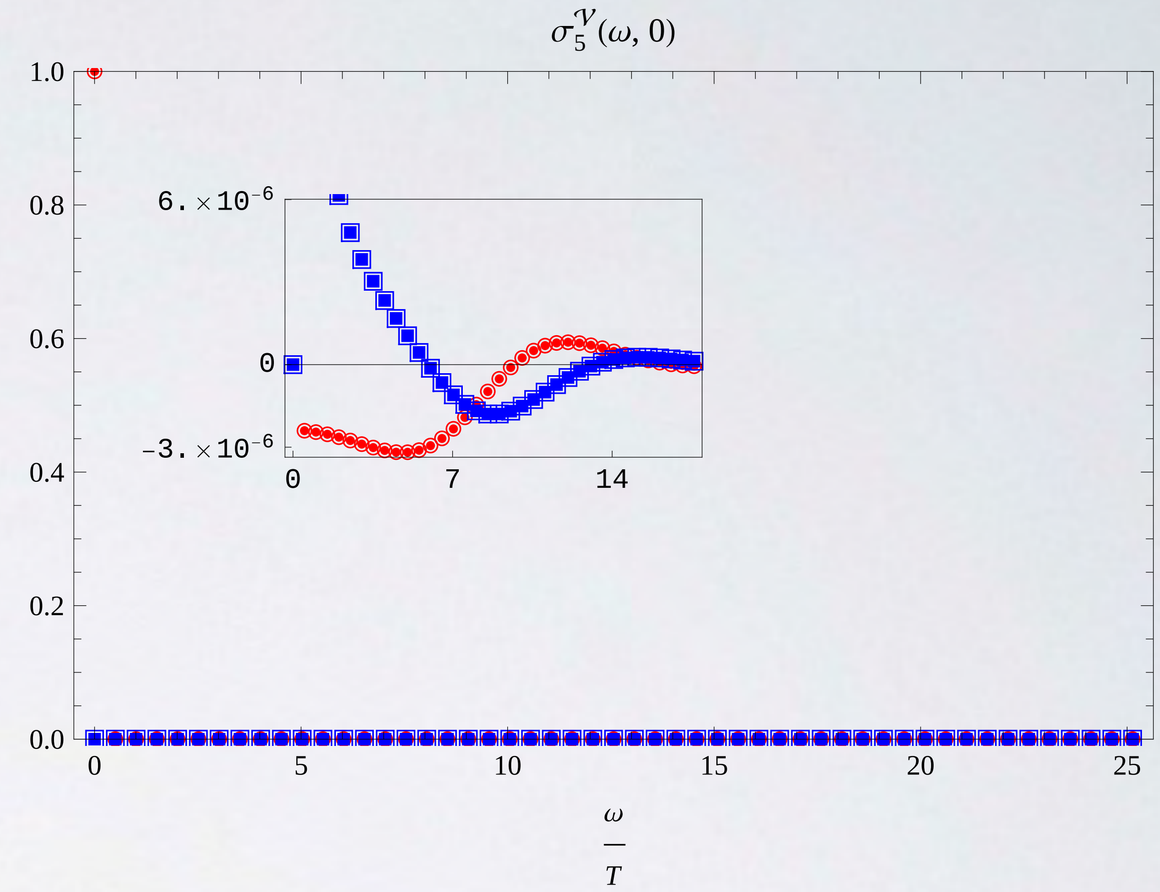
$$Q = \frac{r_H^2}{\sqrt{3}}\mu \qquad Q_5 = \frac{r_H^2}{\sqrt{3}}\mu_5$$

AdS Reissner-Nordström blackhole

vortical conductivities

Numerical vortical conductivity seems to agree with the free case

$$\sigma^V(\omega) = \sigma^V_0(\delta_{\omega,0} + i\omega\delta(\omega))$$



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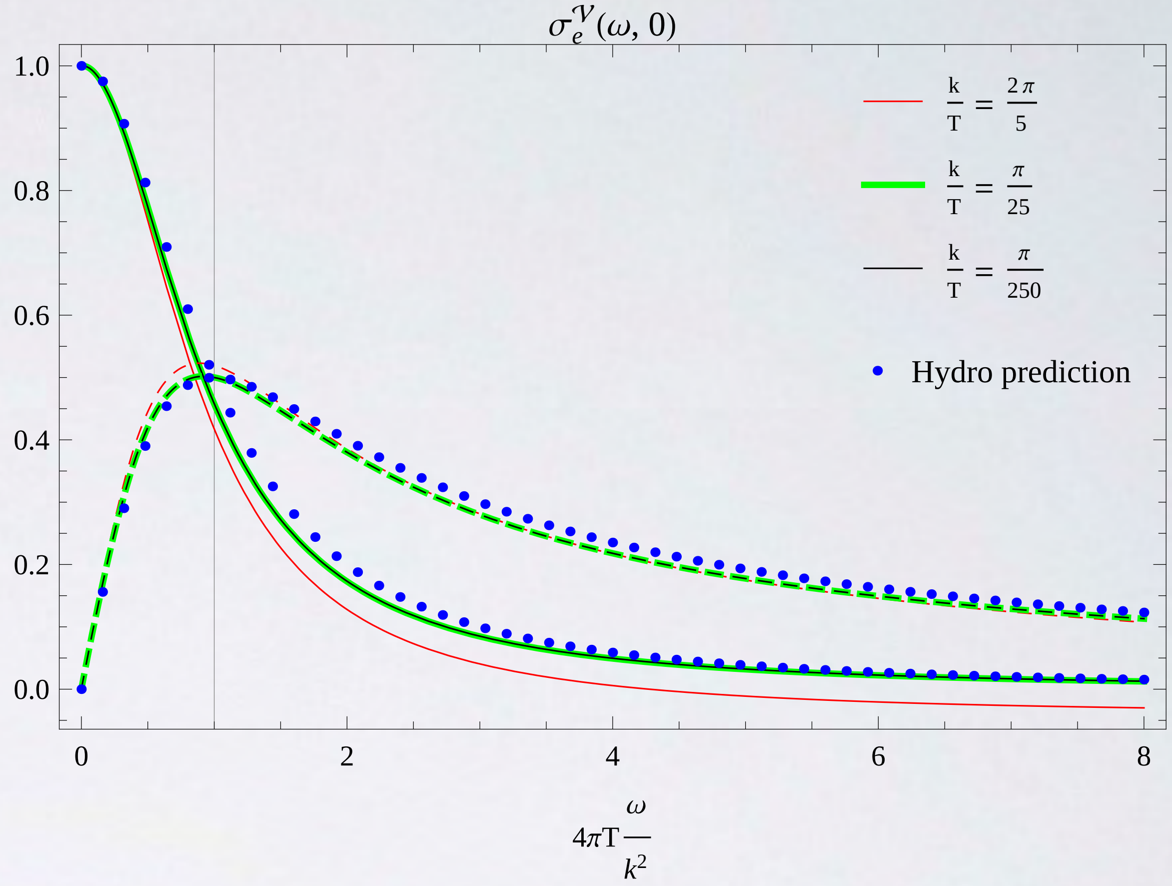


Notice the peak seems to be at

$$\omega \sim \frac{1}{4\pi T} k^2$$

this coefficient coincides with the SYM diffusion constant

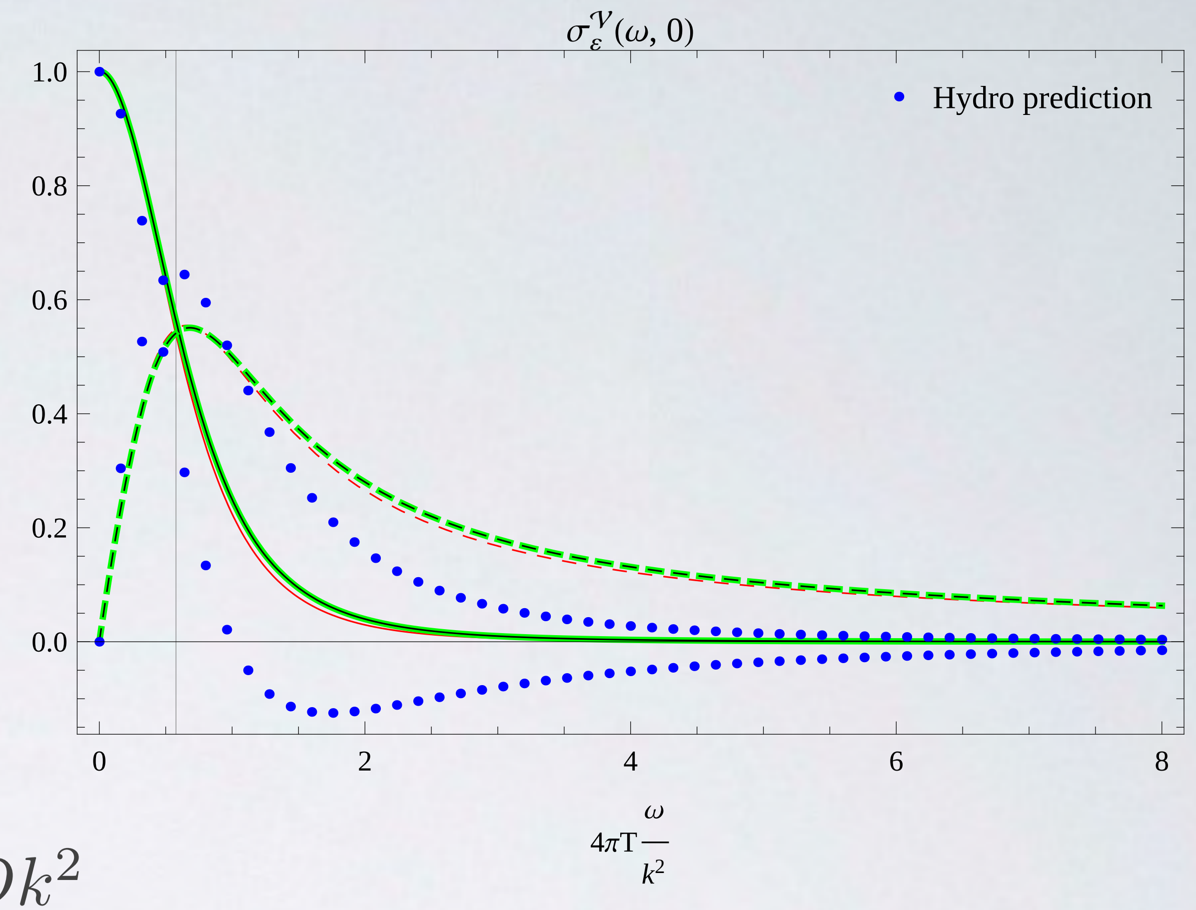
HYDRODYNAMICS

$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \eta\sigma^{\mu\nu} + \sigma^B_\epsilon (B^\mu u^\nu + B^\nu u^\mu) + \sigma^V_\epsilon (\omega^\mu u^\nu + \omega^\nu u^\mu)\,, \\
J^\mu &= \rho u^\mu + \sigma^B B^\mu + \sigma^V \omega^\mu
\end{aligned}
$$

$$
\begin{aligned}
T^{ti} &= (\epsilon + P)v_i + P h_{ti} - ik\epsilon_{ij}\sigma^V_\epsilon (v_j + h_{tj}) - ik\epsilon_{ij}\sigma^B_\epsilon A_j \\
J^i &= \rho v_i - ik\epsilon_{ij}\sigma^V (v_j + h_{tj}) - ik\epsilon_{ij}\sigma^B A_j
\end{aligned}
$$

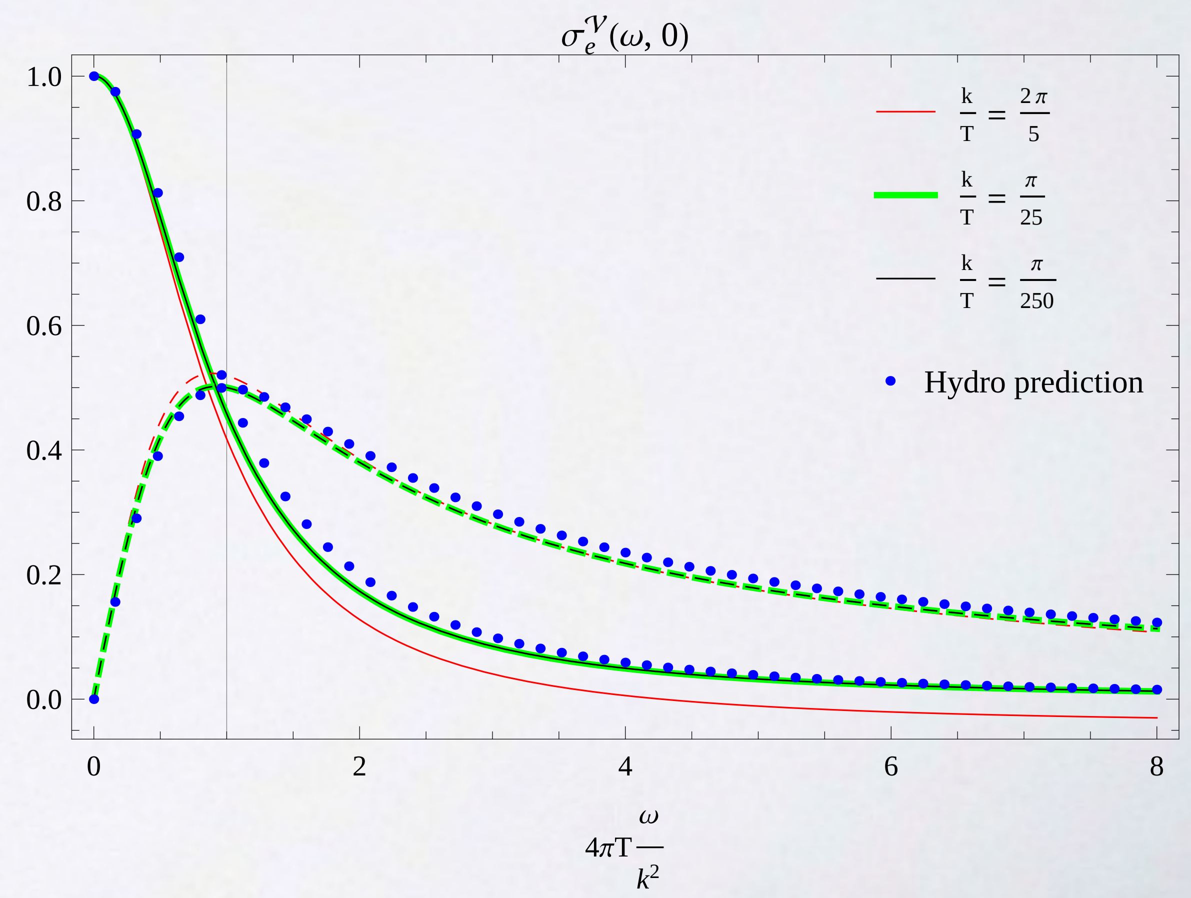
$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu$$

$$-\epsilon_{ij}\sigma^B_\epsilon \omega k A_j + (-i\omega(\epsilon + P) + \eta k^2)v_i - i\omega(P + \epsilon)h_{ti} - \epsilon_{ij}\sigma^V_\epsilon \omega k(h_{tj} + v_j) - \eta\omega k h_{xi} - i\omega A_i \rho = 0$$



$$< T^{ti}T^{tj} > \;=\; -ik\epsilon_{ij}\sigma^V_\epsilon \frac{-D^2k^4}{(\omega+iDk^2)^2}$$

$$< T^{ti}J^j > \;=\; < J^iT^{tj} >= -ik\epsilon_{ij}\sigma^B_\epsilon \frac{iDk^2}{(\omega+iDk^2)}$$



$$\omega_M = \pm\frac{1}{\sqrt{3}}Dk^2 \quad , \quad < T^{ti}T^{tj} >$$

$$\omega_M = \pm Dk^2 \quad , \quad < T^{ti}J^j >$$

ENERGY CONSERVATION

Euclidean effective action

$$\delta W = \int d^4x \left(\frac{\delta W}{\delta g_{\mu\nu}(x)} \delta g_{\mu\nu}(x) + \frac{\delta W}{\delta A_\mu(x)} \delta A_\mu(x) \right) = 0$$

$$\Downarrow$$

$$\nabla_\mu \left(\frac{2}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}(x)} \right) + \nabla_\mu \left(\frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_\mu(x)} \right) A^\nu(x) + \frac{1}{\sqrt{-g}} \frac{\delta W}{\delta A_\mu(x)} F_\mu{}^\nu(x) = 0$$

ENERGY CONSERVATION

$$i(\omega + i\epsilon)\frac{\tilde{\Pi}^{0x,0y}(i\omega_n = \omega + i\epsilon, k_z)}{ik_z} = -\tilde{\Pi}^{0x,zy}(i\omega_n = \omega + i\epsilon, k_z)$$

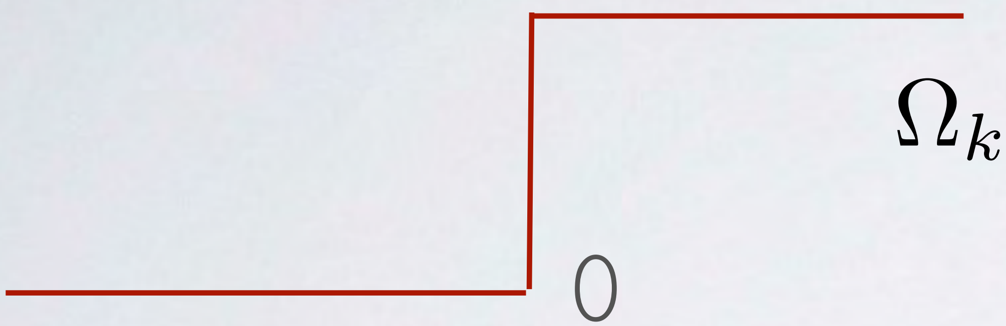
$$\omega\, \sigma_{\epsilon}^{V}(\omega) = 0$$

vanishing at zero momentum because of rotational invariance

$$i(\omega + i\epsilon)\left.\frac{\tilde{G}^{x,0y}(i\omega_n = \omega + i\epsilon, k_z)}{ik_z}\right|_{k_z \to 0} = \tilde{G}^{x,zy}(i\omega_n = \omega + i\epsilon, \vec{k} = 0)$$

$$\omega\, \sigma^{V}(\omega) = 0$$

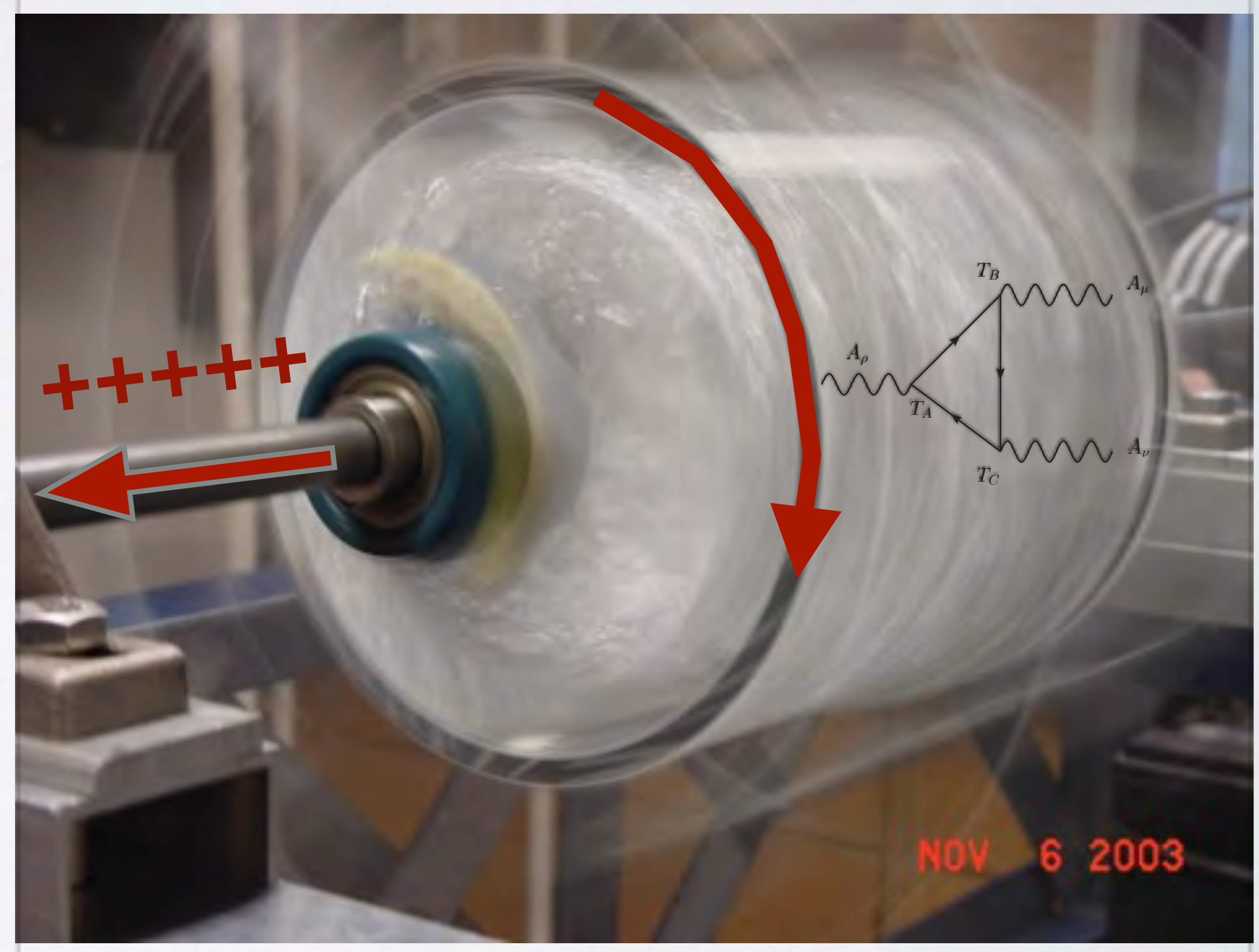
to understand the physical implication of these result,
let's play a game



$$\omega(s) = \frac{i\Omega_k}{s + i\epsilon}$$

$$\sigma^V(s) = \sigma_0^V \frac{iDk^2}{s + iDk^2}$$

$$J(t) = \sigma_0^V \Omega_k (1 - e^{-Dk^2 t})\theta(t)$$



It is a temptation to play with this result and the characteristic numbers of the QGP

$$J(t) = \sigma_0^V \Omega_k (1 - e^{-Dk^2 t}) \theta(t) \qquad \tau = \frac{1}{Dk^2}$$

for the holographic plasma

$$L \sim 10 fm$$

$$T \sim 350 Mev \qquad D = \frac{1}{4\pi T}$$

$$\tau \sim 2100 fm$$

the lifetime of the QGP is of order $\sim 10 fm$

It is a tempta f the QGP

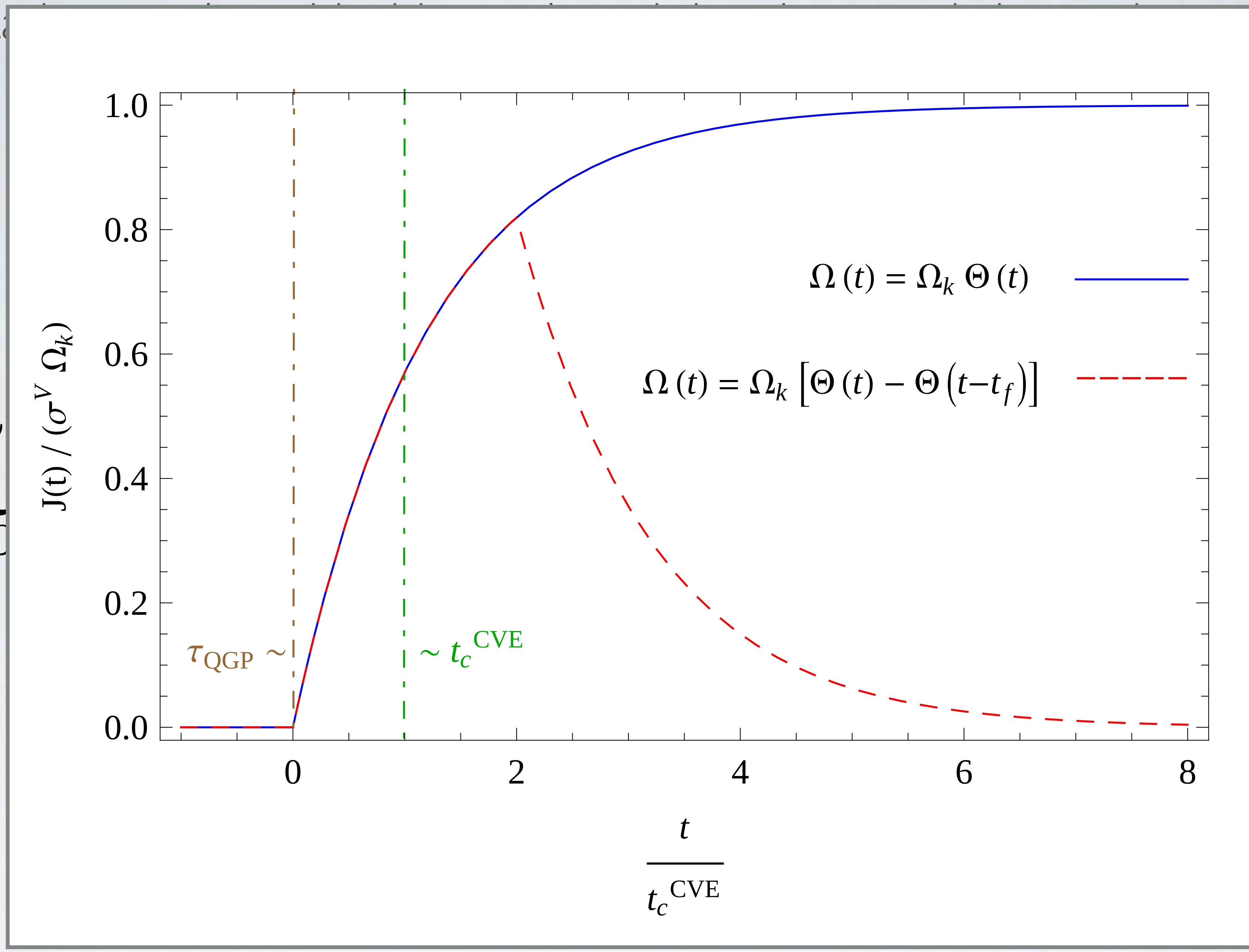
$J(t) =$

asma

$L \sim$

$T \sim 35$

fm



However in the QGP the vorticity is not an external source and the real problem is much more complicated because the vorticity is a dynamical variable!

$$\sigma^{B}_{A,(0)} = \frac{1}{2\pi^2}\begin{cases} \mu_5 \\ \mu \\ \mu\mu_5 \end{cases} , \qquad A = e, 5, \epsilon$$

$$\sigma^{B_5}_{A,(0)} = \frac{1}{2\pi^2}\begin{cases} \mu \\ \mu_5 \\ \frac{\mu^2+\mu_5^2}{2} + \frac{\pi^2 T^2}{6} \end{cases} \qquad \sigma^{V}_{A,(0)} = \frac{1}{2\pi^2}\begin{cases} \mu\mu_5 \\ \frac{\mu^2+\mu_5^2}{2} + \frac{\pi^2 T^2}{6} \\ \mu_5\left(\frac{\mu^2+\mu_5^2}{3} + \frac{\pi^2 T^2}{3}\right) \end{cases}$$

- we need to study more realistic situations because the vorticity is a dynamical quantity
- looking forward for Weyl semimetals!
- are we close to see experimental evidence of the presence of the mixed gravitational anomaly in nature?
- the existence of the anomalous conductivities is well understood already.
- now is necessary to understand the renormalization issues in presence of dynamical gauge fields
- generation of axial chemical potential mechanism in order to be able to do good predictions for QGP

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